Model Predictive Control Design and Implementation

on a 3x3 Model of Shell Heavy Oil Fractionator

by

Madiah Binti Omar

Dissertation submitted in partial fulfilment of
the requirements for the
Bachelor of Engineering (Hons)
(Chemical Engineering)

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CERTIFICATION OF APPROVAL

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Approved by,

____

(Dr. Nooryusmiza B. Yusoff)

CERTIFICATION OF ORIGINALITY

This is certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknoledgemnet, and that the original work contained herein have not been undertaken or done by unspecified sources or person.

MADIAH BINTI OMAR

ABSTRACT

Model Predictive Control (MPC) is the most famous advanced process control method in the industry. MPC refers to a class of computer control algorithms that utilize and explicit process model to predict the future response of the plant. Therefore, we can clearly see that this control strategy has brought a great importance for the industry to control the throughput to meet the requirement. For this purpose, a chemical process model is examined for set point tracking to measure its performance. Different direction of set point is tested for a given model, to measure optimum control horizon for the model and to study whether model is behaved efficiently for MIMO system. This study stated that given model is behaved efficiently for SISO system compared to MIMO system. This may due to modeling error in process gain.

ACKNOWLEDGEMENT

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NOMENCLATURE AND ABBREVIATIONS

MIMO	Multiple Input Multiple Output
SISO	Single Input Single Output
APC	Advanced Process Control
MPC	Model Predictive Control
MVs	Manipulated Variables
CVs	Controlled Variables

CHAPTER 1

1 PROJECT BACKGROUND

1.1 Background of Study

Process control refers to the methods that are used to control process variables when manufacturing a product. Process control technology is the tool that enables manufacturers to keep their operation in specified limits to maximize profitability, ensure quality and safety. One of the technologies is automation, process that corrected any out-of control environment to meet desired throughput. It consists of four-hierarchy layer as shown in **Figure 1.1**. (Zhou, 2001).

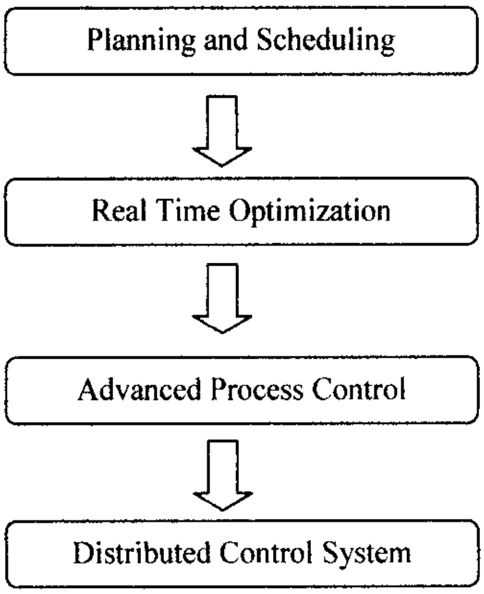


Figure 1.1: *Hierarchy layer of modern control and automation*

The study will focus on the third layer that is Advanced Process Control (APC). The approach of APC used in this research is Model Predictive Control (MPC). MPC refers to a class of computer control algorithms that utilize and explicit process model to predict the future response of the plant (S. Joe Qin and Thomas A.Badgwell, 2003).

The overall objectives of an MPC controller have been summarized by (S. Joe Qin and Thomas A.Badgwell, 2003):

1. Prevent violation of input and output constraints.
2. Drive some output variables to their optimal set points, while maintaining other outputs within specified ranges.
3. Prevent excessive movement of the inputs variables.
4. Control as many process variables as possible when a sensor or actuator is not available.

MPC has been used for more than 30 years mainly in chemical and petrochemical due to its ability for dealing with constraints and multivariable systems (Multiple Input Multiple Output).

Figure 1.2 showed how MPC worked in predicting the projection of output for a given set point.

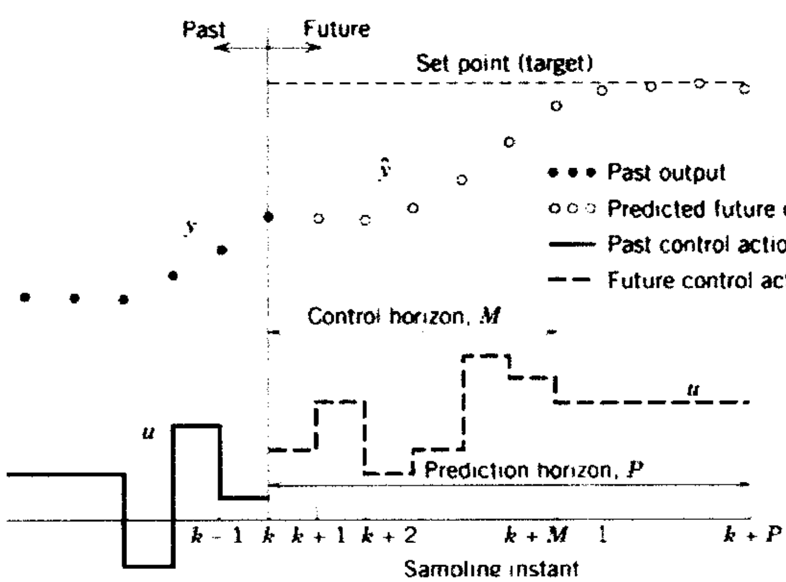


Figure 1.2: *MPC Sampling Prediction*

From **Figure 1.2**, y is the actual output. $\hat{y}$ is the predicted output in the future. Set point or target is determined from optimization calculation from process. The actual input before prediction is u. The next move of step input in the future is derived from control horizon, M. Control horizon is the number of M moves and will determine projection of the y predicted. For *k*-th sampling instant, the values of the manipulated variables, ***u***, at the next control horizon, will be added together at M = 1, 2, 3, M. The input held constant after M moves.

The inputs are calculated so that the set of predicted output reaches set point in optimal manner. The total time for sampling is represented by prediction horizon, P. Area between the set point line and predicted output is the error or deviation from desired output.

1.2 Problem Statement

1.2.1 Problem Identification

The absolute objective of MPC control calculation is to determine a sequence of control moves (manipulated input changes) so that the predicted response moves to the set point in an optimal manner. Therefore, Shell Heavy Oil Fractionator model's performance by setting different set point direction and different control horizon can be determined. It is also importance to measure model efficacy for Single Input Single Output (SISO) and Multiple Input Multiple Output (MIMO) to predict behavior of the plant output.

Case study for this project is divided into SISO and MIMO model for various control horizons for negative and positive set points.

Base case for this project is M = 2 and P = 100 and case study is summarized into **Table 1.1, Table 1.2, Table 1.3** :

- Single Input Single Output (SISO) – only one-step input is moved, other variables remained constant. Set point is changed either positive or negative for every output. Control horizons are manipulated from 2 until 10 according to **Table 1.1.**

Case Study	Output Variables	Set Point	Control Horizon
A	Y1	2	2,4,6,8,10
		-2	2,4,6,8,10
B	Y2	2	2,4,6,8,10
		-2	2,4,6,8,10
C	Y3	2	2,4,6,8,10
		-2	2,4,6,8,10

Table 1.1: *SISO case study.*

- Multiple Input Multiple Output (MIMO) – either two or three step input movements.

Case Study	Output Variables and Set Point	Control Horizon
D	Y1= 2,Y2= 2	2,4,6,8,10
	Y1= -2,Y2= 2	2,4,6,8,10
	Y1= -2,Y2= 2	2,4,6,8,10
	Y1= -2,Y2= -2	2,4,6,8,10
E	Y1= 2,Y3= 2	2,4,6,8,10
	Y1= -2,Y3= 2	2,4,6,8,10
	Y1= -2,Y3= 2	2,4,6,8,10
	Y1= -2,Y3= -2	2,4,6,8,10
F	Y2= 2,Y3= 2	2,4,6,8,10
	Y2= -2,Y3= 2	2,4,6,8,10
	Y2= -2,Y3= 2	2,4,6,8,10
	Y2= -2,Y3= -2	2,4,6,8,10

Table 1.2: *MIMO 2x2 case study.*

Case Study	Output Variables and Set Point	Control Horizon
G	Y1= 2,Y2= 2, Y3= 2	2,4,6,8,10
	Y1= -2,Y2= 2, Y3= 2	2,4,6,8,10
	Y1= 2, Y2= -2, Y3= 2	2,4,6,8,10
	Y1= 2, Y2= 2, Y3= -2	2,4,6,8,10
	Y1= -2, Y2= -2, Y3= 2	2,4,6,8,10
	Y1= 2, Y2= -2, Y3= -2	2,4,6,8,10
	Y1= -2, Y2= -2Y3= 2	2,4,6,8,10
	Y1= -2, Y2= -2, Y3= -2	2,4,6,8,10

Table 1.3: *MIMO 3x3 case study.*

1.2.2 Project Significant

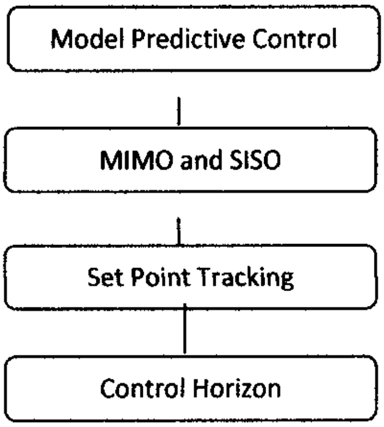
This study is very significant for the MPC development as an approach to determine output projection for various case studies. From this, we can check whether the model can behave efficiently for MIMO case study and optimum control horizon for the model. Finally, error is reduced and increase in profit when we desired throughput is obtained.

1.3 Objective and Scope of Study

Objectives of this study are:

a) To study effect of different direction of set point for given model
b) To measure optimum control horizon for the model
c) To study whether model is behaved efficiently for MIMO

Scope of Study



1.4 Project Relevancy

Nowadays, the petroleum and chemical industries face the unpredictable market condition due to worldwide competition, limitation of resources and strict national and international regulations. In order to achieve the production safety, quality and flexibility, plant automation has become increasingly important for the company.

If the performance of the automation is excellent, we will obtain throughput that is meeting our requirement. This project will provide this desired control performance for the industry.

1.5 Feasibility Study

The project is feasible to be conducted based on these elements:

Time

The time allocated, approximately 20 weeks is sufficient in order to run the MATLAB and analyze the result of the control performance.

Equipment

The tool require are Microsoft Excel and MATLAB which are readily available in the campus.

Cost

The cost for conducting this project is estimated to be minimal. This is because there is no need to use physical complex item like chemical substance or mechanical equipment.

Data

The data for the study will be generated for the given model (model is obtained from (Nafsun, 2010))

References

The references for this project are considered sufficient. The references paper relating this project can be retrieved from **http://www.sciencedirect.com** as UTP already paid for this site.

CHAPTER 2

2 LITERATURE REVIEW

2.1 Introduction

Modern automation control system for processing plant usually consists of a multi-level hierarchy of control layers. The first layer (starting from bottom) is usually Distributed Control System (DCS) which gather all the process measurement. This level will perform simple monitoring and PID-based control of some process variables (such as flow rates, levels, temperatures) to guarantee automation operation of the plant. The second layer is the Advanced Process Control (APC). It performs multivariable model-based constrained control to achieve stable unit operation and maximize the performance for economic benefits. On top of APC is the layer for Real Time Optimization (RTO) followed by Planning and Scheduling (Gabriele Pannocchia, Dec 2007). The time scale for every layer can be observed from **Figure 2.1** (Skogested, 2004):

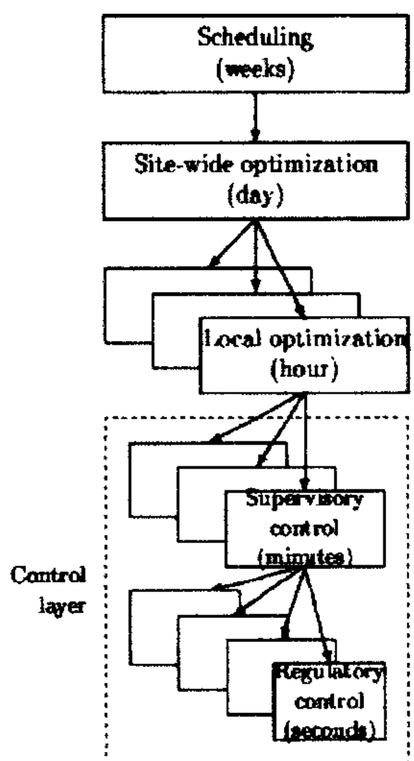


Figure 2.1: *Typical control hierarchy in chemical plant*

APC regulators typically falls within the class of Model Predictive Control (MPC) which the one that will be discussed in this study.

2.2 Model Predictive Control (MPC)

MPC is widely adopted in the process industry as an effective means to deal with large multivariable constrained control problems. The main idea of MPC is to select the control action by online repeated solving of an optimal control problem. MPC has been used in industry for more than 30 years with most commercially available MPC technologies are based on a linear model of the process (S. Joe Qin and Thomas A.Badgwell, 2003).

A block diagram of a model predictive control system is shown in **Figure 1.1** as explained in (Dale E. Seborg, 2004). A process model is used to predict the current values of the output variables. The residual, the differences between the actual and predicted outputs, serve as the feedback signal to *Prediction* block. The predictions are used in two types of MPC calculation sampling that are performed at each sampling instant: set point calculations and control calculations. The set points for the control calculations, also called as *target,* are calculated from plant economic optimization based on steady state model of the process, commonly, a linear model. The optimum values of set points are changed frequently to a varying process condition. This is due to constraint changes in process condition, equipment, instrumentation and economic data. In MPC, set points are typically calculated each time the control calculation are performed.

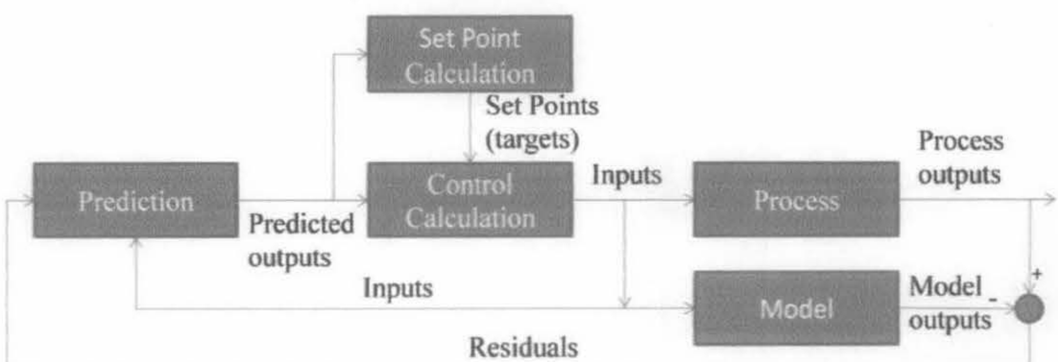


Figure 2.2 *Block diagram for model predictive control*

Control calculations are based on current measurements and prediction of the future values of the outputs. The objective of MPC control calculation is to determine a sequence of control moves (that is, manipulated input changes) so that the predicted response moves to the set point in an optimal manner.

Hydrocarbon processes is large scale and complex, slow dynamic and very high level of disturbances. These characteristics made petrochemical plant suitable for the MPC implementation. In this project, Shell Heavy Oil Fractionator model is selected and is further discussed in the next section.

2.3 Shell Heavy Oil Fractionator Model

The fractionator is shown in **Figure 2.3.** The gaseous feed is entered at the bottom of the column. The fractionator has three product draws and three side circulating duty.

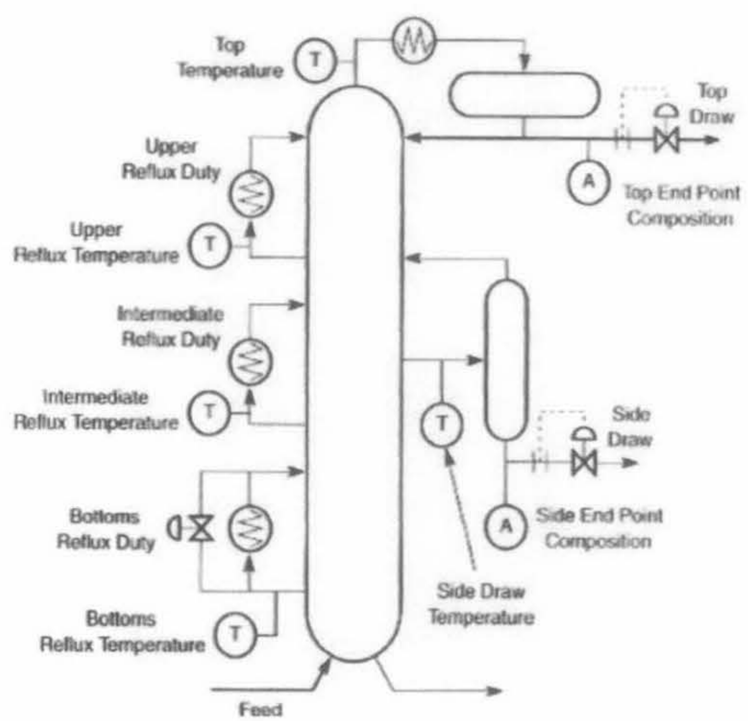


Figure 2.3: *'Shell' Heavy Oil Fractionator.*

Manipulated variables and controlled variables for this model is summarized into **Table 2.1**:

Manipulated variables, Input (U)	Controlled variables, Output (Y)
Top Draw (U1)	Top End Point (Y1)
Side Draw (U2)	Side End Point (Y2)
U3, Bottom Reflux Duty (U3)	Bottom Reflux Temperature (Y3)

Table 2.1: *List of MVs and CVs for 'Shell' Heavy Oil Fractionator*

This model is using first order plus time delay (FOPTD) transfer function as shown:

$$G_m(s) = \frac{k\exp(-\theta s)}{\tau s + 1}$$

Where k = process gain, τ = time constant, θ = time delay

Matrix for this model is developed from

G = Y, U

$$G = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix}$$

Transfer function for three inputs and three outputs is shown as follows (Nafsun, 2010):

$$G = \begin{bmatrix} \frac{4.05e^{-6s}}{50s+1} & \frac{1.77e^{-7s}}{60s+1} & \frac{5.88e^{-6s}}{50s+1} \\ \frac{5.39e^{-4s}}{50s+1} & \frac{5.72e^{-3s}}{60s+1} & \frac{6.9e^{-3s}}{40s+1} \\ \frac{4.38e^{-5s}}{33s+1} & \frac{4.42e^{-5s}}{44s+1} & \frac{7.2}{19s+1} \end{bmatrix}$$

In Simulink environment, the process model is developed to relate between these MVs and CVs as shown in **Figure 2.4**:

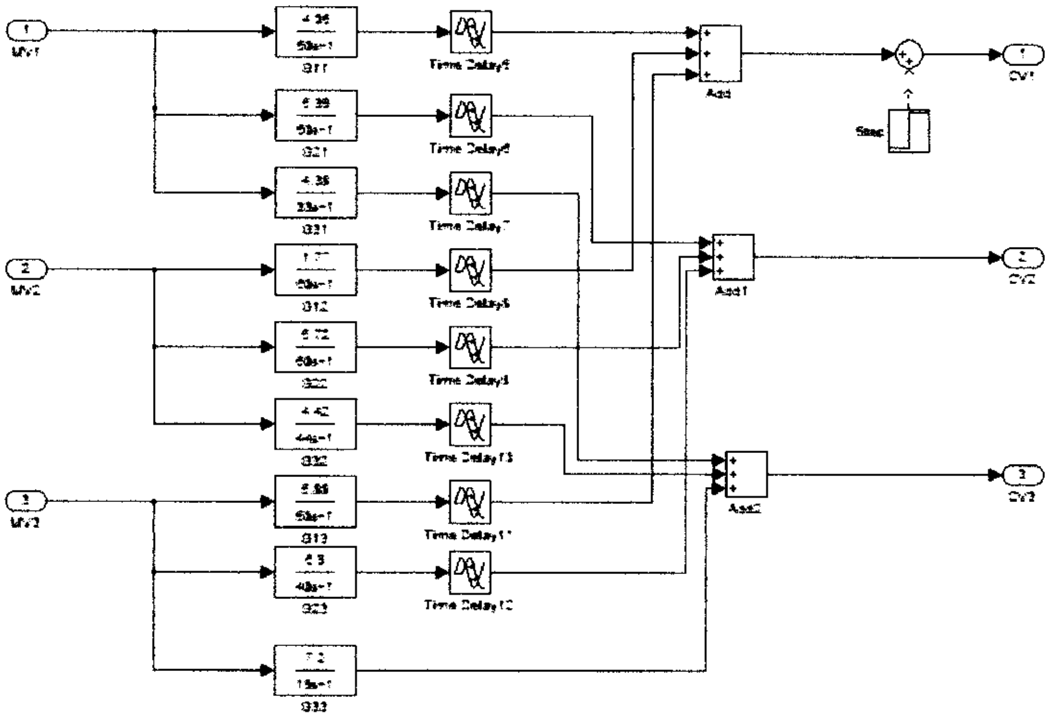


Figure 2.4: ***'Shell' Heavy Oil Fractionator model in Simulink.***

CHAPTER 3

3 METHODOLOGY

3.1 Research Methodology and Activities

1. Shell Heavy Oil Fractionator model is obtained from (Nafsun, 2010). The model is first order plus time delay shown below:

$$G = \begin{bmatrix} \frac{4.05e^{-6s}}{50s+1} & \frac{1.77e^{-7s}}{60s+1} & \frac{5.88e^{-6s}}{50s+1} \\ \frac{5.39e^{-4s}}{50s+1} & \frac{5.72e^{-3s}}{60s+1} & \frac{6.9e^{-3s}}{40s+1} \\ \frac{4.38e^{-5s}}{33s+1} & \frac{4.42e^{-5s}}{44s+1} & \frac{7.2}{19s+1} \end{bmatrix}$$

2. MATLAB Simulink is developed for this dynamic model for set point tracking. MPC layout is designed for three inputs and three outputs as shown in **Appendices.**

3. Different set point and control horizon is entered into the system using MATLAB workspace's coding as shown in **Appendices.**

4. The changes and projection of the set point are displayed in the tables below. Total scenarios to be run are 130 scenarios and every set point and control horizon changes is ran using workspace coding.

- Single Input Single Output (SISO) projection

Case Study	Output Variables	Set Point	Control Horizon
A	Y1	2	2,4,6,8,10
		-2	2,4,6,8,10
B	Y2	2	2,4,6,8,10
		-2	2,4,6,8,10
C	Y3	2	2,4,6,8,10
		-2	2,4,6,8,10

- 2 x 2 Multiple Input Multiple Output (MIMO) projection

Case Study	Output Variables and Set Point	Control Horizon
D	Y1= 2,Y2= 2	2,4,6,8,10
	Y1= -2,Y2= 2	2,4,6,8,10
	Y1= -2,Y2= 2	2,4,6,8,10
	Y1= -2,Y2= -2	2,4,6,8,10
E	Y1= 2,Y3= 2	2,4,6,8,10
	Y1= -2,Y3= 2	2,4,6,8,10
	Y1= -2,Y3= 2	2,4,6,8,10
	Y1= -2,Y3= -2	2,4,6,8,10
F	Y2= 2,Y3= 2	2,4,6,8,10
	Y2= -2,Y3= 2	2,4,6,8,10
	Y2= -2,Y3= 2	2,4,6,8,10
	Y2= -2,Y3= -2	2,4,6,8,10

- 3 x 3 Multiple Input Multiple Output (MIMO) projection

Case Study	Output Variables and Set Point	Control Horizon
G	Y1= 2,Y2= 2, Y3= 2	2,4,6,8,10
	Y1= -2,Y2= 2, Y3= 2	2,4,6,8,10
	Y1= 2, Y2= -2, Y3= 2	2,4,6,8,10
	Y1= 2, Y2= 2, Y3= -2	2,4,6,8,10
	Y1= -2, Y2= -2, Y3= 2	2,4,6,8,10
	Y1= 2, Y2= -2, Y3= -2	2,4,6,8,10
	Y1= -2, Y2= -2Y3= 2	2,4,6,8,10
	Y1= -2, Y2= -2, Y3= -2	2,4,6,8,10

5. For every scenarios, graph of output is examined and error for the case study is determined.

6. The example of the graph for case study A for set point = 2 and control horizon = 2 is shown in **Figure 3.1**:

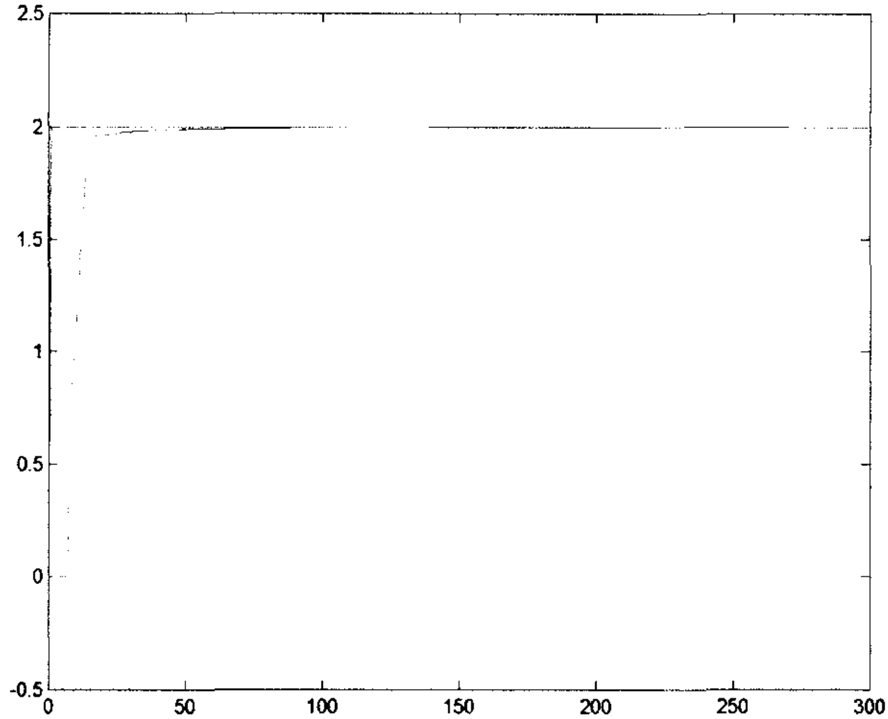


Figure 3.1: *Graph for case study A with control horizon = 2.*

7. Error for this case study is calculated using trapezoidal rule |**yk – ysp**|. yk is representation of the area under the curve and ysp is the area under the set point target. This error is the deviation of the output from desired value. All graph for 7 case studies can be found in **Appendices**.

8. Error for every case study is calculated and summarized into result and discussion section. All recorded error is collected and available in **Appendices**.

3.2 Project Milestone

No	Activities/Weeks	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	Research Continues	■	■	■	■	■	■	■								
2	Model Selection		■	■												
3	Simulink MATLAB				■	■	■	■								
4	Error Calculation								■							
5	Data Gathering and Analysis									■	■					
6	Pre-EDX											■				
7	Submission of Draft Report												■			
8	Submission of dissertation													■		
9	Submission of Technical Paper													■		
10	Oral Presentation														■	
11	Submission Project Dissertation															■

3.3 Tools

1. MATLAB

2. HYSYS

CHAPTER 4

4 RESULT AND DISCUSSION

- Single Input Single Output (SISO) projection

From **Figure 4.1**, step input 2 and -2 is entered into Y1 for different control horizons. All variables are kept constant. It is cleared that whether positive or negative direction of set point, the error for the model is still the same.

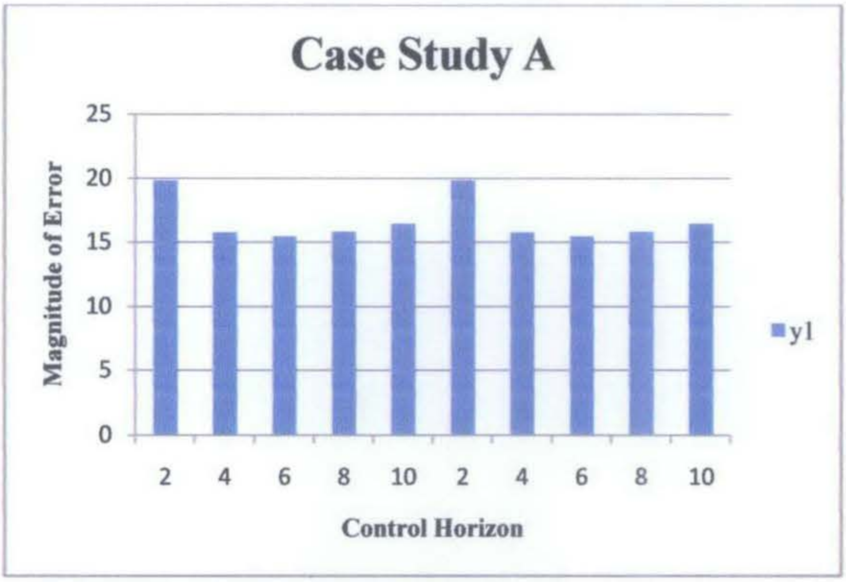


Figure 4.1: *Graph for case study A*

From **Figure 4.2**, step input 2 and -2 is entered into Y2 for different control horizons. All variables are kept constant. It is cleared that whether positive or negative direction of set point, the error for the model is still the same.

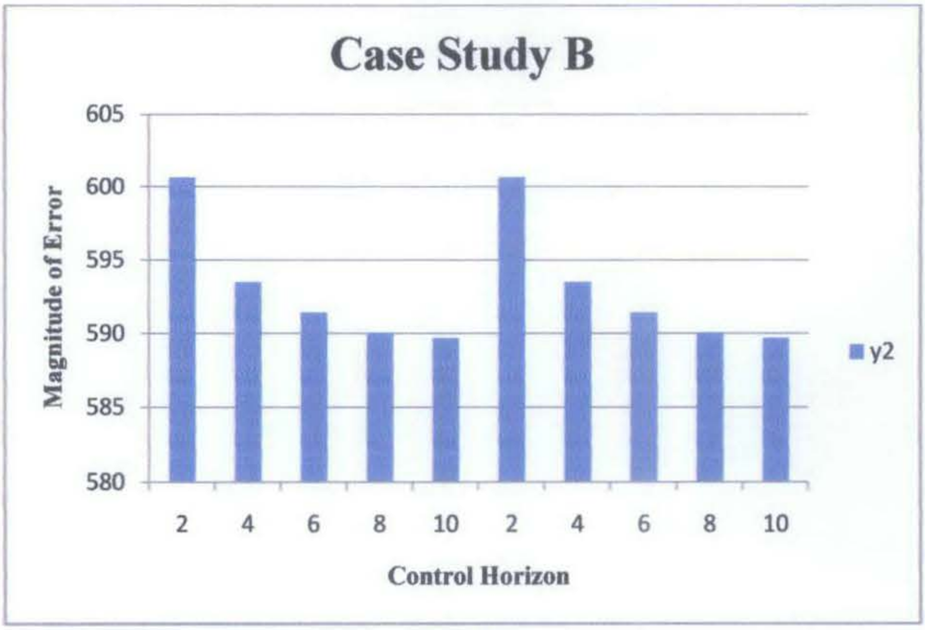


Figure 4.2: *Graph for case study B.*

From **Figure 4.3**, step input 2 and -2 is entered into Y3 for different control horizons. All variables are kept constant. It is cleared that whether positive or negative direction of set point, the error for the model is still the same.

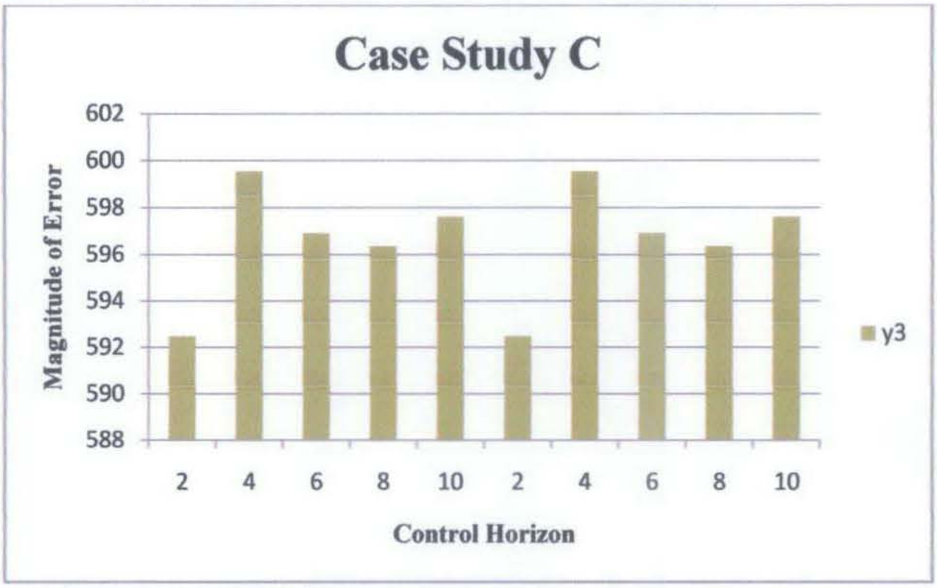


Figure 4.3: *Graph for case study C.*

Therefore, from observation of case study A, B, C it is cleared that within this range (SISO model) we can utilize linear MPC model. This control strategy works when only one control variable manipulated at one time.

- Multiple Input Multiple Output (MIMO)

From **Figure 4.4**, the least error group is when Y1 = 2, Y2 = 2 and when Y1 = -2, Y2 = -2. When the direction of Y1 and Y2 is different, error for the model is very high and near to 1200.

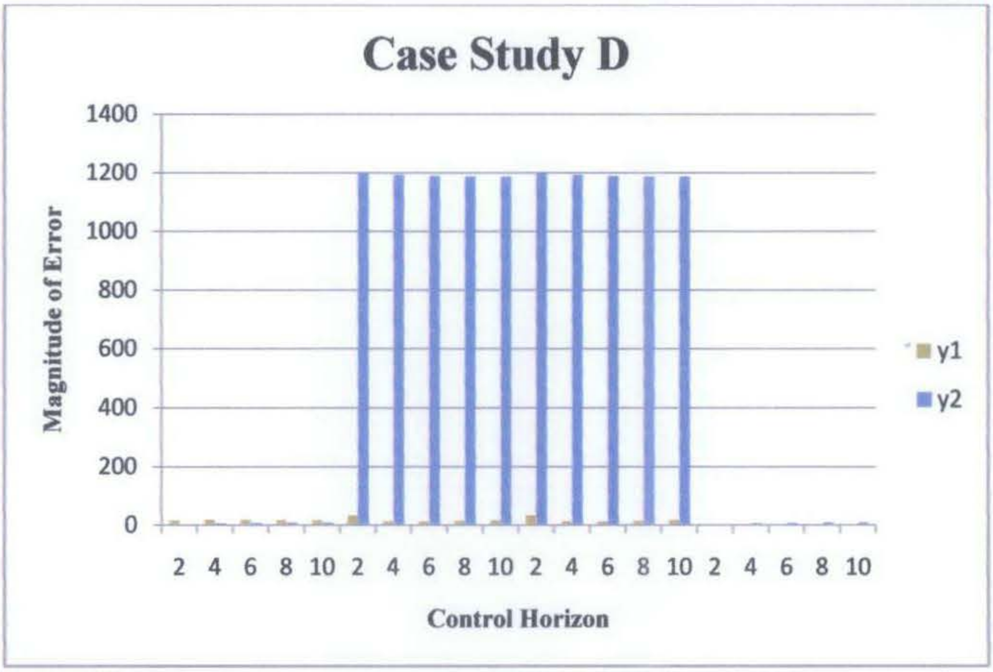


Figure 4.4: *Graph for case study D.*

From **Figure 4.5**, the least error group is when Y1 = 2, Y3 = 2 and when Y1 = -2, Y3 = -2. When the direction of Y1 and Y3 is different, error for the model is very high and near to 1200.

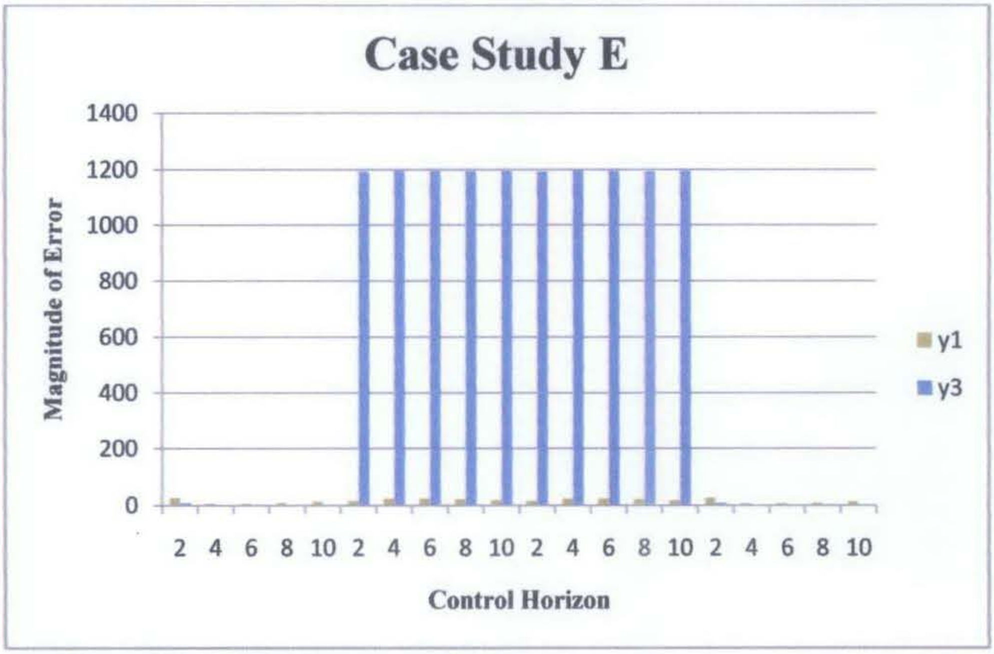


Figure 4.5: *Graph for case study E.*

From **Figure 4.6**, the least error group is when Y2 = 2, Y3 = 2 and when Y2 = -2, Y3 = -2. When the direction of Y2 and Y3 is different, error for the model is very high and near to 630.

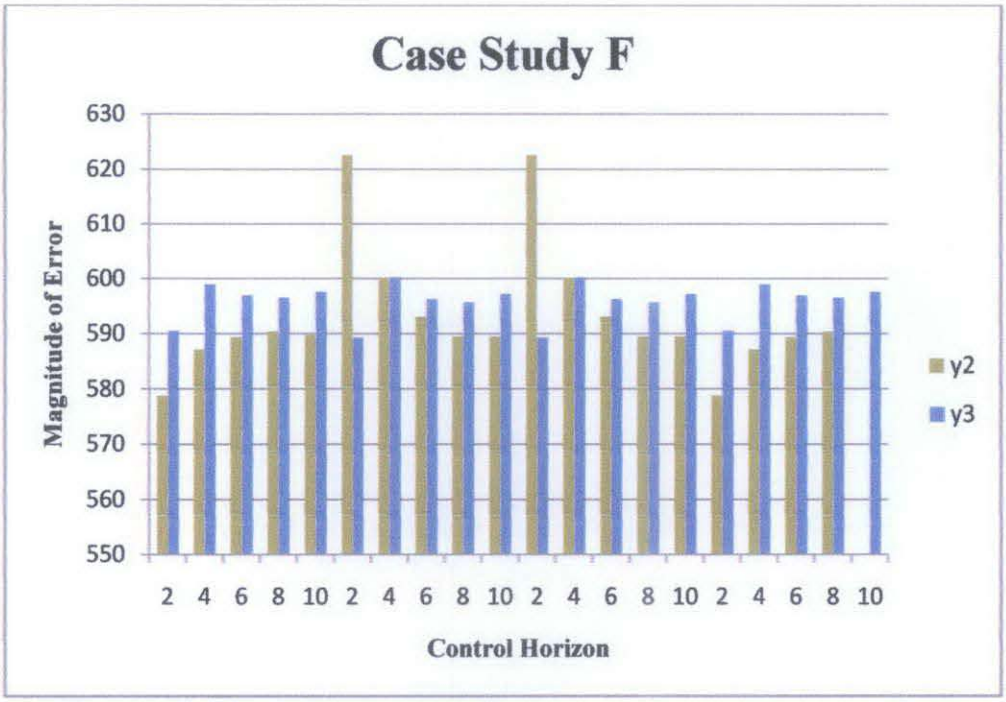


Figure 4.6: *Graph for case study F.*

From **Figure 4.7**, the least error group is when Y1 = 2, Y2 = 2, Y3 = 2 and when Y1= -2, Y2 = -2, Y3 = -2. When the direction of Y1, Y2 and Y3 is different, error for the model is very high and near to 1300.

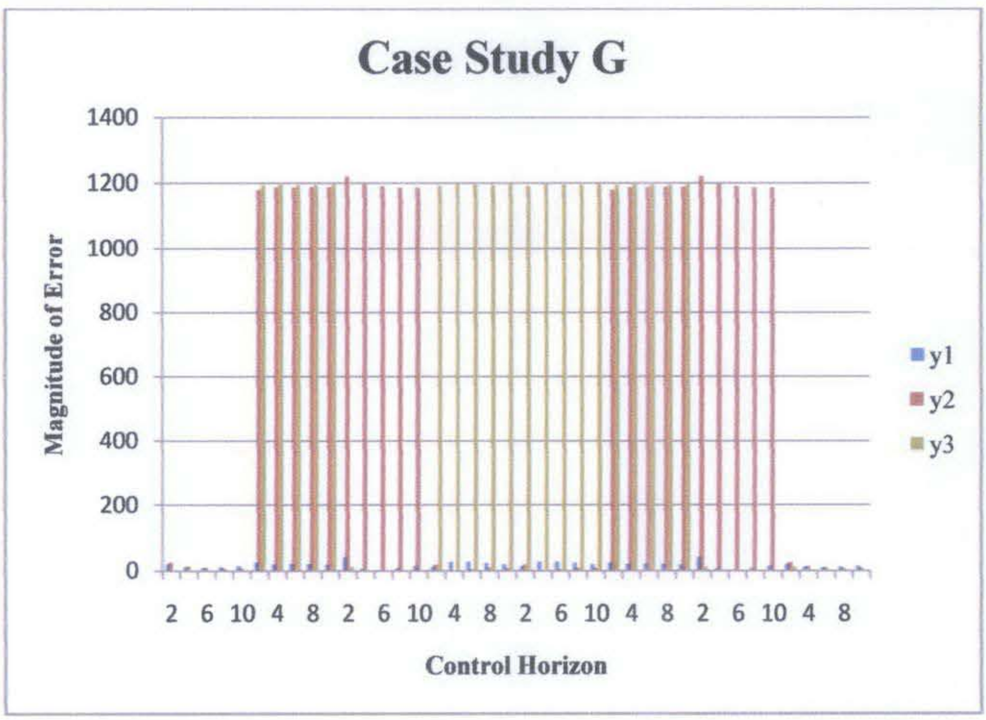


Figure 4.7: *Graph for case study G.*

Therefore, from observation of case study D, E, F, G it is cleared that this model cannot be moved into different direction of set point simultaneously. This can be due to modeling error in the model gain.

- Control Horizon

From **Figure 4.7**, the highest average error is M = 2 and the lowest error or optimum M is 6.

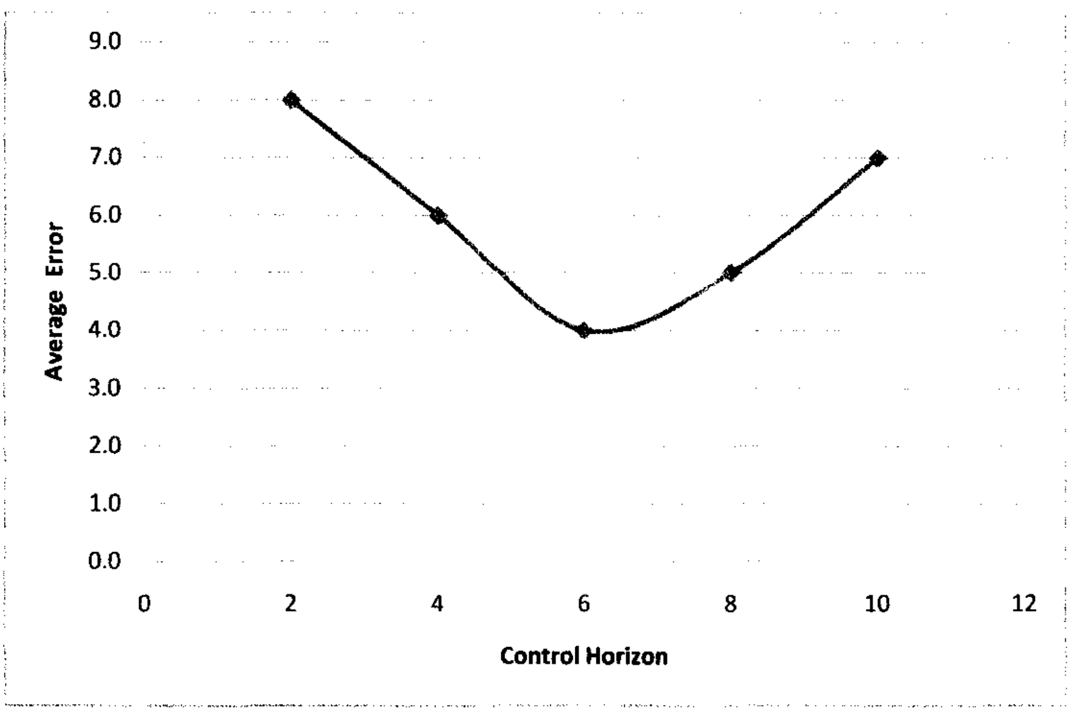


Figure 4.8: *Graph for case study control horizon.*

When control horizon increases, the model has high degree of freedom and it is free to move and reach desired value. However, when control horizon is too high, the model become sensitive and easily disturbed by any changes. Therefore, it is critical for process model to determined optimum control horizon to decrease the deviation in the model.

CHAPTER 5

CONCLUSION

As a conclusion, different direction of set point will produce very high error. The optimum control horizon for this model is when M = 6. 'Shell' Heavy Oil Fractionator model is limited only for SISO model for linear behavior. The error is very high for MIMO system when outputs are drove with different direction. This is due to modeling error in the process gain.

RECOMMENDATION

For future research, besides set point tracking, another method that can measure performance of the model is disturbance rejection. Gaussian input will be entered into the system as a disturbance and degree of the rejection can be measured.

REFERENCES

Dale E. Seborg, *Process Dynamics and Control, 2nd Edition* (pp. 535-536). United States of America: John Wiley & Sons, Inc.

Gabriele Pannocchia, L. G. (Dec 2007). Rigorous Simulation and Model Predictive Control of a Crude Distillation Unit. 636.

Manfred Morari and Jay H. Lee. (2006). Model Predictive Control: Past, Present and Future. 673.

N. Danesh Pour and A. Montazeri. (Nov 2007). Two Case Studies for Applying Model Predictive Controllers on Chemical Processes. 580-585.

Nafsun, A. I. (2010). Detection of Model-plant Mismatch in Model Predictive Control System Using Partial Correlation..

Nicholas Alsop and JoseMaria Ferrer. (n.d.). What Dynamic Simulation brings to a Process Control Engineer. 5.

Nooryusmiza Yusoff, M. Ramasamy. (2009). *A Comparison between MPC and PI Controllers Acting on a Refrigerated Gas Plant* .

Petter Lundstrom and Sigurd Skogestad. (Sept 2008). Opportunities and Difficulties with 5 x 5 distillation control. 249.

R. Khaledi, B. R. Young. (2005). Modelling and Model Predictive Control of Composition an dConversion in an ETBE Reactive Distillation Column. 3134-3144.

S. Joe Qin and Thomas A.Badgwell. (2003). A Survey of Industrial Model Predictive Control Technology. 1.

S. V. Gaikwad, D. E. Rivera. (1994). *Control ID: An integrated Framework For System Identification and Process Control* , 540-544.

Skogested, S. (2004). Control Structure Designfor Complete Chemical Plant.

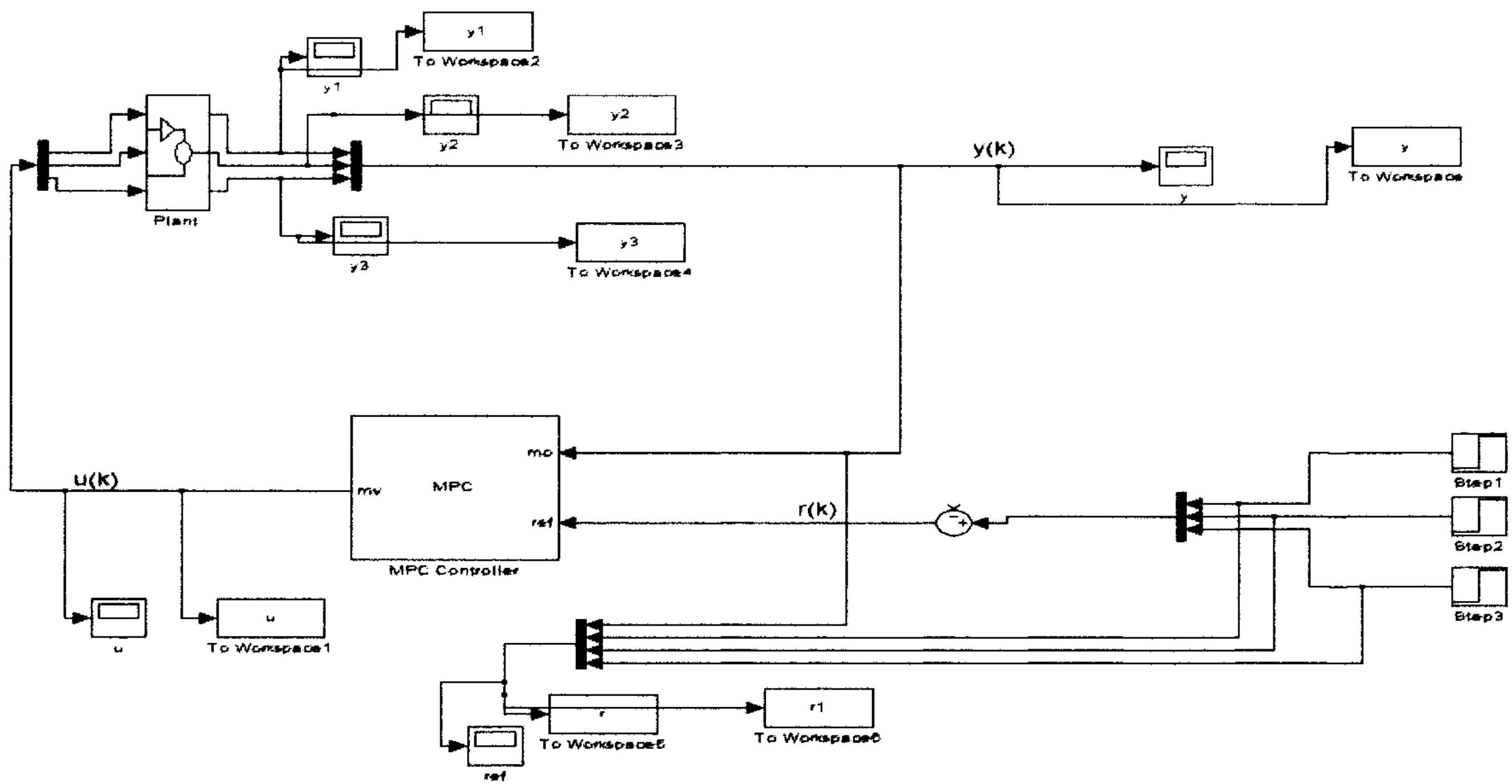
Zhou, Y. (2001). *Multivariable System Identification For Process Control.* United States: Elsevier.

Zhu, Y. (1997). *Multivariable process identification for MPC: the asymptotic method and its applications* , 103.

Zhu, Y. (2007). Tai-Ji Control. *Progress in MPC Identification: A Case Study on Totally Closed-Loop Plant Test* , 1-11.

APPENDICES

Simulink Block Diagram



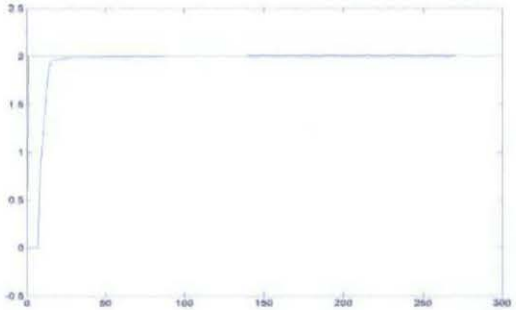
MATLAB Workspace Coding

```
Error=[];
count=5;

while count>0

ysp1=input('Step input y1=');
set_param('base2/Step1','After','ysp1')

ysp2=input('Step input y2=');
set_param('base2/Step2','After','ysp2')

ysp3=input('Step input y3=');
set_param('base2/Step3','After','ysp3')

% Perication and control horizonp=input('Step input y3=');
m=input('Control horizon='); %control horizon
p=100; %prediction horizon


%MPC objective
MPC1=mpc(model,Ts,p,m,Weights,InputSpecs,OutputSpecs);

%MPC state
% xmpc=mpcstate(MPCobj,xp,xd,xn,u)
xmpc=mpcstate(MPC1);

%Simulate

sim('base2')

out1=r(:,1);
out2=r(:,2);
out3=r(:,3);
set=r(:,4);

Error1=trapz(out1)-trapz(set);
Error2=trapz(out2)-trapz(set);
Error3=trapz(out3)-trapz(set);

Error=[Error;Error1 Error2 Error3];

%Plot and save graph


exp=input('Scenario number: ');

%y1 graph
plot(r1.time,r1.signals.values(:,1),r1.time,r1.signals.values(:,4));

file_save=(sprintf('Scenario %d y1',exp))
saveas(gcf,file_save,'tif')

%y2 graph
plot(r1.time,r1.signals.values(:,2),r1.time,r1.signals.values(:,5));
```

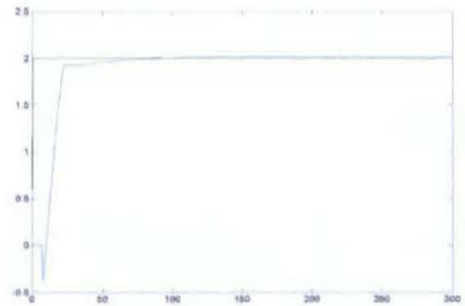
```
file_save=(sprintf('Scenario %d y2',exp))
saveas(gcf,file_save,'tif')

%y3 graph
plot(r1.time,r1.signals.values(:,3),r1.time,r1.signals.values(:,6));

file_save=(sprintf('Scenario %d y3',exp))
saveas(gcf,file_save,'tif')



count=input('continue?');

end
```

4.1.1 Graph of Variables for Various Scenarios

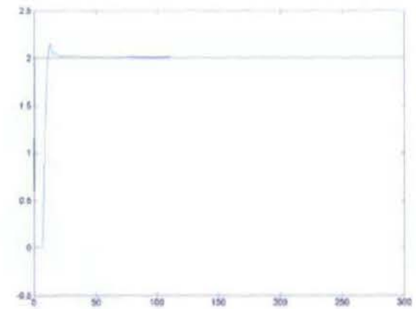
Case Study A



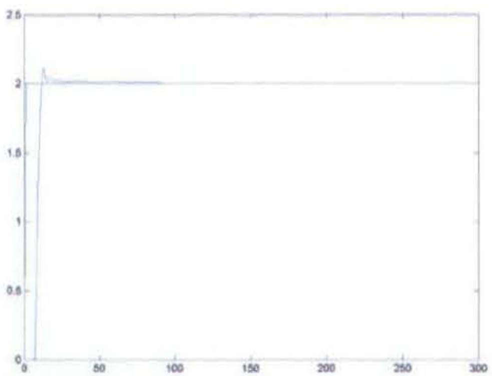
Y1 (2), M=2



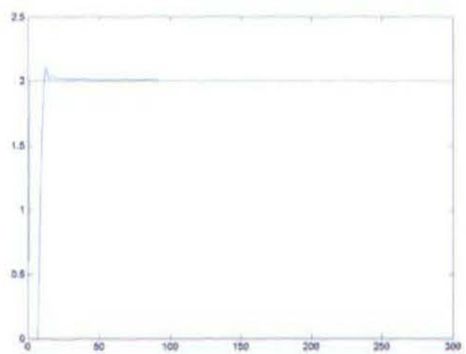
Y1 (2), M=4



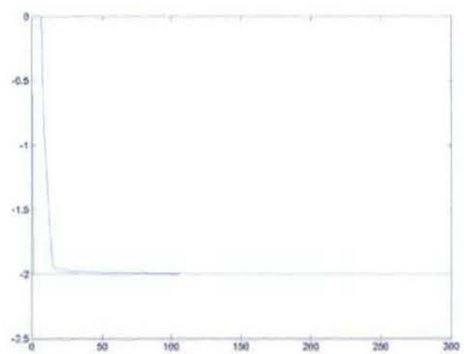
Y1 (2), M=6



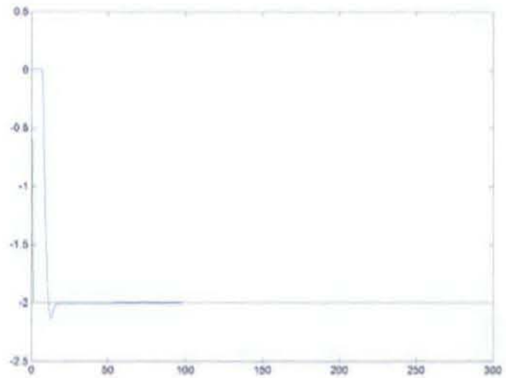
Y1 (2), M=8



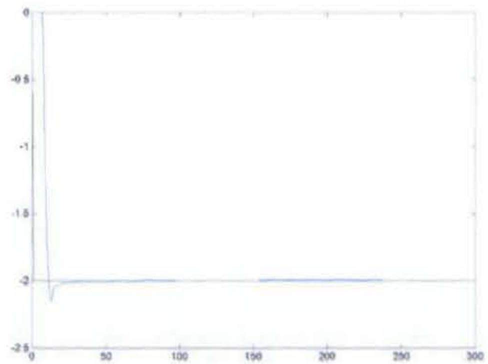
Y1 (2), M=10



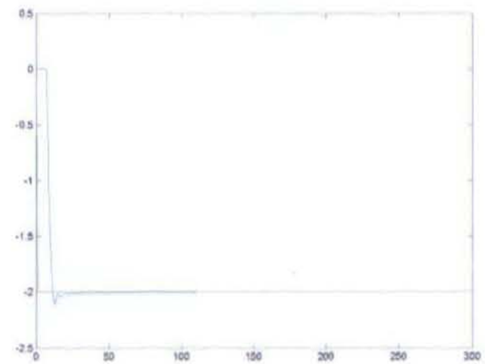
Y1 (-2), M=2



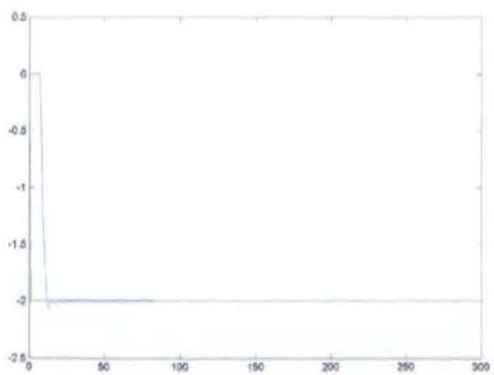
Y1 (-2), M=4



Y1 (-2), M=6



Y1 (-2), M=8



Y1 (-2), M=10

Summary of Error for All Case Studies

		Step/SP	Control Horizon	y1	y2	y3
Scenario 1	Y1	2	2	19.8138	604.0013	601.0189
Scenario 2			4	15.7771	601.4997	596.8384
Scenario 3			6	15.4816	599.4782	598.0925
Scenario 4			8	15.8238	598.6566	598.4982
Scenario 5			10	16.4518	598.8662	598.2266
Scenario 6		-2	2	19.8138	604.0013	601.0189
Scenario 7			4	15.7771	601.4997	596.8384
Scenario 8			6	15.4816	599.4782	598.0925
Scenario 9			8	15.8238	598.6566	598.4982
Scenario 10			10	16.4518	598.8662	598.2266
Scenario 11	Y2	2	2	4.2077	600.6731	0.7252
Scenario 12			4	4.5467	593.5043	0.4809
Scenario 13			6	3.6276	591.4490	0.3117
Scenario 14			8	1.8753	590.0677	0.4652
Scenario 15			10	0.4070	589.7002	0.2070
Scenario 16		-2	2	4.2077	600.6731	0.7252
Scenario 17			4	4.5467	593.5043	0.4809
Scenario 18			6	3.6276	591.4490	0.3117
Scenario 19			8	1.8753	590.0677	0.4652
Scenario 20			10	0.4071	589.7002	0.2070
Scenario 21	Y3	2	2	4.0539	22.5711	592.4767
Scenario 22			4	8.3479	6.8837	599.5563
Scenario 23			6	8.7025	2.5617	596.9288
Scenario 24			8	5.7980	0.1626	596.3634
Scenario 25			10	1.9236	0.4529	597.6196
Scenario 26		-2	2	4.0539	22.5711	592.4767
Scenario 27			4	8.3479	6.8837	599.5563
Scenario 28			6	8.7025	2.5617	596.9288
Scenario 29			8	5.7987	0.1626	596.3634
Scenario 30			10	1.9236	0.4529	597.6196
Scenario 31	Y1+Y2	Y1 (+2) Y2 (+2)	2	16.7000	3.3000	602.1000
Scenario 32			4	20.0000	7.0000	597.9000
Scenario 33			6	19.0000	7.5000	598.1000
Scenario 34			8	17.8000	8.3000	598.1000
Scenario 35			10	17.0000	8.8000	598.1000
Scenario 36		Y1 (-2) Y2 (+2)	2	33.1000	1201.1000	602.2000
Scenario 37			4	13.5000	1192.7000	596.7000
Scenario 38			6	12.4000	1188.9000	599.1000
Scenario 39			8	14.4000	1187.3000	599.5000
Scenario 40			10	16.5000	1187.1000	599.0000
Scenario 41		Y1 (+2)	2	33.1000	1201.1000	602.2000

Scenario 42		Y2 (-2)	4	13.5000	1192.7000	596.7000
Scenario 43			6	12.4000	1188.9000	599.1000
Scenario 44			8	14.4000	1187.3000	599.5000
Scenario 45			10	16.5000	1187.1000	599.0000
Scenario 46		Y1 (-2)	2	16.0000	3.3000	602.1000
Scenario 47		Y2 (-2)	4	20.0000	7.0000	597.9000
Scenario 48			6	19.0000	7.5000	598.1000
Scenario 49			8	17.8000	8.3000	598.1000
Scenario 50			10	17.0000	8.8000	598.1000
Scenario 51	Y1+Y3	Y1 (+2)	2	26.6000	625.5000	10.4000
Scenario 52		Y3(+2)	4	7.2000	607.3000	2.0000
Scenario 53			6	6.3000	601.5000	1.6000
Scenario 54			8	9.6000	598.7000	2.3000
Scenario 55			10	14.0000	599.1000	0.8000
Scenario 56		Y1 (-2)	2	16.3000	581.2000	1192.6000
Scenario 57		Y3(+2)	4	24.4000	594.5000	1196.3000
Scenario 58			6	24.7000	596.9000	1194.8000
Scenario 59			8	22.3000	598.4000	1194.7000
Scenario 60			10	19.1000	598.3000	1195.7000
Scenario 61		Y1 (+2)	2	16.3000	581.2000	1192.6000
Scenario 62		Y3 (-2)	4	24.4000	594.5000	1196.3000
Scenario 63			6	24.7000	596.9000	1194.8000
Scenario 64			8	22.3000	598.4000	1194.7000
Scenario 65			10	19.1000	598.3000	1195.7000
Scenario 66		Y1 (-2)	2	26.6000	625.5000	10.4000
Scenario 67		Y3 (-2)	4	7.2000	607.3000	2.0000
Scenario 68			6	6.3000	601.5000	1.6000
Scenario 69			8	9.6000	598.7000	2.3000
Scenario 70			10	14.0000	599.1000	0.8000
Scenario 71	Y2+Y3	Y2 (+2)	2	1.9139	578.8423	590.5495
Scenario 72		Y3(+2)	4	3.2555	587.1078	598.9975
Scenario 73			6	4.8866	589.3873	597.0346
Scenario 74			8	3.7123	590.4352	596.6347
Scenario 75			10	1.3034	589.7713	597.6260
Scenario 76		Y2(-2)	2	11.0124	622.5652	589.3103
Scenario 77		Y3(+2)	4	11.9770	600.2105	600.2745
Scenario 78			6	12.1077	593.1337	596.2993
Scenario 79			8	7.2888	589.5324	595.7087
Scenario 80			10	2.0089	589.5061	597.2286
Scenario 81		Y2(+2)	2	11.0124	622.5652	589.3103
Scenario 82		Y3(+2)	4	11.9770	600.2105	600.2745
Scenario 83			6	12.1077	593.1337	596.2993
Scenario 84			8	7.2888	589.5324	595.7087
Scenario 85			10	2.0089	589.5061	597.2286
Scenario 86		Y2(-2)	2	1.9139	578.8420	590.5495

Scenario 87		Y3 (-2)	4	3.2555	587.1078	598.9975
Scenario 88			6	4.8866	589.3873	597.0346
Scenario 89			8	3.7123	590.4352	596.6346
Scenario 90			10	1.3034	589.7713	597.6260
Scenario 91	Y1+Y2+Y3	Y1(+2)+Y2(+2)+Y3(+2)	2	20.9000	25.7000	1.2000
Scenario 92			4	11.5000	14.0000	1.0000
Scenario 93			6	10.0000	9.9000	1.9000
Scenario 94			8	11.7000	8.1000	2.5000
Scenario 95			10	14.7000	9.0000	1.3000
Scenario 96		Y1(-2)+Y2(+2)+Y3(+2)	2	24.2000	1180.4000	1194.0000
Scenario 97			4	21.1000	1187.3000	1195.8000
Scenario 98			6	21.5000	1188.2000	1195.1000
Scenario 99			8	20.8000	1188.8000	1194.9000
Scenario 100			10	19.1000	1188.3000	1195.6000
Scenario 101		Y1(+2)+Y2(-2)+Y3(+2)	2	42.6000	1221.5000	11.8000
Scenario 102			4	7.0000	1198.8000	3.2000
Scenario 103			6	4.2000	1190.6000	2.4000
Scenario 104			8	9.1000	1186.4000	3.4000
Scenario 105			10	15.0000	1186.6000	1.6000
Scenario 106		Y1(+2)+Y2(+2)+Y3(-2)	2	13.1000	19.0000	1191.8000
Scenario 107			4	28.8000	0.9000	1196.9000
Scenario 108			6	28.1000	5.7000	1194.5000
Scenario 109			8	24.1000	8.8000	1194.0000
Scenario 110			10	19.4000	9.1000	1195.3000
Scenario 111		Y1(-2)+Y2(-2)+Y3(+2)	2	13.1000	19.0000	1191.8000
Scenario 112			4	28.8000	0.9000	1196.9000
Scenario 113			6	28.1000	5.7000	1194.5000
Scenario 114			8	24.1000	8.8000	1194.0000
Scenario 115			10	19.4000	9.1000	1195.3000
Scenario 116		Y1(+2)+Y2(-2)+Y3(-2)	2	24.2000	1180.4000	1194.0000
Scenario 117			4	21.1000	1187.3000	1195.8000
Scenario 118			6	21.5000	1188.2000	1195.1000
Scenario 119			8	20.8000	1188.8000	1194.9000
Scenario 120			10	19.1000	1188.3000	1195.6000
Scenario 121		Y1(-2)+Y2(+2)+Y3(-2)	2	42.6000	1221.5000	11.8000
Scenario 122			4	7.0000	1198.8000	3.2000
Scenario 123			6	4.2000	1190.6000	2.4000
Scenario 124			8	9.1000	1186.4000	3.4000
Scenario 125			10	15.0000	1186.6000	1.6000
Scenario 126		Y1(-2)+Y2(-2)+Y3(-2)	2	20.9000	25.7000	12.0000
Scenario 127			4	11.5000	14.0000	1.0000
Scenario 128			6	10.0000	9.9000	1.9000
Scenario 129			8	11.7000	8.1000	2.5000
Scenario 130			10	14.7000	9.0000	1.3000