Modeling and Control of Axial and Torsional Stick-Slip Oscillations in Drill String

by

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CERTIFICATION OF APPROVAL

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A project dissertation submitted to the
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Approved by,

____
(Dr. Setyamartana Parman)

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May 2014

CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

(MUHAMMAD FARHAN BIN ALIAS)

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ABSTRACT

A drill string is the transmission component of rotary drill-rig system used for mining petroleum and natural gas resources. The drill string system is essentially a long slender structure whose length can be in kilometers. Additionally, the drill-string is subjected to discontinuous forces from interactions with the wellbore, which can cause erratic torsion oscillations and stick-slip motions. Throughout this report, all the information that is needed to execute this project will be explain in chapter 1 which is introduction that consist of background of the project, the problem statement of the project and the objectives of doing this project. The literature review of this project will be explained after the introduction. All the studies about this project in order to gain insights into the drill string dynamics and reduce order model have been stated in the literature review. In chapter 3, the methodology in doing this project will be explained which consists of project flow, Gantt chart and the key milestone of doing this project. Furthermore, the results and discussion during this Final Year Project have been concluded and the findings are presented in chapter 4 in this report. These findings provide the simulation code in forms of MATLAB code. This report effort provides clues to how the drive speed can be used as a control parameter to move the system out of regions of undesired and how the drill-string motions can be influenced to keep them close to the borehole center.

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ABBREVIATION AND NOMENCLATURES

k	stiffness
c	Torsional damping
φ_r	Angular displacement of rotary top system
φ_b	Angular displacement of the BHA
T_m	Drive torque from electrical motor
T_r, T_b	Dry friction torque
c_r	Damping viscous coefficient
T_{c_r}	Coulomb friction torque
$\overline{\Omega}$	Reference velocity
u	Control input
J_r, J_b	Inertia
T_{max}	Maximum period
T	Period
$\dot{\varphi}_r$	Angular velocity of rotary top system
$\dot{\varphi}_b$	Angular velocity of the BHA

CHAPTER 1

INTRODUCTION

1.1 PROJECT BACKGROUND

Oil and gas wells are predominantly drilled using rotary drilling. A rotary drilling system creates a borehole by means of a rock-cutting tool which is bit. Deep wells for the exploration and production of oil and gas are drilled with a rock-cutting tool driven from the surface. The mechanism used to transfer the torque between the torque generating unit and the cutting tool is typically a series of connected, hollow steel drill pipes called the drill string. Figure 1 shows the configuration of a drill string.

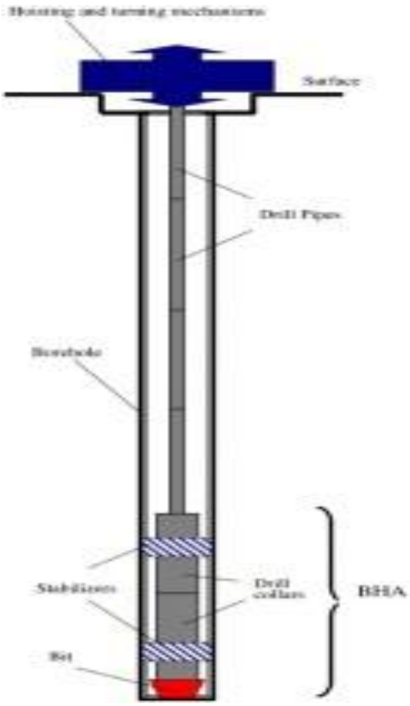


Figure 1: A typical drill string configuration

Each pipe can be 9 meters long, and a threaded connection connects two drill pipes. The drill string is a long, slender structure that is attached to a short heavier segment containing a cutting tool at the free end. This cutting tool is called a drill bit and this heavy segment along with the tool is called the bottom hole assembly (BHA). A short heavy segment may contain what are called stabilizers to minimize lateral motions during a drilling operation.

The drill string is driven in a rotary fashion from the top end, usually, by means of an electric motor and a gearbox unit called the top drive or the torque generating unit or rotary unit, and this string is sent at a prescribed rate through a rotating mass (the rotary) near the ground level. This system is designed to construct a borehole from the earth's surface to a reservoir of oil or gas. The borehole is lined (usually with steel) and the excess in the diameter of this cavity over the diameter of the drill pipe is called the over gauge. This annular gap, which varies along the length of the borehole is necessary for fluid conduction.

The vibrations of the drill string are classified depending on the direction they appear. There are three main types of vibrations of drill strings which are torsional, axial and lateral. Torsional or stick-slip oscillation is the most considerate type of vibration used in drilling. However, failures in drill strings can be significant in the total cost of the of the perforation process. The devices used are complex dynamic systems with many unknown and varying parameters due to the characteristics change as the drilling operation make progress. Torsional or stick-slip oscillations vibrations appear due to the downhole conditions such as significant drag, tight hole and formation characteristics. These will cause the drill bit to stall in the formation while the rotary table continues to rotate. Torsional energy will trapped and when it reaches certain level where the bit can no longer resist, it will become loose rotating and whipping at a very high speed. This behaviour will generate a torsional wave that will travels up to the rotary top system. The high energy of the rotary table act like a fixed end to the drill string and torsional wave will be reflected back to the bit and causes the bit to stall again and the wave cycle repeat. The high speed rotations of the bit can generate axial and lateral vibrations at the bottom-hole assembly (BHA) and causes problems such as drill pipe fatigue, components failure and wellbore instability.

1.2 PROBLEM STATEMENT

Drill string vibrations are classified depending on the direction they appear. One types of the vibration is torsional which is a stick-slip oscillation. Stick-slip oscillation is considered as the most detrimental type of torsional vibration to the service life of the drill string and downhole equipment. However there is consequence of this vibration which is top of the drill string rotates with constant speed whereas the bid rotary speed varies between zero and up to six times the rotary speed measured at the surface. At some point, the rotary speed of the bit will increase and the high speed rotations can generate both severe axial and lateral vibrations at the bottom-hole assembly (BHA). These vibrations will lead to some kind of problems such as drill pipe fatigue problem, drill string components failure and bit damage. Controlling and modeling the stick-slip behaviour have to be done in order to prevent these problems to happen.

1.3 OBJECTIVES

a) To model and control the drill string dynamics considering the effect of friction appeared between drill string components and between the drill string and the formation.

b) To propose a control method to reduce the stick-slip conditions by means of the alignment of different drilling parameters by using existing method of controller.

c) To develop mathematical model of drill string dynamics

d) To perform MATLAB/SIMULINK simulation of drill string dynamics.

1.4 SCOPE OF STUDY

Scope of study is focusing on what is needed to be focused in order for the project to flow in the right ways. Followings are the scope of study that has been focused on.

i) Reviews in drill string vibrations, components of drill string and modeling the drill string and what based model that will be used in modeling the generic drill string.

ii) Modeling and mathematical derivation of drill string dynamics

iii) Formularization of the drill string dynamics with MATLAB/SIMULINK.

iv) Perform simulation of the drill string dynamics in MATLAB/SIMULINK.

CHAPTER 2

LITERATURE REVIEW

2.1 PRINCIPLE OF OIL WELL DRILLING

A rotary drilling system creates a borehole of a rock-cutting tool which is called bit. The energy to drive the bit is generated at the surface by a motor with a mechanical transmission box. The medium to transport the energy to the bit is formed by a drill string which consist of drill pipe about 9m long, coupled with threaded connections, having a typical outside diameter of 127mm and thickness of 9mm.

The bottom-hole assembly (BHA) consists of thick-walled tubulars called drill collars. These drill collars usually have an inner diameter of 64-76 mm and outer diameter of 120-240mm. The drill collars are kept in position by a number of stabilizers.

The drilling process requires a compressive force on the bit of some 10^4-10^6 N. This force is denoted as Weight On Bit (WOB). The drill string rests with the bit on the bottom of the hole and is pulled at the hook by a force called the hookload. The hookload ensures that the drill pipe is kept in tension to avoid buckling. While the drill pipes run in tension, the BHA is loaded in compression.

Torque is transmitted from the rotary table to the drill string. Torque required to drive the bit us referred to as the Torque On Bit (TOB). Fluid called mud is pumped down through the hollow drill-string, through nozzles in the bit and returns to the surface through the annulus between the drill string and the borehole wall.

The drilling process is steered by the hookload, rotary table speed and the flow rate of mud. Standpipe pressure indicates the total pressure drop in the drill string and annulus.

2.2 DOWNHOLE MEASUREMENT

Trafor system is designed to measure downhole and surface data to improve knowledge about drill string dynamics. The Trafor system consists of a downhole measurement device called Televigile, and a surface measurement device known as Survigile. The advantage of Trafor system is the ability to measure both downhole and surface data at real-time. The Televigile is equipped with sensors that measure WOB, downhole torque, downhole accelerations and downhole bending moments.

The measurements are recorded at a full-scale research rig. Various test with different WOB and angular velocity were conducted. Figure 2 shows a time history of the downhole angular velocity, calculated from the magnetometer signals.

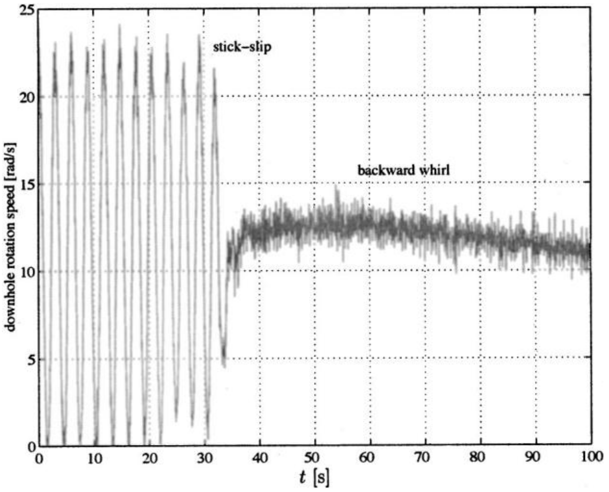


FIGURE 2. Measured downhole angular velocity versus time

The angular velocity at the surface v, WOB and other parameters were almost kept constant during the experiment. The drill string clearly performs stick-slip motion for t<35 s. At t=35 s the stick-slip motion suddenly disappears and backward whirl is prevalent for t>35 s.

2.3 DYNAMIC MODELLING OF A GENERIC DRISLL TRING

Two problems that have to be concern are the problem of modeling the drilling system and the modeling of the rock-bit interaction. The model proposed will be combined with a mechanism in order to maintain the top-drill string rotational speed constant, which is the main goal of the driller. The model used is a simplification of the drill string behaviour that collects the most important phenomena concerning drill string torsional vibrations.

The drill string torsional behaviour described by a simple torsional pendulum and the bit-rock interaction is modeled by a dry friction model. Drill pipes are represented as a linear spring of torsional stiffness k and a torsional damping c which are connected to the inertias J_r and J_b Some assumptions are made, such as the borehole and the drill string are both vertical and straight, no lateral bit motion is present, the rotary top system is supposed to have an angular velocity different from zero the friction in the pipe connections and between the pipes and the borehole are neglected, the drilling mud is simplified by a viscous-type friction element at the bit, the drilling mud fluids orbital motion is considered to be laminar.

2.4 DRILL STRING VIBRATION STUDIES

A drill string undergoes different types of vibration during a drilling operation. These include the following: i) axial or longitudinal vibrations, which are mostly due to the interaction between the drill bit and the rocks, ii) bending or lateral vibrations, often caused by drill-pipe eccentricity, leading to a rotational motion named as drill string whirl, iii) torsion vibrations (sometimes referred to as stick-slip vibrations in the literature because stick-slip interactions are the main source of torsion vibrations), caused by nonlinear interaction between the bit and the rock and/or the drill string with the bore hole wall, and iv) hydraulic vibrations in the circulation system, stemming from pump pressure pulsations. The hydraulic vibrations are not considered to be a main source of drill-string vibrations.

Drill-string vibrations are complicated and coupled. Over the last two decades, an extensive number of modeling, simulation, and experimental studies have been conducted to understand these vibrations. Stick-slip and whirl vibrations of a drill string and the influence of the fluid lubrication on it were studied by Leine, van Campen, and Keultjes (2002). They presented a reduced-order model with two degree-of-freedom, considered contact conditions in detail, and studied bifurcations associated with discontinuities in the system. The work of Leine et al. (2002) illustrated the complexities of torsion drill-string dynamics including interactions between stick-slip and whirl, and the possible instabilities that can be exhibited by such systems. In a broader context, drill string systems are discontinuous systems, instabilities.

CHAPTER 3

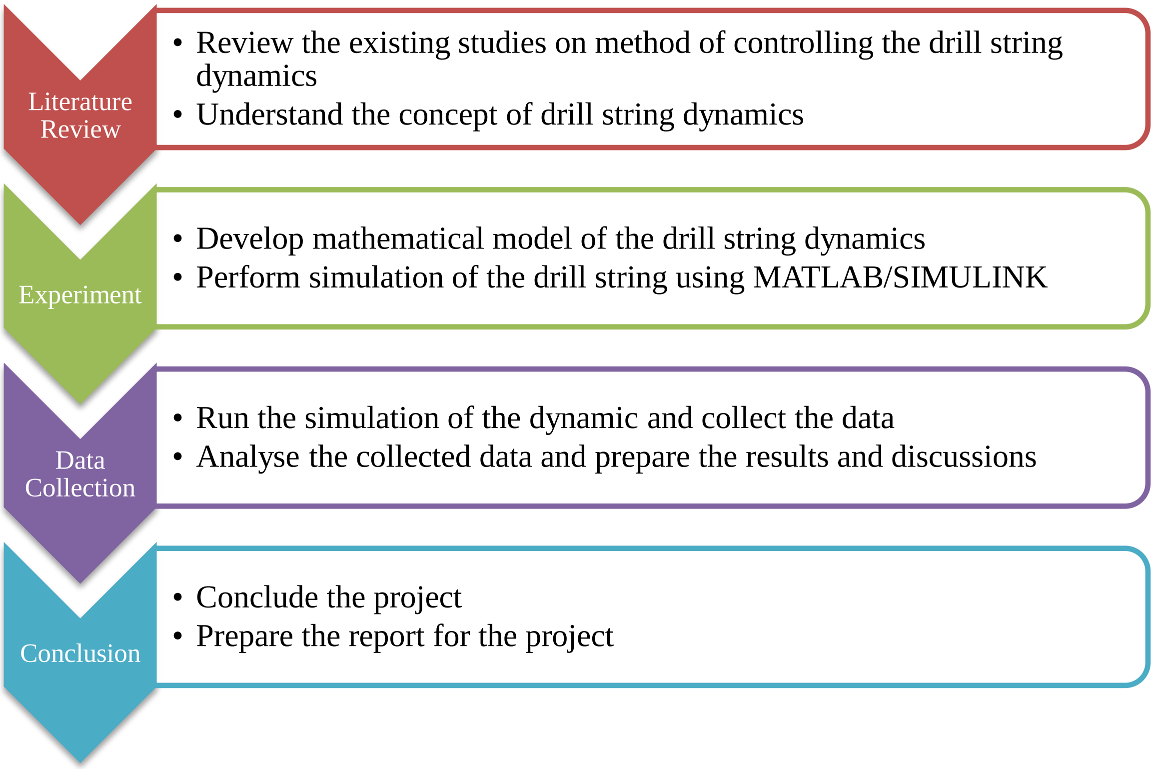
METHODOLOGY

3.1 PROJECT METHODOLOGY

The procedure below briefly summarizes the proposed strategies involved in the problems.

Step 1	Identify the problem of the stick slip oscillation and the consequences of these vibrations
Step 2	Develop a mathematical model of a drill string dynamic to describe the torsional behavior of a generic drill string.
Step 3	Develop an equation of motion based on the model and made assumptions
Step 4	Develop some cases based on the equations.
Step 5	Simulate the model using simulation software to obtain the result.
Step 6	Observe the result and compare with the original findings

The flow chart of the methodology is represented below



- Review the existing studies on method of controlling the drill string dynamics
- Understand the concept of drill string dynamics

- Develop mathematical model of the drill string dynamics
- Perform simulation of the drill string using MATLAB/SIMULINK

- Run the simulation of the dynamic and collect the data
- Analyse the collected data and prepare the results and discussions

- Conclude the project
- Prepare the report for the project

FIGURE 3: Project flow chart

3.2 IDENTIFYING THE PROBLEM (STEP 1)

Drill string vibrations are classified depends on the direction they appear. In stick slip oscillation, some problem will occur. One of the problem is the top of the drill string rotates with a constant rotary speed whereas the bit rotary speed varies between zero and up to six times the rotary speed measured at the surface. The stick slip behavior can generate a torsional wave that can travel to the rotary top system. Other than that, the high speed rotation of the bit in the slip phase will generate severe axial and lateral vibrations at the bottom hole assembly (BHA). These problems can lead to drill pipe fatigue problem that cause drill string connection failures, drill string components failure, wellbore instability and damaging the bit.

3.3 DEVELOPING MATHEMATICAL MODEL (STEP 2)

The drill string consists of BHA and drill pipes screwed end to end to produce a long pipe. Drill string usually includes at the top of the BHA a section of heavy-weight drill pipe. An important element in drilling is the drilling fluid which cleaning, cooling and lubricating the bit. The main problem while modeling has to be distinguished which are the problem of modeling the drilling system and the modeling of the rock-bit interaction. The drill string torsional behavior is described by a simple torsional pendulum driven by an electric motor and the bit-rock interaction is modeled by a dry friction model. Drill pipes are represented as a linear spring of torsional stiffness and a torsional damping which connected to the inertias. A dry friction torque and a viscous damping torque are also considered at the rotary table.

3.4 DEVELOPING EQUATIONS OF MOTION (STEP 3)

Some assumptions are made such as:

i) The borehole and the drill string are both vertical and straight
ii) No lateral bit motion present

iii) The rotary top system have angular velocity

iv) Friction in pipe connection and between the pipes and the borehole are neglected

v) Drilling fluids considered to be laminar

3.5 DEVELOPING CASES (STEP 4)

Based on the equations that have been made, some cases were developed where the value of period and the max period are varies. The variations that has been made is $T = 1,\ T = 2, T = 3, T = 4\ , T = 5,\ T_{max} = 0.8, T_{max} = 0.9, T_{max} = 1.0, T_{max} = 1.1\ and\ T_{max} = 1.2$. Based on these variation, the next step of methodology has been made which is to simulate the values using MATLAB applications.

3.6 SIMULATE (STEP 5)

Some simulations were made using software which is MATLAB to see the result for the model. The model parameters used for the simulation are extracted from the past research. Although the values do not correspond with real parameters, they can be used to describe the behavior of the drill string.

3.7 OBSERVING (STEP 6)

Based on the simulated result, the manipulated result was compared with the results for the model and conclusions were made.

3.8 PROJECT GANTT CHART

No.	Detail/Week	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1.	Title selection and allocation	■	■												
2.	First meeting with supervisor		■												
3.	Preliminary research work and proposal preparation		■	■	■	■	■	■							
4.	Submission of Extended Proposal Defense							■							
5.	Proposal Defense								■						
6.	Commencement of experimental work								■	■	■	■	■		
7.	Project work continues and preparation of Interim Report								■	■	■	■	■	■	
8.	Submission of Interim Draft Report													■	
9.	Submission of Final Interim Report														■

FIGURE 4: FYP1 Gantt chart

No.	Detail/Week	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1.	Project work continues	■	■	■	■	■	■	■							
2.	Submission of progress report														
3.	Project work continues								■	■	■	■	■		
4.	Pre-SEDEX														
5.	Submission of draft final report														
6.	Submission of Dissertation (soft bound)														
7.	Submission of Technical Paper														
8.	Viva														
9.	Submission of Project Dissertation (Hard Bound)														

FIGURE 5: FYP2 Gantt chart

3.9 PROJECT KEY MILESTONE

No.	Detail/Week	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1.	Title selection and allocation														
2.	First meeting with supervisor														
3.	Preliminary research work and proposal preparation														
4.	Submission of Extended Proposal Defense							■							
5.	Proposal Defense														
6.	Commencement of experimental work														
7.	Project work continues and preparation of Interim Report														
8.	Submission of Interim Draft Report													■	
9.	Submission of Final Interim Report														■

FIGURE 6: FYP1 Key Milestone

No.	Detail/Week	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1.	Project work continues															
2.	Submission of progress report							■								
3.	Project work continues															
4.	Pre-SEDEX										■					
5.	Submission of draft final report											■				
6.	Submission of Dissertation (soft bound)												■			
7.	Submission of Technical Paper												■			
8.	Viva													■		
9.	Submission of Project Dissertation (Hard Bound)															■

FIGURE 7: FYP2 Key milestone

CHAPTER 4

RESULTS AND DISCUSSION

4.1 DYNAMIC MODELLING OF A DRILL STRING

The drill string torsional behaviour is described by a simple torsional pendulum driven by an electric motor and the bit-rock interaction is modeled by a dry friction model. The drill pipes are represented as a linear spring of torsional stiffness k and a torsional damping c which are connected to the inertias J_r and J_b, corresponding to the inertia of the rotary table or the top drive and to the inertia of the pipeline.

The mechanical model that describing the torsional behaviour of a drill string has been made by referring to other researches and studies. Figure 6 shows the mechanical model.

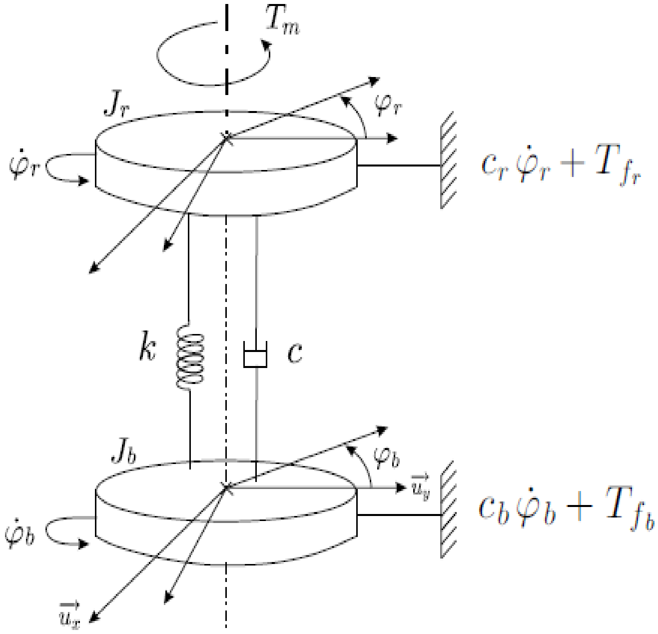


FIGURE 8: Mechanical model describing the torsional behaviour of a dill string

The equation of motions derived from the two degree of freedom model is the following ones :

$$J_r\ddot{\varphi}_r + c(\dot{\varphi}_r - \dot{\varphi}_b) + k(\varphi_r - \varphi_b) = T_m - T_r(\dot{\varphi}_r) \tag{1a}$$

$$J_b\ddot{\varphi}_b - c(\dot{\varphi}_r - \dot{\varphi}_b) - k(\varphi_r - \varphi_b) = -T_b(\dot{\varphi}_r) \tag{1b}$$

Where;

$\varphi_r = angular\ displacement\ of\ the\ rotary\ top\ system$

$\varphi_b = angular\ displacement\ of\ the\ BHA$

$T_m = drive\ torque\ from\ electrical\ motor$

$T_r, T_b = dry\ friction\ torque$

The matrix form of the equations of motions will be as the following:

$$\begin{bmatrix} J_r & 0 \\ 0 & J_b \end{bmatrix}\begin{Bmatrix} \ddot{\varphi}_r \\ \ddot{\varphi}_b \end{Bmatrix} + \begin{bmatrix} c & -c \\ -c & c \end{bmatrix}\begin{Bmatrix} \dot{\varphi}_r \\ \dot{\varphi}_r \end{Bmatrix} + \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}\begin{Bmatrix} \varphi_r \\ \varphi_b \end{Bmatrix} = \begin{Bmatrix} T_m \\ 0 \end{Bmatrix} + \begin{Bmatrix} -T_r(\dot{\varphi}_r) \\ -T_b(\dot{\varphi}_r) \end{Bmatrix}$$

$T_r\ and\ T_b$ represent the dry friction torque plus viscous damping associated with J_r and J_b respectively that is,

$$T_r(\dot{\varphi}_r) = c_r\dot{\varphi}_r + T_{c_r}sgn(\dot{\varphi}_r) \tag{2a}$$

$$T_b(\dot{\varphi}_b) = c_b(\dot{\varphi}_b) + T_{c_b}sgn(\dot{\varphi}_b) \tag{2b}$$

Where c_r and $c_b$are the damping viscous coefficient and T_{c_r} is the Coulomb friction torque. Friction torque leads to a decreasing torque-on-bit $T_b$with increasing bit angular velocity for low velocity which acts as negative damping and is the cause of stick slip vibrations.

4.2 NUMERICAL SIMULATIONS

In the model, the top driving motor dynamics is not considered. It is assumed that arbitrary torques T_m can be applied without taking into account the dynamics generating this torque. Thus,

$$T_m = T_{max} \sin\left(\frac{2\pi}{T} t\right) \qquad (3)$$

Some cases have been made to observe the velocity of the angular velocity of the BHA and the angular velocity of the rotary top system. Some simulation has been made for the model with (3) subjected to the variations of the values of the period, T and the value of maximum period, T_{max} . The model parameters used for the simulation are:

$J_r = 0.518 kgm^2$

$J_b = 0.0318 kgm^2$

$c_r = 0.18 Nms/rad$

$c = 0.0001 Nms/rad$

$c_b = 0.03 Nms/rad$

$T_{s_b} = 8Nm$

$T_{c_r} = 0.7Nm$

$T_{c_b} = 0.4Nm$

$k = 0.073 Nm/rad$

The cases that have been made are as tables below. For these cases, the variation of the value of T_{max} is done. For this, the value of the period, T is assumed constant which is 2 seconds.

	$T_{max}\ (s)$
Case 1	0.8
Case 2	0.9
Case 3	1.0
Case 4	1.1
Case 5	1.2

TABLE 1: The variations of values of T_{max}

From these values, simulations are made by using MATLAB application and the variations are substitute in the equation (3).

Case 1 ($T_{max} = 0.8$)

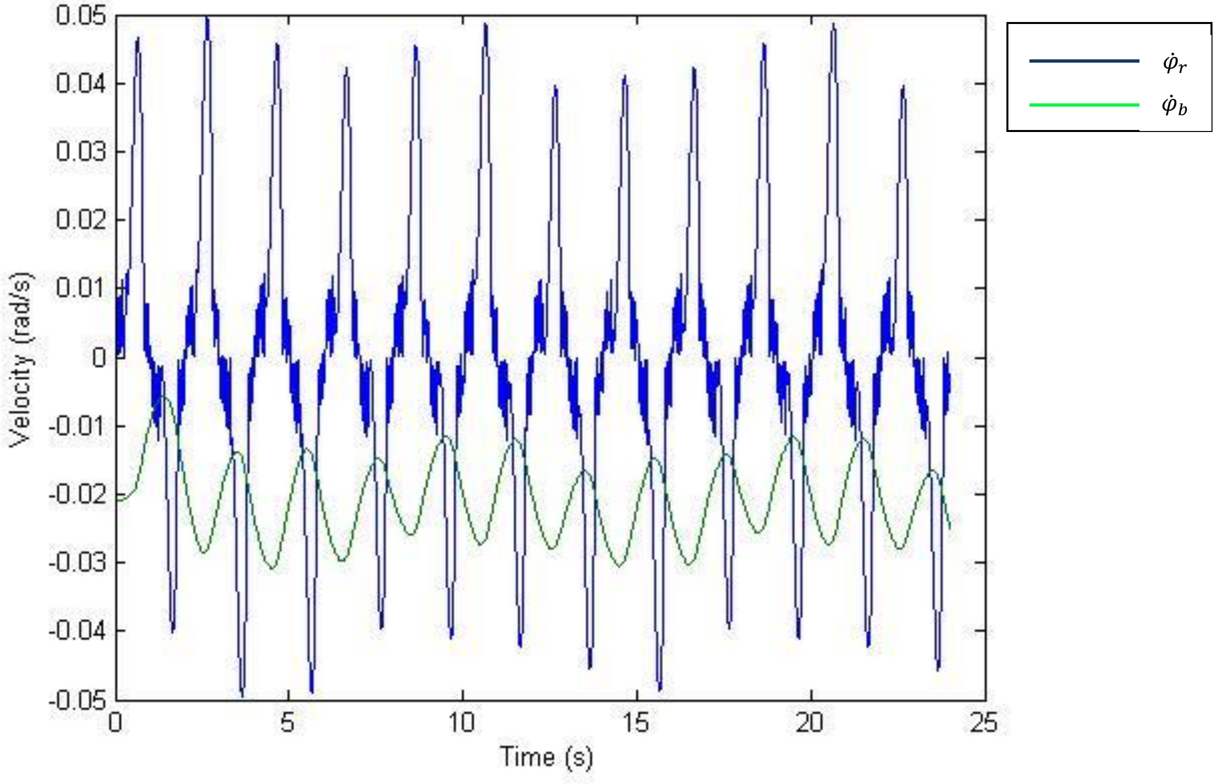


FIGURE 9: Simulations for $T_{max} = 0.8s$

From the above plot, it is shown that the maximum velocity for the angular velocity of the rotary top system is 0.048 rad/s whereas the maximum angular velocity of the BHA is -0.005 rad/s.

Case 2 ($\boldsymbol{T_{max} = 0.9}$)

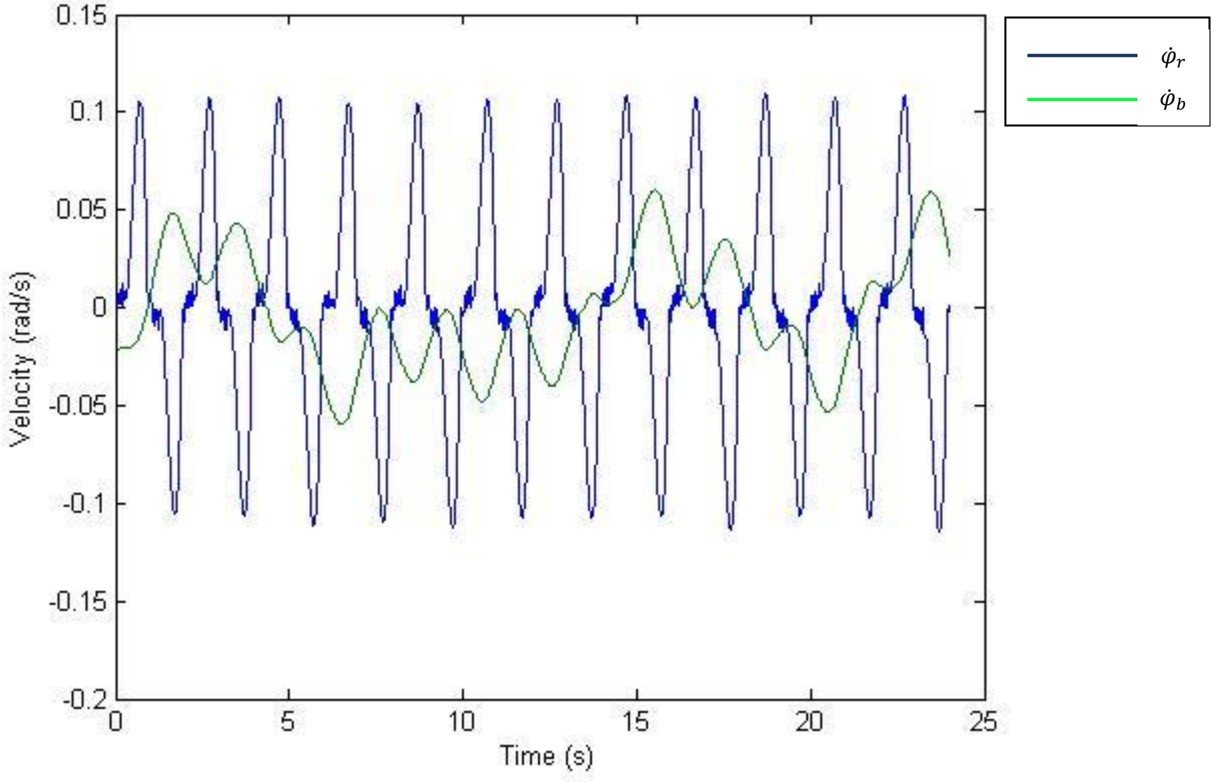


FIGURE 10: Simulations for $T_{max} = 0.9s$

Based on the above plot, it is shown that the maximum velocity for the angular velocity of the rotary top system is 0.11 rad/s whereas the maximum angular velocity of the BHA is 0.06 rad/s.

Case 3 ($T_{max} = 1.0$)

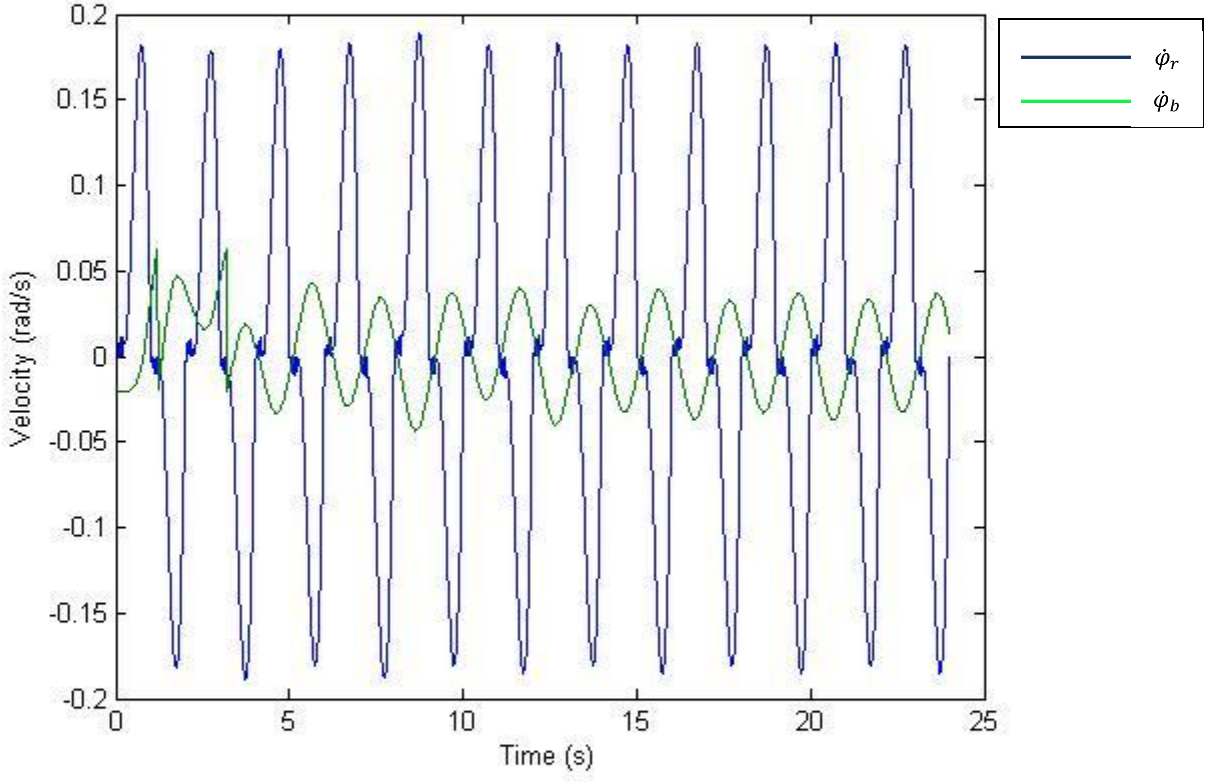


FIGURE 11: Simulations for $T_{max} = 1.0s$

From the above plot, it is shown that the maximum velocity for the angular velocity of the rotary top system is 0.18 rad/s whereas the maximum angular velocity of the BHA is 0.06 rad/s.

Case 4 ($\boldsymbol{T_{max} = 1.1}$)

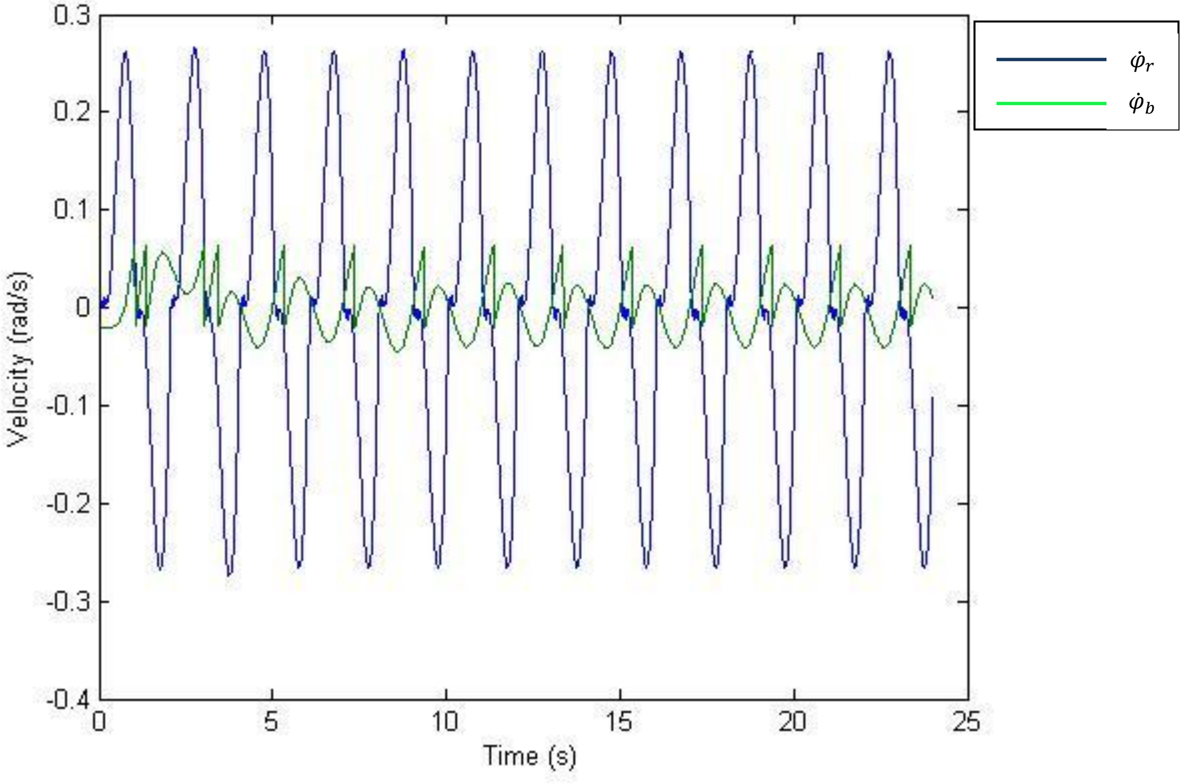


FIGURE 12: Simulations for $T_{max} = 1.1s$

The above plot shows that that the maximum velocity for the angular velocity of the rotary top system is 0.26 rad/s whereas the maximum angular velocity of the BHA is 0.06 rad/s.

Case 5 ($T_{max} = 1.2$)

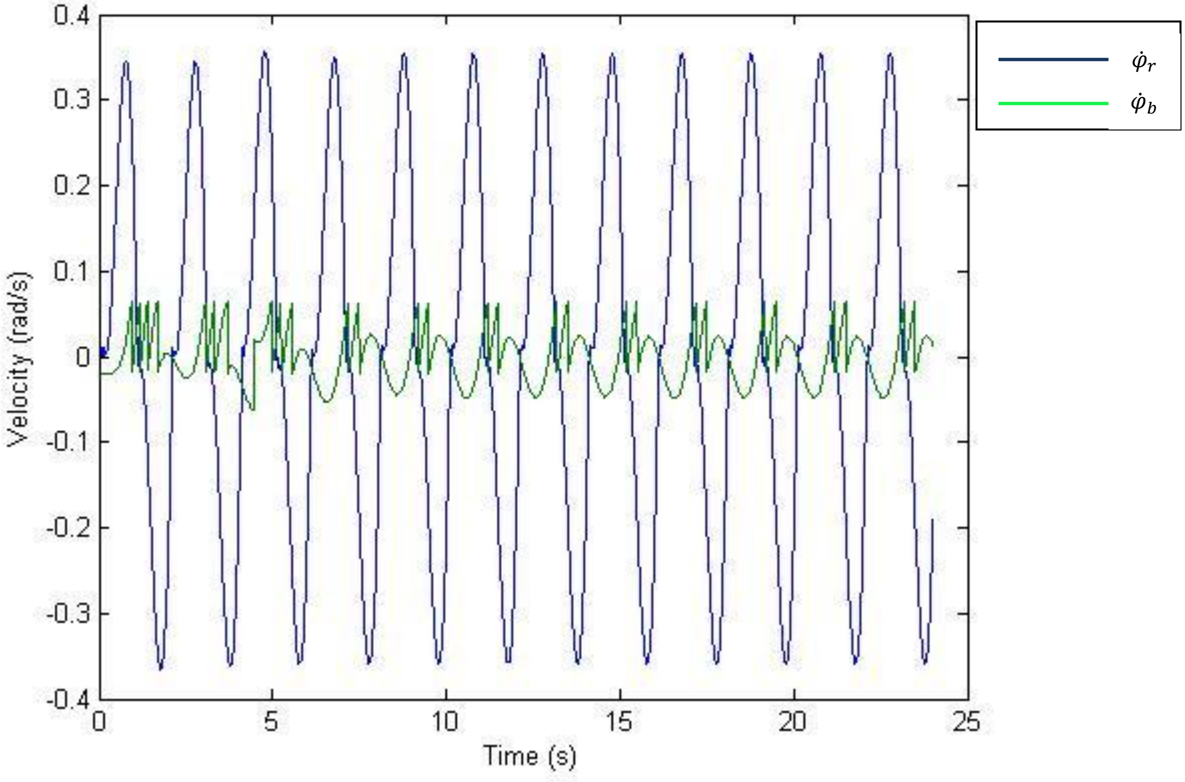


FIGURE 13: Simulations for $T_{max} = 1.2s$

From on the above plot, it is shown that the maximum velocity for the angular velocity of the rotary top system is 0.35 rad/s whereas the maximum angular velocity of the BHA is 0.06 rad/s

Based on these plots, it is shown that as the value of the maximum period, T_{max} increase, the maximum velocity of the angular velocity of the rotary top system will also be increase whereas the value of the angular velocity of the BHA will remain the same.

Other cases have also been made which is the variation of the period, T. As for these cases, the maximum period, T_{max} is assumed constant which is 0.8 seconds. The variation of the values is shown in the table below.

	T (s)
Case 6	1
Case 7	2
Case 8	3
Case 9	4
Case 10	5

TABLE 2: Variations of period, T

From these values, simulations are also made by using MATLAB application and the variations are substitute in the equation (3).

The plots of these variations of period, T is shown below

Case 6 ($T = 1$)

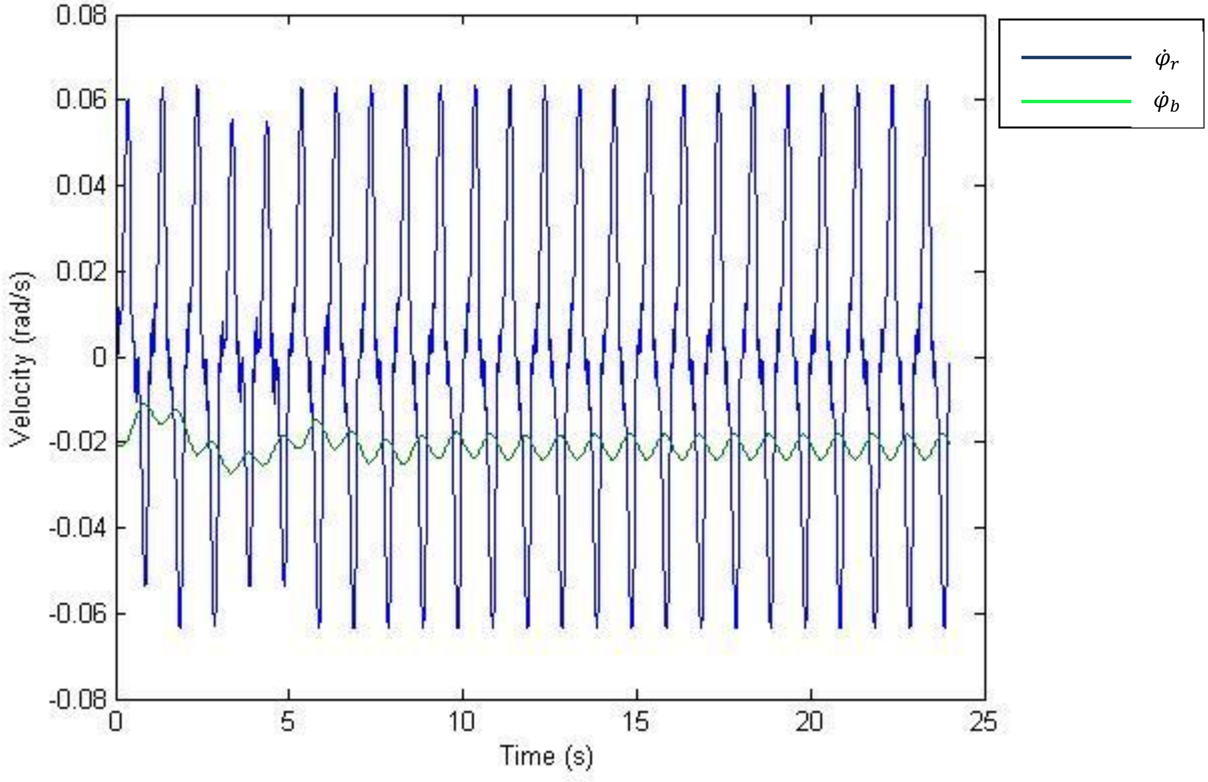


FIGURE 14: Simulations for T=1s

Based on the above plot, it is shown that the maximum velocity for the angular velocity of the rotary top system is 0.61 rad/s whereas the maximum angular velocity of the BHA is -0.01 rad/s.

Case 7 ($T = 2$)

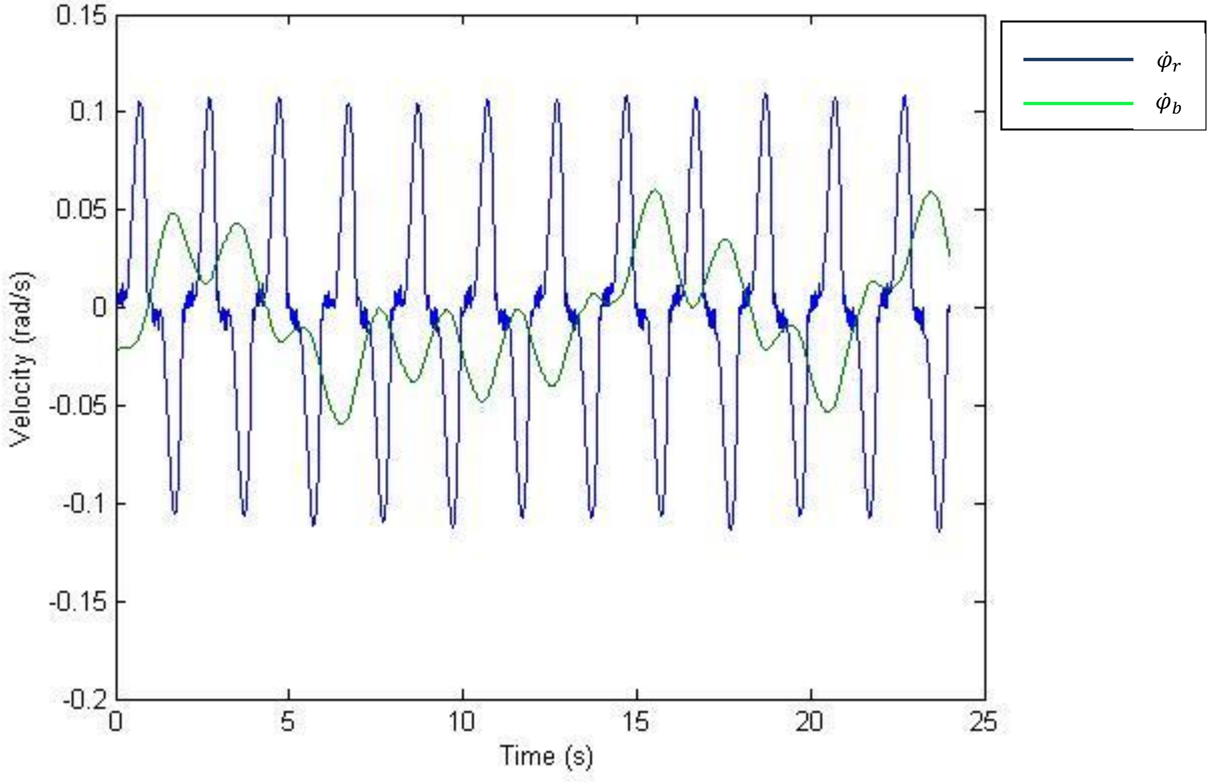


FIGURE 15: Simulations for T=2s

From the plot, it is shown that the maximum velocity for the angular velocity of the rotary top system is 0.1 rad/s whereas the maximum angular velocity of the BHA is 0.06 rad/s.

Case 8 ($T = 3$)

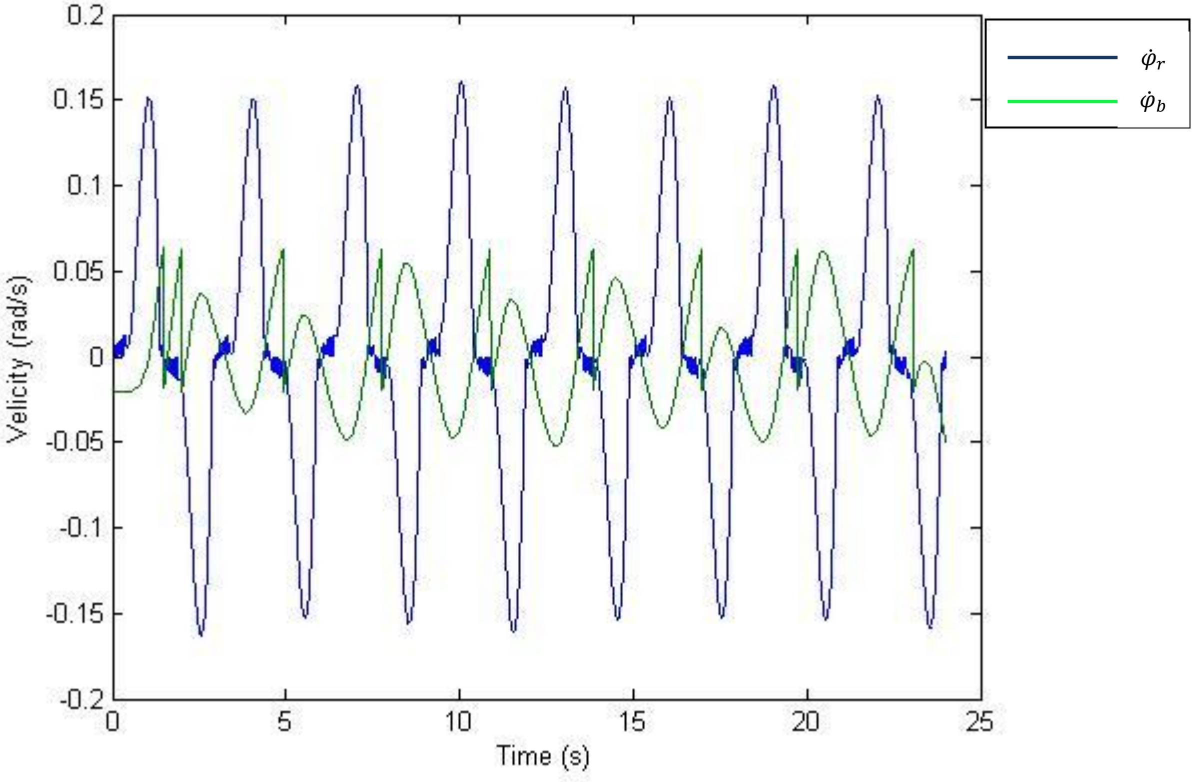


FIGURE 16: Simulations for T=3s

The above plot shown that the maximum velocity for the angular velocity of the rotary top system is 0.15 rad/s whereas the maximum angular velocity of the BHA is 0.06 rad/s.

Case 9 ($T = 4$)

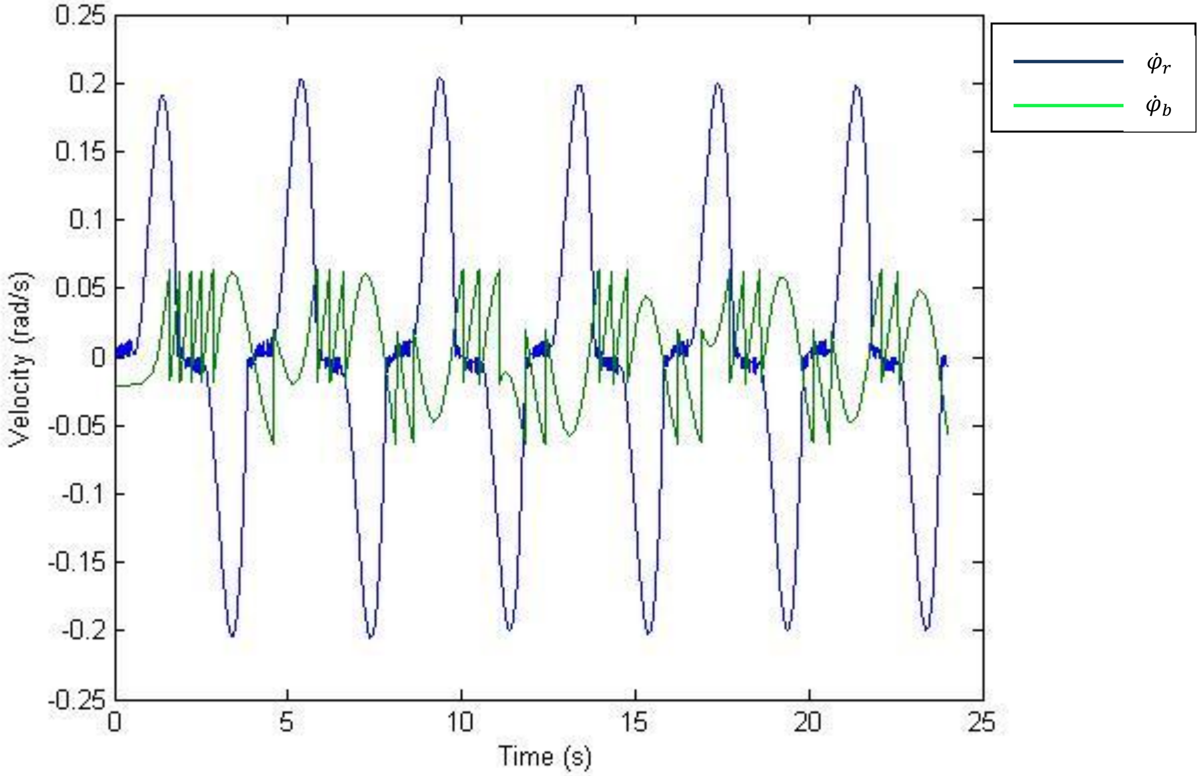


FIGURE 17: Simulations for T=4s

Based on the above plot, it is shown that the maximum velocity for the angular velocity of the rotary top system is 0.2 rad/s whereas the maximum angular velocity of the BHA is 0.06 rad/s.

Case 10 ($T = 5$)

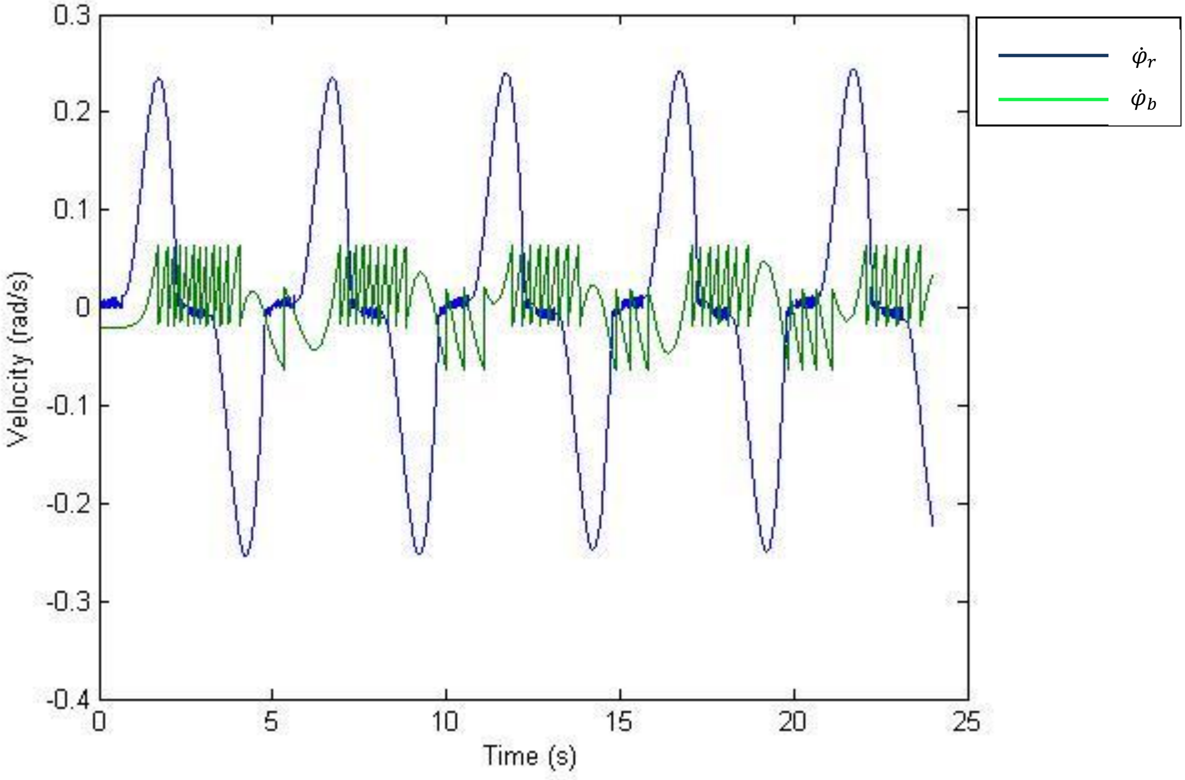


FIGURE 18: Simulations for T=5s

From the T=5s plot, it is shown that the maximum velocity for the angular velocity of the rotary top system is 0.25 rad/s whereas the maximum angular velocity of the BHA is 0.06 rad/s.

Based on the above plots, it can be seen that varying the values of the period will also affect the angular velocity of the rotary top system. As the period, T increase, the value of the maximum angular velocity is also increase and the angular velocity of the BHA will remain the same.

CHAPTER 5

CONCLUSION AND RECOMMENDATION

5.1 CONCLUSION

Within this report, drill string dynamics has been studied through modeling and numerical effort. The dynamics have been studied within the unique long string with contact system. A model describing the torsional behaviour of a generic vertical oil well drill string has been presented. This model is a combination of some previous models proposed in the literature. The problem of modeling is divided in two different problems. First, the problem of modeling the torsional behaviour of the drill string. Second, the problem of modeling the rock bit interaction. To reduce the order models, there are some cases applied based on variation of the values of maximum period, T_{max} and the values of the period, T. A two degree of freedom rotational vibration has been derived to simulate the dynamics of a drill string. In particular, attention has been paid to the contact between the drill string and the outer shell, and the modeling allows to simulate sliding and sticking motions. As the value of the friction coefficient is low, there are bumping motions between the drill string and the outer shell. The characteristic of these interactions changes as the friction coefficient is increased. It is also necessary to make an analysis of the influence of the drill string length, formation properties and bit characteristics in model parameters which would lead to a robust performance analysis. It is conclude that varying the values of the maximum period, T_{max} and the period, T can reduce the stick slip oscillations in the drill string.

5.2 RECOMMENDATION

Some suggestions have been provided in the section for further studies in the following areas: i) model selection ii) nonlinear phenomena and iii) control schemes.

5.2.1 Distibuted parameter models with experiments

Reduced-order models focusing on vertical drill-string dynamics have been studied in this work. For expanding the applications to capture more features such as initial curvature, a distributed-parameter model is needed. Non-dimensional analysis could also be considered to make the predictions have a broad applicability. Experiments focused on horizontal drilling, along with a corresponding distributed-parameter model, can be a bridge between the current work and studies with a full size drill-string system.

5.2.2 More than two degree of freedom model

The continuation of work with the two degree-of-freedom model through numerical simulations and experiments can form another path to study the system. The additional degree-of-freedom, tile angle, can allow the linking of the model to a continuous model for a better description of the whole drill-string system.

5.2.3 Nonlinear phenomena

If feasible, nonlinear analyses can be conducted along the lines. Attention also needs to be paid to possible nonlinear coupling between bending vibrations and torsion vibrations and other modes of vibrations.

5.2.4 Control scheme

Different control schemes can be studied with the experimental apparatus to determine their effectiveness. These schemes can include those that have been previously studied in the active control area and previous efforts related to drill stings.

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APPENDICES

```
function [u]=exforce(rtime,ndof,xsd)
u(1:ndof,1)=0.0;

cb = 0.03;
tcr = 0.7;
tcb = 0.4;
cr = 0.18;
tr = cr*xsd(1,1)+tcr*sign(xsd(1,1));
tb = cb*xsd(2,1)+tcb*sign(xsd(2,1));

T = 2;
tmax = 1.2;

pi=3.1416;
tm = tmax * sin (2*pi/T*rtime);


u(1,1)=tm - tr;
u(2,1)= -tb;
```

```
function [rc]=readdamping(ndof)
% damping matrix components
c=0.0001;
rc(1,1)=c;
rc(1,2)=-c;
% rc(1,3)=0;
% rc(1,4)=0;
rc(2,2)=c;
% rc(2,3)=0;
% rc(2,4)=0;
% rc(3,3)=0;
% rc(3,4)=0;
% rc(4,4)=0;

for i=2:ndof
    for j=1:(i-1)
        rc(i,j)=rc(j,i);
    end
end
```

```
function [xs]= readinit(nss,ndof)

% initial displacement
xs(1:ndof,1)=0;

% initial velocity
xs(ndof+1:nss,1)=0;
```

```
function [rm]=readmass(ndof)
% mass matrix components
rm(1,1)=0.518;
rm(1,2)=0;
% rm(1,3)=0;
% rm(1,4)=0;
rm(2,2)=0.0318;
% rm(2,3)=0;
% rm(2,4)=0;
% rm(3,3)=50;
% rm(3,4)=0;
% rm(4,4)=2;

for i=2:ndof
    for j=1:(i-1)
        rm(i,j)=rm(j,i);
    end
end
```

```
function [rk]=readstiffness(ndof)
% stiffness matrix components
k=0.073;
rk(1,1)=k;
rk(1,2)=-k;
% rk(1,3)=0;
% rk(1,4)=0;
rk(2,2)=k;
% rk(2,3)=-2000;
% rk(2,4)=0;
% rk(3,3)=5000;
% rk(3,4)=-3000;
% rk(4,4)=3000;

for i=2:ndof
    for j=1:(i-1)
        rk(i,j)=rk(j,i);
    end
end
```

```
function [h,tend,trec]= readtime

h = 0.01;
tend = 24;
trec = 0.1;
```

```
% M-file to simulate a 2-DOF flexible system
clear all;
ndof = 2;
nss = 2*ndof;

% defining mass matrix
rm(1:ndof,1:ndof)=0.0;
[rm]=readmass(ndof);

% defining damping matrix
rc(1:ndof,1:ndof)=0.0;
[rc]=readdamping(ndof);

% defining stiffness matrix
rk(1:ndof,1:ndof)=0.0;
[rk]=readstiffness(ndof);

% [X,D] = eig(rk,rm)
% pause

%   xdot = Ax + Bu
% building A
a(1:nss,1:nss)=0;
for i=1:ndof
    a(i,i+ndof)=1;
end
a(ndof+1:nss,1:ndof)=-inv(rm)*rk;
a(ndof+1:nss,ndof+1:nss)=-inv(rm)*rc;

% building B
b(1:nss,1:ndof)=0;
b(ndof+1:nss,1:ndof)=inv(rm);

% reading initial conditions
xs(1:nss,1)=0;
[xs]=readinit(nss,ndof);

t(1:12000)=0; xs1(1:12000)=0; xs2(1:12000)=0; xs3(1:12000)=0;
xs4(1:12000)=0;
[t, xs1,xs2,xs3,xs4]=solve1(a,b,xs,nss,ndof);

% END
```

```
function [t, xs1,xs2,xs3,xs4]= solve1 (a,b,xs,nss,ndof)

us(1:ndof,1)=0.0;
k1(1:nss,1)=0.0;
k2(1:nss,1)=0.0;
k3(1:nss,1)=0.0;
k4(1:nss,1)=0.0;

% xsj(1:nss,1)=xs(1:nss,1);
% xsj1(1:nss,1)=0.0;

[h,tend,trec]=readtime;
ntime = tend/h;

% Runge-Kutta 4th-order

       t(1:ntime+1)=0;
       xs1(1:ntime+1)=0;
       xs2(1:ntime+1)=0;
       xs3(1:ntime+1)=0;
       xs4(1:ntime+1)=0;

for itime = 1:1:ntime+1
    rtime=(itime-1)*h;
    [k1]=xsdot(a,b,xs,nss,ndof,rtime);
    xsh(1:nss,1)=0;
    rtime=(itime-1)*h + h/2;
    xsh(1:nss,1)=xs(1:nss,1)+(h/2)*k1;
    [k2]=xsdot(a,b,xsh,nss,ndof,rtime);
    xsh(1:nss,1)=xs(1:nss,1)+(h/2)*k2;
    [k3]=xsdot(a,b,xsh,nss,ndof,rtime);
    rtime=(itime-1)*h + h;
    xsh(1:nss,1)=xs(1:nss,1)+h*k3;
    [k4]=xsdot(a,b,xsh,nss,ndof,rtime);
    xs(1:nss,1)=xs(1:nss,1)+h*(k1+2*k2+2*k3+k4)/6;

     t(itime)=rtime;
     xs1(itime)=xs(1,1);
     xs2(itime)=xs(2,1);
     xs3(itime)=xs(3,1);
     xs4(itime)=xs(4,1);
end
  plot(t,xs3,t,xs4)
```

```
function [k]=xsdot(a,b,xs,nss,ndof,rtime)

u(1:ndof,1)=0.0;
xsd(1:ndof,1)=0;
xsd(1:ndof,1)=xs(ndof+1:nss,1);
[u]=exforce(rtime,ndof,xsd);
k =a*xs + b*u;
```