

NUR ADILLA MAHADZIR

B.ENG. (HONS) PETROLEUM ENGINEERING

MAY 2013



UNIVERSITI
TEKNOLOGI
PETRONAS

DEVELOPMENT OF MATHEMATICAL MODEL FOR
HYDRAULIC FRACTURING DESIGN

NUR ADILLA BINTI MAHADZIR

PETROLEUM ENGINEERING
UNIVERSITI TEKNOLOGI PETRONAS

MAY 2013

CERTIFICATION OF APPROVAL

Development of Mathematical Model for Hydraulic Fracturing Design

by

Nur Adilla binti Mahadzir

A project dissertation submitted to the
Petroleum Engineering Programme
Universiti Teknologi PETRONAS
in partial fulfillment of the requirement for the
BACHELOR OF ENGINEERING (Hons)
(PETROLEUM ENGINEERING)

Approved by,

(Mr Muhammad Aslam b Md Yusof)

UNIVERSITI TEKNOLOGI PETRONAS
TRONOH, PERAK

May 2013

CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

NUR ADILLA BINTI MAHADZIR

ABSTRACT

Hydraulic fracturing has been proposed as one of the stimulation technique to economically increase oil/gas production. The design of hydraulic fracturing must take into account various parameters, considerations and complicated calculations. Therefore the project entitled “Development of the Mathematical Model for Hydraulic Fracturing Design” has been proposed. The objective of this project was to develop a mathematical model that could determine the calculation of important parameters involved in the design of hydraulic fracturing and to run several case studies to verify and validate the mathematical model. Mathematical model for hydraulic fracturing design was developed using Microsoft Excel Visual Basic for Applications (VBA). In this report, literature review was done which covers the theory and general overview of hydraulic fracturing; the history of hydraulic fracturing, process of hydraulic fracturing as well as hydraulic fracturing design parameters. Research was also done on hydraulic fracturing calculations and equations involve in its design. Project methodology and activities have been developed and the milestones for this project have been planned for Final Year Project 1 and Final Year Project 2. The Gantt Chart and key milestones for FYP I and FYP II were attached in the report in order to make sure that the project run smoothly according to its plan and schedule. Mathematical model for hydraulic fracturing design was completely developed. Case studies were conducted to verify and validate the mathematical model. Mathematical model verifications for hydraulic fracturing design have established a range of 0 to 2.6% of percentage differences between the calculated values in the model and manual hand calculation while mathematical model validation have established a range of 0 to 17% of percentage differences between the calculated values and the field data values.

ACKNOWLEDGEMENT

First of all, I would like to convey my greatest praise and gratitude to Allah the Almighty for His Mercy for giving me the strength and capability to complete this Final Year Project I and II throughout this final year.

My sincere gratitude is extended to my supervisor, Mr Muhammad Aslam bin Md Yusof for his commitment and encouragement throughout the completion of this project. The special thank also goes to my ex-supervisor, AP Aung Kyaw. The supervision, support and advices that he gave truly help the progression and smoothness of this project. The cooperation from both supervisors is much indeed appreciated.

I would like to gratefully acknowledge the support of FYP coordinators and other lecturers for giving excellent advices and guiding me during the progress of this project. I would also like to take this opportunity to acknowledge my internship supervisors, Mr Shahrul Azman bin Zainal Abidin who had giving me exposure and experienced in VBA programming language which helped me a lots in this project.

Deepest thanks and appreciation to my parents and family for their cooperation, encouragement, constructive suggestion and full of support for the project completion, from the beginning till the end. Also thanks to all of my friends and everyone, those have been contributed by supporting my project and help me during the project progress till it is fully completed.

TABLE OF CONTENTS

CERTIFICATION OF APPROVAL	i
CERTIFICATION OF ORIGINALITY.....	ii
ABSTRACT	iii
ACKNOWLEDGEMENT.....	iv
LIST OF FIGURES	vii
LIST OF TABLES	viii
ABBREVIATION & NOMENCLATURES.....	ix
CHAPTER 1:INTRODUCTION.....	1
1.1 Background of Study.....	1
1.2 Problem Statement	2
1.3 Objectives	3
1.4 Scope of Study.....	3
1.5 Relevancy of the Project.....	4
1.6 Feasibility of the Project within the Scope and Time Frame	5
CHAPTER 2:LITERATURE REVIEW AND/OR THEORY.....	6
2.1 History of hydraulic fracturing.....	6
2.2 Hydraulic Fracturing Theory	6
2.3 Hydraulic Fracturing Design	7
2.3.1 Fracture Extent	8
2.3.2 Fracture Area.....	10
2.3.3 Fracturing Fluid Coefficient.....	10
2.3.4 Propping Agent	12
2.3.5 Fracture Hydraulics	14
2.4 Fracture Design Calculation	16
2.5 Mathematical Model.....	17
CHAPTER 3:METHODOLOGY/ PROJECT WORK.....	18
3.1 Research Methodology	18
3.2 Project Activities	19

3.4	Gantt Chart	21
3.5	Program Flow Chart	23
3.6	Calculation Procedures	24
3.7	Tools/ Software	26
CHAPTER 4:RESULTS AND DISCUSSIONS		28
4.1	Data Gathering and Analysis.....	28
4.2	Assumption used in the model	32
4.3	Mathematical Model for Hydraulic Fracturing Design	33
4.4	Mathematical Model Verification and Validation.....	36
4.4.1	Hydraulic Fracturing Mathematical Model Verification	36
4.4.2	Hydraulic Fracturing Mathematical Model Validation	42
4.5	Discussions	50
CHAPTER 5:CONCLUSIONS AND RECOMMENDATIONS		51
5.1	Conclusions	51
5.2	Recommendations	53
REFERENCES.....		54
APPENDICES		56
Appendix 1: Graph of Fracturing Efficiency against Its Function		56
Appendix II: Graph of Estimate Productivity Ratio after Fracturing.....		57
Appendix III: Graph of Effect of Pressure on frac-sand Permeability		58
Appendix IV: Table of Complementary Error Function		59
Appendix V: VBA Coding of Hydraulic Fracturing Design.....		60

LIST OF FIGURES

FIGURE 1: Least Principal Stress is in the Vertical Direction Resulting in a Horizontal Fracture	9
FIGURE 2: Least Principal Stress in the Horizontal Direction in Vertical Fracture	9
FIGURE 3: Process of Mathematical Modeling	17
FIGURE 4: Interface of VBA in Microsoft Excel	26
FIGURE 5: Coding Window in VBA (Assigning data)	33
FIGURE 6: Input Data in Excel	34
FIGURE 7: User manual in the mathematical model	35
FIGURE 8: Results for Case 1	35
FIGURE 9: Graph of Maximum Amount of Sand between HF Mathematical Model and Field Data for All Cases.....	45
FIGURE 10: Graph of Volume of Fracture Fluid between HF Mathematical Model and Field Data for All Cases.....	46
FIGURE 11: Graph of Volume of Fracture Fluid between HF Mathematical Model and Field Data for All Cases.....	47
FIGURE 12: Graph of Hydraulic Horsepower between HF Mathematical Model and Field Data for All Cases.....	48
FIGURE 13: Graph of Productivity Ratio between HF Mathematical Model and Field Data for All Cases.....	49

LIST OF TABLES

TABLE 1: : Key Milestones for FYP I and FYP II	20
TABLE 2: Gantt Chart for FYP I.....	21
TABLE 3: Gantt Chart FYP II.....	22
TABLE 4: Required parameters for Hydraulic Fracturing Design	28
TABLE 5: Field Data for Case Studies.....	31
TABLE 6: Results of mathematical modeling and manual calculation for model verification	40
TABLE 7: Results comparison between mathematical model calculation and field data for Case 1	42
TABLE 8: Results comparison between mathematical model calculation and field data for Case 2	43
TABLE 9: Results comparison between mathematical model calculation and field data for Case 3	43
TABLE 10: Results comparison between mathematical model calculation and field data for Case 4	44
TABLE 11: Results comparison between mathematical model calculation and field data for Case 5	44

ABBREVIATION & NOMENCLATURES

k	<i>Permeability</i>	N_{Re}	<i>Reynolds number</i>
ΔP	<i>Differential pressure across fracture face</i>	f	<i>Friction factor</i>
φ	<i>Porosity</i>	$erfc(x)$	<i>Complementary error function of x</i>
μ	<i>Viscosity</i>	ρ_T	<i>Density of oil at average well temperature</i>
C	<i>Fracturing fluid coefficient</i>	P_T	<i>Fracture treating pressure</i>
m	<i>Slope of fluid loss curve</i>	k_f	<i>Fracture permeability</i>
A_f	<i>Area of filter paper</i>	h	<i>Formation thickness</i>
P_s	<i>Surface injection pressure</i>	r_e	<i>Drainage radius</i>
q_t	<i>Barrels of fluid per minute</i>	r_w	<i>Wellbore radius</i>
ΔP_s	<i>Hydrostatic pressure</i>	LEC	<i>Line efficiency correlation</i>
ΔP_p	<i>Pressure drop through perforation</i>	PR	<i>Productivity Ratio</i>
G_f	<i>Fracture gradient</i>	VBA	<i>Visual Basic for Applications</i>
D	<i>Depth</i>	HF	<i>Hydraulic fracturing</i>
t	<i>Injection time</i>		
W	<i>Fracture width</i>		
Eff	<i>Fracture efficiency</i>		
V	<i>Volume per unit area of fracture</i>		
S	<i>Weight of sand</i>		
φ_{sand}	<i>Porosity of sand</i>		
ρ_{sand}	<i>Sand density</i>		
X	<i>Sand concentration</i>		
v	<i>Average flow velocity in pipe</i>		
q_t	<i>Actual flow rate (including sand)</i>		
d_e	<i>diameter of circular pipe</i>		
γ_T	<i>Specific gravity of oil at average well temperature</i>		

CHAPTER 1

INTRODUCTION

1.1 Background of Study

According to Association of American State Geologists (2012), hydraulic fracturing or known as “fracking”, “hydrofracking” or “fracing” as applied in oil and gas industry is the process of pumping a mixture of water, sand and other chemical additives under high pressure to create fractures originating from the wellbore in a producing formation to provide increased flow channels for production. A viscous fluid containing a proppant such as sand is injected under high pressure until the desired fracturing is achieved. The pressure is then released allowing the fluid to return to the well. The proppant, however, remains in the fractures preventing them from closing. Hydraulic fracturing is usually useful to increase productivity index (PI) of the well especially in low permeability reservoir and increase the flow rate of oil and/or gas from wells that have been damaged. Damage occurs because drilling and/or completion fluids leak into the reservoir and plug up the pores and pore throats. When the pores are plugged, the permeability is reduced, and the fluid flow in this damaged portion of the reservoir may be substantially reduced. To stimulate damaged reservoirs, a short, conductive hydraulic fracture is often the desired solution.

The studies on hydraulic fracturing design are important before conducting any hydraulic fracturing stimulation treatment for the well. A successful hydraulic fracturing stimulation treatment is dependent on many factors. Its design requires a number of considerations such as the prediction of well productivity for various fracture lengths and conductivities, parametric studies on fracture geometry requirement for particular

types of formations, selection of appropriate types of fracture materials and determination of fracture design criteria.

Hydraulic fracturing design models are used today as a prediction tool for the optimization of hydraulic fracturing. In this study, since there are many parameters need to be calculated for the design on hydraulic fracturing, a mathematical model has to be developed. According to Wikipedia, mathematical model is defined as a description of a system using mathematical concepts and language. It is also defined as a representation of the essential aspects of an existing system (or a system to be constructed) which presents knowledge of that system in usable form (Eykhoff, 1974). Decision-theory, queuing theory and linear programming are some of mathematical modeling tools which involve big amounts of number crunching.

For this project, a research study of hydraulic fracturing will be done to investigate the parameters that need to be calculated for hydraulic fracturing design. An alternative method to estimate hydraulic fracturing design parameters has been presented by developing a mathematical model using Microsoft Excel Visual Basic for Applications (VBA). Applicability and validity of the model is then demonstrated by using field data and results.

1.2 Problem Statement

In oil and gas industry, it is important to have an optimized production of oil and gas wells. Production optimization can be defined as an optimum analysis and comprehensive investigation of well production systems to maximize hydrocarbon recovery while minimizing the operating cost and to overcome formation damage. In order to have an optimize production, the whole production systems are needed to be optimized so that they could perform efficiently. Therefore hydraulic fracturing has been proposed as the stimulation technique to economically increase oil/gas production.

The design of hydraulic fracturing must take into account various parameters and considerations. There are many complicated calculations and correlation involves in designing the hydraulic fracturing stimulation technique. Therefore a mathematical model will be developed to ease the calculations for the parameters needed in hydraulic fracturing design. By coding the equations and correlations into Microsoft Excel Visual Basic for Application (VBA), user can insert the input data required and the functions will automate the calculation of various hydraulic fracturing design parameters.

In addition, most company nowadays are using simulation software such as MFrac, StimPlan, Fracpro, GOHFER and etc. for the modeling of hydraulic fracturing design. However these softwares are costly and usually covered more complicated modeling in which only experts who are familiarizing with the software can handle them. Therefore a simple mathematical model could be an alternative tool to design the hydraulic fracturing treatment.

1.3 Objectives

The objectives of this study are:-

- To develop a mathematical model for hydraulic fracturing design.
- To run a case study to verify and validate the equations and coding of hydraulic fracturing design.

1.4 Scope of Study

Hydraulic fracturing is one of the well stimulation methods that can be applied for obtaining more recovery but before applying hydraulic fracturing treatment in particular well, a complicated calculation procedure is required for the treatment. Based on the calculations, the volume of fracturing fluid required, volume of propping agent to be mixed together, required injection pressure and horsepower requirement for the surface

injection pump and fracture area can be calculated and prepared at the well site. By using mathematical model, the calculation can be made within short time and readily applied for the treatment.

The scope of study includes:-

- Understanding on the theory and process of hydraulic fracturing.
- Understanding on the parameters of hydraulic fracturing design.
- Deeper understanding on the developed mathematical correlations and equations involved in hydraulic fracturing design.
- Familiarization with Microsoft Excel Visual Basic for Applications (VBA) in order to develop computer code for the model.
- Analysis of the programmed equations by conducting a case study to verify the equations of hydraulic fracturing design coded.

1.5 Relevancy of the Project

This research will be very relevant judging from certain criteria and circumstances. This project will be focused on the study of hydraulic fracturing design and calculations of important parameters in hydraulic fracturing design. This project will be important and relevant due to the fact that the complicated and lengthy calculation of hydraulic fracturing design parameters can be easily calculated by developing a mathematical model of hydraulic fracturing. User can manually insert the data required and the model will automate the calculations.

Other than that, the author foresee that if more relevant parameters are added to this mathematical model, it will be relevant to be used in oil and gas industry as an alternative tools in designing hydraulic fracturing treatments.

1.6 Feasibility of the Project within the Scope and Time Frame

With careful planning and full dedication in conducting this research, the project are able be completed within the given times of 8 months. During FYP 1, it is required for the author to complete the research on the project topic, the understanding on the mathematical formulation and the familiarization of the programming language in Microsoft Excel Visual Basic for Applications (VBA). For FYP 2, the focus should be on developing the mathematical model and to run a case study to verify and validate the coding and equations of hydraulic fracturing design. The cost for this project is affordable as author only need to use Microsoft Excel with VBA coding function to complete the project. Following section will covers the researches, analysis and interpretation of the results.

CHAPTER 2

LITERATURE REVIEW AND/OR THEORY

2.1 History of hydraulic fracturing

In 1930s, Dow Chemical Company discovered that by applying a large enough downhole fluid pressure, it was possible to deform and fracture the rock formation to have a more effective acid stimulation (Grebe et al. 1935). This is where hydraulic fracturing began in oil and gas industry. In 1947, the first non-acid hydraulic fracturing treatment for well stimulation was introduced on a gas well in Hugoton field, Kansas in order to compare the new technique with the available acidization technology (Veatch et al. 1989). Nowadays, hydraulic fracturing is extensively used to improve oil and gas wells' productivity. Of the production wells drilled in North America since the 1950s, about 70% of gas wells and 50% of oil wells have been hydraulically fractured (Valko and Economides, 1995). Thousands of treatments are implemented each year in a wide range of geological formations which may vary from low permeability gas fields, weakly consolidated offshore sediments, soft coal beds for methane extraction, naturally fractured reservoirs, and geometrically complex structures (Adachi et al. 2007).

2.2 Hydraulic Fracturing Theory

Since the early mid-century, hydraulic fracturing has been proposed as the solution to economically increase oil/gas production from the relatively low pressure, low permeability reservoirs (Murphy & Carney, 1977). Hydraulic Fracturing is the application of pressure through a fluid column to a formation face which is greater than

the least principle stress acting upon the formation in order to crack and fracture the formation rock.

Hydraulic fracture process start with initiating a fracture in the formation with the fracturing fluid, propagating the fracture with fluid, and holding the created fracture open with proppant. The propped fracture becomes the conductivity pathway between the formation and the wellbore for hydrocarbon production. To serve this pathway function, a fracture stimulation design has three principal stages: the pad stage, the slurry stage and the flush stage. The pad stage (without proppant) is used to initiate and propagate the fracture, develop fracture width, and provide suitable fluid for leakoff. The slurry stage (with both fluid and proppant) is used to position the proppant in the fracture so that there is a constant proppant concentration through the length of the fracture at the end of pumping. Finally the treatment is flushed to the perforation.

2.3 Hydraulic Fracturing Design

Ali Daneshy (2010) points out that engineering computation always precede a fracturing treatment which comprises of the calculation of fluid volume and viscosity, injection rate, weight of proppant, volumes of different phases of the job (pre-pad, pad, slurry, and displacement), surface and bottomhole injection pressure, hydraulic horsepower required at the surface, and the mechanical equipment needed for this.

According to H. Jabbari and Z. Zeng (2012), the best hydraulic fracturing design depends very much on the environment in which the fracture treatment will be carried out. The characteristics that define the environment are controllable parameters, such as wellbore casing, tubing and wellhead configurations, wellbore downhole equipment, lateral length, well spacing, perforation location and quantity, fracturing fluid and proppant characteristics, and fracturing treatment rate and pumping schedule.

R. W. Veatch (1983) mentioned on the general treatment design consideration which limited to selecting the appropriate types of materials (e.g., fluids, additives, and proppants), the appropriate volumes of materials, injection rates for pumping these materials, and the schedule for injecting the materials.

2.3.1 Fracture Extent

In Hydraulic Fracturing Operations-Well Construction and Integrity Guideline (2009), the fracture extents are divided into two which consists of horizontal fracture and vertical fracture.

Horizontal fractures are formed in the direction perpendicular to the least stress. Figure 1 illustrates a rock cube as having confining stress exerted on it in three dimensions. Each pair of opposing stresses must be equal in order for the cube to remain stationary in space. The relative size of the arrows represents the magnitude of the confining stress. In Figure 2, the least stress is in the vertical direction. This direction is known as the direction of overburden, referring to the weight of the earth that lies above. The Earth's overburden pressure is the least principal stress only at shallow depth.

Based on experience, horizontal fractures will occur at depths less than 2000 ft. When pressure is applied to the center of this block, the formation will crack or fracture in the horizontal plane as shown, because it will be easier to part the rock in this direction than any other direction. In general, these fractures are parallel to the bedding plane of the formation.

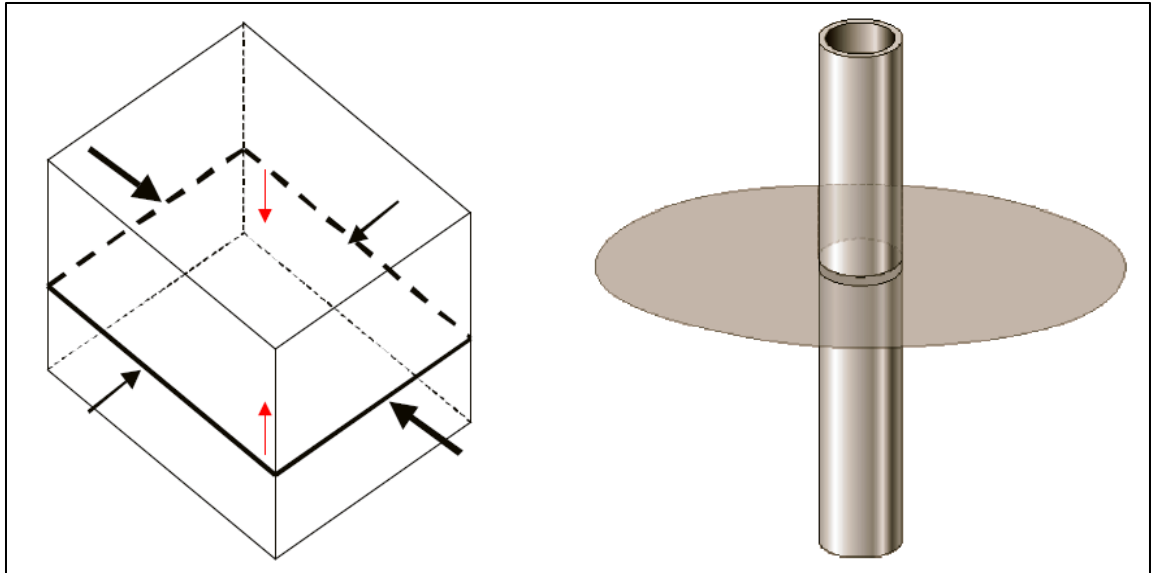


FIGURE 1: Least Principal Stress is in the Vertical Direction Resulting in a Horizontal Fracture (Retrieved from Well construction and Integrity Guideline)

As depth increases, overburden stress in the vertical direction also increases. As the stress in the vertical direction becomes greater with depth, the overburden stress (stress in the vertical direction) becomes the greatest stress. This situation generally occurs at depths greater than 2000 ft. This is represented in Figure 2 by the magnitude of the arrows, where the least stress is represented by the small red horizontal arrows, and the induced fracture will be perpendicular to this stress, or in the vertical orientation.

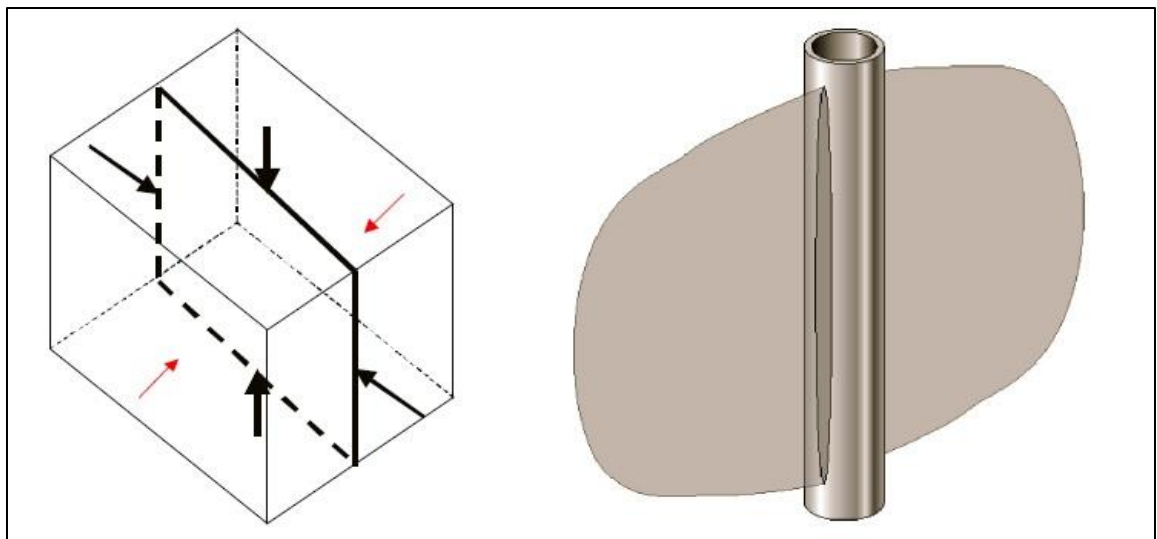


FIGURE 2: Least Principal Stress in the Horizontal Direction in Vertical Fracture (Retrieved from Well Construction and Integrity Guideline)

2.3.2 Fracture Area

According to Howard and Fast (1957), the success of hydraulic fracturing as a production stimulation technique is controlled to a large degree by the depth of penetration of the fracture system. Having pointed out the importance of fracture length on well productivity, the effects of fracturing-fluid characteristics and reservoir-fluid and rock characteristics on the areal extent of a fracture should be considered. The effect of these variables may be illustrated by considering how they influence the calculated areal extent of a fracture. Based on the derivation by R. D. Carter, the equation for estimating the extent of the fractured area in terms of the treating conditions is:

$$A(t) = \frac{q_i W}{4\pi C^2} \left[e^{x^2} \operatorname{erfc}(x) + \frac{2x}{\sqrt{\pi}} - 1 \right] \dots\dots\dots (1)$$

Where $x = 2C\sqrt{\pi t}/W$

$A(t)$ = Area of one face of the fracture at time t

q_i = Injection rate

W = Fracture width

t = Injection time

C = Fracturing fluid coefficient

$\operatorname{erfc}(x)$ = Complementary error function of (x)

2.3.3 Fracturing Fluid Coefficient

To create the fracture, a fluid is pumped into the wellbore at high rate to increase the pressure in the wellbore at the perforations to a value greater than the breakdown pressure of the formation. The ideal fracturing fluid should be compatible with the formation rock, compatible with the formation fluid, generate enough pressure drop down the fracture to create a wide fracture, be able to transport the propping agent in the

fracture, break back to a low viscosity fluid for clean up after the treatment, and be cost effective.

Howard and Fast (1957) stated that since the fracturing fluid properties are reflected in equation (1) through the fracturing fluid coefficient, C, it is important to establish a method for the determination of this factor for various types of fracturing fluids. The fracturing-fluid coefficient, C, defines the three types of linear flow mechanisms countered with fracturing fluids for which equation (1) applies which comprises of viscosity and relative permeability effect, reservoir fluid viscosity-compressibility effects and wall building effects. Although each mechanism is considered as acting alone by this equation, all may act simultaneously in a fracturing treatment so that the mechanisms may complement each other and increase the fluid's effectiveness.

Viscosity controlled fluids includes viscous or semi-viscous fracture fluids in situations where the viscosity controls the amount of fluid loss taking place during fracturing. Where the viscosity of the fracturing fluid is considerably greater than that of the formation fluid, most of the pressure drop during fracturing will occur in invaded zone since the fracturing fluid is less mobile than the formation fluid. Therefore the viscosity of the fracture fluid controls the amount of fluid loss to the formation. Thus, for this case the fracturing fluid coefficient, C_v is defined as:

$$C_v = 0.0469 \left(\frac{k\Delta P\phi}{\mu} \right)^{1/2} \text{ ft}/\sqrt{\text{min}} \dots\dots\dots (2)$$

Reservoir controlled fluid - This category of fracturing fluids has low viscosity and high fluid loss characteristics in which the physical properties identical to those of the reservoir fluid (Craft et al, 1962). Under these conditions the rate of fluid loss would be controlled by the viscosity and the coefficient of compressibility of the fluid being injected and the reservoir fluid (Howard and Fast, 1957). Fracturing fluids which fall into this classification are lease crude and water, which do not contain additives to reduce fluid loss. The fracturing fluid coefficient, C_c can be determined once the

differential treating pressure, formation permeability and porosity, reservoir fluid viscosity and compressibility coefficient are determined as shown in equation (3) below:

$$C_c = 0.374\Delta P \left(\frac{kc_f\phi}{\mu} \right)^{1/2} \text{ ft}/\sqrt{\text{min}} \dots\dots\dots (3)$$

Craft B.C et al (1962) clarified that the use of modern additives to limit fluid loss creates a third class of fracturing fluids which is Wall-building fluids. These fluids build a temporary filter cake or wall on the face of the fracture as it is exposed. In the calculation of the fracturing-fluid coefficient for the two previous flow mechanisms, it was shown that the coefficients could be calculated from known reservoir data and fracturing-fluid viscosity. Equation (4) shows, however, that the fracturing-fluid coefficient for this condition is dependent upon the slope of a filtrate vs. square foot of time plot which must be obtained experimentally. Although many investigators have used filter paper as a means of fracturing-fluid evaluation, it was deemed advisable to conduct tests to determine the fracturing-fluid coefficients on actual formation cores.

The cores were prepared in the form of very thin wafers to insure that the major portion of the flow resistance measured was resulting from the filter cake and not the core itself. It was also believed desirable to study the effects of various pore area distributions on the filter cakes; therefore, various type cores were used. The fracturing fluid coefficient for wall-building fluid, C_w is represented below:

$$C_w = \frac{0.0164m}{A_f} \text{ ft}/\sqrt{\text{min}} \dots\dots\dots (4)$$

2.3.4 Propping Agent

Proppant is a mixture of coarse granular particle, strong enough to keep the fracture open. Propping agents are required to “prop-open” the fracture once the pumps are shut down and the fracture begins to close. According to Holditch (1979) the ideal propping

agent will be strong, resistant to crushing, resistant to corrosion, have a low density, and readily available at low cost. The proppants that best meet these desired traits are silica sand, resin-coated sand, and ceramic proppants.

Generally, silica sand is used to prop open fractures in shallow formations. For coal seam reservoirs, sand is usually the best choice for a propping agent and virtually every fracture treatment in a coal seam reservoir uses sand. Sand is much less expensive per pound than the resin-coated sand or the ceramic proppants.

Resin-coated (epoxy) sand is stronger than sand and is used where more compressive strength is required to minimize proppant crushing. Some resins can be used to form a consolidated sand pack in the fracture, which will help to eliminate proppant flow back into the wellbore. Resin coated sand is more expensive than sand. Resin-coated sand is used where sand flowback is an issue or where more proppant strength and conductivity are needed

Ceramic proppants consist of sintered bauxite, intermediate strength proppant (ISP), and light weight proppant (LWP). The strength of the proppant is proportional to its density. Also, the higher strength proppants, like sintered bauxite, cost more than ISP and LWP. Ceramic proppants are used to stimulate deep (>8,000 ft) wells where large values of in-situ stresses will apply large forces on the propping agent. Ceramic proppants are utilized where higher conductivity and higher strength are required.

For the size of proppant, 100 mesh sand is typically used in the early portion of the job for enhanced distance and height, diversion, etching, and as a propping agent. 40/70 and 40/80 mesh proppants are currently the predominant proppants used in gas shales. 30/50 and 20/40 proppant used in some areas for fracture conductivity enhancement (especially important in the liquids rich plays)

2.3.5 Fracture Hydraulics

The hydraulic power required to pump q_t barrels of fluid per minute into a well with a surface injection pressure of p_s psi is given by the product $p_s q_t$ or, converting to hydraulic horsepower.

$$H_h = \frac{144 \text{ in}^2/\text{ft}^2 \times 5.615 \text{ cuft/bbl } P_s q_t}{33,000 \text{ ft-lb/min/hp}} \dots\dots\dots (5)$$

$$= 0.0245 P_s q_t \text{ hp} \dots\dots\dots (6)$$

Surface Injection Pressure

The pump pressure or surface injection pressure, p_s , is equal to the sum of the bottom-hole fracture treating pressure p_t , the frictional pressure drop in the pipe Δp_f , and the pressure drop through the perforations Δp_p , minus the hydrostatic pressure Δp_s

$$P_s = 0.0245 P_s q_t \text{ hp} \dots\dots\dots (7)$$

Bottomhole Treating Pressure

The bottom-hole treating pressure p_t is determined from the fracture gradient G_f and the depth D to the fracture.

$$P_t = G_f D \dots\dots\dots (8)$$

$$G_f = \frac{P_s + \Delta P_s - \Delta P_f - \Delta P_p}{D} \dots\dots\dots (9)$$

Hydrostatic Pressure

The hydrostatic pressure Δp_s is obtained from the density of the fluid including the propping agent

$$\Delta P_s = \frac{0.433}{8.34} \left(\frac{8.34 \gamma_T + x}{1 + 0.0456x} \right) \dots\dots\dots (10)$$

$$\Delta P_s = 0.052 \rho_T D \dots\dots\dots (11)$$

Where

$$x = \frac{\text{maximum amount of proppant}}{\text{volume of fracture fluid}} \dots\dots\dots (12)$$

$$\rho_T = \frac{8.34\gamma + x}{1 + 0.0456x} \dots\dots\dots (13)$$

$$\gamma_T = \gamma_{60}[1 - \beta(T - 60)] \dots\dots\dots (14)$$

Pressure Drop across Perforations

Pressure drop across the perforations is obtained directly from Bernoulli's equation.

$$\Delta P_p = \frac{\rho q^2}{8090A^2} \dots\dots\dots (15)$$

Frictional Pressure Drop

$$v = 17.16 qd^2 \dots\dots\dots (16)$$

$$N_{Re} = 928dv\rho\mu \dots\dots\dots (17)$$

$$f = \frac{64}{N_{Re}} \text{ for } N_{Re} < 2000 \dots\dots\dots (18)$$

$$\frac{1}{f} = -2\log \left[\frac{\frac{2\varepsilon}{d}}{3.7} - \frac{5.02}{N_{Re}} \log \left(\frac{\frac{2\varepsilon}{d}}{3.7} + \frac{13}{N_{Re}} \right) \right] \text{ for } N_{Re} > 2000 \dots\dots\dots (19)$$

$$\Delta P_f = \frac{(fLv^2)}{25.80d} \dots\dots\dots (20)$$

2.3.6 Productivity Ratio

The productivity ratio is the ratio of the productivity index of the well after fracturing to that of the well before fracturing, J_f/J .

$$PR = \left[\frac{k_f W}{kh} \right] \left[\frac{\left(\frac{kh}{k_f} + 1 \right) \ln \left(\frac{r_e}{r_w} \right)}{\left(\frac{k_f W}{kh} + 1 \right) \ln \left(\frac{r_e}{r_f} \right) + \ln \left(\frac{r_f}{r_w} \right)} \right] \dots\dots\dots (21)$$

2.4 Fracture Design Calculation

The objective of any type of fracture design calculation is to plan the most economical treatment that will result in the desired increase in productivity. This type of calculation involves consideration of the following variables:-

1. Fracturing fluid coefficient
2. Injection rate
3. Total injection volume
4. Area of the fracture
5. Weight of the propping agent
6. Surface injection pressure
7. Horsepower required
8. Productivity ratio of the well.

The values for some of those parameters must be known or assumed before it is possible to determine the others. This suggests two possible methods of solution. The first approach is to assume an injection rate, a size of treatment (total volume of fracture fluid) and a fracture gradient based on previous fracture jobs in the area. From these assumptions it is possible to obtain the surface pressure, horsepower requirements, maximum quantity of propping agent necessary and productivity ratio.

In the second method, all of these parameters can be determined from the desired productivity ratio and an assumed fracture gradient. In the first approach, due to the assumptions made, we will have a design which will be economically feasible; the second approach, however, will yield a design which is practical only if the desired productivity ratio is economically practical.

Therefore in this project, the first method has been chosen to be coded in Microsoft Excel Visual Basic for Application (VBA).

2.5 Mathematical Model

According to Wikipedia, a mathematical model is a description of a system using mathematical concepts and language. A mathematical model uses equations to represent a system. This model is used to guess how a system would work or how the system would react to certain variables. Dr. Gerda de Vries, the assistant professor in Department of Mathematical Sciences, University of Alberta designed the process of mathematical modeling as illustrated in the Figure 3 below:-

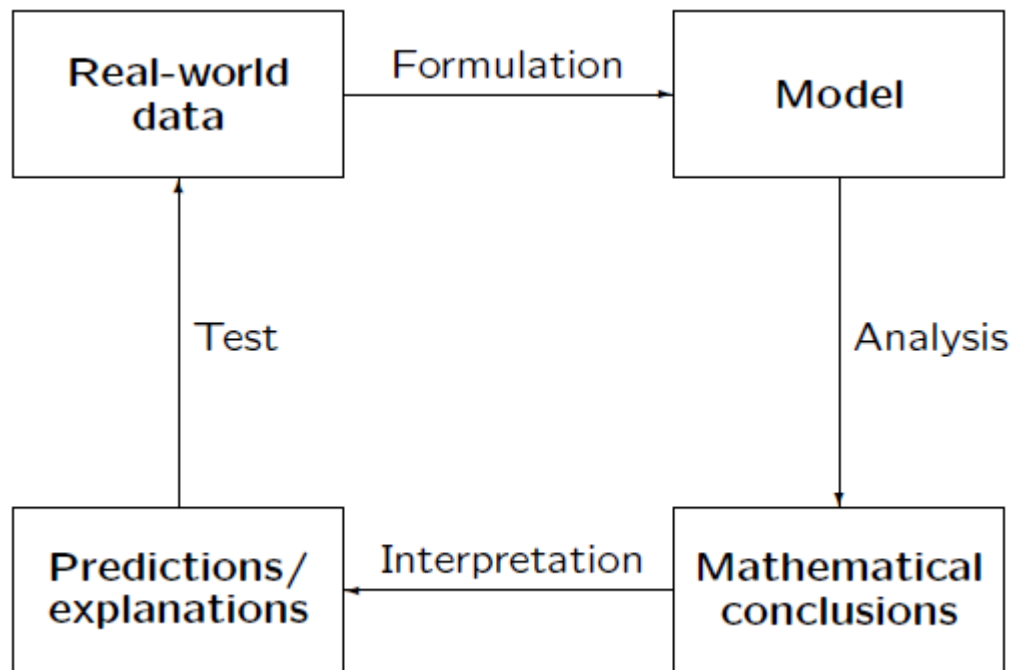


FIGURE 3: Process of Mathematical Modeling

CHAPTER 3

METHODOLOGY/ PROJECT WORK

3.1 Research Methodology

Since the project comprises of two objectives therefore two methodologies has been planned for the project:

1) Objective 1: To develop mathematical model for hydraulic fracturing design

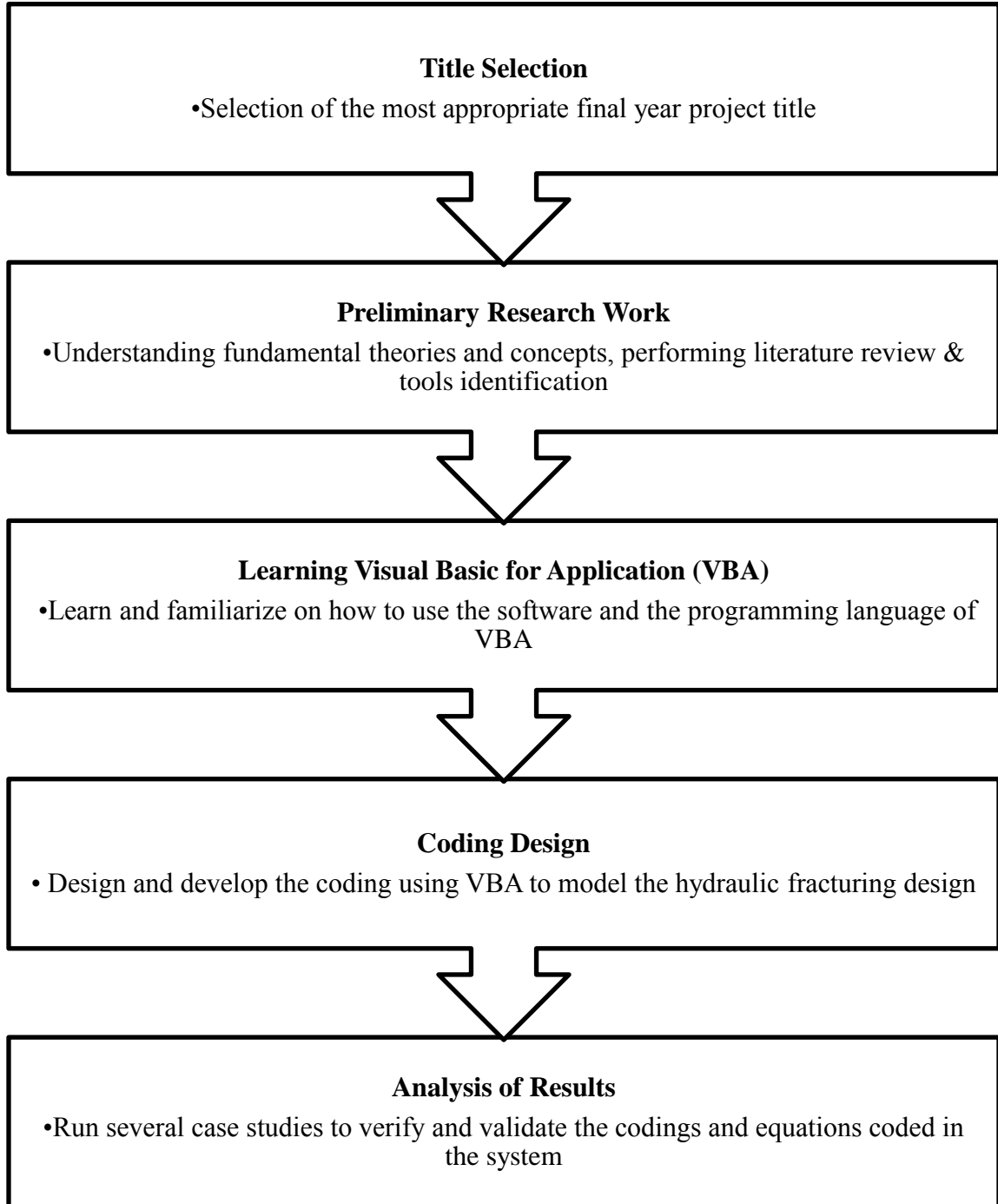
The extensive research and study of hydraulic fracturing has been done in order to have better understanding on the overall concept of hydraulic fracturing design treatment. Detailed research and data gathering has been done to determine the equations, correlations and assumptions on hydraulic fracturing design parameters. In order to verify the equations and coding, manual calculations of hydraulic fracturing need to be done before the equations have been coded in VBA. The coding and programming of hydraulic fracturing calculations will be developed using Microsoft Excel Visual Basic for Applications (VBA). The coding flow of hydraulic fracturing calculation is as illustrated in the flowchart diagram.

2) Objective 2: To run a case study to validate the mathematical model of hydraulic fracturing design.

By using data and results available from the case study which could be obtained from One Petro published papers, reference books and other related research papers and thesis, validation of the mathematical model will be made. The result of mathematical model will be then compared with the field data from published papers to see the variations of the results. Lastly, the results will be interpreted and analyzed.

3.2 Project Activities

The project activities for the whole project are as represented in figures below.



3.3 Key Milestone

Table below shows the key milestone for the whole project (Final Year Project I and Final Year Project II). The milestone of the project is divided into three sections which are early research development, middle research development and final research. Early research development and half of the middle research development was conducted during Final Year Project I while the other half of middle research development and final research was completed during Final Year Project II.

TABLE 1: : Key Milestones for FYP I and FYP II

MILESTONES	WEEK
Early Research Development <ul style="list-style-type: none">• Research background• Scope of studies and Assumptions	1 - 9
Middle Research Development <ul style="list-style-type: none">• Detailed research• Data gathering• Manual calculation• Developing the coding• Testing the coding	10 - 19
Final Research <ul style="list-style-type: none">• Verification and validation of mathematical model for hydraulic fracturing design• Finalizing the coding• Completing the documentation	20 - 26

3.4 Gantt Chart

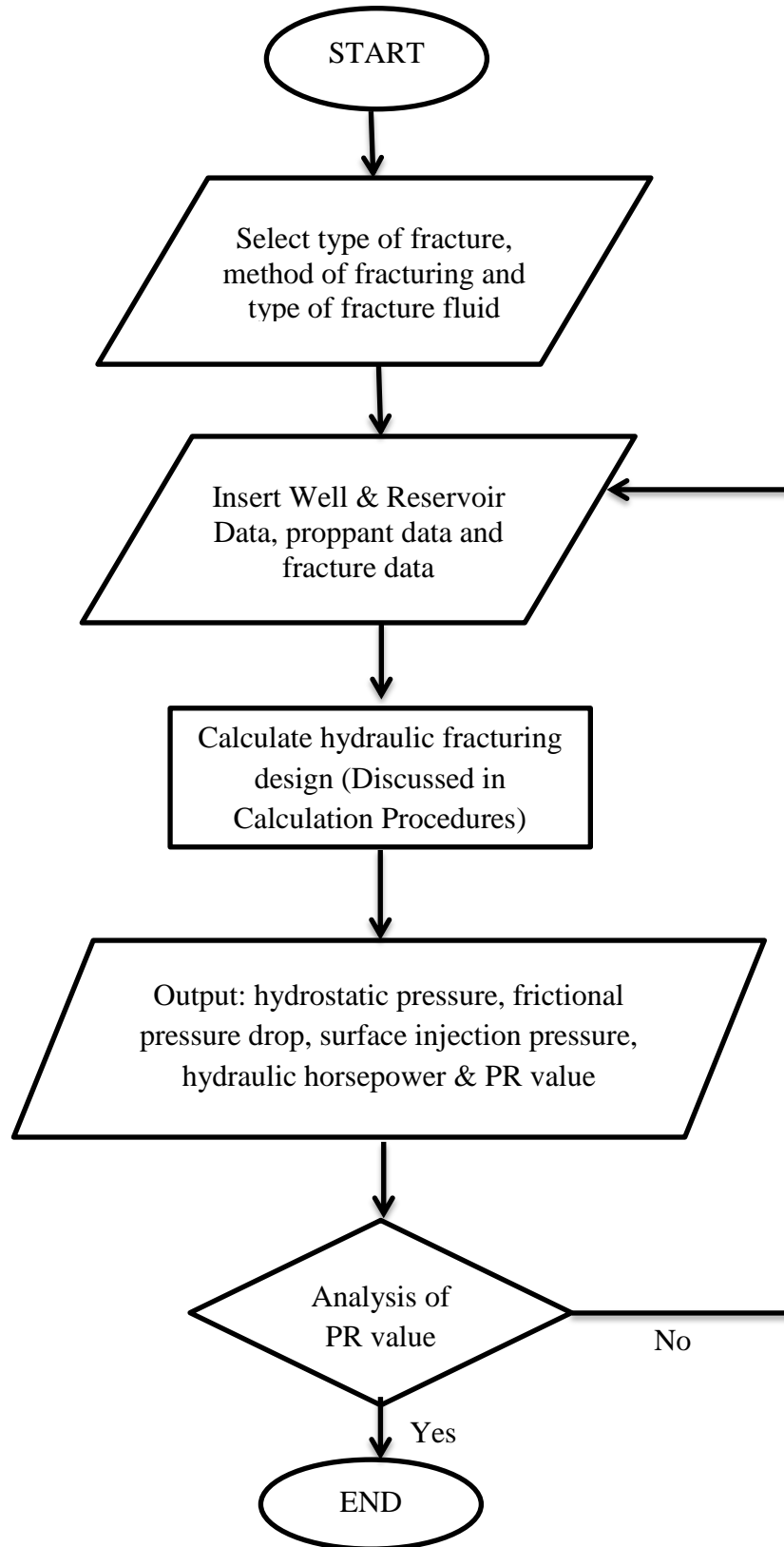
TABLE 2: Gantt Chart for FYP I

ACTIVITIES	WEEKS															
	1	2	3	4	5	6	7	Mid Semester Break	8	9	10	11	12	13	14	
FYP 1 Briefing																
Topic Selection																
Preliminary Research Work: Studies fundamental concept of project																
Extended Proposal Report Submission																
Proposal Defence (Oral Presentation)																
Project Work Continues: In depth studies on Hydraulic Fracturing Design parameters																
Familiarization with Microsoft Excel Visual Basic for Applications (VBA)																
Preparation for Interim Report																
Draft of Interim Report Submission																
Interim Report Submission																

TABLE 3: Gantt Chart FYP II

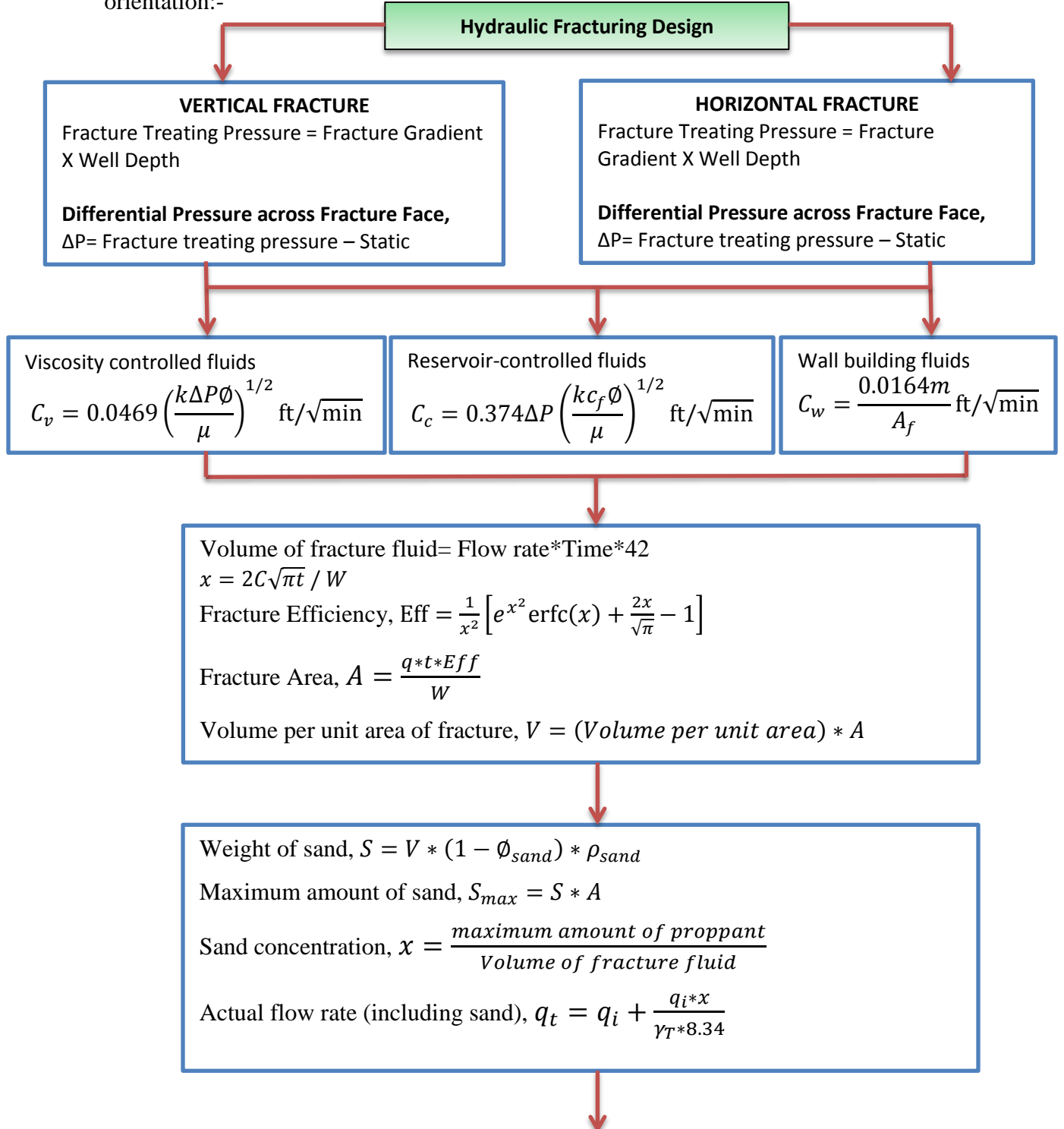
ACTIVITIES	WEEKS															
	1	2	3	4	5	6	7	Mid Semester Break	8	9	10	11	12	13	14	
Manual calculation of HF design																
Prepare the computer code using Microsoft Excel Visual Basic for Applications (VBA)																
FYP II briefing																
Preparation for progress report																
Progress report submission																
Run case studies and analysis of results																
Pre-SEDEX combined with seminar and poster																
Submission of draft report																
Submission of dissertation																
Submission of technical paper																
Final Oral Presentation																
Submission of hardbound copies																

3.5 Program Flow Chart



3.6 Calculation Procedures

The project focused on the calculation of hydraulic fracturing design parameters in vertical and horizontal fracture. Below are the calculation procedures for both fracture orientation:-



Specific gravity of oil at average well temperature, $\gamma_T = \gamma_{60}[1 - \beta(T - 60)]$

$$\text{Density, } \rho_T = \frac{8.34\gamma + x}{1 + 0.0456x}$$

$$\text{Hydrostatic Pressure, } \Delta P_s = 0.052\rho_T D$$

ANNULUS

Calculate diameter of circular pipe using
Crittendon correlation

$$d_e = \frac{\sqrt[4]{d_2^4 - d_1^4 - \frac{(d_2^2 - d_1^2)^2}{\ln(d_2/d_1)}} + \sqrt{d_2^2 - d_1^2}}{2}$$

$$\text{Average flow velocity in pipe, } v = \frac{17.16 * q_t}{d_e^2}$$

$$\text{Reynolds number, } N_{Re} = 928dv\rho/\mu$$

Fanning friction factor:-

$$f = \frac{64}{N_{Re}} \text{ for } N_{Re} < 2000$$

$$\frac{1}{\sqrt{f}} = -2 \log \left[\left(\frac{\varepsilon}{3.715d_e} \right) + \left(\frac{6.943}{N_{Re}} \right)^{0.9} \right]$$

for $N_{Re} > 2000$

$$\text{Frictional pressure drop, } \Delta P_f = \frac{(fL\rho v^2)}{25.80d_e}$$

$$\text{Line Efficiency Correlation, } \text{LEC} = \Delta P_f / (0.90)^2$$

CASING

$$\text{Average flow velocity in casing, } v = \frac{17.16 * q_t}{ID^2}$$

$$\text{Reynolds number, } N_{Re} = 928 * ID * v * \rho / \mu$$

Fanning friction factor:-

$$f = \frac{64}{N_{Re}} \text{ for } N_{Re} < 2000$$

$$\frac{1}{\sqrt{f}} = -2 \log \left[\left(\frac{\varepsilon}{3.715d_e} \right) + \left(\frac{6.943}{N_{Re}} \right)^{0.9} \right]$$

for $N_{Re} > 2000$

$$\text{Frictional pressure drop, } \Delta P_f = \frac{(fL\rho v^2)}{25.80 ID}$$

$$\text{Surface injection pressure, } P_s = P_T + \Delta P_f - \Delta P_s$$

$$\text{Hydraulic Horsepower, } H_h = 0.0245 P_s q_t \text{ hp}$$

Productivity Ratio (Vertical Well)

$$PR = \left[\frac{k_f W}{k} \right] \left[\frac{\left(\frac{k}{k_f} + 1 \right) \ln \left(\frac{r_e}{r_w} \right)}{\left(\frac{k_f W}{k} + 1 \right) \ln \left(\frac{r_e}{r_f} \right) + \ln \left(\frac{r_f}{r_w} \right)} \right]$$

Productivity Ratio (Horizontal Well)

$$PR = \left[\frac{k_f W}{kh} \right] \left[\frac{\left(\frac{kh}{k_f} + 1 \right) \ln \left(\frac{r_e}{r_w} \right)}{\left(\frac{k_f W}{kh} + 1 \right) \ln \left(\frac{r_e}{r_f} \right) + \ln \left(\frac{r_f}{r_w} \right)} \right]$$

3.7 Tools/ Software

Visual Basic for Application (VBA)

Visual Basic for Applications or VBA is a computer programming language which is used to control Microsoft Excel's functionality. VBA allow users to automate report generation in Microsoft Excel, automate chart creation, perform data processing and automate communication between Microsoft Excel and databases. VBA also allows user to perform analysis tasks in Microsoft Excel such as modeling, optimization and etc. In order to command Microsoft Excel effectively using Visual Basic for Applications (VBA), Microsoft Excel's operational capabilities must be well understood along with its program elements.

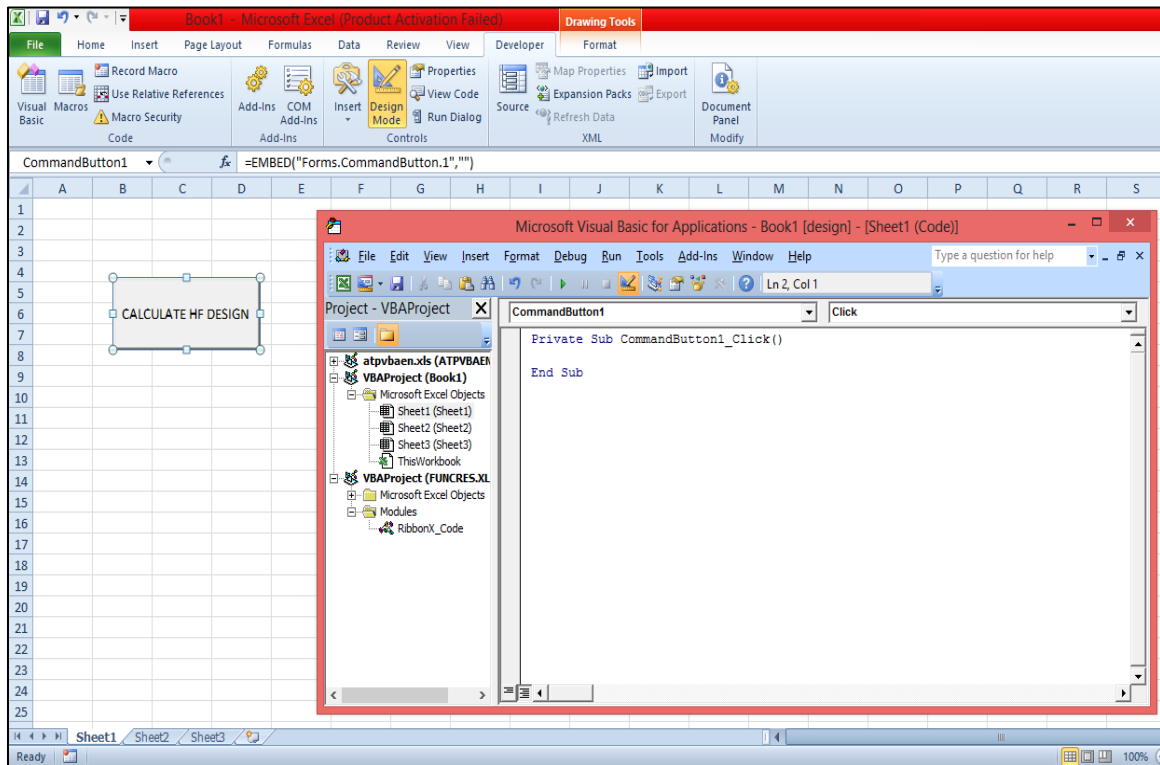


FIGURE 4: Interface of VBA in Microsoft Excel

Microsoft Excel

Microsoft Excel is a spreadsheet tool capable of performing calculations, analyzing data and integrating information from different programs. Microsoft Excel is comprised of organizational units called workbooks. A standard workbook contains worksheets and chart sheets. Worksheets perform calculations, store and organize data, present graphics and controls. A worksheet in turn is comprised of millions of cells. The job of a cell is to store a formula that performs a calculation or communicates with some other application (i.e. program) such as a database. They also store and present data. A chart sheet's job is to present a chart or graph developed from data stored on a worksheet.

CHAPTER 4

RESULTS AND DISCUSSIONS

This chapter will discuss on the results for both objectives of the project which are firstly, to develop a mathematical model for hydraulic fracturing design and secondly, to run a case study to validate the coding of hydraulic fracturing design in VBA.

4.1 Data Gathering and Analysis

Presented in this section, are comprehensive analyses of hydraulic fracturing design data for five field cases which obtained from several published papers. The initial data collection process involves the well and reservoir data, proppant data, fracture data as well as fracturing fluid data, from which we were able to retrieve the necessary information to be implemented in mathematical model of hydraulic fracturing design. Table 4 below recaps these required parameters for the mathematical model of hydraulic fracturing design:-

TABLE 4: Required parameters for Hydraulic Fracturing Design

Well and Reservoir Data	Proppant Data	Fracture Data
Well depth Well Spacing Casing size ID Tubing size ID Tubing size OD Pipe roughness Average permeability Formation thickness Static bottomhole pressure Average Well Temperature Average porosity Drainage radius Wellbore radius	Proppant name Proppant size Specific gravity of sand Sand porosity Sand permeability	Fluid gravity Isothermal compressibility Fluid viscosity Slope of fluid-loss curve Fluid loss pressure Fluid loss temperature Area of filter paper
	Fracture Data	
	Fracture gradient Fracture width Assumed injection rate Assumed injection time	

Table 4 above shows the hypothetical reservoir and well data taken from SPE Monograph and research papers. These data were used for the calculations of hydraulic fracturing parameters developed in Microsoft Excel VBA. Author had chosen to present five field cases where reference data measurements were available. Case 1 was tested in mathematical model for model verification. Five field cases were tested for the validation of mathematical model for hydraulic fracturing design.

Case 1 - The data for first well discussed was obtained from Field Example (Well Design: Drilling and Production by Craft B.C et al, 1962). The well depth is at 7,000 ft and was treated via 4.892 in casing and 2 in tubing. The well was treated with a sand of 20-40 mesh sizes as the type of proppant used. Since the well depth is at 7,000 ft, the fracture extent of the well is assumed as vertical with the fracture gradient of 0.7 ft/psi. The type of fracture fluid used was wall-building fluids with the fluid gravity of 0.876 and the crude viscosity of 7cp. Other related input data for the mathematical model was listed in Table 5. The resulted field data shows that the fracture was treated using 151,000 lb of maximum weight of sand, with 37800 gal of fracture fluid volume. The surface injection pressure used was 3129 psi whereby the hydraulic pump was operated at 2338 hp. The productivity ratio obtained after fracture treatment was 6.5.

Case 2 - The data for Case 2 well was obtained from Field Example in Well Design: Drilling and Production by Craft B.C et al (1962). The well was drilled at depth 10,000 ft and completed with 5.892 in casing and 2 in tubing. The fracture gradient was 0.64 ft/psi and assumed to be vertical well. Wall building fluid was used as the type of fracturing fluid with 0.876 fluid gravity and 7 cp fluid viscosity respectively. Other necessary input data for the mathematical model was listed in Table 5. The resulted field data shows that the fracture was treated using 420,000 lb of maximum weight of sand, with 100,000 gal of fracture fluid volume. The surface injection pressure used was 2500 psi whereby the hydraulic pump operated at 2300 hp. The productivity ratio obtained after fracture treatment was 9.

Case 3 - The data for Case 3 was extracted from Craft B.C et al (1962) Field Example. The well was assumed horizontal as the depth of the well is 2,000 ft with a fracture gradient of 1 psi/ft. The well was treated via 4.892 in casing and 2 inch tubing ID and 2.375 inch tubing OD. Since the fracturing fluid has low viscosity of 4cp, the fracture fluid type is characterized in reservoir controlled fluid. The fracture was done through annulus with the injection rate of 30 bbl/min in 31.7 minutes. Other related input data for the mathematical model was listed in Table 5. The resulted field data indicated that the maximum amount of sand needed for fracturing was 176,600 lb with the fracture fluid volume of 40,000 gal. The injection pressure was at 3263 psi and the hydraulic horsepower used was 2886 hp. The PR of the well after fracturing was 5.

Case 4 - The fourth well was extracted from Hydraulic Fracture Treatment Design Simulation for the Rotliegendes Formation (A. Acharya & C.M Kim, 1987). The type of well is vertical as the well was drilled up to 6500 ft with the fracture gradient of 0.68 psi/ft. The HF treatment used 20-40 mesh sizes sand as the type of proppant. The fracture was done through casing and used reservoir controlled fluids as the type of fracture fluid with the crude viscosity of 4 cp. Other related input data for the mathematical model was listed in Table 5. The resulted data showed the value of 6.4 for productivity ratio after fracture treatment. 226700 gal of fracture fluid was treated at 3550 psi of injection pressure and 2670 hp of hydraulic horsepower. 400,000 gal maximum amount of sand was needed for the successful of the treatment.

Case 5 - For case 5, the data was taken from the paper of A Procedure for Optimal Hydraulic Fracturing Treatment Design by D. K. Poulsen and M. Y. Soliman (1986). The well is at depth of 1600 ft and had a fracture gradient of 1 psi/ft. Sand with mesh size of 20-40 was used for HF treatment. The well was fractured at 20 bbl/min for about 98 minutes. Viscosity controlled fluid was used as the type of fracture fluid with the fluid gravity of 0.84 and viscosity of crude at 460 cp. The resulted HF for the field used 462,000 lb of amount of sand and 84,000 gal of volume of fracture fluid. The well was injected at 2500 psi and PR after fracturing was at the value of 7.4.

For all the cases, user can insert the input data manually for specific reservoir and well. Input data for all cases are attached in the Appendices. The input data will be then linked into Microsoft Excel Visual Basic for Applications (VBA) windows code to calculate the desired hydraulic fracturing parameters. Table 5 summarized and listed the necessary input data of mathematical model for hydraulic fracturing design:-

TABLE 5: Field Data for Case Studies

Cases	1	2	3	4	5
Well & Reservoir Data					
Well Depth	7000	10000	2000	6500	1600
Well Spacing	40	60	40	60	20
Casing Size ID	4.892	5.892	4.892	4.892	4.494
Tubing Size ID	2	3	2	2	
Tubing Size OD	2.375	2.375	2.375	2.375	
Pipe Roughness	0.0006	0.0006	0.0006	0.0006	0.0006
Average Permeability	1	1	0.9	0.87	0.5
Formation Thickness	50	80	50	250	150
Static Bottom Hole Pressure	2500	1900	300	700	343
Average Well Temperature	125	100	80	150	96
Average Porosity	0.135	0.14	0.135	0.15	0.1
Drainage radius	660	750	660	660	500
Wellbore radius	0.408	0.504	0.408	0.475	0.25
Proppant Data					
Specific Gravity of Sand	2.63	2.65	2.63	2.61	2.65
Sand Porosity	0.35	0.36	0.35	0.37	0.4
Sand permeability	13000	30000	60000	60000	54000
Fracture Data					
Fracture gradient	0.7	0.64	1	0.68	1
Fracture width	0.1	0.15	0.1	0.23	0.15
Assumed flow rate	30	25	30	28	20
Assumed injection time	30	95	31.7	190	98
Fracturing Fluid Data					
Fluid gravity	0.876	0.856	0.876	0.876	0.84
Slope of fluid-loss curve	1.2	0.73			
Fluid loss Pressure	1000	1200			
Fluid loss Temperature	125	136			
Viscosity of Crude	7	8	4	4	460

Area of filter paper	22.8	24.7			
Isothermal Compressibility			$10 \times (10^6)$	$10 \times (10^6)$	

4.2 Assumption used in the model

The fracture plane is assumed to be vertical when the fracture gradient is 0.7 psi/ft or less, and horizontal when the fracture gradient is 1.0 psi/ft or greater. For the productivity ratio, it is impossible to predict exactly the productivity ratio of the well, owing to the fact that every fracture pattern is different and unique. However, it is possible to estimate the productivity ratios for vertical and horizontal fractures if the radial pattern of fracture is assumed as uniform. For the case of horizontal fracture, productivity ratio equation can be obtained provided it is assumed that there is zero vertical permeability in the fracture zone.

An expression of fracture area at any time may be derives by using the assumption of; the fracture has uniform width, the flow of fracture fluid into formation is linear and the direction of flow is perpendicular to the fracture face, the velocity of flow into the formation at any point on the fracture face is a function of the time of exposure of the point to flow, the velocity function $v=f(t)$ is the same for every point in the formation, but the zero time for any point is defined as the instant that fracturing fluid first reaches it and the pressure in the fracture is equal to the sandface injection pressure, which is constant.

In this model, injection rate and time of injection will be assumed based on the previous fracture jobs in the area. From these assumptions it is possible to obtain the surface pressure, horsepower requirements, maximum quantity of propping agent needed and also the productivity ratio.

4.3 Mathematical Model for Hydraulic Fracturing Design

This section will focused on the mathematical model for hydraulic fracturing design developed in Microsoft Excel Visual Basic for Applications (VBA). Figure 5 below shows the code of window with the assigned values. In this window, the values of input data will be assigned and linked with Microsoft Excel.

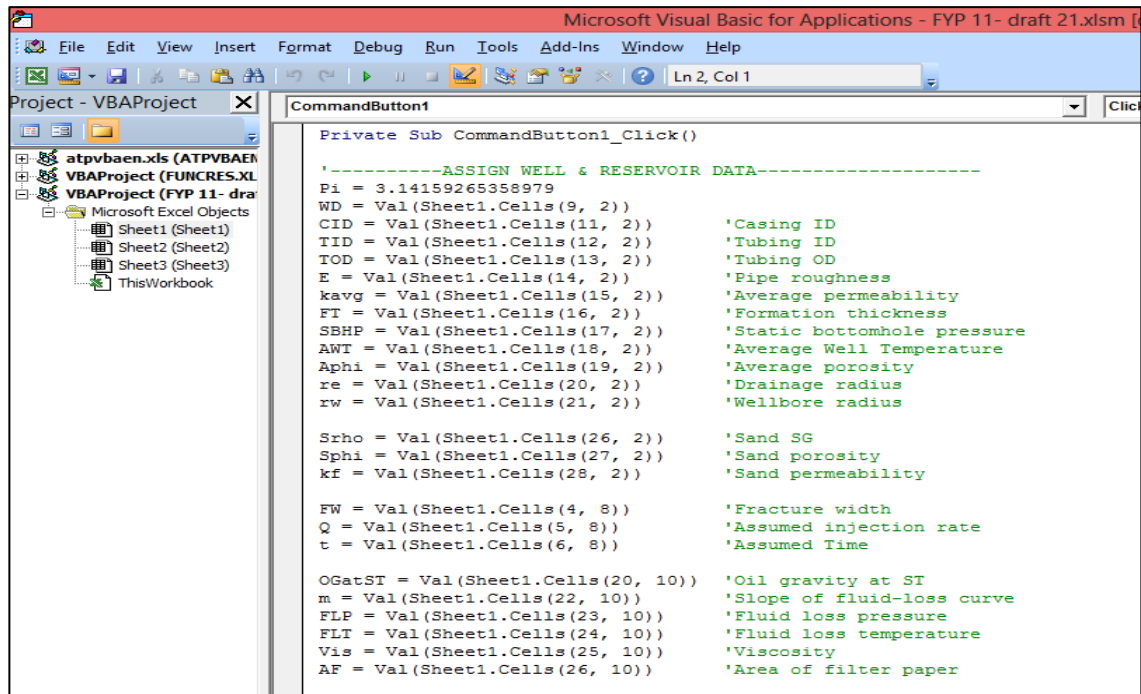


FIGURE 5: Coding Window in VBA (Assigning data)

For this project, the coding of hydraulic fracturing design was done for two different types of fracture extent namely vertical fracture and horizontal fracture. The type of fracture establishes the directional permeability of the formation to be used in calculating the fluid loss during fracturing as well as productivity ratio of the fractured wells. In addition, the type of fracture determines the advisability of using diverting agents.

In the coding of vertical and horizontal fractures, three different types of fracture fluids have been coded in the system which are viscosity controlled fluids, reservoir controlled

fluids and wall-building fluids. Viscosity controlled fluids comprises of viscous or semiviscous fracture fluids, reservoir controlled fluids consists of fracturing fluids with low viscosity and high fluid loss characteristic while wall building fluids build a temporary filter cake. The user can choose either they want to use viscosity controlled fluids, reservoir controlled fluids or wall-building fluids as the types of fracture fluids.

After the amount of sand, hydrostatic pressure drop and volume of fracturing fluid has been calculated, the procedure on the method of fracturing was coded in VBA. The fracture will be done either through casing or annulus. In fracture through annulus, the diameter of circular pipe will be calculated using the Crittendon correlation which has been coded in the window. The diameter then will be used to calculate the average velocity, Reynolds number, frictional pressure drop as well as hydraulic horsepower. For fracture through casing, the internal diameter of the casing will be used for the calculations of average velocity, Reynolds number, frictional pressure drop as well as hydraulic horsepower. Lastly, the equation of productivity ratio has been coded in VBA. The coding of hydraulic fracturing design is as attached in the Appendix V. Figure 6 shows the interface of mathematical model in Excel spreadsheet whereby user can insert the data input manually.

Fracture	
Fracture gradient	0.7 psi/ft
Fracture width	0.1 in
Assumed flow rate	30 bbl/min
Assumed injection time	30 min

Well & Reservoir Data	
Well Depth	7000 ft
Well Spacing	40 acres
Casing Size ID	4.892 in
Tubing Size ID	2 in
Tubing Size OD	2.375 in
Pipe Roughness	0.0008 in
Average Permeability	2 md
Formation Thickness	50 ft
Static Bottom Hole Pressure	2500 psi
Average Well Temperature	125 F
Average Porosity	0.135 %
Drainage radius	660 ft
Wellbore radius	0.408 ft

Proppant	
Proppant name	Sand
Proppant Size	mesh
Specific Gravity of Sand	2.63
Sand Porosity	0.35 %
Sand permeability	13000 md

Types of Fluid (Insert 1/2/3)	
Viscosity controlled fluid (1)/ reservoir controlled fluid(2)/wall building fluid(3)	
Viscosity Controlled Fluid (insert Data for 1)	
Fluid gravity	
Viscosity	cp
Reservoir controlled fluid (insert Data for 2)	
Isothermal Compressibility	psi-1
Fluid viscosity	cp
Wall Building Fluid (insert Data for 3)	
Fluid gravity	0.876
Slope of fluid-loss curve	30 API
Fluid loss Pressure	1.2
Fluid loss Temperature	1000 psi
Viscosity of Crude	125 F
Area of filter paper	7 cp
	22.8

FIGURE 6: Input Data in Excel

The user manual has been prepared in order to guide users to calculate the parameters in hydraulic fracturing design.

User Manual :-

- 1) Insert no 1/2/3 for types of fracture
- 2) Insert no 1/2/3 for fracture procedure
- 3) Insert well and reservoir data
- 4) Insert Proppant data
- 5) Insert fracture data
- 6) Insert no 1/2/3 for types of fluid
- 7) Insert the fluid data
- 8) Click Calculate HF Design :)

FIGURE 7: User manual in the mathematical model

After the command button has been clicked, Microsoft Excel VBA will automate the calculation and produce the output in Microsoft Excel spreadsheet. The results of hydraulic fracturing important parameters (Case 1) are as listed in Figure 8 below.

P	Q	R	S	T	U	V
RESULTS:-						
FTP	Fracture Treating Pressure					4900
delta P	fferential Pressure across fracture fa					2400
V	Volume of fracture fluid					37800
Eff	Fracture Efficiency					0.27697
A	Fracture Area					167957
S	Weight of Sand					0.88894
Smax	Maximum amount of sand needed					149303
Scon	Sand Concentration					3.94983
qt	Actual flow rate including sand					35.4023
Sgawt	SG of oil at average well temperature					0.84753
RhoAWT	Density of oil at average well temper					9.33659
deltaPs	Hydrostatic Pressure					3398.52
Vel	Average Velocity					25.3849
Nre	Reynolds Number					153709
f	Fanning Friction Factor					0.00431
delta Pf	Frcitional Pressure Drop					1438.75
Ps	Surface Injection Pressure					2940.23
Hh	Hydraulic Horsepower					2550.23
PR	Productivity Ratio					6.67539
FINAL RESULTS:-						
	Maximum amount of sand needed					149303
	Volume of fracture fluid					37800
	Surface Injection Pressure					2940.23
	Hydraulic Horsepower					2550.23
	Productivity Ratio					6.67539

FIGURE 8: Results for Case 1

4.4 Mathematical Model Verification and Validation

Model verification and validation are essential parts of the model development process if models to be accepted and used to support decision making (Charles, 2005). Model verification is the process of ensuring that the conceptual description and the solution of the model are implemented correctly. Verification is done to ensure that the model is programmed correctly, the algorithms have been implemented properly and the model does not contain error, oversights or bugs. Verification ensures that the specification is complete and that mistakes have not been made in implementing the model. However verification does not ensure the model solves the problem and correctly reflects the workings of a real world process. Therefore model validation need to be done. The ultimate goal of model validation is to make the model useful in the sense that the model addresses the right problem and provides accurate information about the system being modeled.

4.4.1 Hydraulic Fracturing Mathematical Model Verification

The hydraulic fracturing mathematical model need to be verified to ensure that the codes are free from programming errors and are correctly solving the mathematical equations. A case study of hydraulic fracturing design has been calculated manually before the equations have been coded in VBA. Below are the example of manual calculations for Case 1 (Vertical Fracture, Wall-building fluids and fracturing through casing).

1) Calculation of Fracture Treating Pressure, P_t

$$P_t = \text{Fracture gradient} \times \text{Well Depth}$$
$$P_t = 0.7 \text{ psi/ft} \times 7000 \text{ ft} = 4900 \text{ psi}$$

2) Calculate differential pressure across fracture face

$$\Delta P = \text{Fracture treating pressure} - \text{Static bottomhole pressure}$$
$$\Delta P = 4900 \text{ psi} - 2500 \text{ psi} = 2400 \text{ psi}$$

3) Calculate fracturing fluid coefficient

$$C_w = \frac{0.0164m}{A_f} \text{ ft}/\sqrt{\text{min}}$$
$$C_w = \frac{0.0164 (1.2)}{22.8} \text{ ft}/\sqrt{\text{min}}$$
$$= 0.863 \times 10^{-3} \text{ ft}/\sqrt{\text{min}}$$

4) Calculate actual fracturing fluid coefficient

$$C_{w \text{ act}} = C_w \left(\frac{\Delta P}{P \text{ of fluid loss}} \right)$$
$$C_{w \text{ act}} = (0.863 \times 10^{-3}) \left(\frac{2400}{1000} \right)$$
$$= 1.34 \times 10^{-3} \text{ ft}/\sqrt{\text{min}}$$

5) Calculate volume of fracture fluid

Volume of fracture fluid= Flow rate*Time*42
Volume of fracture fluid= 30 X 30 X 42 = 37,800 gal

6) Calculate the value of x

$$x = 2C\sqrt{\pi t} / W$$
$$x = 2 \left(1.34 \times 10^{-3} \right) \sqrt{\pi(30)} \left(\frac{0.1}{12} \right)$$
$$= 3.12$$

7) Find fracture efficiency from Figure in Appendix I

Eff = 28%

8) Calculate fracture area

$$A = \frac{q*t*Eff}{W}$$
$$A = \frac{(37800)(0.27)}{7.48 \left(\frac{0.1}{12} \right)}$$
$$= 169,797 \text{ ft}^2$$

9) Calculate volume per unit area of fracture

$$V = (\text{Volume per unit area}) * A$$
$$V = (1.0) * \left(\frac{0.1}{12} \right) = 0.00833 \text{ ft}^3/\text{ft}^2$$

10) Calculate weight of sand

$$\begin{aligned} S &= V * (1 - \phi_{sand}) * \rho_{sand} \\ S &= 0.00833(1 - 0.35)(2.63 \times 62.4) \\ &= 0.888 \text{ lb/scf} \end{aligned}$$

11) Calculate maximum amount of sand needed

$$\begin{aligned} S_{max} &= S * A \\ S_{max} &= 0.888(169,797) \\ &= 150,779 \text{ lb} \end{aligned}$$

12) Calculate sand concentration

$$\begin{aligned} x &= \frac{\text{maximum amount of proppant}}{\text{Volume of fracture fluid}} \\ x &= \frac{150779}{37800} \\ &= 3.98 \text{ lb/gal} \end{aligned}$$

13) Calculate actual flow rate (including sand)

$$\begin{aligned} q_t &= q_i + \frac{q_i * x}{\gamma_T * 8.34} \\ q_t &= 30 + \frac{30(3.98)}{(2.63)8.34} \\ &= 35.4 \text{ bbl/min} \end{aligned}$$

14) Calculate specific gravity of oil at average well temperature

$$\begin{aligned} \gamma_T &= \gamma_{60}[1 - \beta(T - 60)] \\ \gamma_{125} &= 0.876[1 - 0.0005(125 - 60)] \\ &= 0.847 \end{aligned}$$

15) Calculate density of oil at average well temperature

$$\begin{aligned} \rho_T &= \frac{8.34\gamma + x}{1 + 0.0456x} \\ \rho_{125} &= \frac{8.34(0.847) + 3.98}{1 + 0.0456(3.98)} \end{aligned}$$

$$= 9.34 \text{ lb/gal}$$

16) Calculate the hydrostatic pressure

$$\Delta P_s = 0.052 \rho_T D$$

$$\Delta P_s = 0.052(9.34)(7000)$$

$$= 3399 \text{ psi}$$

17) Calculate velocity in the casing

$$v = \frac{17.16 * q_t}{ID^2}$$

$$v = \frac{17.16(35.4)}{4.892^2}$$

$$= 25.38 \text{ ft/sec}$$

18) Calculate Reynolds number

$$N_{Re} = 928 * ID * v * \rho / \mu$$

$$N_{Re} = 928 (4.892)(25.3)(7.62)/7 = 153,735 > 2000$$

19) Calculate Fanning Friction Factor

$$F_m = 4 F_f$$

$$\frac{1}{\sqrt{f}} = -2 \log \left[\left(\frac{\varepsilon}{3.715 d_e} \right) + \left(\frac{6.943}{N_{Re}} \right)^{0.9} \right] \text{ for } N_{Re} > 2000$$

$$\frac{1}{\sqrt{f}} = -2 \log \left[\left(\frac{0.0006}{3.715(4.892)} \right) + \left(\frac{6.943}{153735} \right)^{0.9} \right]$$

$$f = 0.00431$$

20) Calculate frictional pressure drop

$$\Delta P_f = \frac{(f L \rho v^2)}{25.80 ID}$$

$$\Delta P_f = \frac{((0.00431)(7000)(9.34)(25.38)^2)}{25.80 (4.892)}$$

$$= 1438 \text{ psi}$$

21) Calculate surface injection pressure

$$P_s = P_T + \Delta P_f - \Delta P_s$$
$$= 4900 + 1438 - 3399 = 2939 \text{ psi}$$

22) Calculate hydraulic horsepower

$$H_h = 0.0245 P_s q_t \text{ hp}$$
$$H_h = 0.0245(2939)(35.4) = 2549 \text{ hp}$$

23) Calculate PR

$$C = k_f W / k = 108$$
$$r_f / r_e = 232 / 660 = 0.35$$

From figure 2, PR = 6.5

The verification of coding has been tested to determine whether the coding will successfully run without any errors. The results of manual calculations and the mathematical model of hydraulic fracturing will then compared and the percentage differences have been calculated. Table 6 below shows the results for both methods in Case 1:-

TABLE 6: Results of mathematical modeling and manual calculation for model verification

RESULTS:-	VBA Coding	Manual Calculation	% diff
Fracture Treating Pressure	4900	4900	0
Differential Pressure across fracture face	2400	2400	0
Volume of fracture fluid	37800	37800	0
Fracture Efficiency	0.276966	0.276966	0
Fracture Area	167956.7	169797	-1.09571
Weight of Sand	0.88894	0.888	0.105744
Maximum amount of sand needed	149303.4	150779	-0.98832
Sand Concentration	3.949826	3.98	-0.76394
Actual flow rate including sand	35.40228	35.4	0.00645
SG of oil at average well temperature	0.84753	0.847	0.062535
Density of oil at average well temperature	9.336593	9.34	-0.03649
Hydrostatic Pressure	3398.52	3399	-0.01413
Average Velocity	25.38491	25.38	0.01935

Reynolds Number	153709.4	153735	-0.01664
Fanning Friction Factor	0.004312	0.004312	0
Frictional Pressure Drop	1438.752	1438	0.052264
Surface Injection Pressure	2940.232	2939	0.041906
Hydraulic Horsepower	2550.228	2549	0.048145
Productivity Ratio	6.675392	6.5	2.627437

Table 6 above shows the results of VBA coding of hydraulic fracturing design and also manual calculation of hydraulic fracturing design for Case 1. From the results, the minimum percentage differences obtain between VBA coding and manual calculations is 0% while the maximum percentage differences obtain between calculation in VBA coding and manual calculation is 2.63%. Therefore, the results conclude that the coding is free from error and viable for data validity. The coding will be further tested and validate for all five cases.

4.4.2 Hydraulic Fracturing Mathematical Model Validation

Hydraulic fracturing design data obtained from five wells have been examined. Of particular interest are the hydraulic fracturing design predicted by several correlations and validity of these HF data to allow meaningful well design calculations such as maximum amount of sand needed, volume of fracture fluid, surface injection pressure, hydraulic horsepower and productivity ratio. All the test data were obtained from different field in published papers. Model validation will present the calculated HF design versus the field results and show the percentage differences between both results. The percentage differences can be obtained by applying the equation below:-

$$\% \text{ diff} = \frac{\text{Field Value} - \text{Calculated Value}}{\text{Field Value}}$$

Tables 7 below shows the results of mathematical model of hydraulic fracturing and the results from field data for all cases:-

TABLE 7: Results comparison between mathematical model calculation and field data for Case 1

FINAL RESULTS:-	Mathematical Model	Published Paper	% difference
	VBA Coding	Field Data & Results	
Maximum amount of sand needed	149303.4	151000	1.123576
Volume of fracture fluid	37800	37800	0
Surface Injection Pressure	2940.232	3129	6.032853
Hydraulic Horsepower	2550.228	2338	9.077331
Productivity Ratio	6.675392	6.5	2.698338

Case study 1 examined the cases of vertical well as the fracture extent, fracturing through casing and used wall-building fluids as the type of fracture fluid. As we can see from the Table 7 above, the hydraulic fracturing design resulted from mathematical model shows minimum percentage difference compared the field data obtained from

published paper. The percentage differences for all parameters of hydraulic fracturing design are less than 10%.

TABLE 8: Results comparison between mathematical model calculation and field data for Case 2

FINAL RESULTS:-	Mathematical Model	Published Paper	% difference
	VBA Coding	Field Data & Results	
Maximum amount of sand needed	421321.7754	420000	0.31470842
Volume of fracture fluid	99750	100000	0.25
Surface Injection Pressure	2932.556862	2500	17.3022744
Hydraulic Horsepower	2139.465632	2300	6.97975514
Productivity Ratio	8.359197406	9	7.12002882

Table 8 shows the result between the calculated hydraulic fracturing design and field data for Case 2. Case 2 covered the vertical fracture extent, fracturing via casing and used wall building fluids as fracturing fluid. As shown in the table, the maximum amount of sand and volume of fracture fluid between the mathematical model and field data shows low amount of percentage differences which are 0.31% and 0.25% respectively. The hydraulic horsepower and productivity ratio also showed slightly small values of percentage differences which are both less than 10%. However, the surface injection pressure between calculated values and field values showed slightly high percentage error compared to other parameters which is 17.3%.

TABLE 9: Results comparison between mathematical model calculation and field data for Case 3

FINAL RESULTS:-	Mathematical Model	Published Paper	% difference
	VBA Coding	Field Data & Results	
Maximum amount of sand needed	177479.3055	176600	0.49790800
Volume of fracture fluid	39942	40000	0.145
Surface Injection Pressure	2786.673781	3263	14.5978001
Hydraulic Horsepower	2463.130212	2886	14.6524528
Productivity Ratio	4.948752758	5	1.02494483

Table 9 shows the result between the calculated hydraulic fracturing design and field data for Case 3 which examined the well with vertical fracture extent, fracturing via annulus and used reservoir controlled fluids as type of fracturing fluid. From the Table 9, the percentage difference between calculated values and field data for maximum amount of sand needed, volume of fracture fluid and productivity ratio shows small values which are 0.498%, 0.145% and 1.02% respectively. However the percentage differences for surface injection pressure and hydraulic horsepower displayed slightly high values which nearly reached 15%.

TABLE 10: Results comparison between mathematical model calculation and field data for Case 4

FINAL RESULTS:-	Mathematical Model	SPE Published Paper	% difference
	VBA Coding	Field Data & Results	
Maximum amount of sand needed	420541.412	400000	5.135352
Volume of fracture fluid	223440	226700	1.438023
Surface Injection Pressure	3074.896692	3550	13.38319
Hydraulic Horsepower	2291.766993	2760	14.16603
Productivity Ratio	7.757409323	7.6	2.071175

Table 10 shows the comparison of result between mathematical model calculation and field case for Case 4. From the table, the percentage difference between calculated values and field data for maximum amount of sand needed, volume of fracture fluid and productivity ratio shows small values which are 5.14%, 1.44% and 2.07% respectively. However the percentage differences for surface injection pressure and hydraulic horsepower displayed slightly high values which are 13.38% and 14.17% respectively.

TABLE 11: Results comparison between mathematical model calculation and field data for Case 5

FINAL RESULTS:-	Mathematical Model	SPE Published Paper	% difference
	VBA Coding	Field Data & Results	
Maximum amount of sand needed	484920.7976	462000	4.961211

Volume of fracture fluid	82320	84000	2
Surface Injection Pressure	2859.120825	2500	14.36483
Hydraulic Horsepower	1774.375911	1945	8.772446
Productivity Ratio	7.07748497	7.4	4.358311

Table 11 shows the result between the calculated hydraulic fracturing design model and field data for Case 5. As shown in the table, all parameters showed slightly minimum values of percentage difference except for maximum amount of sand needed and hydraulic horsepower which are 14.36% and 8.77% respectively.

The resulted parameters from the mathematical model and field data were then converted into bar chart to analyze the difference in the values of hydraulic fracturing design parameters for all cases and analyzed the validity of the equations used to calculate the parameters of hydraulic fracturing design.

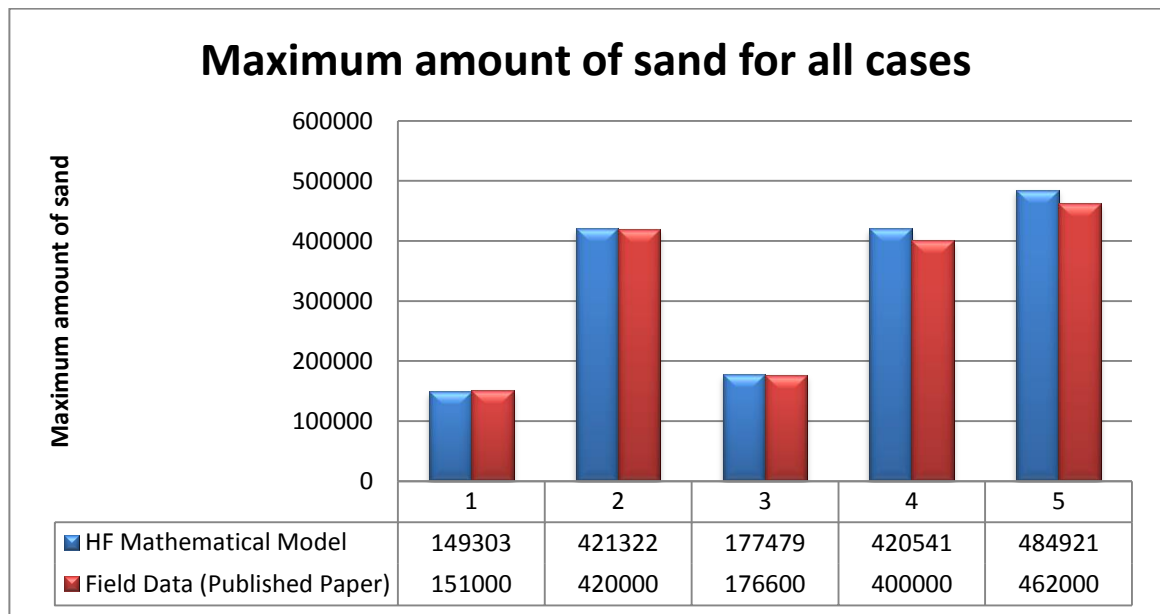


FIGURE 9: Graph of Maximum Amount of Sand between HF Mathematical Model and Field Data for All Cases

Maximum amount of sand for all cases were presented in Figure 9 above. From previous interpretations, the maximum amount of sand for all cases showed slightly differences

between the values of HF mathematical model and field data. For case 1, the percentage differences between the field and calculated values is 1.12% followed by Case 2 which is 0.31%, Case 3 which is 0.49%, Case 4 which is 5.14% and Case 5 which is 4.96%. The percentage differences for all values are less than 5%.

The comparison of volume of fracture fluid for all cases between the calculated model and field data are presented in Figure 10 below.

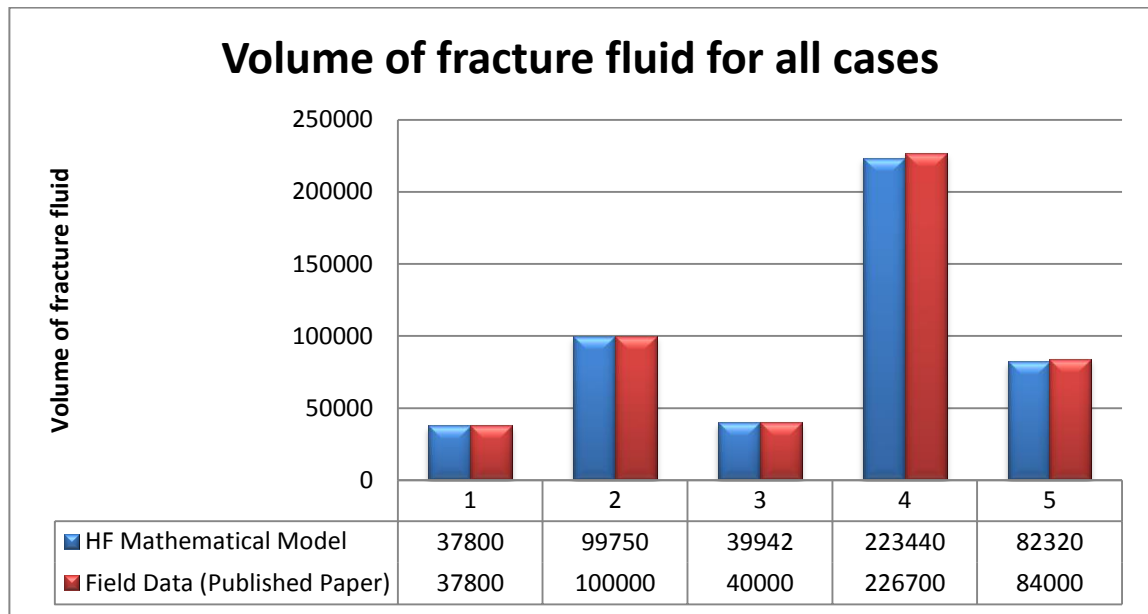


FIGURE 10: Graph of Volume of Fracture Fluid between HF Mathematical Model and Field Data for All Cases

The bar chart shows the volume of fracture fluid between HF mathematical model and field data for all cases. For case 1, there is no change between the field data and calculated volume of fracturing fluid. For the rest of the cases, there are slightly differences in the volume of fracture fluid. Case 2 obtained 0.25% differences, Case 3 obtained the differences of 0.145%, Case 4 for about 1.43% and last but not least, Case 5 obtained 2% of percentage differences in the volume of fracture fluid. To wrap up, the volume of fracture fluid between HF mathematical model and field data for all cases displayed low values of percentage differences which are less than 2%.

The comparison of surface injection pressure for all cases between the calculated model and field data are presented in Figure 11 below.

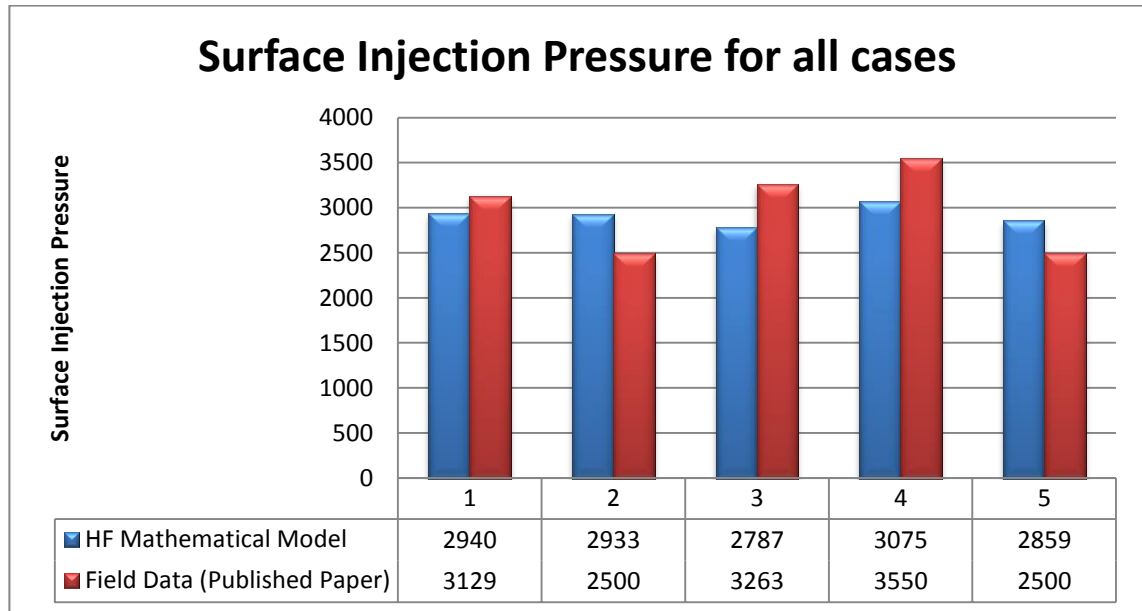


FIGURE 11: Graph of Volume of Fracture Fluid between HF Mathematical Model and Field Data for All Cases

The bar chart shows the surface injection pressure between hydraulic fracturing mathematical model and field data for all cases. For Case 1, the percentage difference between the field and calculated values is slightly high; 6.04%. However the rest of the cases show high percentage differences. The percentage difference Case 2 is 17.32%, followed by Case 3 which is 14.59%, Case 4 which is 13.38% and Case 5 which is 14.36%. Therefore, it can be concluded that the percentage differences are quite high in the values of surface injection pressure between the calculated model and field data.

The comparison of hydraulic horsepower for all cases between the calculated model and field data are presented in Figure 12 below.

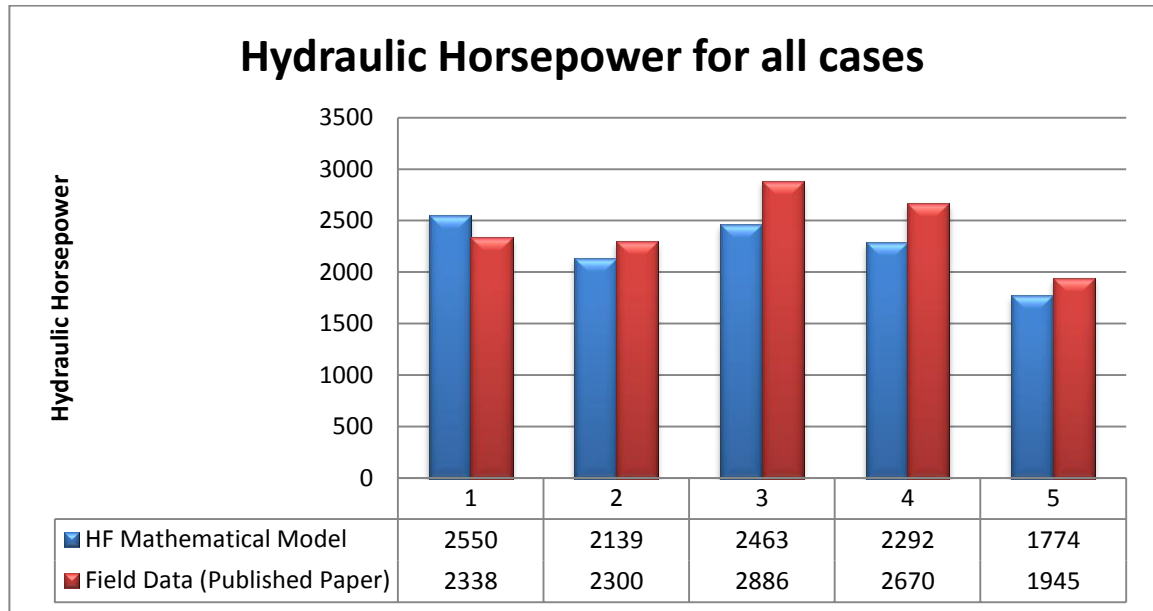


FIGURE 12: Graph of Hydraulic Horsepower between HF Mathematical Model and Field Data for All Cases

The bar chart shows the hydraulic horsepower results between hydraulic fracturing mathematical model and field data for all cases. For Case 1, Case 2 and Case 5, the percentage difference between the field and calculated values is slightly high which are 9.07%, 7% and 8.79% respectively. These values nearly reached 10% but still can be acceptable. However the rest of the cases which are Case 3 and Case 4 show high percentage differences. The percentage difference for Case 3 is 14.66%, and Case 4 is 14.16%, which nearly reached 15%. Therefore, it can be concluded that the percentage differences are quite high in the values of hydraulic horsepower between the calculated model and field data.

The comparison of productivity ratio for all cases between the calculated model and field data are presented in Figure 13 below.

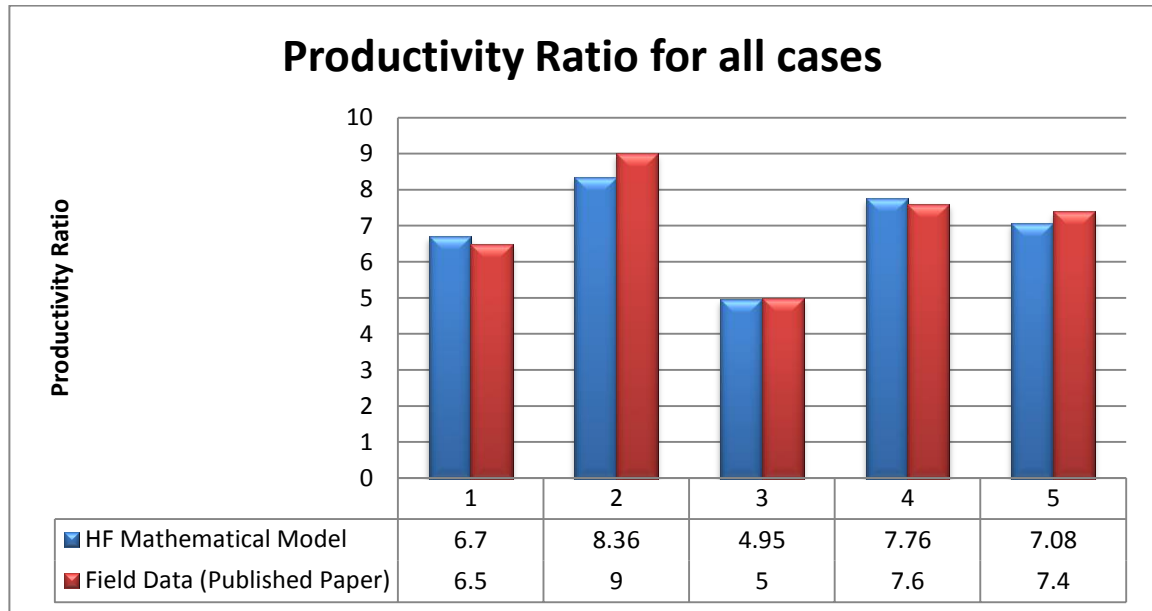


FIGURE 13: Graph of Productivity Ratio between HF Mathematical Model and Field Data for All Cases

The bar chart shows the productivity ratio results between hydraulic fracturing mathematical model and field data for all cases. For Case 1, the percentage difference is 3.07% followed by Case 2 which is 7.11%, Case 3 which is 1 %, Case 4 which is 2.11% and Case 5 which is 4.32%. All cases displayed low amount of percentage differences except for Case 2 in which the percentage difference reached more than 5%.

4.5 Discussions

Maximum amount of sand needed, volume of fracture fluid, surface injection pressure, hydraulic horsepower and productivity ratio after fracturing can be predicted prior to fracture treatment. From the previous result, the maximum amount of sand for all cases showed slightly differences (approximately less than 5%) between the values of HF mathematical model and field data. The volume of fluid for all cases also showed minimum values of percentage differences (approximately less than 2%) between the values of HF mathematical model and field data. The productivity ratio of the well after fracturing displayed the percentage differences range from 1% to 7%. This shows that the equations used to obtain the values of maximum amount of sand and volume of fracture fluid in HF design are valid, practical and workable.

However, the surface injection pressure showed slightly high values which approximately reaching 17% of percentage differences between calculated values and field values for all cases. This may possibly due to the assumption made in this model in which the pressure drop across perforation was assumed negligible because the values usually small compared to other pressure terms. Therefore the surface injection pressure can be obtained from equation $P_s = P_t + \Delta P_f - \Delta P_s$ without accounting the values of pressure drop across perforation. This might be a reason for slightly high percentage differences in the values of surface injection pressure for several case studies.

From the results too, since the values of percentage differences for surface injection pressure is slightly high, the percentage differences for hydraulic horsepower also shows the values reaching 15%. This concerns can be analyzed using the equation of, $H_h = 0.0245 \cdot P_s \cdot q_t$. From the equation, in order to get the value of hydraulic horsepower, surface injection pressure value is required. Therefore as the percentage difference of P_s is high, the percentage difference of H_h will be high too.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

As a conclusion, hydraulic fracturing is a very useful technique for production optimization especially in low permeability wells and wells that have been damaged. Complicated and lengthy governing equations involve in designing hydraulic fracturing treatment. Therefore mathematical model for hydraulic fracturing design will be a very relevance and useful model for the process. By using mathematical model, the calculation can be made within short time and readily applied for the treatment.

The first objective of this project is achieved by developing a mathematical model for hydraulic fracturing design. The mathematical model could run well and there is no error in the coding of the hydraulic fracturing design. The verification result also shows that there is only a slightly percentage difference (ranged from 0 to 2.6 %) between the results from the equations coded in VBA and manual calculation. Therefore it can be concluded that the coding is viable and objective for developing a mathematical model for hydraulic fracturing design was accomplished.

For hydraulic fracturing design, it can be concluded that:-

1. Two fracture extents which are vertically fractured and horizontally fractured have been focused in this mathematical model for hydraulic fracturing design.
2. The three flow mechanisms which control the extension of a hydraulically created fracture are the results of the effect of high fracturing fluid viscosity,

reservoir fluid viscosity and compressibility, and fracturing fluid wall-building characteristics.

The second objective of this project was achieved by running case studies to validate the mathematical model for hydraulic fracturing design. Several case studies were conducted and the field results were compared with the results from mathematical model. The validation result shows low values of percentage differences (ranged from 0 to 17%) between the calculated and field values. Therefore it can be concluded that the overall mathematical model is workable and can be used for further improvement and continuation.

5.2 Recommendations

This study has clearly demonstrated that the mathematical model for hydraulic fracturing design can be successfully applied to problems facing the petroleum industry. However the mathematical model developed is quite general, covering a broad problem domain. Therefore several matters have been listed and recommended for future improvement and continuation of this project.

It is recommended that the 2-Dimensional and 3-Dimensional fracture geometry which consists of PKN, KGD and radial model should be studied and included in the mathematical model of hydraulic fracturing. In HF mathematical model, the calculation of volume of fracture fluid in this model was calculated for the whole process. For further improvement, the mathematical model could be improvise so that it can calculate the volume of fracture fluid in different phase of the job (pre-pad, pad, slurry and displacement). Other than that, in future continuation, it is recommended that the mathematical model covered the economics analysis and could conduct the optimization of hydraulic fracturing treatment.

For the mathematical model itself, it is recommended that the GUI and macro function should be included so that the interface of HF mathematical model will be more users friendly.

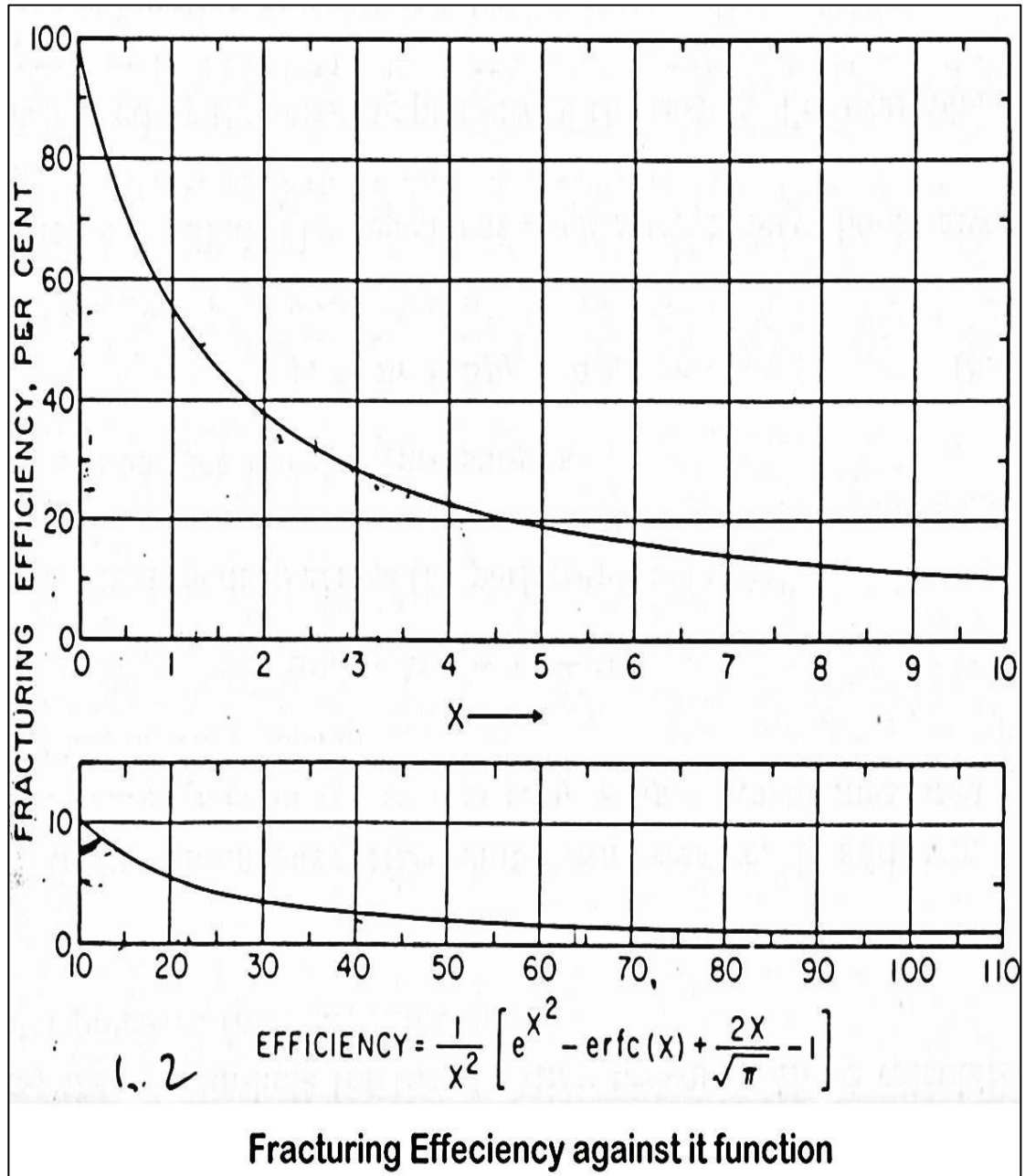
REFERENCES

1. Abou-Sayed, A. S., Ahmed, U., Jones, A. H., Schatz, J. F. 1982, A Step by Step Approach to Hydraulic Fracture Treatment Design, Implementation & Analysis for Tight Gas Sands, Terra Tek, Inc.
2. Adachi, J., Siebrits, E., Peircec, A., and Desroches, J., 2007, Computer simulation of hydraulic fractures, *International Journal of Rock Mechanics & Mining Sciences* 44, pages: 739–757.
3. API, 2009, “Hydraulic Fracturing Operations”, *Well Construction & Integrity Guidelines* (pg. 15-22), NW; Washington DC.
4. Association of American State Geologists (2012) Hydraulic Fracturing, retrieved from www.stategeologists.org
5. Britt L. K. and Jones J. R. 2009, *Design and Appraisal of Hydraulic Fractures*, United States of America, Society of Petroleum Engineer (SPE).
6. Carney M. J. and Murphy D. L. 1977, “Massive FRA- A Second Look”, *Proceedings of the Massive Hydraulic Fracturing Symposium* (pp. 57-75), University of Oklahoma.
7. Carvajal, G., Peries, C., Strobel, M. and Szatny, M. 2012, “Enabling Agile & Responsive Workflow Automation”. A Hydraulic Fracture Design Case Study. Halliburton
8. Charles, M. M. 2005, *Model Verification and Validation*, University of Chicago & Argonne National Laboratory.
9. Craft B. C., Graves E. D. and Holden W. R. 1962, “Hydraulic Fracturing”, *Well Design: Drilling and Production*, New Jersey, Prentice-Hall, Inc, pp 483.

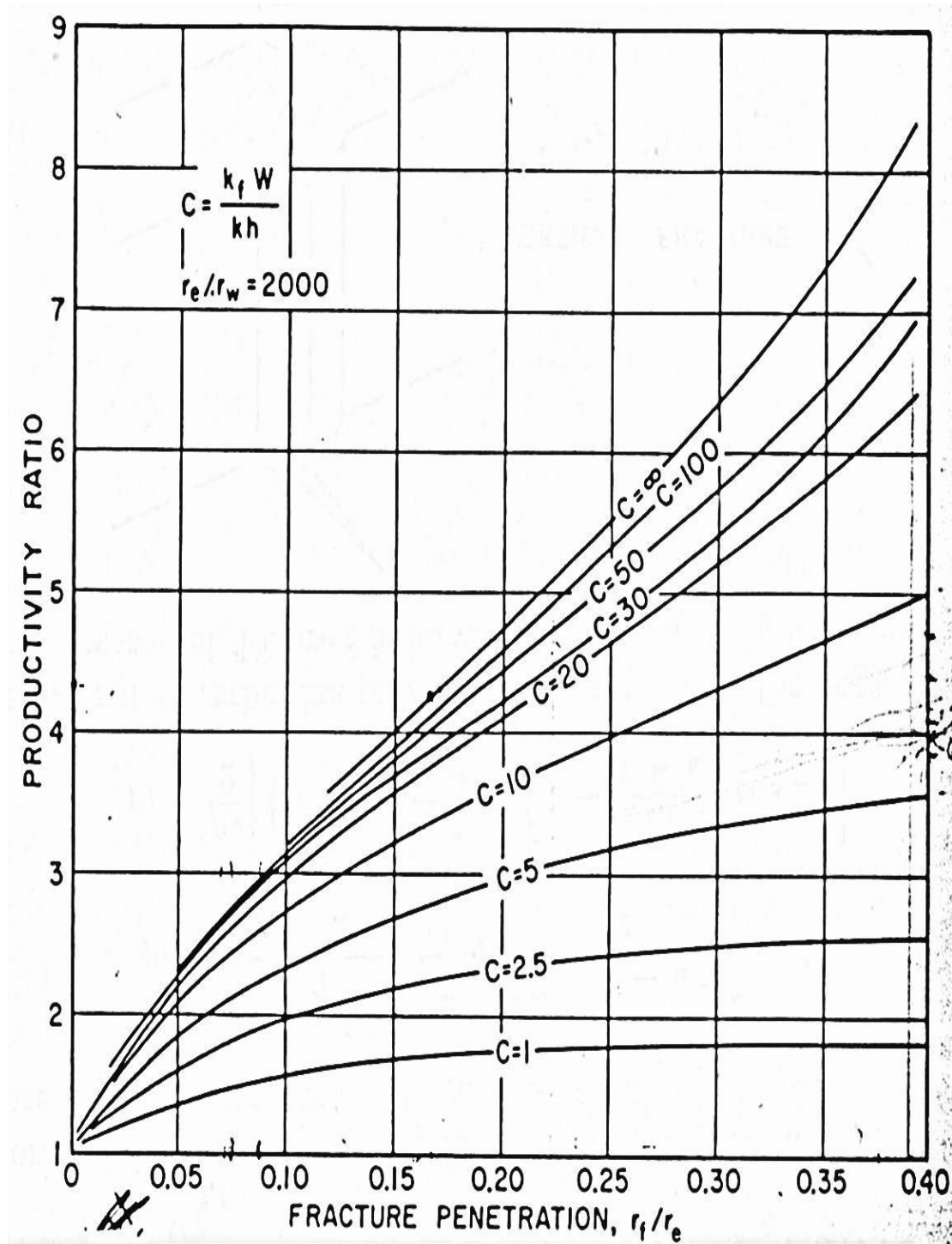
10. Daneshy, A. 2010. *Hydraulic Fracturing to Improve Production*, Daneshy Consultants International.
11. Dinh, T. (ND) How to Verify and Validate Mathematical Model. [Online]. Available: http://www.ehow.com/how_4510835_verify-validate-mathematical-model.html#ixzz2ZmybheaJ
12. Economides M. J. and Valko P. 1995, *Hydraulic Fracture Mechanics*, Texas A&M University USA, John Wiley & Sons.
13. Gidley J. L., Holditch S. A., Nierode D. E. and Veatch Jr. R. W. 1989, *Recent Advances in Hydraulic Fracturing, Monograph Volume 12*, USA, Society of Petroleum Engineer (SPE).
14. Grebe, J.J. and Stoesser, M., 1935, Increasing crude production 20,000,000 bbl. From established fields, *World Petroleum J*, (August), pp. 473–82.
15. Holditch, S. A. 1979, “Criteria of Propping Agent Selection”, prepared for the Norton Company.
16. Howard, G. C. and Fast C. R. 1957, “Optimum Fluid Characteristics for Fracture Extension”, *Drilling and Production Practice* (API) pp. 261-270.
17. Jabbari, H. and Zeng, Z. 2012, *Hydraulic Fracturing Design for Horizontal Wells in the Bakken Formation*. Department of Geology and Geological Engineering, University of North Dakota, Grand Forks, USA.
18. Lu, J. 1998. A Mathematical Model of Horizontal Wells Productivity and Well Testing Analysis, Virginia Polytechnic Institute and State University.
19. Wikipedia, Mathematical Model [Online]
Available: http://en.wikipedia.org/wiki/Mathematical_model

APPENDICES

Appendix 1: Graph of Fracturing Efficiency against Its Function

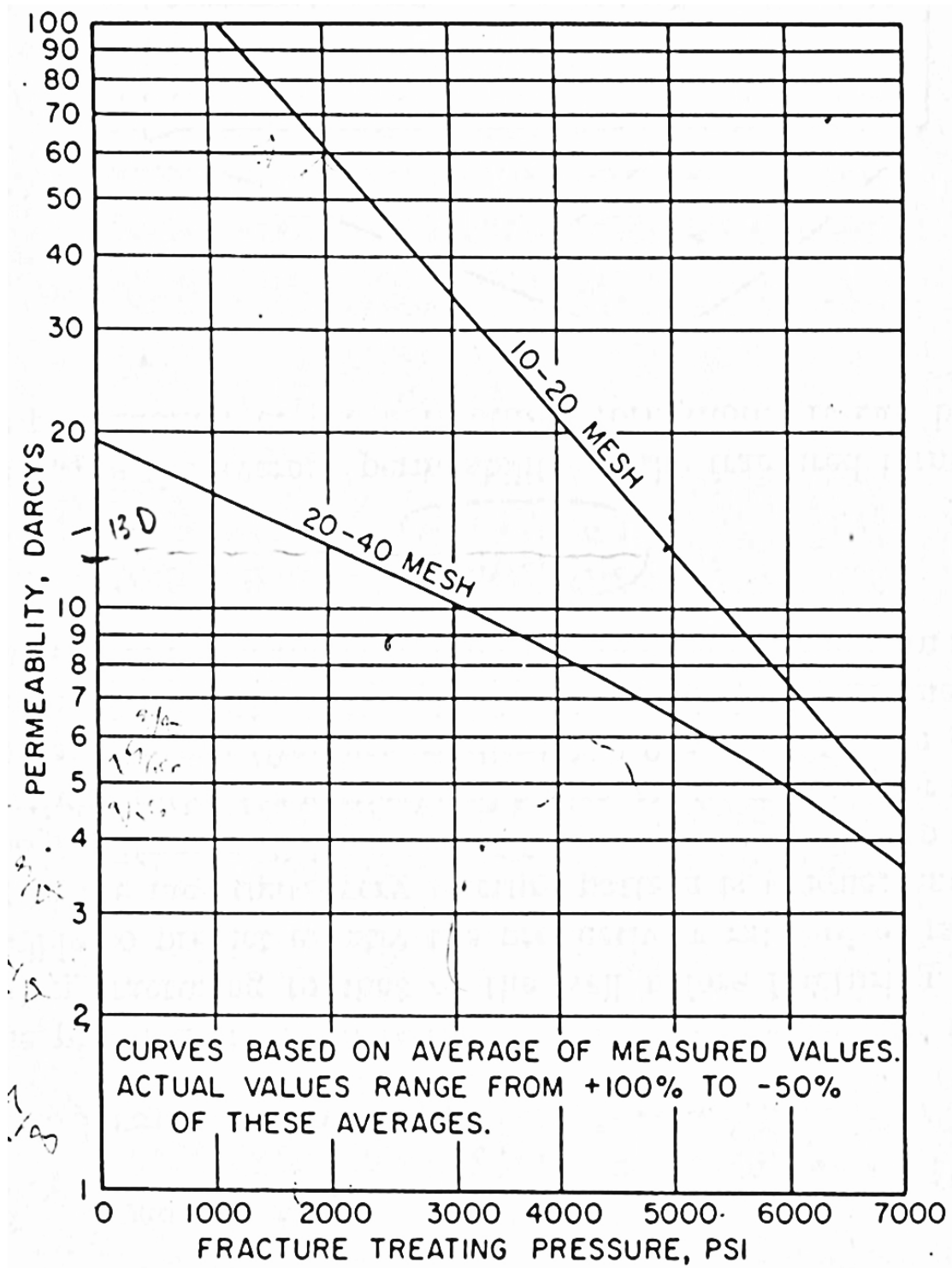


Appendix II: Graph of Estimate Productivity Ratio after Fracturing



Estimate Productivity Ratio after Treatment

Appendix III: Graph of Effect of Pressure on frac-sand Permeability



Effect of Pressure on frac-sand Permeability

Appendix IV: Table of Complementary Error Function

TABLE 7.1.1. COMPLEMENTARY ERROR FUNCTION OF x^a

x	0	1	2	3	4	5	6	7	8	9
0.0	1.0000	0.9887	0.9774	0.9662	0.9549	0.9436	0.9324	0.9211	0.9099	0.8987
0.1	0.8875	0.8764	0.8652	0.8541	0.8431	0.8320	0.8210	0.8100	0.7991	0.7882
0.2	0.7773	0.7665	0.7557	0.7450	0.7343	0.7237	0.7131	0.7026	0.6921	0.6817
0.3	0.6714	0.6611	0.6509	0.6407	0.6306	0.6206	0.6107	0.6008	0.5910	0.5813
0.4	0.5716	0.5620	0.5525	0.5431	0.5338	0.5245	0.5153	0.5063	0.4973	0.4883
0.5	0.4795	0.4708	0.4621	0.4535	0.4451	0.4367	0.4284	0.4202	0.4121	0.4041
0.6	0.3961	0.3883	0.3806	0.3730	0.3654	0.3580	0.3506	0.3434	0.3362	0.3292
0.7	0.3222	0.3153	0.3086	0.3019	0.2953	0.2888	0.2825	0.2762	0.2700	0.2639
0.8	0.2579	0.2520	0.2462	0.2405	0.2349	0.2293	0.2239	0.2186	0.2133	0.2082
0.9	0.2031	0.1981	0.1932	0.1884	0.1837	0.1791	0.1746	0.1701	0.1658	0.1615
1.0	0.1573	0.1532	0.1492	0.1452	0.1414	0.1376	0.1339	0.1302	0.1267	0.1232
1.1	0.1198	0.1165	0.1132	0.1100	0.1069	0.1039	0.1009	0.0980	0.0952	0.0924
1.2	0.0907	0.0870	0.0845	0.0819	0.0795	0.0771	0.0748	0.0725	0.0703	0.0681
1.3	0.0660	0.0639	0.0619	0.0600	0.0581	0.0562	0.0544	0.0527	0.0510	0.0493
1.4	0.0477	0.0461	0.0446	0.0431	0.0417	0.0403	0.0389	0.0376	0.0363	0.0351
1.5	0.0339	0.0327	0.0316	0.0305	0.0294	0.0284	0.0274	0.0264	0.0255	0.0245
1.6	0.0237	0.0228	0.0220	0.0212	0.0204	0.0196	0.0189	0.0182	0.0175	0.0168
1.7	0.0162	0.0156	0.0150	0.0144	0.0139	0.0133	0.0128	0.0123	0.0118	0.0114
1.8	0.0109	0.0105	0.0101	0.0097	0.0093	0.0089	0.0085	0.0082	0.0078	0.0075
1.9	0.0072	0.0069	0.0066	0.0063	0.0061	0.0058	0.0056	0.0053	0.0051	0.0049
2.0	0.00468	0.00448	0.00428	0.00409	0.00391	0.00374	0.00358	0.00342	0.00327	0.00312
2.1	0.00298	0.00285	0.00272	0.00259	0.00247	0.00236	0.00225	0.00215	0.00205	0.00195
2.2	0.00186	0.00178	0.00169	0.00161	0.00154	0.00146	0.00139	0.00133	0.00126	0.00120
2.3	0.00114	0.00109	0.00103	0.00098	0.00094	0.00089	0.00085	0.00080	0.00076	0.00072
2.4	0.00069	0.00065	0.00062	0.00059	0.00056	0.00053	0.00050	0.00048	0.00045	0.00043
2.5	0.00041	0.00039	0.00037	0.00035	0.00033	0.00031	0.00029	0.00028	0.00026	0.00025
2.6	0.00024	0.00022	0.00021	0.00020	0.00019	0.00018	0.00017	0.00016	0.00015	0.00014
2.7	0.00013	0.00013	0.00012	0.00011	0.00011	0.00010	0.00009	0.00009	0.00008	0.00008
2.8	0.000075	0.000071	0.000067	0.000063	0.000059	0.000056	0.000052	0.000049	0.000046	0.000044
2.9	0.000041	0.000039	0.000036	0.000034	0.000032	0.000030	0.000028	0.000027	0.000025	0.000023

Appendix V: VBA Coding of Hydraulic Fracturing Design

Private Sub CommandButton1_Click()

'-----ASSIGN WELL & RESERVOIR DATA-----

Pi = 3.14159265358979

WD = Val(Sheet1.Cells(9, 2)) 'Well Depth

CID = Val(Sheet1.Cells(11, 2)) 'Casing ID

TID = Val(Sheet1.Cells(12, 2)) 'Tubing ID

TOD = Val(Sheet1.Cells(13, 2)) 'Tubing OD

E = Val(Sheet1.Cells(14, 2)) 'Pipe roughness

kavg = Val(Sheet1.Cells(15, 2)) 'Average permeability

FT = Val(Sheet1.Cells(16, 2)) 'Formation thickness

SBHP = Val(Sheet1.Cells(17, 2)) 'Static bottomhole pressure

AWT = Val(Sheet1.Cells(18, 2)) 'Average Well Temperature

Aphi = Val(Sheet1.Cells(19, 2)) 'Average porosity

Re = Val(Sheet1.Cells(20, 2)) 'Drainage radius

rw = Val(Sheet1.Cells(21, 2)) 'Wellbore radius

Srho = Val(Sheet1.Cells(26, 2)) 'Sand SG

Sphi = Val(Sheet1.Cells(27, 2)) 'Sand porosity

kf = Val(Sheet1.Cells(28, 2)) 'Sand permeability

FG = Val(Sheet1.Cells(3, 8)) 'Fracture gradient

FW = Val(Sheet1.Cells(4, 8)) 'Fracture width

Q = Val(Sheet1.Cells(5, 8)) 'Assumed injection rate

t = Val(Sheet1.Cells(6, 8)) 'Assumed Time

CVVis = Val(Sheet1.Cells(13, 10))

CCVis = Val(Sheet1.Cells(17, 10))

OGatST = Val(Sheet1.Cells(20, 10)) 'Oil gravity at ST

m = Val(Sheet1.Cells(22, 10)) 'Slope of fluid-loss curve

FLP = Val(Sheet1.Cells(23, 10)) 'Fluid loss pressure

FLT = Val(Sheet1.Cells(24, 10)) 'Fluid loss temperature

Vis = Val(Sheet1.Cells(25, 10)) 'Viscosity

AF = Val(Sheet1.Cells(26, 10)) 'Area of filter paper

'===== VERTICAL FRACTURE =====

If (Val(Sheet1.Cells(3, 2)) = 1) Then

FTP = FG * WD

V = Q * t * 42

deltaP = FTP - SBHP

Sheet1.Cells(13, 21) = FTP

Sheet1.Cells(15, 21) = V

Sheet1.Cells(14, 21) = deltaP

'----- Fracturing fluid coefficients -----

'Wall building fluids'

If (Val(Sheet1.Cells(9, 13)) = 3) Then

Cw = (0.0164 * m) / AF

Cwa = Cw * ((deltaP / FLP) ^ (1 / 2))

```

x = (2 * Cwa * ((Pi * t) ^ (1 / 2))) / (FW / 12)
JJ = Application.WorksheetFunction.ErfC(x)
KK = Exp(x ^ 2)
Eff = (1 / (x ^ 2)) * ((KK * JJ) + ((2 * x) / (Pi ^ (1 / 2)))) - 1    'Fracture Efficiency

A = (V * Eff) / (7.48 * (FW / 12))                                'Area of fracture

'----- Calculate PR -----
C = (kf * (FW / 12)) / (kavg)
C1 = (kavg) / (kf * (FW / 12))
C2 = (kf * (FW / 12)) / (kavg)
rf = (A / Pi) ^ (1 / 2)
LN1 = Log(Re / rw)
LN2 = Log(Re / rf)
LN3 = Log(rf / rw)
PR = C * (((C1 + 1) * (LN1)) / (((C2 + 1) * LN2) + LN3))            'Productivity Ratio after
fracturing

Vfrac = 1# * (FW / 12)                                            'Volume fracture per unit area
S = ((Vfrac * (1 - Sphi)) * (Srho * 62.4))                        'Weight of sand
Smax = S * A                                                       'Maximum amount of sand
SandCon = Smax / V                                                 'Sand concentration
qt = Q + ((Q * SandCon) / (Srho * 8.34))                          'Total flow rate(including sand)
SGatAWT = OGatST * (1 - (0.0005 * (AWT - 60)))                   'SG at average well T
RhoatAWT = ((8.34 * SGatAWT) + SandCon) / (1 + (0.0456 * SandCon)) 'Density of oil at
average well T
deltaPs = 0.052 * RhoatAWT * WD                                    'Hydrostatic Pressure

'----- Fracture through:-----
'Casing'
If (Val(Sheet1.Cells(6, 2)) = 1) Then
    Vel = (17.16 * qt) / ((CID) ^ 2)                                'Velocity in the casing

    NRe = (928 * CID * RhoatAWT * Vel) / Vis                        'Reynolds number

    If (NRe >= 2000) Then
        FFA = E / (3.715 * CID)
        FFB = (6.943 / NRe) ^ (0.9)
        f = ((1 / (-2 * (Log(FFA + FFB) / Log(10)))) ^ 2) / 4      'Fanning Friction factor if
Re>2000
    Else
        f = 64 / NRe                                                'Fanning friction factor if Re<2000
    End If

    deltaPf = (f * WD * RhoatAWT * (Vel ^ 2)) / (25.8 * CID)        'Frictional pressure drop
    Ps = FTP + deltaPf - deltaPs                                     'Surface injection pressure
    Hh = 0.0245 * Ps * qt
End If

'Annulus'
If (Val(Sheet1.Cells(6, 2)) = 2) Then
    LL = (CID ^ 4) - (TOD ^ 4)
    MM = (CID ^ 2) - (TOD ^ 2)

```



```

NN = Log(CID / TOD)
de = (((LL - ((MM ^ 2) / NN)) ^ (1 / 4)) + (MM ^ (1 / 2))) / 2
Vel = (17.16 * qt) / ((de) ^ 2) 'Velocity in the casing

NRe = (928 * de * RhoatAWT * Vel) / Vis 'Reynolds number

If (NRe >= 2000) Then
    FFA = E / (3.715 * de)
    FFB = (6.943 / NRe) ^ (0.9)
    f = ((1 / (-2 * (Log(FFA + FFB) / Log(10)))) ^ 2) / 4 'Fanning Friction factor if
Re>2000
Else
    f = 64 / NRe 'Fanning friction factor if Re<2000
End If

deltaPf = (f * WD * RhoatAWT * (Vel ^ 2)) / (25.8 * de) 'Frictional pressure drop
Ps = FTP + deltaPf - deltaPs 'Surface injection pressure
Hh = 0.0245 * Ps * qt
End If

Sheet1.Cells(16, 21) = Eff
Sheet1.Cells(17, 21) = A
Sheet1.Cells(18, 21) = S
Sheet1.Cells(19, 21) = Smax
Sheet1.Cells(20, 21) = SandCon
Sheet1.Cells(21, 21) = qt
Sheet1.Cells(22, 21) = SGatAWT
Sheet1.Cells(23, 21) = RhoatAWT
Sheet1.Cells(24, 21) = deltaPs
Sheet1.Cells(25, 21) = Vel
Sheet1.Cells(26, 21) = NRe
Sheet1.Cells(27, 21) = f
Sheet1.Cells(28, 21) = deltaPf
Sheet1.Cells(29, 21) = Ps
Sheet1.Cells(30, 21) = Hh
Sheet1.Cells(31, 21) = PR

Sheet1.Cells(34, 21) = Smax
Sheet1.Cells(35, 21) = V
Sheet1.Cells(36, 21) = Ps
Sheet1.Cells(37, 21) = Hh
Sheet1.Cells(38, 21) = PR
End If

'+++++++ Reservoir controlled fluids +++++++'

If (Val(Sheet1.Cells(9, 13)) = 2) Then
    Cc = 0.0374 * deltaP * (((kavg / 1000) * (10 * (10 ^ (-6))) * Aphi) / CCVis) ^ (1 / 2))

x = (2 * Cc * ((Pi * t) ^ (1 / 2))) / (FW / 12)
JJ = Application.WorksheetFunction.ErfC(x)
KK = Exp(x ^ 2)
Eff = (1 / (x ^ 2)) * ((KK * JJ) + ((2 * x) / (Pi ^ (1 / 2))) - 1)
A = (V * Eff) / (7.48 * (FW / 12))

'----- Calculate PR -----

```



```

C = (kf * (FW / 12)) / (kavg)
C1 = (kavg) / (kf * (FW / 12))
C2 = (kf * (FW / 12)) / (kavg)
rf = (A / Pi) ^ (1 / 2)
LN1 = Log(Re / rw)
LN2 = Log(Re / rf)
LN3 = Log(rf / rw)
PR = C * (((C1 + 1) * (LN1)) / (((C2 + 1) * LN2) + LN3)) 'Productivity Ratio after fracturing

Vfrac = 1# * (FW / 12) 'Volume fracture per unit area
S = ((Vfrac * (1 - Sphi)) * (Srho * 62.4)) 'Weight of sand
Smax = S * A 'Maximum amount of sand
SandCon = Smax / V 'Sand concentration
qt = Q + ((Q * SandCon) / (Srho * 8.34)) 'Total flow rate(including sand)
SGatAWT = OGatST * (1 - (0.0005 * (AWT - 60))) 'SG at average well T
RhoatAWT = ((8.34 * SGatAWT) + SandCon) / (1 + (0.0456 * SandCon)) 'Density of oil at
average well T
deltaPs = 0.052 * RhoatAWT * WD

'----- Fracture through:-----
'Casing'
If (Val(Sheet1.Cells(6, 2)) = 1) Then
    Vel = (17.16 * qt) / ((CID) ^ 2) 'Velocity in the casing

    NRe = (928 * CID * RhoatAWT * Vel) / CCVis 'Reynolds number

    If (NRe >= 2000) Then
        FFA = E / (3.715 * CID)
        FFB = (6.943 / NRe) ^ (0.9)
        f = ((1 / (-2 * (Log(FFA + FFB) / Log(10)))) ^ 2) / 4 'Fanning Friction factor if
Re>2000
    Else
        f = 64 / NRe 'Fanning friction factor if Re<2000
    End If

    deltaPf = (f * WD * RhoatAWT * (Vel ^ 2)) / (25.8 * CID) 'Frictional pressure drop
    Ps = FTP + deltaPf - deltaPs 'Surface injection pressure
    Hh = 0.0245 * Ps * qt
End If

'Annulus'
If (Val(Sheet1.Cells(6, 2)) = 2) Then
    LL = (CID ^ 4) - (TOD ^ 4)
    MM = (CID ^ 2) - (TOD ^ 2)
    NN = Log(CID / TOD)
    de = (((LL - ((MM ^ 2) / NN)) ^ (1 / 4)) + (MM ^ (1 / 2))) / 2
    Vel = (17.16 * qt) / ((de) ^ 2) 'Velocity in the casing

    NRe = (928 * de * RhoatAWT * Vel) / CCVis 'Reynolds number

    If (NRe >= 2000) Then
        FFA = E / (3.715 * de)
        FFB = (6.943 / NRe) ^ (0.9)
        f = ((1 / (-2 * (Log(FFA + FFB) / Log(10)))) ^ 2) / 4 'Fanning Friction factor if
Re>2000
    Else

```

```

        f = 64 / NRe
        End If

        deltaPf = (f * WD * RhoatAWT * (Vel ^ 2)) / (25.8 * de)
        Ps = FTP + deltaPf - deltaPs
        Hh = 0.0245 * Ps * qt
        End If

        Sheet1.Cells(16, 21) = Eff
        Sheet1.Cells(17, 21) = A
        Sheet1.Cells(18, 21) = S
        Sheet1.Cells(19, 21) = Smax
        Sheet1.Cells(20, 21) = SandCon
        Sheet1.Cells(21, 21) = qt
        Sheet1.Cells(22, 21) = SGatAWT
        Sheet1.Cells(23, 21) = RhoatAWT
        Sheet1.Cells(24, 21) = deltaPs
        Sheet1.Cells(25, 21) = Vel
        Sheet1.Cells(26, 21) = NRe
        Sheet1.Cells(27, 21) = f
        Sheet1.Cells(28, 21) = deltaPf
        Sheet1.Cells(29, 21) = Ps
        Sheet1.Cells(30, 21) = Hh
        Sheet1.Cells(31, 21) = PR

        Sheet1.Cells(34, 21) = Smax
        Sheet1.Cells(35, 21) = V
        Sheet1.Cells(36, 21) = Ps
        Sheet1.Cells(37, 21) = Hh
        Sheet1.Cells(38, 21) = PR
    End If

    '+++++++ Viscosity controlled fluids +++++++'

    If (Val(Sheet1.Cells(9, 13)) = 1) Then
        Cv = 0.0469 * (((kavg / 1000) * deltaP * Aphi) / CVVis) ^ (1 / 2)
        x = (2 * Cv * ((Pi * t) ^ (1 / 2))) / (FW / 12)
        JJ = Application.WorksheetFunction.ErfC(x)
        KK = Exp(x ^ 2)
        Eff = (1 / (x ^ 2)) * ((KK * JJ) + ((2 * x) / (Pi ^ (1 / 2))) - 1)
        A = (V * Eff) / (7.48 * (FW / 12))

        '===== Calculate PR =====
        C = (kf * (FW / 12)) / (kavg)
        C1 = (kavg) / (kf * (FW / 12))
        C2 = (kf * (FW / 12)) / (kavg)
        rf = (A / Pi) ^ (1 / 2)
        LN1 = Log(Re / rw)
        LN2 = Log(Re / rf)
        LN3 = Log(rf / rw)
        PR = C * (((C1 + 1) * (LN1)) / (((C2 + 1) * LN2) + LN3))
    End If

    VFrac = 1# * (FW / 12)
    S = ((VFrac * (1 - Sphi)) * (Srho * 62.4))
    Smax = S * A
    SandCon = Smax / V

```

```

qt = Q + ((Q * SandCon) / (Srho * 8.34))          'Total flow rate(including sand)
SGatAWT = OGatST * (1 - (0.0005 * (AWT - 60)))    'SG at average well T
RhoatAWT = ((8.34 * SGatAWT) + SandCon) / (1 + (0.0456 * SandCon)) 'Density of oil at
average well T
deltaPs = 0.052 * RhoatAWT * WD                    'Hydrostatic Pressure

'----- Fracture through:-----
'Casing'
If (Val(Sheet1.Cells(6, 2)) = 1) Then
    Vel = (17.16 * qt) / ((CID) ^ 2)                'Velocity in the casing
    NRe = (928 * CID * RhoatAWT * Vel) / CVVis      'Reynolds number

    If (NRe >= 2000) Then
        FFA = E / (3.715 * CID)
        FFB = (6.943 / NRe) ^ (0.9)
        f = ((1 / (-2 * (Log(FFA + FFB) / Log(10)))) ^ 2) / 4    'Fanning Friction factor if
Re>2000
    Else
        f = 64 / NRe                                            'Fanning friction factor if Re<2000
    End If

    deltaPf = (f * WD * RhoatAWT * (Vel ^ 2)) / (25.8 * CID)    'Frictional pressure drop
    Ps = FTP + deltaPf - deltaPs                                'Surface injection pressure
    Hh = 0.0245 * Ps * qt
End If

'Annulus'
If (Val(Sheet1.Cells(13, 14)) = 2) Then
    LL = (CID ^ 4) - (TOD ^ 4)
    MM = (CID ^ 2) - (TOD ^ 2)
    NN = Log(CID / TOD)
    de = (((LL - (MM ^ 2) / NN)) ^ (1 / 4)) + (MM ^ (1 / 2))) / 2
    Vel = (17.16 * qt) / ((de) ^ 2)                        'Velocity in the casing

    NRe = (928 * de * RhoatAWT * Vel) / CVVis              'Reynolds number

    If (NRe >= 2000) Then
        FFA = E / (3.715 * de)
        FFB = (6.943 / Re) ^ (0.9)
        f = ((1 / (-2 * (Log(FFA + FFB) / Log(10)))) ^ 2) / 4    'Fanning Friction factor if
Re>2000
    Else
        f = 64 / NRe                                            'Fanning friction factor if Re<2000
    End If

    deltaPf = (f * WD * RhoatAWT * (Vel ^ 2)) / (25.8 * de)    'Frictional pressure drop
    Ps = FTP + deltaPf - deltaPs                                'Surface injection pressure
    Hh = 0.0245 * Ps * qt
End If

Sheet1.Cells(16, 21) = Eff
Sheet1.Cells(17, 21) = A
Sheet1.Cells(18, 21) = S
Sheet1.Cells(19, 21) = Smax
Sheet1.Cells(20, 21) = SandCon
Sheet1.Cells(21, 21) = qt

```

```
Sheet1.Cells(34, 21) = Smax
Sheet1.Cells(35, 21) = V
Sheet1.Cells(36, 21) = Ps
Sheet1.Cells(37, 21) = Hh
Sheet1.Cells(38, 21) = PR
End If
```

.....

of:=====

```
Sheet1.Cells(13, 21) = FTP
Sheet1.Cells(15, 21) = V
Sheet1.Cells(14, 21) = deltaP
```

'Wall building fluids'

$$C_w = (0.0164 * m) / AF$$
$$C_{wa} = C_w * ((\Delta P / FLP) ^ (1 / 2))$$
$$\begin{aligned} x &= (2 * Cwa * ((\pi * t)^{(1/2)})) / (FW / 12) \\ JJ &= \text{Application.WorksheetFunction.Erfc}(x) \\ KK &= \exp(x^2) \\ \text{Eff} &= (1 / (x^2)) * ((KK * JJ) + ((2 * x) / (\pi^{(1/2)})) - 1) \quad \text{'Fracture Efficiency'} \end{aligned}$$

'----- Calculate PR -----'

66

```

LN2 = Log(Re / rf)
LN3 = Log(rf / rw)
PR = C * (((C1 + 1) * (LN1)) / (((C2 + 1) * LN2) + LN3))      'Productivity Ratio after
fracturing

VFrac = 1# * (FW / 12)      'Volume fracture per unit area
S = ((VFrac * (1 - Sphi)) * (Srho * 62.4))      'Weight of sand
Smax = S * A      'Maximum amount of sand
SandCon = Smax / V      'Sand concentration
qt = Q + ((Q * SandCon) / (Srho * 8.34))      'Total flow rate(including sand)
SGatAWT = OGatST * (1 - (0.0005 * (AWT - 60)))      'SG at average well T
RhoatAWT = ((8.34 * SGatAWT) + SandCon) / (1 + (0.0456 * SandCon))      'Density of oil at
average well T
deltaPs = 0.052 * RhoatAWT * WD      'Hydrostatic Pressure

'----- Fracture through:-----
'Casing'
If (Val(Sheet1.Cells(6, 2)) = 1) Then
    Vel = (17.16 * qt) / ((CID) ^ 2)      'Velocity in the casing

    NRe = (928 * CID * RhoatAWT * Vel) / Vis      'Reynolds number

    If (NRe >= 2000) Then
        FFA = E / (3.715 * CID)
        FFB = (6.943 / NRe) ^ (0.9)
        f = ((1 / (-2 * (Log(FFA + FFB) / Log(10)))) ^ 2) / 4      'Fanning Friction factor if
Re>2000
    Else
        f = 64 / NRe      'Fanning friction factor if Re<2000
    End If

    deltaPf = (f * WD * RhoatAWT * (Vel ^ 2)) / (25.8 * CID)      'Frictional pressure drop
    Ps = FTP + deltaPf - deltaPs      'Surface injection pressure
    Hh = 0.0245 * Ps * qt
End If

'Annulus'
If (Val(Sheet1.Cells(6, 2)) = 2) Then
    LL = (CID ^ 4) - (TOD ^ 4)
    MM = (CID ^ 2) - (TOD ^ 2)
    NN = Log(CID / TOD)
    de = (((LL - ((MM ^ 2) / NN)) ^ (1 / 4)) + (MM ^ (1 / 2))) / 2
    Vel = (17.16 * qt) / ((de) ^ 2)      'Velocity in the casing

    NRe = (928 * de * RhoatAWT * Vel) / Vis      'Reynolds number

    If (NRe >= 2000) Then
        FFA = E / (3.715 * de)
        FFB = (6.943 / NRe) ^ (0.9)
        f = ((1 / (-2 * (Log(FFA + FFB) / Log(10)))) ^ 2) / 4      'Fanning Friction factor if
Re>2000
    Else
        f = 64 / NRe      'Fanning friction factor if Re<2000
    End If

```

```

    deltaPf = (f * WD * RhoatAWT * (Vel ^ 2)) / (25.8 * de)      'Frictional pressure drop
    Ps = FTP + deltaPf - deltaPs      'Surface injection pressure
    Hh = 0.0245 * Ps * qt
End If

```

```

Sheet1.Cells(16, 21) = Eff
Sheet1.Cells(17, 21) = A
Sheet1.Cells(18, 21) = S
Sheet1.Cells(19, 21) = Smax
Sheet1.Cells(20, 21) = SandCon
Sheet1.Cells(21, 21) = qt
Sheet1.Cells(22, 21) = SGatAWT
Sheet1.Cells(23, 21) = RhoatAWT
Sheet1.Cells(24, 21) = deltaPs
Sheet1.Cells(25, 21) = Vel
Sheet1.Cells(26, 21) = NRe
Sheet1.Cells(27, 21) = f
Sheet1.Cells(28, 21) = deltaPf
Sheet1.Cells(29, 21) = Ps
Sheet1.Cells(30, 21) = Hh
Sheet1.Cells(31, 21) = PR

```

```

Sheet1.Cells(34, 21) = Smax
Sheet1.Cells(35, 21) = V
Sheet1.Cells(36, 21) = Ps
Sheet1.Cells(37, 21) = Hh
Sheet1.Cells(38, 21) = PR

```

```
End If
```

```
'+++++++ Reservoir controlled fluids +++++++'
```

```

If (Val(Sheet1.Cells(9, 13)) = 2) Then
    Cc = 0.0374 * deltaP * (((kavg / 1000) * (10 * (10 ^ (-6))) * Aphi) / CCVis) ^ (1 / 2))

```

```

x = (2 * Cc * ((Pi * t) ^ (1 / 2))) / (FW / 12)
JJ = Application.WorksheetFunction.ErfC(x)
KK = Exp(x ^ 2)
Eff = (1 / (x ^ 2)) * ((KK * JJ) + ((2 * x) / (Pi ^ (1 / 2))) - 1)
A = (V * Eff) / (7.48 * (FW / 12))

```

```
'----- Calculate PR -----
```

```

C = (kf * (FW / 12)) / (kavg * FT)
C1 = (kavg * FT) / (kf * (FW / 12))
C2 = (kf * (FW / 12)) / (kavg * FT)
rf = (A / Pi) ^ (1 / 2)
LN1 = Log(Re / rw)
LN2 = Log(Re / rf)
LN3 = Log(rf / rw)
PR = C * (((C1 + 1) * (LN1)) / (((C2 + 1) * LN2) + LN3))      'Productivity Ratio after fracturing

```

```

VFrac = 1# * (FW / 12)      'Volume fracture per unit area
S = ((VFrac * (1 - Sphi)) * (Srho * 62.4))      'Weight of sand
Smax = S * A      'Maximum amount of sand
SandCon = Smax / V      'Sand concentration
qt = Q + ((Q * SandCon) / (Srho * 8.34))      'Total flow rate(including sand)

```

```

SGatAWT = OGatST * (1 - (0.0005 * (AWT - 60)))      'SG at average well T
RhoatAWT = ((8.34 * SGatAWT) + SandCon) / (1 + (0.0456 * SandCon))  'Density of oil at
average well T
deltaPs = 0.052 * RhoatAWT * WD

'----- Fracture through:-----
'Casing'
If (Val(Sheet1.Cells(6, 2)) = 1) Then
    Vel = (17.16 * qt) / ((CID) ^ 2)                'Velocity in the casing

    NRe = (928 * CID * RhoatAWT * Vel) / CCVis      'Reynolds number

    If (NRe >= 2000) Then
        FFA = E / (3.715 * CID)
        FFB = (6.943 / NRe) ^ (0.9)
        f = ((1 / (-2 * (Log(FFA + FFB) / Log(10)))) ^ 2) / 4      'Fanning Friction factor if
Re>2000
    Else
        f = 64 / NRe                                          'Fanning friction factor if Re<2000
    End If

    deltaPf = (f * WD * RhoatAWT * (Vel ^ 2)) / (25.8 * CID)      'Frictional pressure drop
    Ps = FTP + deltaPf - deltaPs                                  'Surface injection pressure
    Hh = 0.0245 * Ps * qt
End If

'Annulus'
If (Val(Sheet1.Cells(6, 2)) = 2) Then
    LL = (CID ^ 4) - (TOD ^ 4)
    MM = (CID ^ 2) - (TOD ^ 2)
    NN = Log(CID / TOD)
    de = (((LL - ((MM ^ 2) / NN)) ^ (1 / 4)) + (MM ^ (1 / 2))) / 2
    Vel = (17.16 * qt) / ((de) ^ 2)                          'Velocity in the casing

    NRe = (928 * de * RhoatAWT * Vel) / CCVis                'Reynolds number

    If (NRe >= 2000) Then
        FFA = E / (3.715 * de)
        FFB = (6.943 / NRe) ^ (0.9)
        f = ((1 / (-2 * (Log(FFA + FFB) / Log(10)))) ^ 2) / 4      'Fanning Friction factor if
Re>2000
    Else
        f = 64 / NRe                                          'Fanning friction factor if Re<2000
    End If

    deltaPf = (f * WD * RhoatAWT * (Vel ^ 2)) / (25.8 * de)      'Frictional pressure drop
    Ps = FTP + deltaPf - deltaPs                                  'Surface injection pressure
    Hh = 0.0245 * Ps * qt
End If

Sheet1.Cells(16, 21) = Eff
Sheet1.Cells(17, 21) = A
Sheet1.Cells(18, 21) = S
Sheet1.Cells(19, 21) = Smax
Sheet1.Cells(20, 21) = SandCon
Sheet1.Cells(21, 21) = qt

```

```

Sheet1.Cells(22, 21) = SGatAWT
Sheet1.Cells(23, 21) = RhoatAWT
Sheet1.Cells(24, 21) = deltaPs
Sheet1.Cells(25, 21) = Vel
Sheet1.Cells(26, 21) = NRe
Sheet1.Cells(27, 21) = f
Sheet1.Cells(28, 21) = deltaPf
Sheet1.Cells(29, 21) = Ps
Sheet1.Cells(30, 21) = Hh
Sheet1.Cells(31, 21) = PR

Sheet1.Cells(34, 21) = Smax
Sheet1.Cells(35, 21) = V
Sheet1.Cells(36, 21) = Ps
Sheet1.Cells(37, 21) = Hh
Sheet1.Cells(38, 21) = PR
End If

'+++++++ Viscosity controlled fluids ++++++'

If (Val(Sheet1.Cells(9, 13)) = 1) Then
    Cv = 0.0469 * (((kavg / 1000) * deltaP * Aphi) / CVVis) ^ (1 / 2)
    x = (2 * Cv * ((Pi * t) ^ (1 / 2))) / (FW / 12)
    JJ = Application.WorksheetFunction.ErfC(x)
    KK = Exp(x ^ 2)
    Eff = (1 / (x ^ 2)) * ((KK * JJ) + ((2 * x) / (Pi ^ (1 / 2))) - 1)
    A = (V * Eff) / (7.48 * (FW / 12))

'===== Calculate PR =====
C = (kf * (FW / 12)) / (kavg * FT)
C1 = (kavg * FT) / (kf * (FW / 12))
C2 = (kf * (FW / 12)) / (kavg * FT)
rf = (A / Pi) ^ (1 / 2)
LN1 = Log(Re / rw)
LN2 = Log(Re / rf)
LN3 = Log(rf / rw)
PR = C * (((C1 + 1) * (LN1)) / (((C2 + 1) * LN2) + LN3)) 'Productivity Ratio after fracturing

Vfrac = 1# * (FW / 12) 'Volume fracture per unit area
S = ((Vfrac * (1 - Sphi)) * (Srho * 62.4)) 'Weight of sand
Smax = S * A 'Maximum amount of sand
SandCon = Smax / V 'Sand concentration
qt = Q + ((Q * SandCon) / (Srho * 8.34)) 'Total flow rate(including sand)
SGatAWT = OGatST * (1 - (0.0005 * (AWT - 60))) 'SG at average well T
RhoatAWT = ((8.34 * SGatAWT) + SandCon) / (1 + (0.0456 * SandCon)) 'Density of oil at
average well T
deltaPs = 0.052 * RhoatAWT * WD 'Hydrostatic Pressure

'----- Fracture through:-----
'Casing'
If (Val(Sheet1.Cells(6, 2)) = 1) Then
    Vel = (17.16 * qt) / ((CID) ^ 2) 'Velocity in the casing
    NRe = (928 * CID * RhoatAWT * Vel) / CVVis 'Reynolds number

    If (NRe >= 2000) Then
        FFA = E / (3.715 * CID)

```



```

        FFB = (6.943 / NRe) ^ (0.9)
        f = ((1 / (-2 * (Log(FFA + FFB) / Log(10)))) ^ 2) / 4      'Fanning Friction factor if
Re>2000
    Else
        f = 64 / NRe      'Fanning friction factor if Re<2000
    End If

    deltaPf = (f * WD * RhoatAWT * (Vel ^ 2)) / (25.8 * CID)      'Frictional pressure drop
    Ps = FTP + deltaPf - deltaPs      'Surface injection pressure
    Hh = 0.0245 * Ps * qt
End If

'Annulus'
If (Val(Sheet1.Cells(13, 14)) = 2) Then
    LL = (CID ^ 4) - (TOD ^ 4)
    MM = (CID ^ 2) - (TOD ^ 2)
    NN = Log(CID / TOD)
    de = (((LL - ((MM ^ 2) / NN)) ^ (1 / 4)) + (MM ^ (1 / 2))) / 2
    Vel = (17.16 * qt) / ((de) ^ 2)      'Velocity in the casing

    NRe = (928 * de * RhoatAWT * Vel) / CVVis      'Reynolds number

    If (NRe >= 2000) Then
        FFA = E / (3.715 * de)
        FFB = (6.943 / Re) ^ (0.9)
        f = ((1 / (-2 * (Log(FFA + FFB) / Log(10)))) ^ 2) / 4      'Fanning Friction factor if
Re>2000
    Else
        f = 64 / NRe      'Fanning friction factor if Re<2000
    End If

    deltaPf = (f * WD * RhoatAWT * (Vel ^ 2)) / (25.8 * de)      'Frictional pressure drop
    Ps = FTP + deltaPf - deltaPs      'Surface injection pressure
    Hh = 0.0245 * Ps * qt
End If

Sheet1.Cells(16, 21) = Eff
Sheet1.Cells(17, 21) = A
Sheet1.Cells(18, 21) = S
Sheet1.Cells(19, 21) = Smax
Sheet1.Cells(20, 21) = SandCon
Sheet1.Cells(21, 21) = qt
Sheet1.Cells(22, 21) = SGatAWT
Sheet1.Cells(23, 21) = RhoatAWT
Sheet1.Cells(24, 21) = deltaPs
Sheet1.Cells(25, 21) = Vel
Sheet1.Cells(26, 21) = NRe
Sheet1.Cells(27, 21) = f
Sheet1.Cells(28, 21) = deltaPf
Sheet1.Cells(29, 21) = Ps
Sheet1.Cells(30, 21) = Hh
Sheet1.Cells(31, 21) = PR

Sheet1.Cells(34, 21) = Smax
Sheet1.Cells(35, 21) = V
Sheet1.Cells(36, 21) = Ps

```

```
Sheet1.Cells(37, 21) = Hh  
Sheet1.Cells(38, 21) = PR  
End If
```

```
End If
```

```
End Sub
```