Three-Dimensional Analysis on Thermal Stresses in A Perforated Disc Brake of Vehicle

by

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Dissertation submitted in partial fulfillment of the requirements for the Bachelor of Engineering (Hons) (Mechanical Engineering)

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CERTIFICATION OF APPROVAL

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by Lee Jin Ming

A project dissertation submitted to the Mechanical Engineering Programme Universiti Teknologi PETRONAS in partial fulfilment of the requirement for the BACHELOR OF ENGINEERING (Hons) (MECHANICAL ENGINEERING)

Approved by,

(Dr. Khairul Fuad)

UNIVERSITI TEKNOLOGI PETRONAS TRONOH, PERAK January 2010

CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

LEE JIN MING

ABSTRACT

This report provides background information on small vehicle's perforated disc brake which is subjected to high frictional force during braking. This frictional action will consequently converts the car's kinetic energy into thermal energy in order to slow the car down. On the other hand, the thermal energy will be the source of thermal stresses in the perforated disc brake. The objectives of this project are to analyze and simulate three dimensional temperature and thermal stresses distribution within the perforated disc brake and to analyze the effect of having perforated holes by comparing the results with a non-perforated disc brake. The methodologies used in achieving these objectives are through numerical calculation and finite elements analysis by ANSYS. The temperature distributions obtain from the simulation show that maximum temperature produced by perforated and non perforated disc brakes is almost the same but perforated holes in perforated disc brake contributed to better temperature distribution within its cross section, giving better cooling effect. While for the thermal stress distribution, the generated Von Mises stress for perforated disc brake is lower than non perforated disc brake but high magnitude of Von Mises stress exists around the perforated holes. Furthermore, overall top surface Von Mises stress distribution shows that perforated disc brake has better stress distribution with extra lower stress region. Thus, it can be concluded through this project that perforated disc brake is better in increasing the performance of vehicle but the weighting are between high stress concentration at the perforated zones and better cooling ability as compared to conventional disc brake.

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CHAPTER 1 INTRODUCTION

1.1 BACKGROUND OF STUDY

Braking system in a vehicle is one of the most crucial components because apart from functioning as a mechanism to slow down the vehicle, it also affects the handling and safety performance of the vehicle. Two main components of the brake are the pad and the disc brake. The pad is essentially a piece of slim frictional material used to create a rubbing force against the disc brake surface in order to convert the vehicle's kinetic energy into thermal energy to slow and ultimately stops the vehicle. Furthermore, disc brake is being used to dissipate thermal heat produced during braking by providing sufficient surface for thermal convection and radiation. In sudden braking condition, high thermal heat is generated from the frictional force between the pad and disc brake. Thus, the materials used for both of these components have to be able to sustain high temperature without any failure. In the market, commonly used raw material for the disk brake is cast iron because it has high melting temperature of 1200 °C, easy to manufacture and relatively cheaper than other materials such as carbon disc.

As most of the generated thermal heat will be transferred to the disc brake, it will then functions as a small heat sink reservoir. In cases such as emergency braking, the disc brake might not be able to accumulate an infinite quantity of heat generated from continuous braking. Thus, the heat needs to be dissipated by having a good circulation of air which will be heated up when it is in contact with the hot surface of the disc and this keeps the disc brake from severe temperature that contributes to mechanical failure, or cracking. In order to increase the efficiency of heat dissipation from the disc brake, some disc brakes are vented by connecting "bridges" between two disc brakes to allow air flow in between them. Therefore, by having larger in contact surface with the air, heat is dissipated more effectively. Example of single disc and ventilated disc brake is shown in Figure 1 and Figure 2 below.



Figure 1.1: Single Plate Disc Brake



Figure 1.2: Ventilated Disc Brake

As for high performance vehicles such as racing car, conventional disc brake may not be sufficient enough in dissipating high generated heat, because of this perforated disc brake is being used instant of conventional disc brake. Perforated disc brake is also known as high performance or drilled disc brake as shown in Figure 1.3 below. Furthermore, perforated disc is commonly used in high performance vehicles due to its higher braking power especially during high speed travelling. But associated problem is that the brake pads will outgas and create boundary layer of gas between the pad and the disc if it is under use, hurting braking performance. Due to that, perforations are created to provide the gas someplace to escape. Although modern brake pads seldom suffer from out gassing problems, water residue may also build up after a vehicle passes through a puddle and impede braking performance.

However, perforated discs do have positive effect in wet condition because the perforated holes prevent a film of water building up between the disc and the pads as explained in Wikipedia.org [1]. Besides, the drilled holes also give more bite and allow air current (eddies) to blow through the disc brake to assist cooling and ventilating air so that more heat can be release by the disc surface onto the air as described by www.howstuffworks.com [2].



Figure 1.3: Perforated Disc Brake

1.2 PROBLEM STATEMENT

Light vehicle's disc brake experiences high temperature and forces especially during severe braking condition. Under such condition, the disc brake's surface temperature is much higher than its internal temperature due to the contact between the disc surface and the pad. This thermal shock due to the fast surge in temperature will contribute to the creation of thermal stress from uneven expansion of the disc brake between its surface volume and internal volume after a large number of cycles. These cycles of thermal stress arose from the braking may induce a number of unfavorable conditions such as material fatigue, surface cracking and permanent distortions. Ji-Hoon and In Lee [3] explained that frictional heating, thermal deformation and elastic contact in sliding contact systems affect the contact pressure and temperature on friction surfaces. These coupled thermal and mechanical behaviors can be unstable and lead to localized high temperature contact regions known as "hot spot" if the sliding speed is extremely high. These hot spots can cause material damage and ultimately thermal crack. Cracks are very much a big concern to the driver because they affect not only the braking performance of the vehicle but will also lead to braking failure.

For perforated disc brake, it poses a major setback in its application due to higher chance of cracking because the perforated holes are a source of stress cracks or zones with larger concentration of thermal stress under severe conditions as shown in Figure 1.4. According to www.carbibles.com [4], the drilling or perforation weakens the rotors and typically results in micro fractures to the rotor. The significant of this project is to carry out analysis on stresses arose due to high temperature or better known as thermal stresses and to examine how the perforation on the disc brake affects its performance. Furthermore, this project will verify the claim that the perforated holes are the main cause of cracking due to high concentration of stress around the perforated holes.



Figure 1.4: Cracking of Perforated Disc Brake

1.3 OBJECTIVES

The objectives of this project are stated as below:

- 1. To analyze and simulate three dimensional temperature and thermal stress distribution in a perforated brake disc of small vehicle during braking using ANSYS.
- 2. To compare the temperature and thermal stress distribution between perforated and non-perforated brake disc.

1.4 SCOPE OF STUDY

In this project, a pair of perforated and non-perforated disc brakes and brake pads will be considered for further analysis as per stated in the objectives. It involves the contact between the brake pad and disc brake to create sufficient frictional force in order to slow the wheel. Thermal stresses arise due to this high frictional force and temperature. So it is important to perform the analysis on thermal stress as stress will ultimately leads to thermal cracking of the disc brake and stress analysis is one of the crucial aspects in designing sliding contact systems.

In the analysis, the pressure distribution imposed by the brake pad onto the disc brake is assumed to be uniform throughout the contact area for the ease of analysis and also it is assumed that the frictional heat flux at the contact area is constant along the surface. Besides, this project involved a lot of applications and knowledge from various mechanical engineering fields of thermodynamic, finite element analysis, computer aided design and engineering and stress analysis.

CHAPTER 2 LITERATURE REVIEW

There are three types of mechanical stresses subjected by the disc brake as shown in Figure 2.1. The first one is the traction force created by the centrifugal effect due to the rotational of the disc brake when the wheel is rotating and no braking force is applied to the disc. During braking operation, there are another two additional forces experienced by the disc brake. Firstly, compression force is created as the result of the force exerted by the brake pad pressing perpendicular onto the surface of the disc to slow it down. Secondly, the braking action due to the rubbing of the brake pad against the surface of the disc brake is translated into frictional or traction force on the disc surface which acts in the opposite direction of the disc rotation. It is due to this frictional force that heat is generated, increases the temperature of the disc brake and contributes to the rise of thermal stresses. Thermal stresses will exist in a material whenever temperature gradients are present within it.



Figure 2.1: Mechanical Stresses on Disc Brake

The different in temperatures produces different expansions and subject materials to internal stress. Mohd Adam (2009, p.5) [5] points out that the major sources of thermal stress are restrained thermal expansion or contraction and temperature gradient established during heating or cooling process. The thermal stress which developed if a structure or material is completely constrained is the product of the coefficient of linear expansion (α), the temperature change (ΔT) and Young's modulus (*E*) for the material where the relationship is given by:

$$\sigma = \alpha \left(\Delta T \right) E \tag{2-1}$$

As referring to Robert W. Soutas-Little (1999, p. 78-80) [6], the effect of temperature change can be introduced into the theory of elasticity by examining the strain energy function, W which is based on Hooke's Law. For a hyperelastic material, a strain energy function W is assumed to exist, with property of:

$$\sigma_{ij} = \frac{\delta W}{\delta \epsilon_{ij}} \tag{2-2}$$

The existence of this function can be shown to place symmetry constraints on the constants involved in the generalized Hooke's Law.

$$\sigma_{ij} = C_{ijrs} \in_{rs} \tag{2-3}$$

If the material obeys Hooke's law, the stain energy function can be written as:

$$W = \frac{1}{2} C_{ijkl} \in_{kl}$$
(2-4)

In accounting the effect of temperature change in the theory of elasticity, W is assumed to be in the form of a power series expansion in strain, giving:

$$W = C_o + C'_{ij} \in_{ij} + \frac{1}{2} C_{ijkl} \in_{ij} \in_{kl} + \dots$$
(2-5)

The stress tensor may be related to the strain tensor by the relationship of:

$$\sigma_{ij} = \frac{\delta W}{\delta \epsilon_{ij}} = C'_{ij} + C_{ijkl} \epsilon_{kl}$$
(2-6)

The second term leads to the generalized Hooke's law and the first term implies that stresses may present in a body which is in a state of zero strain where such stresses may arise if a liner expansion occurs due to temperature changes. The simplest assumption is to assume that C'_{ij} is linearly proportional to the temperature change $(T-T_o)$ and equal in all directions for isotropic material:

$$C'_{ij} = -\alpha \left(3\lambda + 2\mu\right) \left(\mathbf{T} - T_o\right) \delta_{ij} \tag{2-7}$$

where α is coefficient of linear thermal expansion.

The stress-strain relation then becomes:

$$\sigma_{ij} = \lambda \,\delta_{ij} \,\epsilon_{kk} + 2\mu\epsilon_{ij} - \alpha \,(3\lambda + 2\mu) \,(\mathrm{T} - T_o)\delta_{ij} \tag{2-8}$$

Above equation is known as the Duhamel – Neumann generalization of Hooke's law. The thermal strain equation can be obtained by adding a thermal expansion term to the strain equation:

$$\epsilon_{ij} = \frac{1}{2\mu} \left[\sigma_{ij} - \frac{\lambda}{3\lambda + 2\mu} \sigma_{kk} \delta_{ij} \right] + \alpha \,\delta_{ij} \,(\text{T-}T_o) \tag{2-9}$$

According to D.J. Kim, Y.M. Lee, J.S. Park and C.S. Seok (2008, p. 456-459) [7], thermal stresses are generated by the cyclic frictional heat and thermal fatigue cracks are generated by the thermal stress on a frictional plate. Therefore, in order to secure the braking stability and to improve the fatigue life of a disk brake, it is necessary to research the thermal stress on a disk brake. Furthermore, J.H. Choi and I. Lee (2004, p. 47–58) [3] mentioned that thermal stresses due to high temperatures may induce a number of unfavorable conditions such as surface cracks and permanent distortions. Frictional heating, thermal deformation and elastic contact in sliding contact systems affect the contact pressure and temperature on the friction surfaces. Based on journey by Thomas J. Mackin and group (2002, p.63-76) [8], thermal cracking in disc brake rotors is a low cycle thermo-mechanical fatigue problem and there are three ways to eliminate thermal cracking in brake rotors: (1) increase the yield and fatigue strength of the rotor material; (2) decrease the braking temperatures; and/or (3) re-design the hub–rotor unit to eliminate constraint stresses.

Although the perforated holes are initially designed to provide better grip to the disc brake and increases its frictional property, the holes themselves might appear to be the localized high temperature contact area. This area will experiences higher metal fatigue and initiates the thermal crack. Further analysis by using finite element method will be used to compute the temperature and thermal stresses distribution on the surface of the perforated disc brake. Finite element method is known as a powerful tool for many engineering simulations was widely used to compute the distributions of thermal elastic, elastic–plastic, residual stresses and thermal stresses and finite element method is a numerical technique for finding approximate solutions of partial differential equations as well as of integral equations as described in Wikipedia.com [9]. For this reason, ANSYS, a general purpose code, was preferred in the analysis of the thermal stress problem. Moreover, finite element solution to be carried out by ANSYS will divides the modeled three dimension disc brake volume into smaller elements and nodes by mesh generation process to evaluate the thermal stress distribution within the disc brake.

CHAPTER 3 METHODOLOGY

3.1 METHODOLOGY

The three dimensional model of disc brake will be modeled by using computer design software from CATIA. This model can be further analyzed by performing finite element analysis method with the help of engineering software ANSYS capable of running the simulation and analysis for required conditions. Firstly, calculated heat flux generation will be applied to the disc brake model with the help of ANSYS to investigate the resulting temperature distribution. Next, the acquired heat distribution will be use as the input to further investigating the corresponding thermal stresses on the disc brake. The methodology used is presented in Figure 3.1 below. Please refer to Appendix G for project's Gantt Charts.



Figure 3.1: Methodology

3.2 DATA GATHERING

3.2.1 Vehicle Specifications

Table 3.1 contains the vehicle's specifications that are used in the analysis. Please refer to Appendix A for the full calculations of each vehicle's specification.

| No. | Parameter | Specification |
|-----|--|--|
| 1 | Engine capacity | 2.4 liter (2400 cc) |
| 2 | Maximum car weight | 1950kg |
| 3 | Front and rear axle weight rating | a) Front gross axle weight rating: 1060kg b) Rear gross axle weight rating: 915kg |
| 4 | Weight ratio | a) Front ratio $= \frac{1060kg}{1950kg} = 0.54$ b) Rear ratio $= \frac{915kg}{1950kg} = 0.46$ |
| 5 | Considering maximum car weight for analysis | 1950kg |
| 6 | Front and rear wheel applied | Front wheel applied weight = 1053kg |
| | weight | Rear wheel applied weight = 897kg |
| 7 | Weight applied on each disc brake surface (heavier weight applied to front wheel is considered) | 263.25 kg |
| 8 | Wheel Diameter | 658 mm |

Table 3.1: Vehicle Specifications

3.2.2 Disc Brake and Brake Pad Specifications

Actual disc brake dimensions and specifications are obtained for further analysis in order to ensure that the result for temperature and thermal stresses distribution is as close to the real application condition as possible. The specifications for disc brake and brake pad are shown in Table 3.2 and Table 3.3 as following. Besides, the material of the disc brake is of Grey Cast Iron (ASTM A48) and its properties are shown in Table 3.4.



Figure 3.2 Perforated Disc Brake



| No. | Parameter | Dimension (mm) |
|-----|-----------------------|----------------|
| 1 | Outer diameter | 328 |
| 2 | Inner diameter | 220 |
| 3 | Individual disc | 5.75 |
| | thickness | |
| 4 | Ventilation thickness | 16.0 |
| 5 | Ventilation wide | 6.0 |
| 6 | Perforation diameter | 8.0 |



Figure 3.3 Brake Pad

No.ParameterDimension (mm)

| 1 | Thickness | 17.5 |
|---|--------------|------|
| 2 | Total length | 186 |
| 3 | Height | 52.5 |

Table 3.3 Brake Pad Specifications

| Table 3.4. | Properties | of Grev | Cast Iron | (ASTM A48) |) |
|--------------------------|------------|---------|-----------|------------|---|
| 1 auto 5. 4 . | roperties | of Oldy | Cast non | (DDIMD+0) | , |

| No. | Property | Value |
|-----|--|-------------------------------------|
| 1 | Specific heat, Cp | 544 J/kg°C |
| 2 | Thermal conductivity, k | 55 W/m°C |
| 3 | Density, p | $7150 \text{kg}/m^2$ |
| 4 | Coefficient of convection | $50 \text{ W/}m^2$ °C |
| 5 | Coefficient of thermal expansion, α | $11.8 \ge 10^{-6} \text{ °/}C^{-1}$ |
| 6 | Young's modulus of elasticity, E | 9.239 x10 ¹⁰ Pa |
| 7 | Poisson's ratio, v | 0.26 |

3.3 DISC BRAKE DYNAMIC AND HEAT ANALYSIS

In this analysis, several assumptions and conditions are set to simplify the analysis and calculation as following:

- During the vehicle deceleration, the disc brake is assumed to be stationary while the brake pad rotates in counter-clockwise direction around it.
- Uniform pressure distribution by the brake pad onto the disc brake surface.
- Since a ventilated disc brake is being used as the sample with one side of the disc plate is asymmetrical to the other side, only one side of the disc plate is used in the analysis.
- The vehicle kinetic energy is 100% converted to heat energy with 95% of the total heat being absorbed by the disc while another 5% by the brake pad.
- Force distributed on one disc brake is equal to the total frictional force applied on the rubbing surface.
- Constant deceleration of the vehicle from initial speed of 150km/hr to stopping at 0.7 Gravity.

Based on the initial conditions and assumptions made, the value of the heat flux and time interval of each pad rotational movement or step are calculated to be set as input for ANSYS simulation. In the calculation, first of all the total stopping time and travelled distance is determined from the initial vehicle condition. Next, the disc brake initial angular velocity and acceleration is obtained to calculate the initial angular velocity of the next brake pad rotational movement. Then, the time interval between two pad rotational positions is calculated followed by the heat absorbed by the disc brake based on the vehicle kinetic energy calculation. Finally, the heat flux or heat transfer per unit area is obtained. These calculations will continue until the angular velocity of the rotating brake pad reached 0 rad/s. An example of the calculation is shown in Appendix B for the first brake pad movement and the following calculations till the final movement of brake pad is done with the help of Microsoft Excel as shown in Appendix C.

3.3.1 Justification of Selected Parameter for Analysis Condition

In the analysis of the thermal stresses that arise within the vehicle's disc brake due to the braking mechanism, it is assumed that the constant deceleration experienced by the vehicle prior to stopping to be of 0.7 gravity. Throughout the analysis in this project, the parameters chosen including the dimensions of disc brake and brake pad, vehicle specifications and also the vehicle operating conditions are referred to the real available parameters in order to ensure the accuracy of the analysis to be as close to the real working conditions. This also includes the optimum choice of the vehicle constant deceleration of 0.7 gravity based on the maximum braking specification of the selected vehicle model to create a worst case braking scenario in order to study the maximum effect of braking on the development of thermal stresses. The justification of the chosen constant deceleration value can be referred to the following estimation.

Maximum deceleration capability of vehicle obtained from two testing sources:

- i. <u>Testing one (referred to www.carwale.com/Research)</u> Minimum vehicle stopping distance = 52.87m, initial speed at 100km/hr. $v^2 = u^2 + 2$ a s $0^2 = (27.78m/s)^2 + 2$ a (52.87m) $a = -7.2972 \text{ m/s}^2$ in gravity fraction = $\frac{-7.2972 \text{ m/s}^2}{-9.81 \text{ m/s}^2} = 0.744 \text{ gravity}$
- ii. <u>Testing Two (referred to http://productsearch.rediff.com/productdetail)</u> Minimum vehicle stopping distance = 49.10m, initial speed at 100km/hr. $v^2 = u^2 + 2$ a s $0^2 = (27.78m/s)^2 + 2$ a (49.10m) $a = -7.875 \text{ m/s}^2$ in gravity fraction = $\frac{-7.875m/s^2}{-9.81m/s^2} = 0.80 \text{ gravity}$

Thus, from the analysis, vehicle's constant deceleration is justified by choosing value of 0.7 gravity as the initial condition input because this value is slightly lower than the maximum deceleration that this vehicle can achieve in reality.

3.3.2 Dynamic and Heat Energy Analysis

From the dynamic calculation of the brake pad rotational movements around the disc brake until it stopped, an angular velocity versus time graph is plotted as shown in Figure 3.4 below. It shows the time taken by the vehicle starting with the initial angular velocity of the brake pad at 126.66 rad/s decreasing to 0 rad/s in 6.01 seconds. In other word, this is the time taken for complete conversion of the vehicle's kinetic energy into heat energy by the first law of thermodynamics. Furthermore, from the dynamic calculation, the brake pad experienced 367 rotational steps before it came to complete stop. Also from the graph, the angular velocity of the brake pad is decreasing linearly with time due to the constant deceleration of the vehicle at 0.7 Gravity.



Figure 3.4: Angular Velocity versus Time

As the vehicle kinetic energy decrease due to the decreasing vehicle speed, heat energy is slowly building up from the conversion of the kinetic energy to heat energy by the frictional force between the surfaces of disc brake and pad. In this analysis, all the kinetic energy is assumed to be converted into heat energy with 95% of it is absorbed by the disc brake and another 5% by the brake pad. Following Figure 3.5 shown the graph of accumulative heat energy versus the time taken by the vehicle to stop. The plotted curve gave a parabolic response with moderate increase of the heat energy generation at the beginning and decreases with time where the total accumulated heat energy is 217121.05 Joule.



Figure 3.5: Accumulative Heat Energy versus Time

3.3.3 Heat Flux Generation

Heat flux is the measurement of heat transfer per unit area. It is the amount of heat absorbed by the disc brake surface per unit time and Figure 3.6 shown the corresponding heat flux versus time graph from the start of the braking force application of the pad onto the disc until the disc stopped rotating. The graph shown the highest heat flux value of 9226731.6 KW/ m^2 at the beginning of the analysis due to the higher angular velocity of the disc brake at that moment, and heat flux value linearly decreases with time as the vehicle decelerated constantly at 0.7 Gravity.



Figure 3.6: Heat Flux versus Time

3.4 DISC BRAKE MODELING

The three dimensional model of the disc brake is generated by using CATIA, a computer aided design software according to the specific disc brake dimensions. For the thermal stresses analysis, only one plate of the disc brake will be used because both disc plates are asymmetrical to each other and assumption is being made that both sides of the disc plate experience same amount of force by the brake pad and thus having the same temperature distribution allowing asymmetric analysis. In order to ensure that the analysis results are as accurate as the real condition, the vents which connect two disc plates together are also included into the modeled disc brake.

This model is transferred to ANSYS for finite element analysis where heat flux will be applied on one side of this disc brake as the brake pad rotates around the disc. The heat flux is transfer to the surface of the disc brake according to the rotary position of the pad and the brake pad covers 60° of the disc brake surface area at a time. Thus, the area covered by the pad will experiences the transfer of heat flux to the surface while leaving the rest of 300° without any heat flux application but convection to the surrounding. Furthermore, there will be no heat transfer at the surface of the sliced vents where an adiabatic condition is assumed. In order for 60° covered area to be selected in the ANSYS for heat flux application, the disc brake models have been divided into 6 individual parts in the CATIA and are glued into one single volume in ANSYS.

In order for comparisons to be made between perforated and non-perforated disc brakes, both types of disc brakes are being modeled and similar tests of temperature and thermal stresses distribution are carried out in order to investigate the trend for each disc brake and further conclude on how perforation will affects the performance of the disc brake and the vehicle in general. The non-perforated disc brake model is modified based on the dimensions of perforated disc brake with the different in the absence of perforation holes. The isometric, top and bottom views of the modeled perforated disc brake volume are shown is Figure 3.7, 3.8 and 3.9 respectively while the isometric, top and bottom views of the modeled non-perforated disc brake are shown in Figure 3.10, 3.11 and 3.12 respectively.



Figure 3.7: Isometric View of Perforated Disc Brake

Figure 3.10: Isometric View of Non-Perforated



Figure 3.8: Top View of Perforated Disc



Figure 3.11: Top View of Non-Perforated Disc Brake



Figure 3.9: Bottom View of Perforated Disc Brake Figure 3.12: Bottom V



Figure 3.12: Bottom View of Non-Perforated Disc

3.5 FINITE ELEMENT MODELING

Finite element analysis is the best method that can be used in this project to investigate the behaviors of thermal stresses of the disc brake. This is because finite element method is a good choice for solving partial differential equations over complicated shapes or subjects such as the perforated disc brake, where the subject will reacts with changes in its physical properties. Besides, finite element method allows precise simulation and computation over the entire body of the subjects. In the investigations of the disc brake behaviors under the influences of heat flux, the modeled disc brakes are meshed by using the ANSYS software to prepare them for finite element analysis by dividing the model volume into smaller elements.

Since the simulation is being carried out with the use of three dimensional models, element type of SOLID 87 is chosen to give the best meshing attributes to the disc brake and the material properties are defined as the properties of grey cast iron in shown in Table 3.4. This is because the quality of the volume meshing will directly affects the accuracy of the finite element simulation results. While for the analysis of thermal stress, element type of BRICK SOLID 186 is chosen. Both the volumes of perforated and non-perforated disc brakes are meshed in ANSYS with 39747 elements and 35986 elements generated for perforated and non-perforated disc brake respectively. The meshing patterns of the perforated and non perforated disc brake are shown in following figures in section 3.5.1 and 3.5.2.

3.5.1 Meshing of Perforated Disc Brake



Figure 3.13: Isometric view of Perforated Disc Brake Meshing



Figure 3.14: Detailed view of Perforated Disc Brake Meshing



Figure 3.15: Cross Section view of Perforated Disc Brake Meshing

3.5.2 Meshing of Non-Perforated Disc Brake



Figure 3.16: Isometric view of Non-Perforated Disc Brake Meshing



Figure 3.17: Detailed view of Non-Perforated Disc Brake Meshing



Figure 3.18: Cross Section View of Non-Perforated Disc Brake Meshing

3.6 SIMULATION INITIAL CONDITIONS AND PARAMETERS

Before simulation processes can be run to test the disc brakes, there are few initial conditions and parameters that need to be defined on the finite element volumes, some of the parameters and assumption made are:

- Initial temperature of the disc brake volume is 30°C.
- Ambient temperature is taken to be 25°C room temperature.
- Coefficient of convection is 50 W/ m^{2} °C.
- The bottom surface of the ventilation joints (vents) is assumed to be adiabatic.
- Heat flux applied to the disc brake's surface is uniform throughout the area.
- No constrain will be defined for the disc brake on the temperature distribution analysis but for thermal stress and thermal expansion analysis, constrain is defined at the inner diameter surface of the disc brake.

As calculated in the disc brake kinetic calculation section, there are 367 thermal load steps before the disc brake come to complete stop. Since the pad only covers an area of 60° from the whole disc brake area, there will be 367 rotating motion of the break pad with each pad position having different values of heat flux and time interval as per calculated. On the other hand, for area which is not covered by the pad, cooling process through heat convection will take place. Each thermal load step will be solved by ANSYS simulation for temperature distribution analysis and the result from the previous load step will be used as the initial condition input for the next thermal load step. Next, for thermal stress analysis, the generated temperature distribution will be used as the input for their analysis. This process will be repeated for 367 times until the pad stops.

Since the finite element analysis using ANSYS requires 367 moving load steps to be defined and being fed as inputs, ANSYS command language is generated using C++ programming software in order to simplify these steps. Besides, this method allow correction to be make onto the command language without having to repeat the whole input defining process manually and it saves time. The example of the written programming codes in generating ANSYS command language is shown in Appendix D. The total simulation time is 5.99 s.

CHAPTER 4 RESULT AND DISCUSSION 4.1 DISC BRAKE TEMPERATURE DISTRIBUTION

Disc brake surface and body temperature increases due to the heat flux generated by the frictional contact with the brake pad. In order to investigate the increase of disc brake temperature at different thickness, four nodes labeled as A, B, C and D have been chosen along its thickness. Node A is situated at the contact surface between the disc and the pad while node D is situated at its bottom surface with node B and C come in between. The plotted nodes temperatures again the running time of 5.99 s for perforated and non-perforated disc brake are shown in Figure 4.1 and 4.2.



Figure 4.1: Temperature History at Nodes A, B & C (Perforated Disc Brake)



Figure 4.2: Temperature History at Nodes A, B, C & D (Non-Perforated Disc Brake)

For both perforated and non perforated disc brake, the temperature is highest at node A because node A located at the frictional surface between the pad and disc brake where heat flux is at its maximum. The fluctuation of temperature throughout the analysis time is due to the repeating heating (heat flux) and cooling (convection) processes at the disc brake's surface. This fluctuating pattern shows similarity to the fluctuating surface temperature of disc brake in the research done by C.H. Gao & X.Z. Lin [10]. Furthermore, the fluctuating pattern is more obvious for node A because this node is situated at the surface which experienced thermal convection with the chance to release greater amount of heat compared to other nodes position. By referring to Figure 4.1 above, the highest temperature achieved by the perforated disc brake is 256.4 °C at time 4.722 of node A. While the highest temperature achieved by node B and C are 233.8 °C and 231.2 °C respectively. The temperature of node B and C are lower than node A because they are situated further from thermal source which is the heat flux generated by the frictional surface.

By referring to Figure 4.2, the highest temperature achieved by the non - perforated disc brake is 256.4 °C at time 4.686 seconds of node A. While the highest temperature achieved by node B, C and D are 233.7 °C, 231.9 °C and 231.2 °C respectively. By comparing both disc brakes, the maximum temperature produced is almost the same which can be verified by referring to the experiment by M.Eltoukhy [11] regarding simulation of heat generation problem. Thus attention will be further focusing on the temperature distribution within disc brake body.

4.1.1 Temperature Distribution along Frictional Surface

Temperature distribution can also be investigated by looking into its distribution along the frictional surface between disc brake and brake pad. Figure 4.3 shows the A - A' path along the radial distance of the disc brake's surface. This path is selected along the surface of solid cross section for the perforated disc brake. The temperature at different time during the simulation period is recorded in order to investigate the temperature distribution at these particular times. Temperature distribution along the frictional surface for perforated disc brake is shown in Figure 4.4 and non perforated disc brake in Figure 4.5. The temperature distribution along the surface increases with the time until a maximum value at time just before the end of simulation period. This is due to the increment in the accumulated frictional heat flux to the disc brake's surface.



Figure 4.3: A – A' Path along the Radial Distance of Disc Brake



Figure 4.4: Temperature Distribution along Path A – A' for Perforated Disc Brake at Different times



Figure 4.5: Temperature Distribution along Path A – A' for Non Perforated Disc Brake at Different times

A more detailed temperature distributions are obtained by specifying path A – A', B – B' and C – C' which are the paths along the disc brake's surface, mid way along of disc brake's thickness and along bottom of the disc brake respectively as shown in Figure 4.6. Further comparison is made for both disc brakes at the time where the temperature reached its maximum for both cases, 4.72 seconds for perforated disc and 4.68 seconds for non perforated disc as shown in Figure 4.7.

The plotted temperature distribution curves show that the distribution patterns for all three paths for perforated and non perforated discs are almost the same pattern with small decrease in temperature at the outer radial distance. This is largely because they are all situated perpendicular to the heat source with path A - A' received the highest magnitude of frictional heat flux at the surface followed by path B - B' and path C - C', which explain why the temperature distribution for path A - A' is the highest, path C - C' the lowest and path B - B' comes in between. Through the comparison of temperature distribution for perforated and non perforated disc brake, both show similar distribution for all three paths.



Figure 4.6: A - A', B - B' and C - C' Paths along the Radial Distance of Disc Brake



Figure 4.7: Temperature Distribution along Path A-A', B-B' and C-C' for Perforated and Non Perforated Disc at Time 4.72 Seconds and 4.68 Seconds Respectively

4.1.2 Temperature Distribution across the Cross Section

Temperature distribution within the disc brake body is analyzed by creating a slicing plane along the perforated holes to compare the temperature distribution between perforated and non perforated disc brake. Figure 4.8 and 4.9 illustrate how the temperature is being produced and distributed at the end of braking process, after 5.99 seconds.



Figure 4.8: Cross Section Temperature Distribution (Perforated Disc Brake), Time 5.99s



Figure 4.9: Cross Section Temperature Distribution (Non Perforated Disc Brake), Time 5.99s

Both Figure 4.8 and 4.9 provide a better illustration of temperature distribution across the disc brake and by comparing both disc brakes, the perforated disc seen to provide better results as far as the temperature distribution is concerned as compared to non perforated disc. Although earlier analysis shown that the maximum temperature produced by both disc brakes is almost the same, the temperature distribution varies for both disc brake. Figure 4.8 shows that perforated holes contributed to better temperature distribution by having less red to yellow colored region within its cross section, giving better cooling effect although the maximum temperature is the same for both cases. Refer to Figures 4.10 and 4.11 from Section 4.1.2.1 and 4.1.2.2 respectively for cross section temperature distribution for various braking period.


Figure 4.10: Temperature Distribution across Cross Section for Perforated Disc at Different Set of Time





Figure 4.11: Temperature Distribution across Cross Section for Non Perforated Disc at Different Set of Time

4.1.3 Temperature Contour

Another interesting observation is the overall surface temperature distribution of both disc brakes. It gives a bigger picture of the effects of perforation on the temperature distribution. Figure 4.12 below shows the top surface temperature distribution at time 5.99 seconds. Refer to Appendix E for temperature distribution (disc brake's top view) in various braking period.



Figure 4.12: Top Surface Temperature Distribution, Time 5.99s

The overall top surface temperature distribution also shows that the perforated disc brake has better temperature distribution because it has less high temperature region (red colored) over the whole surface as compared to non perforated disc brake. And it is noticed that the high temperature regions concentrated around the outer parameter of both disc brakes. Besides, it can also be concluded that the vents connecting two disc plates for ventilated disc brake eventually helped in the cooling process of the disc brake as shown Figure 4.13. The vents (blue colored regions) provide larger surface area for heat convection to the ambient environment and thus lower the surface temperature.



Figure 4.13: Bottom Surface Temperature Distribution, Time 5.99s

4.2 DISC BRAKE THERMAL STRESS DISTRIBUTION

The temperature rise on the contact surface between brake pad and disc brake will contribute to the development of thermal stress on the disc brake body. This thermal stress arises due to the compression from the forbidden expansion of the disc brake when the internal surface of the disc is constrained as illustrated in Figure 4.14.



Figure 4.14: Disc Brake Constrained Area

The thermal stress on disc brake surface and body increases due to the increase in the temperature caused by absorbed heat flux. In order to investigate the increase of disc brake's thermal stress at different thickness of the disc, four nodes labeled as A, B, C and D have been chosen along its thickness, similar to temperature distribution analysis. Node A is situated at the contact surface between the disc and the pad while node D is situated at its bottom surface with node B and C come in between. The plotted thermal stress history for perforated and non-perforated disc brake are shown in Figure 4.15 and 4.16.



Figure 4.15: Von Mises Thermal Stress History (Perforated Disc Brake)



Figure 4.16: Von Mises Thermal Stress History (Non-Perforated Disc Brake)

For both perforated and non perforated disc brake, the thermal stress is the highest at node A because node A is located at the frictional surface between the pad and disc brake where heat flux and subsequently temperature rise is at their maximum. The fluctuation pattern of thermal stress especially at node A as shown in Figure 4.15 and 4.16 is due to the repeating heating (heat flux) and cooling (convection) processes at the disc brake's surface, which is similar to the fluctuation of the node A's temperature in Figure 4.1 and 4.2. This thermal stress fluctuating pattern shows similarity to the fluctuation of disc brake surface's thermal stress in a research done by Pier Francesco Gotowicki, Prof. Vinzenco Nigrelli & their team [14]. As the result of frictional action on the disc brake surface, disc brake's temperature will increase causing the disc brake to expand. But due to the applied constrain, stress is developed. Since the inner part of the disc brake surface is constrained as shown in Figure 4.14, the material of the disc tends to expand toward the outer side of the disc. Thus, Node A will experiences expansion force while node D experiences compaction force.

For perforated disc, the highest thermal stress achieved is 470.67 MPa at node A, while the highest thermal stress achieved by node B and C are 398.68 MPa and 429.18 MPa respectively. The thermal stress of node B and C are lower than node A because they are situated further from thermal source but at the end of the braking

time the thermal stress value for node C increase closing to node A's thermal stress value due to the increase of the compaction stress at node C. While for the non perforated disc brake, the highest thermal stress achieved is 531.14 MPa at node A. And the highest thermal stress achieved by node B, C and D are 449.54 MPa, 425.13 MPa and 480.45 MPa respectively. The thermal stress of node B, C and D are lower than node A because they are situated further from thermal source but at the end of the braking time the thermal stress value for node D increase closing to node A 's thermal stress value due to the increase of the compaction stress at node D. By comparing both disc brakes, the maximum thermal stress produced by perforated disc brake is lower than the non perforated disc. Thus attention will be further focusing on the thermal stress distribution within disc brake body to compare both disc brakes by looking at their overall thermal stress distribution instead of at few selected nodes.

4.2.1 Surface Thermal Stress Distribution

Isometric view for the disc brake surface can gives a bigger picture of thermal stress distribution as shown in Figure 4.17 and 4.18 below for perforated and non perforated disc brakes. Although previous graph show that the highest thermal stress for perforated disc brake is 470.67 MPa at surface node near the inner part of the disc, isometric view show that the highest thermal stress actually occurred at the perforated hole with magnitude of 596 MPa at the end of the braking time. While the highest thermal stress value for non perforated disc is 514 MPa at the inner part of the disc at the end of the braking.



Figure 4.17: Isometric View of Perforated Disc

Figure 4.18: Isometric View of Non Perforated Disc

4.2.2 Thermal Stress Distribution along Frictional Surface

Thermal stress distribution can be investigated by studying its distribution along the frictional surface between disc brake and brake pad. Figure 4.19 shows the A-A' path along the radial distance of the disc brake's surface. This path is selected along the surface of solid cross section for the perforated disc brake. Thermal stress generated at different time during the simulation period is recorded in order to investigate the thermal stress distribution at these particular times. Thermal stress distribution along the frictional surface for perforated disc brake is shown in Figure 4.20 and non perforated disc brake in Figure 4.21. From both figures, the thermal stress distribution along the surface increases with the time until a maximum value at time just before the end of simulation period. This is due to the increment in the accumulated frictional heat flux to the disc brake's surface, leading to the increment of surface temperature.



Figure 4.19: A – A' Path along the Radial Distance of Disc Brake



Figure 4.20: Thermal Stress Distribution along Path A – A' for Perforated Disc Brake at Different Times



Figure 4.21: Thermal Stress Distribution along Path A – A' for Non Perforated Disc Brake at Different Times

A more detailed thermal stress distributions are obtained by specifying path A - A', B - B' and C - C' which are the paths along the disc brake's surface, mid way along of disc brake's thickness and along bottom of the disc brake respectively as shown in Figure 4.22. These paths are similar to the one specified for temperature distribution in Figure 4.6. Further comparison is made for both disc brakes at the time when the temperature reached its maximum for both cases, 5.35 seconds for perforated disc and 5.29 seconds for non perforated disc as shown in Figure 4.23.

The plotted thermal stress distribution curves show that the distribution patterns for all three paths have almost the same pattern for both perforated and non perforated disc. This is largely because they are all situated perpendicular to the heat source with path A - A' received the highest magnitude of frictional heat flux at the surface followed by path B - B' and path C - C', which explain why the thermal stress distribution for path A - A' is the highest, path C - C' the lowest and path B - B' comes in between. Through the comparison of thermal stress distribution for path A - A' is track, it can be concluded that the perforated disc brake has a generally lower thermal stress distribution as compared to non perforated disc for all three different paths.





Figure 4.23: Thermal Stress Distribution along Path A-A', B-B' and C-C' for Perforated and Non Perforated Disc at Time 5.35 Seconds and 5.29 Seconds Respectively

4.2.3Thermal Stress Distribution across the Cross Section

Thermal stress distribution within the disc brake body is analyzed by creating a slicing plane along the perforated holes to compare the thermal stress distribution between perforated and non perforated disc brake. Figure 4.24 and 4.25 illustrate the thermal stress distribution at the end of braking process, at 5.99 seconds.



Figure 4.24: Cross Section Von Mises Stress (Perforated Disc Brake), Time 5.99s



Figure 4.25: Cross Section Von Mises Stress (Non Perforated Disc Brake), Time 5.99s

Although the Von Mises stress generated at perforated disc's surface is lower than non-perforated disc brake, high magnitude of Von Mises stresses (red colored region) are generated around the perforated holes for perforated disc brake as illustrated in Figure 4.24 as compared to non perforated disc brake where high thermal stress region is situated at the inner part of the disc only. Refer to Figure 4.26 and 4.27 in Section 4.2.3.1 and 4.2.3.2 respectively for the cross section thermal stress distribution for various braking period.

4.2.3.1 Von Mises Thermal Stress Distribution across Cross Section for



Figure 4.26: Von Mises Thermal Stress Distribution across Cross Section for Perforated Disc at Different Set of Time

4.2.3.2 Von Mises Thermal Stress Distribution across Cross Section for Non

Perforated Disc

(a) Time = 0.51814 s





4.2.4 Thermal Stress Contour

Another interesting observation is the surface's thermal stress distribution of the whole disc brake. It gives a bigger picture of the effects of perforation on the thermal stress distribution. Figure 4.28 below shows the top surface's thermal stress distribution at time 5.99 seconds. Refer to Appendix F for thermal stress distribution (disc brake's top view) in various braking period.



Figure 4.28: Top Surface Thermal Stress Distribution, Time 5.99s

The overall top surface thermal stress distribution shows that the perforated disc brake has better stress distribution with more distribution of lower stress region (blue colored) over the whole surface as compared to non perforated disc brake. This observation is also true for the bottom surface of both disc brakes where perforated disc brake also has better stress distribution with more distribution of lower stress region (blue colored) as shown in Figure 4.29.



Figure 4.29: Bottom Surface Thermal Stress Distribution, Time 5.99s

CHAPTER 5 CONCLUSION AND RECOMMENDATIONS

This project is critical and practical in analyzing the thermal stresses that arise from the high temperature distribution along the perforated brake disc surface, where the perforated holes are often referred to as the stress concentration zones that will contribute to the cracking of the disc in the end of severe usage. This phenomenon is confirmed through the simulation performed on the perforated disc brake in this project where the results clearly showed highly concentrated thermal stress exists around the area of perforated hole's circumference. On the other hand, another interesting area of analysis in this project is to investigate the effects of the perforated holes to the overall performance of the disc brake. This is done by comparing the overall thermal stress distribution between perforated and non perforated disc brake where the results show that the perforated disc brake has a better thermal stress distribution over the surface and body as compared to non perforated disc brake in general. Conclusion can draw that the perforated disc brake is better in increasing the performance of the vehicle but the weighting are between the high stress concentration at the perforated zones and better cooling ability of the perforated brake disc in comparison to the conventional disc brake.

There are three recommendations for future work expansion related to perforated disc brake which can be carry out to further understand the effects of having perforated holes under various designs. The recommendations are as following:

- 1. Analyze the effects of different perforated holes patterns on disc brake's temperature and thermal stress distributions.
- Analyze the effects of different density of perforated holes on disc brake's temperature and thermal stress distributions.
- Analyze the effects of different disc brake's materials on its temperature and thermal stress distributions, such as cast iron, reinforced carbon-carbon and ceramic matrix composites.

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Appendix A. Vehicle Specifications

- 1) Engine capacity: 2.4 liter (2400 cc)
- 2) Vehicle weight: 1465kg
- 3) Maximum car weight: 1950kg
 - a) Front gross axle weight rating: 1060kg
 - b) Rear gross axle weight rating: 915kg
- 4) Weight ratio:

a) Front ratio
$$= \frac{1060kg}{1950kg} = 0.54$$

b) Rear ratio =
$$\frac{915kg}{1950kg} = 0.46$$

5) Considering maximum car weight for further analysis:

Car weight = 1950kg

a) Front wheel applied weight = 1950kg * front weight ratio

$$= 1950 \text{kg} * 0.54 \approx 1053 \text{kg}$$

b) Rear wheel applied weight = 1950kg * rear weight ratio

$$= 1950 \text{kg} * 0.46 \approx 897 \text{kg}$$

- 6) Weight applied on each disc brake surface
 - a) By taking the weight on a more severe/higher weight side of the vehicle, attention is focused to the front wheel applied weight of 1053kg.
 - b) There are two brakes at the left and right hand side of the front wheel where the applied weight is equally distributed:

Weight on one side of the front wheel = $\frac{1053kg}{2}$ = 526.5 kg

c) For thermal stress analysis purpose, only one side of the disc brake will be considered. (Asymmetrical analysis). Thus, the weight on each disc brake surface:

Weight on each disc brake surface =
$$\frac{526.5kg}{2} = \frac{263.25 \text{ kg}}{2}$$

Appendix B. Disc Brake Kinetic Calculation

1) Time taken to stop vehicle: $t = \frac{v - u}{a}$ u = 150 km/hr = 41.67 m/s v = 0 m/s $a = -0.7 \text{ Gravity} = -6.867 \text{ m/s}^2$ $t = \frac{0 - 41.67 \text{ m/s}}{-6.867 \text{ m/s}^2}$ $= \underline{6.068 \text{ s}}$

2) Distance travelled before stopping:

$$v^2 = u^2 + 2$$
 as
 $s = \frac{1}{2a} (v^2 - u^2)$
 $= \frac{1}{2(-6.867)} (0 - (41.67^2))$
 $= \frac{126.43 \text{ m}}{2}$

3) Pad contact area:
Perimeter of disc brake =
$$2\pi r_{disc}$$

= $2\pi (0.164 \text{ m})$
= 1.030 m
Length of pad = 0.186 m
Degree of covered area = $\frac{0.186 \text{ m}}{1.030 \text{ m}} \ge 360^{\circ}$
= 65°

For simplification of analysis, we take the covered area as 60°

Thus, pad contact area =
$$[(\pi r_{out}^2) - (\pi r_{in}^2)] \ge \frac{60^\circ}{360^\circ}$$

= $[(\pi 0.164^2) - (\pi 0.11^2)] \ge \frac{60^\circ}{360^\circ}$
= $\underline{0.00775} m^2$

4) Disc initial angular velocity:

 $\omega_{tire} = \omega_{disc}$

$$v_{tire} = r_{tire} \omega_{tire}$$

$$41.67 \text{m/s} = \frac{0.658 \text{ m}}{2} \omega_{tire}$$

$$\omega_{tire} = \frac{126.66 \text{ rad/s}}{2}$$

And,
$$v_{disc} = r_{disc}\omega_{disc}$$

= $\frac{0.328 m}{2}$ (126.66)
= $20.77 m/s$

5) Disc initial angular acceleration

$$\alpha_{tire} = \alpha_{disc}$$

$$a_{tire} = r_{tire} \alpha_{tire}$$
$$-6.867 \text{m/s}^2 = \frac{0.658 \text{ m}}{2} \alpha_{tire}$$

$$\alpha_{tire} = \underline{-20.87 \text{ rad/}s^2}$$

And,
$$a_{disc} = r_{disc} \alpha_{disc}$$

= $\frac{0.328 m}{2}$ (-20.87)
= $-3.423 m/s^2$

6) Initial angular velocity of next pad rotational: $\omega_j^2 = \omega_i^2 + 2 \alpha \theta$

$$\theta = \frac{s}{r} = \frac{2 \pi (0.164 \text{m}) \times \frac{60^{\circ}}{360^{\circ}}}{0.164 \text{ m}}$$

= 1.0471
$$\omega_j = [(126.66^2) + 2(-20.87) (1.0471)]^{1/2}$$

= 126.48 rad/s

7) Time interval between 2 pad rotational positions:

$$\omega_j = \omega_i + \alpha t$$
$$\Delta t = \frac{\omega_j - \omega_i}{\alpha}$$
$$= \frac{126.48 - 126.66}{-20.87}$$
$$= 0.00862 s$$

- 8) Heat energy absorbed by the disc brake:
 - 95 % of heat absorbed by disc and 5% by pad
 - Load, m = 263.25 kg
 - $v_i = 41.67 \text{ m/s}$
 - $v_j = r_{tire}\omega_j$ = 0.329 m x 126.48 rad/s = 41.612 m/s

 $\Delta Q = 0.95$ (Kinetic energy)

$$= 0.95 \left[\frac{1}{2} \times m \times (v_i^2 - v_j^2)\right]$$
$$= 0.95 \left[\frac{1}{2} \times 263.25 \times (41.67^2 - 41.612^2)\right]$$
$$= 604.0 \text{ Joule}$$

9) Heat flux at contact area (heat transfer per unit area):

Heat flux,
$$\dot{q} = \frac{\dot{Q}}{A} = \frac{\Delta Q}{A t}$$

= $\frac{604.0 J}{0.00862 (0.00775 m^2)}$
= 9.041 MW/m^2

Appendix C. Disc Brake Kinetic Calculations

| | Time | | Angular | | Heat | | |
|------------|----------|------------|----------|----------|-------------|--------------|-------------|
| No. of pad | interval | Total time | velocity | Velocity | absorbed by | Accumulative | Heat flux |
| rotational | (s) | (s) | (rad/s) | (m/s) | disc (J) | heat (J) | (W/m2) |
| 0 | 0 | 0 | 126.660 | 41.671 | 0 | 0 | 0 |
| 1 | 0.00827 | 0.00827 | 126.487 | 41.614 | 591.6105 | 591.6105 | 9226731.626 |
| 2 | 0.00828 | 0.01656 | 126.314 | 41.557 | 591.6105 | 1183.2210 | 9214136.284 |
| 3 | 0.00830 | 0.02485 | 126.141 | 41.500 | 591.6105 | 1774.8315 | 9201523.7 |
| 4 | 0.00831 | 0.03316 | 125.968 | 41.443 | 591.6105 | 2366.4419 | 9188893.805 |
| 5 | 0.00832 | 0.04148 | 125.794 | 41.386 | 591.6105 | 2958.0524 | 9176246.526 |
| 6 | 0.00833 | 0.04981 | 125.620 | 41.329 | 591.6105 | 3549.6629 | 9163581.792 |
| 7 | 0.00834 | 0.05815 | 125.446 | 41.272 | 591.6105 | 4141.2734 | 9150899.53 |
| 8 | 0.00835 | 0.06651 | 125.272 | 41.214 | 591.6105 | 4732.8839 | 9138199.668 |
| 9 | 0.00837 | 0.07487 | 125.097 | 41.157 | 591.6105 | 5324.4944 | 9125482.131 |
| 10 | 0.00838 | 0.08325 | 124.923 | 41.100 | 591.6105 | 5916.1049 | 9112746.845 |
| 11 | 0.00839 | 0.09164 | 124.748 | 41.042 | 591.6105 | 6507.7154 | 9099993.737 |
| 12 | 0.00840 | 0.10004 | 124.572 | 40.984 | 591.6105 | 7099.3258 | 9087222.731 |
| 13 | 0.00841 | 0.10845 | 124.397 | 40.926 | 591.6105 | 7690.9363 | 9074433.751 |
| 14 | 0.00842 | 0.11687 | 124.221 | 40.869 | 591.6105 | 8282.5468 | 9061626.722 |
| 15 | 0.00844 | 0.12531 | 124.045 | 40.811 | 591.6105 | 8874.1573 | 9048801.567 |
| 16 | 0.00845 | 0.13376 | 123.868 | 40.753 | 591.6105 | 9465.7678 | 9035958.208 |
| 17 | 0.00846 | 0.14222 | 123.692 | 40.695 | 591.6105 | 10057.3783 | 9023096.569 |
| 18 | 0.00847 | 0.15069 | 123.515 | 40.636 | 591.6105 | 10648.9888 | 9010216.57 |
| 19 | 0.00848 | 0.15918 | 123.338 | 40.578 | 591.6105 | 11240.5992 | 8997318.132 |
| 20 | 0.00850 | 0.16767 | 123.161 | 40.520 | 591.6105 | 11832.2097 | 8984401.178 |
| 21 | 0.00851 | 0.17618 | 122.983 | 40.461 | 591.6105 | 12423.8202 | 8971465.625 |
| 22 | 0.00852 | 0.18470 | 122.805 | 40.403 | 591.6105 | 13015.4307 | 8958511.394 |
| 23 | 0.00853 | 0.19324 | 122.627 | 40.344 | 591.6105 | 13607.0412 | 8945538.404 |
| 24 | 0.00855 | 0.20178 | 122.449 | 40.286 | 591.6105 | 14198.6517 | 8932546.573 |
| 25 | 0.00856 | 0.21034 | 122.270 | 40.227 | 591.6105 | 14790.2622 | 8919535.819 |
| 26 | 0.00857 | 0.21891 | 122.091 | 40.168 | 591.6105 | 15381.8727 | 8906506.058 |
| 27 | 0.00858 | 0.22749 | 121.912 | 40.109 | 591.6105 | 15973.4831 | 8893457.207 |
| 28 | 0.00860 | 0.23609 | 121.733 | 40.050 | 591.6105 | 16565.0936 | 8880389.183 |
| 29 | 0.00861 | 0.24470 | 121.553 | 39.991 | 591.6105 | 17156.7041 | 8867301.899 |
| 30 | 0.00862 | 0.25332 | 121.373 | 39.932 | 591.6105 | 17748.3146 | 8854195.272 |
| 31 | 0.00863 | 0.26196 | 121.193 | 39.872 | 591.6105 | 18339.9251 | 8841069.214 |
| 32 | 0.00865 | 0.27060 | 121.013 | 39.813 | 591.6105 | 18931.5356 | 8827923.639 |
| 33 | 0.00866 | 0.27926 | 120.832 | 39.754 | 591.6105 | 19523.1461 | 8814758.46 |
| 34 | 0.00867 | 0.28794 | 120.651 | 39.694 | 591.6105 | 20114.7566 | 8801573.589 |
| 35 | 0.00869 | 0.29662 | 120.470 | 39.634 | 591.6105 | 20706.3670 | 8788368.937 |
| 36 | 0.00870 | 0.27926 | 120.288 | 39.575 | 591.6105 | 19523.1461 | 8775144.415 |
| 37 | 0.00871 | 0.31403 | 120.106 | 39.515 | 591.6105 | 21889.5880 | 8761899.933 |
| 38 | 0.00873 | 0.32276 | 119.924 | 39.455 | 591.6105 | 22481.1985 | 8748635.4 |

| 39 | 0.00874 | 0.33150 | 119.742 | 39.395 | 591.6105 | 23072.8090 | 8735350.725 |
|----|---------|---------|---------|--------|----------|------------|-------------|
| 40 | 0.00875 | 0.34025 | 119.559 | 39.335 | 591.6105 | 23664.4195 | 8722045.816 |
| 41 | 0.00877 | 0.34902 | 119.376 | 39.275 | 591.6105 | 24256.0300 | 8708720.58 |
| 42 | 0.00878 | 0.35779 | 119.193 | 39.214 | 591.6105 | 24847.6404 | 8695374.924 |
| 43 | 0.00879 | 0.36659 | 119.009 | 39.154 | 591.6105 | 25439.2509 | 8682008.753 |
| 44 | 0.00881 | 0.37539 | 118.826 | 39.094 | 591.6105 | 26030.8614 | 8668621.973 |
| 45 | 0.00882 | 0.38421 | 118.641 | 39.033 | 591.6105 | 26622.4719 | 8655214.488 |
| 46 | 0.00883 | 0.39305 | 118.457 | 38.972 | 591.6105 | 27214.0824 | 8641786.202 |
| 47 | 0.00885 | 0.40189 | 118.272 | 38.912 | 591.6105 | 27805.6929 | 8628337.017 |
| 48 | 0.00886 | 0.41075 | 118.088 | 38.851 | 591.6105 | 28397.3034 | 8614866.836 |
| 49 | 0.00887 | 0.41963 | 117.902 | 38.790 | 591.6105 | 28988.9139 | 8601375.56 |
| 50 | 0.00889 | 0.42852 | 117.717 | 38.729 | 591.6105 | 29580.5243 | 8587863.09 |
| 51 | 0.00890 | 0.43742 | 117.531 | 38.668 | 591.6105 | 30172.1348 | 8574329.324 |
| 52 | 0.00892 | 0.44634 | 117.345 | 38.606 | 591.6105 | 30763.7453 | 8560774.164 |
| 53 | 0.00893 | 0.45527 | 117.159 | 38.545 | 591.6105 | 31355.3558 | 8547197.505 |
| 54 | 0.00895 | 0.46422 | 116.972 | 38.484 | 591.6105 | 31946.9663 | 8533599.247 |
| 55 | 0.00896 | 0.47317 | 116.785 | 38.422 | 591.6105 | 32538.5768 | 8519979.286 |
| 56 | 0.00897 | 0.48215 | 116.598 | 38.361 | 591.6105 | 33130.1873 | 8506337.516 |
| 57 | 0.00899 | 0.49114 | 116.410 | 38.299 | 591.6105 | 33721.7977 | 8492673.834 |
| 58 | 0.00900 | 0.50014 | 116.222 | 38.237 | 591.6105 | 34313.4082 | 8478988.133 |
| 59 | 0.00902 | 0.50916 | 116.034 | 38.175 | 591.6105 | 34905.0187 | 8465280.307 |
| 60 | 0.00903 | 0.51819 | 115.845 | 38.113 | 591.6105 | 35496.6292 | 8451550.247 |
| 61 | 0.00905 | 0.52724 | 115.657 | 38.051 | 591.6105 | 36088.2397 | 8437797.846 |
| 62 | 0.00906 | 0.53630 | 115.467 | 37.989 | 591.6105 | 36679.8502 | 8424022.994 |
| 63 | 0.00908 | 0.54538 | 115.278 | 37.926 | 591.6105 | 37271.4607 | 8410225.58 |
| 64 | 0.00909 | 0.55447 | 115.088 | 37.864 | 591.6105 | 37863.0712 | 8396405.493 |
| 65 | 0.00911 | 0.56357 | 114.898 | 37.802 | 591.6105 | 38454.6816 | 8382562.622 |
| 66 | 0.00912 | 0.57270 | 114.708 | 37.739 | 591.6105 | 39046.2921 | 8368696.853 |
| 67 | 0.00914 | 0.58183 | 114.517 | 37.676 | 591.6105 | 39637.9026 | 8354808.072 |
| 68 | 0.00915 | 0.59099 | 114.326 | 37.613 | 591.6105 | 40229.5131 | 8340896.164 |
| 69 | 0.00917 | 0.60015 | 114.135 | 37.550 | 591.6105 | 40821.1236 | 8326961.014 |
| 70 | 0.00918 | 0.60934 | 113.943 | 37.487 | 591.6105 | 41412.7341 | 8313002.503 |
| 71 | 0.00920 | 0.61853 | 113.751 | 37.424 | 591.6105 | 42004.3446 | 8299020.516 |
| 72 | 0.00921 | 0.62775 | 113.559 | 37.361 | 591.6105 | 42595.9551 | 8285014.931 |
| 73 | 0.00923 | 0.63698 | 113.366 | 37.298 | 591.6105 | 43187.5655 | 8270985.631 |
| 74 | 0.00925 | 0.64622 | 113.173 | 37.234 | 591.6105 | 43779.1760 | 8256932.494 |
| 75 | 0.00926 | 0.65548 | 112.980 | 37.170 | 591.6105 | 44370.7865 | 8242855.397 |
| 76 | 0.00928 | 0.66476 | 112.786 | 37.107 | 591.6105 | 44962.3970 | 8228754.219 |
| 77 | 0.00929 | 0.67405 | 112.593 | 37.043 | 591.6105 | 45554.0075 | 8214628.834 |
| 78 | 0.00931 | 0.68336 | 112.398 | 36.979 | 591.6105 | 46145.6180 | 8200479.118 |
| 79 | 0.00932 | 0.69269 | 112.204 | 36.915 | 591.6105 | 46737.2285 | 8186304.945 |
| 80 | 0.00934 | 0.70203 | 112.009 | 36.851 | 591.6105 | 47328.8389 | 8172106.188 |
| 81 | 0.00936 | 0.71139 | 111.813 | 36.787 | 591.6105 | 47920.4494 | 8157882.718 |
| 82 | 0.00937 | 0.72076 | 111.618 | 36.722 | 591.6105 | 48512.0599 | 8143634.405 |
| 83 | 0.00939 | 0.73015 | 111.422 | 36.658 | 591.6105 | 49103.6704 | 8129361.119 |

| 84 | 0.00941 | 0.73956 | 111,225 | 36,593 | 591,6105 | 49695,2809 | 8115062,728 |
|-----|---------|---------|---------|--------|----------|------------|-------------|
| 85 | 0.00942 | 0.74898 | 111.029 | 36.528 | 591,6105 | 50286.8914 | 8100739.1 |
| 00 | 0.00014 | 0.75942 | 110.023 | 26.464 | 591.0105 | 50200.0311 | 8086200.1 |
| 80 | 0.00944 | 0.75842 | 110.832 | 30.404 | 591.6105 | 50878.5019 | 8086390.1 |
| 87 | 0.00946 | 0.76788 | 110.634 | 36.399 | 591.6105 | 51470.1124 | 8072015.592 |
| 88 | 0.00947 | 0.77735 | 110.437 | 36.334 | 591.6105 | 52061.7228 | 8057615.441 |
| 89 | 0.00949 | 0.78684 | 110.239 | 36.269 | 591.6105 | 52653.3333 | 8043189.509 |
| 90 | 0.00951 | 0.79635 | 110.040 | 36.203 | 591.6105 | 53244.9438 | 8028/3/.656 |
| 91 | 0.00953 | 0.80587 | 109.841 | 36.138 | 591.6105 | 53836.5543 | 8014259.743 |
| 92 | 0.00954 | 0.81542 | 109.642 | 36.072 | 591.6105 | 54428.1648 | 7999755.627 |
| 93 | 0.00956 | 0.82498 | 109.443 | 36.007 | 591.6105 | 55019.7753 | 7985225.167 |
| 94 | 0.00958 | 0.83455 | 109.243 | 35.941 | 591.6105 | 55611.3858 | 7970668.218 |
| 95 | 0.00959 | 0.84415 | 109.043 | 35.875 | 591.6105 | 56202.9962 | 7956084.634 |
| 96 | 0.00961 | 0.85376 | 108.842 | 35.809 | 591.6105 | 56794.6067 | 7941474.269 |
| 97 | 0.00963 | 0.86339 | 108.641 | 35.743 | 591.6105 | 57386.2172 | 7926836.975 |
| 98 | 0.00965 | 0.87304 | 108.440 | 35.677 | 591.6105 | 57977.8277 | 7912172.603 |
| 99 | 0.00967 | 0.88271 | 108.238 | 35.610 | 591.6105 | 58569.4382 | 7897481.001 |
| 100 | 0.00968 | 0.89239 | 108.036 | 35.544 | 591.6105 | 59161.0487 | 7882762.017 |
| 101 | 0.00970 | 0.90209 | 107.833 | 35.477 | 591.6105 | 59752.6592 | 7868015.498 |
| 102 | 0.00972 | 0.91181 | 107.630 | 35.410 | 591.6105 | 60344.2697 | 7853241.289 |
| 103 | 0.00974 | 0.92155 | 107.427 | 35.344 | 591.6105 | 60935.8801 | 7838439.232 |
| 104 | 0.00976 | 0.93131 | 107.224 | 35.277 | 591.6105 | 61527.4906 | 7823609.17 |
| 105 | 0.00978 | 0.94108 | 107.020 | 35.209 | 591.6105 | 62119.1011 | 7808750.944 |
| 106 | 0.00979 | 0.95088 | 106.815 | 35.142 | 591.6105 | 62710.7116 | 7793864.391 |
| 107 | 0.00981 | 0.96069 | 106.610 | 35.075 | 591.6105 | 63302.3221 | 7778949.351 |
| 108 | 0.00983 | 0.97052 | 106.405 | 35.007 | 591.6105 | 63893.9326 | 7764005.657 |
| 109 | 0.00985 | 0.98037 | 106.200 | 34.940 | 591.6105 | 64485.5431 | 7749033.146 |
| 110 | 0.00987 | 0.99024 | 105.994 | 34.872 | 591.6105 | 65077.1536 | 7734031.648 |
| 111 | 0.00989 | 1.00013 | 105.787 | 34.804 | 591.6105 | 65668.7640 | 7719000.996 |
| 112 | 0.00991 | 1.01004 | 105.580 | 34.736 | 591.6105 | 66260.3745 | 7703941.019 |
| 113 | 0.00993 | 1.01997 | 105.373 | 34.668 | 591.6105 | 66851.9850 | 7688851.544 |
| 114 | 0.00995 | 1.02992 | 105.166 | 34.599 | 591.6105 | 67443.5955 | 7673732.397 |
| 115 | 0.00997 | 1.03989 | 104.958 | 34.531 | 591.6105 | 68035.2060 | 7658583.402 |
| 116 | 0.00999 | 1.04987 | 104.749 | 34.462 | 591.6105 | 68626.8165 | 7643404.383 |
| 117 | 0.01001 | 1.05988 | 104.540 | 34.394 | 591.6105 | 69218.4270 | 7628195.16 |
| 118 | 0.01003 | 1.06991 | 104.331 | 34.325 | 591.6105 | 69810.0374 | 7612955.551 |
| 119 | 0.01005 | 1.07996 | 104.121 | 34.256 | 591.6105 | 70401.6479 | 7597685.375 |
| 120 | 0.01007 | 1.09002 | 103.911 | 34.187 | 591.6105 | 70993.2584 | 7582384.446 |
| 121 | 0.01009 | 1.10011 | 103.701 | 34.118 | 591.6105 | 71584.8689 | 7567052.577 |
| 122 | 0.01011 | 1.11022 | 103.490 | 34.048 | 591.6105 | 72176.4794 | 7551689.581 |
| 123 | 0.01013 | 1.12035 | 103.278 | 33.979 | 591.6105 | 72768.0899 | 7536295.267 |
| 124 | 0.01015 | 1.13050 | 103.066 | 33.909 | 591.6105 | 73359.7004 | 7520869.442 |
| 125 | 0.01017 | 1.14067 | 102.854 | 33.839 | 591.6105 | 73951.3109 | 7505411.913 |
| 126 | 0.01019 | 1.15086 | 102.642 | 33.769 | 591.6105 | 74542.9213 | 7489922.483 |
| 127 | 0.01021 | 1.16108 | 102.428 | 33.699 | 591.6105 | 75134.5318 | 7474400.953 |

| 128 | 0.01023 | 1.17131 | 102.215 | 33.629 | 591.6105 | 75726.1423 | 7458847.124 |
|-----|---------|---------|---------|--------|----------|-------------|-------------|
| 129 | 0.01026 | 1.18157 | 102.001 | 33.558 | 591.6105 | 76317.7528 | 7443260.793 |
| 130 | 0.01028 | 1.19184 | 101.786 | 33.488 | 591.6105 | 76909.3633 | 7427641.755 |
| 131 | 0.01030 | 1.20214 | 101.571 | 33.417 | 591.6105 | 77500.9738 | 7411989.803 |
| 132 | 0.01032 | 1.21246 | 101.356 | 33.346 | 591.6105 | 78092.5843 | 7396304.729 |
| 133 | 0.01034 | 1.22281 | 101.140 | 33.275 | 591.6105 | 78684.1947 | 7380586.321 |
| 134 | 0.01037 | 1.23317 | 100.924 | 33.204 | 591.6105 | 79275.8052 | 7364834.366 |
| 135 | 0.01039 | 1.24356 | 100.707 | 33.133 | 591.6105 | 79867.4157 | 7349048.648 |
| 136 | 0.01041 | 1.25397 | 100.490 | 33.061 | 591.6105 | 80459.0262 | 7333228.95 |
| 137 | 0.01043 | 1.26440 | 100.272 | 32.989 | 591.6105 | 81050.6367 | 7317375.05 |
| 138 | 0.01045 | 1.27486 | 100.054 | 32.918 | 591.6105 | 81642.2472 | 7301486.726 |
| 139 | 0.01048 | 1.28533 | 99.835 | 32.846 | 591.6105 | 82233.8577 | 7285563.753 |
| 140 | 0.01050 | 1.29583 | 99.616 | 32.774 | 591.6105 | 82825.4682 | 7269605.902 |
| 141 | 0.01052 | 1.30636 | 99.396 | 32.701 | 591.6105 | 83417.0786 | 7253612.945 |
| 142 | 0.01055 | 1.31690 | 99.176 | 32.629 | 591.6105 | 84008.6891 | 7237584.648 |
| 143 | 0.01057 | 1.32748 | 98.956 | 32.556 | 591.6105 | 84600.2996 | 7221520.775 |
| 144 | 0.01059 | 1.33807 | 98.734 | 32.484 | 591.6105 | 85191.9101 | 7205421.09 |
| 145 | 0.01062 | 1.34869 | 98.513 | 32.411 | 591.6105 | 85783.5206 | 7189285.351 |
| 146 | 0.01064 | 1.35933 | 98.291 | 32.338 | 591.6105 | 86375.1311 | 7173113.314 |
| 147 | 0.01067 | 1.37000 | 98.068 | 32.264 | 591.6105 | 86966.7416 | 7156904.735 |
| 148 | 0.01069 | 1.38069 | 97.845 | 32.191 | 591.6105 | 87558.3521 | 7140659.364 |
| 149 | 0.01071 | 1.39140 | 97.621 | 32.117 | 591.6105 | 88149.9625 | 7124376.949 |
| 150 | 0.01074 | 1.40214 | 97.397 | 32.044 | 591.6105 | 88741.5730 | 7108057.236 |
| 151 | 0.01076 | 1.41291 | 97.173 | 31.970 | 591.6105 | 89333.1835 | 7091699.967 |
| 152 | 0.01079 | 1.42369 | 96.947 | 31.896 | 591.6105 | 89924.7940 | 7075304.882 |
| 153 | 0.01081 | 1.43451 | 96.722 | 31.821 | 591.6105 | 90516.4045 | 7058871.717 |
| 154 | 0.01084 | 1.44535 | 96.496 | 31.747 | 591.6105 | 91108.0150 | 7042400.206 |
| 155 | 0.01087 | 1.45621 | 96.269 | 31.672 | 591.6105 | 91699.6255 | 7025890.079 |
| 156 | 0.01089 | 1.46710 | 96.042 | 31.598 | 591.6105 | 92291.2359 | 7009341.064 |
| 157 | 0.01092 | 1.47802 | 95.814 | 31.523 | 591.6105 | 92882.8464 | 6992752.883 |
| 158 | 0.01094 | 1.48896 | 95.585 | 31.448 | 591.6105 | 93474.4569 | 6976125.258 |
| 159 | 0.01097 | 1.49993 | 95.356 | 31.372 | 591.6105 | 94066.0674 | 6959457.906 |
| 160 | 0.01100 | 1.51093 | 95.127 | 31.297 | 591.6105 | 94657.6779 | 6942750.541 |
| 161 | 0.01102 | 1.52195 | 94.897 | 31.221 | 591.6105 | 95249.2884 | 6926002.873 |
| 162 | 0.01105 | 1.53300 | 94.666 | 31.145 | 591.6105 | 95840.8989 | 6909214.609 |
| 163 | 0.01108 | 1.54407 | 94.435 | 31.069 | 591.6105 | 96432.5094 | 6892385.453 |
| 164 | 0.01110 | 1.55518 | 94.203 | 30.993 | 591.6105 | 97024.1198 | 6875515.104 |
| 165 | 0.01113 | 1.56631 | 93.971 | 30.917 | 591.6105 | 97615.7303 | 6858603.259 |
| 166 | 0.01116 | 1.57746 | 93.738 | 30.840 | 591.6105 | 98207.3408 | 6841649.609 |
| 167 | 0.01119 | 1.58865 | 93.505 | 30.763 | 591.6105 | 98798.9513 | 6824653.843 |
| 168 | 0.01121 | 1.59986 | 93.271 | 30.686 | 591.6105 | 99390.5618 | 6807615.645 |
| 169 | 0.01124 | 1.61110 | 93.036 | 30.609 | 591.6105 | 99982.1723 | 6790534.697 |
| 170 | 0.01127 | 1.62237 | 92.801 | 30.532 | 591.6105 | 100573.7828 | 6773410.674 |
| 171 | 0.01130 | 1.63367 | 92.565 | 30.454 | 591.6105 | 101165.3932 | 6756243.249 |

| 172 | 0.01133 | 1.64500 | 92.329 | 30.376 | 591.6105 | 101757.0037 | 6739032.092 |
|-----|---------|---------|--------|--------|----------|-------------|-------------|
| 173 | 0.01136 | 1.65636 | 92.092 | 30.298 | 591.6105 | 102348.6142 | 6721776.864 |
| 174 | 0.01139 | 1.66774 | 91.854 | 30.220 | 591.6105 | 102940.2247 | 6704477.227 |
| 175 | 0.01142 | 1.67916 | 91.616 | 30.142 | 591.6105 | 103531.8352 | 6687132.836 |
| 176 | 0.01145 | 1.69060 | 91.377 | 30.063 | 591.6105 | 104123.4457 | 6669743.341 |
| 177 | 0.01148 | 1.70208 | 91.138 | 29.984 | 591.6105 | 104715.0562 | 6652308.388 |
| 178 | 0.01151 | 1.71358 | 90.897 | 29.905 | 591.6105 | 105306.6667 | 6634827.621 |
| 179 | 0.01154 | 1.72512 | 90.657 | 29.826 | 591.6105 | 105898.2771 | 6617300.674 |
| 180 | 0.01157 | 1.73669 | 90.415 | 29.747 | 591.6105 | 106489.8876 | 6599727.181 |
| 181 | 0.01160 | 1.74829 | 90.173 | 29.667 | 591.6105 | 107081.4981 | 6582106.768 |
| 182 | 0.01163 | 1.75991 | 89.931 | 29.587 | 591.6105 | 107673.1086 | 6564439.059 |
| 183 | 0.01166 | 1.77157 | 89.687 | 29.507 | 591.6105 | 108264.7191 | 6546723.668 |
| 184 | 0.01169 | 1.78327 | 89.443 | 29.427 | 591.6105 | 108856.3296 | 6528960.21 |
| 185 | 0.01172 | 1.79499 | 89.199 | 29.346 | 591.6105 | 109447.9401 | 6511148.29 |
| 186 | 0.01176 | 1.80675 | 88.953 | 29.266 | 591.6105 | 110039.5506 | 6493287.509 |
| 187 | 0.01179 | 1.81854 | 88.707 | 29.185 | 591.6105 | 110631.1610 | 6475377.464 |
| 188 | 0.01182 | 1.83036 | 88.460 | 29.103 | 591.6105 | 111222.7715 | 6457417.743 |
| 189 | 0.01185 | 1.84221 | 88.213 | 29.022 | 591.6105 | 111814.3820 | 6439407.933 |
| 190 | 0.01189 | 1.85410 | 87.965 | 28.940 | 591.6105 | 112405.9925 | 6421347.61 |
| 191 | 0.01192 | 1.86602 | 87.716 | 28.859 | 591.6105 | 112997.6030 | 6403236.348 |
| 192 | 0.01196 | 1.87798 | 87.467 | 28.777 | 591.6105 | 113589.2135 | 6385073.713 |
| 193 | 0.01199 | 1.88997 | 87.216 | 28.694 | 591.6105 | 114180.8240 | 6366859.266 |
| 194 | 0.01202 | 1.90199 | 86.965 | 28.612 | 591.6105 | 114772.4344 | 6348592.56 |
| 195 | 0.01206 | 1.91405 | 86.714 | 28.529 | 591.6105 | 115364.0449 | 6330273.143 |
| 196 | 0.01209 | 1.92614 | 86.461 | 28.446 | 591.6105 | 115955.6554 | 6311900.557 |
| 197 | 0.01213 | 1.93827 | 86.208 | 28.363 | 591.6105 | 116547.2659 | 6293474.335 |
| 198 | 0.01217 | 1.95044 | 85.954 | 28.279 | 591.6105 | 117138.8764 | 6274994.005 |
| 199 | 0.01220 | 1.96264 | 85.700 | 28.195 | 591.6105 | 117730.4869 | 6256459.088 |
| 200 | 0.01224 | 1.97488 | 85.444 | 28.111 | 591.6105 | 118322.0974 | 6237869.097 |
| 201 | 0.01227 | 1.98715 | 85.188 | 28.027 | 591.6105 | 118913.7079 | 6219223.538 |
| 202 | 0.01231 | 1.99946 | 84.931 | 27.942 | 591.6105 | 119505.3183 | 6200521.909 |
| 203 | 0.01235 | 2.01181 | 84.673 | 27.858 | 591.6105 | 120096.9288 | 6181763.703 |
| 204 | 0.01239 | 2.02420 | 84.415 | 27.773 | 591.6105 | 120688.5393 | 6162948.401 |
| 205 | 0.01242 | 2.03662 | 84.156 | 27.687 | 591.6105 | 121280.1498 | 6144075.481 |
| 206 | 0.01246 | 2.04909 | 83.896 | 27.602 | 591.6105 | 121871.7603 | 6125144.408 |
| 207 | 0.01250 | 2.06159 | 83.635 | 27.516 | 591.6105 | 122463.3708 | 6106154.643 |
| 208 | 0.01254 | 2.07413 | 83.373 | 27.430 | 591.6105 | 123054.9813 | 6087105.636 |
| 209 | 0.01258 | 2.08671 | 83.110 | 27.343 | 591.6105 | 123646.5917 | 6067996.828 |
| 210 | 0.01262 | 2.09933 | 82.847 | 27.257 | 591.6105 | 124238.2022 | 6048827.654 |
| 211 | 0.01266 | 2.11199 | 82.583 | 27.170 | 591.6105 | 124829.8127 | 6029597.538 |
| 212 | 0.01270 | 2.12469 | 82.318 | 27.083 | 591.6105 | 125421.4232 | 6010305.893 |
| 213 | 0.01274 | 2.13743 | 82.052 | 26.995 | 591.6105 | 126013.0337 | 5990952.127 |
| 214 | 0.01278 | 2.15022 | 81.785 | 26.907 | 591.6105 | 126604.6442 | 5971535.635 |
| 215 | 0.01283 | 2.16304 | 81.517 | 26.819 | 591.6105 | 127196.2547 | 5952055.803 |

| 216 | 0.01287 | 2.17591 | 81.249 | 26.731 | 591.6105 | 127787.8652 | 5932512.007 |
|-----|---------|---------|--------|--------|----------|-------------|-------------|
| 217 | 0.01291 | 2.18882 | 80.979 | 26.642 | 591.6105 | 128379.4756 | 5912903.614 |
| 218 | 0.01295 | 2.20177 | 80.709 | 26.553 | 591.6105 | 128971.0861 | 5893229.977 |
| 219 | 0.01300 | 2.21477 | 80.438 | 26.464 | 591.6105 | 129562.6966 | 5873490.442 |
| 220 | 0.01304 | 2.22781 | 80.166 | 26.374 | 591.6105 | 130154.3071 | 5853684.341 |
| 221 | 0.01309 | 2.24089 | 79.893 | 26.285 | 591.6105 | 130745.9176 | 5833810.998 |
| 222 | 0.01313 | 2.25402 | 79.619 | 26.194 | 591.6105 | 131337.5281 | 5813869.722 |
| 223 | 0.01318 | 2.26720 | 79.344 | 26.104 | 591.6105 | 131929.1386 | 5793859.811 |
| 224 | 0.01322 | 2.28042 | 79.068 | 26.013 | 591.6105 | 132520.7490 | 5773780.553 |
| 225 | 0.01327 | 2.29369 | 78.791 | 25.922 | 591.6105 | 133112.3595 | 5753631.221 |
| 226 | 0.01331 | 2.30700 | 78.513 | 25.831 | 591.6105 | 133703.9700 | 5733411.076 |
| 227 | 0.01336 | 2.32036 | 78.234 | 25.739 | 591.6105 | 134295.5805 | 5713119.367 |
| 228 | 0.01341 | 2.33377 | 77.954 | 25.647 | 591.6105 | 134887.1910 | 5692755.328 |
| 229 | 0.01346 | 2.34723 | 77.673 | 25.555 | 591.6105 | 135478.8015 | 5672318.18 |
| 230 | 0.01351 | 2.36074 | 77.391 | 25.462 | 591.6105 | 136070.4120 | 5651807.13 |
| 231 | 0.01356 | 2.37429 | 77.108 | 25.369 | 591.6105 | 136662.0225 | 5631221.371 |
| 232 | 0.01361 | 2.38790 | 76.825 | 25.275 | 591.6105 | 137253.6329 | 5610560.079 |
| 233 | 0.01366 | 2.40156 | 76.540 | 25.181 | 591.6105 | 137845.2434 | 5589822.419 |
| 234 | 0.01371 | 2.41526 | 76.253 | 25.087 | 591.6105 | 138436.8539 | 5569007.535 |
| 235 | 0.01376 | 2.42902 | 75.966 | 24.993 | 591.6105 | 139028.4644 | 5548114.56 |
| 236 | 0.01381 | 2.44283 | 75.678 | 24.898 | 591.6105 | 139620.0749 | 5527142.607 |
| 237 | 0.01386 | 2.45670 | 75.389 | 24.803 | 591.6105 | 140211.6854 | 5506090.774 |
| 238 | 0.01392 | 2.47062 | 75.098 | 24.707 | 591.6105 | 140803.2959 | 5484958.142 |
| 239 | 0.01397 | 2.48459 | 74.807 | 24.611 | 591.6105 | 141394.9064 | 5463743.772 |
| 240 | 0.01403 | 2.49861 | 74.514 | 24.515 | 591.6105 | 141986.5168 | 5442446.709 |
| 241 | 0.01408 | 2.51270 | 74.220 | 24.418 | 591.6105 | 142578.1273 | 5421065.979 |
| 242 | 0.01414 | 2.52683 | 73.925 | 24.321 | 591.6105 | 143169.7378 | 5399600.587 |
| 243 | 0.01419 | 2.54103 | 73.629 | 24.224 | 591.6105 | 143761.3483 | 5378049.519 |
| 244 | 0.01425 | 2.55528 | 73.331 | 24.126 | 591.6105 | 144352.9588 | 5356411.742 |
| 245 | 0.01431 | 2.56959 | 73.033 | 24.028 | 591.6105 | 144944.5693 | 5334686.2 |
| 246 | 0.01437 | 2.58396 | 72.733 | 23.929 | 591.6105 | 145536.1798 | 5312871.816 |
| 247 | 0.01443 | 2.59838 | 72.432 | 23.830 | 591.6105 | 146127.7902 | 5290967.493 |
| 248 | 0.01449 | 2.61287 | 72.129 | 23.731 | 591.6105 | 146719.4007 | 5268972.107 |
| 249 | 0.01455 | 2.62742 | 71.826 | 23.631 | 591.6105 | 147311.0112 | 5246884.514 |
| 250 | 0.01461 | 2.64203 | 71.521 | 23.530 | 591.6105 | 147902.6217 | 5224703.544 |
| 251 | 0.01467 | 2.65671 | 71.215 | 23.430 | 591.6105 | 148494.2322 | 5202428.003 |
| 252 | 0.01474 | 2.67144 | 70.907 | 23.328 | 591.6105 | 149085.8427 | 5180056.67 |
| 253 | 0.01480 | 2.68624 | 70.598 | 23.227 | 591.6105 | 149677.4532 | 5157588.3 |
| 254 | 0.01487 | 2.70111 | 70.288 | 23.125 | 591.6105 | 150269.0637 | 5135021.618 |
| 255 | 0.01493 | 2.71604 | 69.976 | 23.022 | 591.6105 | 150860.6741 | 5112355.323 |
| 256 | 0.01500 | 2.73104 | 69.663 | 22.919 | 591.6105 | 151452.2846 | 5089588.083 |
| 257 | 0.01507 | 2.74611 | 69.349 | 22.816 | 591.6105 | 152043.8951 | 5066718.537 |
| 258 | 0.01513 | 2.76124 | 69.033 | 22.712 | 591.6105 | 152635.5056 | 5043745.295 |
| 259 | 0.01520 | 2.77645 | 68.716 | 22.607 | 591.6105 | 153227.1161 | 5020666.932 |

| 260 | 0.01528 | 2.79172 | 68.397 | 22.503 | 591.6105 | 153818.7266 | 4997481.992 |
|-----|---------|---------|--------|--------|----------|-------------|-------------|
| 261 | 0.01535 | 2.80707 | 68.077 | 22.397 | 591.6105 | 154410.3371 | 4974188.984 |
| 262 | 0.01542 | 2.82249 | 67.755 | 22.291 | 591.6105 | 155001.9475 | 4950786.384 |
| 263 | 0.01549 | 2.83798 | 67.431 | 22.185 | 591.6105 | 155593.5580 | 4927272.63 |
| 264 | 0.01557 | 2.85355 | 67.106 | 22.078 | 591.6105 | 156185.1685 | 4903646.122 |
| 265 | 0.01564 | 2.86919 | 66.780 | 21.971 | 591.6105 | 156776.7790 | 4879905.223 |
| 266 | 0.01572 | 2.88491 | 66.452 | 21.863 | 591.6105 | 157368.3895 | 4856048.255 |
| 267 | 0.01580 | 2.90071 | 66.122 | 21.754 | 591.6105 | 157960.0000 | 4832073.498 |
| 268 | 0.01588 | 2.91658 | 65.791 | 21.645 | 591.6105 | 158551.6105 | 4807979.191 |
| 269 | 0.01596 | 2.93254 | 65.458 | 21.536 | 591.6105 | 159143.2210 | 4783763.528 |
| 270 | 0.01604 | 2.94858 | 65.123 | 21.426 | 591.6105 | 159734.8314 | 4759424.654 |
| 271 | 0.01612 | 2.96470 | 64.787 | 21.315 | 591.6105 | 160326.4419 | 4734960.671 |
| 272 | 0.01621 | 2.98091 | 64.448 | 21.204 | 591.6105 | 160918.0524 | 4710369.63 |
| 273 | 0.01629 | 2.99720 | 64.108 | 21.092 | 591.6105 | 161509.6629 | 4685649.528 |
| 274 | 0.01638 | 3.01358 | 63.767 | 20.979 | 591.6105 | 162101.2734 | 4660798.314 |
| 275 | 0.01647 | 3.03005 | 63.423 | 20.866 | 591.6105 | 162692.8839 | 4635813.878 |
| 276 | 0.01656 | 3.04660 | 63.077 | 20.752 | 591.6105 | 163284.4944 | 4610694.054 |
| 277 | 0.01665 | 3.06325 | 62.730 | 20.638 | 591.6105 | 163876.1049 | 4585436.617 |
| 278 | 0.01674 | 3.07999 | 62.381 | 20.523 | 591.6105 | 164467.7153 | 4560039.28 |
| 279 | 0.01683 | 3.09682 | 62.029 | 20.408 | 591.6105 | 165059.3258 | 4534499.693 |
| 280 | 0.01693 | 3.11376 | 61.676 | 20.291 | 591.6105 | 165650.9363 | 4508815.438 |
| 281 | 0.01703 | 3.13078 | 61.321 | 20.174 | 591.6105 | 166242.5468 | 4482984.029 |
| 282 | 0.01713 | 3.14791 | 60.963 | 20.057 | 591.6105 | 166834.1573 | 4457002.906 |
| 283 | 0.01723 | 3.16514 | 60.604 | 19.939 | 591.6105 | 167425.7678 | 4430869.436 |
| 284 | 0.01733 | 3.18247 | 60.242 | 19.820 | 591.6105 | 168017.3783 | 4404580.907 |
| 285 | 0.01744 | 3.19991 | 59.878 | 19.700 | 591.6105 | 168608.9887 | 4378134.525 |
| 286 | 0.01754 | 3.21745 | 59.512 | 19.579 | 591.6105 | 169200.5992 | 4351527.413 |
| 287 | 0.01765 | 3.23510 | 59.143 | 19.458 | 591.6105 | 169792.2097 | 4324756.603 |
| 288 | 0.01776 | 3.25286 | 58.773 | 19.336 | 591.6105 | 170383.8202 | 4297819.036 |
| 289 | 0.01787 | 3.27074 | 58.400 | 19.214 | 591.6105 | 170975.4307 | 4270711.558 |
| 290 | 0.01799 | 3.28873 | 58.024 | 19.090 | 591.6105 | 171567.0412 | 4243430.91 |
| 291 | 0.01811 | 3.30683 | 57.646 | 18.966 | 591.6105 | 172158.6517 | 4215973.731 |
| 292 | 0.01823 | 3.32506 | 57.266 | 18.841 | 591.6105 | 172750.2622 | 4188336.549 |
| 293 | 0.01835 | 3.34341 | 56.883 | 18.715 | 591.6105 | 173341.8726 | 4160515.778 |
| 294 | 0.01847 | 3.36188 | 56.498 | 18.588 | 591.6105 | 173933.4831 | 4132507.707 |
| 295 | 0.01860 | 3.38048 | 56.109 | 18.460 | 591.6105 | 174525.0936 | 4104308.503 |
| 296 | 0.01873 | 3.39921 | 55.719 | 18.331 | 591.6105 | 175116.7041 | 4075914.198 |
| 297 | 0.01886 | 3.41807 | 55.325 | 18.202 | 591.6105 | 175708.3146 | 4047320.686 |
| 298 | 0.01900 | 3.43706 | 54.928 | 18.071 | 591.6105 | 176299.9251 | 4018523.714 |
| 299 | 0.01913 | 3.45620 | 54.529 | 17.940 | 591.6105 | 176891.5356 | 3989518.875 |
| 300 | 0.01928 | 3.47547 | 54.127 | 17.808 | 591.6105 | 177483.1460 | 3960301.602 |
| 301 | 0.01942 | 3.49489 | 53.722 | 17.674 | 591.6105 | 178074.7565 | 3930867.157 |
| 302 | 0.01957 | 3.51446 | 53.313 | 17.540 | 591.6105 | 178666.3670 | 3901210.625 |
| 303 | 0.01972 | 3.53418 | 52.902 | 17.405 | 591.6105 | 179257.9775 | 3871326.9 |
| 304 | 0.01987 | 3.55405 | 52.487 | 17.268 | 591.6105 | 179849.5880 | 3841210.68 |

| 305 | 0.02003 | 3.57408 | 52.069 | 17.131 | 591.6105 | 180441.1985 | 3810856.452 |
|-----|---------|---------|--------|--------|----------|-------------|-------------|
| 306 | 0.02019 | 3.59428 | 51.647 | 16.992 | 591.6105 | 181032.8090 | 3780258.482 |
| 307 | 0.02036 | 3.61464 | 51.223 | 16.852 | 591.6105 | 181624.4195 | 3749410.801 |
| 308 | 0.02053 | 3.63517 | 50.794 | 16.711 | 591.6105 | 182216.0299 | 3718307.193 |
| 309 | 0.02070 | 3.65587 | 50.362 | 16.569 | 591.6105 | 182807.6404 | 3686941.182 |
| 310 | 0.02088 | 3.67676 | 49.926 | 16.426 | 591.6105 | 183399.2509 | 3655306.01 |
| 311 | 0.02107 | 3.69782 | 49.486 | 16.281 | 591.6105 | 183990.8614 | 3623394.628 |
| 312 | 0.02126 | 3.71908 | 49.043 | 16.135 | 591.6105 | 184582.4719 | 3591199.669 |
| 313 | 0.02145 | 3.74053 | 48.595 | 15.988 | 591.6105 | 185174.0824 | 3558713.438 |
| 314 | 0.02165 | 3.76218 | 48.143 | 15.839 | 591.6105 | 185765.6929 | 3525927.881 |
| 315 | 0.02186 | 3.78404 | 47.687 | 15.689 | 591.6105 | 186357.3034 | 3492834.568 |
| 316 | 0.02207 | 3.80610 | 47.227 | 15.538 | 591.6105 | 186948.9138 | 3459424.666 |
| 317 | 0.02228 | 3.82839 | 46.762 | 15.385 | 591.6105 | 187540.5243 | 3425688.909 |
| 318 | 0.02251 | 3.85089 | 46.292 | 15.230 | 591.6105 | 188132.1348 | 3391617.573 |
| 319 | 0.02274 | 3.87363 | 45.817 | 15.074 | 591.6105 | 188723.7453 | 3357200.439 |
| 320 | 0.02298 | 3.89661 | 45.338 | 14.916 | 591.6105 | 189315.3558 | 3322426.756 |
| 321 | 0.02322 | 3.91983 | 44.853 | 14.757 | 591.6105 | 189906.9663 | 3287285.21 |
| 322 | 0.02348 | 3.94331 | 44.363 | 14.595 | 591.6105 | 190498.5768 | 3251763.869 |
| 323 | 0.02374 | 3.96704 | 43.868 | 14.433 | 591.6105 | 191090.1872 | 3215850.145 |
| 324 | 0.02401 | 3.99105 | 43.367 | 14.268 | 591.6105 | 191681.7977 | 3179530.74 |
| 325 | 0.02429 | 4.01534 | 42.860 | 14.101 | 591.6105 | 192273.4082 | 3142791.585 |
| 326 | 0.02458 | 4.03992 | 42.347 | 13.932 | 591.6105 | 192865.0187 | 3105617.779 |
| 327 | 0.02488 | 4.06480 | 41.828 | 13.761 | 591.6105 | 193456.6292 | 3067993.518 |
| 328 | 0.02519 | 4.09000 | 41.302 | 13.588 | 591.6105 | 194048.2397 | 3029902.015 |
| 329 | 0.02552 | 4.11552 | 40.769 | 13.413 | 591.6105 | 194639.8502 | 2991325.417 |
| 330 | 0.02586 | 4.14137 | 40.230 | 13.236 | 591.6105 | 195231.4607 | 2952244.7 |
| 331 | 0.02621 | 4.16758 | 39.683 | 13.056 | 591.6105 | 195823.0711 | 2912639.566 |
| 332 | 0.02658 | 4.19416 | 39.128 | 12.873 | 591.6105 | 196414.6816 | 2872488.314 |
| 333 | 0.02696 | 4.22112 | 38.565 | 12.688 | 591.6105 | 197006.2921 | 2831767.706 |
| 334 | 0.02736 | 4.24847 | 37.994 | 12.500 | 591.6105 | 197597.9026 | 2790452.805 |
| 335 | 0.02777 | 4.27625 | 37.415 | 12.309 | 591.6105 | 198189.5131 | 2748516.801 |
| 336 | 0.02821 | 4.30446 | 36.826 | 12.116 | 591.6105 | 198781.1236 | 2705930.803 |
| 337 | 0.02867 | 4.33313 | 36.228 | 11.919 | 591.6105 | 199372.7341 | 2662663.607 |
| 338 | 0.02915 | 4.36228 | 35.619 | 11.719 | 591.6105 | 199964.3445 | 2618681.431 |
| 339 | 0.02966 | 4.39193 | 35.000 | 11.515 | 591.6105 | 200555.9550 | 2573947.602 |
| 340 | 0.03019 | 4.42213 | 34.370 | 11.308 | 591.6105 | 201147.5655 | 2528422.202 |
| 341 | 0.03076 | 4.45288 | 33.728 | 11.097 | 591.6105 | 201739.1760 | 2482061.644 |
| 342 | 0.03135 | 4.48423 | 33.074 | 10.881 | 591.6105 | 202330.7865 | 2434818.191 |
| 343 | 0.03199 | 4.51622 | 32.407 | 10.662 | 591.6105 | 202922.3970 | 2386639.372 |
| 344 | 0.03266 | 4.54888 | 31.725 | 10.438 | 591.6105 | 203514.0075 | 2337467.301 |
| 345 | 0.03338 | 4.58225 | 31.028 | 10.208 | 591.6105 | 204105.6180 | 2287237.864 |
| 346 | 0.03414 | 4.61639 | 30.316 | 9.974 | 591.6105 | 204697.2284 | 2235879.728 |
| 347 | 0.03496 | 4.65136 | 29.586 | 9.734 | 591.6105 | 205288.8389 | 2183313.159 |
| 348 | 0.03585 | 4.68721 | 28.838 | 9.488 | 591.6105 | 205880.4494 | 2129448.562 |

| 349 | 0.03680 | 4.72401 | 28.070 | 9.235 | 591.6105 | 206472.0599 | 2074184.68 |
|-----|---------|---------|--------|-------|----------|-------------|-------------|
| 350 | 0.03784 | 4.76185 | 27.280 | 8.975 | 591.6105 | 207063.6704 | 2017406.358 |
| 351 | 0.03897 | 4.80082 | 26.467 | 8.708 | 591.6105 | 207655.2809 | 1958981.706 |
| 352 | 0.04020 | 4.84102 | 25.628 | 8.432 | 591.6105 | 208246.8914 | 1898758.48 |
| 353 | 0.04157 | 4.88258 | 24.760 | 8.146 | 591.6105 | 208838.5019 | 1836559.394 |
| 354 | 0.04308 | 4.92566 | 23.861 | 7.850 | 591.6105 | 209430.1123 | 1772175.927 |
| 355 | 0.04476 | 4.97042 | 22.927 | 7.543 | 591.6105 | 210021.7228 | 1705360.021 |
| 356 | 0.04667 | 5.01709 | 21.953 | 7.223 | 591.6105 | 210613.3333 | 1635812.699 |
| 357 | 0.04883 | 5.06592 | 20.934 | 6.887 | 591.6105 | 211204.9438 | 1563168.068 |
| 358 | 0.05134 | 5.11726 | 19.863 | 6.535 | 591.6105 | 211796.5543 | 1486970.216 |
| 359 | 0.05427 | 5.17153 | 18.730 | 6.162 | 591.6105 | 212388.1648 | 1406638.665 |
| 360 | 0.05777 | 5.22930 | 17.525 | 5.766 | 591.6105 | 212979.7753 | 1321414.582 |
| 361 | 0.06205 | 5.29135 | 16.230 | 5.340 | 591.6105 | 213571.3857 | 1230272.675 |
| 362 | 0.06745 | 5.35880 | 14.822 | 4.876 | 591.6105 | 214162.9962 | 1131767.315 |
| 363 | 0.07457 | 5.43336 | 13.266 | 4.364 | 591.6105 | 214754.6067 | 1023739.936 |
| 364 | 0.08457 | 5.51793 | 11.501 | 3.784 | 591.6105 | 215346.2172 | 902692.545 |
| 365 | 0.10016 | 5.61808 | 9.411 | 3.096 | 591.6105 | 215937.8277 | 762180.1332 |
| 366 | 0.13003 | 5.74811 | 6.697 | 2.203 | 591.6105 | 216529.4382 | 587087.0576 |
| 367 | 0.26976 | 6.01787 | 1.067 | 0.351 | 591.6105 | 217121.0487 | 282982.2419 |
| END | | | | 0.000 | 0.00000 | | |

Appendix D. C++ Codes for Generation of ANSYS command Language

<u>C++ Code for Generation of ANSYS Command Language (Thermal Analysis)</u>

```
#include <stdio.h>
#include <string.h>
#include <conio.h>
#include <math.h>
int main ()
{
FILE *input;
input= fopen("c:\\ANSYS_ProgmCodes.txt","w");
float v=0, u=41.67, a=-6.867, A=0.00775;
float alpha=-20.87, teta=1.0471;
float delta_t=0.0, accum_t=0.0;
float w0=126.66, w1, vi, vj, Q, q;
int i,j=1;
for(i=1;i<368;i++)
{
w1=sqrt((w0*w0)+(2*alpha*teta));
delta_t=(w1-w0)/alpha;
vi = (0.329 * w0);
v_j = (0.329 * w_1);
Q = 0.95*((0.5*238.25)*((vi*vi)-(vj*vj)));
q=(Q)/(A*delta_t);
fprintf(input,"!Load Step %d\n\n",i);
fprintf(input,"ASEL,ALL\n");
fprintf(input,"SFA,ALL,1,CONV,50,25\n\n");
fprintf(input,"ASEL,S,AREA,,%d\n",j);
fprintf(input,"CM,FLUX%d,AREA\n",j);
fprintf(input,"ASEL,,AREA,,FLUX%d\n",j);
fprintf(input, "SFA,ALL,1,HFLUX,%f\n\n",q);
```

accum_t+=delta_t; fprintf(input,"TIME,%f\n",accum_t); fprintf(input,"AUTOTS,-1\n");

```
fprintf(input,"NSUBST,5,,,1\n");
fprintf(input,"KBC,1\n");
fprintf(input,"OUTRES,BASIC,LAST\n");
fprintf(input, "OUTPR,BASIC\n\n");
```

```
fprintf(input,"LSWRITE,%d\n\n",i);
```

```
fprintf(input,"ASEL,ALL\n");
fprintf(input,"SFADELE,ALL,1,ALL\n\n\n\n\n");
```

```
fprintf(input,"!***\n");
printf("%3d %f %f\n",i,q,accum_t);
```

```
w0=w1;
if(w0<0) break;
j++;
if(j==7) j=1;
}
printf("\nCOMPLETED!\n\nPress Any Key to Exit.. ");
getch();
```

}

<u>C++ Code for Generation of ANSYS Command Language (Thermal Stress</u> <u>Analysis)</u>

```
#include <stdio.h>
#include <stdio.h>
#include <conio.h>
#include <conio.h>
#include <math.h>

int main ()
{
FILE *input;
input= fopen("d:\\Structural_thermal_stress.txt","w");
float v=0, u=41.67, a=-6.867, A=0.00775;
float alpha=-20.87, teta=1.0471;
float delta_t=0.0, accum_t=0.0;
float w0=126.66, w1, vi, vj, Q, q;
int i;
```

```
for(i=1;i<368;i++)
{
w1=sqrt((w0*w0)+(2*alpha*teta));
delta_t=(w1-w0)/alpha;
vi= (0.329*w0);
vj= (0.329*w1);
Q= 0.95*((0.5*238.25)*((vi*vi)-(vj*vj)));
q=(Q)/(A*delta_t);
```

fprintf(input,"!Load Step %d\n\n",i);

accum_t+=delta_t;

fprintf(input,"LDREAD,TEMP,,,%f,,Simulation_27_perforated_vented_model_glue d,rth\n",accum_t);

```
fprintf(input,"TIME,%f\n",accum_t);
fprintf(input,"AUTOTS,-1\n");
fprintf(input,"KBC,1\n");
fprintf(input,"OUTRES,BASIC,LAST\n");
fprintf(input, "OUTPR,BASIC\n\n");
```

fprintf(input,"LSWRITE,%d\n\n",i);

```
w0=w1;
if(w0<0) break;
}
printf("\nCOMPLETED!\n\nPress Any Key to Exit.. ");
getch();
```

```
}
```



Perforated Disc Brake (Top View Temperature Distribution)



(b) Time =1.09s



















Non Perforated Disc Brake (Top View Temperature Distribution)



(b) Time =1.09s


















Appendix F. Top View Von Mises Thermal Stress Distribution

Perforated Disc Brake (Top View Von Mises Thermal Stress Distribution)





(b) Time =1.09s



















Appendix G. Project Gantt Chart

Final Year Project 1 Gantt Chart

| No | Activities / Week | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|----|---|---|---|---|---|---|---|---|---|---|----|----|----|----|----|
| 1 | Selection of project topic | | | | | | | | | | | | | | |
| 2 | Preliminary Research Work | | | | | | | | | | | | | | |
| 3 | Familiarization of softwares: CATIA and ANSYS | | | | | | | | | | | | | | |
| 4 | Submission of preliminary report | | | | | | | | | | | | | | |
| 5 | Data gathering and calculation of initial input (energy) | | | | | | | | | | | | | | |
| 6 | CAD modeling | | | | | | | | | | | | | | |
| 7 | Submission of progress report | | | | | | | | | | | | | | |
| 8 | Seminar | | | | | | | | | | | | | | |
| 9 | Transferring CAD model to ANSYS and creating C++ command text for ANSYS for analysis work | | | | | | | | | | | | | | |
| 10 | Simulation on temperature development and distribution | | | | | | | | | | | | | | |
| 11 | Submission of interim report final draft | | | | | | | | | | | | | | |
| 12 | Oral presentation | | | | | | | | | | | | | | |

Final Year Project 2 Gantt Chart

| No | Activities / Week | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|----|--|---|---|---|---|---|---|---|---|---|----|----|----|----|----|
| 1 | Simulation of thermal stress development and distribution | | | | | | | | | | | | | | |
| 2 | Submission of progress report 1 | | | | | | | | | | | | | | |
| 3 | Simulation of thermal stress development and distribution continues | | | | | | | | | | | | | | |
| 4 | Submission of progress report 2 | | | | | | | | | | | | | | |
| 5 | Seminar | | | | | | | | | | | | | | |
| 6 | Review of project objectives and results | | | | | | | | | | | | | | |
| 7 | Final report preparation | | | | | | | | | | | | | | |
| 8 | Poster exhibition | | | | | | | | | | | | | | |
| 9 | Submission of dissertation final draft | | | | | | | | | | | | | | |
| 10 | Oral presentation | | | | | | | | | | | | | | |
| 11 | Submission of dissertation (hard bound) | | | | | | | | | | | | | | |