CHAPTER 1

1.0. INTRODUCTION

1.1. Background Study

From large central storage facilities oil moved through large diameter, long distance trunk lines comprise a wide variety of pipe sizes and capacities. Pumps are required at the beginning of the trunk line, and pumping stations must also be spaced along the pipeline to maintain pipeline pressure at the level required overcoming friction, changes in elevation and other looses. Crude trunk lines operate at higher pressure than field-gathering system and are also made of steel. Individual sectioned are joined by welding.

Long cylindrical tubes and pipes will usually bend. Bending in the presence of external pressure is experienced by pipelines during their installation and also subsequently during their operation. The pipeline is bent under relatively high external pressure as it conforms to surface undulations on the seafloor. Bending at the sea floor can be experienced due to snaking resulting from pipeline expansion cause by the passage of hot hydrocarbons. It is also a condition that can develop in case of upheaval buckling of a section of a buried pipe. Bending ovalizes the pipe cross section, which course reduces its resistance to external pressure.

1.2. Problem Statement

Buckling is a failure mode characterized by sudden failure of a structural member subjected to high compressive stresses, where the actual compressive stress at the point of failure is less than the ultimate compressive stresses that the material is capable of withstanding. This mode of failure is also described as failure due to elastic instability. Thus in designing structure it is important to determine the critical buckling load. The critical buckling load is the limit for stability of structure before it become unstable.

1.3. Objective

- To simulate the degree of freedom, that is the maximum deflection of hollow cylinder geometry provided that the critical buckling load would be identified from theoretical formula for column buckling.
- By varying the length, thickness, material properties and percentage of horizontal force with respect to critical buckling load of hollow cylinder geometry, the degree of freedom for the model will be analyzed.

1.4. Scope of Study

The project is focuses on relationship between critical buckling load of hollow cylinder with maximum deflection. The failure is due to elastic buckling. For the purpose of this project few assumptions have been made which are the material element is homogeneous and isotropic, Hooks Law holds (stress linearly proportional to strain), the maximum stress the hollow cylinder can handle is equivalent to yield stress of material used which are steel and aluminum, the loads and the bending moment act in a plane are passing through a principal axis of inertia of cross section and the deflection are small compared to cross sectional dimension.

1.5. The relevancy of the project

This project is relevant by applying theoretical formula for eccentric loading on hollow cylinder. The stress distribution acting over the cross sectional area of the hollow cylinder is determined from both the axial force and bending moment. The bending moment and disturbance force introduced as an imperfection to the hollow cylinder.

CHAPTER 2
LITERATURE REVIEW

2.0. LITERATURE REVIEW

2.1. The Study of Elastic Stability of a Structure

The first study on elastic stability is attributed to Leonhard Euler [1707–1783], who used the theory of calculus of variations to obtain the equilibrium equation and buckling load of a compressed elastic column. This work was published in the appendix “De curvis elasticis” of his book titled *Methodus inveniendi lineas curvas maximi minimive proprietate gaudentes*, Lausanne and Geneva, 1744. Joseph-Louis Lagrange [1736–1813] developed the energy approach that is more general than Newton’s vector approach for the study of mechanics problems.

This led naturally to the fundamental energy theorem of minimum total potential energy being sufficient for stability. Jules Henry Poincar´e [1854–1912] is known as the founder of bifurcation theory and the classification of the singularities. On the other hand, Aleksandr Mikhailovich Liapunov [1857–1918] gave the basic definitions of stability and introduced the generalized energy functions that bear his name, Liapunov functions. Furthermore, Lev Semenovich Pontryagin [1908–1988] introduced, with A. A. Andronov, the important topological concept of structural stability.

This work has led to the well known classification theory presented in a treatise, *Stabilite structurelle et morphogenese: Essai d’une theorie generale des modeles (Structural Stability and Morphogenesis: An Outline of General Theory of Models)* by R. Thom. Theodore von K´arm´an [1881–1963] began his work on inelastic buckling of columns. He devised a model to explain hysteresis loops and conducted research on plastic deformation of beams. Warner Tjardus Koiter

[1914–1997] initiated the classical nonlinear bifurcation theory in his dissertation, “Over de Stabiliteit van het Elastisch Evenwicht”, at Delft. Budiansky and his colleagues (1946, 1948) gave a modern account of the nonlinear branching of continuous elastic structures under conservative loads. Furthermore, Hutchinson (1973a, b) made an important contribution to the nonlinear branching theory of structures loaded in the plastic range.

Pioneering research by a number of other individuals is also significant and some of them are: F. Engesser and S. P. Timoshenko on buckling of shear–flexural buckling of columns; A. Considere, F. Engesser and F. R. Shanley on inelastic buckling of columns; G. R. Kirchhoff on buckling of elastica; J. A. Haringx on buckling of springs; V. Vlasov on torsional buckling; L. Prandtl, A. G. M. Michell, S. P. Timoshenko,H. Wagner and N. S. Trahair on flexural–torsional buckling of beams (see Trahair and Bradford, 1991).

B. W. James, R. K. Livesly and D.B. Chandler, R. von Mises and J. Ratzersdorfer, and E. Chwalla on buckling of frames; H. Lamb, J. Boussinesq, C. B. Biezeno and J. J.Koch on buckling of rings and arches; E. Hurlbrink, E. Chwalla, E. L.Nicolai, I. J. Steuermann, A. N. Dinnik and K. Federhofer on arches; G.H. Bryan, S. P. Timoshenko, T. von Karman, E. Trefftz, A. Kromm, K.Marguerre and G. Herrmann on buckling (and postbuckling) of plates;G. H. Handelmann, W. Prager, E. I. Stowell, S. B. Batdorf, F. Bleich and P. P. Bijlaard on plastic buckling of plates; R. Lorentz, R. von Mises, S. P. Timoshenko, R. V. Southwell, T. von K´arm´an and H. S. Tsien on cylindrical shells under combined axial and lateral pressure; L. H. Donnell, K. M. Marguerre and K. M. Mushtari on the postbuckling of shells; A. Pfl¨uger on buckling of conical shells; and R. Zoelly and E. Schwerin on buckling of spherical shells. Additional references can be found in the book by Timoshenko and Gere (1961) and the survey article by Baˇzant (2000).

2.2 The Study of Pipeline Buckling

When studying the elastic stability of geometric properties for the purpose of this project the author has reviewed on the real situation of buckling involving Oil pipeline which uses the same geometric model properties that is hollow cylinder. In the case of Pipeline Buckling [8] temperature and pressure effects create expansion effective forces which may cause a pipeline to buckle globally. Pipelines installed on the seabed and left exposed have a potential to buckle globally and change configuration while a buried pipeline is designed to stay in place being restricted by the surrounding soil reaction forces. The driving force for global buckling of the pipeline is the effective axial force, which represents the combined action of pipe wall force, and internal and external pressures.

For a certain expansion force, the pipeline will buckle globally. For a partially displacement controlled condition, this implies that it will find a new equilibrium by moving perpendicular to the pipe axis at the same time as the pipe will move axially, feed in, from both sides towards the buckle. The level of axial force to initiate this global buckling depends on pipe cross section properties, lateral resistance, out of straightness in the pipeline and lateral triggering force for example trawling. A straight column will buckle according to the classical Euler buckling formulation. As the out-of-straightness in the column increases, the level of axial force required to buckle it will be reduces. This effect, away from the buckle, is illustrated in Figure 1

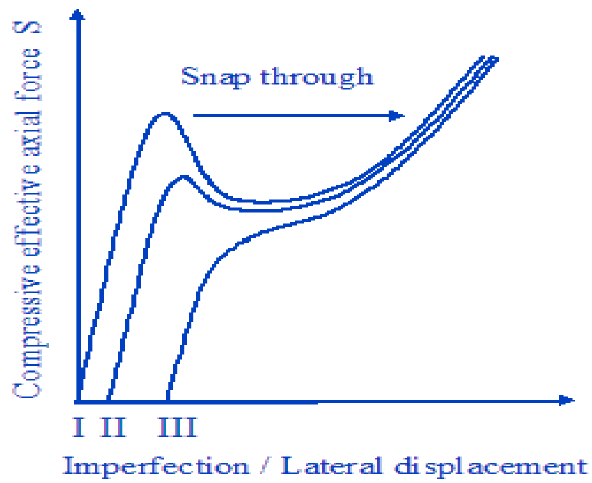


Figure 1

The out-of-straightness may be caused by small imperfections on the seabed like the pipeline resting on rocks, global imperfections as uneven seabed and curvature in the horizontal plane purposely made or random from installation.

2.3 The Study of Hollow Buckling Cylinder

2.3.1 Hollow Cylinder is fixed at the bottom and free at the top

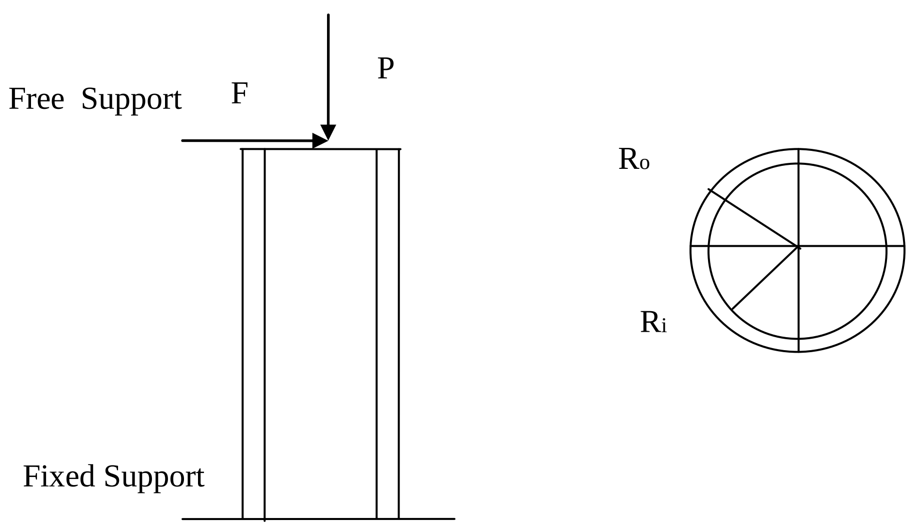


Figure 2

The stress distribution acting over the cross section area of the hollow cylinder column shown in Figure 2 is determined by both the axial force P, and the bending moment M, In particular, the maximum compressive stress is:

$\sigma \;\; max =$ (P/A) + (Mc/I)   (1)

Where F = Horizontal force

By taking into consideration that in reality column never suddenly buckles, instead they begin to bend, although ever so slightly, immediately upon application of the load. The horizontal force to cause bending will result to the out-of-straightness in the column which function as an initial imperfection to the column. As a result, the actual criterion for load application will be limited either to a specified deflection of the column or by not allowing the maximum stress in the column to exceed yield stress of material.

2.3.2 Hollow Cylinder is Fixed at the bottom and pinned at the top

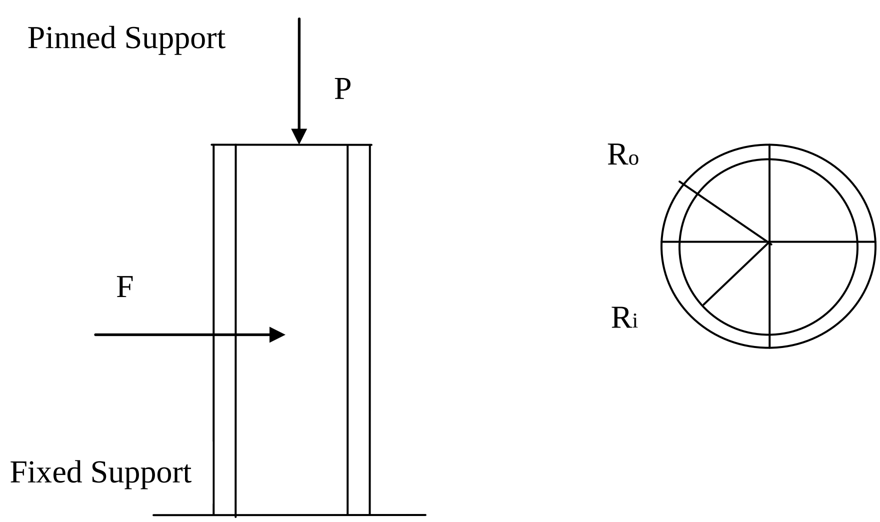


Figure 3

Since an ideal column is straight, theoretically the axial load P could be increased until failure occurs by either fracture or yielding of material .However, when critical load, P_{cr} is reached; the column is on the verge of becoming unstable, so that small lateral force F, will cause the column to remain in the deflected position when F is removed. Any slight reduction in the axial load P from P_{cr} will allow the column to straighten out and any slight increase in P beyond P_{cr} will further increase in lateral deflection.

$$P_{cr} = \pi^2 EI / (KL) \qquad (2)$$

Where F = Lateral force

CHAPTER 3

METHODOLOGY

3.0 METHODOLOGY

3.1 Technique of Analysis

3.1.1 Preprocessing: Defining the Problem

3.1.1.1 Create Key points

Preprocessor > Modeling > Create > Key points > In Active CS
K, #, X, Y

Table 1: Keypoint versus Coordinate

Key point	Coordinate (x,y)
1	(0,0)
2	(0,4,500)

3.1.1.2 Define Lines

Preprocessor > Modeling > Create > Lines > Lines > Straight Line >Create a line between point 1 and key point 2.

3.1.1.3 Define Element Types

Preprocessor > Element Type > Add/Edit/Delete...
The BEAM3 (Beam 2D elastic) element will be selected. This element has 3 degrees of freedom (translation along the X and Y axis's, and rotation about the Z axis). With only 3 degrees of freedom, the BEAM3 element can only be used in 2D analysis.

3.1.1.4 Define cross sectional area

Sections > Beam > Common Section >Beam tool

Table 2: Internal and Outer Radius

R_i (mm)	70
R_o (mm)	75

3.1.1.5 Define Real Constants

Preprocessor > Real Constants... > Add...

In the 'Real Constants for BEAM3' window, enter the following geometric properties:

i. Cross-sectional area AREA: 2276 (mm^2)
ii. Area Moment of Inertia IZZ: 0.598 e7 (mm^4)
iii. Total beam height HEIGHT: 4500 (mm)

3.1.1.6 Define Element Material Properties

Preprocessor > Material Props > Material Models > Structural > Linear > Elastic > Isotropic.

In the window that appears, enter the following geometric properties for steel:

i. Young's modulus EX: 2.0e5 Pa
ii. Poisson's Ratio PRXY: 0.3

3.1.1.7 Define Mesh Size

Preprocessor > Meshing > Size Cntrls > Lines > All Lines...

For this example we will specify an element edge length of 1 mm (100 element divisions along the line).

3.1.1.8 Mesh the frame

Preprocessor > Meshing > Mesh > Lines > click 'Pick All'

3.1.2 Solution: Assigning Loads and Solving

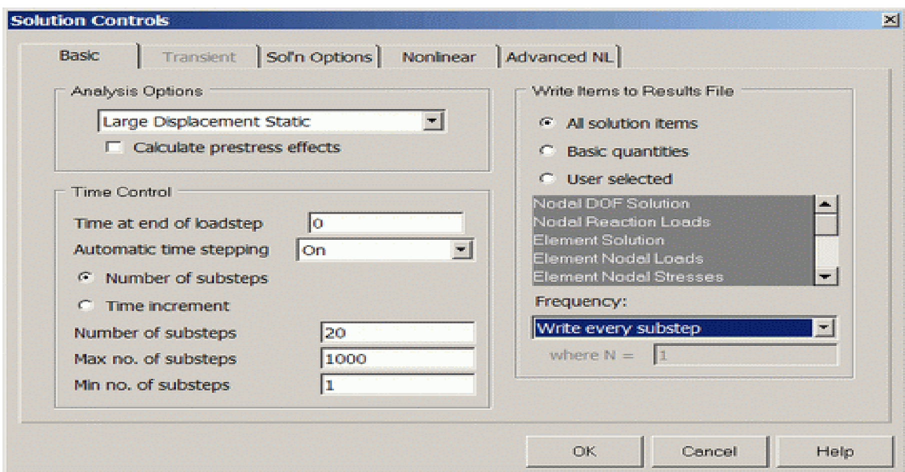
3.1.2.1 Define Analysis Type

Solution > New Analysis > Static

3.1.2.2 Set Solution Controls

Select Solution > Analysis Type > Sol'n Control...

Figure 4: Transient Solution Controls



Ensure the following selections are made under the 'Basic' tab (as shown above)

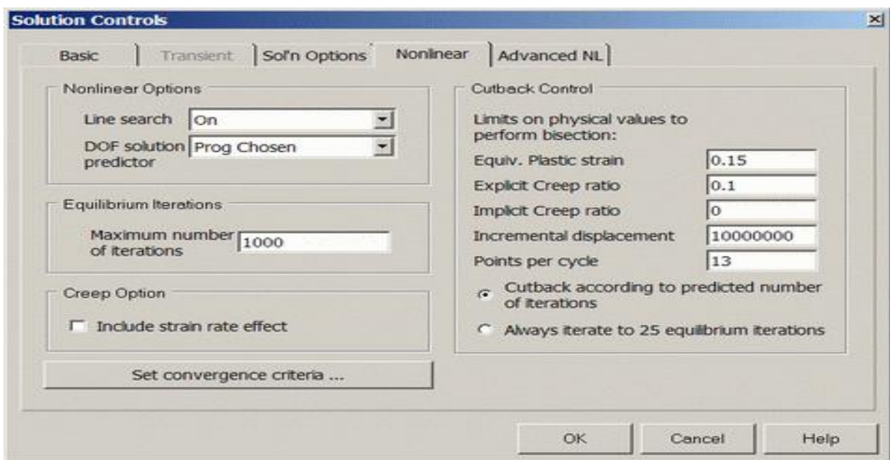
A. Ensure Large Static Displacements are permitted
B. Ensure Automatic time stepping is on.

C. Enter 20 as the number of sub steps.
D. Enter a maximum number of sub steps of 1000.
E. Enter a minimum number of substeps of 1.
F. Ensure all solution items are written to a results file.

Ensure the following selection is made under the 'Nonlinear' tab (as shown below)

A. Ensure Line Search is 'On'. This option is used to help the Newton-Raphson solver converge.
B. Ensure Maximum Number of Iterations is set to 1000.

Figure 5: Solution Option for Solution Controls



3.1.2.3 Apply Constraints

Solution > Define Loads > Apply > Structural > Displacement > On Key points.

Fix Key point 1 (ie all DOFs constrained).

3.1.2.4 Apply Load

Solution > Define Loads > Apply > Structural > Force/Moment > On Key points.

For supporting condition column is free at the top and fixed at the bottom that is subjected to both an axial load P and moment, M:

- Place a -41118 N load in the FY direction on the top of the beam (Key point 2). Also apply a 4112 N load in the FX direction on Key point 2. This horizontal load will persuade the beam to buckle at the minimum buckling load.

For supporting condition column is pinned at its top and fixed at the bottom and subjected to an axial force, P and disturbance force, F

- Place -1192230N load in the FY direction on the top of the beam (keypoint 2).Also applied a 119223N load in the FX direction on Key point 2 at the center of the model.

Figure 6: Supporting Condition: Hollow cylinder free at the top and fixed at the bottom

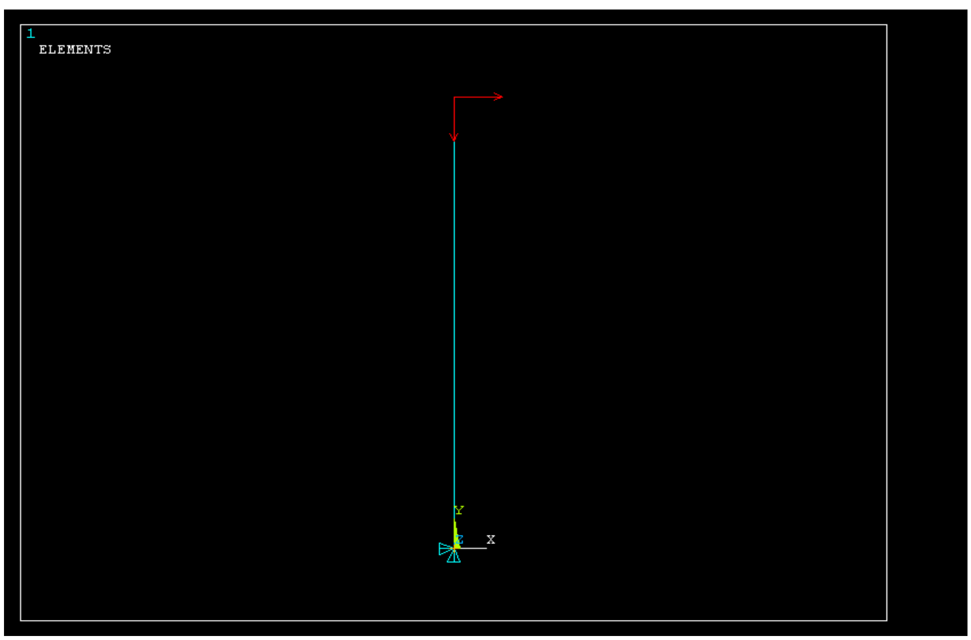
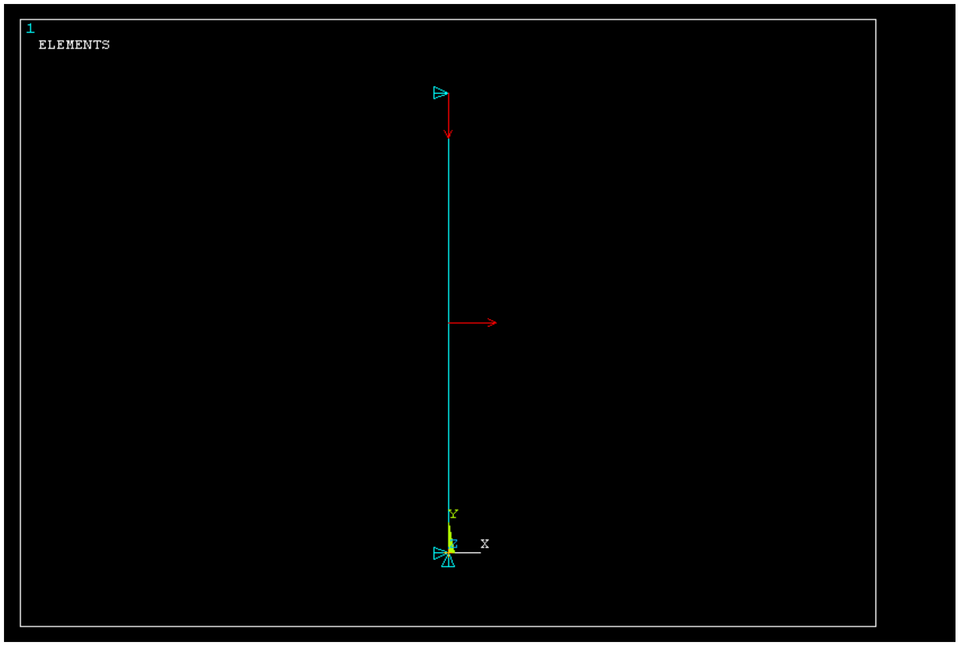


Figure 7: Supporting Condition: Hollow cylinder pinned at the top and fixed at the bottom



3.1.2.5 Solve the System

Solution > Solve > Current LS.

3.1.3 General Postprocessor: View Result

Plot Result > Deformed+Undeformed.

3.1.4 Time History Postprocessor: View deflection over time

CHAPTER 4

RESULTS AND DISCUSSION

4.0 RESULT AND DISCUSSION

4.1 View Simulation Results

Figure 8: Supporting Condition: Hollow cylinder free at the top and fixed at the bottom

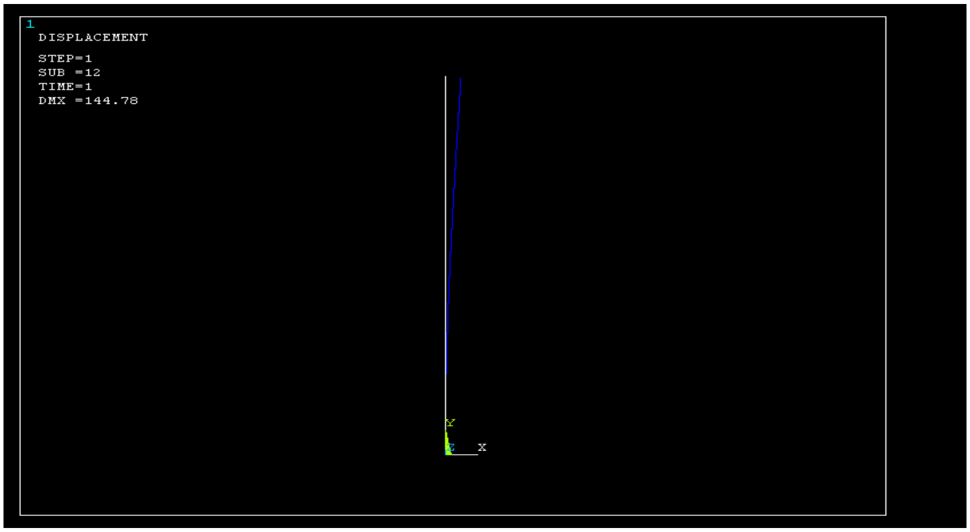


Figure 9: Graf deflection versus critical buckling load for hollow cylinder free at the top and fixed at the bottom

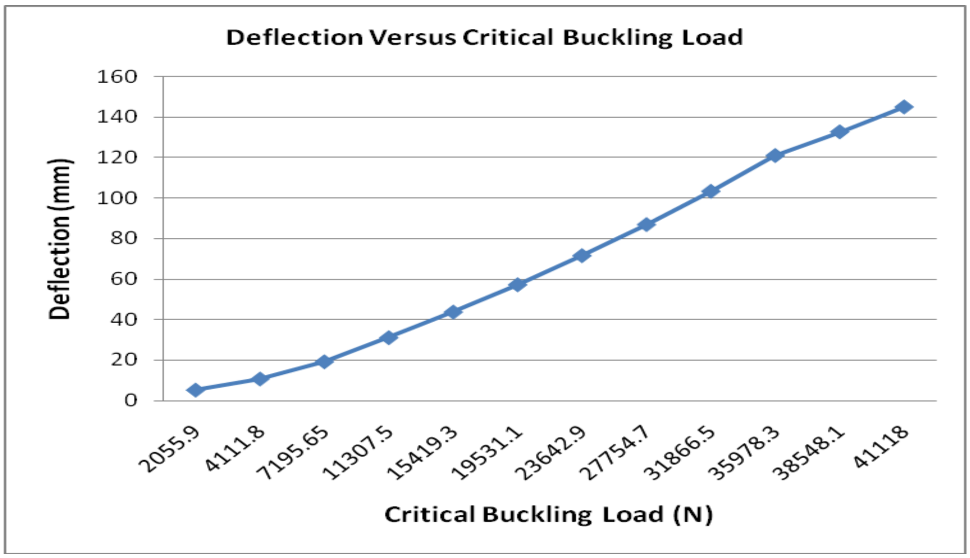


Figure 10: Deformed + undeformed shape for Hollow Cylinder pinned at the top and fixed at the bottom

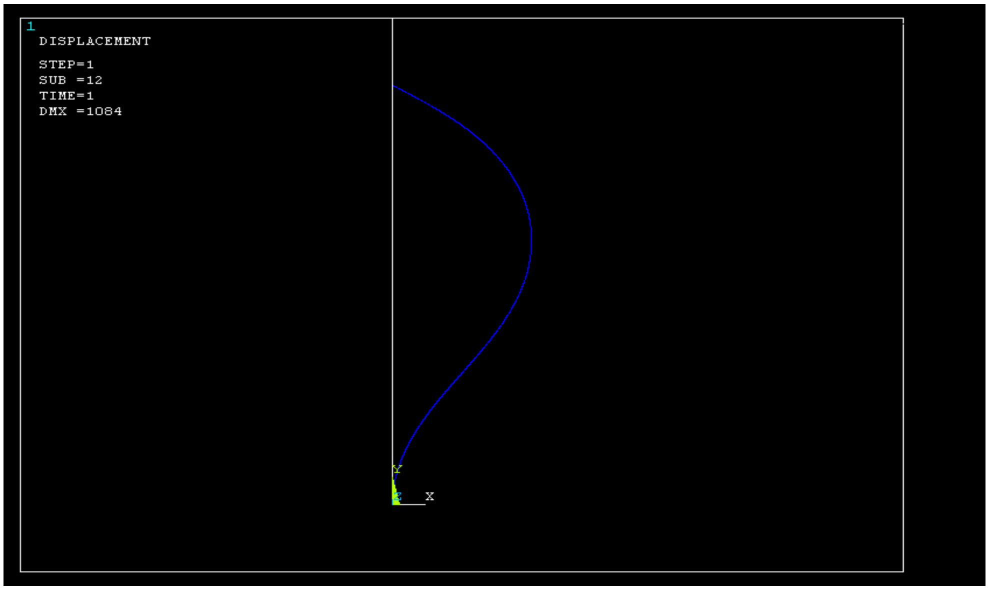
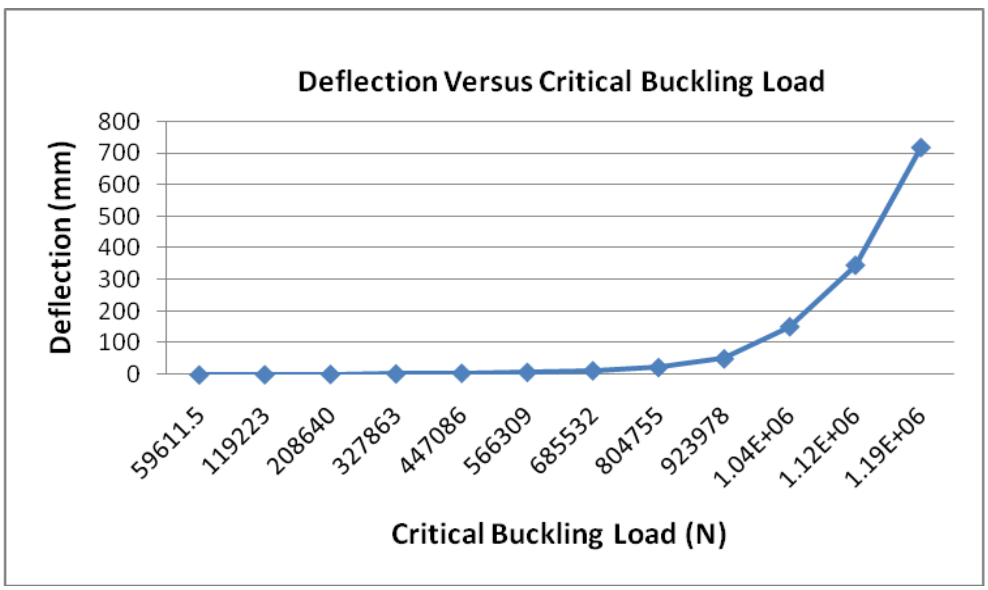


Figure 11: Graf deflection versus critical buckling load for Hollow Cylinder pinned at the top and fixed at the bottom



4.2 View Result in Tables

Table 3: Increase length of hollow cylinder maintain the thickness

Inner radius (mm)	Outer Radius (mm)	Moment Of Inertia (mm^4)	Cross Sectional Area (mm^2)	Length (mm)	Critical Stress (MPa)	Critical Buckling Load, P_{cr} (KN)	Horizontal Load (KN)	Maximum Deflection (mm)
70	75	5993079	2276	4500	250	41.118	4.112	144.744
70	75	5993079	2276	5000	250	37.336	3.734	189.102
70	75	5993079	2276	5500	250	36.321	3.632	266.182

Table 4: Increase length of hollow cylinder increase the thickness

Inner radius (mm)	Outer Radius (mm)	Moment Of Inertia (mm^4)	Cross Sectional Area (mm^2)	Length (mm)	Critical Stress (MPa)	Critical Buckling Load, P_{cr} (KN)	Horizontal Load (KN)	Maximum Deflection (mm)
70	75	5993079	2276	4500	250	41.118	4.112	144.744
70	80	13312499	4712	5000	250	77.720	7.772	171.999
70	90	32672564	10053	5500	250	154.849	15.485	184.333

Table 5: Increase thickness of hollow cylinder maintain the length

Inner radius (mm)	Outer Radius (mm)	Moment Of Inertia (mm^4)	Cross Sectional Area (mm^2)	Length (mm)	Critical Stress (MPa)	Critical Buckling Load, P_{cr} (KN)	Horizontal Load (KN)	Maximum Deflection (mm)
70	75	5993079	2276	4500	250	41.118	4.112	144.744
70	80	13312499	4712	4500	250	85.734	8.573	132.496
70	90	32672564	10053	4500	250	186.701	18.670	113.251

Table 6: Decrease length of hollow cylinder and increase thickness

Inner radius (mm)	Outer Radius (mm)	Moment Of Inertia (mm^4)	Cross Sectional Area (mm^2)	Length (mm)	Critical Stress (MPa)	Critical Buckling Load, P_{cr} (KN)	Horizontal Load (KN)	Maximum Deflection (mm)
70	75	5993079	2276	4500	250	41.118	4.112	144.744
70	80	13312499	4712	4000	250	95.575	9.5575	99.546
70	90	32672564	10053	3500	250	235.055	23.506	62.577

4.3 Discussion

4.3.1 Critical Buckling Load and Horizontal Load

The transition between stable and unstable conditions occurs at value of the axial force called the critical Load, Pcr.The critical buckling load can be found by considering the structure in the disturbed position and consider equilibrium. At this value the structure is in equilibrium regardless of the magnitude of small deflection it created to the column on lateral direction. P_{cr} represents the load for which the mechanism is on the verge of buckling and becoming unstable, so that a small horizontal load will cause the column to remain in the deflected position.

Any slight reduction in the axial load P from P_{cr} will allow column to straighten out, and any slight increase in P beyond P_{cr} will further increase in lateral. As the load increases, deflections and slopes of the median line of the column increase and yielding progresses from the base towards the free end for the case column fixed at the bottom and free at top.

For the First supporting condition where column is free at the top and fixed at the bottom eccentric load is applied at the centroid on the top which means there will be a bending moment and a curvature from the start. With any larger load, the least horizontal load would cause the column to buckle in a mode shape of a quarter sine wave with an indefinitely large displacement.

4.3.2 Increase length while maintaining thickness of Hollow Cylinder

When increase length of hollow cylinder while maintaining the thickness, critical buckling load decrease followed by increasing in maximum deflection .At length 4500 mm,the critical buckling load is 41.118 KN followed by 37.336 KN at length 5000 mm and 36.321KN at length 5500 mm.The maximum deflection along X-axis is increased from 144.744 mm at length 4500 mm followed by

189.102 mm at length 5000 mm and 266.182 mm at length 5500 mm.Based on critical buckling load formula for elastic stability limit given by Euler formula:

$$F_{cr} = \frac{EI\pi^2}{L^2} \quad (3)$$

Where E = young modulus of column material,I is moment of inertia of cross section,L is length of column.Critical buckling load decreases with the square of column length.

4.3.3 Increase Thickness and Length of Hollow Cylinder

When the thickness of hollow cylinder is increased as well as increasing length, the critical buckling load is increasing. At length 5000 mm, cross sectional area 4712 mm^2 , the critical buckling load is 77.720 KN followed by the critical load of 154.849 KN at area 10053 mm^2 and length of 5500 mm.

4.3.4 Increase thickness while maintaining length of Hollow Cylinder

When the thickness of the hollow cylinder is increasing while maintaining the length, the critical buckling load is increasing. For supporting condition fixed at the bottom and free at the end, at cross sectional area 2276 mm^2,the critical buckling load is 41.118 KN, followed by at 4712 mm^2 the critical buckling load is 85.734 KN and 186.701 KN is the critical buckling load at cross sectional 10053 mm^2.Ibrahim A. Assakkaf and Jaime F. Cárdenas-García [6] in his experiment proposed a reliability design of the doubler plates which are dominantly found in ship structures. A doubler plate is nothing but a plate that is added to top of the defective area and welded around the plate's perimeter.

Critical buckling strength of damaged column structure was estimated thorough Finite-difference and finite-element analysis and further evaluation was made on the buckling strength of the unstiffened panel doubler plate structure by placing the doubler plate at different locations within the unstiffened panel. They studied the effect of doubler location on the critical buckling strength of unstiffened plate (doubler on one side only).In the study, it was noted that presence of doublers on the plates has considerable effect on the buckling strength of the plate. Usually for larger doubler thickness, the buckling strength tends to increase. When the thickness of the doubler is less than the thickness of corrosion feature on the plate, the buckling strength is reduced.

4.3.5 Horizontal Load increased to 30% of Critical Buckling Load

As for the condition where the horizontal Load is increased to 30% of the critical buckling load, the result shows the same pattern as in the first condition when the geometric properties is varied. Taking an example of the result of critical buckling load when the thickness of hollow cylinder model is increased while maintaining length, the critical buckling load shows an increment but then the increase is small compared to the same condition without increasing the horizontal load to 30%.As for the maximum deflection the increment is small compared to when horizontal load is 10% of critical buckling load.

4.3.6 Hollow Cylinder fixed at the bottom and pinned at the top

Besides that, for hollow cylinder that is pinned at the top and fixed at bottom, the result shows the same pattern as in the first condition when different supporting condition is used. (see Appendix).When the length of hollow cylinder is increased while maintaining the thickness, the critical buckling load is decreasing. At length 4500 mm, critical buckling load is 1192.226 KN while maximum deflection is 718.603 mm. At length 5000 mm, critical buckling load is 96.570 KN with maximum deflection of 792.642 mm.At length 5500 mm, critical buckling load is 798.10 KN with maximum deflection of 874.844 mm.When increase the percentage of horizontal load to 30%, maximum deflection is higher than using 10% of horizontal load.

4.3.7 Using Aluminum as Material Properties

The critical Buckling load is decreasing for all condition when varying the geometric properties as well as increasing the critical buckling load to 30% of horizontal load. This is applicable for both supporting condition.Since Critical buckling load depends on stiffness of material. Stiffness is depending on Modulus of Elasticity. Aluminum has Modulus of elasticity which is 70GPa that is much lower compared to Steel which is 200 GPa.However for the supporting condition fixed at the bottom and pinned at the top, the maximum deflection similar to the first supporting condition when horizontal load is increased to 30% of critical buckling load by using Steel as Material properties.

4.3.8 Maximum Deflection Decrease with decrease in Slenderness Ratio

In terms of maximum deflection, the result shows when increasing the thickness maintaining the length at cross section 4712 mm^2 the maximum deflection is 132.496 mm, at 10053 mm^2 the maximum deflection is 113.251 mm.Maximum deflection decreases with decrease in slenderness ratio. Slender is a geometric concept of a two-dimensional area that is quantified by the radius of gyration. The radius of gyration, r, has the units of length and describes the way in which the area of a cross-section is distributed around its centroidal axis. If the area is concentrated far from the centroidal axis it will have a greater value of r and a greater resistance to buckling. The section tends to buckle around the axis with the smallest value. The radius of gyration is defined as:

$$r = \sqrt{(I / A)} \quad (4)$$

Where r = radius of gyration, I = area moment of inertia, and A = area of the cross-section.

CHAPTER 5

5.0 CONCLUSION AND RECOMMENDATION

5.1 Conclusion

The buckling of compressively-loaded members is one of the most important factors limiting the overall strength and stability of a structure. Normally when structural members are in compression, they will not fail except by crushing (exceeding their compressive yield strength), and fatigue does not occur for elements in compression. However if the geometry of the member is such that it is a column or hollow cylinder then buckling can occur. Buckling is particularly dangerous because it is a catastrophic failure that gives no warning. That is, the structural system collapses often resulting in total destruction of the system and unlike yielding failures, there may be no signs that the collapse is about to occur. Thus design engineers must be constantly on vigil against buckling failure.

5.2 Recommendation

In many cases, especially in aerospace applications, it may be more beneficial and sometimes absolutely necessary to resort to active methods to enhance the buckling load of slender structures. Active control can increase the load carrying capacity of a structure and piezoelectric materials are good actuators to provide this active control. In literature, Thomas Bailey and James E. Hubbard [9] have reported that distributed piezoelectric polymers bonded on the surface of the structure can be used to control the vibration of a cantilever beam. Thomson and Loughlan [10] have carried out experiments on composite column strips fabricated from commercially available carbon-epoxy pre-impregnated sheets and have demonstrated that an increase in load carrying capability of the order of about 20% to 37% is possible in slender columns. Rao and Singh [11] have proposed an enhancement of buckling load of a column by introducing a follower force paradigm and have shown that there is theoretically a possibility of increasing the buckling load by a factor of up to 3.5 in the case of a uniform cantilever column.

.

REFERENCES

[1] The Seventh SI Edition of Hibbler's Mechanics of Materials, SI Conversion by S.C.FAN, Nanyang Technological University

[2] http://www.mece.ualberta.ca/tutorials/ansys/CL/CIT/Buckling/Print.pdf

[3] Finite Element Analysis of Buried Pipeline Subjected to Buckling, Luis Bortolan Neto, Roberto Dalledone Machado, Mildred Ballin Hecke, Numerical Method in Engineering Graduation Programme

[4] Exact Solutions for Buckling of Structural Members C. M. Wang, C. Y. Wang, and J. N. Reddy CRC Press 2004

[5] Structures: a conspectus of main results," *ZAMM*, **80**(11–12), 709–732.Baˇzant, Z. P. and Cedolin, L. (1991), *Stability Structures. Elastic, Inelastic Fracture and Damage Theories,* Oxford University Press, New York.

[6] Ibrahim A. Assakkaf and Jaime F. Cárdenas-García, 2003, 'Reliability based design of the doubler plates for ship structures', Proceedings of the Fourth International Symposium on Uncertainty Modeling and Analysis.

[7] Budiansky, B. and Hu, P. C. (1946), "The Lagrangian multiplier method for finding upper and lower limits to critical stresses of clamped plates,"NACA Report No. 848, Washington, DC.

[8] DNV-RP-F110 Global Buckling of Submarine Pipeline-Structural Design due to high Temperature/high pressure

[9] Thomas Bailey and James E. Hubbard Jr, "Distributed Piezoelectric-Polymer Active Vibration Control of a Cantilever Beam," J. GUID-ANCE, vol. 8, no.5, SEPT.-OCT. 1985.

[10] S.P. Thompson and J. Loughlan, The active buckling control of composite column strips using piezoelectric actuators. *Compos. Struct.* **32** 1–4 (1995), pp. 59–67

[11] G.Venkateswara Rao and A.Gajbir Singh,A Smart Structure Concept for the Buckling Load Enhancement of Column, Smart Materials and Structure,Vol 10,843-845,2001

[12] Lee, B. K. and Mo, J. M., "Free vibrations and buckling loads of tapered beam-columns of regular

[13] Todhunter, I. and Pearson, K., A History of the Theory of Elasticity and of the Strength of Materials, Cambridge Press, 1886.

[14] Timoshenko, S.P. and Gere, J. M., Theory of Elastic Stability, McGraw-Hill, 1961.

[15] Lee, B.K. and S.J. Oh, "Elastica and buckling load of simple tapered column with constant volume," *International Journal of Solids and Structures,* Vol. 37, No. 18 2000, pp. 2507-2518.

[16] Tadjbakhsh, I. and Keller, J. B., "Strongest columns and isoperimetric inequalities for eigenvalues," *Journal of Applied mechanics, ASME*, Vol. 29, 1962, pp. 159-164.

[17] Keller, J. B., "The shape of the strongest column," *Archive for Rational Mechanics and Analysis*, Vol. 5, 1960, pp. 275-285.

[18] Cox, S. J. and Overton, M. I., "On the optimal design of columns against Buckling," *SIAM Journal on Mathematical Analysis*, Vol. 23, 1992, pp. 287-325.

[19]
http://inside.mines.edu/~jsteele/411/Lectures/L04StressStrain/L43Buckling.pdf

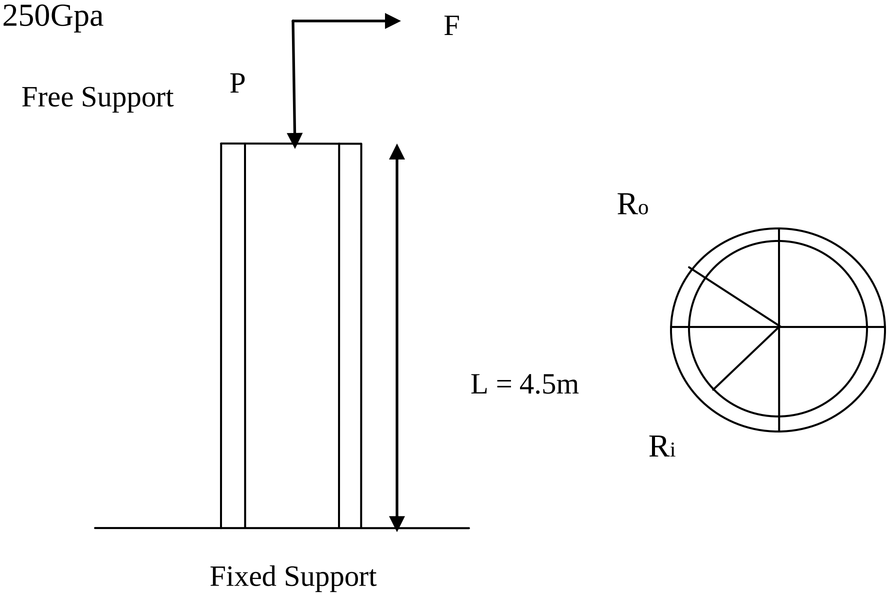
APPENDIX

Appendix A

Hollow Cylinder: Free at the top Fixed at the bottom

Inner radius: 70 mm   Outer Radius: 75 mm   Length: 4500 mm Elastic Modulus = 200GPa

Yield Strength = 250Gpa



Take F = 0.1P

$\sigma\, max$ = (P/A) + MC/I

= P/ $\pi(R0^2 - Ri^2)$ + [(FL) R0]/ [0.25 $\pi(R0^4 - Ri^4)$]

=P/ $\pi(0.075^2\text{m} - 0.070^2\text{ m})$ + [(FL) R0]/ [0.25 $\pi(0.075^4\text{ m} - 0.0070^4\text{m})$]

=P/ [0.002276] + [0.1P X 4.5 X0.075]/[5.993079E-6]

=439P+5631P=6070P

$\sigma\ cr = \sigma\, Y = 250Mpa$

$\sigma\ cr$ =250 =6070 P

P_{cr} = 41.186KN

Thus F = 4.1186KN

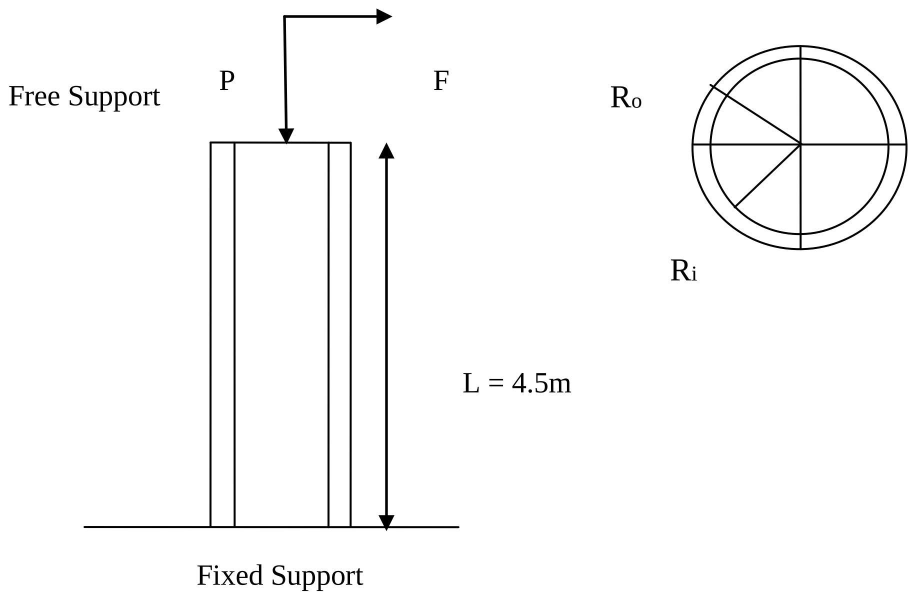
Appendix B

Hollow Cylinder: Free at the top Fixed at the bottom

Increase horizontal load 30% to critical buckling load

Inner radius: 70mm   Outer Radius: 75mm   Length: 4500 mm Elastic Modulus = 200GPa

Yield Strength = 250Gpa



Take F = 0.3P

$\sigma\ max$ = (P/A) + MC/I

= P/ $\pi(R0^2 - Ri^2)$ + [(FL) R0]/ [0.25 $\pi(R0^4 - Ri^4)$]

=P/ $\pi(0.075^2\text{m} - 0.070^2\text{ m})$ + [(FL) R0]/ [0.25 $\pi(0.075^4\text{ m} - 0.0070^4\text{m})$]

=P/ [0.002276] + [0.3P X 4.5 X0.075]/[5.993079E-6]

=439P+16894P = 17333P

$\sigma\ cr = \sigma\ Y = 250Mpa$

$\sigma\ cr$ =250 =17333 P

P_{cr} = 14.423KN

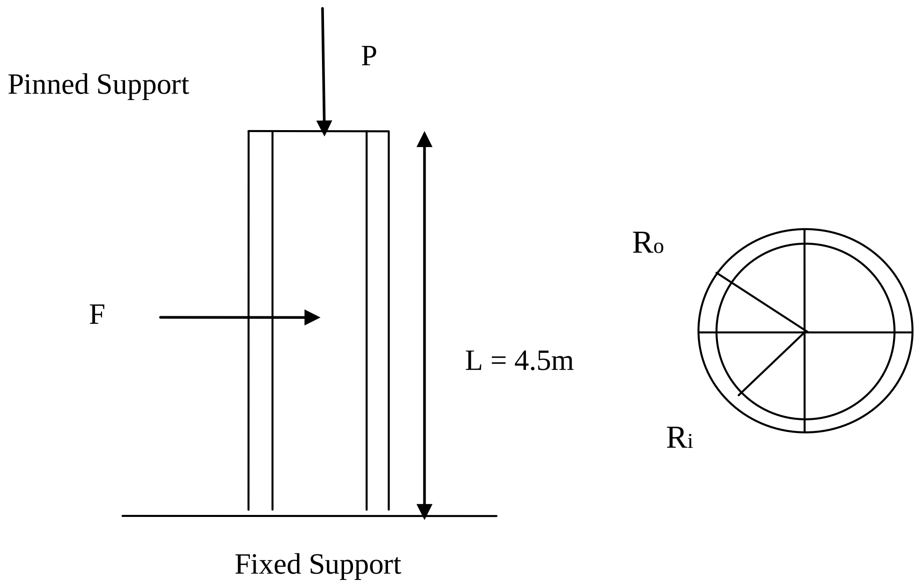
Thus F = 4.327KN

Appendix C

Hollow Cylinder: Pinned at the top Fixed at the bottom

Inner radius: 70 mm   Outer Radius: 75 mm   Length: 4500 mm Elastic Modulus = 200GPa

Yield Strength = 250Gpa



Take F = 0.1P

P_{cr} $= \pi^2 EI / (KL)^2$

$= \pi^2(200(10)^3(5993079)/ (0.7 \text{ X } 4500)^2$

=1192.226KN

F = 119.2226KN

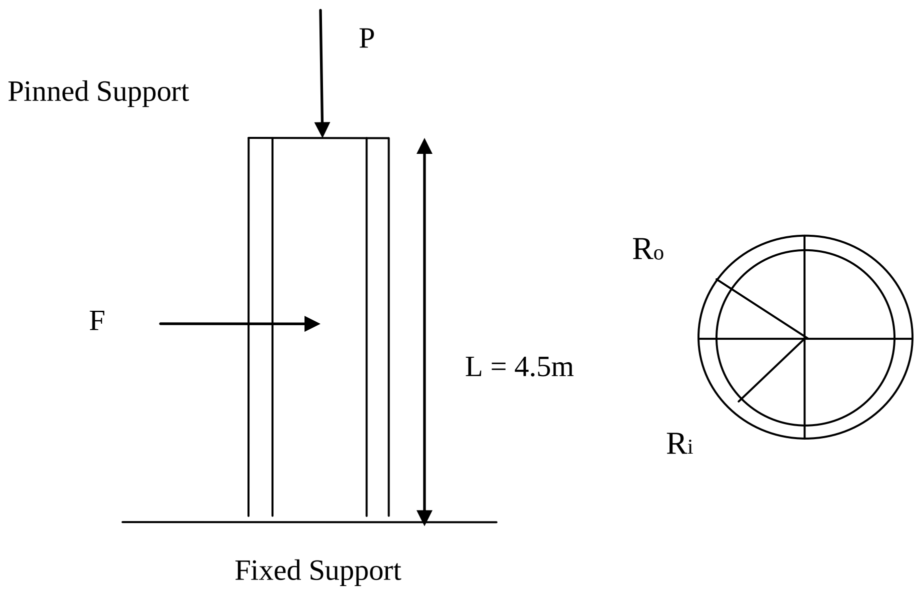
Appendix D

Hollow Cylinder: Pinned at the top Fixed at the bottom

Increase horizontal load 30% to critical buckling load

Inner radius: 70 mm   Outer Radius: 75 mm   Length: 4500 mm Elastic Modulus = 200GPa

Yield Strength = 250Gpa



Take F = 0.3P

P_{cr} = π^2 EI / $(KL)^2$

= π^2(200$(10)^3$(5993079)/ (0.7 X 4500)^2

=1192.226KN

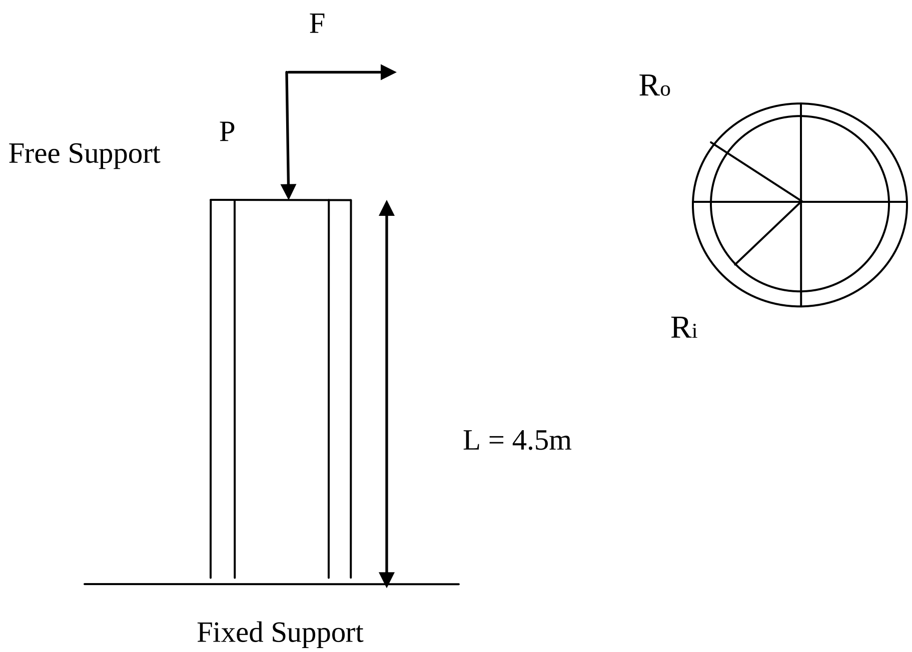
F = 357.668KN

Appendix E

Hollow Cylinder: Free at the top Fixed at the bottom

Inner radius: 70mm   Outer Radius: 75mm   Length: 4500 mm Elastic Modulus = 70GPa

Yield Strength = 190Gpa



Take F = 0.1P

$\sigma\ max$ = (P/A) + MC/I

= P/ $\pi(R0^2 - Ri^2)$ + [(FL) R0]/ [0.25 $\pi(R0^4 - Ri^4)$]

=P/ $\pi(0.075^2\text{m} - 0.070^2\text{ m})$ + [(FL) R0]/ [0.25 $\pi(0.075^4\text{ m} - 0.0070^4\text{m})$]

=P/ [0.002276] + [0.1P X 4.5 X0.075]/[5.993079E-6]

=439P+5631P = 6070P

$\sigma\ cr = \sigma Y = 190\ Mpa$

$\sigma\ cr$ =250 =6070 P

P_{cr} = 31.301KN

Thus F = 3.1301KN

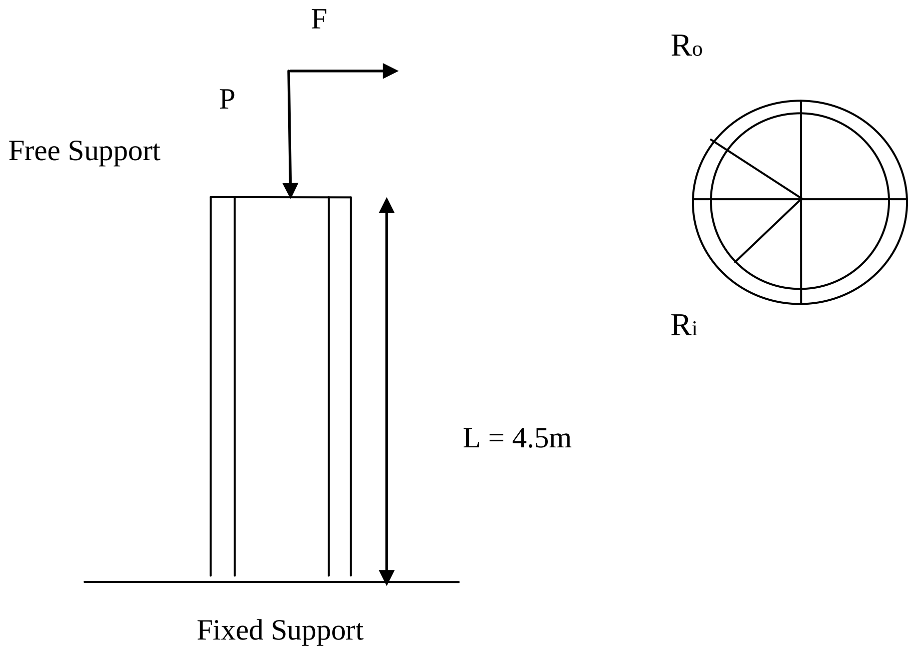
Appendix F

Hollow Cylinder: Free at the top Fixed at the bottom

Increase horizontal load 30% to critical buckling load

Inner radius: 70 mm Outer Radius: 75mm Length: 4500 mm Elastic Modulus = 70GPa

Yield Strength = 190 Gpa



Take F = 0.3P

$\sigma\ max$ = (P/A) + MC/I

= P/ $\pi(R0^2 - Ri^2)$ + [(FL) R0]/ [0.25 $\pi(R0^4 - Ri^4)$]

=P/ $\pi(0.075^2\text{m} - 0.070^2\text{ m})$ + [(FL) R0]/ [0.25 $\pi(0.075^4\text{ m} - 0.0070^4\text{m})$]

=P/ [0.002276] + [0.3P X 4.5 X0.075]/[5.993079E-6]

=439P+16894P = 17333P

$\sigma\ cr = \sigma Y = 190\ Mpa$

$\sigma\ cr$ =190 =17333 P

P_{cr} = 10.962KN

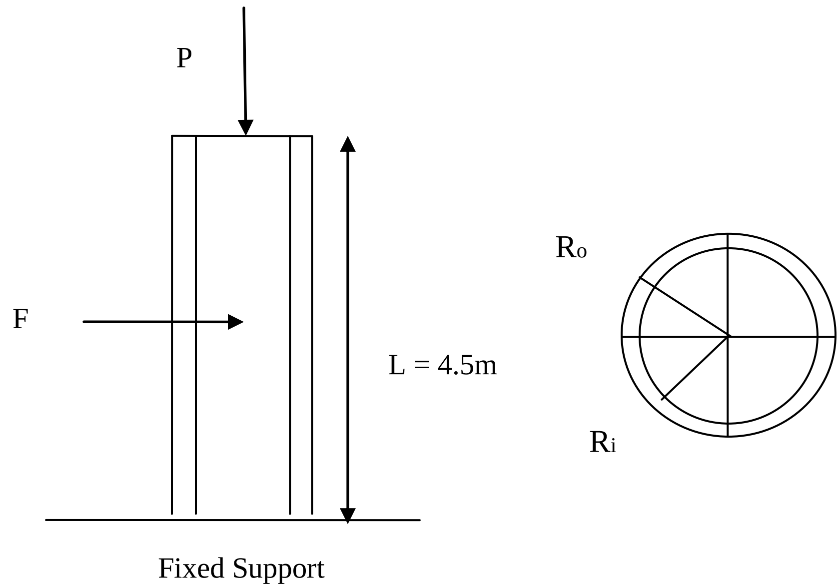
Thus F = 3.289KN

Appendix G

Hollow Cylinder: Pinned at the top Fixed at the bottom

Inner radius: 70mm   Outer Radius: 75mm   Length: 4500 mm Elastic Modulus = 70GPa

Yield Strength = 190Gpa



Take F = 0.1P

P_{cr} = $\pi^2 EI / (KL)^2$

= $\pi^2(70(10)^3(5993079)/ (0.7 \text{ X } 4500)^2$

=417.279KN

F = 41.7279KN

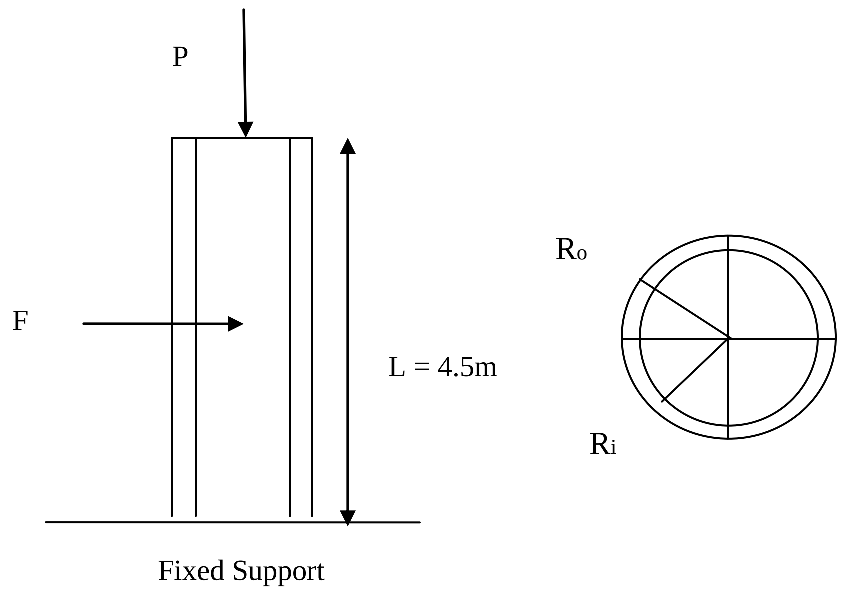
Appendix H

Hollow Cylinder: Pinned at the top Fixed at the bottom

Increase horizontal load 30% to critical buckling load

Inner radius: 70mm Outer Radius: 75mm Length: 4500 mm Elastic Modulus = 70GPa

Yield Strength = 190Gpa



Take F = 0.3P

$P_{cr} = \pi^2 EI / (KL)^2$

$= \pi^2 (70(10)^3 (5993079) / (0.7 \text{ X } 4.5)^2$

$= 417.279 KN$

F = 125.184KN

Appendix I

Hollow Cylinder with supporting condition fixed at the bottom and free at the top, material properties used is steel with Elastic Modulus =200GPa

Increase percentage of horizontal Load to 30% of critical buckling load

Table A 1: Increase length of hollow cylinder maintain the thickness

Inner radius (mm)	Outer Radius (mm)	Moment Of Inertia (mm^4)	Cross Sectional Area (mm^2)	Length (mm)	Critical Stress (MPa)	Critical Buckling Load, P_{cr} (KN)	Horizontal Load (KN)	Maximum Deflection (mm)
70	75	5993079	2276	4500	250	14.423	4.327	121.704
70	75	5993079	2276	5000	250	13.013	3.904	152.478
70	75	5993079	2276	5500	250	11.855	3.557	187.188

Table A 2: Increase thickness of hollow cylinder maintain length

Inner radius (mm)	Outer Radius (mm)	Moment Of Inertia (mm^4)	Cross Sectional Area (mm^2)	Length (mm)	Critical Stress (MPa)	Critical Buckling Load, P_{cr} (KN)	Horizontal Load (KN)	Maximum Deflection (mm)
70	75	5993079	2276	4500	250	14.423	4.327	121.704
70	80	13312499	4712	4500	250	30.031	9.009	113.167
70	90	32672564	10053	4500	250	65.476	19.643	99.567

Table A 3: Increase thickness of hollow cylinder and increase lenght

Inner radius (mm)	Outer Radius (mm)	Moment Of Inertia (mm^4)	Cross Sectional Area (mm^2)	Length (mm)	Critical Stress (MPa)	Critical Buckling Load, Pcr (KN)	Horizontal Load (KN)	Maximum Deflection (mm)
70	75	5993079	2276	4500	250	14.423	4.327	121.704
70	80	13312499	4712	5000	250	27.097	8.129	141.672
70	90	32672564	10053	5500	250	53.826	16.148	91.644

Table A 4: Decrease length of hollow cylinder and increase thickness

Inner radius (mm)	Outer Radius (mm)	Moment Of Inertia (mm^4)	Cross Sectional Area (mm^2)	Length (mm)	Critical Stress (MPa)	Critical Buckling Load, Pcr (KN)	Horizontal Load (KN)	Maximum Deflection (mm)
70	75	5993079	2276	4500	250	14.423	4.327	121.704
70	80	13312499	4712	4000	250	33.678	10.103	88.134
70	90	32672564	10053	3500	250	83.562	25.069	58.615

Hollow Cylinder with supporting condition fixed at the bottom and pinned at the top, material properties used is steel with Elastic Modulus =200GPa

Percentage of horizontal Load 10% of critical buckling load

Table A 5: Increase length of hollow cylinder maintain the thickness

Inner radius (mm)	Outer Radius (mm)	Moment Of Inertia (mm^4)	Cross Sectional Area (mm^2)	Length (mm)	Critical Buckling Load, Pcr (KN)	Horizontal Load (KN)	Maximum Deflection (mm)
70	75	5993079	2276	4500	1192.226	119.223	718.603
70	75	5993079	2276	5000	965.703	96.570	796.642
70	75	5993079	2276	5500	798.102	79.810	874.844

Table A 6: Increase thickness of hollow cylinder maintain length

Inner radius (mm)	Outer Radius (mm)	Moment Of Inertia (mm^4)	Cross Sectional Area (mm^2)	Length (mm)	Critical Buckling Load, Pcr (KN)	Horizontal Load (KN)	Maximum Deflection (mm)
70	75	5993079	2276	4500	1192.226	119.223	718.603
70	80	13312499	4712	4500	2648.306	264.831	710.214
70	90	32672564	10053	4500	6499.678	649.968	720.892

Table A 7 : Increase thickness of hollow cylinder and increase lenght

Inner radius (mm)	Outer Radius (mm)	Moment Of Inertia (mm^4)	Cross Sectional Area (mm^2)	Length (mm)	Critical Buckling Load, Pcr (KN)	Horizontal Load (KN)	Maximum Deflection (mm)
70	75	5993079	2276	4500	1192.226	119.223	718.603
70	80	13312499	4712	5000	2145.128	214.513	787.166
70	90	32672564	10053	5500	4351.024	435.102	876.826

Table A 8: Decrease length of hollow cylinder and increase thickness

Inner radius (mm)	Outer Radius (mm)	Moment Of Inertia (mm^4)	Cross Sectional Area (mm^2)	Length (mm)	Critical Buckling Load, Pcr (KN)	Horizontal Load (KN)	Maximum Deflection (mm)
70	75	5993079	2276	4500	1192.226	119.223	718.603
70	80	13312499	4712	4000	3351.763	335.176	633.460
70	90	32672564	10053	3500	10744.365	1074.437	566.049

Percentage of horizontal Load 30% of critical buckling load

Table A 9: Increase length of hollow cylinder maintain the thickness

Inner radius (mm)	Outer Radius (mm)	Moment Of Inertia (mm^4)	Cross Sectional Area (mm^2)	Length (mm)	Critical Buckling Load, Pcr (KN)	Horizontal Load (KN)	Maximum Deflection (mm)
70	75	5993079	2276	4500	1192.226	357.668	1303.000
70	75	5993079	2276	5000	965.703	289.711	1447.000
70	75	5993079	2276	5500	798.102	239.431	1590.000

Table A 10: Increase thickness of hollow cylinder maintain length

Inner radius (mm)	Outer Radius (mm)	Moment Of Inertia (mm^4)	Cross Sectional Area (mm^2)	Length (mm)	Critical Buckling Load, Pcr (KN)	Horizontal Load (KN)	Maximum Deflection (mm)
70	75	5993079	2276	4500	1192.226	357.668	1303.000
70	80	13312499	4712	4500	2648.306	794.492	1296.000
70	90	32672564	10053	4500	6499.678	1949.903	1305.000

Table A 11: Increase thickness of hollow cylinder and increase lenght

Inner radius (mm)	Outer Radius (mm)	Moment Of Inertia (mm^4)	Sectiona l Area (mm^2)	Length (mm)	Critical Buckling Load, Pcr (KN)	Horizontal Load (KN)	Maximum Deflection (mm)
70	75	5993079	2276	4500	1192.226	357.668	1303.000
70	80	13312499	4712	5000	2145.128	643.538	1438.000
70	90	32672564	10053	5500	4351.024	1305.307	1592.000

Table A 12: Decrease length of hollow cylinder and increase thickness

Inner radius (mm)	Outer Radius (mm)	Moment Of Inertia (mm^4)	Cross Sectiona l Area (mm^2)	Length (mm)	Critical Buckling Load, Pcr (KN)	Horizontal Load (KN)	Maximum Deflection (mm)
70	75	5993079	2276	4500	1192.226	357.668	1303.000
70	80	13312499	4712	4000	3351.763	1005.529	1153.000
70	90	32672564	10053	3500	10744.365	3223.310	1019.000

Hollow Cylinder with supporting condition fixed at the bottom and free at the top, material properties used is steel with Elastic Modulus =70 GPa

Percentage of horizontal Load 10% of critical buckling load

Table A 13: Increase length of hollow cylinder maintain the thickness

Inner radius (mm)	Outer Radius (mm)	Moment Of Inertia (mm^4)	Cross Sectional Area (mm^2)	Length (mm)	Critical Stress (MPa)	Critical Buckling Load, Pcr (KN)	Horizontal Load (KN)	Maximum Deflection (mm)
70	75	5993079	2276	4500	190	31.301	3.130	570.309
70	75	5993079	2276	5000	190	28.375	2.838	655.289
70	75	5993079	2276	5500	190	25.949	2.595	732.331

Table A 14: Increase thickness of hollow cylinder maintain length

Inner radius (mm)	Outer Radius (mm)	Moment Of Inertia (mm^4)	Cross Sectional Area (mm^2)	Length (mm)	Critical Stress (MPa)	Critical Buckling Load, Pcr (KN)	Horizontal Load (KN)	Maximum Deflection (mm)
70	75	5993079	2276	4500	190	31.301	3.130	570.309
70	80	13312499	4712	4500	190	65.158	6.516	488.082
70	90	32672564	10053	4500	190	141.953	14.1953	380.056

Table A 15: Increase thickness of hollow cylinder and increase lenght

Inner radius (mm)	Outer Radius (mm)	Moment Of Inertia (mm^4)	Cross Sectional Area (mm^2)	Length (mm)	Critical Stress (MPa)	Critical Buckling Load, Pcr (KN)	Horizontal Load (KN)	Maximum Deflection (mm)
70	75	5993079	2276	4500	190	31.301	3.130	570.309
70	80	13312499	4712	5000	190	59.061	5.906	709.599
70	90	32672564	10053	5500	190	49.771	4.977	813.562

Table A 16: Decrease length of hollow cylinder and increase thickness

Inner radius (mm)	Outer Radius (mm)	Moment Of Inertia (mm^4)	Cross Sectional Area (mm^2)	Length (mm)	Critical Stress (MPa)	Critical Buckling Load, Pcr (KN)	Horizontal Load (KN)	Maximum Deflection (mm)
70	75	5993079	2276	4500	190	31.301	3.130	570.309
70	80	13312499	4712	4000	190	72.630	7.263	331.751
70	90	32672564	10053	3500	190	178.642	17.864	181.508

Increase Percentage of horizontal Load to 30% of critical buckling load

Table A 17: Increase length of hollow cylinder maintain the thickness

Inner radius (mm)	Outer Radius (mm)	Moment Of Inertia (mm^4)	Cross Sectional Area (mm^2)	Length (mm)	Critical Stress (MPa)	Critical Buckling Load, Pcr (KN)	Horizontal Load (KN)	Maximum Deflection (mm)
70	75	5993079	2276	4500	190	10.961	3.288	301.741
70	75	5993079	2276	5000	190	9.890	0.989	352.173
70	75	5993079	2276	5500	190	9.010	0.901	445.321

Table A 18: Increase thickness of hollow cylinder maintain length

Inner radius (mm)	Outer Radius (mm)	Moment Of Inertia (mm^4)	Cross Sectional Area (mm^2)	Length (mm)	Critical Stress (MPa)	Critical Buckling Load, Pcr (KN)	Horizontal Load (KN)	Maximum Deflection (mm)
70	75	5993079	2276	4500	190	10.961	3.288	301.741
70	80	13312499	4712	4500	190	22.823	6.847	277.911
70	90	32672564	10053	4500	190	49.758	14.927	240.777

Table A 19: Increase thickness of hollow cylinder and increase lenght

Inner radius (mm)	Outer Radius (mm)	Moment Of Inertia (mm^4)	Cross Sectional Area (mm^2)	Length (mm)	Critical Stress (MPa)	Critical Buckling Load, Pcr (KN)	Horizontal Load (KN)	Maximum Deflection (mm)
70	75	5993079	2276	4500	190	10.961	3.288	301.741
70	80	13312499	4712	5000	190	20.594	6.178	353.618
70	90	32672564	10053	5500	190	40.909	12.273	379.450

Table A 20: Decrease length of hollow cylinder and increase thickness

Inner radius (mm)	Outer Radius (mm)	Moment Of Inertia (mm^4)	Cross Sectional Area (mm^2)	Length (mm)	Critical Stress (MPa)	Critical Buckling Load, Pcr (KN)	Horizontal Load (KN)	Maximum Deflection (mm)
70	75	5993079	2276	4500	190	10.961	3.288	301.741
70	80	13312499	4712	4000	190	25.596	7.679	213.066
70	90	32672564	10053	3500	190	63.514	19.054	154.442

Hollow Cylinder with supporting condition fixed at the bottom and pinned at the top, material properties used is steel with Elastic Modulus =70 GPa

Percentage of horizontal Load 10% of critical buckling load

Table A 21: Increase length of hollow cylinder maintain the thickness

Inner radius (mm)	Outer Radius (mm)	Moment Of Inertia (mm^4)	Cross Sectional Area (mm^2)	Length (mm)	Critical Buckling Load, Pcr (KN)	Horizontal Load (KN)	Maximum Deflection (mm)
70	75	5993079	2276	4500	417.279	41.728	718.601
70	75	5993079	2276	5000	337.996	33.800	796.647
70	75	5993079	2276	5500	279.336	27.9336	874.861

Table A 22: Increase thickness of hollow cylinder maintain length

Inner radius (mm)	Outer Radius (mm)	Moment Of Inertia (mm^4)	Cross Sectional Area (mm^2)	Length (mm)	Critical Buckling Load, Pcr (KN)	Horizontal Load (KN)	Maximum Deflection (mm)
70	75	5993079	2276	4500	417.279	41.728	718.601
70	80	13312499	4712	4500	926.907	92.691	710.204
70	90	32672564	10053	4500	2274.887	227.489	720.883

Table A 23: Increase thickness of hollow cylinder and increase lenght

Inner radius (mm)	Outer Radius (mm)	Moment Of Inertia (mm^4)	Cross Sectional Area (mm^2)	Length (mm)	Critical Buckling Load, Pcr (KN)	Horizontal Load (KN)	Maximum Deflection (mm)
70	75	5993079	2276	4500	417.279	41.728	718.601
70	80	13312499	4712	5000	750.794	75.080	787.161
70	90	32672564	10053	5500	1522.858	152.286	876.817

Table A 24: Decrease length of hollow cylinder and increase thickness

Inner radius (mm)	Outer Radius (mm)	Moment Of Inertia (mm^4)	Cross Sectional Area (mm^2)	Length (mm)	Critical Buckling Load, Pcr (KN)	Horizontal Load (KN)	Maximum Deflection (mm)
70	75	5993079	2276	4500	417.279	41.728	718.601
70	80	13312499	4712	4000	1173.116	117.312	633.456
70	90	32672564	10053	3500	3760.528	376.053	566.039

Increase Percentage of horizontal Load to 30% of critical buckling load

Table A 25: Increase length of hollow cylinder maintain the thickness

Inner radius (mm)	Outer Radius (mm)	Moment Of Inertia (mm^4)	Cross Sectional Area (mm^2)	Length (mm)	Critical Buckling Load, Pcr (KN)	Horizontal Load (KN)	Maximum Deflection (mm)
70	75	5993079	2276	4500	417.279	125.184	1303.000
70	75	5993079	2276	5000	337.996	101.400	1447.000
70	75	5993079	2276	5500	279.336	83.801	1591.000

Table A 26 : Increase thickness of hollow cylinder maintain length

Inner radius (mm)	Outer Radius (mm)	Moment Of Inertia (mm^4)	Cross Sectional Area (mm^2)	Length (mm)	Critical Buckling Load, Pcr (KN)	Horizontal Load (KN)	Maximum Deflection (mm)
70	75	5993079	2276	4500	417.279	125.184	1303.000
70	80	13312499	4712	4500	926.907	278.072	1296.000
70	90	32672564	10053	4500	2274.887	682.466	1305.000

Table A 27: Increase thickness of hollow cylinder and increase lenght

Inner radius (mm)	Outer Radius (mm)	Moment Of Inertia (mm^4)	Cross Sectiona l Area (mm^2)	Length (mm)	Critical Buckling Load, Pcr (KN)	Horizontal Load (KN)	Maximum Deflection (mm)
70	75	5993079	2276	4500	417.279	125.184	1303.000
70	80	13312499	4712	5000	750.794	225.238	1439.000
70	90	32672564	10053	5500	1522.858	456.856	1594.000

Table A 28: Decrease length of hollow cylinder and increase thickness

Inner radius (mm)	Outer Radius (mm)	Moment Of Inertia (mm^4)	Cross Sectional Area (mm^2)	Length (mm)	Critical Buckling Load, Pcr (KN)	Horizontal Load (KN)	Maximum Deflection (mm)
70	75	5993079	2276	4500	417.279	125.184	1303.000
70	80	13312499	4712	4000	1173.116	351.935	1154.000
70	90	32672564	10053	3500	3760.528	1128.158	1019.000