Design and Synthesis of Advanced Chebyshev Filters

By

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FINAL PROJECT REPORT

Submitted to the Electrical & Electronics Engineering Programme
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CERTIFICATION OF APPROVAL

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Approved:

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Project Supervisor

UNIVERSITI TEKNOLOGI PETRONAS
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August 2014

CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

ELDEN ZEE CHIEN PHENG

ABSTRACT

The increasing complexity of microwave filter designs demands higher performance tools to cope with. In this paper, the capability of software to design advanced Chebyshev type filter is discussed. Emphasis is placed on the synthesis part that is done. The challenge posted and discussed here is the ability of marketed software to synthesize a ladder prototype network containing transmission zeroes and its usability in handling complex and modern microwave filters designs. This paper also discussed the in depth method to perform pole-zero extraction for the resonant elements that has impedance function and S_{11} functions derived from combination of finite and infinite placed zeroes.

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LIST OF ABBREVIATIONS

LP	Low-Pass
HP	High-Pass
BP	Band-Pass
BS	Band-Stop
CAD	Computer-Aided Design
MATLAB	Matrix Laboratory
GUI	Graphical User Interface
ADS	Advanced Design System
Z	Impedance
L	Inductance
C	Capacitance

CHAPTER 1
INTRODUCTION

1.1 Background

Microwave filter designs are an area that had been constantly improving and enhancing. To summarize the definition of microwave filters, they are used to isolate wanted signals from a wide range of signals [1] and are mainly categorized into Low-Pass (LP), High-Pass (HP), Band-Pass (BP) and Band-Stop (BS). The discoveries of filters had contributed greatly to the advancement of our modern society [1].

Microwave filters can be designed using two approaches. One approach is through lumped elements, which uses inductors, capacitors and resistors to synthesize the circuit [2]. However, higher order circuits using these topologies would result in higher loss rate due to the increasing amount of elements used which would also make it bulky [3]. Therefore, another approach was used which utilized the usage of distributed elements. According to Ian Hunter, distributed elements consist of high Q-factor, which is the quality of signals transmitted using materials that had interconnections between them that act as a substitute of the lumped elements.

New topologies and methods are researched and discovered from time to time. One area that it research had been focusing on is to incorporate filter designs into algorithms and application software.

There are at least 47 Computer-Aided Design (CAD) that is available on the internet for filter designing [4]. CADs are used extensively both in the academic world as well as the industrial world for learning purposes as well as designing purposes. The applications ranged from free to expensive, basic to sophisticate. It is difficult to specify CAD due to its wide domain [5]. However, in a common approach to filter designing, it can be split into analysis and synthesis.

1.2 Problem Statement

Due to the increasingly complicated design of filters, it is found out that the available CAD tools are insufficient in the designing process. By considering the lumped elements approach, most available software could not sufficiently satisfy the designing demand which would requires capability to synthesize virtually a circuit based on available components in the market.

Besides, most of the available CAD tools require strong programming command to be able to effectively utilize [6]. And most commercially available tools that are sophisticated enough for users to effectively use it had very expensive licenses.

Another challenge that is posted currently is the lack of filter designs tools that are able to handle synthesis with transmission zeroes. According to [9], there are a few software that are more developed for filter designing process which are LADDER, FILTER, FILTERD and FIESTA. FILTER, FIESTA and LADDER had a major setback where it can only operate in the DOS settings. To add on, most of the above mentioned software is not able to produce ladder network model synthesis which is crucial in some of the filter designs. Currently, FILTER and FIESTA offered only designing usability for cascade type designs while LADDER did not offer active ladder network designs. There are certain packages that are available in MATLAB that are able to perform these, but currently, there is no design software in the market that is able to handle designing process with zeroes included in the transfer function [9].

In addition, the complexity of filters synthesis is obvious in more advanced filters such as advanced chebyshevs or elliptic filters. This is evident because these models requires more than the continuous fraction method to extract its resonant elements values. The existence of the combination of finite and infinite zeroes had caused the complexity to arise and by using normal conventional method, it will be hard for both academicians and people in the industry to effectively perform synthesis.

Therefore, it is necessary to come out with a solution that would results in an easy to use tool that are able to meet the standards of both by academicians and industrial designers. This project will aim at creating a source-file by using

MATLAB that is capable of performing the tedious pole-zero extraction part of the synthesis which would help lessen the work load for people desiring to use finite-zeroes related filter models in the future.

1.3 Objective

1. To study on the synthesis model involving filters containing a combination of finite and infinite zeroes.
2. To generalize the pole-zero extraction technique so to be able to be utilized in various filter models of different orders.
3. To come out with a simple and straight-forward mathematical model that could enable the different kinds of responses for the synthesis of the ladder network.

1.4 Scope of Study

In this project, knowledge on microwave filter designing would be required. It is important to understand the basics fundamentals that contribute the design parameters. It also requires the understanding of different analytical models to help in the analysis process. Mainly the two port network model and the S-parameters model.

Another aspect of the scope of study is also the understanding of the synthesis of ladder network. The functionality of the ladder network and the methods to synthesize it is studied. This project will also emphasize on researching the steps that are needed to perform ladder network synthesis with transmission zeroes.

Therefore, the suitable pole-zero extraction technique is also studied or derived to be implemented in this project. This is because in order to incorporate the synthesis model not only in MATLAB but in other programming language of lower computational power, a direct, and non-complex theoretically model must be implemented. If needed, a simple approach to complex mathematical operation will also be researched and covered in this project so as to implement it in coding algorithm.

The research also requires understanding in MATLAB which will be used to come out with the code that would be transferred to an object-oriented programming language for the creation of the GUI. Understanding in Maple13 software is also needed. This software will be used to verify and to support the designing of the model using MATLAB by making complex calculations easy and thus saving the time.

1.5 Relevancy and Feasibility

This project is relevant and feasible. Its objective can be achieved provided that ample research had been done to form the mathematical models for the ladder network synthesis. After the exploration of the mathematical models, the ladder synthesis algorithm can be formed using MATLAB which is a common tool used for complex coding since it is powerful enough.

This project is also relevant to the course. The two main areas in this project are:

1. Exploration into the synthesis of microwaves filters
2. Exploration into the algorithm based on MATLAB

It employs genuine pole-extraction technique that is used to extract the poles and zeros in the form of lumped elements in the form of ladder network. This is relevant to the electrical and electronic course offered by the university.

CHAPTER 2
LITERATURE REVIEW

2.1 Kinds of filters

There are four kinds of microwave filters, 1) Low Pass 2) High Pass 3) Bandpass 4) Bandstop.

Low pass – Isolate and allows frequency bands that is below the cut-off frequency.

High pass – Isolate and allows frequency bands that is above the cut-off frequencies

Bandpass – Isolate and allows a bandwidth of frequencies. It can be a combination of Low and High pass filters

Bandstop – Isolate and cut-off a bandwidth of frequencies. It can be a combination of Low and High pass filters.

2.2 Frequency Transformation

It is possible to perform frequency transformation to convert a filter from a low-pass to another kind [7]. Frequency Transformation because in initial phase of the designing, it is easier if the frequency is set to 1 radian which helps simplifies designing. Besides frequency, the impedance is also normally set at 1**Ω**. It is also possible to scale impedance.

Lowpass to Lowpass : $\omega = \frac{\omega}{\omega_c}$

Lowpass to Highpass : $\omega = -\frac{\omega_c}{\omega}$

Lowpass to Bandpass : $\omega = \alpha(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega})$

Lowpass to Bandstop : $\omega = -\frac{1}{\alpha(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega})}$

- α is the bandwidth scaling factor [7]

We can further expand on the information above and use it to transform the elements values [1]:

Lowpass -> Lowpass

Lowpass filters prototype mainted the same circuit representation with shifting in frequencies done.

Inductance can be transformed by using $L = \frac{L}{w_c}$

Capacitance can be transformed by using $C = \frac{C}{w_c}$

Lowpass -> Highpass

In highpass, inductance will be replaced with capacitance while capacitance is replaced with inductance.

Inductance can be transformed by using $C' = \frac{1}{w_c L}$

Capacitance can be transformed by using $L' = \frac{1}{w_c C}$

Lowpass -> Bandpass

In bandpass transformation, inductance will produce a series combination of inductance and capacitance while capacitance will produce a parallel combination of inductance and capacitance

Inductance can be transformed by using $L' = \frac{\propto L}{w_o}$, $C' = \frac{1}{w_c \propto L}$

Capacitance can be transformed by using $L' = \frac{1}{w_o \propto C}$, $C' = \frac{\propto C}{w_c}$

Lowpass -> Bandstop

In bandstop transformation, inductance will produce a parallel combination of inductance and capacitance while capacitance will produce a series combination of inductance and capacitance

Inductance can be transformed by using $L' = \frac{1}{w_o \propto C}$, $C' = \frac{\propto C}{w_c}$

Capacitance can be transformed by using $L' = \frac{\propto L}{w_o}$, $C' = \frac{1}{w_c \propto L}$

2.3 Different types of filters

There are different types of filters that are developed with some of them being common. Maximally flat filters, or also known as Butterworth filters are filters that had the closest to ideal response in terms of its bandwidth and bandstop. It does not contains equiripples and is considered the simplest to design [1] compared to other designs and approximations. All its zeroes are found in infinities due to its transfer function

$$|S_{12}(jw)|^2 = \frac{1}{1 + \omega^{2N}}$$

$|S_{12}(jw)|^2$ is known as the transducer power gain according to Ian Hunter [page 37] which translates as the signal output strength computed using the Scattering Parameters. Without its square, it is known as the output reflection coefficients with S_{11} known as the input reflection Coefficients. Normally, both of their square products are considered equally due to the assumption that both are ideal responses without any losses between them.

N is the order of the equations and is normally used to determine the selectivity and in case of lumped elements synthesis, the number of elements that are to be used. According to the online Oxford Dictionary, selectivity is defined as “the ability of a device to respond to a particular frequency without interference from others”. When a cut-off frequency is designated, selectivity states how capable the filter can isolate the bandpass and the rejected band.

For Chebyshev filters, selectivity is improved compared to Butterworth filters. It is due to its transfer function:

$$|S_{12}(jw)|^2 = \frac{1}{1 + \epsilon^2 T_N^2(\omega)}$$

$T_N^2(\omega) = \cos[Ncos^{-1}(\omega)]$ which translates into higher selectivity which requires less elements and thus less orders/. However due to the existence of the ripple factor, ϵ, Chebyshev contains ripples on its passband. The common feature of both Chebyshev and Butterworth is that its transmission zeroes are located at infinities.

However, a modification to the Chebyshev prototype would give the advantage of placing arbitrary zeroes at finite frequencies [1]. This would enables greater manipulation in designing as designers could control the cut-off gradient at the stop band by placing zeroes (which are roots in the transfer function) to control the response. This is known as the Generalized Chebyshev function.

2.4 Generalized Chebyshev Filters

Generalized Chebyshev filters is unique in its design mainly due to its ability to allow users to arbitrarily place zeroes in its responses. In the previously discussed filters such as the Butterworth, Chebyshev or Elliptic filters, the S_{21} response had its transmission zeroes located at infinity. Transmission zeroes is defined as the location in the range of frequencies that the output of the filter is completely cut-off, producing zero output. This means that, with the transmission zeroes located at infinity, the S_{21} response will gradually approach to zero when cut-off.

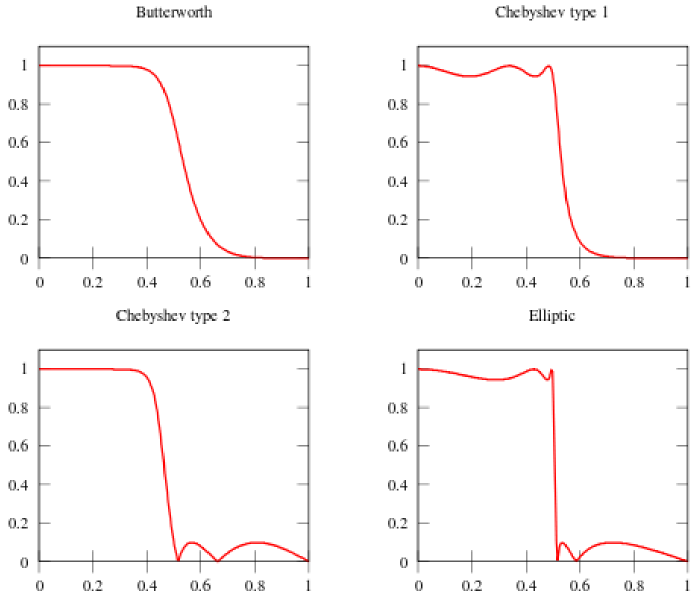


Figure 1: Different responses of filters.

As we can see from figure 1, for Butterworth and Chebyshev type 1 filters, the zeroes are placed at infinity while for type 2 and Elliptic, the zeroes can be also placed at finite frequencies. However, the setback is these zeroes are not flexible and cannot be defined by the users. In Generalized Chebyshev, these transmission zeroes can be predefined by users and can either be placed at the real or complex plane. Transmission zeroes at the real plane will results in a total cut-off at the particular frequency while a zero at complex plane may not result in a complete cut-off at a particular frequency.

2.5 Ladder Synthesis

Apart from the modelling of the transfer functions and plotting the responses graphs, perhaps the most important expect in filter designing is the synthesis of the filter itself. In the lump element approach, synthesis prototypes can be categorized as cascade prototypes and ladder prototypes [10]. In this project, we will be looking into the ladder prototypes. The common method of synthesis would means that a lowpass prototype is derived. Frequency transformations will then be applied if it is needed to convert the lowpass to either highpass, bandpass or bandstop. In the ladder network synthesis, the synthesis process can be generally categorized into 2 different groups.

The first group is the synthesis without arbitrary zeroes in the responses which means that all zeroes are at infinity. Generally, the method applied when there are no transmission zeroes in the transfer function is to use the continued fraction of expansion method. This method is simple and easy to implement and thus provide little difficulties.

The second group is the synthesis of responses with zeroes. There are a few ways to approach in this kind of synthesis. The common continuous fraction method cannot be applied due to the presence of zeroes which translates as resonant circuits in the network as shown in Figure 2.

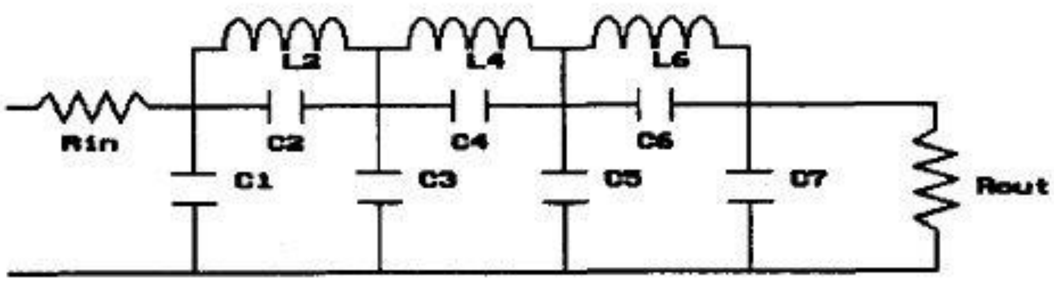


Figure 2: A Ladder network circuit with resonant circuits.

The most common approach is to use filter tables to avoid complex synthesis as posted by the numerical approach of synthesizing the network. However, filter tables offers a few setbacks. Firstly, it is less flexible with constraints on limited designs as offered by the filter tables [10].

Order	Pole position				
2	$(-1{\cdot}0186 \pm j1{\cdot}1347)$				
3	$(-1{\cdot}0686 + j0)$	$(-4{\cdot}7002 \pm j1{\cdot}0802)$			
4	$(-0{\cdot}8028 \pm j0{\cdot}3496)$	$(-0{\cdot}2683 \pm j1{\cdot}0460)$			
5	$(-0{\cdot}8447 + j0)$	$(-0{\cdot}5590 \pm j0{\cdot}5145)$	$(-0{\cdot}1727 \pm j1{\cdot}0292)$		
6	$(-0{\cdot}7439 \pm j0{\cdot}1649)$	$(-0{\cdot}3877 \pm j0{\cdot}6327)$	$(-0{\cdot}1202 \pm j1{\cdot}0201)$		
7	$(-0{\cdot}7788 + j0)$	$(-0{\cdot}5977 \pm j0{\cdot}2706)$	$(-0{\cdot}2799 \pm j0{\cdot}7204)$	$(-0{\cdot}885 \pm j1{\cdot}0147)$	
8	$(-0{\cdot}7258 \pm j0{\cdot}0944)$	$(-0{\cdot}4505 \pm j0{\cdot}3673)$	$(-0{\cdot}2114 \pm j0{\cdot}7828)$	$(-0{\cdot}6777 \pm j1{\cdot}0112)$	
9	$(-0{\cdot}7509 + j0)$	$(-0{\cdot}6319 \pm j0{\cdot}1607)$	$(-0{\cdot}3324 \pm j0{\cdot}4682)$	$(-0{\cdot}1654 \pm j0{\cdot}8272)$	$(-0{\cdot}5357 \pm j1{\cdot}0088)$
10	$(-0{\cdot}7184 \pm j0{\cdot}6085)$	$(-0{\cdot}5194 \pm j0{\cdot}2204)$	$(-0{\cdot}2525 \pm j0{\cdot}5602)$	$(-0{\cdot}1331 \pm j0{\cdot}8596)$	$(-0{\cdot}4341 \pm j1{\cdot}0071)$

Figure 3: A filter table example taken from [11]

Besides, according to [8], the filter tables also post some challenges. With the setbacks of these methods as well as lack of capable software that are able to perform synthesis with zeroes [9], an algorithm to perform such a synthesis should be looked into.

2.6 Existing pole-zero extraction with arbitrary placed zeroes.

Currently, exploration is done into 2 methods shown in [9]. In both methods, the objective is to synthesize the ladder network based on the impedance function which can be obtained by:

$$Z(p) = \frac{1 + S_{11}}{1 - S_{11}}$$

Where p=jω. The S_{11} is known as the input reflection coefficient. With the impedance function obtained, the poles and zeros can be extracted from the equation in the form of elements (inductors or capacitors). In non-zeros impedance functions where the zeros are all located at infinity, continuous fraction method is used to extract the elements. However, advanced filters models contained arbitrary zeros which rendered the classical method incapable of accurately extracting the elements. The both methods currently explored would provide an insight on how to extract the resonant elements from the impedance function before continuing with the classical method when all zeros are extracted.

Below is the first method's steps and equations that are needed in designing the MATLAB synthesis algorithm model:

1. $Z_B = Z_A - \frac{1}{pC_1}$

2. Shunt capacitor extracted using: $\frac{b_1 z^{N-1} - b_3 z^{N-3} + b_5 z^{N-5} - \ldots\ldots}{a_0 z^{N-1} - a_2 z^{N-3} + a_4 z^{N-5} - \ldots\ldots}$ where b is the denominator for Z function and a the numerator.

3. Split and extract Numerator and Denominator of Z_B

4. $D_C = \frac{D_C}{z^2 p^2 + 1}$

5. $L_R = \frac{N_B}{pD_c}$, where p -> $\frac{j}{z}$

6. $Z_C = \frac{pL_{11}}{z^2 p^2 + 1} - Z_B$

7. $C_R = \frac{1}{z^2 L_R}$

8. Step 1-7 is repeated for each zeroes

9. The remainder after all zeroes (resonant circuits) are extracted is synthesized using the continuous fraction method.

The second method which is also elaborated in [12] is done through zero shifting. Note that in both methods, it is assumed that the impedance function is lossless. Below is the explored method for method 2:

1. Choose a zero pair to obtain shunt capacitor using $y_2 = \frac{1}{Z_{11}(p)} - C_1 p$
2. The following term to be removed from $\frac{1}{y_2}$(which is $Z_{B)}$ is $\frac{kp}{p^2+z^2}$ where k is the coefficient of the pole at the origin from the Foster's expansion method [12].
3. The resonant capacitor can be removed by: $k\left(\frac{1}{C}\right) = Z_B(p^2 + z^2)|_{p=jz}$
4. The resonant inductor is obtained through $\frac{1}{z^2 C}$
5. The above steps are repeated until all zeros elements are removed. With all zeros removed, the classical continuous fraction method is used to find the remaining ladder network values.

CHAPTER 3
METHODOLOGY

3.1 Project Flow and Milestones

The project is a continuation of a pervious student work. In this project, collaboration is done with a software company in Penang. The company in Penang will be responsible in creating the GUI using C# programming language.

For the university side, we will be responsible of coming out with the mathematical model in MATLAB coding and to provide the necessary mathematics and industrial standards information to the programmers to be integrated into their software.

The end product will be commercialized software robust enough to be used for filter designing in both academic and industrial purposes.

Currently, the milestones achieved are as follow:

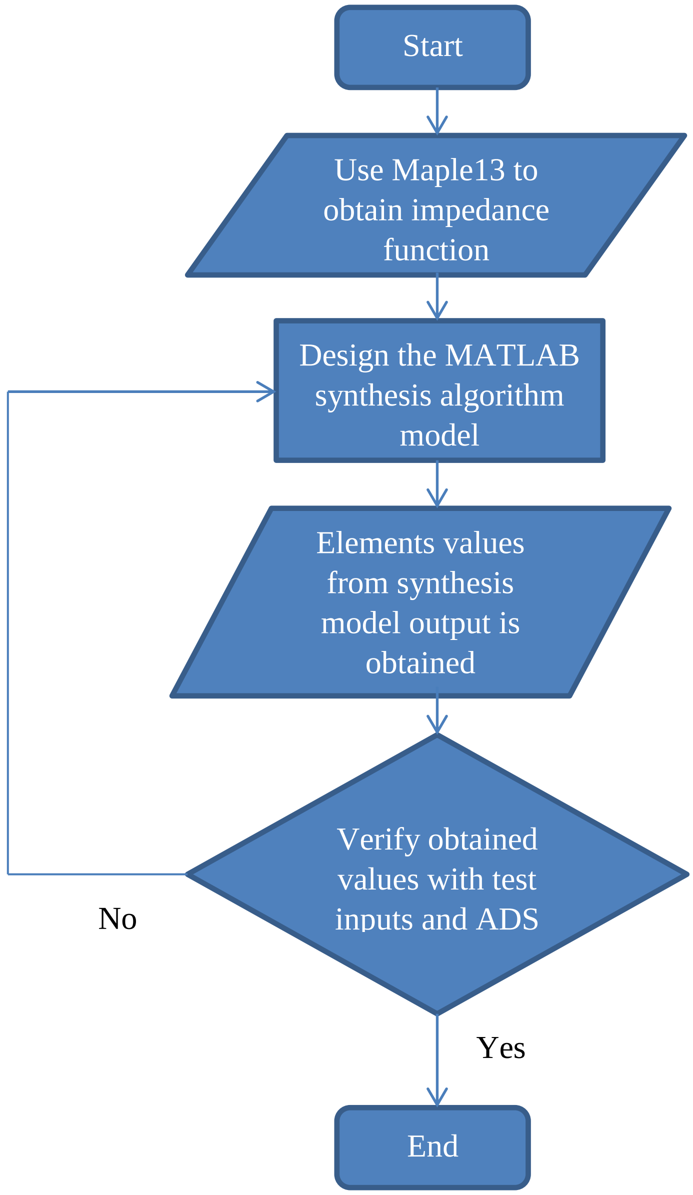
1. Completed MATLAB programming of design for Butterworth Filter.
2. Completed MATLAB programming of design for Chebyshev Filter.

In my project, I would be continuing the research by the previous student. There are one milestone that is set:

1. To come out with the synthesis model and pole-extraction technique in MATLAB programming for advanced Chebyshev Filter.

Currently, progress had been made in MATLAB to come out with the algorithm for the synthesis model for the advanced Chebyshev filter. The MATLAB software and Maple 13 software will be used together in designing and verification of the product. MATLAB will be used in the designing phase of the algorithm while Maple 13 is used to create the Generalized Chebyshev mathematical model that is able to produce the S_{11} and S_{21} responses which will then be used to find the impedance function that is used in the synthesis algorithm. Maple 13 will also be used to verify the output of the MATLAB algorithm. ADS will be used to construct the circuit to evaluate the accuracy of the values obtained by plotting and observing the frequency responses.

Presented here is the flowchart of the progress flow:



Below is the tentative Gantt Chart based on current understanding of the progress:

	FYP I Semester														FYP II Semester													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
Study fundamentals Of microwave filters	/	/	/	/	/	/	/																					
Understand characteristics of Generalized Chebyshev filters					/	/	/	/																				
Research on pole/zero extraction technique							/	/	/																			
Programming synthesis algorithm for Generalized Chebyshev network									/	/	/																	
Extending usability for advanced Chebyshev filter design											/	/	/	/														
Troubleshooting and enhancing synthesis model															/	/	/	/	/	/	/	/	/	/	/	/	/	/

Table 1: Gantt Chart of Progress

3.2 Style of programming in MATLAB

In this project, due to the dynamic content and values that is used to synthesize the code, for example, an impedance function that is of 13th order, MATLAB may produce inaccurate results due to the computational method it used. Converting the impedance function to symbolic variables in MATLAB to form the equations and reverting back to polynomial form for calculating would cause a lot of inaccuracies to be generated in calculations.

Therefore, to reduce the probability of errors in computation, a rule of thumb is to calculate the numerator and denominator separately in array form. For example, a symbolic expression of:

$$X = 4p^4 + 2p^3 + p^2 + 12p + 5$$

Can be computed in MATLAB array form as

$$X = [4\ 2\ 1\ 12\ 5]$$

Throughout the progress of the project, this method of dividing for computation would reduce the tendency of errors in MATLAB significantly.

CHAPTER 4
RESULTS AND DISCUSSIONS

4.1 Ladder Network Model

Below is a diagram of the expected generated ladder network circuit:

This diagram depicts the arrangement of the ladder networks with the orientation of the resonant circuits.

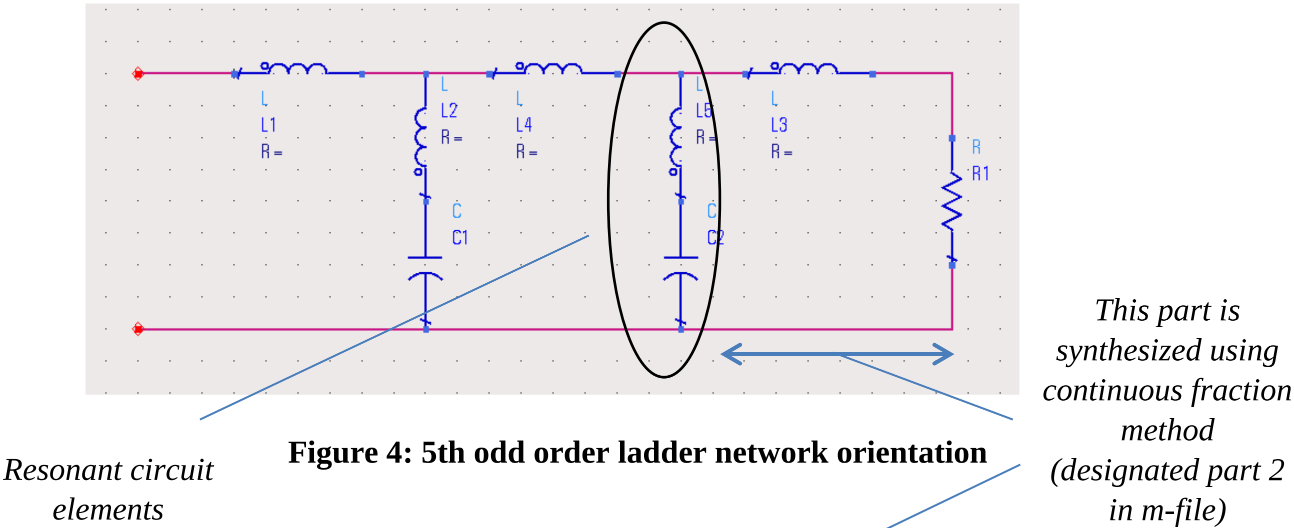


Figure 4: 5th odd order ladder network orientation

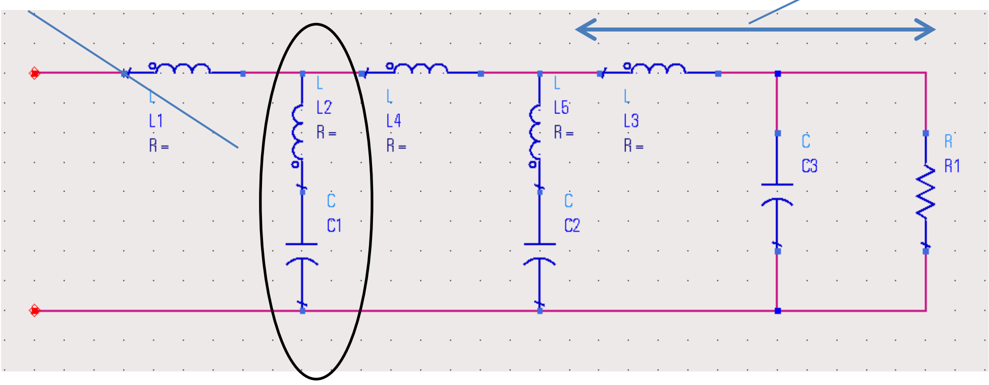
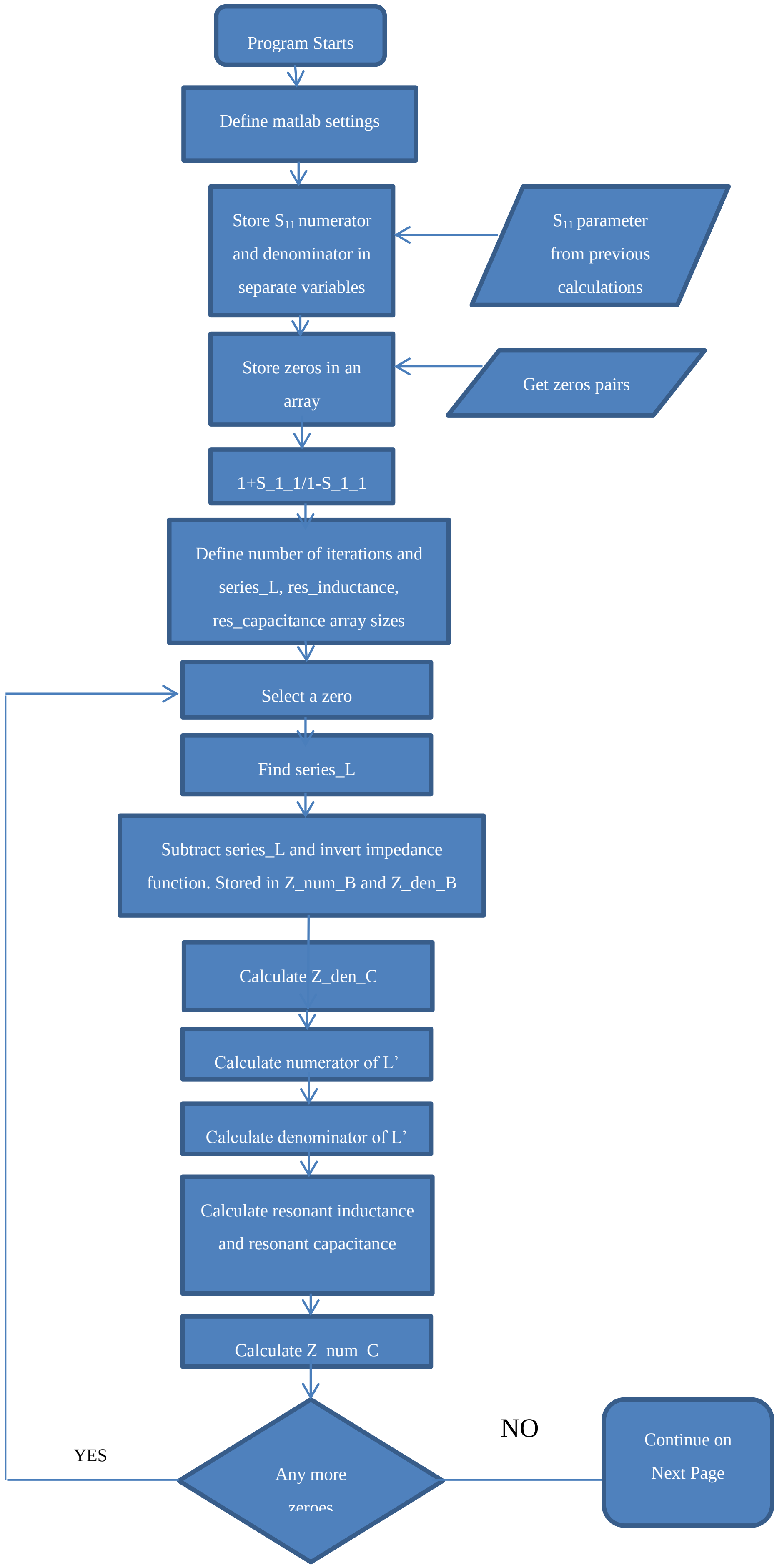
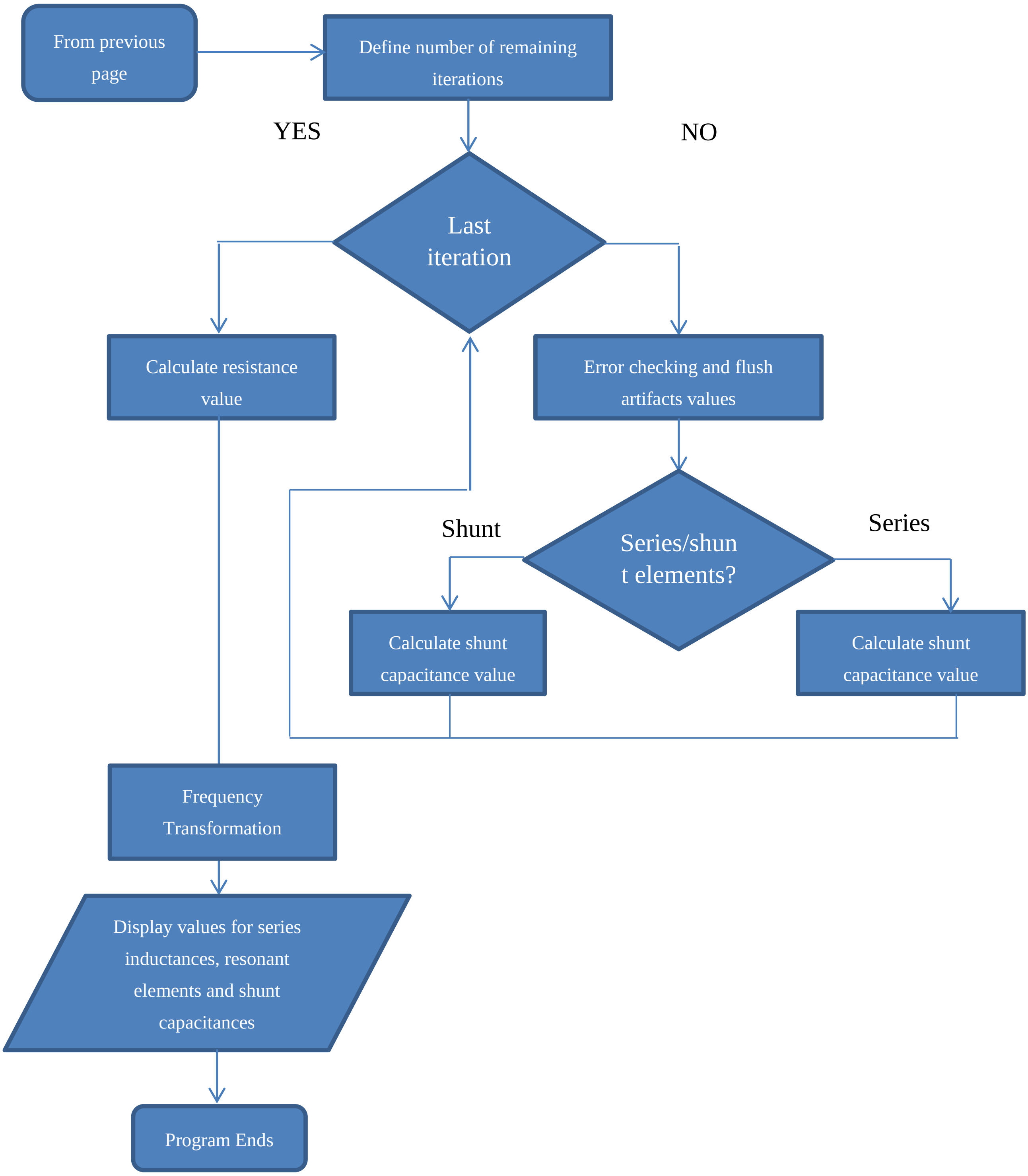


Figure 5: 6th even order ladder network orientation

4.2 Algorithm and Program Flow

In order to explain the flow of the m-file, a flow diagram is used to better describe the program flow.




From previous page
Define number of remaining iterations
YES
NO
Last iteration
Calculate resistance value
Error checking and flush artifacts values
Shunt
Series
Series/shunt elements?
Calculate shunt capacitance value
Calculate shunt capacitance value
Frequency Transformation
Display values for series inductances, resonant elements and shunt capacitances
Program Ends

4.3 The theory and working behind the program

The synthesis m-file can generally be split into 4 section.

1. Finding the series inductance (finite zeroes)
2. Finding the resonant elements (finite zeroes)
3. Finding the remaining elements (infinite zeroes)
4. Frequency Transformation and display

4.3.1 In-depth explanation to obtain series inductance

In general, with this pre-set ladder network structure the impedance function obtained from $Z = \frac{1+S_{11}}{1-S_{11}}$ can be written in a continuous fraction form.

$$Z_A = pL_1 + \cfrac{1}{\cfrac{pC_2}{1+(\frac{p}{w_1})^2} + \cfrac{1}{pL_3 + \cfrac{1}{\cfrac{pC_4}{1+(\frac{p}{w_2})^2} + \cfrac{1}{pL_5+1}}}}$$

To obtain the series inductance, we substitute the p =jw_1 where w_1 is the zero we will use in finding the series inductance as well as the resonant elements. Lets find L_1 in this case, which is the series inductance. By replacing p=jw_1, we notice that $(\frac{p}{w_1})^2 = -1$ and thus $\frac{pC_2}{1+(\frac{p}{w_1})^2}$ would become infinity. Further calculation would show that

$$Z_A = jw_1L_1$$

We will be able to obtain L_1 after some simple calculations. In a ladder network, when calculating a shunt element, the impedance function will be converted to its admittance form. Therefore after every series inductance calculation, Z_A must be inverted.

4.3.2 In-depth explanation on finding resonant elements

Assuming a 5th order circuit with 2 zero pairs:

$$Z_A = pL_1 + \cfrac{1}{\cfrac{pC_2}{1+(\frac{p}{w_1})^2} + \cfrac{1}{pL_3 + \cfrac{1}{\cfrac{pC_4}{1+(\frac{p}{w_2})^2} + \cfrac{1}{pL_5+1}}}}$$

The "pL" signifies the series inductance elements. The $\frac{pC}{1+(\frac{p}{w})^2}$ is used to extract the resonant elements. After some expansion, it is found out that $\frac{pC}{1+(\frac{p}{w})^2}$ can also be written as:

$$\frac{pw^2C}{w^2+p^2}$$

The relation between the 'w' and the resonant 'L ' and 'C' can be written as

$$w = \frac{1}{\sqrt{LC}}$$

After some expansion, we can safely say that $w^2C = \frac{1}{L} = L'$.

From the equation of Z_{in}, it is noted that the impedance function after the cancellation of the 1st element (series inductance) is

$$Z_B = \cfrac{1}{\cfrac{pC_2}{1+(\frac{p}{w_1})^2} + \cfrac{1}{pL_3 + \cfrac{1}{\cfrac{pC_4}{1+(\frac{p}{w_2})^2} + \cfrac{1}{pL_5+1}}}}$$

To obtain Z_B, it can be written as the subtraction of the series inductance before inverting the whole function:

$$Z_B = \frac{1}{Z_A - pL_1}$$

And since Z_B is obtained after the series inductance the extracted, Z_C can be obtained from the extraction of $\frac{pw^2C}{w^2+p^2}$ which is the resonant element obtained from Z_B.

$$Z_B = \frac{pL'}{w^2 + p^2} + Z_C$$

Lets assume Z_B as having a 6th order numerator and a 7th order denominator. It should have a form of

$$Z_B(p) = \frac{a_6p^6 + a_5p^5 + a_4p^4 + a_3p^3 + a_2p^2 + a_1p + a_0}{b_7p^7 + b_6p^6 + b_5p^5 + b_4p^4 + b_3p^3 + b_2p^2 + b_1p + b_0}$$

Separating out the resonant elements with partial fraction will make Z_B to be in this form:

$$Z_B(p) = \frac{pL'}{w^2 + p^2} + \frac{a'_4p^4 + a'_3p^3 + a'_2p^2 + a'_1p + a'_0}{b'_5p^5 + b'_4p^4 + b'_3p^3 + b'_2p^2 + b'_1p + b'_0}$$

The above equation is the form that is used to come out with the synthesis algorithm in the code.

There are few things that need to be found:

1. Numerator of Z_C
2. Denominator of Z_C
3. L'

The numerator of Z_B can be found by

$$(a'_4p^4 + a'_3p^3 + a'_2p^2 + a'_1p + a'_0)(w^2 + p^2)$$

$$+(b'_5p^5 + b'_4p^4 + b'_3p^3 + b'_2p^2 + b'_1p + b'_0)(pL')$$

If we split this into a few equations based on the number of coefficients, we will get:

$$a_6 = a'_4 + b'_5L' \quad \text{--- 1}$$

$$a_5 = a'_3 + b'_4L' \quad \text{--- 2}$$

$$a_4 = a'_2 + a'_4w^2 + b'_3L' \quad \text{--- 3}$$

$$a_3 = a'_1 + a'_3w^2 + b'_2L' \quad \text{--- 4}$$

$$a_2 = a'_0 + a'_2w^2 + b'_1L' \quad \text{--- 5}$$

$$a_1 = a'_1w^2 + b'_0L' \quad \text{--- 6}$$

$$a_0 = a'_0 w^2 \quad \text{--- 7}$$

For the denominator of Z_B , it can be written as

$$(b'_5 p^5 + b'_4 p^4 + b'_3 p^3 + b'_2 p^2 + b'_1 p + b'_0)(w^2 + p^2)$$

The equations that can be derived from this are:

$$b_7 = b'_5 \quad \text{--- 1}$$

$$b_6 = b'_4 \quad \text{--- 2}$$

$$b_5 = b'_3 + b'_5 w^2 \quad \text{--- 3}$$

$$b_4 = b'_2 + b'_4 w^2 \quad \text{--- 4}$$

$$b_3 = b'_1 + b'_3 w^2 \quad \text{--- 5}$$

$$b_2 = b'_0 + b'_2 w^2 \quad \text{--- 6}$$

$$b_1 = b'_1 w^2 \quad \text{--- 7}$$

$$b_0 = b'_0 w^2 \quad \text{--- 8}$$

From observation, it is noticeable that the denominator's values can be easily calculated from calculating (N-2) equations where N is the number of coefficients available in $Z_{B,}$ starting from the 1st equation. However, obtaining the numerator value might be complex. Therefore we resort to finding the unknown of L' before calculating the numerator. Lets do some modification to the simultaneous equations of the numerators:

$$a'_4 = a_6 - b'_5 L' \quad \text{--- 1}$$

$$a'_3 = a_5 - b'_4 L' \quad \text{--- 2}$$

$$a'_2 = a_4 - a'_4 w^2 - b'_3 L' \quad \text{--- 3}$$

$$a_3 = a'_1 + a'_3 w^2 + b'_2 L' \quad \text{--- 4}$$

$$a_2 = a'_0 + a'_2 w^2 + b'_1 L' \quad \text{--- 5}$$

$$a_1 = a'_1 w^2 + b'_0 L' \quad \text{--- 6}$$

$$a_0 = a'_0 w^2 \quad \text{--- 7}$$

Solving the simultaneous equations (using the odd numbered equations) with respect to L' will yield

$$L' = \frac{Z_N - Z_5 w^2 + Z_3 w^4 - Z_1 w^6}{-b'_5 w^6 + b'_3 w^4 - b'_1 w^2}$$

Where N is the number of coefficients in the numerator of Z_B. Interestingly, 8^{th} order derivation is found to be

$$L' = \frac{Z_N - Z_7 w^2 + Z_5 w^4 - Z_3 w^6 + Z_1 w^8}{b'_7 w^8 - b'_5 w^6 + b'_3 w^4 - b'_1 w^2}$$

While 4^{th} order produces

$$L' = \frac{Z_N - Z_3 w^2 + Z_1 w^4}{b'_3 w^4 - b'_1 w^2}$$

Therefore, from these observations, we can derive and say that

$$L' = \frac{Z_N - Z_{N-1} w^2 + Z_{N-3} w^4 - Z_{N-5} w^6 + \cdots +/- Z_1 w^N}{-b'_{N-1} w^N + b'_{N-3} w^{N-2} - b'_{N-5} w^{N-4} + \cdots - b'_1 w^2}$$

Where N is the number of coefficients in numerator of Z_B. Finding the L' (note from the flow of program that L' numerator and denominator are found in separate computation for accuracy purposes) will allow us to be able to calculate both of the values of the resonant elements. Besides, it will also enable us to solve for a'. Finding the b' and a' allows us to construct the numerator for Z_C which is then used to find the next series inductance using another zero and so forth.

Take note that when calculating the next series inductance, the remaining impedance function (Z_C) must be inverted.

4.3.3 Continuous fraction to find remaining zeroes

In this part, the highest order numerator's coefficient is used to divide the highest order denominator's coefficient. The value would either be series inductance or shunt capacitance, depending on whether the impedance is used or the admittance is used. The rule is that the numerator must always be higher than the denominator. This is called the continuous fraction method where the p -> ∞. By assuming the limit of p as infinity, generally we would cancel the all the lower order elements, leaving only the highest order. The operation would be:

$$\left. \frac{Z(p)}{p} \right|_{p \to \infty}$$

During experimenting with test inputs, it is found out that due to the precision of MATLAB, some artefact values might exist after the subtraction of the highest order element.

```
Z_den =

                    0   0.000000006989512   0.017193778649458   0.016573473765039   0.011194373148765


Z_den =

                    0  -0.000000004543050   0.011613350616491   0.011194373168849


Z_den =

                    0  -0.000000001086509   0.011194373148765
```

Figure 6: artefact values in calculation

As seen in figure 6 (each line indicates after a cancellation of an element from the function), we can observe that there exist some remaining artefact values that are supposed to be cancelled out. This may jeopardize the flow of the program due to the program taking the 1st element in the numerator array to divide by the 1st element in the denominator array. Due to this problem, the division might be wrong and produce value at infinity.

Therefore, an error checking sequence is derived to counter this problem. Below is the code snippet of the sequence:

```
%void invalid/artifacts array values to prevent failure in
calculating the elements
        counter = 1;
        state = 1;

        while (state == 1)
            if ( Z_den(counter) < 0.00001)
                Z_den(counter) = [];
            else
                %no more errors
                state = 0;
            end
        end
```

This method would take the array and check every element against a condition which in this case is a value that is lesser than 0.00001. If the condition is fulfilled, that element is voided out and thus ensuring the 1st element in the array will always be of the expected order.

The current synthesis m-file would be able to robustly perform pole-zero extraction from advanced filter models with order as high as 20 degree. This program also is flexible enough to run perfectly with either maximum or minimum user inputs on the positions on zeroes in ladder network synthesis. However, the current progress is only capable of computing for low-pass filters. Frequency transformation rules would need to be stated in this program for it to be able to compute for high-pass, bandpass or band-stop filters.

4.3.4 Frequency Transformation and display

After the elements values are extracted, it is noted that they are normalized to 1 in the lowpass prototype. Therefore, frequency shifting and transformation would need to be done. The frequency transformation as stated in section 2.2 is applied.

Below are some of the results tested with their graph responses plotted using ADS using the values obtained from the algorithm:

Even Order Lowpass Prototype

```
Choose which prototype (LP/HP/BP/BS): lp
Enter the cut-off frequency: 1e9
**************elements extracted from finite zeros**************
the series inductances are:   1.0e-009 *

   0.053213662097620   0.188272730970069   0.206234775868585

the resonant inductances are:   1.0e-009 *

   0.195431921392044   0.092654335742059   0.093605654857239

the resonant capacitances are:   1.0e-009 *

   0.090008239933087   0.161766210457975   0.138064525933324

**************elements extracted from infinite zeros**************
the series inductances are:     9.435899053274721e-011

the load resistance value is:    0.999923299668322
```

Figure 7: Element values for 5th order normalized lowpass prototype

Below are the circuit constructed in ADS and the frequency response. As displayed, it is observed that the response based on the obtained values is valid.

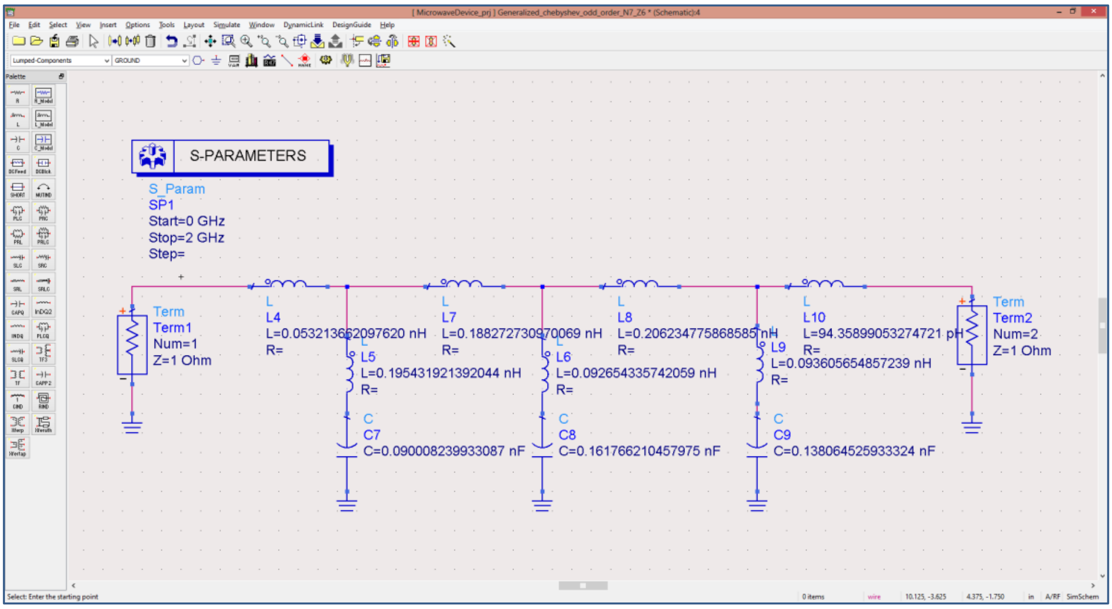


Figure 8: Circuit representation of even order lowpass prototype example

Figure 9: Frequency Response of even order lowpass prototype example

Odd Order Highpass Prototype

```
Choose which prototype (LP/HP/BP/BS): hp
Enter the cut-off frequency: 1e9
**************elements extracted from finite zeros**************
the series capacitance are:   1.0e-009 *

   0.476011139096500   0.134540439181346   0.122822621955500

the resonant capacitance are:   1.0e-009 *

   0.129611865503645   0.273384895673978   0.270606470829315

the resonant inductance are:   1.0e-009 *

   0.281421966804543   0.156585827404079   0.183467083520189

**************elements extracted from infinite zeros**************
the series capacitance are:    2.684460247780373e-010

the load resistance value is:   0.999923299668322
```

Figure 10: Element values for 5th order normalized highpass prototype

Below are the circuit constructed in ADS and the frequency response. As displayed, it is observed that the response based on the obtained values is valid.

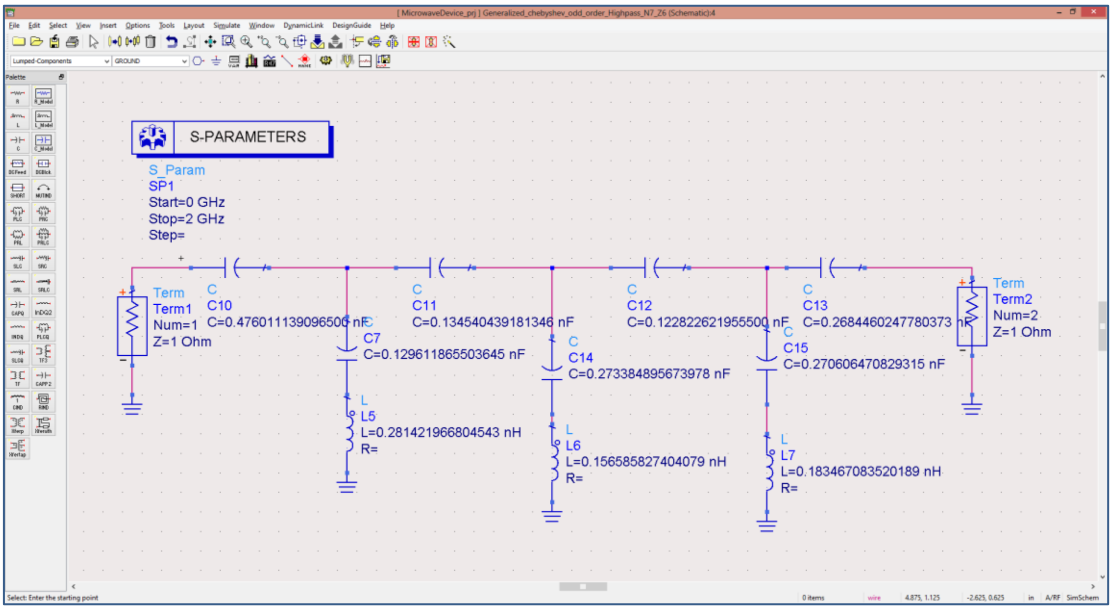


Figure 11: Circuit representation of odd order highpass prototype example

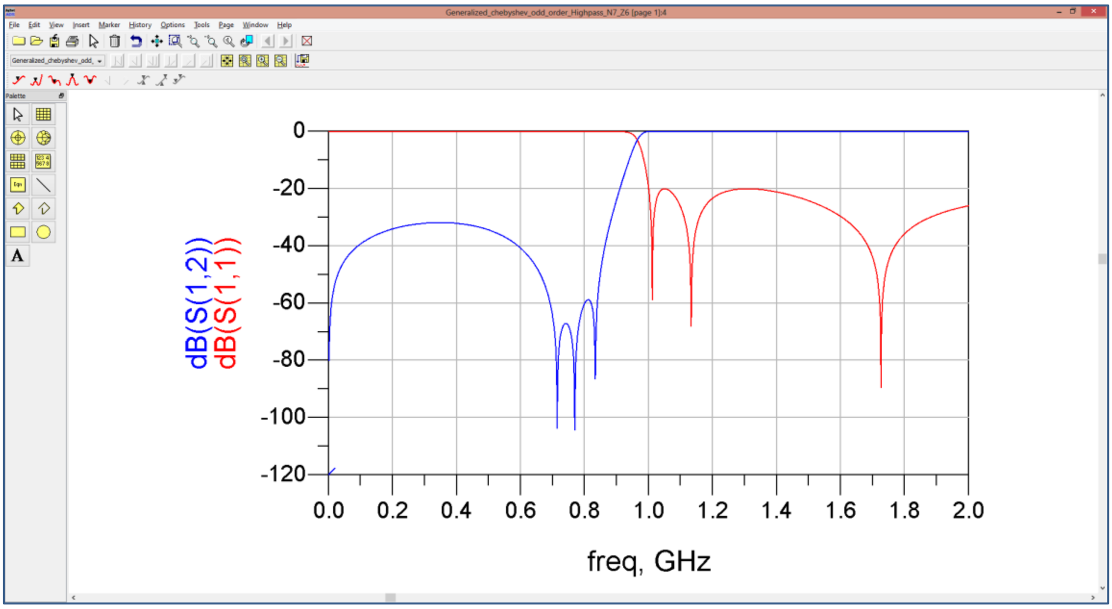


Figure 12: Frequency Response of odd order lowpass prototype example

CHAPTER 5
CONCLUSION AND RECOMMENDATIONS

The main objective of this project is to research on a direct and easy to compute algorithm in performing the synthesis process. The pole-zero extraction process had proven to be a major challenge in attempts to put the synthesis algorithm into high-level programming codes. By using MATLAB, this project had proven that it is possible to perform pole-extraction without the usage of complex mathematical operations. The direct and straight forward way of computing the elements for a ladder network circuits with arbitrary zeroes could suggest future expansion and usability in other programming language like C#.

Understanding the need for a low-cost user friendly program in the academic and industrial world, this project will provide a key component in the development process of coming out with a filter designing software. The simple theory that is explained in this project would be easy for engineers in the development process to easily utilize this as a synthesis component in their program. This synthesis module would be able to cover key advanced filter models such as Generalized Chebyshevs, Inverse Chebyshevs and Elliptic filters model which utilized either user input or pre-determined arbitrary zeroes.

Current progress in this project had revelaed that bandstop prototype would required a slightly altered general formula. Expansion into a more concrete and generalized frequency transformation can be done. Programming in other high-level language is also a recommendation to find out the usability of this algorithm produced.

CHAPTER 6
REFERENCE

[1] Hunter, I, *Theory and Design of Microwave Filters*. United Kingdom:IET, 2001.

[2] Radhakrishnan, V.; Ravikumar, S., "Quick Fil: A Microwave Filter Design Tool Using C#.Net for Teaching Purposes," *Computer Modelling and Simulation (UKSim), 2013 UKSim 15th International Conference on* , vol., no., pp.380,384, 10-12 April 2013

[3] Sedra, A.S. and Smith, K. C., *Microelectronic Circuits Theory and Applications.* New Delhi: Oxford University Press, 2004, pp. 933.

[4] 101 Digital Signal Processing, 101science.com [online] 1998, http://101science.com/dsp.htm (Accessed: 20 November 2000).

[5] Chen, W.K., *The Circuits and Filters Handbook, Second Edition.* Washington, DC: CRC Press, 2002, pp. 1247.

[6] Djengomemgoto, G. "Development of Graphical User Interface for Microwave Filter Design," B.C.(Hons) dissertation, Dept. Elect. Eng., UTP., Perak, Malaysia, 2013.

[7] Hunter, I, *Theory and Design of Microwave Filters*. United Kingdom:IET, 2001, pp 101-125.

[8] Koller, R.D.; Wilamowski, B.M., "A ladder prototype synthesis algorithm," Circuits and Systems, 1992., Proceedings of the 35th Midwest Symposium on , vol., no., pp.730,732 vol.1, 9-12 Aug 1992

[9] Wilamowski, B.M., Gottiparthy, R. "Active and Passive Filter Synthesis using MATLAB*", Vol. 21, No. 4, pp. 561-571, 2005

[10] Koller, R.D.; Wilamowski B.M., "Simulation of Analog Filters Using Ladder Prototypes" Modeling and Simulation., Proceedings of the 23rd Annual Pittsburgh Conference, 30 April – 1 May, 1992

[11] Wood, R.A.; Jones, N.B., "Generalised Chebyshev lowpass filters," Electronics Letters , vol.4, no.8, pp.158,159, April 19 1968

[12] A Su, K.L., *Analog Filters,* 2nd. Ed.,Massachusetts: Kluwer Academic Publishers, 2002, pp. 154-158.