Final Year Project II

Dissertation

Stochastic Programming with Economic and Operational Risk Management in Petroleum Refinery Planning under Uncertainty

by

Yam Chee Wai

10477

Chemical Engineering

Supervisor: Khor Cheng Seong

Universiti Teknologi PETRONAS

Bandar Seri Iskandar

31750 Tronoh

Perak Darul Ridzuan

CERTIFICATION OF APPROVAL

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10477

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____

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CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

Yam Chee Wai

ABSTRACT

Rising crude oil price and global energy concerns have revived great interests in the oil and gas industry, including the optimization of oil refinery operations. However, the economic environment of the refining industry is typically one of low margins with intense competition. This state of the industry calls for a continuous improvement in operating efficiency by reducing costs through business-driven engineering strategies. These strategies are derived based on an acute understanding of the world energy market and business processes, with the incorporation of advanced financial modeling and computational tools. With regards to this present situation, this work proposes the application of the two-stage stochastic programming approach with fixed recourse to effectively account for both economic and operational risk management in the planning of oil refineries under uncertainty. The scenario analysis approach is adopted to consider uncertainty in three parameters: prices of crude oil and commercial products, market demand for products, and production yields. However, a large number of scenarios are required to capture the probabilistic nature of the problem. Therefore, to circumvent the problem posed by the resulting large-scale model, a Monte Carlo simulation approach is implemented based on the sample average approximation (SAA) technique. The SAA technique enables the determination of the minimum number of scenarios required yet still able to compute the true optimal solution of the problem for a desired level of accuracy within the specified confidence intervals. We consider Conditional Value-at-Risk (CVaR) as the risk metric to hedge against the three parameters of uncertainty, which affords a framework that also involves the use of the Value-at-Risk (VaR) measure. We adopt two approaches in formulating appropriate two-stage stochastic programs with mean–CVaR objective function. The first approach is by using the conventional definition of CVaR that leads to a linear optimization model approximation coupled with a graphical-based solution strategy to determine the value of VaR using SAA in order to arrive at the optimal solution. The second approach is to utilize auxiliary variables to formulate a suite of stochastic linear programs with CVaR-based constraints. We conduct computational studies on a representative refinery

planning problem to investigate the various model formulations using GAMS/CPLEX and offer some remarks about the merits of these formulations.

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ABBREVIATIONS AND NOTATIONS

Sets and Indices

I'	set of materials or products i
J	set of processes j
T	set of time periods t
S	set of scenarios s
K	set of products with yield uncertainty k

Deterministic Parameters

$d_{i,t,s}$, $d^{\mathrm{L}}_{i,t,s}$, $d^{\mathrm{U}}_{i,t,s}$	demand for product i in period t per realization of scenario s, with its corresponding constant lower (superscript L) and upper (superscript U) bounds
P_t	amount of crude oil purchased in period t
p^{L}_t, p^{U}_t	lower and upper bounds of the availability of crude oil during period t
$I^{\mathrm{fmin}}_{i,t}$, $I^{\mathrm{fmax}}_{i,t}$	minimum and maximum required amount of inventory for material i at the end of period t
$b_{i,j}$	yield coefficient for material i in process j
$\gamma_{i,t}$	unit sales price of product type i in period t
$\lambda_{i,t}$	unit purchase price of crude oil type i in period t
$\tilde{\lambda}_{i,t}$	value of the starting inventory of material i in period t
$\tilde{\gamma}_{i,t}$	value of the final inventory of material i in period t
$C_{j,t}$	operating cost of process j in period t
$h_{i,t}$	unit cost of subcontracting or outsourcing the production of product type i in period t
r_t, o_t	cost per man-hour of regular and overtime labour in period t, respectively
$\alpha_{j,t}$	variable-size cost coefficient for the investment cost of capacity expansion of process j in period t
$\beta_{j,t}$	fixed-cost charge for the investment cost of capacity expansion of process j in period t
θ_1, θ_2, θ_3	risk factors or weighting factors (weights) for multiobjective optimization procedure

Stochastic Parameters

p_s	probability of scenario s
$\gamma_{i,s,t}$	unit sales price of product type i in period t per realization of scenario s
λ_t	unit purchase price of crude oil in period t per realization of scenario s
$d_{i,s,t}$	demand for product i in time period t per realization of scenario s

Recourse Parameters

c^{+}_{i}	fixed penalty cost per unit demand $d_{i,s}$ of underproduction (shortfall) of product i per realization of scenario s (also the cost of lost demand)
c^{-}_{i}	fixed penalty cost per unit demand $d_{i,s}$ of overproduction (surplus) of product i per realization of scenario s (also the cost of inventory to store production surplus)
$q^{+}_{i,k}$	fixed unit penalty cost for shortage in yields from material i for product k
$q^{-}_{i,k}$	fixed unit penalty cost for excess in yields from material i for product k

Deterministic Variables (First-Stage Decision Variables)

$x_{j,t}$	production capacity of process j during period t
$x_{j,t-1}$	production capacity of process j during period $t - 1$
$CE_{j,t}$, $CE^{\mathrm{L}}_{j,t}$, $CE^{\mathrm{U}}_{j,t}$	capacity expansion of the plant for process j that is installed in period t, with its corresponding constant lower (superscript L) and upper (superscript U) bounds

$y_{j,t}$	binary decision variable that equals one (1) if there is an expansion for process *j* at the beginning of period *t*, and zero (0) otherwise
$S_{i,t}$	amount of product *i* sold in period *t*
$L_{i,t}$	amount of lost demand for product *i* in period *t*
$P_{i,t}$	amount of crude oil purchased in period *t*
$I_{i,t}^{\mathrm{s}}$, $I_{i,t}^{\mathrm{f}}$	initial and final amount of inventory of material *i* in period *t*
$H_{i,t}$	amount of product type *i* to be subcontracted or outsourced in period *t*
R_t, O_t	regular and overtime working or production hours in period *t*, respectively

Stochastic Recourse Variables (Second-Stage Decision Variables)

$z_{i,s}^{+}$	amount of unsatisfied demand for product *i* due to underproduction per realization of scenario *s*
$z_{i,s}^{-}$	amount of excess product *i* due to overproduction per realization of scenario *s*
$y_{i,k,s}^{+}$	amount of shortage in yields from material *i* for product type *k* per realization of scenario *s*
$y_{i,k,s}^{-}$	amount of excess in yields from material *i* for product type *k* per realization of scenario *s*
$\mathrm{VaR}_{\mathrm{p}}$	Value-at-Risk for uncertainty due to price
$u_{\mathrm{p},s}$	auxiliary variable for uncertainty due to price for scenario *s*
δ_{p}	user-specified risk aversion parameter for uncertainty due to price
c_i	fixed penalty cost of product *i*
x_{i}	Product *i* flowrate for price
$\mathrm{VaR}_{\mathrm{d\text{-}y}}$	Value-at-Risk for uncertainty due to demand and yield
$u_{\mathrm{d\text{-}y},s}$	auxiliary variable for uncertainty due to demand and yield for scenario *s*
$\delta_{\mathrm{d\text{-}y}}$	user-specified risk aversion parameter for uncertainty due to demand yield
$d_{i,}$	Product *i* flowrate for demand
a_{i}	Product *i* flowrate for yield
$c_{i,s}$	fixed penalty cost in price of product *i* per realization of scenario *s*
$q_{i,k}$	fixed unit penalty cost in yields from material *i* for product *k*
$z_{i,s,k}$	Product *i* flowrate for demand per realization of scenario *s* for product *k*
$r_{i,m}$	fixed unit penalty cost in demand from material *i* for product *m*
$y_{i,s,m}$	Product *i* flowrate for demand per realization of scenario *s* for product m
$\mathrm{VaR}_{\mathrm{d}}^{k_1}$	Value-at-Risk for uncertainty due to demand shortfall
$\mathrm{VaR}_{\mathrm{d}}^{k_2}$	Value-at-Risk for uncertainty due to demand surplus
$\mathrm{VaR}_{\mathrm{y}}^{m_1}$	Value-at-Risk for uncertainty due to yield shortfall
$\mathrm{VaR}_{\mathrm{y}}^{m_2}$	Value-at-Risk for uncertainty due to yield surplus
q_{i,k_1}	fixed unit penalty cost in yields shortfall from material *i* for product *k*
q_{i,k_2}	fixed unit penalty cost in yields surplus from material *i* for product *k*
z_{i,s,k_1}	Product *i* flowrate for demand shortfall per realization of scenario *s* for product *k*
z_{i,s,k_2}	Product *i* flowrate for demand surplus per realization of scenario *s* for product *k*
r_{i,m_1}	fixed unit penalty cost in demand shortfall from material *i* for product *m*
y_{i,s,m_1}	Product *i* flowrate for demand shortfall per realization of scenario *s* for product m
r_{i,m_2}	fixed unit penalty cost in demand surplus from material *i* for product *m*
y_{i,s,m_2}	Product *i* flowrate for demand surplus per realization of scenario *s* for product m
$u_{\mathrm{d}}^{k_1}$	auxiliary variable for uncertainty due to demand shortfall for scenario *s*

$u_{\mathrm{d}}^{k_2}$	auxiliary variable for uncertainty due to demand surplus for scenario s
$u_{\mathrm{y}}^{m_1}$	auxiliary variable for uncertainty due to yield shortfall for scenario s
$u_{\mathrm{y}}^{m_2}$	auxiliary variable for uncertainty due to yield surplus for scenario s

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CHAPTER 1

INTRODUCTION

1.1 BACKGROUND OF STUDY

It is commonly known that the problem in Chemical process industry and oil and gas industry are subjected to uncertainties by random events such as raw materials and products price variations, market demand fluctuations and chemical production yield. Therefore, the application of the information technology and information systems in the industries is important to enhance the operating flexibility and resiliency of petroleum refineries. To be particular, to optimize petroleum refinery under uncertainties a two-stage stochastic model with fixed recourse via scenario analysis and incorporation of risk management is developed. Recourse model is corrective action made after a random event has taken place. Two-stage Stochastic Programming aims to serve the optimization purpose of a process by minimizing uncertainties and maximizing profit.

1.2 PROBLEM STATEMENT

The midterm refinery production planning problem addressed in this paper can be stated as follows. Given the following information:

- The available process units and their capacities;
- Cost of crude oil and refinery products;
- Market demand of products

Our goal is to determine the amount of materials that are processed in each process stream of each process unit by considering the following uncertain parameters whose stochastic data (including probabilities) are available or obtainable:

- Market demand for products, that is, production amounts of desired products;

- Prices of crude oil and the saleable products; and
- Product (or production) yields of crude oil from chemical reactions in the crude distillation unit

It is assumed that:

- The uncertain parameters of prices, costs, and demands are externally imposed, that is, they are exogenous uncertainties;
- Further, the uncertain parameters are random variables that exhibit the behavior and properties of discrete probability distribution functions; and
- The physical resources of the plant are fixed.

1.3 OBJECTIVE AND SCOPE OF STUDY

We strive to meet this goal of computing the optimal flow-rates by considering the risk involved through the use of the risk measure known as Conditional Value-at-Risk (CVaR), which is a convex and thus computationally-attractive metric that has gained wide attention in computational finance. Further, we wish to develop a computationally-efficient framework for applying CVaR in a refinery planning problem that utilizes minimum computational time in the solution of the refinery planning model. The scenario analysis approach is adopted to represent uncertainties in three classes of stochastic parameters, namely prices of crude oil and commercial products, market demands, and production yields. However, a large number of scenarios are required to capture the stochasticity of the problem. Therefore, to circumvent the problem of the resulting large-scale model, we implement a Monte Carlo simulation approach based on the sample average approximation (SAA) technique to generate the scenarios. A statistical-based scenario reduction strategy is applied to determine the minimum number of scenarios required yet still able to compute the true optimal solution for a desired level of accuracy within the specified confidence intervals. In this study paper, there are large numbers of scenarios that create difficulty to handle various circumstances. For example, there may be more than thousands of cases happening. It is hard to predict and control numerous scenarios. Therefore, it is necessary to find the minimum number of scenarios to capture all the circumstances. Monte Carlo simulation approach based on the sample average approximation (SAA) technique is applied in this thesis to generate the minimum number of scenarios which present for thousands cases.

CHAPTER 2

LITERATURE REVIEW

2.1 BACKGROUND OF STOCHASTIC PROGRAMMING; OPTIMIZATION CONSIDERING UNDER UNCERTAINTIES MATTERS

Stochastic Programs are more difficult to for formulate and solve than deterministic mathematical. The purpose of it, in a given availability of post-optimally analysis, it can be tempting to ease the process by relying on sensitivity analysis to investigate the impact of uncertainty by in introducing recourse problems or in other words including the risk term in the optimization model of maximizing the profit.

The risk terms in Khor (2007) are handled using the metric mean-absolute deviation. After obtaining the first model with MAD as risk measurement, the second model is developed in which the risk terms are performed by CVaR. A comparison between the two models to assess which of these two risk measures is superior, both computationally and conceptually, in capturing the economic and operating risk in the planning of a refinery. Khor model (2007) is expressed:

$$\max z = E[z_o] - \theta_1 V(z_o) - E_{s'} - \theta_2 W \tag{1}$$

Where:

$E[z_o]$: Expected deterministic profit (crude oil and saleable products)

θ_1, $\theta_3 \in$ (0, 1]: weight the components of the objective function

$V(z_o)$: Variance of price uncertainty

$E_{s'}$: Expected penalty of demand and yield uncertainties

W : MAD of demand and yield penalty

Apply MAD as risk measure:

$$\max\ z = E\left[z_0\right] - \theta_1 \mathrm{MAD}_{z_o} - E[\xi] - \theta_3 \mathrm{MAD}_{z_o} \tag{2}$$

Apply CVaR as risk measure:

$$\max\ z = E\left[z_0\right] - \theta_1 CVaR_{\xi} - E[\xi] - \theta_3 CVaR_{\xi} \tag{3}$$

2.2 GENERAL FORMULATION OF TWO-STAGE STOCHASTIC PROGRAMMING

Two-stage Stochastic Programming aims to serve the optimization purpose of a process by minimizing uncertainties and maximizing profit. A stochastic program (SP) was first introduced by George Dantzig in the 1950's. SP is gaining recognition as a viable approach for large scale models of decisions under certainty. The classic form of the stochastic programming (SP) approaches for an optimal midterm refinery planning can be represent in the seminal works of Dantzig (1955) and Beale (1955) and has the following general form:

$$\begin{aligned} \min\ & c^T x + E_{\xi}\left[Q\left(x, \xi(\omega)\right)\right] \\ & \text{s.t. to } Ax = b \\ & x \in X \geq 0 \end{aligned} \tag{4}$$

where $\quad Q\left(x,\xi(\omega)\right)=\min\ q^{T}(\omega)\,y$

$$\begin{aligned} &\text{s.t. to } W(\omega)\,y = h(\omega) - T(\omega)x \\ &\qquad y \geq 0 \end{aligned} \tag{5}$$

With the notation:

$x \in R^{n}$	: Vector of first-stage decision variables, size $(1\times n)$
c	: First-stage column vector of cost coefficient, sizes $(n\times 1)$
A	: First-stage coefficient matrix, size $(m \times n)$
b	: Corresponding right-hand side vectors, size $(m\times 1)$
$\omega \in \Omega$	: Random events or scenario
$\xi(\omega)$	: Random vector
$q(\omega)$	: Second stage vector of recourse cost coefficient vectors size $(k\times 1)$
$h(\omega)$	: Second stage right-hand side vectors, size $(l\times 1)$
$T(\omega)$	: Matrix that ties the two stages together, size $(l\times k)$
$W(\omega)$	: Random recourse coefficient matrix, size $(l\times k)$
y	: Vector of second-stage decision variables, size $(k\times 1)$

$c^{T}x$ is known as the first stage or "here and now" decision, x does not response to ω. In contrast, y presents second stage variable with $Q\left(x,\xi(\omega)\right)$ is "wait and see" and is determined after observation regarding ω has been obtained.

2.2 TWO-STAGE STOCHASTIC PROGRAMMING WITH SIMPLE RECOURSE SUB-PROBLEM

Simple recourse model is a special case of recourse model when recourse coefficients in the second stage, W, form an identity matrix. In general, we have:

$$Q\left(x, \xi(\omega)\right) = \sum_{i \in I} Q_i\left(x, \xi(\omega)\right)$$

Where:

$$Q_i\left(x, \xi(\omega)\right) = \min \quad q^+_{\omega i} y^+_i + q^-_{\omega i} y^-_i$$

$$\text{s.t.} \quad I\, y^+_i - I\, y^-_i = h(\omega_i) - (T(\omega)x)_i \qquad (6)$$

$$y^+_i \text{ , } y^-_i \geq 0$$

$h(\omega) - T(\omega)x$, a feasible solution to (3) is easily determined by setting y^+ and y^- accordingly. Moreover, if the i[th] component of q^+_ω - q^-_ω >0, this feasible solution is optimal.

Example of simple recourse is that when a target profit in one company is determined, the company will try to reduce the deviation from profit.

2.3. TWO-STAGE STOCHASTIC PROGRAMMING WITH FIXED RECOURSE SUB-PROBLEM

Fixed recourse model is the model that the constraint matrix in the recourse sub-problem is fixed (not subject to uncertainty). (6) is written as:

$$Q\left(x,\xi(\omega)\right) = \min q^T(\omega)\, y$$
$$\text{s.t. } Wy = h(\omega) - T(\omega)x \qquad (7)$$
$$y \geq 0$$

When second stage objective coefficients are also fixed, the recourse subproblem can be written as:

$$Q\left(x,\xi(\omega)\right) = \min \quad \pi^{\mathrm{T}}\,(\,h(\omega) - T(\omega)x\,) \qquad (8)$$
$$\text{s.t.} \qquad \pi^{\mathrm{T}}\, W \leq q^{\mathrm{T}}$$
$$\pi \geq 0$$

2.4 TWO-STAGE STOCHASTIC PROGRAMMING WITH COMPLETE RECOURSE SUB-PROBLEM

A problem is said to have complete recourse if $Y(\omega,\chi) = \left\{ y \mid W_{\omega} y \geq \chi \right\}$ is nonempty for any value of χ and the recourse function is necessary finite, $Q\left(x,\xi(\omega)\right) = \infty$. Moreover, relatively complete recourse results if $Y(\omega,\chi)$ is nonempty for all $\chi \in \left\{ h(\omega) - T(\omega)x \mid (\omega, x) \in \Omega \times X \right\}$.

With the complete recourse problem, model (4) becomes:

$$Q\left(x,\xi(\omega)\right) = \text{Min } q^T(\omega)\, y + \mathrm{M}\, \mathrm{e}^{\mathrm{T}}\, \mathrm{z}$$

$$\text{s.t } Wy + z \geq h(\omega) - T(\omega)x\,,\ \ y, z \geq 0 \qquad (9)$$

With M: large constant and e: appropriately dimensioned vector of ones.

2.5 FINANCIAL MATHEMATICS, RISK MEASUREMENT, AND RISK MANAGEMENT

The development of the theory of probability as a companion to statistics is well chronicled in Bernstein (23). In the nineties, there was considerable activity in the fields

- Three broad classes of risk are studied at present.
- The first, *market risk*, attempts to determine the uncertainty in the prices of an object that is traded in a liquid market. The second, *credit risk*, attempts to place a value on the uncertainty associated with an account receivable. How should we account for the possibility that a debtor may default on an obligation? The third, *operational risk*, basically tries to handle everything else. It considers the full set of other risks that a business must conventionally/typically face, including the risk of catastrophic political events, weather-related risk, and risk of criminal activity.

Table 1: Period, nature of risk and risk metric

Period/Time	Nature of Risk	Risk Metric/Measure
Short term (< 1 month)	Operational	Earnings
Intermediate/Medium/Midterm (1 month–1 year)	Financial/Trading	Value-at-risk, cash flow, earnings, credit risk
Long term (> 1 year)	Asset valuation	Equity

2.6 ECONOMICAL/ECONOMIC RISK

(Al-Sharrah. Ghanima. Planning the petrochemical industry in Kuwait using economic and safety objectives. *PhD Thesis*. Loughborough University, 2006.)

- Economic risk is perceived by business people in two ways.
- The first is risk of not achieving the targeted financial objective.
- The second is the risk of variation in the results (Park and Sharp-Bette, 1990).
- The first type of risk may be caused by a number of causes whether economic, political, technical, or the like (of it), and can be represented as the probability of not achieving the financial objective. This type of risk has been employed with planning activities by Barbaro and Bagajewicz (2003).

- The second type of risk can be well-handled by variance techniques such as the variance of Expected Monetary Value (EMV) (Bush and Johnson, 1998) or risk-adjusted return family of methods such as Sharpe ratio (Jones, 1998).
- Applequist et al. (2000) has adopted a risk premium defined as an increase in the expected return in exchange for a given amount of variance in order to evaluate risk and uncertainty for chemical manufacturing plants.

2.7 VALUE-AT-RISK (VAR) AND CONDITIONAL-VALUE-AT-RISK (CVAR)

Figure 1 expresses the idea about VaR and CVaR.

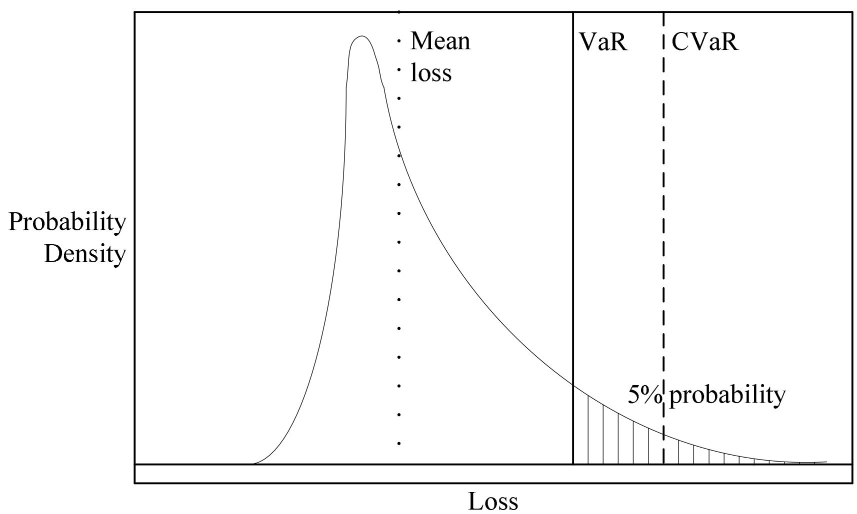

Figure 1: VaR and CVaR illustration

According to Rockafellar and Uryasev, 2002:

Informally, Value at risk (VaR) can be defined as a maximum loss in a specified period with some confidence level, except α (e.g., confidence level = 95%, period = 1 week). Formally, α -VaR is the α-percentile of the loss distribution: α-VaR is a smallest value such that probability that loss exceeds or equals to this value is bigger or equals to α. It suffers, however, from being unstable and difficult to work with numerically when losses are not normally distributed.

CVaR (Mean Excess Loss, Mean Shortfall or Tail VaR) is risk assessment technique used to reduce the probability a portfolio will incur high losses. CVaR is performed by taking the likelihood (at a specific confidence level, example, 0.95 or 0.99, etc…) that a specific loss will exceed the value at risk (VaR). In mathematical point of view, CVaR is derived by taking a weighted average between the VaR and losses exceeding the VaR. CVaR maintains consistency with VaR by yielding the same results in limited settings where VaR computations are tractable, i.e., for normal distribution. Most importantly for applications, CVaR can be expressed by a remarkable minimization formula. This formula can readily be incorporated into problems of optimization with respect to $x \in X$ that are designed to minimize risk or shape in within the bounds. Significant shortcuts are thereby achieved while preserving crucial problem features like convexity.

2.8 MONTE CARLO SIMULATION APPROACH BASED ON SAMPLE AVERAGE APPROXIMATION (SAA)

In the study paper of Risk Management for a Global Supply Chain Planning under Uncertainty: Models and Algorithms, You et al used Monte Carlo method to determine the minimum number of scenarios using the formula $N = \left[\frac{z_{\alpha/2} S_n}{H}\right]^2$ with first estimate the value of sampling estimator S(n) by using formula $S_n = \sqrt{\frac{\sum_{s=1}^{S}\left(E[\mathrm{Cost}] - \mathrm{Cost}_s\right)^2}{n-1}}$ and then using the formula $\left[E_z - \frac{z_{\alpha/2} S(n)}{\sqrt{S}}, E_z + \frac{z_{\alpha/2} S(n)}{\sqrt{S}}\right]$ to determine the required number of scenarios for a desired confidence interval.

In this work, we adopt the Monte Carlo simulation approach for scenario generation based on the Sample Average Approximation (SAA) method (Shapiro, 2000; Shapiro and Homem-de-Mello , 1998; You, Wassick, and Grossmann, 2008). The procedure involved is as follows:

Step1: Generate M independent samples each of size N. For each sample solve the corresponding SAA problem

$$\min_{y \in Y} \left\{ c^T y + \frac{1}{N} \sum_{n=1}^{N} Q(y, \xi_j^{\,n}) \right\}$$

Step 2: Compute minimum number of scenario

Step 3: Apply risk measure into the model

(Referred section 3.3 in chapter 3 for more detail about the mathematical equation)

2.9 SOLVING VAR USING PLOT OF CUMULATIVE DISTRIBUTION FUNCTION AGAINST THE SORTED DETERMINISTIC LOSSES.

In the Mekong 2007 paper by Webby et al giving an idea of using pseudo random sampling from a normal distribution is used to generated an empirical distribution for loss and the results are ranked in order to find VaR. In other words, each resulted losses will be assigned to a random probability, then a graph of cumulative distribution function is plot against the resulted losses, after that reading the value of VaR at 0.95 confidence level.

2.10 OPTIMIZATION PROBLEMS WITH CONSTRAINTS ON RISK AS REPRESENTED BY CVAR

By consider the following linear programming formulation that utilizes the auxiliary real variables us for scenarios s = 1, …, S in order to determine the numerical values for VaR1 of price uncertainty and VaR2 of demand and yield uncertainty, which is based on the formulation proposed by Rockafellar and Uryasev (2000) and which has been applied by Krokhmal et al. (2001). The proposed formulation is as below:

$$\min \ \mathrm{VaR} + \frac{1}{s(1-\beta)} \sum_{s=1}^{S} \left(f(x, y) - \mathrm{VaR} \right)$$

$$\Rightarrow \min \ \mathrm{VaR} + \frac{1}{s(1-\beta)} \sum_{s=1}^{S} u_s$$

where

$$u_s = f(x, y) - \mathrm{VaR}$$

$$f(x, y) = -[x_1 y_1 + \cdots + x_n y_n] = -x^{\mathrm{T}} y$$

Subject to the following linear constraints:

$$u_s \geq 0 \qquad \forall s \in S$$

$$u_s \geq f(x, y) - \mathrm{VaR} \qquad \forall s \in S$$

$$\Rightarrow -x^T y_s + \mathrm{VaR} + u_s \geq 0 \qquad \forall s \in S$$

Furthermore, in the "Risk management in the oil supply chain: A CVaR approach" paper Carneiro et al. (2009) has modified the Krokhmal et al. (2001) original equation as stated above to suit its own objective function which

Maximizing $\sum_{i=1}^{N} x_i \mu_i$

Subject to

$$\alpha + \frac{1}{s(1-\beta)} \sum_{s=1}^{S} u_s \leq K$$

$$u_s \geq 0 \qquad S = 1, \ldots, S$$

$$\sum_{i=1}^{N} x_i r_{is} + \alpha + u_s \geq 0 \qquad S = 1, \ldots, S$$

$$\sum_{i=1}^{N} x_i = 1$$

$$x_i \geq 0 \qquad i = 1, \ldots N$$

Where

K – is an upper bound on the portfolio's CVaR.
N – number of candidate assets
x_i – capital fraction applied on candidate asset i
u_i – return expected of the ith candidate asset
α – variable that provides the portfolio's VaR and CVaR at confidence level of $\beta\%$
β – confidence level to compute the CVaR measure of risk
S – number of scenarios

The constraints of the this paper have taken into our research consideration because the constraints are applicable to our stochastic model as the Carneiro et al. (2009) objective function is the same as our objective function, maximizing the objective function.

2.11 OPTIMIZATION PROBLEMS WITH VERDERAME AND FLOUDAS (2010) ON DIFFERENT APPROACH ON MONTE CARLO SAA AND CVAR

Formulation based on Verderame and Floudas (2010):

$$\tilde{F}_{\omega}(x,\xi) = \text{VaR} + \frac{1}{(1-\beta)|S|} \sum_{s} z_s \leq \text{max\_risk}$$

$$z_s \geq f_{x,s} - \text{VaR} \quad \forall s$$

Where

$\tilde{F}_{\omega}(x,\xi)$ = approximation function for CVaR

β = confidence level

$|S|$ = no. of scenarios

z_s = auxiliary variable

max_risk = threshold value of tolerance of risk, i.e., maximum level of risk acceptable

In the formulation of max_risk of Verderame and Floudas (2010) has introduced δ whereuser-specified risk aversion parameter. The following figure 2 shows the steps of Sample average approximation algorithm used in Verderame and Floudas (2010).

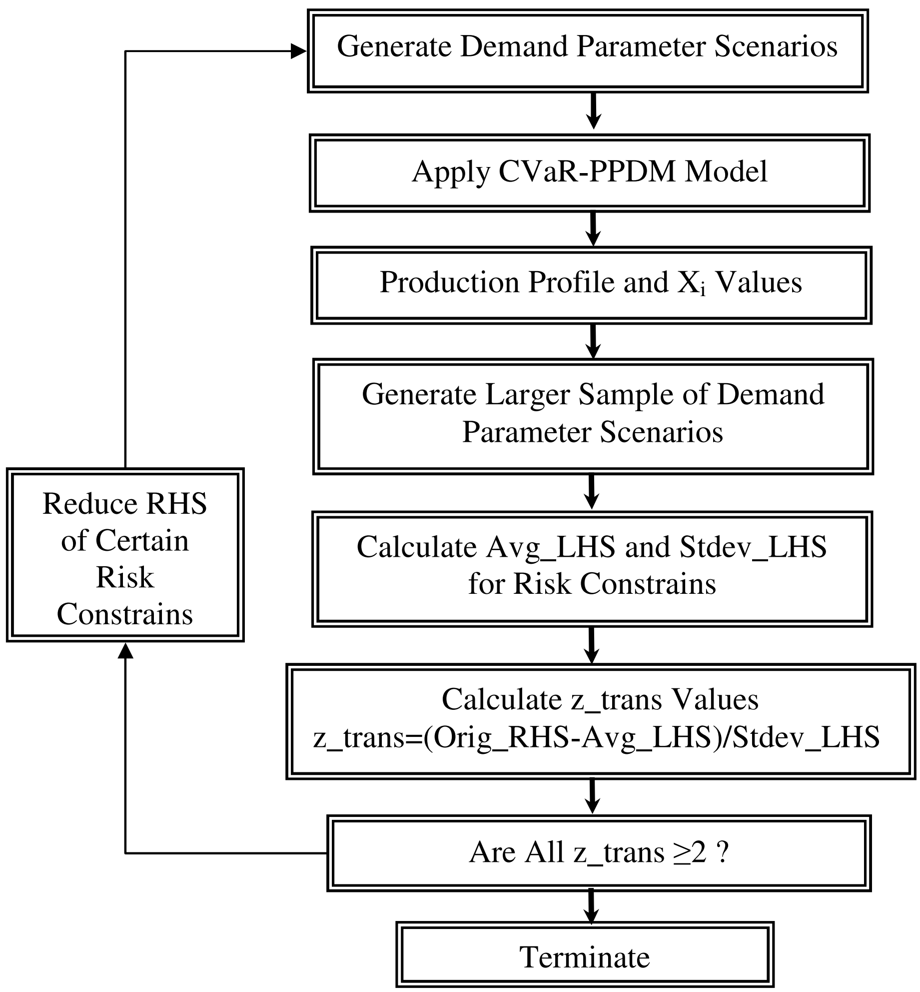

Figure 2: Verderame and Floudas (2010) of Sample average approximation algorithm

CHAPTER 3

METHODOLOGY

Research Methodology

The general methodology of the Stochastic Programming with Economic and Operational Risk Management in Petroleum Refinery Planning under Uncertainties is as below:

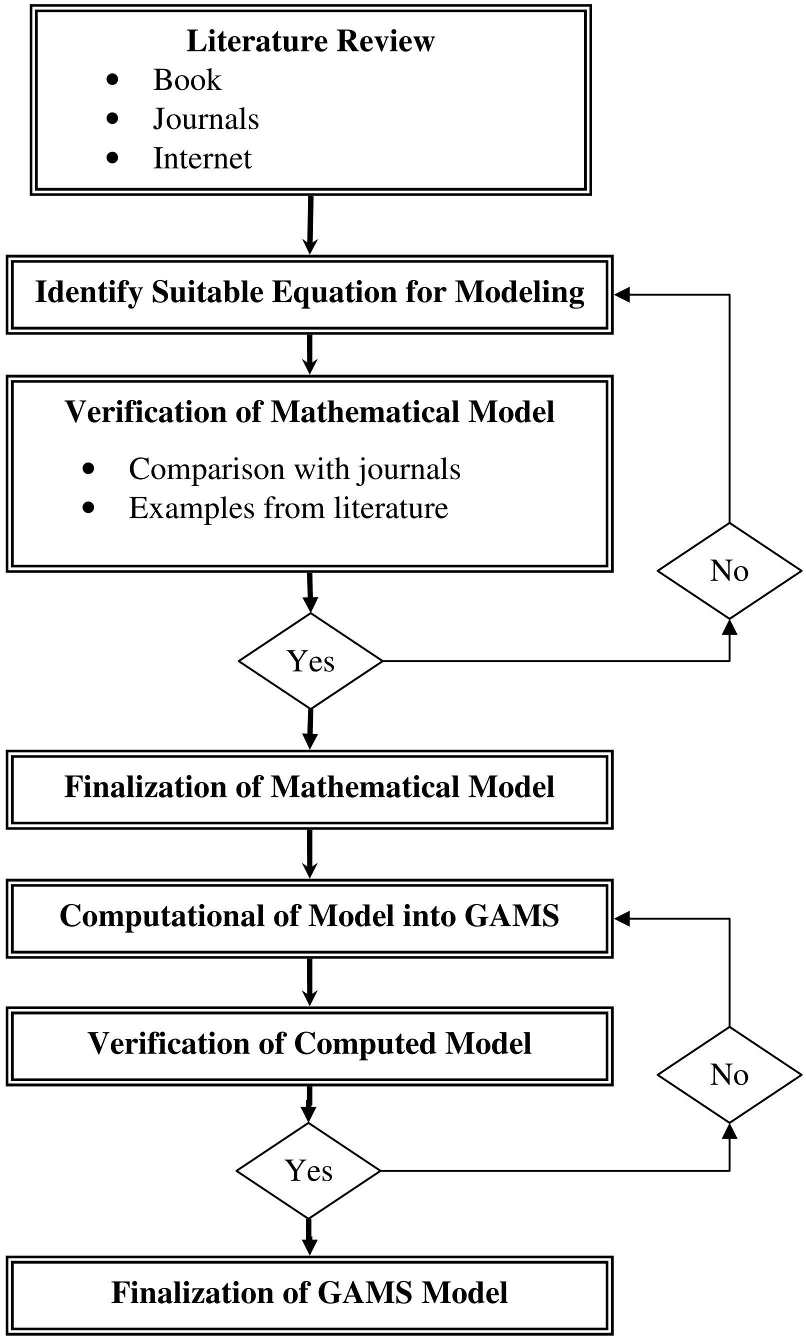

Figure 3: Research Methodology flow chart

Project Activities

The project activities in this work are listed in the Gantt Chart as shown in Figure 3. (● - Milestones)

No.	Detail/ Week	1	2	3	4	5	6	7	8	9	10		11	12	13	14	18-19
1	Briefing &update on the progress		●									Mid semester break					
2	More Research and Literature Review				●												
3	Submission of Progress report 1					●											
4	Project Work continues									●							
6	Submission of Progress Report 2												●				
7	Pre-EDX/ Poster Exhibition Progress Reporting												●				
8	Project Work continues / EDX													●			
9	Dissertation Report submission															●	
10	Final oral presentation																●

Figure 4: Gantt Chart of FYP II

3.1 OPTIMIZATION MODEL FORMULATION

Using CVaR as the risk metric yields the following form of the main objective function is shown below.

Our model formulation involves maximization of profit whereas in the Rockafellar and Uryasev (2000) their problem mainly involves minimization of losses.

$$\max \; z = E[z_0] - \theta_1 \underbrace{\mathrm{CVaR}_{z0}}_{\substack{\text{uncertainty} \\ \text{in profit}}} - E_{s'} - \theta_3 \underbrace{\mathrm{CVaR}_{\xi}}_{\substack{\text{uncertainty} \\ \text{in demands} \\ \text{and yields}}} \tag{10}$$

Where

i: material/product i

s: scenario

k: shortfall, surplus notation for demand uncertainty

m: shortfall/surplus notation for yield uncertainty

c: unit price of material/saleable products

x: material flow-rate or saleable products

z: objective value (profit) in ($/day)

ξ: Monetary value of demand and yield penalty

θ1, θ2: weight factor

3.2 MONTE CARLO SIMULATION APPROACH BASED ON SAMPLE AVERAGE APPROXIMATION (SAA)

In this research work, we adopt the Monte Carlo simulation approach for scenario generation based on the Sample Average Approximation (SAA) method (Shapiro, 2000; Shapiro and Homem-de-Mello , 1998; You and Grossmann, 2008) proposed by Santoso et al. (2005). The procedure involved is as follows:

Monte Carlo Step 1:

A relatively small number of scenarios (for example, 50 scenarios) with their associated probabilities are randomly and independently generated for the uncertain parameters of prices, demands, and yields. (This data is otherwise obtained from plant historical data.) The resulting stochastic model (a linear program) with the objective function given in (37) is solved to determine the optimal stochastic profit with its corresponding material flow-rates.

$$\text{max profit= E[z]} = \underbrace{\sum_{i\in I}\sum_{s\in S} p_s c_{i,s} x_i}_{E[z_o]} - \underbrace{\sum_{i\in I}\sum_{k\in K}\sum_{s\in S} p_s\left[\left(c_i^+ z_{i,s}^+ + c_i^- z_{i,s}^-\right) + \left(q_{i,j}^+ y_{i,k,s}^+ + q_{i,j}^- y_{i,k,s}^-\right)\right]}_{E[\xi]}$$

(for further development and information of the formulation, please refer to appendix I)

$$E[\text{z}] = E[z_o] - E[\xi] \tag{11}$$

Monte Carlo Step 2:

The Monte Carlo sampling variance estimator is determined using the optimal stochastic profit and flow-rates computed in Monte Carlo step 1.

From You et al. (2009):

$$S_n = \sqrt{\frac{\sum_{s=1}^{S}\left(E[\text{Cost}]-\text{Cost}_s\right)^2}{n-1}} \tag{12}$$

But the objective function of our model formulation involves maximization of profit (instead of cost minimization), therefore equation (12) may need to be adapted/reformulated for a profit maximization objective function:

$$\max\ (\text{profit}) = -\min(\text{cost})$$

so:

$$E[\text{Cost}] = -E[\text{profit}]$$

$$E[\text{Cost}]-\text{Cost}_s = -\left(E[\text{Profit}]-\text{Profit}_s\right)$$

$$\left(E[\text{Cost}]-\text{Cost}_s\right)^2 = \left(-\left(E[\text{Profit}]-\text{Profit}_s\right)\right)^2 = E[\text{Profit}]-\text{Profit}_s$$

Hence, the form of eq. (12) that is applicable in our case s given by:

$$S_n = \sqrt{\frac{\sum_{s=1}^{S}\left(E[\text{Profit}]-\text{Profit}_s\right)^2}{n-1}} \tag{13}$$

$$S_n = \sqrt{\frac{\sum_{s=1}^{S}\left(E_z - z_{i,s}\right)^2}{n-1}} \qquad \text{where } z_{i,s} = \sum_{i\in I}\left(c_{i,s}x_i + \xi_{i,s}\right)$$

Where

- Confidence interval H of 1-α is given as:

$$\left[E[\text{z}] - \frac{z_{\alpha/2}S(n)}{\sqrt{S}}, E[\text{z}] + \frac{z_{\alpha/2}S(n)}{\sqrt{S}}\right] \tag{14}$$

Consider confidence interval 95%, that is:

$$
\begin{aligned}
1-\alpha &= 95\% \\
&\Rightarrow \alpha = 5\%,\ \alpha/2 = 2.5\% \\
1-\frac{\alpha}{2} &= 1-2.5\% \\
&= 0.975 \\
\Pr\left(z \le z_{\alpha/2}\right) &= 0.975 \\
z_{\alpha/2} &= 1.96
\end{aligned}
$$

- The minimum number of scenarios N that is theoretically required to obtain an optimal solution is determined using the relation below:

$$N = \left[\frac{z_{\alpha/2} S(n)}{H}\right]^2 \tag{15}$$

Numerical experiments indicate that well controlled choice of the sample sizes can significantly reduce the computational time and improve the accuracy of obtained solutions.

Monte Carlo Step 3:

Risk measure using the metrics of CVaR is incorporated in a new stochastic model with the scenarios given by the minimum number of scenarios N, in which the N number of scenarios are generated as a new set of independent random samples of the uncertain parameters.

A new stochastic model is formulated based on/with minimum number of scenarios N with the incorporation of the risk measure of CVaR, respectively,

3.3 GENERAL FORMULATION OF STOCHASTIC REFINERY PLANNING WITH RISK EXPRESSED IN TERMS OF CVAR

CVaR in conjunction with VaR is a powerful tool to measure risk. Rockafellar and Uryasev (2002) define Conditional Value-at-Risk for continuous distribution function:

$$F_{\alpha}(x,VaR) = VaR + \frac{1}{1-\alpha}\frac{1}{S}\sum_{i\in I}\sum_{s\in S}\ (f(x, y_{i,s}) - VaR) \tag{16}$$

With probability distribution in y, CVaR is written as:

$$F_{\alpha}(x,VaR) = VaR + \frac{1}{1-\alpha}\sum_{i\in I}\sum_{s\in S} p_s (f(x, y_{i,s}) - VaR) \tag{17}$$

<u>Applying The Concept Of Cvar Into The Recourse Terms:</u>

$$\max z = E[z_o] - \theta_1 \text{CVaR}_{z_o} - E_{s'} - \theta_3 \text{CVaR}_{\xi}$$

a) $CVaR_{z_o}$: Risk measure for uncertainty in price of crude oil and refinery products

$$\text{CVaR}(z_o) = \text{VaR}_1 + \frac{1}{1-\alpha}\sum_{s}\sum_{i} p_s \left(c_{i,s}x_{i,s} - \text{VaR}_1\right) \tag{18}$$

b) $CVaR_{\xi}$: Risk measure for uncertainty in market demand and production yield.

$$\text{CVaR}\left(\xi\right) = \text{VaR}_2 + \frac{1}{1-\alpha}\sum_{i\in I}\sum_{k\in K}\sum_{s\in S} p_s \left[\left(c_i^{+} z_{i,s}^{+} + c_i^{-} z_{i,s}^{-}\right) + \left(q_{i,j}^{+} y_{i,k,s}^{+} + q_{i,j}^{-} y_{i,k,s}^{-}\right) - \text{VaR}_2\right] \tag{19}$$

Then put the equation (18) and (19) back into the main objective function (10), we achieve the stochastic model by maximize the profit in which risk elements are expressed in term of CVaR.

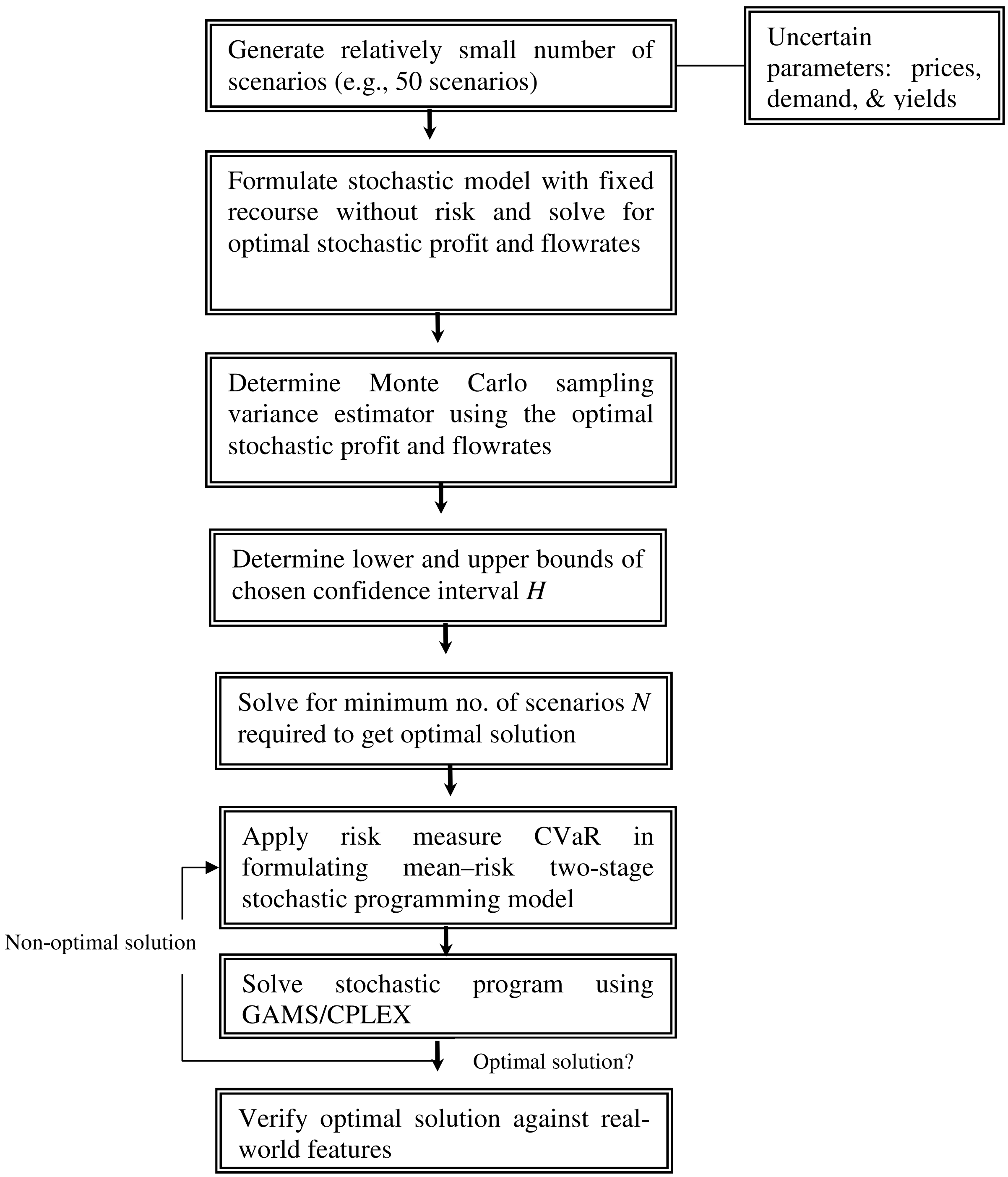

Figure 5 presents an overview of the method of solving the objective function by using Monte Carlo SAA algorithm and CVaR

CHAPTER 4

RESULT AND DISCUSSION

4.1. OPTIMIZATION MODEL FORMULATION

Objective function:

$$\sum_{i \in I} c_i x_i \tag{20}$$

Production demand requirements:

$$x_i \le d_i \quad \forall i \in I \tag{21}$$

Demand uncertainty:

$$x_i + z_{i,s,k_1} - z_{i,s,k_2} = d_{s,i}, \quad \forall i \in I, \forall s \in S \tag{22}$$

Production yield:

$$\sum_{i \in I} a_{i,j} x_i = 0, \quad \forall j \in J \tag{23}$$

Yield uncertainty:

$$-a_{s,i,j} x_{i_1} + x_i + y_{i,s,m_1} - y_{i,s,m_2} = 0, \quad \forall i \in I, \forall s \in S \tag{24}$$

Variable bounds:

$$x_i^{\mathrm{L}} \le x_i \le x_i^{\mathrm{U}} \tag{25}$$

Non-negativity constraints:

$$x_i, z_{i,s,k_1}, z_{i,s,k_2}, y_{i,s,m_1}\, y_{i,s,m_2} \ge 0 \tag{26}$$

4.1.1. Process Flow Network

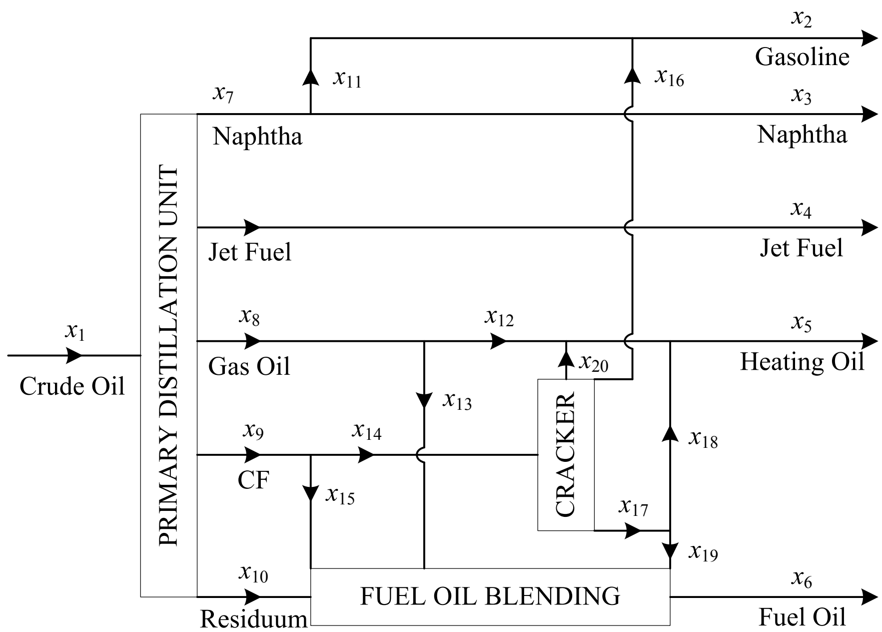

Figure 6: Process flow network

We adapt Khor et al process flow network as this modeling research example in considering the problem of petroleum refinery planning under uncertainties.

Mass balance constraints are in the form of equalities. There are three types of such constraints: fixed plant yield, fixed blends or splits, and unrestricted balances. Except in some special situations such as planned shutdown of the plant or storage movements, the right hand-side of balance constraints is always zero. For the purpose of consistency, flow into the plant or stream junction has negative coefficients and flows out have positive coefficients, (Adapted from Khor et al, 2008). The constraints are as follows:

For the primary distillation unit:

$$-0.13x_1 + x_7 = 0 \tag{27}$$

$$-0.15x_1 + x_4 = 0 \tag{28}$$

$$-0.22x_1 + x_8 = 0 \tag{29}$$

$$-0.20x_1 + x_9 = 0 \tag{30}$$

$$-0.30x_1 + x_{10} = 0 \tag{31}$$

For the cracker:

$$-0.05x_{14} + x_{20} = 0 \tag{32}$$

$$-0.40x_{14} + x_{16} = 0 \tag{33}$$

$$-0.55x_{14} + x_{17} = 0 \tag{34}$$

For gasoline blending:

$$0.5x_2 - x_{11} = 0 \tag{35}$$

$$0.5x_2 - x_{16} = 0 \tag{36}$$

For heating oil blending:

$$0.75x_5 - x_{12} = 0 \tag{37}$$

$$0.25x_5 - x_{18} = 0 \tag{38}$$

Naphtha: $$-x_7 + x_3 + x_{11} = 0 \tag{39}$$

Gas oil: $$-x_8 + x_{12} + x_{13} = 0 \tag{40}$$

Cracker feed: $$-x_9 + x_{14} + x_{15} = 0 \tag{41}$$

Cracked oil: $$-x_{17} + x_{18} + x_{19} = 0 \tag{42}$$

Fuel oil: $$-x_{10} - x_{13} - x_{15} - x_{19} + x_6 = 0 \quad (43)$$

Materials and saleable products are divided into three groups

- Demand uncertainty (X_{ID}): X_2, X_3, X_4, X_5, X_6
- Yield uncertainty (X_{IY}): X_4, X_7, X_8, X_9, X_{10}
- Price uncertainty (X_{IP}): X_1, X_2, X_3, X_4, X_5, X_6, X_{14}

The constraints considered so far are concerned with the physical plant. Constraints are also needed relating to external factors such as the availability of raw materials and product requirements over a time period. For this example, there are no restrictions on crude oil availability or the minimum production required. The maximum production requirement constraints (in t/d) are as follows:

Gasoline: $$x_2 \leq 2700 \quad (44)$$

Naphtha: $$x_3 \leq 1100 \quad (45)$$

Jet fuel: $$x_4 \leq 2300 \quad (46)$$

Heating oil: $$x_5 \leq 1700 \quad (47)$$

Fuel oil: $$x_6 \leq 9500 \quad (48)$$

Overall constrained equations of the model

$$X_1 \le 15,000 \tag{49}$$

$$X_{14} \le 2,500 \tag{50}$$

4.1.2 Optimization Model Formulation with CVaR Constraints Using Auxiliary Variables

Based on the formulation presented in Section 2.10, initially we investigate a formulation for the profit maximization problem in our case that utilizes the following customized constraints:

$$\mathrm{VaR}_1 + \frac{1}{1-\alpha}\sum_{i\in I} p_s\left(c_{i,s}x_{i,s} - \mathrm{VaR}_1\right) \ge 0 \qquad \forall s \in S \tag{51}$$

$$\mathrm{VaR}_2 + \frac{1}{1-\alpha}\sum_{i\in I} p_s\left(\sum_{i\in I}\sum_{k\in K} d_{i,k}z_{i,s,k} + \sum_{i\in I}\sum_{m\in M} q_{i,m}y_{i,s,m} - \mathrm{VaR}_2\right) \ge 0 \qquad \forall s \in S \tag{52}$$

$$\sum_{i\in I} p_s c_{i,s}x_{i,s} + \mathrm{VaR}_1 + \frac{1}{1-\alpha}\sum_{i\in I} p_s\left(c_{i,s}x_{i,s} - \mathrm{VaR}_1\right) \ge 0 \qquad \forall s \in S \tag{53}$$

$$\sum_{i\in I} p_s c_{i,s}x_{i,s} + \mathrm{VaR}_2 + \frac{1}{1-\alpha}\sum_{i\in I} p_s\left(\sum_{i\in I}\sum_{k\in K} d_{i,k}z_{i,s,k} + \sum_{i\in I}\sum_{m\in M} q_{i,m}y_{i,s,m} - \mathrm{VaR}_2\right) \ge 0 \qquad \forall s \in S \tag{54}$$

Eventually the research based on the formulation presented in Section 2.11; Verderame and Floudas(2010) , we have investigated the formulation for the profit maximization problem in our case that utilizes the following customized constraints by replacing the original formulation with our variables and terms:

Constraint On Auxiliary Variables

Based on Verderame and Floudas (2010), the CVaR constraint is given by:

$$\mathrm{VaR} + \frac{1}{(1-\beta)|S|}\sum_{s\in S} u_s \le \text{risk\_aversion} \tag{55}$$

The CVaR constraint applied in our model is expressed as follows:

Constraint 1:

$$\mathrm{VaR}_{\text{price}} + \frac{1}{(1-\beta)|S|}\sum_{s\in S} u_{\text{price},s} \le \underbrace{\delta_{\text{price}} \sum_{i\in I_{\mathrm{P}}} c_{\text{deterministic},i}\, x_i}_{\substack{\text{risk aversion due to}\\ \text{price uncertainty}}} \tag{56}$$

$$\mathrm{VaR}_{\text{demand\_and\_yield}} + \frac{1}{(1-\beta)|S|}\sum_{s\in S} u_{\text{demand\_and\_yield},s} \le \underbrace{\delta_{\text{demand\_and\_yield}}\left(\sum_{i\in I_{\mathrm{D}}} d_{\text{deterministic},i,k_1} z_i + \sum_{i\in I_{\mathrm{Y}}} q_{\text{deterministic},i}\, y_i\right)}_{\substack{\text{risk aversion due to}\\ \text{demand and yield uncertainty}}}$$

$$\Rightarrow \mathrm{VaR}_{\text{demand, yield}} + \frac{1}{(1-\beta)|S|}\sum_{s\in S} u_{\text{demand, yield},s} \le \delta_{\text{demand, yield}}\left(\sum_{s\in S}\sum_{i\in I_{\mathrm{D}}}\sum_{k\in K} d_{i,k}\, z_{i,s,k} + \sum_{s\in S}\sum_{i\in I_{\mathrm{Y}}}\sum_{m\in M} q_{i,m}\, y_{i,s,m}\right) \tag{57}$$

Constraint 2:

$$f_{x,s} + \mathrm{VaR} + u_s \ge 0 \qquad \forall s \tag{58}$$

Rearranging the terms yield:

$$u_s \ge -f_{x,s} - \mathrm{VaR} \qquad \forall s \tag{59}$$

Constraint 3:

$$u_s \ge 0 \quad \forall s \in S \tag{60}$$

Formulation Of Loss Function $F_{x,S}$:

Loss functions for price uncertainty:

$$f_{x,s} = \sum_{s \in S} \sum_{i \in I} c_{i,s} x_{i,s} = \sum_{s,i} c_{i,s} x_{i,s}$$

Loss functions for demand and yield uncertainty:

$$\begin{aligned} f_{x,s} &= \sum_{s \in S} \sum_{i \in I} \sum_{k \in K} d_{i,k} z_{i,s,k} + \sum_{s \in S} \sum_{i \in I} \sum_{m \in M} q_{i,m} y_{i,s,m} \\ &= d_{i,k} z_{i,s,\text{shortfall}} + d_{i,k} z_{i,s,\text{surplus}} + q_{i,m} y_{i,s,\text{shortfall}} + q_{i,m} y_{i,s,\text{surplus}} \\ &= d_{i,k} \left(\text{demand}_{i,s} - x_i \right) \end{aligned}$$

In our problem, we consider uncertainty in three parameters: prices, demand, and yields loss function variable for scenario *s*

$$\text{VaR}_{\text{price}} + \frac{1}{(1-\beta)|S|} \sum_{s \in S} u_{\text{price},s} \leq \delta\left(\max\_\, risk\right)$$

for demand and yield uncertainty:

Where

max_risk = threshold value of tolerance of risk, i.e., maximum level of risk acceptable

u_s = auxiliary real variables

β: confidence level

|S| = no. of scenarios

δ = user-specified risk aversion parameter

4.2 MODEL DATA AND ANALYSIS

Lower and upper bounds of all the materials and products flow-rate is summarize in the table below:

Table 2: Lower bound and upper bound of all material and products flow rate

X_i	Lower bound	Upper bound
X_1	12,500	15000
X_2	2000	2700
X_3	625	1100
X_4	1875	2300
X_5	1700	1700
X_6	6175	9500
X_7	1625	1950
X_8	2750	3300
X_9	2500	3000
X_{10}	3750	3000
X_{11}	1000	1350
X_{12}	1275	1275
X_{13}	1475	3300
X_{14}	2500	2500
X_{15}	0	3000
X_{16}	1000	1200
X_{17}	1375	1650
X_{18}	425	425
X_{19}	950	1650
X_{20}	125	150

We illustrate the risk modeling approach proposed in this paper on the numerical example taken from Khor et al. (2008) and provide major details on the implementation using GAMS/CONOPT3 solver in a hardware platform with 2GB memory and a 1.8 GHz processor. An optimal flow-rates corresponding to each respective materials and determining minimum number of scenarios by Monte Carlo simulation approach based on the sample average approximation (SAA) technique to generate the scenarios have obtained. For further GAMS modeling code please refer appendices II and III.

Table 3: Flow-rates of crude oil and saleable products

X_1	7574	X_{11}	1000
X_2	2000	X_{12}	1274
X_3	950	X_{13}	1877
X_4	2300	X_{14}	2500
X_5	1698	X_{15}	500
X_6	6327	X_{16}	1000
X_7	1950	X_{17}	1375
X_8	3151	X_{18}	424.6
X_9	3000	X_{19}	950.4
X_{10}	3000	X_{20}	125

The table above shows all the optimal flow-rates of crude oil and saleable products generate by GAMS. Each of the optimal flow-rates are fulfilled all the constraints as stated or in other words it would not exceed the given upper bound. The upper bound is measured by maximum production constraints. The proposed initial value is measure by an appropriate marginal deterministic value. Quality of the data/quality of industrial data or deterministic value used/ensure that data of high quality is used. Besides that, another observation is that some of the optimal flow-rates will exceed, equal, or less than the proposed initial values, the value changes it is because GAMS will auto iterate the proposed initial values until it reach the optimal flow-rates with constraints.

Table 4: Summary of computational results

Monte Carlo sampling variance estimator $S(n)$	489.4
Lower bound of confidence interval H	965.3
Upper bound of confidence interval H	1237

Range of confidence interval *H*	271.3
Minimum number of scenarios *N*	13

In the table 4, the GAMS show that the Monte Carlo sampling variance estimator S(n) is 489.4 and the minimum number of scenarios *N* is 13.

The usage of generated sampling variance S(n) is to calculate the lower- and upper-confidence limits of the 95% confidence interval H of 1–α are computed as follows:

$$\left[E_z - \frac{z_{\alpha/2}S(n)}{\sqrt{S}}, E_z + \frac{z_{\alpha/2}S(n)}{\sqrt{S}}\right]$$

The result of the lower and upper bound of confidence interval *H* are listed in the table 5. The minimum number of scenarios N which is 13 is required to obtain an optimal solution is determined using the relation below numerical experiments indicate that well controlled choice of the sample sizes can significantly reduce the computational time and improve the accuracy of obtained solutions. Moreover, if we know the minimum number of scenarios, we know that we only require that minimum amount of data to obtain the optimal solution. For instance, in our case, since 13 is the minimum number of scenarios, we do not have to unnecessarily collect more than that amount of data, hence we could save the costs that would have been otherwise incurred if we were to collect more than the data of those 13 scenarios.

Monte Carlo simulation approach based on Sample Average Approximation (SAA) is a powerful method to calculate minimum number of scenario because it can capture the entire possible scenario and becomes preventative for all scenario. Therefore, it saves time and convenient.

Furthermore, a research has been done to determine the difference between a separate model in different GAMS files and a combine GAMS files of determine the optimal flow-rates and minimum number of scenario. The differences are listed down in the table 5. As a result the Combined Model shows a more optimal solution than the separated one, this is because the combined model improves the time taken to solve a computational model and become a more efficient model structure.

Table 5: Difference between a separate GAMS files and a combined GAMS files

	Combined Model	Separated Model
Ez	1100.917	1101.133
S(n)	488.833	489.429
Min. scenario	13	13

When we proceed to the methodology section 3.3 the General formulation of stochastic refinery planning with risk expressed in terms of CVaR. There are two methods that we adopt to calculate the values of the VaR variables.

The first method is using pseudorandom sampling from a normal distribution to generate an empirical distribution for a profit function. Subsequently, the computed optimal deterministic profit values for each scenario are ranked or sorted in ascending order to determine the value of VaR. In other words, the computed values are assigned a random probability, and then a graph of cumulative distribution function is plotted against the profit values. Finally, we then read off the value of VaR from the profit distribution plot for a specified confidence interval.

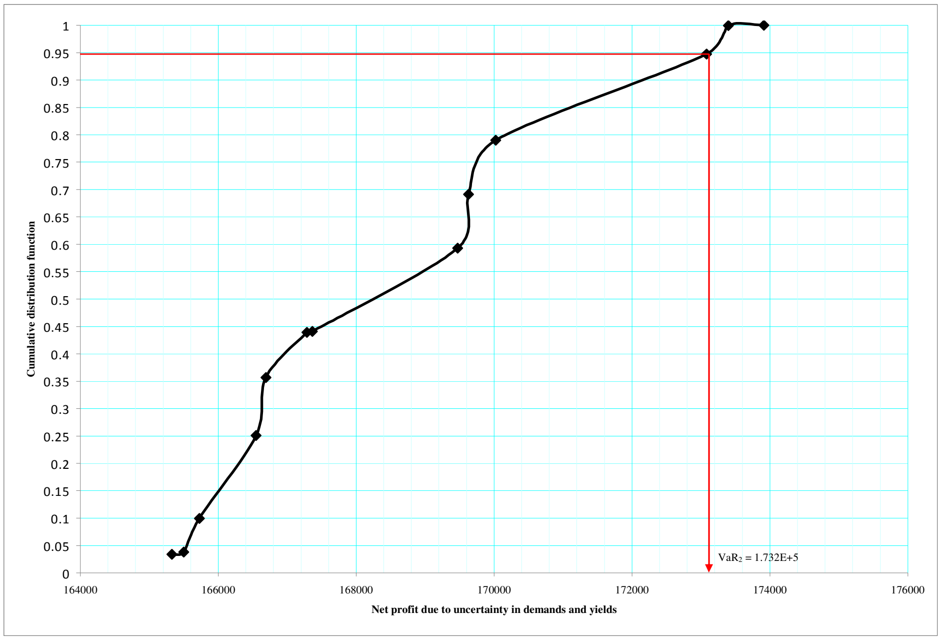

Figure 7: Loss distribution to determine VaR_2 as given by the cumulative distribution function versus the multiplied values of penalty costs due to shortfalls or surpluses for both demands and yields

In the first approach, each computed optimal deterministic profit value is assigned to a random probability based on a Monte-Carlo-simulation-based method. Table 6 lists important results from this approach.

Table 6: Result of VaR1, VaR2 and CVaR using graphical method

VaR1	7.235E+4
VaR2	1.731E+5
Optimal solution for model with CVaR	$20 800.66/day

Approach 2: Model Formulations of CVaR-Based Risk Management Models Using Auxiliary Variables

Our second approach considers the formulation in **section 2.10**. The second CVaR-based risk model formulation presents an alternative to the use of Monte-Carlo-based random probabilities. We consider three major model formulations conveniently referred to as CVaR1a, CVaR1b, and CVaR2 in our computational study. Table 7 summarizes the main parameters considered in our computational study for the three major model formulations that have been developed.

Table 7: Comparison of the three major model formulations of CVaR1a, CVaR1b, and CVaR2 for β = 0.95, $\delta_p = \delta_{d\text{-}y} = 0.5$

Model (with CVaR constraints)	Important feature of formulation	Objective function value ($/day)	Deterministic maximum profit	Stochastic maximum profit	VaR	Production rate
CVaR1a (with static relative risk factors)	Considers static relative risk factors	2938.095	71 558.302	71 524.726	VaR_p = 25.000 $VaR_{d\text{-}y}$ = 84579.102	x_1 = 7574.363, x_2 = 2000.000, x_3 = 950.000, x_4 = 2300.000, x_5 = 1693.951, x_6 = 6331.820, x_7 = 1950.000, x_8 = 3150.770, x_9 = 3000.000, x_{10} = 3000.000 x_{11} = 1000.000, x_{12} = 1270.463, x_{13} = 1880.307, x_{14} = 2500.000, x_{15} = 500.000 x_{16} = 1000.000, x_{17} = 1375.000, x_{18} = 423.488, x_{19} = 951.512, x_{20} = 125.000
CVaR1b	• Effects of risk are represented by aggregated auxiliary variables • Considers dynamic relative risk factors (via loop function in GAMS)	532 235.537	59.305	8.952	VaR_p = 23.882 $VaR_{d\text{-}y}$ = 84579.102	x_1 = 14801.363, x_2 = 2000.000, x_3 = 950.000, x_4 = 2285.617, x_6 = 8175.000, x_7 = 1950.000, x_8 = 3300.000, x_9 = 3000.000, x_{10} = 3000.000,

						x_{11} = 1000.000, x_{13} = 3300.000, x_{14} = 2500.000, x_{15} = 500.000, x_{16} = 1000.000, x_{17} = 1375.000, x_{19} = 1375.000, x_{20} = 125.000
CVaR2:	• Effects of risk are distributed via explicitly disaggregated auxiliary variables • Considers dynamic relative risk factors	3.136981E+7 (with lower bound of all auxiliary variables set as 0.2)	248.471	198.156	$\mathrm{VaR_p}$ = 19.885 $\mathrm{VaR_d^{k_1}}$ = 84571.000 $\mathrm{VaR_d^{k_2}}$ = 84571.000 $\mathrm{VaR_y^{m_1}}$ = 0.103 $\mathrm{VaR_y^{m_2}}$ = 0.103	x_1 = 14801.363, x_2 = 2000.000, x_3 = 950.000, x_4 = 2285.617, x_6 = 8175.000, x_7 = 1950.000, x_8 = 3300.000, x_9 = 3000.000, x_{10} = 3000.000, x_{11} = 1000.000, x_{13} = 3300.000, x_{14} = 2500.000, x_{15} = 500.000, x_{16} = 1000.000, x_{17} = 1375.000, x_{19} = 1375.000, x_{20} = 125.000

Remarks:

- This deterministic maximum profit value for CVaR1b and CVaR2is lower than the deterministic model solution (as reported in Khor et al. (2008)). One of the reasons is because when risk is considered, the optimal solution computed specifies that one of the major products (heating oil) is not to be produced. (Its negative shadow price (reduced costs) (in the solution of the linear program) implies a lower profit when its value is increased.)
- A higher risk corresponds to a higher profit. CVaR2 registers a profit that is four (4) times greater because of the greater risks taken as represented by the greater number of disaggregated auxiliary variables. Also, since the risk is evaluated as separate components as related to the individual uncertainties in prices, demands, and yields, the profit may tend to be higher. This is similar to distributing risks throughout a portfolio of investments rather than a single investment.

Mean–CVaR model based on aggregated auxiliary variables with static relative risk factors:

$$
\begin{aligned}
\max \quad & \underbrace{\sum_{i\in I}\sum_{s\in S}\left(p_s \text{price}_{i,s}^{\mathrm{T}} x_i - p_s \text{cost}_{i,s}^{\mathrm{T}} x_i\right)}_{\text{profit due to price uncertainty}} - \theta_1\left(\mathrm{VaR}_1 + \frac{1}{1-\alpha}\sum_{s}\sum_{i} p_s\left(c_{i,s}x_{i,s} - \mathrm{VaR}_1\right)\right) \\
& - \underbrace{\sum_{i\in I}\sum_{s\in S}\sum_{k\in K} p_s\left(c_{i,k} z_{i,s,k}\right)}_{\substack{\text{recourse penalty due to}\\ \text{demand uncertainty}}} - \underbrace{\sum_{i\in I}\sum_{s\in S}\sum_{m\in M} p_s\left(q_{i,j,m} y_{i,s,m}\right)}_{\substack{\text{recourse penalty due to}\\ \text{yield uncertainty}}} \\
& - \theta_2\left(\mathrm{VaR}_2 + \frac{1}{1-\alpha}\sum_{i\in I}\sum_{s\in S} p_s\left(\sum_{k\in K}\left(c_{i,k}\cdot z_{i,s,k}\right) + \sum_{m\in M}\left(q_{i,j,m}\cdot y_{i,s,m}\right) - \mathrm{VaR}_2\right)\right) \\
\text{s.t} \quad & \text{constraints } (22)-(26) \\
& \mathrm{VaR}_{\mathrm{p}} + \frac{1}{(1-\beta)|S|}\sum_{s\in S} u_{\mathrm{p},s} \le \delta_{\mathrm{p}} \sum_{i\in I_{\mathrm{P}}} c_i x_i \\
& \mathrm{VaR}_{\text{d-y}} + \frac{1}{(1-\beta)|S|}\sum_{s\in S} u_{\text{d-y},s} \le \delta_{\text{d-y}}\left(\sum_{i\in I_{\mathrm{D}}} c_i d_i + \sum_{i\in I_{\mathrm{Y}}} c_i a_i\right) \\
& \sum_{i\in I_{\mathrm{P}}} p_s c_{i,s} x_i + \mathrm{VaR}_{\mathrm{p}} + u_{\mathrm{p},s} \ge 0 \quad \forall s\in S, \quad p_s = \frac{1}{|S|} \\
& \left(\sum_{\substack{i\in I_D\\ k}} p_s q_{i,k} z_{i,s,k} + \sum_{\substack{i\in I_Y\\ m}} p_s r_{i,m} y_{i,s,m}\right) + \mathrm{VaR}_{\text{d-y}} + u_{\text{d-y},s} \ge 0 \quad \forall s\in S, \quad p_s = \frac{1}{|S|} \qquad \textbf{(CVaR1a)} \\
& u_{\mathrm{p},s}, u_{\text{d-y},s} \ge 0 \quad \forall s\in S
\end{aligned}
$$

Mean–CVaR model based on aggregated auxiliary variables with dynamic relative risk factors:

$$
\begin{aligned}
\max \quad & \sum_{i\in I}\sum_{s\in S}\left(p_s \text{price}_{i,s}^{\mathrm{T}} x_i - p_s \text{cost}_{i,s}^{\mathrm{T}} x_i\right) - \theta_{1,j}\left(\mathrm{VaR}_1 + \frac{1}{1-\alpha}\sum_{s}\sum_{i} p_s\left(c_{i,s}x_{i,s} - \mathrm{VaR}_1\right)\right) - \sum_{i\in I}\sum_{s\in S}\sum_{k\in K} p_s\left(c_{i,k} z_{i,s,k}\right) \\
& - \sum_{i\in I}\sum_{s\in S}\sum_{m\in M} p_s\left(q_{i,j,m} y_{i,s,m}\right) - \theta_{2,j}\left(\mathrm{VaR}_2 + \frac{1}{1-\alpha}\sum_{i\in I}\sum_{s\in S} p_s\left(\sum_{k\in K}\left(c_{i,k}\cdot z_{i,s,k}\right) + \sum_{m\in M}\left(q_{i,s,m}\cdot y_{i,s,m}\right) - \mathrm{VaR}_2\right)\right) \\
\text{s.t} \quad & \text{constraints } (22)-(26) \\
& \mathrm{VaR}_{\mathrm{p}} + \frac{1}{(1-\beta)|S|}\sum_{s\in S} u_{\mathrm{p},s} \le \delta_{\mathrm{p}} \sum_{i\in I_{\mathrm{P}}} c_i x_i \\
& \mathrm{VaR}_{\text{d-y}} + \frac{1}{(1-\beta)|S|}\sum_{s\in S} u_{\text{d-y},s} \le \delta_{\text{d-y}}\left(\sum_{i\in I_{\mathrm{D}}} c_i d_i + \sum_{i\in I_{\mathrm{Y}}} c_i a_i\right) \\
& \sum_{i\in I_{\mathrm{P}}} p_s c_{i,s} x_i + \mathrm{VaR}_{\mathrm{p}} + u_{\mathrm{p},s} \ge 0 \quad \forall s\in S, \quad p_s = \frac{1}{|S|} \\
& \left(\sum_{\substack{i\in I_D \\ k}} p_s q_{i,k} z_{i,s,k} + \sum_{\substack{i\in I_Y \\ m}} p_s r_{i,m} y_{i,s,m}\right) + \mathrm{VaR}_{\text{d-y}} + u_{\text{d-y},s} \ge 0 \quad \forall s\in S, \quad p_s = \frac{1}{|S|} \\
& u_{\mathrm{p},s}, u_{\text{d-y},s} \ge 0 \quad \forall s\in S
\end{aligned}
\qquad \textbf{(CVaR1b)}
$$

Mean–CVaR model based on disaggregated auxiliary variables:

$$
\begin{aligned}
\max \quad & \sum_{\substack{i \in I_{\mathrm{P}} \\ s}} p_s c_{i,s} x_i - \theta_{\mathrm{p}} \left(\mathrm{VaR}_{\mathrm{p}} + \frac{1}{(1-\beta)} \sum_{\substack{i \in I_{\mathrm{P}} \\ s}} p_s \left(c_{i,s} x_i - \mathrm{VaR}_{\mathrm{p}} \right) \right) - \sum_{\substack{i \in \in I_{\mathrm{P}} \\ s}} p_s \left(\sum_{\substack{i \in I_{\mathrm{D}} \\ k}} q_{i,k} z_{i,s,k} + \sum_{\substack{i \in I_Y \\ m}} r_{i,m} y_{i,s,m} \right) \\
& - \theta_{\mathrm{d}}^{k_1} \left(\mathrm{VaR}_{\mathrm{d}}^{k_1} + \frac{1}{(1-\beta)} \sum_{s \in S} p_s \left(q_{i,k_1} z_{i,s,k_1} - \mathrm{VaR}_{\mathrm{d}}^{k_1} \right) \right) - \theta_{\mathrm{d}}^{k_2} \left(\mathrm{VaR}_{\mathrm{d}}^{k_2} + \frac{1}{(1-\beta)} \sum_{s \in S} p_s \left(q_{i,k_2} z_{i,s,k_2} - \mathrm{VaR}_{\mathrm{d}}^{k_2} \right) \right) \\
& - \theta_{\mathrm{d}}^{m_1} \left(\mathrm{VaR}_{\mathrm{d}}^{m_1} + \frac{1}{(1-\beta)} \sum_{s \in S} p_s \left(r_{i,m_1} y_{i,s,m_1} - \mathrm{VaR}_{\mathrm{y}}^{m_1} \right) \right) - \theta_{\mathrm{d}}^{m_2} \left(\mathrm{VaR}_{\mathrm{d}}^{m_2} + \frac{1}{(1-\beta)} \sum_{s \in S} p_s \left(r_{i,m_2} y_{i,s,m_2} - \mathrm{VaR}_{\mathrm{y}}^{m_2} \right) \right) \\
\text{s.t.} \quad & \text{constraints } (22)-(26) \\
& \left.
\begin{aligned}
& \mathrm{VaR}_{\mathrm{p}} + \frac{1}{(1-\beta)|S|} \sum_{s \in S} u_{\mathrm{p},s} \le \delta_{\mathrm{p}} \sum_{i \in I_{\mathrm{P}}} c_i x_i, \quad \delta_{\mathrm{p}} = 0.5 \\
& \mathrm{VaR}_{\mathrm{d}}^{k_1} + \frac{1}{(1-\beta)|S|} \sum_{s \in S} u_{\mathrm{d}}^{k_1} \le \delta_{\mathrm{d}} \sum_{i \in I_{\mathrm{D}}} c_i d_i, \quad \delta_{\mathrm{d}} = 0.5 \\
& \mathrm{VaR}_{\mathrm{d}}^{k_2} + \frac{1}{(1-\beta)|S|} \sum_{s \in S} u_{\mathrm{d}}^{k_2} \le \delta_{\mathrm{d}} \sum_{i \in I_{\mathrm{D}}} c_i d_i, \quad \delta_{\mathrm{d}} = 0.5 \\
& \mathrm{VaR}_{\mathrm{d}}^{m_1} + \frac{1}{(1-\beta)|S|} \sum_{s \in S} u_{\mathrm{y}}^{m_1} \le \delta_{\mathrm{y}} \sum_{i \in I_{\mathrm{Y}}} c_i a_i, \quad \delta_{\mathrm{y}} = 0.5 \\
& \mathrm{VaR}_{\mathrm{d}}^{m_2} + \frac{1}{(1-\beta)|S|} \sum_{s \in S} u_{\mathrm{y}}^{m_2} \le \delta_{\mathrm{y}} \sum_{i \in I_{\mathrm{Y}}} c_i a_i, \quad \delta_{\mathrm{y}} = 0.5
\end{aligned}
\right\} \text{(Verderame and Floudas, 2010)} \\
& \left.
\begin{aligned}
& \sum_{i \in I_{\mathrm{P}}} p_s c_{i,s} x_i + \mathrm{VaR}_{\mathrm{p}} + u_{\mathrm{p},s} \ge 0 \quad \forall s \in S, \quad p_s = \frac{1}{|S|} \\
& \sum_{i \in I_D} p_s q_{i,k_1} z_{i,s,k_1} + \mathrm{VaR}_{\mathrm{d}}^{k_1} u_{\mathrm{d}}^{k_1} \ge 0 \quad \forall s \in S, \quad p_s = \frac{1}{|S|} \\
& \sum_{i \in I_D} p_s q_{i,k_2} z_{i,s,k_2} + \mathrm{VaR}_{\mathrm{d}}^{k_2} + u_{\mathrm{d}}^{k_2} \ge 0 \quad \forall s \in S, \quad p_s = \frac{1}{|S|} \\
& \sum_{i \in I_D} p_s r_{i,m_1} y_{i,s,m_1} + \mathrm{VaR}_{\mathrm{y}}^{m_1} + u_{\mathrm{y}}^{m_1} \ge 0 \quad \forall s \in S, \quad p_s = \frac{1}{|S|} \\
& \sum_{i \in I_D} p_s r_{i,m_2} y_{i,s,m_2} + \mathrm{VaR}_{\mathrm{y}}^{m_2} + u_{\mathrm{y}}^{m_2} \ge 0 \quad \forall s \in S, \quad p_s = \frac{1}{|S|}
\end{aligned}
\right\} \text{(Carneiro et al., in press)} \qquad \textbf{(CVaR2)} \\
& u_{\mathrm{p},s}, u_{\mathrm{d}}^{k_1}, u_{\mathrm{d}}^{k_2}, u_{\mathrm{y}}^{m_1}, u_{\mathrm{y}}^{m_2} \ge 0 \quad \forall s \in S
\end{aligned}
$$

The Effects of Relative Risk Factors

From Table 8, it is observed that with greater values of θ_1 and θ_2, the objective function value tends to increase whereas the deterministic profit and stochastic profit tend to decrease.

Table 8: Trend of θ_1 and θ_2 vs. . $E[z]$, z_{det}, and z_{stoc} for $\delta_p = \delta_{d\text{-}y\text{-}2} = 0.5$ for CVaR1 with dynamic risk factors and $\beta = 0.95$

θ_1	θ_2	**Objective function value ($/day)**	**Deterministic profit ($/day)**	**Stochastic profit ($/day)**
0	0	1100.917	71 596.422	71 562.839
0	0.3	135 869.895	38 113.727	38 076.204
0.3	0	-50 378.197	59.276	8.926
0.3	0.3	123 746.366	59.305	8.952
0.3	0.6	297 871.003	59.305	8.952
0.6	0.9	472 937.791	59.305	8.952
0.9	0.6	299 755.304	59.305	8.952
0.6	1	530 979.337	59.305	8.952
0.8	1	531 607.437	59.305	8.952
1	1	532 235.537	59.305	8.952

It is noteworthy that in our computational study, the flowrate of heating oil (material i_5) vanishes or goes to zero at certain values of θ_1 and θ_2.

The Effects of Confidence Level β

Table 9 compares the trend of confidence level β against the objective function value $E[z]$, deterministic profit z_{det}, and stochastic profit z_{stoc} for $\theta_1 = \theta_2 = 1$ for the CVaR1 model with dynamic risk factors and user-specified risk aversion parameter of 0.5. From Figure 8, we observe that for higher β, the returns are larger as given by the values of $E[z]$, z_{det}, and z_{stoc}. As β increases, $E[z]$ increases exponentially while both z_{det} and z_{stoc} increases at a high rate for smaller β and gradually increases at a lower rate towards larger β.

Table 9: Trend of β vs. $E[z]$, z_{det}, and z_{stoc} for CVaR1b model (with dynamic risk factors) for $\theta_1 = \theta_2 = 1$ and $\delta_p = \delta_{d-y\_2} = 0.5$

Confidence level β	Objective function value ($/day)	Deterministic profit ($/day)	Stochastic profit ($/day)
99	3 201 566.966	96.971	46.631
97	977 010.114	81.131	30.786
95	532 235.537	59.305	8.952
93	341 643.741	27.021	-23.343

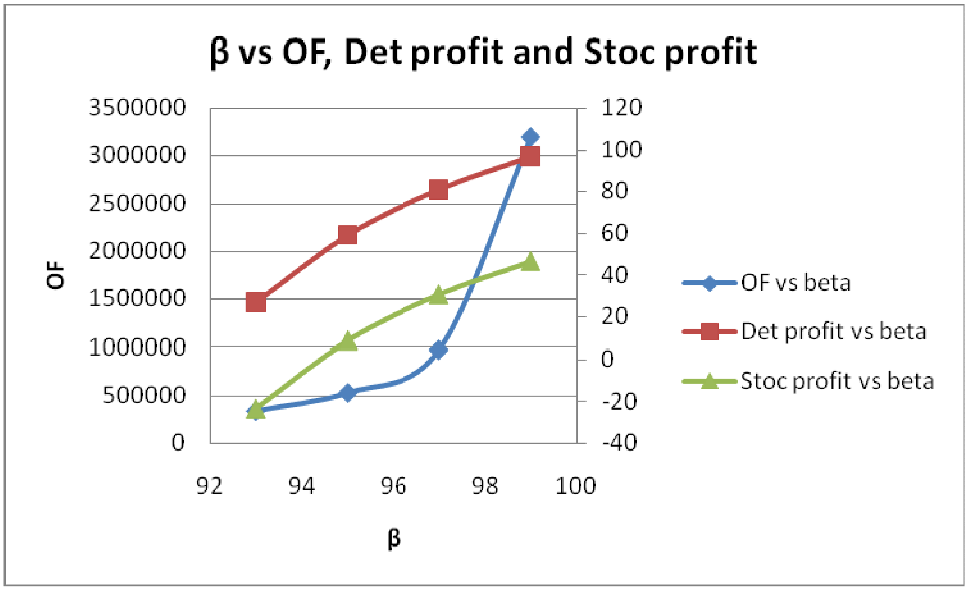

Figure 8: Relation of β vs $E[z]$, z_{det}, and z_{stoc}

The Effects of User-Specified Risk Aversion Parameter δ

Table 10 tabulates the trends of variation in the parameters of returns of $E[z]$, z_{det}, and z_{stoc} against $\delta_p = \delta_{d-y}$ for CVaR1b model (with dynamic risk factors) for $\theta_1 = \theta_2 = 1$ and β = 0.95. In general, we observe that higher values of δ_p and δ_{d-y} correspond to higher $E[z]$ but lower values of z_{det}, and z_{stoc}. We also observe that δ_{d-y} has a greater impact on $E[z]$ than δ_p, in which for larger δ_{d-y}, we obtain larger $E[z]$. On the overall, $E[z]$ increases with δ_p and δ_{d-y}, that is, the higher the risk taken by the investor, the higher is the expected profit (Note that the computational study of model CVaR1a uses the arbitrary values of $\delta_p = \delta_{d-y} = 0.5$.)

Table 10: Trends of $\delta_p = \delta_{d\text{-}y}$ vs. $E[z]$, z_{det}, and z_{stoc}, for CVaR1b model (with dynamic risk factors) for $\theta_1 = \theta_2 = 1$ and $\beta = 0.95$

δ_p	$\delta_{d\text{-}y}$	**Objective function value ($/day)**	**Deterministic profit ($/day)**	**Stochastic profit ($/day)**
0	0	-1.10486E+6	1336.098	1286.176
0	0.3	-140660.716	1336.098	1286.176
0.3	0	-1.08658E+6	556.164	505.980
0.5	0.5	532 235.537	59.305	8.952
0.5	0.9.	1 817 837.895	59.305	8.952
0.9	0.5	554 045.760	-938.582	-988.966
1	1	2 164 350.001	-1179.915	-1230.238

Below table shows the comparison of each model the three major model formulations of CVaR1a, CVaR1b, and CVaR2 statistics

Table 11: Model size and computational statistics

	CVaR1a	CVaR1b	CVaR2
Model type	LP	LP	LP
Solver	GAMS/CONOPT3	GAMS/CONOPT3	GAMS/CONOPT3
No. of continuous variables	1123	1123	1276
No. of constraints	717	617	770
CPU time (s)	trival	trival	trival

CHAPTER 5

CONCLUSION AND RECOMMENDATION

<u>Recommendation</u>

Work closely and research on the Verderame and Floudas,(2010) paper, where considering the proposed termination criterion using Z-transformation formulation to improve the feasibility of the stochastic model solution.

<u>Conclusion:</u>

The Combined of two initial GAMS file of calculating the flow-rate of crude oil saleable product and minimum number of scenarios to one GAMS file. This will improve the time taken to solve a computational model and become a more efficient model structure.

Stochastic programming is one of the ultimate operation research models for optimization that involves uncertainties. The input values such as materials flow-rate, shortfall and surplus of demand and yield penalty are determined maximize the profit.

Monte Carlo simulation approach based on Sample Average Approximation (SAA) is a powerful method to calculate minimum number of scenario because it can capture the entire possible scenario and becomes preventative for all scenario and decease computational time. Therefore, it saves time and convenient. Nevermore, the usage of the Monte Carlo simulation approach based on Sample Average Approximation (SAA) is used to ensure that the a production profile that is feasible and tight upper bound on the production capacity of the plant in question.

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APPENDIX I: MODEL FORMULATION

$$\max\ \text{profit} = \max z$$

$$\max z =$$

$$\max E[z] = \underbrace{\sum_{i\in I}\sum_{s\in S}\left(p_s \text{price}^{\mathrm{T}}_{i,s} x_i - p_s \text{cost}^{\mathrm{T}}_{i,s} x_i\right)}_{\text{profit due to price uncertainty}} - \underbrace{\sum_{i\in I}\sum_{s\in S}\begin{pmatrix} p_s c_{\text{shortfall},i} z_{\text{shortfall},i} \\ + p_s c_{\text{surplus},i} z_{\text{surplus},i}\end{pmatrix}}_{\substack{\text{recourse penalty due to}\\ \text{demand uncertainty}}} - \underbrace{\sum_{i\in I}\sum_{s\in S}\begin{pmatrix} p_s q_{\text{yield}_{\text{shortfall}},i}\, y_{i,s,\text{yield}_{\text{shortfall}}} \\ + p_s q_{\text{yield}_{\text{surplus}},i}\, y_{i,s,\text{yield}_{\text{surplus}}}\end{pmatrix}}_{\substack{\text{recourse penalty due to}\\ \text{yield uncertainty}}}$$

$$= \sum_{i\in I}\sum_{s\in S} p_s\left(\text{price}^{\mathrm{T}}_{i,s} x_i - p_s \text{cost}^{\mathrm{T}}_{i,s} x_i\right) - \sum_{i\in I}\sum_{s\in S} p_s\left[\begin{pmatrix} c_{\text{shortfall},i} z_{\text{shortfall},i} \\ + c_{\text{surplus},i} z_{\text{surplus},i}\end{pmatrix} + \begin{pmatrix} q_{\text{yield}_{\text{shortfall}},i}\, y_{i,s,\text{yield}_{\text{shortfall}}} \\ + q_{\text{yield}_{\text{surplus}},i}\, y_{i,s,\text{yield}_{\text{surplus}}}\end{pmatrix}\right]$$

$$= \sum_{i\in I}\sum_{s\in S} p_s c_{i,s} x_i - \left(\sum_{i\in I_D}\sum_{s\in S}\sum_{k\in K} p_s c_{i,k} z_{i,s,k} + \sum_{i\in I_Y}\sum_{s\in S}\sum_{m\in M} p_s q_{i,m} y_{i,s,m}\right)$$

$$= \sum_{i\in I}\sum_{s\in S} p_s c_{i,s} x_i - \sum_{i\in I}\sum_{s\in S} p_s \xi_{i,s}$$

$$\max E[z] = E[z_0] - E[\xi]$$

$$x_{\text{gasoline}} + z_{\text{shortfall}} = \text{demand}_{\text{gasoline},s1} = \{200 \text{ bpd}, 180, 220, 199, 205, 187\}$$

$$x_{\text{gasoline}} = 200$$

$$200 = 200$$

$$200 - \underbrace{20}_{z_{\text{surplus}}} = 180$$

$$200 + \underbrace{20}_{z_{\text{shortfall}}} = 220$$

$$x_i + z_{\text{shortfall},i} - z_{\text{surplus},i} \le \text{demand}_{i,s}$$

Modeling uncertainty in production yield:

$$-\text{yield}_{i,s} x_i + x_{i'} - y_{i,s,\text{yield}_{\text{shortfall}}} + y_{i,s,\text{yield}_{\text{surplus}}} = 0$$

$$\max\ \text{profit} = \max E[z] = \max\left(E[z_0] - E_\xi\right)$$

where $E[z_0] = \sum_{i\in I}\sum_{s\in S} p_s c_{i,s} x_i$

Recourse penalty function:

$$E_{\xi} = \sum_{i \in I} \sum_{s \in S} p_s \xi_{i,s} = \sum_{i \in I_D} \sum_{s \in S} \sum_{k \in K} p_s c_{i,k} z_{i,s,k} + \sum_{i \in I_Y} \sum_{s \in S} \sum_{m \in M} p_s q_{i,m} y_{i,s,m}$$

APPENDIX II: GAMS MODELING CODE (CVaR1a)

```
$TITLE  Find flow rate
$EOLCOM #

SETS

I     material streams   /I1*I20/

K     DEMAND SHORTFALL OR SURPLUS     /K1, K2/

M     yield shortfall and surplus     /M1, M2/


$ontext
IY1('I1',IY)
/
I1.(I7, I4, I8, I9, I10, I20, I16, I17)
/
$offtext
;


$onecho >taskin.txt
dset=S rng=Sheet4!A16:A65 rdim=1
dset=ID rng=Sheet4!B15:F15 cdim=1
dset=IY rng=Sheet4!H15:L15 cdim=1
dset=IP rng=Sheet4!Q15:W15 cdim=1
par=D rng=Sheet4!A15:F65 cdim=1 rdim=1
par=Yield rng=Sheet4!G15:L65  cdim=1 rdim=1
par=Price rng=Sheet4!P15:W65 cdim=1 rdim=1
$offecho

$call gdxxrw.exe WStep1.xls @taskin.txt

$gdxin WStep1.gdx

Sets

S(*)      scenario
IP(I)     material with price uncertainty
ID(I)     material with demand uncertainty
IY(I)     material with yield uncertainty;

$load S ID IY IP
display S, ID, IY, IP;

ALIAS (S,S1)
;

SCALAR

Z_ALPHA /1.96/
DELTA_PRICE     /0.5/
DELTA_DEMAND_YIELD     /0.5/


PARAMETERS

D(S,ID)       demand
N
Yield(S,IY)   yield
Price(S,IP)   price
ACTUAL_ACTUAL_OBJ_FNC_VALUE_DETERMINISTIC
ACTUAL_OBJ_FNC_VALUE_STOCHASTIC

$load D Yield Price
display D, Yield, Price;
```

```
$gdxin

PARAMETERS

DETERMINISTIC_PRICE(IP)
/
I1     -8
I2     18.5
I3     8.0
I4     12.5
I5     14.5
I6     6
I14    -1.5
/

PRICE_DEMAND(ID)
/
I2     18.5
I3     8.0
I4     12.5
I5     14.5
I6     6
/


DETERMINISTIC_DEMAND(ID)
/
I2     2700
I3     1100
I4     2300
I5     1700
I6     9500
/

PRICE_YIELD(IY)
/
I7     8.0      #(ASSUME price of naphtha(I7) to be the same as naphtha(I3))
I4     12.5
I8     14.5      #(ASSUME price same as heating oil (I5))
I9     1.5      #(ASSUME price same as cracker feed(I14) but the positive value)
I10    6        #(ASSUME price same as fuel oil (I6))
/

DETERMINISTIC_YIELD(IY)
/
I7     0.13
I4     0.15
I8     0.22
I9     0.20
I10    0.30
/

Table Penalty_Demand(ID,K) TABLE OF PENALTY DEMAND
      K1        K2
I2     25        20
I3     17        13
I4     5         4
I5     6         5
I6     10        8;

Table Penalty_Yield(IY,M)  TABLE OF PENALTY YIELD
      M1        M2
I7      5         3
I4      5         4
I8      5         3
I9      5         3
I10     5         3;
```

```
FREE Variables

OBJ        OBJECTIVE FUNCTION
Sn
OBJ_RISK
U(S)
;


Positive Variables

X(I)       MATERIAL FLOWRATE
Y(IY,S,M)  SHORT FALL OR SURPLUS FOR YEILD
Z(ID,S,K)  SHORT FALL OR SURPLUS FOR DEMAND
VaR1
VaR2
U1(S)
U2(S)

;


Equations

OBJFNC
Sn_eqn
OBJFNC_RISK

DEMAND(S,ID) Demand,
YIELD_CON(S,IY)    Yield,
Feed1,
Feed14,
PDU_14_16,
PDU_14_17,
PDU_14_20,
FB_2_11,
FB_2_16,
FB_5_12,
FB_5_18,
UB_8,
UB_14,
UB_17,
UB_18,
UB_6

*CVaR constraints
CONSTRAINT_1_PRICE_UNCERTAINTY
CONSTRAINT_1_DEMAND_YIELD_UNCERTAINTY
CONSTRAINT_2_PRICE_UNCERTAINTY
CONSTRAINT_2_DEMAND_YIELD_UNCERTAINTY
CONSTRAINT_3_PRICE_UNCERTAINTY
CONSTRAINT_3_DEMAND_YIELD_UNCERTAINTY
;


OBJFNC..    OBJ    =e=    SUM((S,IP),(CARD(S)**(-1))*PRICE(S,IP)*X(IP))    -(SUM((ID,S,K),(CARD(S)**(-
1))*(Penalty_Demand(ID,K)*Z(ID,S,K))) + SUM((IY,S,M),(CARD(S)**(-1))*(Penalty_Yield(IY,M)*Y(IY,S,M)))) ;

*Nga:    OBJFNC..    OBJ    =e=    SUM((S,IP),(CARD(S)**(-1))*PRICE(S,IP)*X(IP))    -(SUM((ID,S,K),(CARD(S)**(-
1))*(Penalty_Demand(ID,K)*Z(ID,S,K))) + SUM((IY,S,M),(CARD(S)**(-1))*(Penalty_Yield(IY,M)*Y(IY,S,M)))) ;


**LIMITATIONS OF PLANT CAPACITY
Feed1..     X('I1') =L= 15000;
Feed14..    X('I14') =L= 2500;

*mbl..X('1')-(X('2')+X('3')+X('4')+X('5')+X('6'))=E=0  ;

*Reformulated stochastic constraints to account for uncertain yield coefficient
```

```
YIELD_CON(S,IY).. -YIELD(S,IY)*X('I1') + X(IY) + Y(IY,S,'M1') - Y(IY,S,'M2') =E= 0;


$ontext
e1..     -0.13*X('I1') + X('I7') - Y(IY,S,'M1') + Y(IY,S,'M2') =E= 0;
e1(IY,'I1',S) $ IY1('I1',IY)..     -YIELD(IY,S)*X('I1') + X(IY) - Y(IY,S,'M1') + Y(IY,S,'M2') =E= 0;
e1(IY,'I14',S)..     -YIELD(IY,S)*X('I14') + X(IY) - Y(IY,S,'M1') + Y(IY,S,'M2') =E= 0;

X('I1') = 100
X('I7') = 14

-13 + 14 - Y(IY,S,'M1') + Y(IY,S,'M2') =E= 0
Y(IY,S,'M1') = 1
Y(IY,S,'M2') = 0

-0.05*X('I14') + X('I20') =E= 0
;


e2..     -0.15*X('I1') + X('I4') =E= 0;
$offtext



**************************************************************************************************
*********
*FIXED YIELDS FOR CRACKER (deterministic constraints)
**************************************************************************************************
*********

PDU_14_20..  -0.05*X('I14') + X('I20') =E= 0;
PDU_14_16..  -0.40*X('I14') + X('I16') =E= 0;
PDU_14_17..  -0.55*X('I14') + X('I17') =E= 0;
FB_2_11..    0.5*X('I2') - X('I11') =E= 0;
FB_2_16..    0.5*X('I2') - X('I16') =E= 0;
FB_5_12..    0.75*X('I5') - X('I12') =E= 0;
FB_5_18..    0.25*X('I5')- X('I18') =E= 0;
UB_8..       -X('I7') + X('I3') + X('I11') =E= 0;
UB_14..      -X('I8') + X('I12') + X('I13') =E= 0;
UB_17..      -X('I9') + X('I14') + X('I15') =E= 0;
UB_18..      -X('I17') + X('I18') +X('I19') =E= 0;
UB_6..       -X('I10') - X('I13') - X('I15') - X('I19') + X('I6') =E= 0;


**************************************************************************************************
*********
**CONSTRAINTS ON PRODUCTION DEMANDS
**************************************************************************************************
*********

DEMAND(S,ID).. X(ID) + Z(ID,S,'K1')-Z(ID,S,'K2') =E= D(S,ID);


Sn_eqn..                                    Sn          =E=          SQRT(SUM(S,(ABS(1100.911-(SUM(IP,PRICE(S,IP)*X(IP))-
(SUM((ID,K),Penalty_Demand(ID,K)*Z(ID,S,K)) + SUM((IY,M),Penalty_Yield(IY,M)*Y(IY,S,M))))))**2)/49 )  ;


*for price uncertainty:


**************************************************************************************************
*********
*CVaR
**************************************************************************************************
*********

OBJFNC_RISK.. OBJ_RISK =e= SUM ( (S,IP), (1/CARD(S))*PRICE(S,IP)*X(IP) )
                         - 0.0001 * ( VaR1 + ( 1 /(1 - 0.95)) * SUM ( (S,IP),(1/CARD(S)) * ( PRICE(S,IP)*X(IP) - VaR1 ) ) )
                         -                         SUM(S,(1/CARD(S))*(SUM((ID,K),Penalty_Demand(ID,K)*Z(ID,S,K))+SUM((IY,M),
Penalty_Yield(IY,M)*Y(IY,S,M))))
```

```
- 0.01 * ( VaR2 + ( 1 / (1 - 0.95) ) * SUM ( S, (1/CARD(S)) * ( SUM ( (ID,K), Penalty_Demand(ID,K)*Z(ID,S,K) ) + SUM ( (IY,M), Penalty_Yield(IY,M)*Y(IY,S,M)) - VaR2 ) ) );



CONSTRAINT_1_PRICE_UNCERTAINTY..
       VaR1 + ( 1 /((1 - 0.95)*CARD(S)) ) * SUM ( S, U1(S) ) =L= DELTA_PRICE*SUM (IP, DETERMINISTIC_PRICE(IP) )
;

CONSTRAINT_1_DEMAND_YIELD_UNCERTAINTY..
       VaR2 + ( 1 /((1 - 0.95)*CARD(S)) ) * SUM ( S, U2(S) ) =L= DELTA_DEMAND_YIELD*( SUM ( ID, PRICE_DEMAND(ID)*DETERMINISTIC_DEMAND(ID) ) + SUM ( IY, PRICE_YIELD(IY)*DETERMINISTIC_YIELD(IY) ) )
;
*problem here is: there are two expressions for U(S) which

CONSTRAINT_2_PRICE_UNCERTAINTY(S)..     U1(S) =G= - SUM ( IP, PRICE(S,IP)*X(IP) ) - VaR1
;

CONSTRAINT_2_DEMAND_YIELD_UNCERTAINTY(S)..     U2(S) =G= - SUM ( (ID,K), Penalty_Demand(ID,K)*Z(ID,S,K) )
+ SUM ( (IY,M), Penalty_Yield(IY,M)*Y(IY,S,M) ) - VaR2
;

CONSTRAINT_3_PRICE_UNCERTAINTY(S)..     U1(S) =G= 0;

CONSTRAINT_3_DEMAND_YIELD_UNCERTAINTY(S)..     U2(S) =G= 0;



* Nga's solution from cumumlative density function:
*Var1 = 72400;
*Var2 = 173200;

$ontext
*CVaR for price uncertainty
CVaR1_constraint_1..     CVaR1 =E= Var1 + (1/(1-0.95)) * SUM ( (S,IP), P(S) * ( PRICE(S,IP)*X(IP) - Var1 ) )
;

*CVaR for demand and yield uncertainty
CVaR2_constraint_1..     CVaR2 =E= Var2 + (1/(1-0.95)) * SUM ( (S,IP), P(S)*(PRICE(S,IP)*X(IP) - Var2 ) )
;

*AUXILIARY VARIABLES

CVaR1_constraint_2..    VaR1 + ( (1 - 0.95)**(-1) * SUM ( S, P(S)*U(S) ) ) =G= CVaR1.LO
;
CVaR2_constraint_2..    VaR2 + ( (1 - 0.95)**(-1) * SUM ( S, P(S)*U(S) ) ) =G= CVaR2.LO
;

CVaR_constraint_1(S)..    U(S) =L= 0
;

*not sure what the following constraints are and where they are obtained
CVaR1_constraint_3(S)..     U(S) =L= PRICE(S,'1')*X('1') - VaR1
;
CVaR2_constraint_3(S)..     U(S) =L= PRICE(S,'1')*X('1') - VaR2
;

CVaR1.L = 0
;
CVaR2.L = 0
;
$offtext


*DECISION  VARIABLE BOUNDS
*X.UP(I) = 12100;
Y.UP(IY,S,M) = 1500;
*VaR1.UP = 1E5;
*VaR2.UP = 1E5;
```

```
*Initial values
X.L('I1') = 12500;
*X.L('I2') = 2700;
X.L('I3') = 625;
X.L('I4') = 1875;
X.L('I5') = 1700;
X.L('I6') = 6175;
X.L('I7') = 1625;
X.L('I8') = 2750;
X.L('I9') = 2500;
X.L('I10') = 3750;
*X.L('I11') = 1000;
X.L('I12') = 1275;
X.L('I13') = 1475;
X.L('I14') = 2500;
X.L('I15') = 0;
*X.L('I16') = 1000;
X.L('I17') = 1375;
X.L('I18') = 425;
X.L('I19') = 950;
X.L('I20') = 125;

U1.L(S) = 100;
U2.L(S) = 100;

* Upper bounds of variables
X.UP('I1') = 15000;

*original: X.UP('I2') = 2700;
X.UP('I2') = 3000;

X.UP('I3') = 1100;
X.UP('I4') = 2300;
X.UP('I5') = 1700;
X.UP('I6') = 9500;
X.UP('I7') = 1950;
X.UP('I8') = 3300;
X.UP('I9') = 3000;
X.UP('I10') = 3000;

*original: X.UP('I11') = 1350;
X.UP('I11') = 2000;

X.UP('I12') = 1275;
X.UP('I13') = 3300;
X.UP('I14') = 2500;
X.UP('I15') = 3000;

*original: X.UP('I16') = 1200;
X.UP('I16') = 2000;

X.UP('I17') = 1650;
X.UP('I18') = 425;
X.UP('I19') = 1650;
X.UP('I20') = 150;


MODEL WStep1 to determine stochastic profit
*/ALL/;
/
OBJFNC
DEMAND
YIELD_CON
Feed1,
Feed14,
PDU_14_16,
PDU_14_17,
PDU_14_20,
FB_2_11,
FB_2_16,
```

```
FB_5_12,
FB_5_18,
UB_8,
UB_14,
UB_17,
UB_18,
UB_6
*Sn_eqn
/;


OPTION LIMROW = 100000;
OPTION LIMCOL = 100000;

*SOLVE WStep1 USING DNLP MAXIMIZING OBJ;
SOLVE WStep1 USING DNLP MAXIMIZING OBJ;

EXECUTE_UNLOAD 'WStep1.GDX', Sn;
EXECUTE 'GDXXRW.EXE WStep1.GDX O=WStep1.XLS VAR=Sn RNG=SHEET4!A68';


MODEL WStep2
*/ALL/;
/
Sn_eqn
*OBJFNC
DEMAND
YIELD_CON
Feed1,
Feed14,
PDU_14_16,
PDU_14_17,
PDU_14_20,
FB_2_11,
FB_2_16,
FB_5_12,
FB_5_18,
UB_8,
UB_14,
UB_17,
UB_18,
UB_6
/
;

SOLVE WStep2 USING DNLP MINIMIZING Sn;

N = ( Z_ALPHA*Sn.L/( (OBJ.L + Z_ALPHA*Sn.L/SQRT(CARD(S)) ) - (OBJ.L - Z_ALPHA*Sn.L/SQRT(CARD(S)) ) ) )**2


MODEL CVaR
/
OBJFNC_RISK
DEMAND
YIELD_CON
Feed1,
Feed14,
PDU_14_16,
PDU_14_17,
PDU_14_20,
FB_2_11,
FB_2_16,
FB_5_12,
FB_5_18,
UB_8,
UB_14,
UB_17,
UB_18
UB_6
```

```
CONSTRAINT_1_PRICE_UNCERTAINTY
CONSTRAINT_1_DEMAND_YIELD_UNCERTAINTY
CONSTRAINT_2_PRICE_UNCERTAINTY
CONSTRAINT_2_DEMAND_YIELD_UNCERTAINTY
CONSTRAINT_3_PRICE_UNCERTAINTY
CONSTRAINT_3_DEMAND_YIELD_UNCERTAINTY
/
;

SOLVE CVaR USING LP MAXIMIZING OBJ_RISK;

ACTUAL_ACTUAL_OBJ_FNC_VALUE_DETERMINISTIC = SUM ( IP, DETERMINISTIC_PRICE(IP)*X.L(IP) );
ACTUAL_OBJ_FNC_VALUE_STOCHASTIC = SUM ( (S,IP), (1/CARD(S))*PRICE(S,IP)*X.L(IP) );

*DISPLAY X.L, OBJ.L, Y.L , Z.L;
DISPLAY X.L, Y.L , Z.L, N, U1.L,U2.L, VaR1.L, VaR2.L, OBJ_RISK.L,
ACTUAL_ACTUAL_OBJ_FNC_VALUE_DETERMINISTIC, ACTUAL_OBJ_FNC_VALUE_STOCHASTIC;
```

APPENDIX III: GAMS MODELING CODE (CVaR1b)

```
$TITLE  Find flow rate
$EOLCOM #

SETS

I     material streams   /I1*I20/

K     DEMAND SHORTFALL OR SURPLUS     /K1, K2/

M     yield shortfall and surplus     /M1, M2/

COUNTER     /1*10/

$ontext
IY1('I1',IY)
/
I1.(I7, I4, I8, I9, I10, I20, I16, I17)
/
$offtext
;


$onecho >taskin.txt
dset=S rng=Sheet4!A16:A65 rdim=1
dset=ID rng=Sheet4!B15:F15 cdim=1
dset=IY rng=Sheet4!H15:L15 cdim=1
dset=IP rng=Sheet4!Q15:W15 cdim=1
par=D rng=Sheet4!A15:F65 cdim=1 rdim=1
par=Yield rng=Sheet4!G15:L65  cdim=1 rdim=1
par=Price rng=Sheet4!P15:W65 cdim=1 rdim=1
$offecho

$call gdxxrw.exe WStep1.xls @taskin.txt

$gdxin WStep1.gdx

Sets

S(*)      scenario
IP(I)     material with price uncertainty
ID(I)     material with demand uncertainty
IY(I)     material with yield uncertainty;

$load S ID IY IP
```

```
display S, ID, IY, IP;

ALIAS (S,S1)
;

SCALAR

Z_ALPHA /1.96/
DELTA_PRICE     /0.5/
DELTA_DEMAND_YIELD     /0.5/

WEIGHT1     /0/
WEIGHT2     /0/


PARAMETERS

D(S,ID)       demand
N
Yield(S,IY)   yield
Price(S,IP)   price
ACTUAL_ACTUAL_OBJ_FNC_VALUE_DETERMINISTI
C
ACTUAL_OBJ_FNC_VALUE_STOCHASTIC

$load D Yield Price
display D, Yield, Price;
$gdxin


PARAMETERS

DETERMINISTIC_PRICE(IP)
/
I1     -8
I2     18.5
I3     8.0
I4     12.5
I5     14.5
I6     6
I14    -1.5
/

PRICE_DEMAND(ID)
/
I2     18.5
I3     8.0
I4     12.5
I5     14.5
I6     6
/


DETERMINISTIC_DEMAND(ID)
/
I2     2700
I3     1100
I4     2300
I5     1700
I6     9500
/

PRICE_YIELD(IY)
/
I7      8.0       #(ASSUME price of naphtha(I7) to be the same
as naphtha(I3))
I4     12.5
I8     14.5     #(ASSUME price same as heating oil (I5))
I9      1.5       #(ASSUME price same as cracker feed(I14) but
the positive value)
```

```
I10    6        #(ASSUME price same as fuel oil (I6))
/

DETERMINISTIC_YIELD(IY)
/
I7     0.13
I4     0.15
I8     0.22
I9     0.20
I10    0.30
/

Table    Penalty_Demand(ID,K)    TABLE    OF    PENALTY
DEMAND
       K1        K2
I2     25        20
I3     17        13
I4     5         4
I5     6         5
I6     10        8;

Table Penalty_Yield(IY,M)  TABLE OF PENALTY YIELD
       M1        M2
I7      5         3
I4      5         4
I8      5         3
I9      5         3
I10     5         3;



FREE Variables

OBJ         OBJECTIVE FUNCTION
Sn
OBJ_RISK
U(S)
;


Positive Variables

X(I)        MATERIAL FLOWRATE
Y(IY,S,M)  SHORT FALL OR SURPLUS FOR YEILD
Z(ID,S,K)   SHORT FALL OR SURPLUS FOR DEMAND
VaR1
VaR2
U1(S)
U2(S)
;

*Initial values
X.L('I1') = 12500;
*X.L('I2') = 2700;
X.L('I3') = 625;
X.L('I4') = 1875;
X.L('I5') = 1700;
X.L('I6') = 6175;
X.L('I7') = 1625;
X.L('I8') = 2750;
X.L('I9') = 2500;
X.L('I10') = 3750;
*X.L('I11') = 1000;
X.L('I12') = 1275;
X.L('I13') = 1475;
X.L('I14') = 2500;
X.L('I15') = 0;
*X.L('I16') = 1000;
X.L('I17') = 1375;
X.L('I18') = 425;
```

```
X.L('I19') = 950;
X.L('I20') = 125;

Equations

OBJFNC
Sn_eqn
OBJFNC_RISK

DEMAND(S,ID) Demand,
YIELD_CON(S,IY)    Yield,
Feed1,
Feed14,
PDU_14_16,
PDU_14_17,
PDU_14_20,
FB_2_11,
FB_2_16,
FB_5_12,
FB_5_18,
UB_8,
UB_14,
UB_17,
UB_18,
UB_6

*CVaR constraints
CONSTRAINT_1_PRICE_UNCERTAINTY
CONSTRAINT_1_DEMAND_YIELD_UNCERTAINTY
CONSTRAINT_2_PRICE_UNCERTAINTY
CONSTRAINT_2_DEMAND_YIELD_UNCERTAINTY

;


OBJFNC..    OBJ    =e=    SUM((S,IP),(CARD(S)**(-
1))*PRICE(S,IP)*X(IP))    -(SUM((ID,S,K),(CARD(S)**(-
1))*(Penalty_Demand(ID,K)*Z(ID,S,K)))    +
SUM((IY,S,M),(CARD(S)**(-
1))*(Penalty_Yield(IY,M)*Y(IY,S,M)))) ;

*Nga:  OBJFNC..  OBJ  =e=  SUM((S,IP),(CARD(S)**(-
1))*PRICE(S,IP)*X(IP))    -(SUM((ID,S,K),(CARD(S)**(-
1))*(Penalty_Demand(ID,K)*Z(ID,S,K)))    +
SUM((IY,S,M),(CARD(S)**(-
1))*(Penalty_Yield(IY,M)*Y(IY,S,M)))) ;


**LIMITATIONS OF PLANT CAPACITY
Feed1..    X('I1') =L= 15000;
Feed14..    X('I14') =L= 2500;

*mbl..X('1')-(X('2')+X('3')+X('4')+X('5')+X('6'))=E=0  ;

*Reformulated stochastic constraints to account for uncertain
yield coefficient
YIELD_CON(S,IY)..  -YIELD(S,IY)*X('I1')  +  X(IY)  +
Y(IY,S,'M1') - Y(IY,S,'M2') =E= 0;


$ontext
e1..    -0.13*X('I1') + X('I7') - Y(IY,S,'M1') + Y(IY,S,'M2')
=E= 0;
e1(IY,'I1',S)  $  IY1('I1',IY)..     -YIELD(IY,S)*X('I1')  +
X(IY) - Y(IY,S,'M1') + Y(IY,S,'M2') =E= 0;
e1(IY,'I14',S)..      -YIELD(IY,S)*X('I14')  +  X(IY)  -
Y(IY,S,'M1') + Y(IY,S,'M2') =E= 0;

X('I1') = 100
X('I7') = 14
```

```
-13 + 14 - Y(IY,S,'M1') + Y(IY,S,'M2') =E= 0
Y(IY,S,'M1') = 1
Y(IY,S,'M2') = 0

-0.05*X('I14') + X('I20') =E= 0
;


e2..     -0.15*X('I1') + X('I4') =E= 0;
$offtext



**************************************************
**************************************************
*****************
*FIXED YIELDS FOR CRACKER (deterministic constraints)
**************************************************
**************************************************
*****************

PDU_14_20..  -0.05*X('I14') + X('I20') =E= 0;
PDU_14_16..  -0.40*X('I14') + X('I16') =E= 0;
PDU_14_17..  -0.55*X('I14') + X('I17') =E= 0;
FB_2_11..    0.5*X('I2') - X('I11') =E= 0;
FB_2_16..    0.5*X('I2') - X('I16') =E= 0;
FB_5_12..    0.75*X('I5') - X('I12') =E= 0;
FB_5_18..    0.25*X('I5')- X('I18') =E= 0;
UB_8..       -X('I7') + X('I3') + X('I11') =E= 0;
UB_14..      -X('I8') + X('I12') + X('I13') =E= 0;
UB_17..      -X('I9') + X('I14') + X('I15') =E= 0;
UB_18..      -X('I17') + X('I18') +X('I19') =E= 0;
UB_6..       -X('I10') - X('I13') - X('I15') - X('I19') + X('I6')
=E= 0;


**************************************************
**************************************************
*****************
**CONSTRAINTS ON PRODUCTION DEMANDS
**************************************************
**************************************************
*****************

DEMAND(S,ID)..  X(ID)  +  Z(ID,S,'K1')-Z(ID,S,'K2')  =E=
D(S,ID);


Sn_eqn..            Sn   =E=   SQRT(SUM(S,(ABS(1100.911-
(SUM(IP,PRICE(S,IP)*X(IP))-
(SUM((ID,K),Penalty_Demand(ID,K)*Z(ID,S,K))               +
SUM((IY,M),Penalty_Yield(IY,M)*Y(IY,S,M))))))**2)/49  )
;


*for price uncertainty:


**************************************************
**************************************************
*****************
*CVaR
**************************************************
**************************************************
*****************

OBJFNC_RISK..   OBJ_RISK   =e=   SUM   (   (S,IP),
(1/CARD(S))*PRICE(S,IP)*X(IP) )
```

```
                    - 1 * ( VaR1 + ( 1 /(1 - 0.95)) * SUM (
(S,IP),(1/CARD(S)) * ( PRICE(S,IP)*X(IP) - VaR1 ) ) )
                    -
SUM(S,(1/CARD(S))*(SUM((ID,K),Penalty_Demand(ID,K)
*Z(ID,S,K))+SUM((IY,M),
Penalty_Yield(IY,M)*Y(IY,S,M))))
                    - 1 * ( VaR2 + ( 1 / (1 - 0.95) ) * SUM (
S, (1/CARD(S)) * ( SUM ( (ID,K),
Penalty_Demand(ID,K)*Z(ID,S,K) ) + SUM ( (IY,M),
Penalty_Yield(IY,M)*Y(IY,S,M)) - VaR2 ) ) );



CONSTRAINT_1_PRICE_UNCERTAINTY..
      VaR1 + ( 1 /((1 - 0.95)*CARD(S)) ) * SUM ( S, U1(S)
) =L= DELTA_PRICE*SUM (IP,
DETERMINISTIC_PRICE(IP) )
*     VaR1 + ( 1 /((1 - 0.95)*CARD(S)) ) * SUM ( S,
U1(S) ) =L= DELTA_PRICE*SUM (IP,
DETERMINISTIC_PRICE(IP)*X(IP) )
;

CONSTRAINT_1_DEMAND_YIELD_UNCERTAINTY..
      VaR2 + ( 1 /((1 - 0.95)*CARD(S)) ) * SUM ( S, U2(S)
) =L= DELTA_DEMAND_YIELD*( SUM ( ID,
PRICE_DEMAND(ID)*DETERMINISTIC_DEMAND(ID)
) + SUM ( IY,
PRICE_YIELD(IY)*DETERMINISTIC_YIELD(IY) ) )
*     VaR2 + ( 1 /((1 - 0.95)*CARD(S)) ) * SUM ( S,
U2(S) ) =L= DELTA_DEMAND_YIELD*( SUM (
(ID,S,K),
PRICE_DEMAND(ID)*DETERMINISTIC_DEMAND(ID)*
Z(ID,S,K) ) + SUM ( (IY,S,M),
PRICE_YIELD(IY)*DETERMINISTIC_YIELD(IY)*Y(IY,
S,M) ) )
;
*problem here is: there are two expressions for U(S) which

CONSTRAINT_2_PRICE_UNCERTAINTY(S)..      U1(S)
=G= - SUM ( IP, (1/CARD(S))*PRICE(S,IP)*X(IP) ) - VaR1
;

CONSTRAINT_2_DEMAND_YIELD_UNCERTAINTY(S)
..          U2(S) =G= - SUM ( (ID,K),
(1/CARD(S))*Penalty_Demand(ID,K)*Z(ID,S,K) ) - SUM (
(IY,M), (1/CARD(S))*Penalty_Yield(IY,M)*Y(IY,S,M) ) -
VaR2
;




* Nga's solution from cumumlative density function:
*Var1 = 72400;
*Var2 = 173200;

$ontext
*CVaR for price uncertainty
CVaR1_constraint_1..     CVaR1 =E= Var1 + (1/(1-0.95)) *
SUM ( (S,IP), P(S) * ( PRICE(S,IP)*X(IP) - Var1 ) )
;

*CVaR for demand and yield uncertainty
CVaR2_constraint_1..     CVaR2 =E= Var2 + (1/(1-0.95)) *
SUM ( (S,IP), P(S)*(PRICE(S,IP)*X(IP) - Var2 ) )
;

*AUXILIARY VARIABLES
```

```
CVaR1_constraint_2..    VaR1 + ( (1 - 0.95)**(-1) * SUM (
S, P(S)*U(S) ) ) =G= CVaR1.LO
;
CVaR2_constraint_2..    VaR2 + ( (1 - 0.95)**(-1) * SUM (
S, P(S)*U(S) ) ) =G= CVaR2.LO
;

CVaR_constraint_1(S)..    U(S) =L= 0
;

*not sure what the following constraints are and where they
are obtained
CVaR1_constraint_3(S)..      U(S) =L= PRICE(S,'1')*X('1') -
VaR1
;
CVaR2_constraint_3(S)..      U(S) =L= PRICE(S,'1')*X('1') -
VaR2
;

CVaR1.L = 0
;
CVaR2.L = 0
;
$offtext


*DECISION  VARIABLE BOUNDS
*X.UP(I) = 12100;
Y.UP(IY,S,M) = 1500;
*VaR1.UP = 1E5;
*VaR2.UP = 1E5;

U1.L(S) = 100;
U2.L(S) = 100;

* Upper bounds of variables
X.UP('I1') = 15000;

*original: X.UP('I2') = 2700;
X.UP('I2') = 3000;

X.UP('I3') = 1100;
X.UP('I4') = 2300;
X.UP('I5') = 1700;
X.UP('I6') = 9500;
X.UP('I7') = 1950;
X.UP('I8') = 3300;
X.UP('I9') = 3000;
X.UP('I10') = 3000;

*original: X.UP('I11') = 1350;
X.UP('I11') = 2000;

X.UP('I12') = 1275;
X.UP('I13') = 3300;
X.UP('I14') = 2500;
X.UP('I15') = 3000;

*original: X.UP('I16') = 1200;
X.UP('I16') = 2000;

X.UP('I17') = 1650;
X.UP('I18') = 425;
X.UP('I19') = 1650;
X.UP('I20') = 150;


MODEL WStep1 to determine stochastic profit
*/ALL/;
/
```

```
OBJFNC
DEMAND
YIELD_CON
Feed1,
Feed14,
PDU_14_16,
PDU_14_17,
PDU_14_20,
FB_2_11,
FB_2_16,
FB_5_12,
FB_5_18,
UB_8,
UB_14,
UB_17,
UB_18,
UB_6
*Sn_eqn
/;


OPTION LIMROW = 100000;
OPTION LIMCOL = 100000;

*SOLVE WStep1 USING DNLP MAXIMIZING OBJ;
SOLVE WStep1 USING DNLP MAXIMIZING OBJ;

EXECUTE_UNLOAD 'WStep1.GDX', Sn;
EXECUTE 'GDXXRW.EXE WStep1.GDX O=WStep1.XLS
VAR=Sn RNG=SHEET4!A68';


MODEL WStep2
*/ALL/;
/
Sn_eqn
*OBJFNC
DEMAND
YIELD_CON
Feed1,
Feed14,
PDU_14_16,
PDU_14_17,
PDU_14_20,
FB_2_11,
FB_2_16,
FB_5_12,
FB_5_18,
UB_8,
UB_14,
UB_17,
UB_18,
UB_6
/
;

SOLVE WStep2 USING DNLP MINIMIZING Sn;

N = ( Z_ALPHA*Sn.L/( (OBJ.L +
Z_ALPHA*Sn.L/SQRT(CARD(S)) ) - (OBJ.L -
Z_ALPHA*Sn.L/SQRT(CARD(S)) ) ) )**2


MODEL CVaR
/
OBJFNC_RISK
DEMAND
YIELD_CON
Feed1,
Feed14,
```

```
PDU_14_16,
PDU_14_17,
PDU_14_20,
FB_2_11,
FB_2_16,
FB_5_12,
FB_5_18,
UB_8,
UB_14,
UB_17,
UB_18
UB_6

CONSTRAINT_1_PRICE_UNCERTAINTY
CONSTRAINT_1_DEMAND_YIELD_UNCERTAINTY
CONSTRAINT_2_PRICE_UNCERTAINTY
CONSTRAINT_2_DEMAND_YIELD_UNCERTAINTY
/
;

LOOP ( COUNTER,
WEIGHT1 = WEIGHT1 + 0.1;
WEIGHT2 = WEIGHT2 + 0.1;


SOLVE CVaR USING LP MAXIMIZING OBJ_RISK;

ACTUAL_ACTUAL_OBJ_FNC_VALUE_DETERMINISTI
C = SUM ( IP, DETERMINISTIC_PRICE(IP)*X.L(IP) );
ACTUAL_OBJ_FNC_VALUE_STOCHASTIC = SUM (
(S,IP), (1/CARD(S))*PRICE(S,IP)*X.L(IP) );

*DISPLAY X.L, OBJ.L, Y.L , Z.L;
DISPLAY X.L, Y.L , Z.L, N, U1.L,U2.L, VaR1.L, VaR2.L,
OBJ_RISK.L,
ACTUAL_ACTUAL_OBJ_FNC_VALUE_DETERMINISTI
C, ACTUAL_OBJ_FNC_VALUE_STOCHASTIC;

);    # end of LOOP
```

APPENDIX IV: GAMS MODELING CODE (CVaR2)

```
$TITLE  Find flow rate
$EOLCOM #

SETS

I     material streams   /I1*I20/

K     DEMAND SHORTFALL OR SURPLUS
/
K1     demand shortfall
K2     demand surplus
/

M     yield shortfall and surplus     /M1, M2/

COUNTER     /1*20/

$ontext
IY1('I1',IY)
/
I1.(I7, I4, I8, I9, I10, I20, I16, I17)
/
$offtext
;
```

```
$onecho >taskin.txt
dset=S rng=Sheet4!A16:A65 rdim=1
dset=ID rng=Sheet4!B15:F15 cdim=1
dset=IY rng=Sheet4!H15:L15 cdim=1
dset=IP rng=Sheet4!Q15:W15 cdim=1
par=D rng=Sheet4!A15:F65 cdim=1 rdim=1
par=Yield rng=Sheet4!G15:L65  cdim=1 rdim=1
par=Price rng=Sheet4!P15:W65 cdim=1 rdim=1
$offecho

$call gdxxrw.exe WStep1.xls @taskin.txt

$gdxin WStep1.gdx

Sets

S(*)      scenario
IP(I)     material with price uncertainty
ID(I)     material with demand uncertainty
IY(I)     material with yield uncertainty;

$load S ID IY IP
display S, ID, IY, IP;

ALIAS (S,S1)
;

SCALARS

Z_ALPHA /1.96/
DELTA_PRICE     /0.5/
DELTA_DEMAND    /0.5/
DELTA_YIELD    /0.5/
WEIGHT1 /0/
WEIGHT2 /0/
WEIGHT3 /0/
WEIGHT4 /0/
WEIGHT5 /0/
;

PARAMETERS

D(S,ID)       demand
N
ACTUAL_OBJ_FNC_VALUE_DETERMINISTIC
ACTUAL_OBJ_FNC_VALUE_STOCHASTIC

Yield(S,IY)   yield
Price(S,IP)   price
$load D Yield Price
display D, Yield, Price;
$gdxin


PARAMETERS

DETERMINISTIC_PRICE(IP)
/
I1     -8
I2     18.5
I3     8.0
I4     12.5
I5     14.5
I6     6
I14    -1.5
/

PRICE_DEMAND(ID)
/
```

```
I2     18.5
I3     8.0
I4     12.5
I5     14.5
I6     6
/


DETERMINISTIC_DEMAND(ID)
/
I2     2700
I3     1100
I4     2300
I5     1700
I6     9500
/

PRICE_YIELD(IY)
/
I7      8.0       #(ASSUME price of naphtha(I7) to be the same
as naphtha(I3))
I4     12.5
I8     14.5     #(ASSUME price same as heating oil (I5))
I9      1.5       #(ASSUME price same as cracker feed(I14) but
the positive value)
I10    6        #(ASSUME price same as fuel oil (I6))
/

DETERMINISTIC_YIELD(IY)
/
I7     0.13
I4     0.15
I8     0.22
I9     0.20
I10    0.30
/

Table    Penalty_Demand(ID,K)    TABLE    OF    PENALTY
DEMAND
      K1        K2
I2     25        20
I3     17        13
I4     5         4
I5     6         5
I6     10        8;

Table Penalty_Yield(IY,M)  TABLE OF PENALTY YIELD
      M1        M2
I7      5         3
I4      5         4
I8      5         3
I9      5         3
I10     5         3;



FREE Variables

OBJ         OBJECTIVE FUNCTION
Sn
OBJ_RISK
;


Positive Variables

X(I)        MATERIAL FLOWRATE
Y(IY,S,M)  SHORT FALL OR SURPLUS FOR YEILD
Z(ID,S,K)   SHORT FALL OR SURPLUS FOR DEMAND
VaR1
```

```
VaR21
VaR22
VaR31
VaR32

U1(S)
U21(S)
U22(S)
U31(S)
U32(S)

;


Equations

OBJFNC
Sn_eqn
OBJFNC_RISK

DEMAND(S,ID) Demand,
YIELD_CON(S,IY)    Yield,
Feed1,
Feed14,
PDU_14_16,
PDU_14_17,
PDU_14_20,
FB_2_11,
FB_2_16,
FB_5_12,
FB_5_18,
UB_8,
UB_14,
UB_17,
UB_18,
UB_6

*CVaR constraints
CONSTRAINT_1_PRICE_UNCERTAINTY
CONSTRAINT_1_DEMAND_UNCERTAINTY_SHORTF
ALL
CONSTRAINT_1_DEMAND_UNCERTAINTY_SURPLUS
CONSTRAINT_1_YIELD_UNCERTAINTY_SHORTFALL
CONSTRAINT_1_YIELD_UNCERTAINTY_SURPLUS

CONSTRAINT_2_PRICE_UNCERTAINTY
CONSTRAINT_2_DEMAND_UNCERTAINTY_SHORTF
ALL
CONSTRAINT_2_DEMAND_UNCERTAINTY_SURPLUS
CONSTRAINT_2_YIELD_UNCERTAINTY_SHORTFALL
CONSTRAINT_2_YIELD_UNCERTAINTY_SURPLUS


;


OBJFNC..    OBJ    =e=    SUM((S,IP),(CARD(S)**(-
1))*PRICE(S,IP)*X(IP))    -(SUM((ID,S,K),(CARD(S)**(-
1))*(Penalty_Demand(ID,K)*Z(ID,S,K)))              +
SUM((IY,S,M),(CARD(S)**(-
1))*(Penalty_Yield(IY,M)*Y(IY,S,M)))) ;

*Nga:  OBJFNC..  OBJ  =e=  SUM((S,IP),(CARD(S)**(-
1))*PRICE(S,IP)*X(IP))    -(SUM((ID,S,K),(CARD(S)**(-
1))*(Penalty_Demand(ID,K)*Z(ID,S,K)))              +
SUM((IY,S,M),(CARD(S)**(-
1))*(Penalty_Yield(IY,M)*Y(IY,S,M)))) ;


**LIMITATIONS OF PLANT CAPACITY
```

```
Feed1..      X('I1') =L= 15000;
Feed14..     X('I14') =L= 2500;

*mbl..X('1')-(X('2')+X('3')+X('4')+X('5')+X('6'))=E=0  ;

*Reformulated stochastic constraints to account for uncertain
yield coefficient
YIELD_CON(S,IY)..   -YIELD(S,IY)*X('I1')   +   X(IY)   +
Y(IY,S,'M1') - Y(IY,S,'M2') =E= 0;


$ontext
e1..      -0.13*X('I1') + X('I7') - Y(IY,S,'M1') + Y(IY,S,'M2')
=E= 0;
e1(IY,'I1',S)  $  IY1('I1',IY)..          -YIELD(IY,S)*X('I1')  +
X(IY) - Y(IY,S,'M1') + Y(IY,S,'M2') =E= 0;
e1(IY,'I14',S)..             -YIELD(IY,S)*X('I14')   +   X(IY)   -
Y(IY,S,'M1') + Y(IY,S,'M2') =E= 0;

X('I1') = 100
X('I7') = 14

-13 + 14 - Y(IY,S,'M1') + Y(IY,S,'M2') =E= 0
Y(IY,S,'M1') = 1
Y(IY,S,'M2') = 0

-0.05*X('I14') + X('I20') =E= 0
;


e2..     -0.15*X('I1') + X('I4') =E= 0;
$offtext



****************************************************
****************************************************
*****************
*FIXED    YIELDS    FOR    CRACKER    (deterministic
constraints)
****************************************************
****************************************************
*****************

PDU_14_20..  -0.05*X('I14') + X('I20') =E= 0;
PDU_14_16..  -0.40*X('I14') + X('I16') =E= 0;
PDU_14_17..  -0.55*X('I14') + X('I17') =E= 0;
FB_2_11..    0.5*X('I2') - X('I11') =E= 0;
FB_2_16..    0.5*X('I2') - X('I16') =E= 0;
FB_5_12..    0.75*X('I5') - X('I12') =E= 0;
FB_5_18..    0.25*X('I5')- X('I18') =E= 0;
UB_8..       -X('I7') + X('I3') + X('I11') =E= 0;
UB_14..      -X('I8') + X('I12') + X('I13') =E= 0;
UB_17..      -X('I9') + X('I14') + X('I15') =E= 0;
UB_18..      -X('I17') + X('I18') +X('I19') =E= 0;
UB_6..        -X('I10') - X('I13') - X('I15') - X('I19') + X('I6')
=E= 0;


****************************************************
****************************************************
*****************
**CONSTRAINTS ON PRODUCTION DEMANDS
****************************************************
****************************************************
*****************

DEMAND(S,ID)..  X(ID)  +  Z(ID,S,'K1')-Z(ID,S,'K2')  =E=
D(S,ID);
```

```
Sn_eqn..        Sn  =E=  SQRT(SUM(S,(ABS(1100.911-
(SUM(IP,PRICE(S,IP)*X(IP))-
(SUM((ID,K),Penalty_Demand(ID,K)*Z(ID,S,K))          +
SUM((IY,M),Penalty_Yield(IY,M)*Y(IY,S,M))))))**2)/49  )
;


*for price uncertainty:


************************************************
************************************************
*****************
*CVaR
************************************************
************************************************
*****************

$ontext
OBJFNC_RISK..  OBJ_RISK  =e=  SUM  (  (S,IP),
(1/CARD(S))*PRICE(S,IP)*X(IP) )
                - 0.0001 * ( VaR1 + ( 1 /(1 - 0.95)) *
SUM ( (S,IP),(1/CARD(S)) * ( PRICE(S,IP)*X(IP) - VaR1 )
) )
                -
SUM(S,(1/CARD(S))*(SUM((ID,K),Penalty_Demand(ID,K)
*Z(ID,S,K))            +            SUM((IY,M),
Penalty_Yield(IY,M)*Y(IY,S,M))))
                - 0.01 * ( VaR21 + ( 1 / (1 - 0.95) ) *
SUM  (  S,  (1/CARD(S))  *  SUM  (  (ID),
Penalty_Demand(ID,'K1')*Z(ID,S,'K1') - VaR21 ) ) )
                - 0.01 * ( VaR22 + ( 1 / (1 - 0.95) ) *
SUM  (  S,  (1/CARD(S))  *  SUM  (  (ID),
Penalty_Demand(ID,'K2')*Z(ID,S,'K2') - VaR22 ) ) )
                - 0.01 * ( VaR31 + ( 1 / (1 - 0.95) ) *
SUM  (  S,  (1/CARD(S))  *  SUM  (  (IY),
Penalty_Yield(IY,'M1')*Y(IY,S,'M1') - VaR31 ) ) )
                - 0.01 * ( VaR32 + ( 1 / (1 - 0.95) ) *
SUM  (  S,  (1/CARD(S))  *  SUM  (  (IY),
Penalty_Yield(IY,'M1')*Y(IY,S,'M2') - VaR32 ) ) )
;


*try different values of theta:
OBJFNC_RISK..  OBJ_RISK  =e=  SUM  (  (S,IP),
(1/CARD(S))*PRICE(S,IP)*X(IP) )
                - WEIGHT1 * ( VaR1 + ( 1 /(1 - 0.95)) *
SUM ( (S,IP),(1/CARD(S)) * ( PRICE(S,IP)*X(IP) - VaR1 )
) )
                -
SUM(S,(1/CARD(S))*(SUM((ID,K),Penalty_Demand(ID,K)
*Z(ID,S,K))            +            SUM((IY,M),
Penalty_Yield(IY,M)*Y(IY,S,M))))
                - WEIGHT2 * ( VaR21 + ( 1 / (1 - 0.95)
)  *  SUM  (  S,  (1/CARD(S))  *  SUM  (  (ID),
Penalty_Demand(ID,'K1')*Z(ID,S,'K1') - VaR21 ) ) )
                - WEIGHT3 * ( VaR22 + ( 1 / (1 - 0.95)
)  *  SUM  (  S,  (1/CARD(S))  *  SUM  (  (ID),
Penalty_Demand(ID,'K2')*Z(ID,S,'K2') - VaR22 ) ) )
                - WEIGHT4 * ( VaR31 + ( 1 / (1 - 0.95)
)  *  SUM  (  S,  (1/CARD(S))  *  SUM  (  (IY),
Penalty_Yield(IY,'M1')*Y(IY,S,'M1') - VaR31 ) ) )
                - WEIGHT5 * ( VaR32 + ( 1 / (1 - 0.95)
)  *  SUM  (  S,  (1/CARD(S))  *  SUM  (  (IY),
Penalty_Yield(IY,'M1')*Y(IY,S,'M2') - VaR32 ) ) )
;
$offtext
```

```
* 19apr10: with clear separation between the 1st stage and
2nd stage
OBJFNC_RISK.. OBJ_RISK =E=
* 1st stage:
*SUM ( IP, DETERMINISTIC_PRICE(IP)*X(IP) ) - SUM (
ID,
PRICE_DEMAND(ID)*DETERMINISTIC_DEMAND(ID)
)            -            SUM            (            IY,
PRICE_YIELD(IY)*DETERMINISTIC_YIELD(IY) )
* 2nd stage:
SUM ( (S,IP), (1/CARD(S))*PRICE(S,IP)*X(IP) )
                        - WEIGHT1 * ( VaR1 + ( 1 /(1 - 0.95)) *
SUM ( (S,IP),(1/CARD(S)) * ( PRICE(S,IP)*X(IP) - VaR1 )
) )
                        -
SUM(S,(1/CARD(S))*(SUM((ID,K),Penalty_Demand(ID,K)
*Z(ID,S,K))            +            SUM((IY,M),
Penalty_Yield(IY,M)*Y(IY,S,M))))
                        - WEIGHT2 * ( VaR21 + ( 1 / (1 - 0.95)
)  *  SUM  (  S,  (1/CARD(S))  *  SUM  (  (ID),
Penalty_Demand(ID,'K1')*Z(ID,S,'K1') - VaR21 ) ) )
                        - WEIGHT3 * ( VaR22 + ( 1 / (1 - 0.95)
)  *  SUM  (  S,  (1/CARD(S))  *  SUM  (  (ID),
Penalty_Demand(ID,'K2')*Z(ID,S,'K2') - VaR22 ) ) )
                        - WEIGHT4 * ( VaR31 + ( 1 / (1 - 0.95)
)  *  SUM  (  S,  (1/CARD(S))  *  SUM  (  (IY),
Penalty_Yield(IY,'M1')*Y(IY,S,'M1') - VaR31 ) ) )
                        - WEIGHT5 * ( VaR32 + ( 1 / (1 - 0.95)
)  *  SUM  (  S,  (1/CARD(S))  *  SUM  (  (IY),
Penalty_Yield(IY,'M1')*Y(IY,S,'M2') - VaR32 ) ) )
;


CONSTRAINT_1_PRICE_UNCERTAINTY..
       VaR1 + ( 1 /((1 - 0.95)*CARD(S)) ) * SUM ( S, U1(S)
)        =L=        DELTA_PRICE*SUM        (IP,
DETERMINISTIC_PRICE(IP) )
;

CONSTRAINT_1_DEMAND_UNCERTAINTY_SHORTF
ALL..
       VaR21 + ( 1 /((1 - 0.95)*CARD(S)) ) * SUM ( S,
U21(S)  )  =L=  DELTA_DEMAND*(  SUM  (  ID,
PRICE_DEMAND(ID)*DETERMINISTIC_DEMAND(ID)
) )
;
CONSTRAINT_1_DEMAND_UNCERTAINTY_SURPLUS
..
       VaR22 + ( 1 /((1 - 0.95)*CARD(S)) ) * SUM ( S,
U22(S)  )  =L=  DELTA_DEMAND*(  SUM  (  ID,
PRICE_DEMAND(ID)*DETERMINISTIC_DEMAND(ID)
) )
;
CONSTRAINT_1_YIELD_UNCERTAINTY_SHORTFALL
..
       VaR31 + ( 1 /((1 - 0.95)*CARD(S)) ) * SUM ( S,
U31(S)  )  =L=  DELTA_YIELD*  (SUM  (  IY,
PRICE_YIELD(IY)*DETERMINISTIC_YIELD(IY) ) )
;
CONSTRAINT_1_YIELD_UNCERTAINTY_SURPLUS..
       VaR32 + ( 1 /((1 - 0.95)*CARD(S)) ) * SUM ( S,
U32(S)  )  =L=  DELTA_YIELD*  (SUM  (  IY,
PRICE_YIELD(IY)*DETERMINISTIC_YIELD(IY) ) )
;

CONSTRAINT_2_PRICE_UNCERTAINTY(S)..      U1(S)
=G= - SUM ( IP, (1/CARD(S))*PRICE(S,IP)*X(IP) ) - VaR1
;
```

```
CONSTRAINT_2_DEMAND_UNCERTAINTY_SHORTF
ALL(S)..                U21(S)   =G=   -   SUM  (   (ID),
(1/CARD(S))*Penalty_Demand(ID,'K1')*Z(ID,S,'K1')   )   -
VaR21
;
CONSTRAINT_2_DEMAND_UNCERTAINTY_SURPLUS
(S)..               U22(S)   =G=   -   SUM  (   (ID),
(1/CARD(S))*Penalty_Demand(ID,'K2')*Z(ID,S,'K2')   )   -
VaR22
;

CONSTRAINT_2_YIELD_UNCERTAINTY_SHORTFALL
(S)..               U31(S)   =G=   -    SUM  (   (IY),
(1/CARD(S))*Penalty_Yield(IY,'M1')*Y(IY,S,'M1')    )   -
VaR31
;
CONSTRAINT_2_YIELD_UNCERTAINTY_SURPLUS(S)
..               U32(S)   =G=   -    SUM  (   (IY),
(1/CARD(S))*Penalty_Yield(IY,'M2')*Y(IY,S,'M2')    )   -
VaR32
;


* Nga's solution from cumumlative density function:
*Var1 = 72400;
*Var2 = 173200;

$ontext
*CVaR for price uncertainty
CVaR1_constraint_1..     CVaR1 =E= Var1 + (1/(1-0.95)) *
SUM ( (S,IP), P(S) * ( PRICE(S,IP)*X(IP) - Var1 ) )
;

*CVaR for demand and yield uncertainty
CVaR2_constraint_1..     CVaR2 =E= Var2 + (1/(1-0.95)) *
SUM ( (S,IP), P(S)*(PRICE(S,IP)*X(IP) - Var2 ) )
;

*AUXILIARY VARIABLES

CVaR1_constraint_2..    VaR1 + ( (1 - 0.95)**(-1) * SUM (
S, P(S)*U(S) ) ) =G= CVaR1.LO
;
CVaR2_constraint_2..    VaR2 + ( (1 - 0.95)**(-1) * SUM (
S, P(S)*U(S) ) ) =G= CVaR2.LO
;

CVaR_constraint_1(S)..   U(S) =L= 0
;

*not sure what the following constraints are and where they
are obtained
CVaR1_constraint_3(S)..     U(S) =L= PRICE(S,'1')*X('1') -
VaR1
;
CVaR2_constraint_3(S)..     U(S) =L= PRICE(S,'1')*X('1') -
VaR2
;

CVaR1.L = 0
;
CVaR2.L = 0
;
$offtext


*DECISION  VARIABLE BOUNDS
*X.UP(I) = 12100;
Y.UP(IY,S,M) = 1500;
*VaR1.UP = 1E5;
```

```
*VaR2.UP = 1E5;

*Initial values
X.L('I1') = 12500;
*X.L('I2') = 2700;
X.L('I3') = 625;
X.L('I4') = 1875;
X.L('I5') = 1700;
X.L('I6') = 6175;
X.L('I7') = 1625;
X.L('I8') = 2750;
X.L('I9') = 2500;
X.L('I10') = 3750;
*X.L('I11') = 1000;
X.L('I12') = 1275;
X.L('I13') = 1475;
X.L('I14') = 2500;
X.L('I15') = 0;
*X.L('I16') = 1000;
X.L('I17') = 1375;
X.L('I18') = 425;
X.L('I19') = 950;
X.L('I20') = 125;

U1.L(S) = 100;
U21.L(S) = 100;

* Upper bounds of variables
X.UP('I1') = 15000;

*original: X.UP('I2') = 2700;
X.UP('I2') = 3000;

X.UP('I3') = 1100;
X.UP('I4') = 2300;
X.UP('I5') = 1700;
X.UP('I6') = 9500;
X.UP('I7') = 1950;
X.UP('I8') = 3300;
X.UP('I9') = 3000;
X.UP('I10') = 3000;

*original: X.UP('I11') = 1350;
X.UP('I11') = 2000;

X.UP('I12') = 1275;
X.UP('I13') = 3300;
X.UP('I14') = 2500;
X.UP('I15') = 3000;

*original: X.UP('I16') = 1200;
X.UP('I16') = 2000;

X.UP('I17') = 1650;
X.UP('I18') = 425;
X.UP('I19') = 1650;
X.UP('I20') = 150;

* lower bounds on auxiliary variables
U1.LO(S) = 0.2;
U21.LO(S) = 0.2;
U22.LO(S) = 0.2;
U31.LO(S) = 0.2;
U32.LO(S) = 0.2;

* initial values on auxiliary variables
*U1.L(S) = 0.1;
```

```
MODEL WStep1 to determine stochastic profit
*/ALL/;
/
OBJFNC
DEMAND
YIELD_CON
Feed1,
Feed14,
PDU_14_16,
PDU_14_17,
PDU_14_20,
FB_2_11,
FB_2_16,
FB_5_12,
FB_5_18,
UB_8,
UB_14,
UB_17,
UB_18,
UB_6
*Sn_eqn
/;


OPTION LIMROW = 100000;
OPTION LIMCOL = 100000;

*SOLVE WStep1 USING DNLP MAXIMIZING OBJ;
SOLVE WStep1 USING DNLP MAXIMIZING OBJ;

EXECUTE_UNLOAD 'WStep1.GDX', Sn;
EXECUTE 'GDXXRW.EXE WStep1.GDX O=WStep1.XLS
VAR=Sn RNG=SHEET4!A68';


MODEL WStep2
*/ALL/;
/
Sn_eqn
*OBJFNC
DEMAND
YIELD_CON
Feed1,
Feed14,
PDU_14_16,
PDU_14_17,
PDU_14_20,
FB_2_11,
FB_2_16,
FB_5_12,
FB_5_18,
UB_8,
UB_14,
UB_17,
UB_18,
UB_6
/
;

SOLVE WStep2 USING DNLP MINIMIZING Sn;

N = ( Z_ALPHA*Sn.L/( (OBJ.L +
Z_ALPHA*Sn.L/SQRT(CARD(S)) ) - (OBJ.L -
Z_ALPHA*Sn.L/SQRT(CARD(S)) ) ) )**2

DISPLAY N;

MODEL CVaR
/
OBJFNC_RISK
```

```
DEMAND
YIELD_CON
Feed1,
Feed14,
PDU_14_16,
PDU_14_17,
PDU_14_20,
FB_2_11,
FB_2_16,
FB_5_12,
FB_5_18,
UB_8,
UB_14,
UB_17,
UB_18
UB_6

CONSTRAINT_1_PRICE_UNCERTAINTY

CONSTRAINT_1_DEMAND_UNCERTAINTY_SHORTF
ALL
CONSTRAINT_1_DEMAND_UNCERTAINTY_SURPLUS
CONSTRAINT_1_YIELD_UNCERTAINTY_SHORTFALL
CONSTRAINT_1_YIELD_UNCERTAINTY_SURPLUS

CONSTRAINT_2_PRICE_UNCERTAINTY
CONSTRAINT_2_DEMAND_UNCERTAINTY_SHORTF
ALL
CONSTRAINT_2_DEMAND_UNCERTAINTY_SURPLUS
CONSTRAINT_2_YIELD_UNCERTAINTY_SHORTFALL
CONSTRAINT_2_YIELD_UNCERTAINTY_SURPLUS


/
;

LOOP( COUNTER,
WEIGHT1 = WEIGHT1 + 0.1;
WEIGHT2 = WEIGHT2 + 0.1;
WEIGHT3 = WEIGHT3 + 0.1;
WEIGHT4 = WEIGHT4 + 0.1;
WEIGHT5 = WEIGHT5 + 0.1;

SOLVE CVaR USING LP MAXIMIZING OBJ_RISK;

ACTUAL_OBJ_FNC_VALUE_DETERMINISTIC = SUM (
IP, DETERMINISTIC_PRICE(IP)*X.L(IP) );
ACTUAL_OBJ_FNC_VALUE_STOCHASTIC = SUM (
(S,IP), (1/CARD(S))*PRICE(S,IP)*X.L(IP) );

DISPLAY X.L, Y.L , Z.L, N, U1.L,U21.L, U22.L, U31.L,
U32.L, VaR1.L, VaR21.L, VaR22.L, VaR31.L, VaR32.L,
OBJ_RISK.L,
ACTUAL_OBJ_FNC_VALUE_DETERMINISTIC,
ACTUAL_OBJ_FNC_VALUE_STOCHASTIC;

);    # end of LOOP
```