Remaining Fatigue Life Assessment by Using Crack Growth Model

by

Nuratiqah binti Ab Manaf

Dissertation submitted in partial fulfilment of
the requirements for the
Bachelor of Engineering (Hons)
(Mechanical Engineering)

DECEMBER 2013

University Teknologi PETRONAS
Bandar Seri Iskandar
31750 Tronoh
Perak Darul Ridzuan

CERTIFICATION OF APPROVAL

Remaining Fatigue Life Assessment by Using Crack Growth Model

by

Nuratiqah binti Ab Manaf

A project dissertation submitted to the
Mechanical Engineering Programme
Universiti Teknologi PETRONAS
in partial fulfillment of the requirement for the
BACHELOR OF ENGINEERING (Hons.)
(MECHANICAL ENGINEERING)

Approved by,

____

(DR. AINUL AKMAR BINTI MOKHTAR)

UNIVERSITI TEKNOLOGI PETRONAS

TRONOH, PERAK

December 2013

CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

____

NURATIQAH BINTI AB MANAF

ABSTRACT

Fatigue has become major causes of deterioration among mechanical equipments. Study shows that 90 percent of all mechanical services failure was contributed by fatigue. For this reason, conducting fatigue reliability analysis was essential in maintaining the availability and reliability of the system. The Remaining Fatigue Life Assessment by using Crack Growth Model (CGM) has been conducted as an initiative to prevent fatigue failure. The objective of the study is to estimate the fatigue crack growth over the time period, and develop the Crack Growth Model by using spreadsheet that assists the engineers in preventing fatigue damage. In this project, the analysis was focused at the crack initiation and propagation on the material surface. The crack advance was depending on the number of work cycles and year of services. As soon as the applied load exceeding fatigue limit depending on the material properties, the possibility of sudden fracture is very high. The framework is prototyped into an analyzing tool using Microsoft Excel and Microsoft Visual Basic. Monte Carlo Simulation has been majorly applied in this analysis. It was referred as deterministic method by using series of random numbers. This project work would be expected to contribute in assisting engineers in predicting the next possible failure and monitoring the reliability level of the equipments. This studies help in improving reliability, efficiency, and productivity of the system.

ACKNOWLEDGEMENT

Praise to Allah the Almighty that in His will, I successfully completed The Remaining Fatigue Life Assessment by Using Crack Growth Model Project within the allocated eight months in Universiti Teknologi PETRONAS (UTP).

First and foremost, deepest thank goes to my parents, Mr. Ab Manaf bin Mendan and Madam Anum binti Samsudin for their constant love and support; emotionally and financially throughout my journey in becoming a Mechanical Engineer. I would like to dedicate a sincere gratitude to my brother and sisters for their continually and conveyed a spirit of adventure in completing my studies together with this final year project. Deepest thanks to my beautiful supervisor, Dr Ainul Akmar binti Mokhtar for her guidance, advice, knowledge and criticism in guiding me during the progression of this project. My sincere appreciation also goes to UTP's Graduate Assistance, Mr. Mohamad Mohaimin bin Mokhlis, for his assistance in conducting simulations using complicated software for the project. Besides, I would like to thank all UTP lecturers and staff who involve in completing the project either directly or indirectly.

Not to forget, deepest thanks go to all my friends for making this university life enjoyable and valuable experience for me. Thank you very much.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

ABBREVIATION AND NOMENCLATURE

a	crack depth
α	traffic increase rate per year
ADTT	average daily truck transfer
C	fatigue coefficient
CGM	Crack Growth Model
F_e	crack shape factor
F_s	free surface effect factor
F_s	finite width factor
F_g	non-uniform stress factor
FRA	Fatigue Reliability Analysis
G(a)	non-dimensional function
ΔK	stress intensity factor
K_{tm}	stress concentration factor
m	fatigue exponent/ material constant
MATLAB	Matrix Laboratory
n_i	number of observation
N	number of cycles
N_{total}	total number of observation up to the kth range
PoD	Probability Of Detection Model
S_{reff}	effective stress range
S_{ri}	stress range bin
t_f	flange thickness
t_{cp}	cover plate thickness
y	number of year
z	weld size

CHAPTER 1

INTRODUCTION

1.1 BACKGROUND OF THE STUDY

This project was conducted mainly for fatigue reliability analysis based on crack growth model. This chapter would explanation on fatigue mechanism, history of the discovery, and the impact of fatigue failure.

1.1.1 Fatigue Phenomenon

Fatigue is defined as the weakening of a material caused by repeated applied loads. In engineering perspective, fatigue is used to describe damage due to repeated loading and unloading application and its effect on the strength and structural integrity of a structural member (Ayyub et al, n.d.). Normally both metallic and polymeric materials are susceptible to fatigue failure, while ceramics and concretes tend to provide more resistance.

Fatigue damage is occurred when the mechanical stress applied is above a limiting value known as the fatigue limit. As the applied stress exceeding the fatigue limit, microcracks will accumulate on the surface tension of the material. O'connor (2012) described the formation of fatigue crack was resulting from the energy transferred to the crystal boundaries by the cyclic stress. The initiation and propagation of the cracks varies depending on the surface and internal condition of the material.

According to the Xiong (2011) in Fatigue and Fracture Reliability Engineering, there are three stages of fatigue damage.

- Crack initiation – crack formation resulting from the plastic stresses in localized area.
- Crack propagation – crack growth along these lines of weakness, which act as the stress concentrator.
- Final fracture – unstable crack advance which results in sudden fracture.

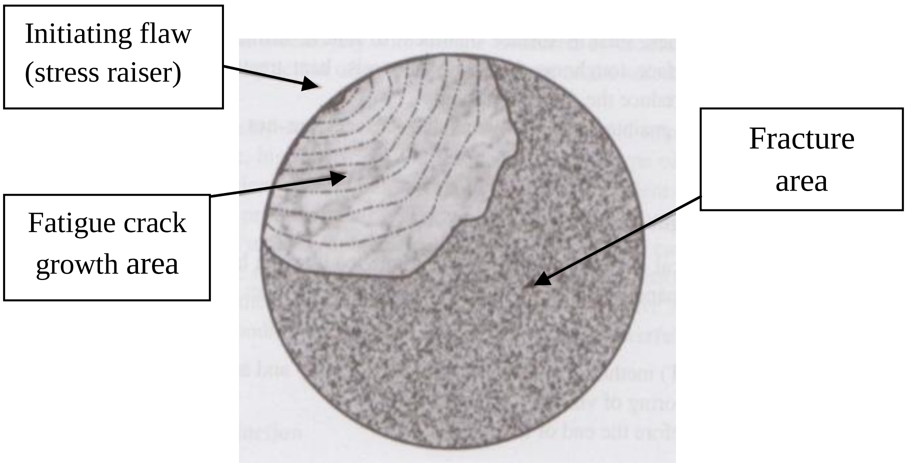


FIGURE 1.1: Typical fatigue failure (schematic)

The schematic diagram of crack propagation as shown in Figure 1.1 illustrates the stages of fatigue damage. Over period of time and constant reciprocating work load, the propagation of crack will take place that lead to sudden fracture. Without proper maintenance action, this incident may cause catastrophic damage to the system and fatality to the surrounding people. Figure 1.2 shows fatigue damage on the crack arm of a bicycle.

FIGURE 1.2: Fatigue on a bicycle crank spider arm

The black mark formed on the cross sectional area of the crack arm indicates the slow crack growth pattern of the crack initiation and propagation. When the applied stress exceeds the material fatigue limit, the crack propagation will become unstable and resulting in sudden fracture.

FIGURE 1.3 : The upper part of Boeing 737-297 after the incident

There are various catastrophic accidents emanating from fatigue failure. Figure 1.3 shows an air craft accident back in 1998. A Boeing 737-297 suffered extensive damage where a large section of the roof was tore off from the airplane while travelling from Hilo to Honolulu. Fortunately, the airplane was able to land safely at Kahului Airport on Maui. There was only a fatality was reported while others 94 passengers and crews were survived. The investigation found the main cause of the accident was due to the fatigue cracking (Walter, 2001). A few fatigue cracks were found among many rivet holes from the right wing.

FIGURE 1.4 : Capsized Alexander L. Kielland oil platform in Norwegian

Similarly, Alexander L. Kielland oil platform capsized incident in March 1980, as shown in Figure 1.4. The pentagon-type, semi-submersible drilling rig capsized while working in the poor weather in the Ekofisk Field for Phillips Petroleum. There were 212 men aboard and only 89 survived the accident. The investigations concluded that fatigue fracture had developed in the lower horizontal bracing. After the failure, the remaining five braces attached to the leg failed to support the quick succession causing the leg to break off (House, 2006).

These incidents have shown what fatigue is capable off. Although the crack size is very small and unnoticeable, it may lead to catastrophic destruction to the system. Therefore, the failure that caused by fatigue mechanism should not be taken lightly.

1.1.2 History of Fatigue

Fatigue is not a new phenomenon in the engineering field. For the past 200 years, various experiments and analysis were performed in order to reveal its true nature. Sir William Fairbairn (1789-1874) and August Wohler (1819-1914) were among the earliest analysts to investigate the fatigue failure. The history of fatigue begins in 1837 where Wilhelm Albert published the very first fatigue-test results. Albert constructed a test machine for the conveyor chains which had failed to perform its task in the Clausthal mines. In that particular time, the hemp road that used to replaced the conveyor chain only available in high cost which resulting in the discovery of wire rope (Schutz, 1996).

In 1839, Jean-Victor Poncelet has explained fatigue failure as the material becoming ‘tired’ and no longer able to withstand the design load. Four years later, Rankine; better known from thermodynamics by the “Rankine process”, and Glynn were the first to identify crack growth as the key mechanism to fatigue failure. The finding was made while they were reviewing the catastrophic Versailles rail accident in Paris.

Then, the term “fatigue” was first mentioned by Braithwaite in 1854 (Collin, 2008). In 1870, August Wohler provided the first schematic investigation of S-N curve. He concludes that the cyclic stress range is more important than peak stress and introduces the concept of endurance limit. All of these efforts in investigating fatigue phenomenon continue to this day. Table 1.1 lists the chronology of the recent knowledge advancement in understanding and modelling fatigue as a mechanism of failure.

TABLE 1.1: Chronology of the recent development in fatigue mechanism

Year	Investigator(s)	Development
1945	Miner	Damage accumulation models
1951	Zappfe and Worden	Identification of fatigue striations
1954	Coffin-Manson	Strain-based characterisation of fatigue
1957	Irwin	linear elastic fracture mechanics approach
1961	Paris, Gomez and Anderson	
1962	Laird and Smith	Conceptual and quantitative modelling
1966	Weertman	
1969	Newman	
1970	Elber	Fatigue crack closure
1975	Pearson	
1984	Suresh and Ritchie	
1990	Bannantine, Comer and Handrock (review)	Total life approaches

1.1.3 A Case Study of Fatigue Failure on Mechanical Equipment - Piston

Studies show that approximately 90 percents of all mechanical service failure was contributed by fatigue (ASM International, 2008). The remaining 10 percents was occupied by other failure mechanism such as corrosion, insulation, installation, maintenance failure and etc. Fatigue mechanism is more common with any moving parts such as gear, pistons, and turbine.

A case study on engine piston has been carried out by F.S. Silva (2006) from the University of Minho, Guimarães, Portugal. The objective of the study is to identified the causes and consequences of fatigue on a mechanical equipment which to be specific; an engine piston.

Engine pistons are one of the most complex components in automotive parts. It is the most important part in an engine. There are lots of research have been done on the geometry, material and manufacturing technique that contribute to the continuous improvements of the pistons. However, the number of damage pistons still considered to be huge. Figure 1.5 and 1.6 shows the crack initiation on the petrol and diesel engine piston respectively. The crack initiated from one side of the pin hole to the piston head.

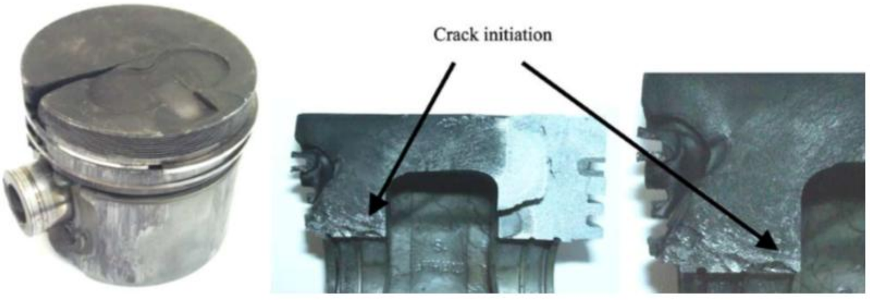


FIGURE 1.5: Crack initiations on the petrol engine piston

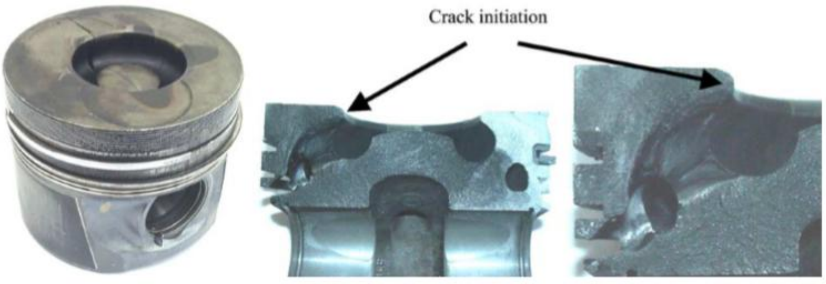


FIGURE 1.6: Crack initiations on the diesel engine piston

The metallographic analysis was made according two different classifications; a) mechanical and high temperature mechanical fatigue, and b) thermal-mechanical fatigue. Mechanical and high temperature mechanical fatigue analysis was based on the crack initiation and propagation in a critical stressed area due to the piston movement. Figure 1.7 shows the typical stress distribution area on an engine piston. The heat generated from the motion also responsible in reducing fatigue resistance of the material. Thus, promote fatigue failure on the engine piston.

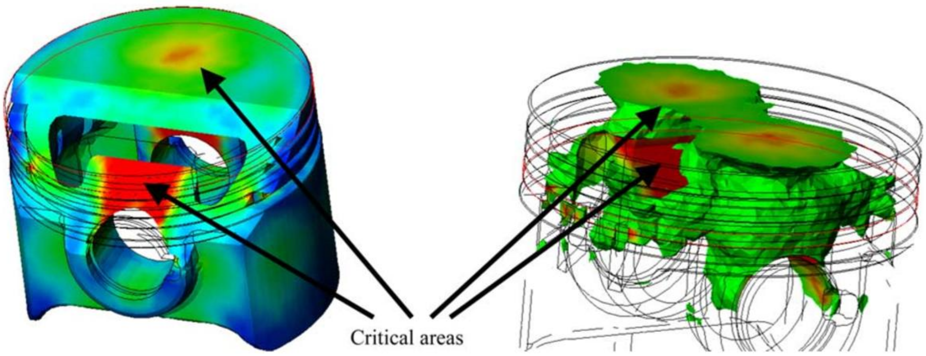


FIGURE 1.7: Typical stress distributions on the engine piston

Thermal-mechanical fatigue is the overlay of a cyclical mechanical loading, which create thermal gradients in the engine piston. There are two ways through which thermal gradients act on the piston; thermal stresses due to the vertical distribution of the temperature along the piston, and thermal stresses due to the difference temperatures at the head of the piston due to the flow of the hot air or fuel impingement. Form this case study, both analyses clearly had shown the presence of fatigue mechanism in any mechanical components and fatigue damage has been proved to be the reason to major parts of the mechanical service failure.

1.1.4 A Case Study of Fatigue Failure on a Ship Structural

According to Ayyub et al (n.d.), from University of Maryland, marine and offshore structures are dominant in fatigue failure due to the action of seawater waves and its environments. A research has been conducted on the reliability-based design for fatigue of a ship structure. The main objective was to present the accurate guideline in conducting fatigue reliability analysis. There were two major approaches used for predicting fatigue life; a) crack growth model (CGM), and b) the S-N curve approach.

The crack growth model approach is based on the crack initiation and propagation on the subjected structure. This particular approach would help in predicting fatigue life by determining the number of cycles required to grow the crack to a certain unstable growth; the final fracture phase. The next approach, S-N curve is based on the experimental measurement on fatigue life by the number of cycles to failure for different loading levels and specimen geometries. Both of the approaches had been proven to provide accurate and reliable results for fatigue reliability analysis.

1.2 PROBLEM STATEMENT

After a few years of services, every mechanical equipments or components will experiences deterioration. It is important to understand the courses of failure and the reliability level of the components.

Fatigue damage might occur as stated earlier where 90 percent of the mechanical failure was due to fatigue mechanism. By performing fatigue life analysis, the next failure of the component can be estimate. Thus, maintenance action can be taken to prevent the failure. Besides, this analysis would helps in identifying component with low reliability level. This fatigue reliability analysis not only forecasting and preventing fatigue damage, it also helps in reducing the total maintenance cost of the company.

However, the means to carry out the fatigue reliability analysis is very complicated and time consuming. Due to this matter, failure due to fatigue mechanism has been taken for granted.

1.3 OBJECTIVE OF THE STUDY

These are the objective of this study:

2 To estimate the fatigue crack growth over the time period.

3 To develop the Crack Growth Model by using spreadsheet that assists the engineers in preventing fatigue damage.

1.4 SCOPE OF THE STUDY

In this project, the crack growth model will be the only approach used to perform the fatigue reliability analysis. The basic theory that governs the crack growth model is given by the Paris law that will be discussed further throughout this report.

There are two methods involves in conduction the analysis; theoretical and analytical. The theoretical analysis was conducted by using Monte Carlo simulation. The simulator was fully built by the author using Microsoft Excel. In analytical analysis, the results were generated by software named MatLab. The difference between the theoretical and analytical methods will be observed.

Once the crack growth model is proven to be accurate and reliable, the development of the fatigue analysis tool will be started. The analyzer will be using Microsoft Excel format so that applicable for every user.

1.5 RELEVANCY OF THE PROJECT

The project is useful for the industry as it is a tool in determining reliability level of any mechanical equipments and structures. It also provides necessary information in forecasting the next possible failure based on the crack size. Therefore, mitigation action can be executed before damage was happened. Thus, increase the system reliability level, keep maintenance cost at the optimum level and improve the safety environment at the work place.

1.6 FEASIBILITY OF THE PROJECT

With the given time frame of approximately 7 months, the project was implemented to its best potential. The main source of information which is a journal of Bridge Fatigue Assessment and Management Using Reliability-Based Crack Growth is readily available online. Books and encyclopaedias related to fatigue reliability analysis are accessible from the library. The tools used for building the framework and designing the programming guidelines such as Microsoft Excel and MatLab are also easily access.

CHAPTER 2

THEORY AND LITERATURE REVIEW

2.1 THEORY

2.1.1 Paris Law

Paris law is used as the governing equations for the crack growth model. It was first published by Paris et al, (1961), centred on the life prediction for fatigue cracks. It is a set of linear partial differential equation that described the fatigue crack extension, da/dN, in a function of stress intensity factor range, ΔK. The Paris law can be expressed in the equation below;

$$\frac{da}{dN} = C\,.\,(\Delta K)^m \tag{1}$$

where a is the crack size, N is the number of cycles, C is the fatigue coefficient and m is the material constant, respectively. The stress intensity factor range; also known as Irwin stress, is the loading parameter for on a crack. Figure 2.1 illustrates a typical fatigue-panel configuration that used in determining the stress intensity factor. For a crack with the size of 2a, subjected to remote stress, σ, the stress intensity factor produced is (Suo, 2013);

$$K = Y.\,\sigma.\sqrt{\pi.\,a} \tag{2}$$

where Y is the dimensionless parameter that depends on the physical geometry and σ is the uniform tensile stress perpendicular to the crack plane. L is the length of the specimen and b is half of the panel width.

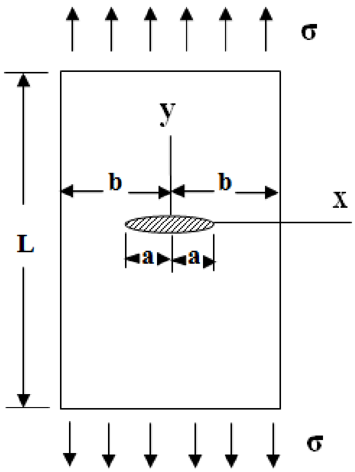


FIGURE 2.1: Typical fatigue-panel configurations

The stress intensity factor range, ΔK, is the difference between the stress intensity factor at maximum and minimum level. In relation to the Irwin stress intensity factor, Paris made a hypothesis, that the crack extension per cycle, da/dN, is a function of instantaneous K_{max} and K_{min} .

$$\frac{da}{dN} = f(\Delta K, R) \tag{3}$$

$$\Delta K = K_{max} - K_{min} = \sigma_{max} . \sqrt{\pi . a} - \sigma_{min} . \sqrt{\pi . a} \tag{4}$$

$$R = K_{max} / K_{min} \tag{5}$$

To prove this hypothesis, Paris et al, (1961) plotted the experimental data of da/dN vs. ΔK from three different investigators for two different materials. FIGURE 2.2 and 2.3 show the results of the experiments for 2024-T3 and 7075-T6 aluminium alloy. According to Paris et al., (1961), the experiments were conducted using many specimen sizes, i.e., widths from 1.8 to 12 in., thickness from 0.032 to 0.102 in., and lengths from 5 to 35 in. The maximum stresses on the gross area ranging from 6 to 30 ksi and the testing frequencies ranging from 50 to 2000 cpm. On each graph, the materials are both clad metals and bare metals. Therefore, the correlation shown is more than a coincidental.

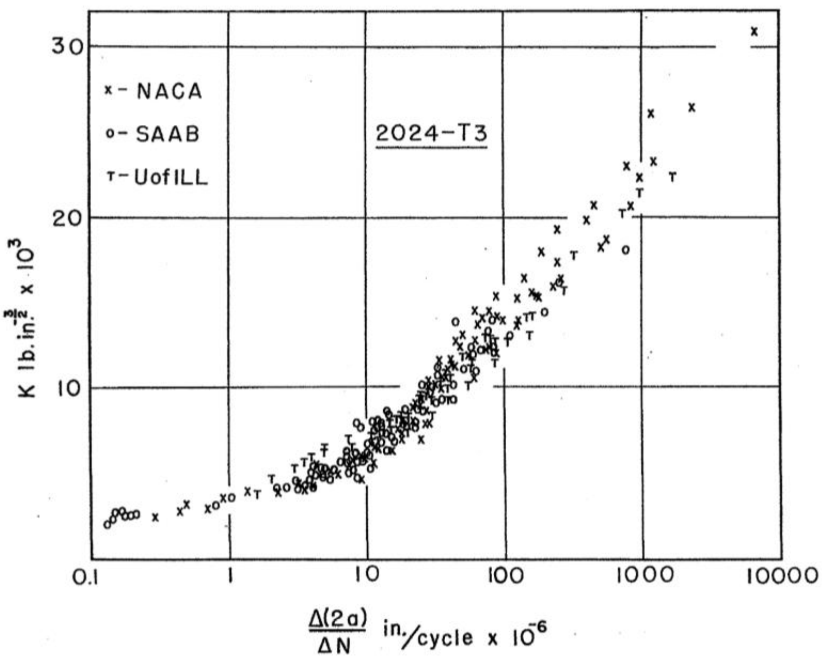


FIGURE 2.2: Crack extension rate data on 2024-T3 aluminium alloy

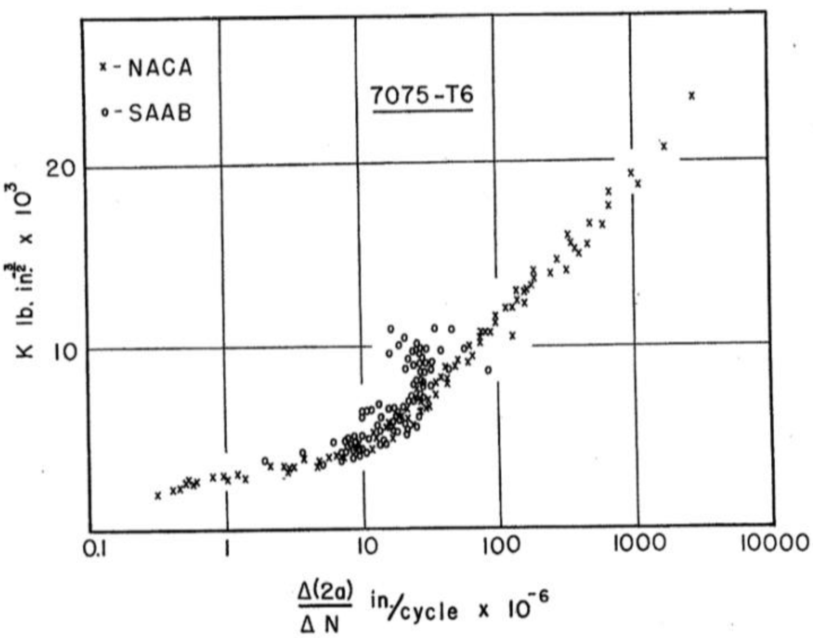


FIGURE 2.3: Crack extension rate data on 7075-T6 aluminium alloy

Based on the results, both materials showed similar growth pattern. The crack extension rates per cycle growth slowly at the earlier plot before rapidly increase with the stress intensity factor.

With these experimental data, Paris also plotted the da/dN vs. ΔK graph on a double logarithmic basis (Suo, 2013) as shown in Figure 2.4. The straight line of the plotted graph advocates that the experimental data fits the power law.

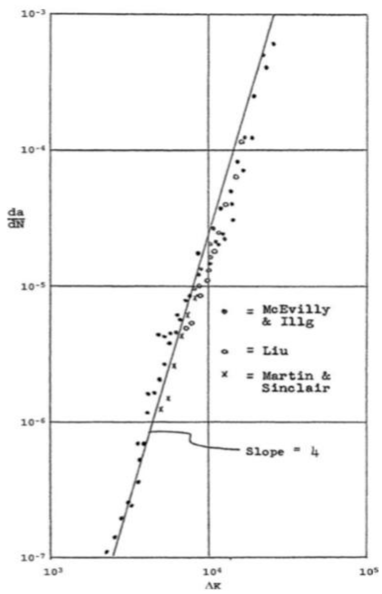


FIGURE 2.4: The double log da/dN vs. ΔK graph

Further studies on the double log da/dN vs. ΔK data had improvised the graph to be the graph as shown in Figure 2.5. The graph was divided into three phase which are the threshold, middle and fast fracture phase. Threshold phase indicates the crack initiation stage of fatigue failure. It is followed with the crack propagation in the middle phase and lastly, the final fracture phase where the where the facture subject to occur.

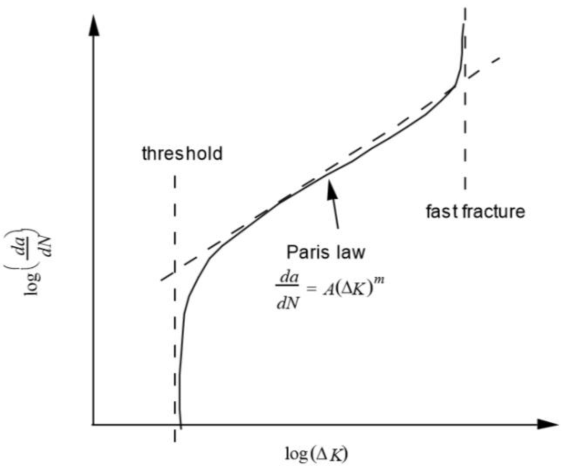


FIGURE 2.5: Paris law for crack growth rate

2.2 LITERATURE REVIEW

2.2.1 Crack Growth Model

Crack growth model is a life prediction module for fatigue mechanism. According to Righiniotis and Chryssanthopoulus (2003), this crack growth module have all relied on the standard Paris law approximation to measure crack growth pattern with respect to the number of applied cycles. Figure 2.6 shows the crack pattern for most often on the planes, i.e., planes perpendicular for maximum-principle tension stress (Paris et al., 1961).

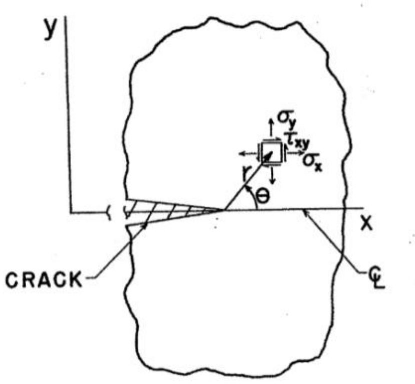


FIGURE 2.6: Coordinate used to describe stress near a crack tip

There are several reasons in selecting crack growth model as the main approach of the studies. Firstly, this Remaining Fatigue Life Assessment project was the continuation on a previous case study; Bridge Fatigue Assessment and Management Using Reliability-based Crack Growth and Probability of Detection Models. The case study was performed by Kihyon Kwon and Dan M. Frangopol at ATLSS Engineering Research Center, Lehigh University, USA.

According to Kwon (2011), the study was focused on conducting lifetime performance assessment and management of aging steel bridges under fatigue mechanism. It used the combination of three different approaches in conducting the analysis which are; a) the fatigue reliability model, b) the crack growth model, and c) the probability of detection model. Figure 2.7 demonstrates the work flow of the combined approaches.

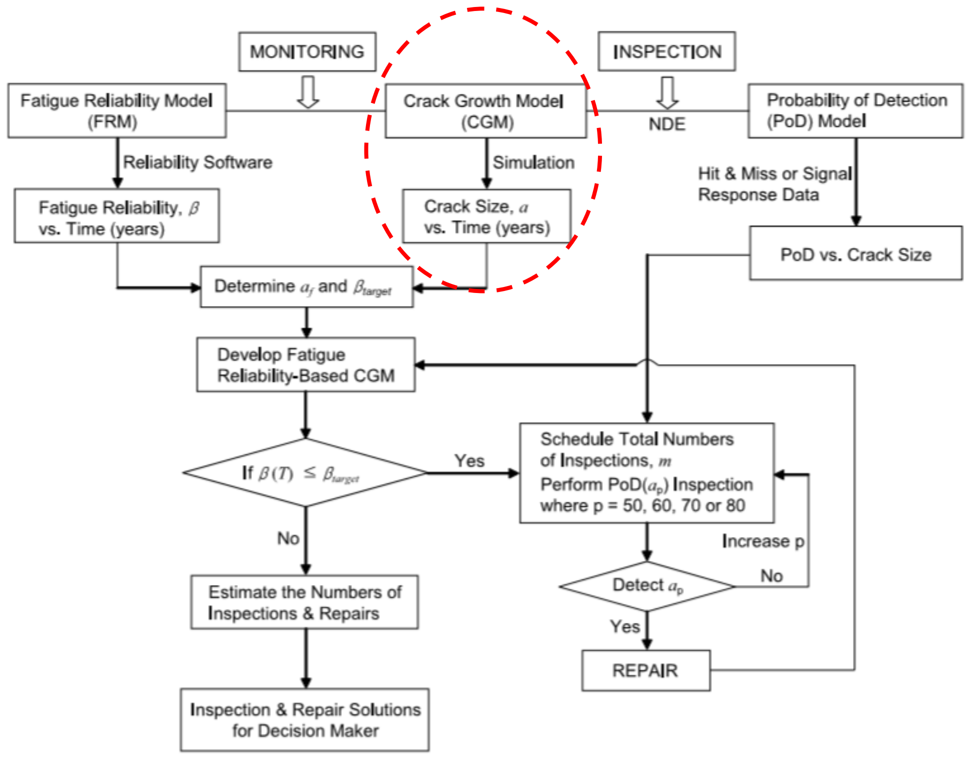


FIGURE 2.7: Flowchart of the combined approach

As the continuation on the research, this Remaining Fatigue Life Assessment project was purposely focused on the crack growth model. The red circle shows the expected outcome of the project which is the crack size verses time graph.

Secondly, the crack growth model was selected due to its capability in predicting possible failure by using cracks propagation. According to Pungo et al., (2006), since the fracture mechanics was found, more research has been done, i.e., to predict or at least understand the propagation of cracks. Studies show that the crack propagation speed was far from being constant in time. Besides, the crack advance was larger for increasing stress amplitudes. These complex parameters were only applicable in the crack growth model approach. Therefore, it is found to be more effective and suitable in current engineering practice.

In this crack growth model approach, the possibility of crack-like defects being present at the start of service life has to be recognized. Even if the component is not cracked to start with, the remaining life after initiation may be significant or possibly dominant with respect to the total life.

In order to support the Paris law, there are various parameters and equations related to crack size was included in crack growth model (Kwon, 2011). The stress intensity factor range, ΔK, used in the studies is;

$$\Delta K(a) = G(a).\,Sreff.\sqrt{\pi.a} \qquad (6)$$

where a is the crack size, Sreff is the effective stress range, and G(a) is the dimensionless function of the geometry including various factors, i.e., finite width factor, non-uniform stress factor, free surface effect factor, and crack shape factor. In the previous ΔK equation, the value of Sreff and G(a) were represent by σ and Y, respectively. The effective stress range, Sreff can be measured based on equation (7);

$$S_{reff} = \left(\sum_{i=1}^{k} n_i / N_{total}\ .\ S_{ri}^{m}\right)^{1/m} \qquad (7)$$

where n_i in the number of observations in the *i*th predefined stress range bin, S_{ri}, and N_{total} is total number of observations up to the *k*th range during the monitoring period. The dimensionless function, G(a) can be expressed in terms of crack size as;

$$G(a) = F_e(a).\,F_s(a).\,F_w(a).\,F_g(a) \qquad (8)$$

$$F_g(a) = K_{tm}.\,[1 + 6.7889(a/t_f)^{0.4348}]^{-1} \qquad (9)$$

$$K_{tm} = -3.539\ln\,(z/t_f) + 1.981\ln\,(t_{cp}/t_f) + 5.798 \qquad (10)$$

where $F_e(a)$ is the crack shape factor, $F_s(a)$ is the free surface effect factor = 1.211-0.186.$\sqrt{\pi.c}$, for c = 5.462.$a^{1.133}$, $F_w(a)$ is the finite width factor, and $F_g(a)$ is the non uniform stress factor. The parameter z, t_f and t_{cp} are the weld size, flange thickness, and cover plate thickness, respectively. These deterministic parameters are gathered by using inspection. When the number of cycle has been obtained, the number of service year was calculated by;

$$y = \ln[N(a)\ .\ \ln(1+\alpha) + 365\ .\ ADTT] - \ln(365\ .\ ADTT) - \ln(1+\alpha) \qquad (11)$$

where y is the number of service years, N(a) is the number of cycles, α is the traffic increase rate per year, and ADTT is the is the average daily truck traffic considering a single stress cycle per truck passage (cycle per day). The derivation of the equations was further explained in the Appendix A.

2.2.2 Monte Carlo Simulation

Monte Carlo Simulation was the main part of the analysis. By using the combination of Equation (6) to (10) with Paris Law, the function of N has ben generated as shown in Equation(12). Due to its complexiy, it is impossible to solve the equation by using normal integration. Therefore, Monte Carlo simulation was used as the solving medium.

$$N(a) = \frac{1}{C.S_{reff}^{m}} \int_{a_i}^{a_f} \frac{1}{[F_e(a).1.211-0.186\sqrt{a/c}.F_w(a).K_{tm}.[1+6.7889\left({}^{a}/_{t_f}\right)^{0.4348}]^{-1}.\sqrt{\pi.a}]^m}\, da \qquad (12)$$

Monte Carlo Simulation is a method that uses random sampling to study properties of system with component that behave in random and deterministic pattern. This method provides approximate solutions to a vearity of mathematical problems by performing statistical sampling experimentations. It is very useful when the system complexity make the formulation of exact models essentially impossible.

According to Pilana, The modern Monte Carlo method was found by Stanislaw Ulam in 1944. It was named after the famous casino in the Monaco principality because of roulette. Stanislaw Ulam is a mathematician from Poland who participated in the study of thermonuclear weapon. He has implemented the Monte Carlo method for evaluate complicated mathematical integrals in the nuclear chain reactions theory. It is then led to the more systematic development by John Von Neumann. Figure 2.8 shows the picture of Stanislaw Ulam and John Von Neumann, respectively.

FIGURE 2.8: Pictures of the Pioneers of Monte Carlo Method

The real purpose of Monte Carlo method is to perform direct simulation of the probabilistic problems concerned with random neutron diffusion of the atomic bomb during the World War II. In 1970s, research on Monte Carlo method has been continued by John Von Neumann. The newly developed electronic computing techniques began to provide a more precise and persuasive rationale for Monte Carlo method. The implementation of Monte Carlo can be seen in the determination of the value of π.

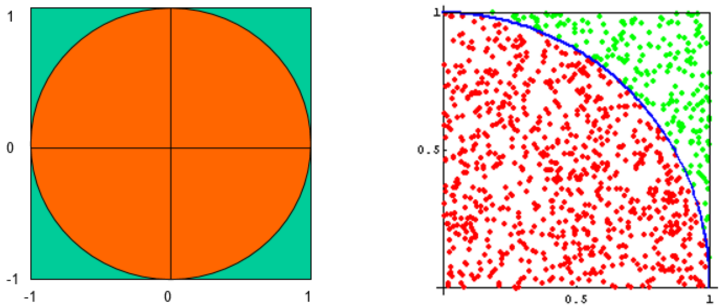


FIGURE 2.9: The area used in determining π value

A square that compasses a circle with the radius of 1 cm has been constructed as shown in FIGURE 2.9. The area of the square is 4 cm2, and the area of the circle is π cm2. Then, series random points had been generated in one of the region. The total number of points that hit within the square, the number of points that hit the shaded part (circle quadrant) is proportional to the area of that part. In other words;

$$\frac{\text{Number of points in the circle quadrant}}{\text{Number of points inside the square}} = \frac{\text{Area of shaded area}}{\text{Area of square}}$$

$$\frac{\text{Number of points in the circle quadrant}}{\text{Number of points inside the square}} = \frac{1/4\pi r^2}{r^2} = \frac{1}{4}\pi$$

$$\pi = 4\,\frac{\text{Number of points in the circle quadrant}}{\text{Number of points inside the square}}$$

By using Monte Carlo simulation, 1000 random points has been taken as the sample size. There were 787 points under the $x_i^2 + y_i^2 \leq 1$ curve. From this data, the value of π can be obtain;

$$\pi = 4\frac{787}{1000} = 3.148$$

The value generated from Monte Carlo simulation, 3.148 is closed to the standard π value which is 3.142. Therefore, the Monte Carlo probabilistic approach by using random numbers is approved to be acceptable and reliable.

The simulator has to be build personally based on the case study. It required full understanding on the concep. FIGURE 2.10 shows the commen steps taken in conducting the Monte Calo simulation as expressed by O'connor (2012).

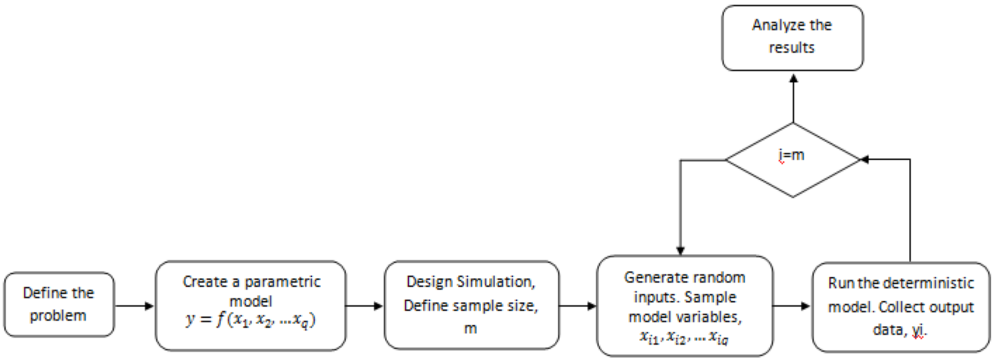


FIGURE 2.10: Monte Carlo simulation process

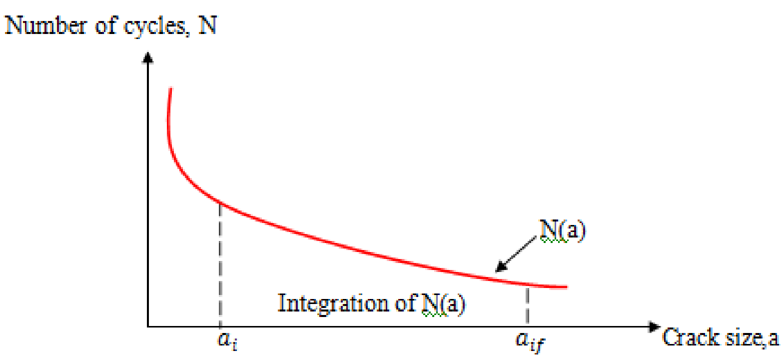


FIGURE 2.11: The integration of N(a) equation

For fatigue reliability analysis, the Monte Carlo approach has been illustrated in Figure 2.11. The graph shows the number of cycle, N against crack size, a graph. The curve is the relationship function of N(a) with respect of crack size. Area under the curve indicates the deterministic value of N after integration. Monte Carlo simulation used probabilistic methods by counting random crack sizes that lays under the curve region. Then, the number of years taken for every crack advance was determined based on the number of cycles.

In ensuring the accuracy of the results, another simulation was conducted by using MatLab simulator. MatLab (Matrix Laboratory) Simulation is a numerical computing environment that developed in proggraming language. Its allow matrix manipulations, plotting of function and data , implimentation of algorithms,and creation of user interfaces. Results obtained from the Monte Carlo simulation will be compared and discussed againts the results from MatLab simulation.

In order to measure accuracy and reliability of the results, the simulation have been carried out for several time. The results gathered will be discussed further throughout the report.

CHAPTER 3

METHODOLOGY

3.1 THE ANALYSIS FRAMEWORK

3.1.1 Crack Growth Model

Based on the observation and analysis performed for the remaining fatigue life assessment project, a Crack Growth Model (CGM) was developed. Figure 3 shows the flowchart for fatigue reliability evaluation by using CGM. As discussed in the project background, most of the case studies implemented crack growth model as their main approach. CGM was applicable in any mechanical components and structure.

The first step taken was defining the governing equations. Observations were made on various case studies relevant to CGM. It is proven that Paris law is able to measure the crack growth rate by using the number of applied cycles. After that, Monte Carlo simulation was conducted to solve the theoretical problems. The probabilistic analysis was conducted by using spreadsheet. Further explanations on Monte Carlo simulation were made on the sub-chapter.

In order to validate the results, MatLab simulation has been carried out. The same governing equations and parameters were inputted into the software. This computerised method had also generated the crack growth graph with respect to the number of applied cycles. Both results then will be compared and analysed for its similarities. When the analysis procedure is considered correct, this crack growth model it will be published as the Fatigue Reliability Analyzer (FRA).

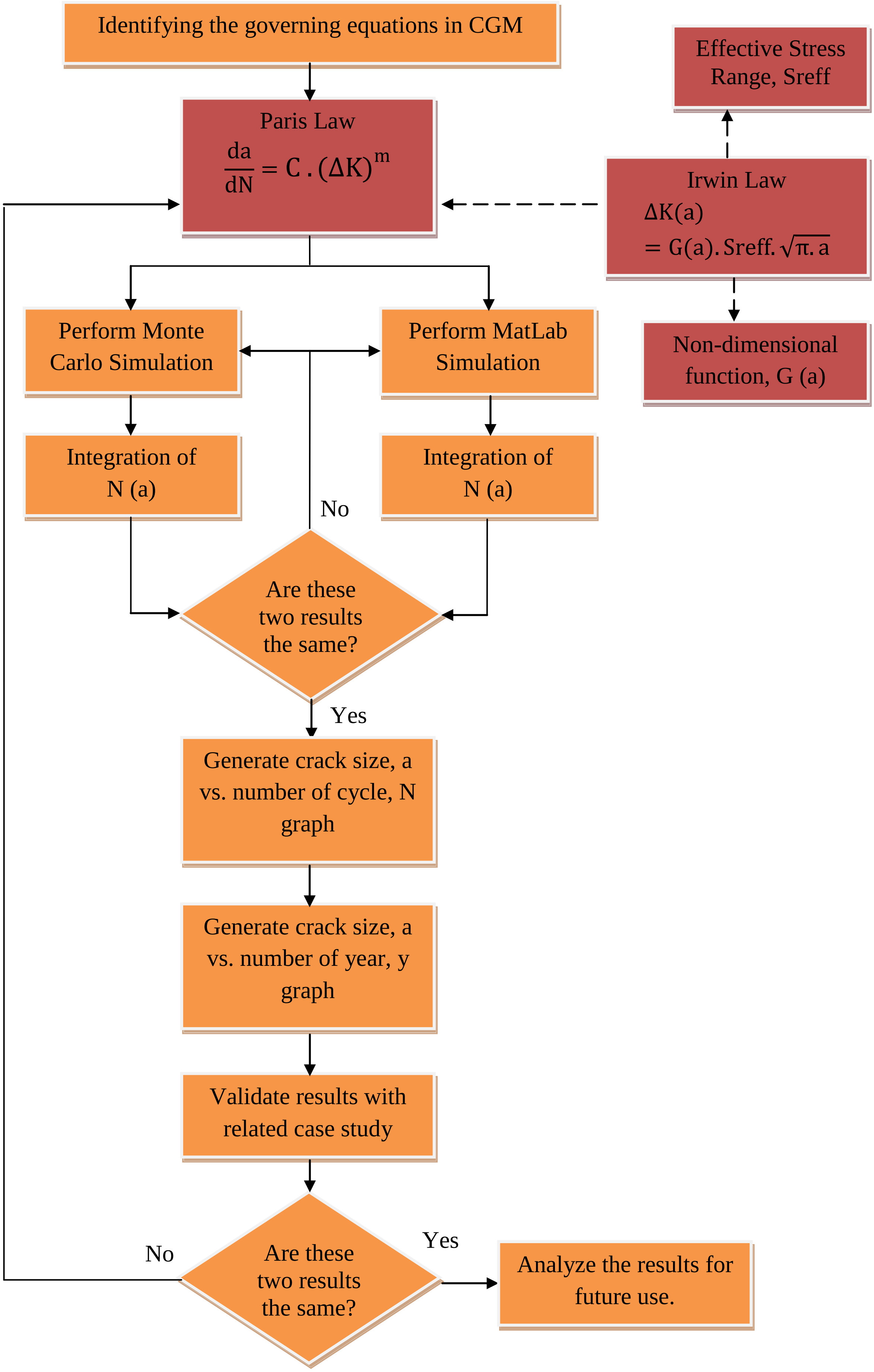
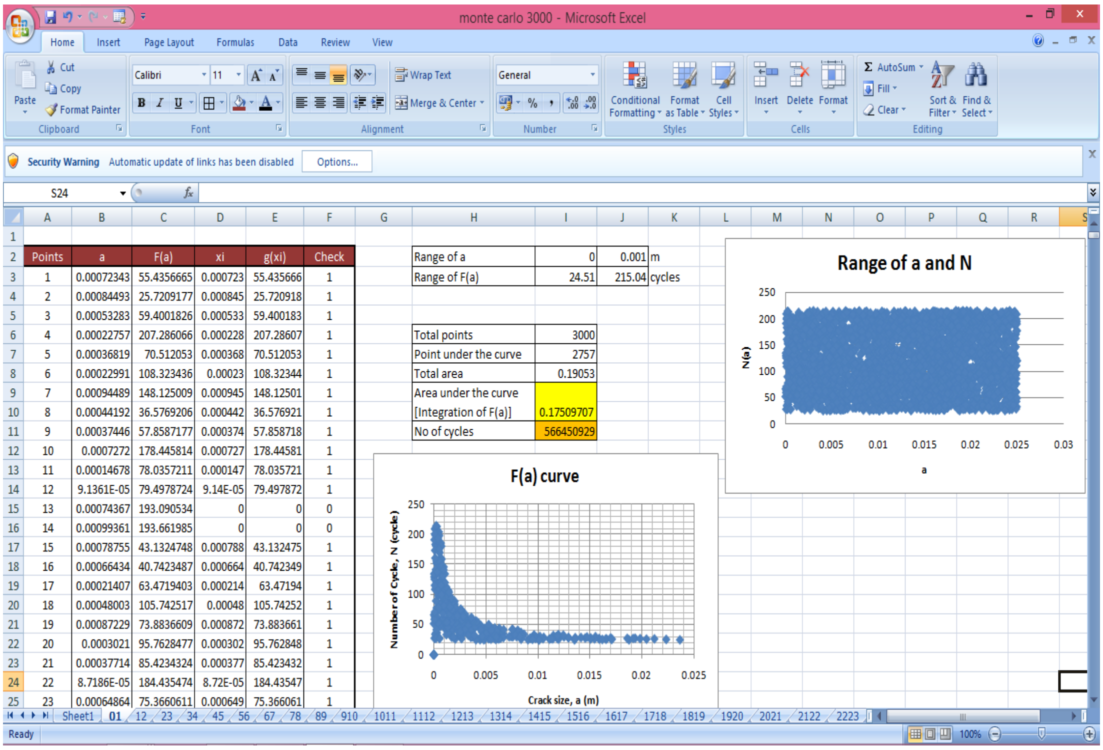


Figure 3.1: Flowchart of Crack Growth Model

3.1.2 The Construction Of Monte Carlo Simulator

Monte Carlo simulation is used to solve the integration in the Paris law. The simulation was conducted by using Microsoft Excel. The range of cracks size, a, has been assumed to be from 0 to 25 mm. From there, the range of number of cycle, N, was generated. Figure 3.2 shows the layout of the spreadsheet.



Points	a	F(a)	xi	g(xi)	Check
1	0.00072343	55.4356665	0.000723	55.435666	1
2	0.00084493	25.7209177	0.000845	25.720918	1
3	0.00053283	59.4001826	0.000533	59.400183	1
4	0.00022757	207.286066	0.000228	207.28607	1
5	0.00036819	70.512053	0.000368	70.512053	1
6	0.00022991	108.323436	0.00023	108.32344	1
7	0.00094489	148.125009	0.000945	148.12501	1
8	0.00044192	36.5769206	0.000442	36.576921	1
9	0.00037446	57.8587177	0.000374	57.858718	1
10	0.0007272	178.445814	0.000727	178.44581	1
11	0.00014678	78.0357211	0.000147	78.035721	1
12	9.1361E-05	79.4978724	9.14E-05	79.497872	1
13	0.00074367	193.090534	0	0	0
14	0.00099361	193.661985	0	0	0
15	0.00078755	43.1324748	0.000788	43.132475	1
16	0.00066434	40.7423487	0.000664	40.742349	1
17	0.00021407	63.4719403	0.000214	63.47194	1
18	0.00048003	105.742517	0.00048	105.74252	1
19	0.00087229	73.8836609	0.000872	73.883661	1
20	0.0003021	95.7628477	0.000302	95.762848	1
21	0.00037714	85.4234324	0.000377	85.423432	1
22	8.7186E-05	184.435474	8.72E-05	184.43547	1
23	0.00064864	75.3660611	0.000649	75.366061	1

Range of a	0	0.001	m
Range of F(a)	24.51	215.04	cycles

Total points	3000
Point under the curve	2757
Total area	0.19053
Area under the curve [Integration of F(a)]	0.17509707
No of cycles	566450929

FIGURE 3.2: Layout of Monte Carlo simulator

More than 3000 points of random numbers were generated within the range. Column B and C shows the coordinate of random points for x-axis and y-axis respectively. These random points will be subjected to the Paris equation that was inserted in column D. While, column E and F will compute the criteria for accept and reject of the points according to the equation.

The equation will act as the curve live. The random point's lies under the curve will be accepted while the points that lie above the curve will be rejected. The accepted points show the area under the graph, which indicates the integration value of the Paris equation. For further explanation regarding Monte Carlo simulation, please refer to Appendix C.

3.2 THE PROJECT MANAGEMENT FRAMEWORK

Methodology can be defined as steps and procedure taken in the process of completing this project. This project plan has been clarified into four main stages; Project Drafting, Project Execution, Project Outcome and Project Close Out.

In Project Drafting, the first step taken was defining the objective of the project. Subsequently, a proposal has been submitted to the supervisor describing the objective, problem statement, relevancy and possible outcome of the project. During this stage, most of the time was spent in collecting and reading useful information and data from any related journal and textbooks. Once the proposal has been approved, the process moves to the next stage.

During Project Execution, analyzing process undergoes by highlighting the important data in the case study. The exact process in conducting fatigue analysis by using CGM first has been identified. Then the theoretical calculation has been performed to order to validate the fatigue analysis process. If the results obtained are similar to the case studies, the fatigue analysis process is considered truthful and reliable. The analysis process is further confirmed by specific software such as MATLAB. Matrix Laboratory (MATLAB) is a high-level language and interactive environment for numerical computation, visualization, and programming. Its helps in analyze data, develop algorithms, and create models and applications. The language, tools, and built-in math functions enable you to explore multiple approaches and reach a solution faster than with spreadsheets or traditional programming languages, such as C or C++ [SS].

In Project Outcome stage, the fatigue analysis process is expressed in a simple tool y using Microsoft Excel. This tool will help the engineer to identify the reliability level of any mechanical equipment. Thus, helps in forecasting the next failure based on the CGM. The analysis can easily perform and the results obtained are reliable by using this tool. The tool is the product of this Remaining Fatigue Life Assessment Project.

Finally, the project documentation was taking place in the Project Close Out stage. Every single data and steps taken in completing this project is recorded in a form of final project report. It will be submitted to the supervisor for learning reference of the future students. Figure 3.3 below is the illustration of the project plan as discussed above.

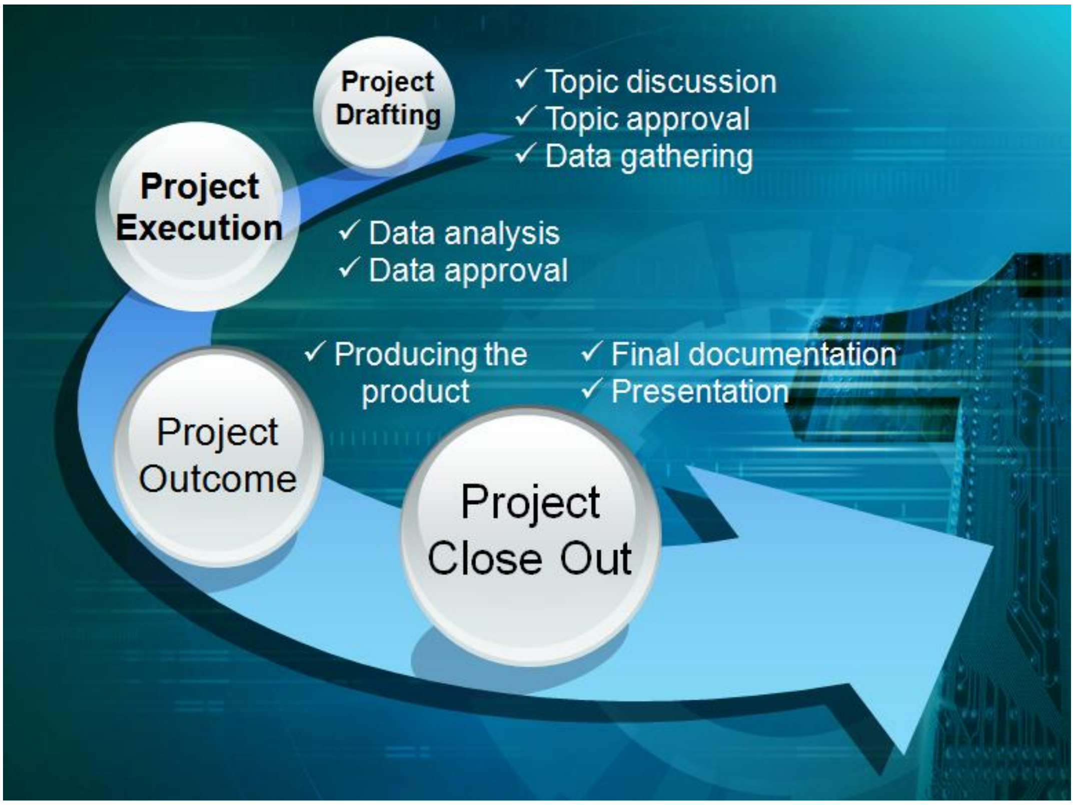


FIGURE 3.3: Flow of the project

3.2.1 Key Milestone

The total duration of this project is precisely 29 weeks. 15 weeks ware allocate for Final Year Project I and 14 weeks for Final Year Project II. Therefore, a key milestone has been made to have an optimum scheduling while undergoing the project.

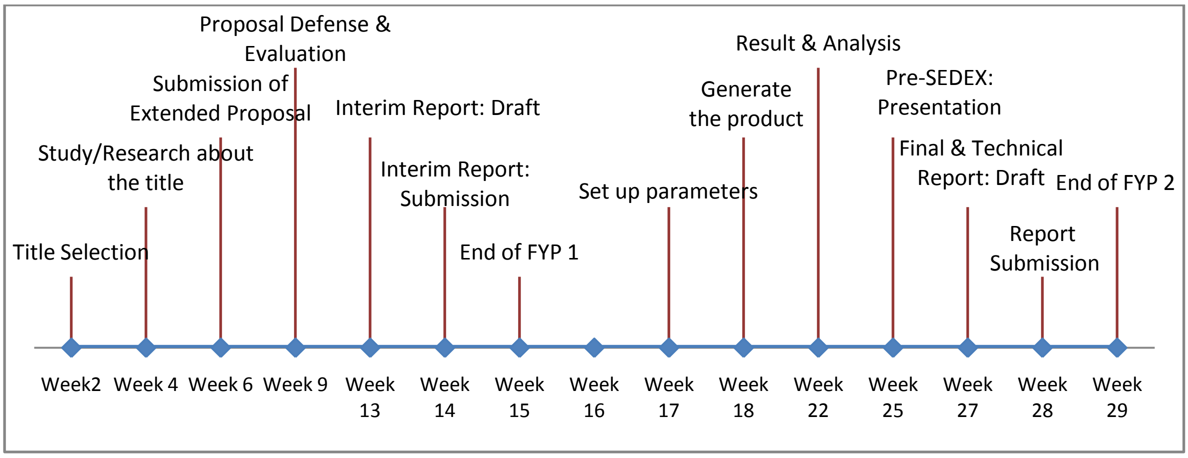


FIGURE 3.4: Key Milestone of the Project

3.2.2 Gantt chart

Activities	Number of Weeks																												
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
FYP I																													
Title Slection	☆	☆																											
Literature Review			☆	☆	☆	☆																							
Extended Proposal Submission						☆	☆																						
Proposal Defence								☆	☆																				
Interim Report: Draft										☆	☆	☆	☆																
Interim Report: Submisstion														☆	☆														
FYP II																													
Setup Parameter																☆	☆												
Process Results and Analysis																		☆	☆	☆	☆								
Product Generation																						☆	☆						
Pre-SEDEX Presentation																									☆				
Final & Technical Report: Draft																										☆	☆	☆	
Final & Technical Report: Submission																													☆

FIGURE 3.5: Gantt chart of The Project

CHAPTER 4

RESULTS AND DISCUSSION

4.1 MONTE CARLO SIMULATION

The results from the simulation have been collected and analyzed throughout this chapter. Figure 4.1 below show the generated graph from Monte Carlo simulation. The shaded area shows the integration of function N(a) with crack size ranged from 0 to 1mm. Hence, every simulation will only provide single value of N; the process has been repeated with different range of crack size. The maximum value of crack size is set at 25mm. Therefore, the simulation has been conducted for 25 times. Most of the graphs generated were similar with the graph in Figure 4.1.

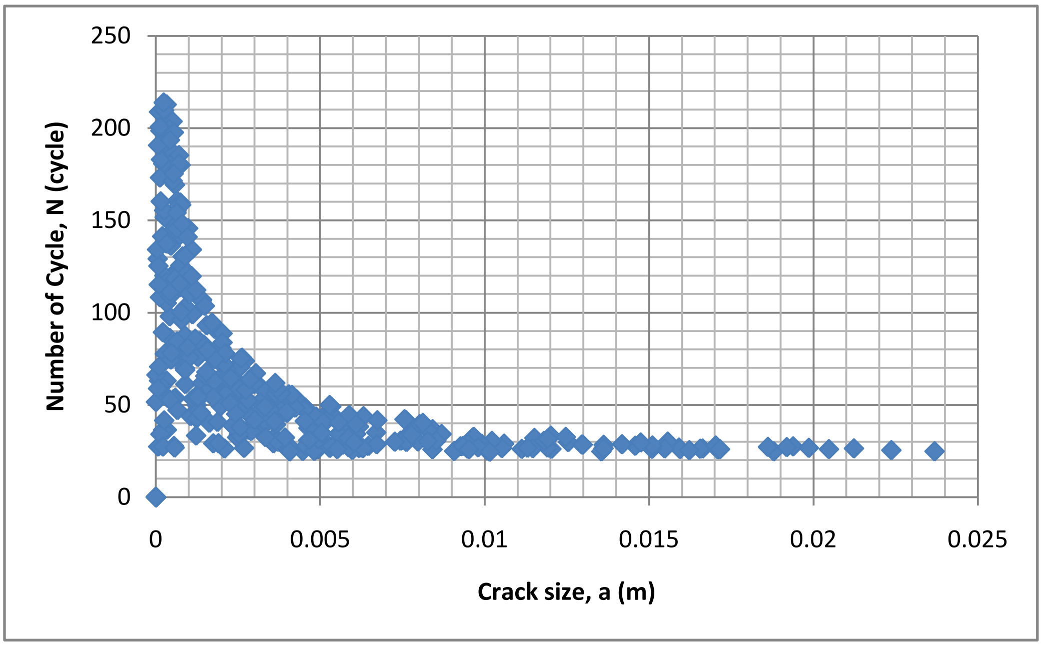


FIGURE 4.1: The number of cycle, N against crack size, a graph by Monte Carlo Simulation

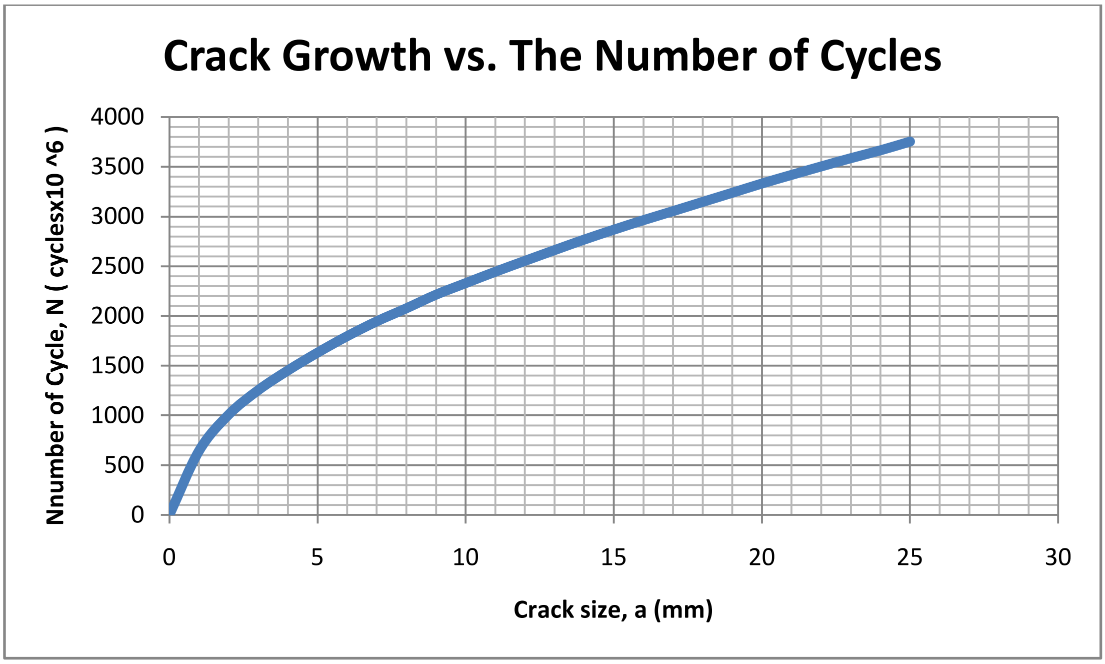


FIGURE 4.2: The crack size, a, against number of cycle, N graph for 0 to 25 mm crack size range

Based on the previous data, the cumulative number of cycle, N has been generated as shown in Figure 4.2. The crack growth pattern shows the slow movement of crack propagation at the beginning of failure. As the number of applied cycle is gradually increased, the 'speed' of the crack propagation is increased rapidly. The rapid crack propagation will continue until the fracture occurs. This crack growth pattern is due to the high strength of the material at the early onset of fatigue. As the applied load increase, the crack size will also increase. When the applied load is greater than the fatigue limit, sudden fracture will occur.

According to Kwon, (2011), the crack growth pattern can be represented with the number of years based on the equation (11). The derivation of equation (11) can be referred to Appendix A.

Hence the equation was mainly build for bridge life assessment; the parameters were focusing on the daily movement of vehicles. The average daily truck transfer (ADTT) and traffic increase rate per year (α) are the parameters used. Therefore, this equation is not applicable to other mechanical equipments and structures that have different features with a bridge.

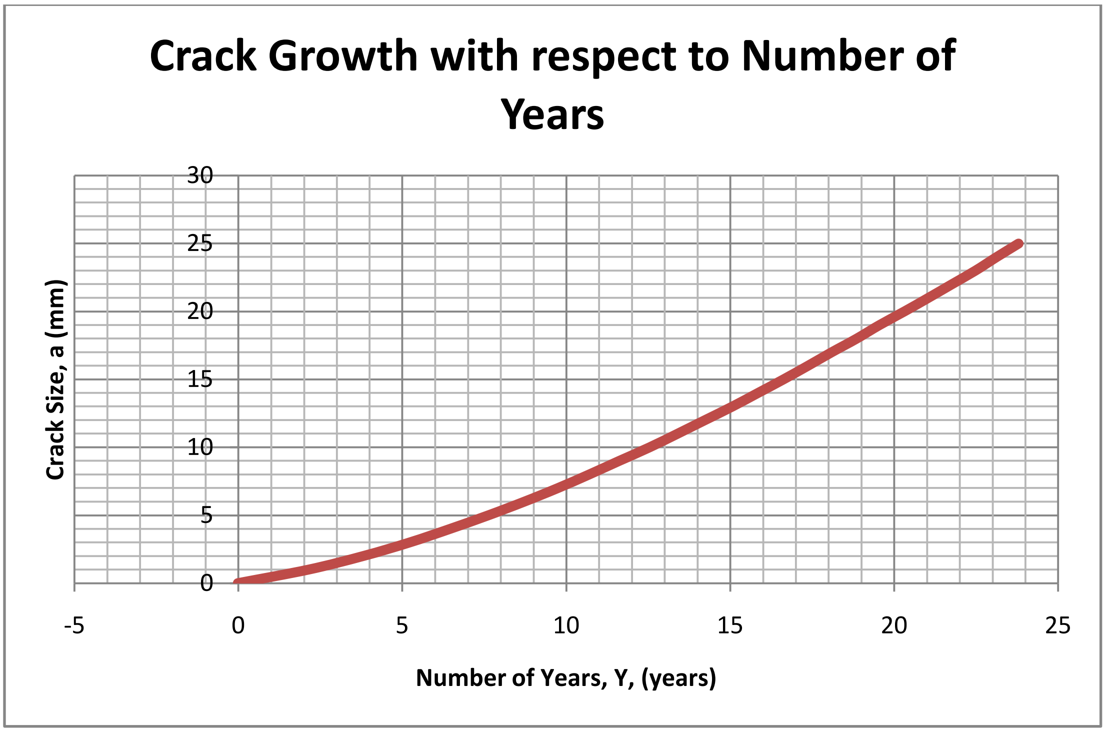


FIGURE 4.3: The crack size, a, against year, y graph

Figure 4.3 illustrates the crack growth curve against the operational years for a bridge structure. It shows that the crack was propagating linearly with the number of years. The bridge is estimated to be functioning up to 23 years after the crack initiation. In order to prevent the failure, maintenance action should be applied 2 or 3 year before the predicted period.

Once the maintenance action has been implemented, the new fatigue reliability analysis should be carried out. Figure 4.4 shows the relationship between the number of years and cycles.

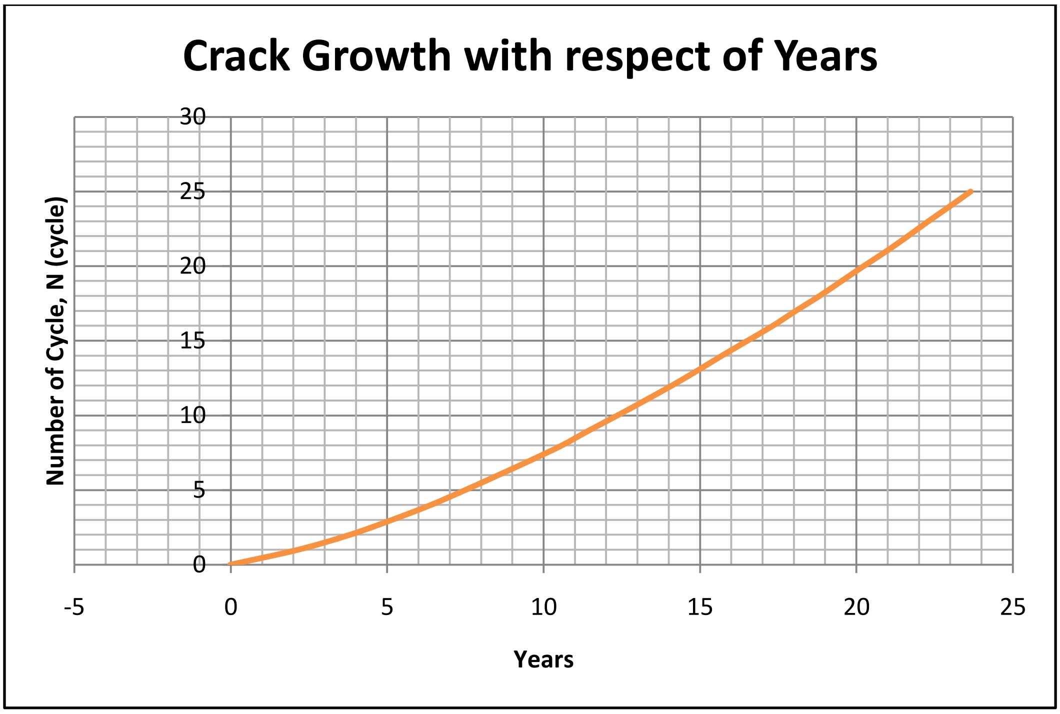


FIGURE 4.4: The number of cycles, N, against the number of years, y graph

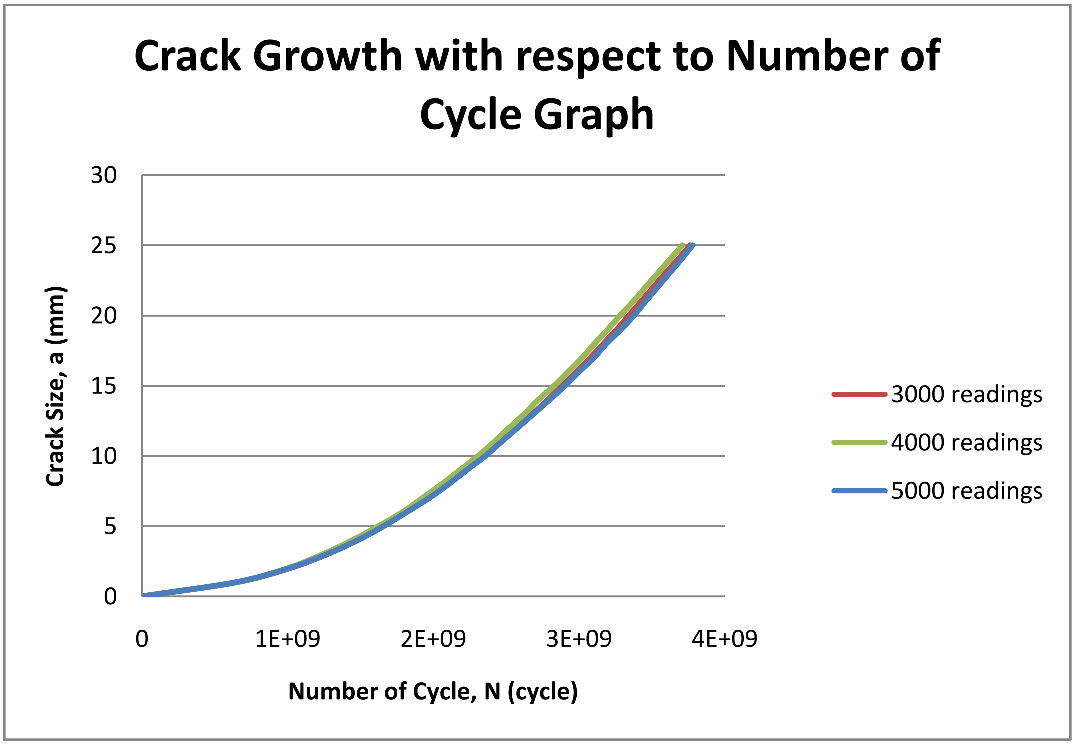


FIGURE 4.5: The number of cycle, N against crack size, a graph for 3 readings

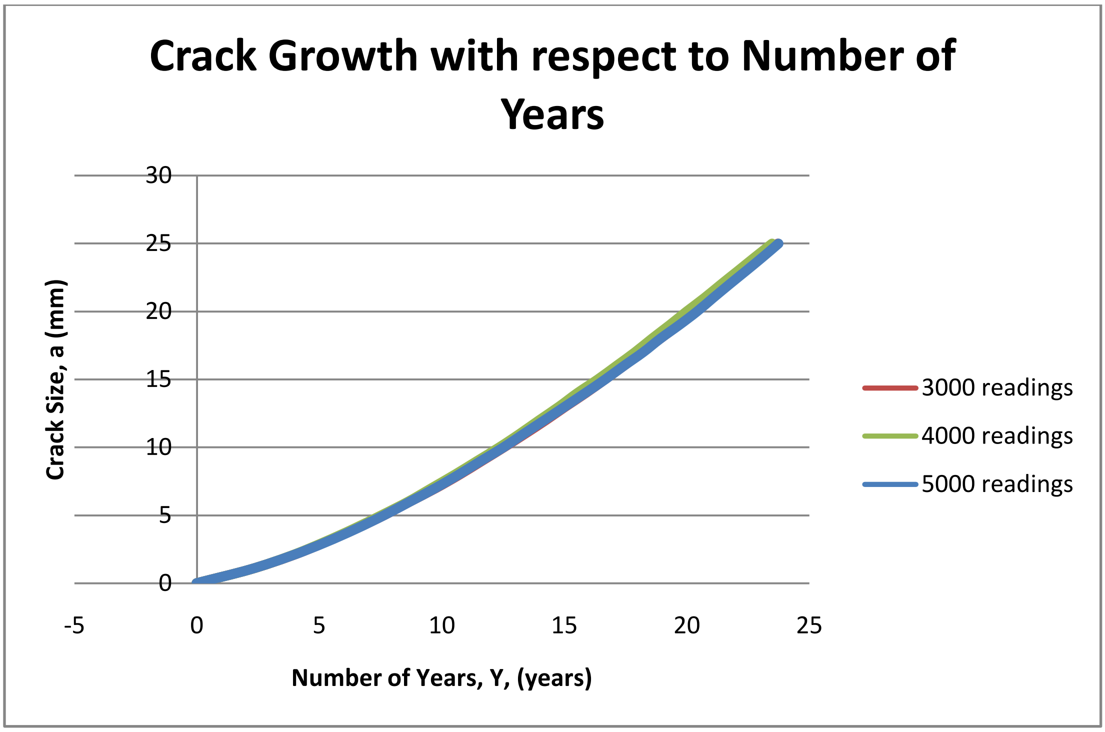


FIGURE 4.6: Crack growth graph with respect of years for 3 readings

In order to measure the accuracy and consistency of the results, the simulation has to be carried out by using various number of sample size. The bigger the sample size, the more accurate the results would be. For this project, the minimum sample size used is set to be at 3000 points. The similar simulations then performed by using 4000 and 5000 points of sample size, respectively.

If the results obtained were similar for every simulations, the concept of Monte Carlo is achieved and the procedure is consider correct. Figure 4.5 and 4.6 above shows the crack growth curve generated by 3000, 4000 and 5000 random numbers respectively. Both graphs show consistency in every reading. Therefore, Monte Carlo simulation method in building this deterministic model was considered correct.

4.2 MATLAB SIMULATION

There is another method in approving the accuracy of Monte Carlo simulation. MatLab is the computerized software used to perform mathematical analysis. Figure 4.7 and 4.8 shows the working area of the software.

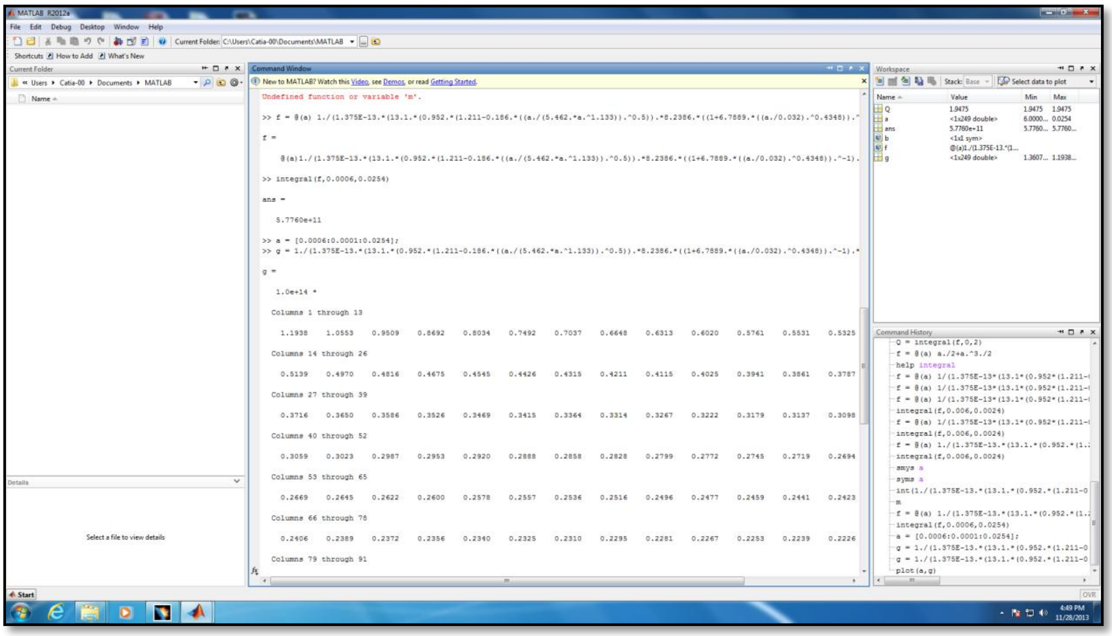


FIGURE 4.7: The input command for function N (a)

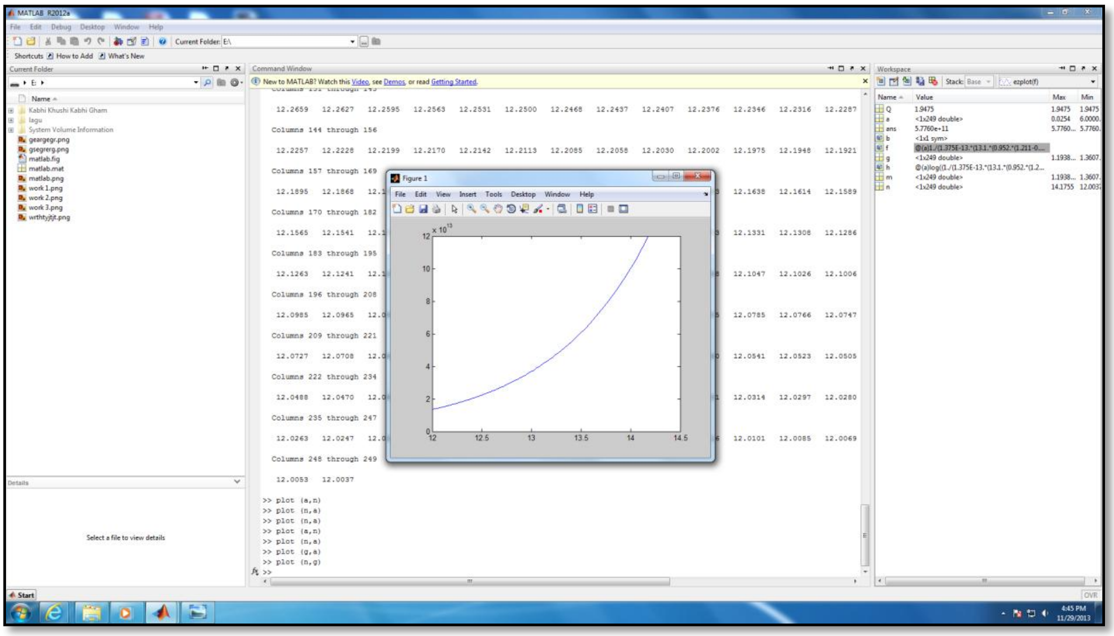


FIGURE 4.8: The generated graph for the function

MatLab simulation was conducted to validate the previous results generated by Monte Carlo simulation. As show in the Figure 4.9 and 4.10; both graphs show the outcome of the MatLab simulation.. Figure 4.9 was the crack size, a, against number of load cycle, N graph. It shows high similarity rates with the previous results from Monte Carlo simulation as shows in Figure 4.1.

However, due to some constraint, the software was unable to generate the crack growth with respect of the number of years. The gathered information was limited to Paris equation only.

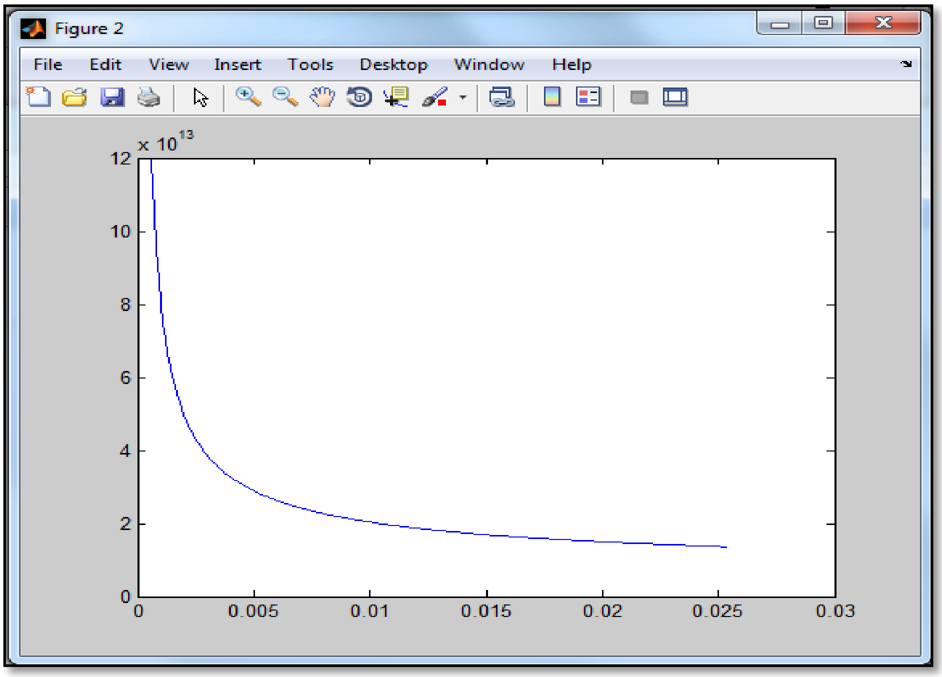


FIGURE 4.9: The number of cycles, N against crack size, a graph by MatLab simulation

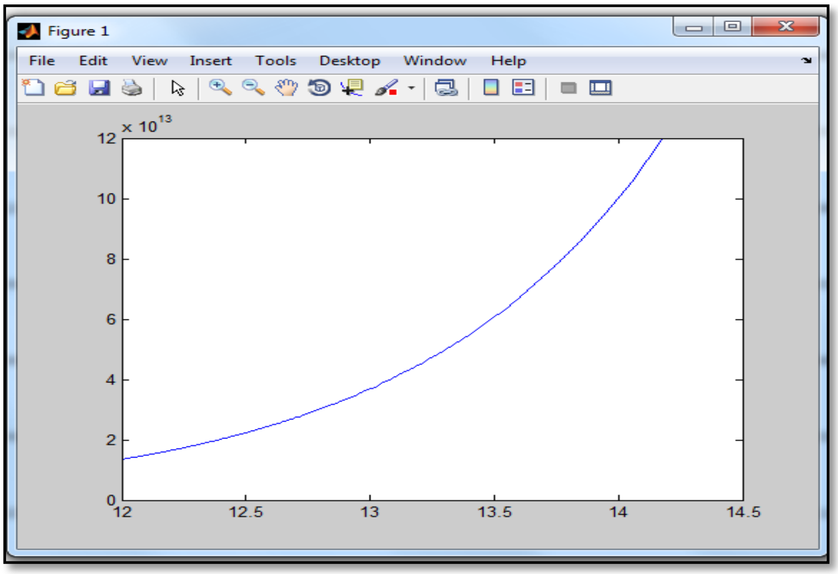


FIGURE 4.10: The number of cycles, N against number of year, y graph by MatLab simulation

Based on the observations, both results show significant similarities especially in their structural behaviour. In Figure 4.1 and 4.9, the crack growth curve was decreases along with the crack propagation. The numbers of applied cycles were decreases on each additional crack. This shows that the crack propagation occur more rapidly at the last moments before the fracture.

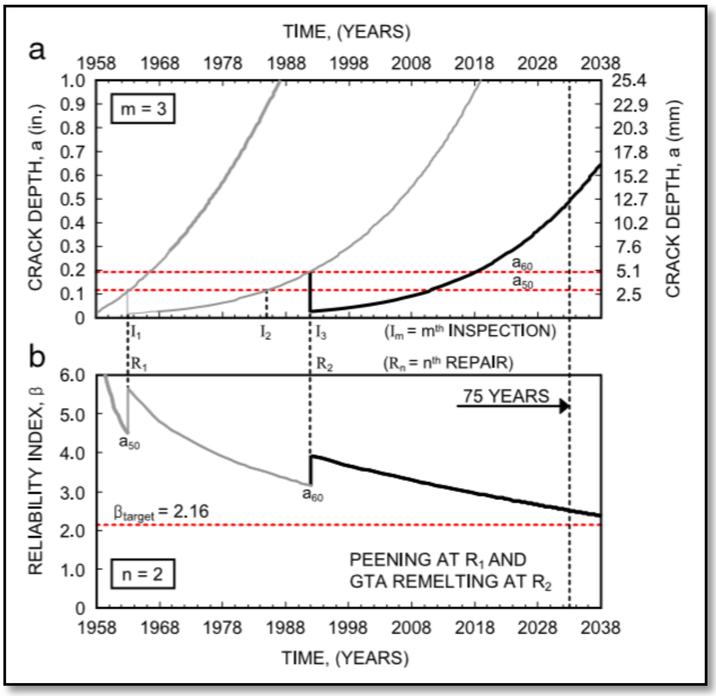


FIGURE 4.11: Crack growth curve from the previous case study

Figure 4.11 is the results from the previous case study on Bridge Fatigue Life Assessment (Kwon, 2011). Figure 4.11 (a) shows the crack growth curve from Crack Growth Model while (b) is the reliability growth curve from Fatigue Reliability Model approach. These results were used as the references in determining the accuracy of the developed CGM.

As shown in Figure 4.2 and 4.11(a), both crack growth curve with respect to the number of years graphs shows prominent similarities. The graph is increased very slowly at the beginning and increased quite dramatically toward the end. This is because of the subjected material was loosening strength in line with the increases of applied stress.

Besides, there are slightly different encountered in the results. The expected lifetime of the bridge from both results shows a difference of 20 years. According to Kwon (2011), the bridge is expected to survive for the next 45 years after the crack initiation. However, this fatigue life assessment solitary by crack growth model estimated the bridge will only be survived for the next 25 years. This large difference might be due to the dissimilar approached used from both of the case studies. Bridge Fatigue Life Assessment has combined three different approaches; Fatigue Reliability Model, Crack Growth Model and Probability of Detection model, in performing the analysis. The integration of the approaches might have affecting the outcome results.

Nevertheless, the behaviour and the structure of the results are the same. Therefore, the Monte Carlo simulation process for crack growth model was considered to be correct.

CHAPTER 5

CONCLUSION AND RECOMMENDATION

5.1 CONCLUSION

As the conclusion, this remaining fatigue life assessment project helps in preventing damages due to fatigue failure. The present study introduces a useful technique in predicting life cycle of any mechanical equipment and civil structure. It is beneficial especially for the process industries in making maintenance plan and strategies to be more effective. Based on the results obtained, the crack propagation is very fast when the material reached its fatigue limits. However, proper inspection and maintenance action should be taken as early as the cracks were found. It would helps in prolong the life spend of the equipments and system.

Apart from that, this analysis may help reducing maintenance cost and prevent major lost due to sudden failure. When the maintenance action was taken just before the failure to happened, the chances of the accident is very low. Besides, the cost in maintaining the existing system is more reasonable than installing new equipments. The generated prototype uses Monte Carlo simulation as the foundation for its complex analysis. This prototype would help facilitate the engineers in monitoring the fatigue crack growth and forecasting the possible next failure to happen. Thus, maintenance action could be taken.

5.2 RECOMMENDATION

The proposed model can be improved further by continuing the studies specifically on mechanical equipments or components. It would assist in identifying components that are more susceptible toward fatigue mechanism. This effort will help complement the existing system of crack growth model. Besides that, to further enhance the effectiveness of the model, integration of CGM with other elements, such as the material properties, physical geometry and design specification should be included in future work. There are many possibilities of fatigue failure that yet to be discovered. Furthermore, there are various methods available in conducting fatigue reliability analysis. Every method would provide assisting in preventing fatigue failure depends on the system situation and requirements. These recommendations would improve the result's accuracy, efficiency and reliability level.

REFERENCES

[1] ASM International, 2008, "Fatigue", *Elements of Metallurgy and Engineering Alloys*, United Kingdom

[2] Ayyub, B.M., Assakkaf, I, A., Khil. D.P., and Sieve. M. W,, 2000, "Reliability-based design guidelines for fatigue of ship structure", University of Maryland, United States

[3] Beden, S. M., Abdullah, S., and Ariffin, A. K., 2009, "Modeling of fatigue crack growth under constant and variable amplitude loading", *International Journal of Mechanical and Materials Engineering (IJMME)*, 4(2):131-135

[4] Branco, R, Antunes, F.V., Ferreira, J. A. M., Silva, J. M., 2009 "Determination of Paris law constants with a reverse engineering technique", *Journal of Engineering Failure Analysis*, **16**:.631-638

[5] Colin, R. G.and Lewis, P. R., 2008, "In-service fatigue failure of engineered products and structures – case study review", *Journal of Engineering Failure Analysis*, **16**: 1775-1793

[6] Crack Growth Analysis Theory, <http://www.mscsoftware.com/training_videos/patran/Reverb_help/index.html#page/Fatigue%2520Users%2520Guide/fat_growth.08.9.html>

[7] Dong, W., Moan, T., and Gao, Z., 2012, "Fatigue Reliability Analysis of the jacket support structure for offshore wind turbine considering the effect of corrosion and inspection", *Journal of Reliability Engineering and System Safety*, **106**: 11-27

[8] European Commission Directorate-General Joint Research Centre, "Introduction to the Monte Carlo Method", Nucleonica

[9] House, O., 2006, *Review of issues associated with the stability of semi-submersibles*, BMT Fluid Mechanics Limited, United Kingdom

[10] Kwon, K. and Frangopol, D. M., 2011, "Journal of Bridge fatigue assessment and management using reliability-based crack growth and probability of detection model," *Journal of Probability Engineering Mechanics*, **26**: 473-474

[11] Moan, T., Hovde, G. O., and Blanker, A. M., 1993, "Reliability-based fatigue design criteria off offshore structure considering the effect of inspection and repair", In *Proceeding of Offshore Technology Conference*, The Norwegian Inst. of Technology, 591-600.

[12] Nguyen, O., Repetto, E. O., Ortiz, M., and Radovitzky, R. A., 2001, "A cohesive model for crack growth", *International Journal of Fracture*, **10**: 351-369.

[13] O'connor, P. D. T., and Kleyner, A., 2012, *Practical Reliability Engineering,* United Kingdom, John Wiley & Sons

[14] Paris, P. C., Gomez, M. P., and Anderson, W.E., 1961, "A rational analysis theory of fatigue", *The Trend in Engineering,* 13: 9-14

[15] Pilana, S., <http://web.student.tuwien.ac.at/~e9527412/history.html>

[16] Pugno, N., Ciavarella, M., Cornetti, P., and Carpinteri, A., 2006, "A generalized Paris law for fatigue crack growh", *Journal of the Mechanics and Physics of Solids,*54:1333-1349

[17] Righiniotis, T. D., and Chryssanthopoulos, M. K., 2003, "Probabilistic fatigue analysis under constant amplitude loading", *Journal of Constructional Steel Reaserch,* **59**:867-886

[18] S. Walter, "The Plane Was Disintegrating", Time Megazine, June 24,2001 <http://content.time.com/time/magazine/article/0,9171,149181,00.html>

[19] Schutz, W., 1996, "A History of Fatigue", *Journal of Engineering Fracture Mechanics* **54(2):** 263-300

[20] Silva, F. S., 2004, "Fatigue on Engine Pistons – A Compendium of Case Studies," *Journal of Engineering Failure Analysis,* **13**: 480-484

[21] Suo, Z., 2010, "Fatigue",, <http://imechanica.org/node/7448>

[22] TCR Engineering Services Technical Team, 2004, *Investigating material and Component Failure,* TCR Engineering Services Pvt. Ltd.

[23] Xiang, Y., Lu, Z., and Liu, Y., 2009, “Crack growth-based fatigue life prediction using an equivalent initial flaw model. Part1: Uniaxial Loading”, *International Journal of Fatigue,* New York, USA

[24] Xiong, J. J., and Shenoi, R. A., 2001, “Deterministic Theorem on Fatigue and Fracture,” in *Fatigue and Fracture Reliability Engineering,* New York: 2-4

APPENDIX A

The Derivation of Paris Equation

Values of the parameters for this analysis were taken from the previous case study; Bridge Fatigue Assessment (Kwon, 2011). Some of the deterministic parameters were taken based on the non-destructive test, while other random variables were from lognormal probability density function (PDF). Table A.1 shows the relevant parameters used for the analysis.

Table A.1

NO	PARAMETERS		VALUE	UNIT
1	a_i	Initial crack size	0.6	mm
2	a_f	Final crack size	25.4	mm
3	N	Number of cycles	-	
4	ΔK	Stress intensity factor	-	
5	G(a)	Non-dimensional function	-	
6	C	Fatigue coefficient	1.375E-13	MPa $\sqrt{m}$
7	m	Fatigue exponent/material constant	3	
8	F_e(a)	Crack shape factor	0.952	
9	F_s	Free surface effect factor	-	
10	F_w	Finite width factor	1.0	
11	F_g	Non-uniform stress factor	-	
12	Z	Weld size	16.0	mm
13	K_{tm}	Stress concentration factor	-	
14	t_f	Flange thickness	32.0	mm
15	t_{cp}	Cover plate thickness	31.8	mm
16	S_{reff}	Effective stress range	13.1	MPa
17	n_i	Number of observation	-	
18	N_{total}	Total number of observation up to the kth range	-	
19	S_{ri}	Stress range bin	-	
20	y	Number of years	-	
21	α	Traffic increase rate per year	2	%
22	ADTT	Average daily truck traffic	4430	cycle per day

Defining number of cycle, N

$$\frac{da}{dN} = C.(\Delta K)^m$$

$$dN = \frac{1}{C.(\Delta K)^m}\, da$$

$$dN = \frac{1}{C.[\,G(a).S_{reff}\,.\sqrt{\pi.a}\,]^m}\, da$$

$$N(a) = \int \frac{1}{C.[\,G(a).S_{reff}\,.\sqrt{\pi.a}\,]^m}\, da$$

$$N(a) = \frac{1}{C.S_{reff}^m} \int_{a_i}^{a_f} \frac{1}{[G(a).\sqrt{\pi.a}]^m}\, da$$

$$N(a) = \frac{1}{C.S_{reff}^m} \int_{a_i}^{a_f} \frac{1}{[F_e(a).F_s(a).F_w(a).F_g(a).\sqrt{\pi.a}]^m}\, da$$

$$N(a) = \frac{1}{C.S_{reff}^m} \int_{a_i}^{a_f} \frac{1}{[F_e(a).F_s(a).F_w(a).K_{tm}\,.[1+6.7889\left({}^{a}/_{t_f}\right)^{0.4348}]^{-1}.\sqrt{\pi.a}]^m}\, da$$

$$N(a) =$$

$$\frac{1}{C.S_{reff}^m} \int_{a_i}^{a_f} \frac{1}{[F_e(a).1.211-0.186\sqrt{a/c}.F_w(a).K_{tm}\,.[1+6.7889\left({}^{a}/_{t_f}\right)^{0.4348}]^{-1}.\sqrt{\pi.a}]^m}\, da$$

$$N(a) =$$

$$\frac{1}{C.S_{reff}^m} \int_{a_i}^{a_f} \frac{1}{[F_e(a).1.211-0.186\sqrt{a/c}.F_w(a).K_{tm}\,.[1+6.7889\left({}^{a}/_{t_f}\right)^{0.4348}]^{-1}.\sqrt{\pi.a}]^m}\, da$$

Defining number of year, Y

$$N(y) = 365.\mathrm{ADTT} \int_0^y (1+\alpha)^y \; dy$$

$$N = 365.\mathrm{ADTT} \, [\frac{(1+\alpha)^y}{\ln\,(1+\alpha)}]_0^y$$

$$N = 365.\mathrm{ADTT} \left[\frac{(1+\alpha)^y}{\ln\,(1+\alpha)} - \frac{1}{\ln\,(1+\alpha)}\right]$$

$$N = 365.\mathrm{ADTT} \left[\frac{(1+\alpha)^y}{\ln\,(1+\alpha)} - \frac{1}{\ln\,(1+\alpha)}\right]$$

$$N = 365.\mathrm{ADTT} \left[\frac{(1+\alpha)^y - 1}{\ln\,(1+\alpha)}\right]$$

$$\frac{\mathrm{N.ln}\,(1+\alpha)}{365.\mathrm{ADTT}} = (1+\alpha)^y - 1$$

$$1 + \frac{\mathrm{N.ln}\,(1+\alpha)}{365.\mathrm{ADTT}} = (1+\alpha)^y$$

$$y = \log_{(1+\alpha)} \left[1 + \frac{\mathrm{N}\,.\,\ln(1+\alpha)}{365.\mathrm{ADTT}}\right]$$

$$y = \frac{1}{\ln(1+\alpha)} . \ln\left[1 + \frac{\mathrm{N}\,.\;\ln(1+\alpha)}{365.\mathrm{ADTT}}\right]$$

$$y = \ln[(365\,.\;\mathrm{ADTT}) + (\mathrm{N}\,.\ln(1+\alpha)] - \ln(365\,.\;\mathrm{ADTT}) - \ln(1+\alpha)$$

APPENDIX C

The Fatigue Reliability Analyzer - Monte Carlo Simulation

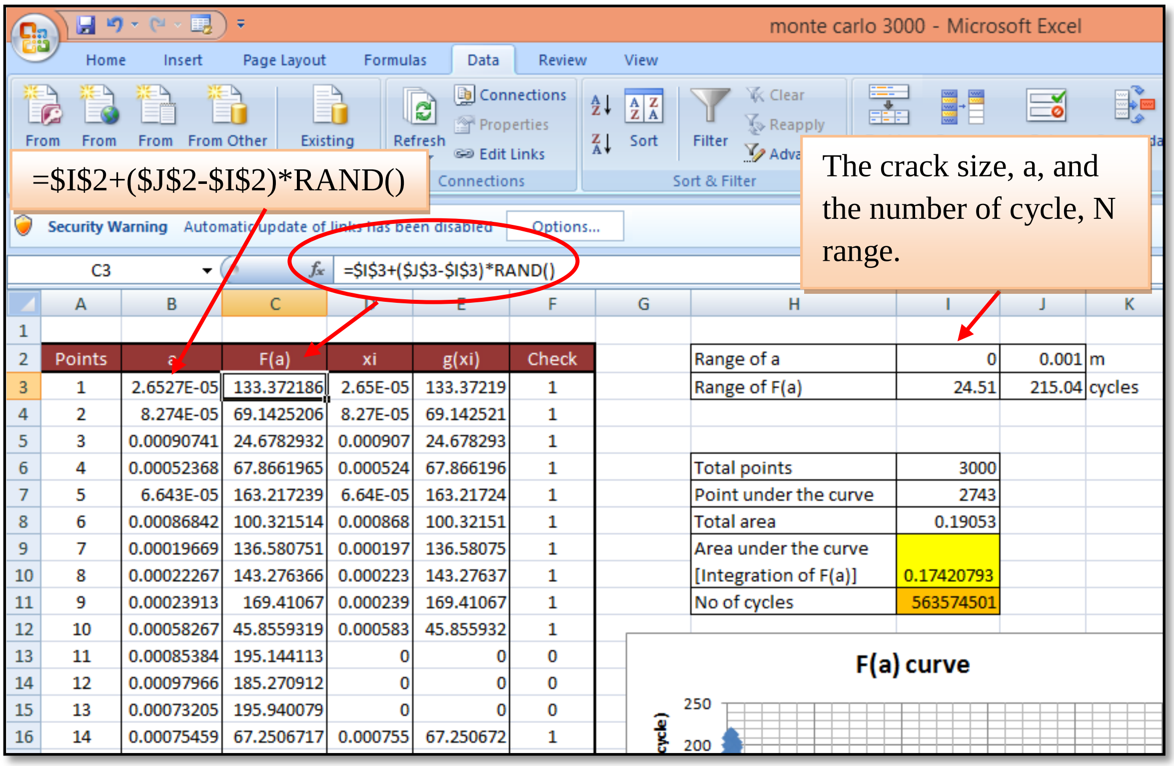
- This is the main page of the analyzer. It requires the user to insert new data with respect to the failure. The data then will be link to another Excel sheet to perform the analysis. The user need to provide data gathered during inspection for the yellow boxes.

FATIGUE RELIABILITY ANALYSIS		
DATA INPUT		
Year of Inspection		year
Inspected Crack Size		mm
Effective Stress Range, Sreff		
Weld Size, Z		mm
Flange Thickness, tf		mm
Cover Plate Thickness, tcp		mm
Stress Concentration Factor, Ktm		
CONSTANT PARAMETERS		
Crack Shape Factor, Fe	0.952	
Finite Width Factor	1	
Fatigue Coefficient	1.38E-13	MPa√m

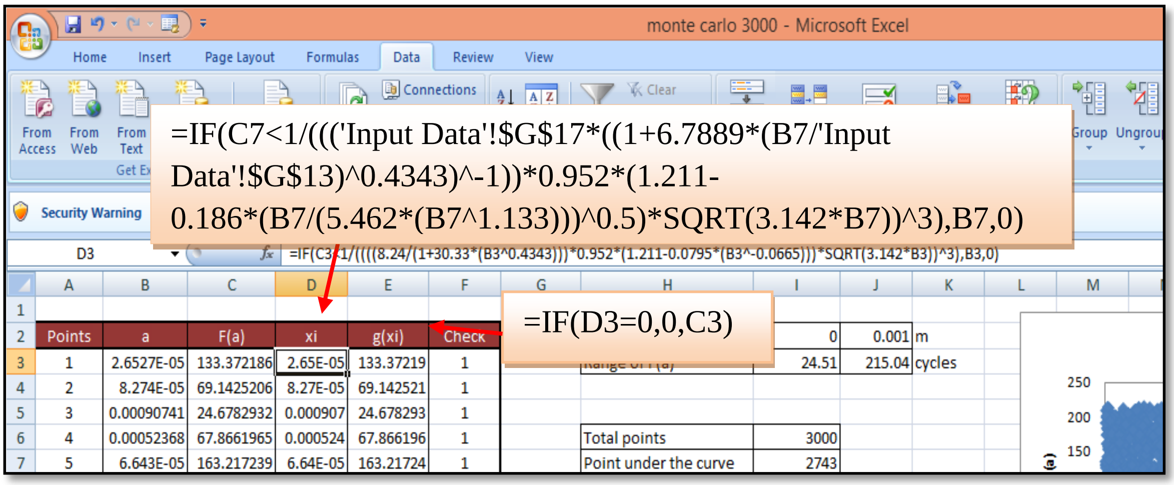
The layout of the FRA to import new data into the system analyzer

➢ The objective of this analysis is to determine the number of applied cycle with respect to crack size. The next figures will show the steps taken in building the Monte Carlo simulator.

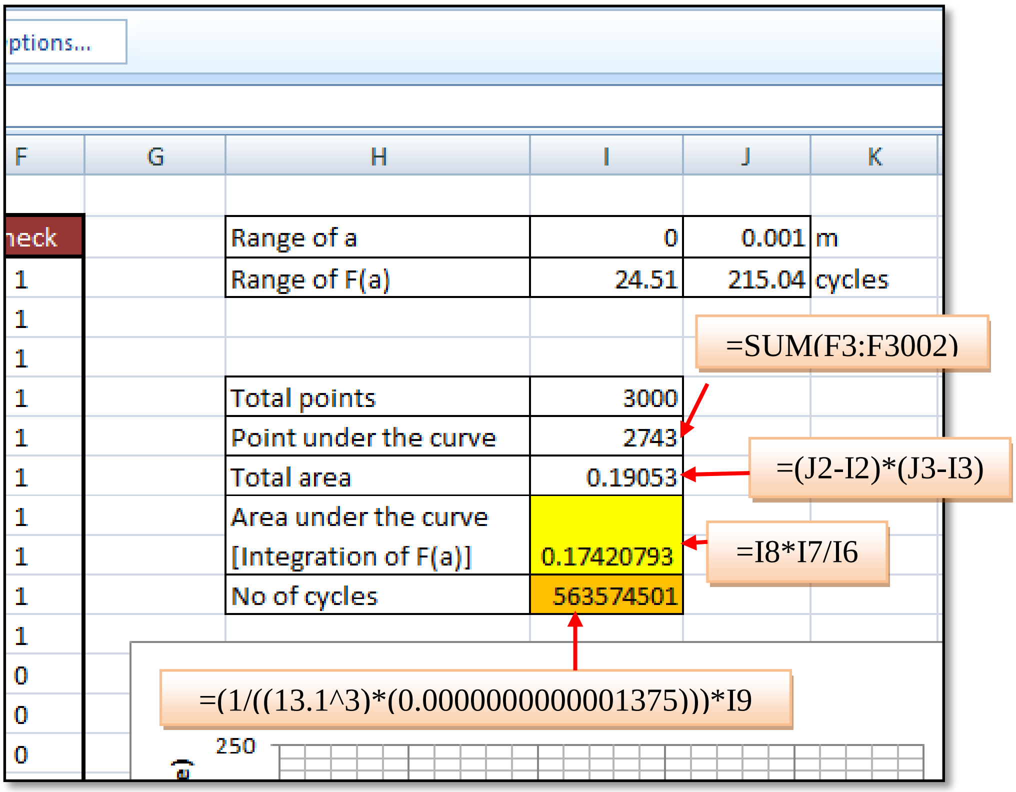
i. Build the range of the system by setting the maximum and minimum value of the crack size and the number of cycle.



ii. Computing the accept or reject function.



iii. Calculate the area under the graph.



3000 Readings						
a (mm)	Integration of F(a)	No of Cycle, N(a)	Cumulative of N	No of Years, Y	Cumulative of Y	Cumulative Y
0	0.00000	0	0	-0.0198	-0.019802627	1992
1	0.17472	565218174.4	565218174.4	2.049863	2.030060065	1994
2	0.08764	283533653.5	848751827.9	1.478124	3.508184563	1996
3	0.05646	182653201.4	1031405029	1.154825	4.663009064	1997
4	0.03988	129028358.2	1160433388	0.928063	5.591072417	1998
5	0.03252	105195094.6	1265628482	0.808013	6.399085072	1998
6	0.02363	76430810.93	1342059293	0.640842	7.039927409	1999
7	0.02140	0.02140287	1342059293	-0.0198	7.020124782	1999
8	0.01848	59788618.23	1401847911	0.529604	7.549729046	2000
9	0.01569	50748414.79	1452596326	0.463556	8.013284747	2000
10	0.01315	42530048.02	1495126374	0.399474	8.412758944	2000
11	0.01137	36777191.28	1531903565	0.352041	8.764800273	2001
12	0.01035	33489844.58	1565393410	0.323891	9.088691476	2001
13	0.00851	27531528.67	1592924939	0.270757	9.359448716	2001
14	0.00737	23833263.62	1616758202	0.236299	9.595747421	2002
15	0.00648	20956835.26	1637715038	0.208652	9.804399253	2002
16	0.00438	14176682.67	1651891720	0.140291	9.944689987	2002
17	0.00603	19518621.07	1671410341	0.194536	10.13922637	2002
18	0.00394	12738468.49	1684148810	0.125169	10.26439536	2002
19	0.00305	9862040.12	1694010850	0.094222	10.3586171	2002
20	0.00273	8834744.275	1702845594	0.082933	10.44154997	2002
21	0.00197	6369234.244	1709214828	0.055308	10.49685795	2003
22	0.00171	5547397.568	1714762226	0.045927	10.54278542	2003
23	0.00095	3081887.538	1717844113	0.017246	10.5600315	2003
24	0.00051	1643673.353	1719487787	0.000127	10.56015885	2003
25	0.00057	1849132.523	1721336919	0.002591	10.56274975	2003

Tabulated results of the analysis