Development of Spread sheet Based Simulation Model for Failure Prediction of Repairable System

By

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13917

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CERTIFICATION OF APPROVAL

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A project dissertation submitted to the
Mechanical Engineering Programme
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in partial fulfilment of the requirement for the
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(MECHANICAL)

Approved by,

____
(Dr. Masdi Bin Muhammad)

UNIVERSITI TEKNOLOGI PETRONAS
TRONOH, PERAK
May 2014

CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

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(SURAYA NABILAH BINTI ZAINI)

ABSTRACT

A repairable system is usually defined as a system that able to return to its function after each failure. Optimal maintenance strategies necessitate the estimation of the reliability of complex repairable systems accurately and numerous models and methods have been developed for system reliability prediction.

Thus, this project shall come out with an Excel spread sheet that used Monte Carlo method of using random number to solve for the number of failure and time to failure of a repairable system. Where, the developed spread sheet shall only focuses on the four types of main distributions which are Normal distribution, Lognormal distribution, Weibull distribution, and exponential distribution. Besides that, the developed spread sheet is developed with aim of improving the accuracy and applicability prediction of repairable system, the Monte Carlo solution developed shall later be compared with the evaluation made by the existing reliability software.

ACKNOWLEDGEMENTS

"In the name of Allah, the most Beneficient, the Most Merciful"

Alhamdullilah. All praises are to Allah who has made this work possible.

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CHAPTER 1

INTRODUCTION

1. PROJECT BACKGROUND

In daily life, risk analysis is considered as part of every decision being made and due to it, people always faced with uncertainty, ambiguity and variability in financial, project management, cost and other forecasting models. With limited access to information, it is impossible to accurately predict the future. Therefore, a computerize mathematical technique, Monte Carlo become handy as it allow people to account for risk in quantitative analysis by allowing people all possible outcome and access the impact risk; thus, allowing better decision making under uncertainty could be made for repairable system.

This paper shall develop an Excel spread sheet with Monte Carlo solution that able to evaluate and solve the reliability of repairable system by focusing of four types of main distributions which are Normal distribution, Lognormal distribution, Weibull distribution and exponential distribution. With the aim of improving the accuracy and applicability prediction of repairable system, the Monte Carlo solution developed shall later be compare with the evaluation made by the existing reliability software.

2. PROBLEM STATEMENT

Previously, estimating the uncertainties in the simulations was done on statistical sampling and deterministic problem methods. Then research was done and reliability software was developed to make things easier. However, the existing reliability software could cost quite a fortune to the user and the reliability solution done could not be simply share with others that do not have the specific reliability software. Besides, the probabilistic models made were done in exponential manner where it only manages to predict the average of the repairable system to fail and no practical and accurate approach exists to address the probability of repairable system without getting any of the software.

While on the other hand, Monte Carlo solution shall come out with a range of possible values instead of a single guess. When a model is based on ranges of estimates, the output of the model will also be a range and allow users to have more realistic probability outcomes. Thus, this project had come out to basically cater such problems where by the end of this project the Monte Carlo solution will be produce using an Excel spread sheet.

3. OBJECTIVES

The objectives to be obtain from this project is to develop an Excel spread sheet based on Monte Carlo model for estimating number of failure of repairable system depending on the failure distribution models. This is done through solving the renewal function for selected distributions using Monte Carlo method. Second objective is to make a comparison analysis of reliability analysis done by the developed Excel with Monte Carlo method and the one done using reliability software; Weibull ++.

4. SCOPE OF STUDY

Here, reliability analysis on repairable system shall be done by using the Monte Carlo method. Therefore, one of the scope in this project is to develop and use an Excel spread sheet where the calculation of the reliability analysis prepared in it as the repairable system shall be evaluate and remodel in graphical manner. Furthermore, this project shall also focus on the four types of distributions to be used in the Monte Carlo solution.

CHAPTER 2

LITERITURE REVIEW

1. RELIABILITY

Reliability definition is agreed by many scholars as a measure of probability of success where it conveys the successful performance and absence of failure within a certain period of time while at the same time operating under Normal or as stated environmental condition (Andrew & Moss, 2002) (Blischke, 2000). Furthermore, the probability could be divided into qualitative and quantitative measures (Birolini, 2004). In general, the reliability of a system with constant rate of failure can be represented in simple expression of $R(t) = e^{-\lambda t}$ (Andrew & Moss, 2002).Where the *R(t)* is the probability of successful operation for a period of time and λ is the failure rate.

Reliability and risk assessments method have been developed years ago from variety of different initiatives. It is stated that initially the assessment of reliability is meant for system of components that assumed to be in steady state or useful life phase only (Andrew & Moss, 2002). However, the reliability modelling deals with interdisciplinary where it shall use probability and statistic as it deals with model building in order to obtain a solution to problem in estimating or optimizing the survival performance of unreliable system, (Blischke, 2000). In order to reduce time spend extendedly on probability analysis, efficient sampling routing must be use. This is because it is the nature of the reliability probability where the occurrence of failure event is rare.

This project was meant for repairable system, the analysis of the repairable system should consider the effect of successive repair action. Therefore, the Monte Carlo solution shall be developed to solve the renewal function since it does not have trend in the system data and shall be treated good-as-new (Wang, 2005). They also explained that times between failures for renewal function can be distributes according to any lifetime.

2. MONTE CARLO SOLUTION

A repairable system is defined as a system that able to return to its function after failure regardless is returning to its original state, “As Good As New” or even worse than before, “As Bad As Old” - ABAO. (Basile. O., 2004) It is also mentioned that treating a repairable system is suggested to count the event which include the failure and its repair. (Basile. O., 2004) Basile had developed a Monte Carlo simulation for both repairable and no repairable system and he aims to apply general model to Weibull Distribution a renewal process ABAO to correspond to non-homogeneous Poisson process. (Basile. O., 2004) as only a very small fraction of a system part is plausible to be assume that the system’s reliability once repair is was immediately the same before failure.

Monte Carlo been defined as a computerized mathematical simulation that working in repetitive manner by duplicating the behaviour of the system and study the interaction and among its components. The output this kind of solution usually presented in term of selected measure of system performance (Andrew & Moss, 2002). Although there are several specific and more intensive software are available for analysis and usually assumed preferable to other users. However, another author stated otherwise as which aver analytical methods that able to provide satisfactory answers are more preferable (Birolini, 2004). For this case, it is the Monte Carlo solution as it is known for it able to provide accurate and more general solution besides it is way cheaper than any other software (Birolini, 2004). Furthermore, it is also been supported that Monte Carlo also able to be used in for systems that require more complex model and can’t be addressed by analytical method (Ebeling & Munson, 1997). There are several types of distribution associates with Monte Carlo simulation and only four out many will be focus on, Normal, Lognormal, exponential and Weibull distribution.

Normal distribution is a commonly use probability distribution where all the population of normally distributed are within the standard deviation of the mean and its density of function also symmetric to the mean (Andrew & Moss, 2002). On the other hand, Lognormal distributions allow having a negative lifetime or repair time with condition that the probability is a non-zero (Dubi, 2000). Moreover, the research had also stated that the Normal distribution is used for log of lifetime or repair time (Andrew & Moss, 2002).

An exponential distribution can be obtained with assumption that the past has no effect on the future as it is independent to the time (Dubi, 2000). However, it does not differ much from the Weibull distribution as random samples could also be obtain in the same manner of the exponential distribution (Andrew & Moss, 2002). Regardless, Weibull distribution able to make it so versatile as it has no fixed characteristic shape (Andrew & Moss, 2002).

3. IMPLEMENTATION OF MONTE CARLO

There had been several research and thesis papers that had shown great deal of using Monte Carlo solution for different proposes. However, Monte Carlo solution had been clearly use whenever there is need of generating numerous random numbers for its solution. This is because Monte Carlo solution able to provide an Excellent method for evaluating estimators and goodness-of-fit statistics under a variety of conditions, including sample size, non-normality, dichotomous or ordinal variables, model complexity, and model misspecification. (al, 2001).

An advance Monte Carlo Solution namely Subset Solution had been used by expressing the functional failure probability as a product of conditional probabilities of some chosen intermediate events which later a sequence of simulations were performed to tackle the problem of evaluating the probability of functional failure (Zio, 2012).

4. SENSITIVITY ANALYSIS

Sensitivity analysis is somewhat a step that shall assist the user to identify the importance and contribution of the parameters to be used in the Monte Carlo simulation and also helps in quantification of the importance of the uncertain parameters which affecting the performance of the system (Zio, 2012). As for example, the sensitivity analysis can also be seen as a statistical measure of the range of each investment asset's performance volatility where the higher the standard deviation of an asset the higher the volatility of its return later. (NaviPlan). Sensitivity analysis is actually aims instead at identifying the contribution to the output uncertainty of the various model parameters and hypotheses made earlier (E, 2009).

In addition, sensitivity analysis also requires multiple evaluations of different combinations of system input which most of the approaches is divided into two as it can be either local or global (Zio, 2012). For instance a sensitivity analysis had been done to a model of thermal-hydraulic passive system had come out with local approach where the sensitivity measure of the contribution of a generic uncertain input parameter to the uncertainty of the output is the *partial derivative* of the output with respect to the input parameter itself calculated around the nominal values of the input parameters (Zio, 2012).

It actually used to identify the critical parameters that able to produce the most variation in the output. On the other hand, the global approach is use to determine which of the uncertain input parameters shall influence the output most. One example of methods for global sensitivity analysis is including the variance-based technique where it had been done by nuclear passive system failure research by using the Monte Carlo solution method namely Subset simulation and Line sampling (Zio, 2012). In several areas of applications, this sensitivity analysis can be great aid in detecting where to reduce effectively the output uncertainty shown through the model generated (E, 2009).

CHAPTER 3

METHODOLOGY

1. RESEARCH METHODOLOGY

In order to complete this project, a thorough study shall be done on the Monte Carlo solution and the four types of distributions which later will be simplified and used when developing the Excel spread sheet. The understanding of analytical method is important as it will provide a conceptual basis to Monte Carlo theory and application. The developed Monte Carlo solution in the Excel spread sheet, shall solve renewal function of set of data obtained Then from the result obtain, a comparative analysis shall be done with the result of reliability analysis done with Weibull ++ software, in order to check the effectiveness of developed solution compared to existing software.

There are few tools to be used in order to develop a reliability solution using Monte Carlo method especially four type of distributions; Normal distribution, Lognormal distribution, exponential distribution and weibull distribution. Other than that, for this particular project, Macro Excel shall be used, where it is a collection of command and allowed to perform repetitive task. Typically, Monte Carlo calculates the model hundreds, thousands times in repetitive manners by using different random values for each time.

Later, after developing the Excel spread sheet Weibull ++ software will be used to compare both reliability analysis done by the two methods as a benchmarking for solutions develop so that it is acceptable to be used widely by others.

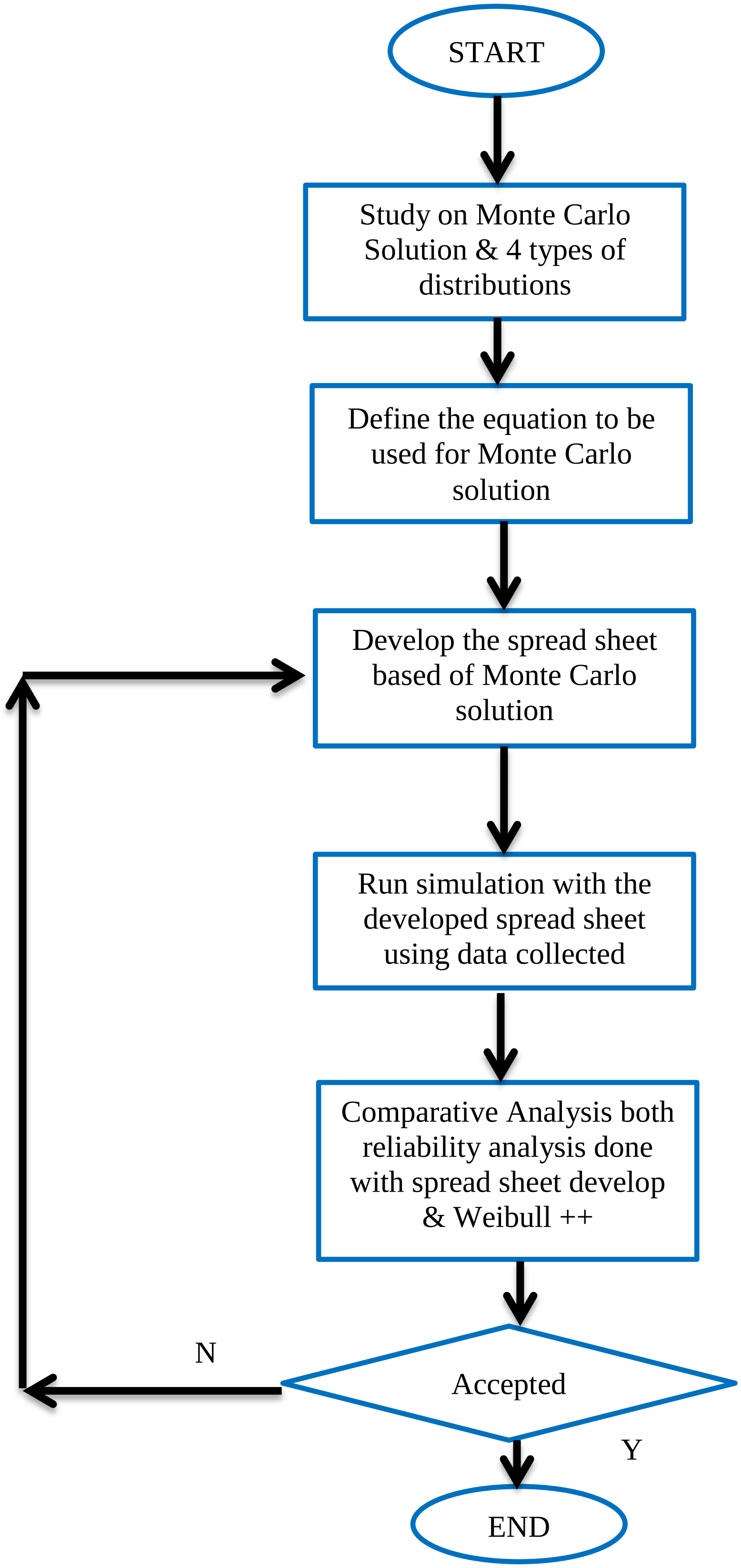


Figure 1: Research Methodology Flow diagram

2. KEY MILESTONES & GANTT CHART

All projects have a work plan that lays out the specific steps and actions that are necessary to complete the project. Project milestones shall help to keep track of the project progress and a guidance to identify which actions are required before the next step is possible. On the other hand, Gantt chart illustrates the start and finish dates of the terminal elements and summary elements of the project. Typically some tasks are dependent upon another.

Table 1: Milestones of 1st Semester

ACTIVITIES	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12	Week 13	Week 14	Week 15
FYP 1 :Project Framing (Jan –May)															
Briefing on Project Title	■														
Preliminary Research work		■	■	■	■										
Extended Proposal preparation					■	■									
Extended Proposal Submission							✸								
Proposal Defence								✸							
Data Collection									■	■					
Data Analysis										■	■	■			
Submission of Interim Report														✸	

Table 2: Milestones of 2nd Semester

ACTIVITIES	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12	Week 13	Week 14	Week 15
FYP 2 :Project Execution (May- Sept)															
Extensive Studies	■	■	■												
Project Execution				■	■	■	■	■	■	■	■				
Submission of Progress Report							★								
Preparation for Pre-SEDEX									■						
Pre-SEDEX Presentation										■					
Submission Draft of Dissertation												★			
Submission of Dissertation (Technical Paper)													★		
Viva														■	
Submission of Dissertation (Hard bound)															★
END															

VBS	Tasks	Task Lead	Start	End	Duration (Days)	% Complete	Working Days	Days Complete	Days Remaining
1	**FRAMING II**	[Name]	1/13/14	1/26/14	14	100%	10	14	0
1.1	FYP 1st Briefing		1/15/14	1/15/14	1	100%	1	1	0
1.2	Consolidation of FYP Titles		1/13/14	1/26/14	14	100%	10	14	0
1.2.1	Selection of project		1/13/14	1/19/14	7	100%	5	7	0
1.2.2	Approval of project		1/20/14	1/24/14	5	100%	5	5	0
1.3	Topic assignment to students		1/25/14	1/26/14	2	100%	0	2	0
2	**FRAMING II**	[Name]	1/27/14	2/23/14	28	97%	20	27	1
2.1	FYP 2nd Briefing		1/29/14	1/29/14	1	100%	1	1	0
2.2	Conduct preliminary research work		1/27/14	2/9/14	7	90%	10	6	1
2.3	Prepare extended proposal		2/10/14	2/16/14	7	100%	5	7	0
2.4	Submit extended proposal		2/17/14	2/23/14	7	100%	5	7	0
3	**FRAMING III**	[Name]	2/24/14	4/13/14	49	0%	35	0	49
3.1	Prepare proposal defence		2/24/14	3/2/14	7	0%	5	0	7
3.2	Present proposal defence		3/3/14	3/16/14	14	0%	10	0	14
3.3	Prepare extensive literiture review		3/31/14	4/6/14	7	0%	5	0	7
3.4	Prepare the project detail methodology		3/31/14	4/13/14	7	0%	10	0	7
3.5	Submit interim report		4/7/14	4/20/14	7	0%	10	0	7
4	**EXECUTION**	[Name]	5/11/14	9/8/14	121	0%	86	0	121
4.1	Extensive study on types of distributions		5/11/14	5/30/14	7	0%	15	0	7
4.2	Defining equation & Parameters		5/24/14	5/30/14	7	0%	5	0	7
4.3	Data gathering		6/2/14	6/10/14	8	0%	7	0	8
4.4	Developing excel spread sheet		6/10/14	6/24/14	15	0%	11	0	15
4.5	Comparative analysis		6/25/14	7/6/14	14	0%	8	0	14
5	**COMPILATION**	[Name]	7/6/14	9/8/14	65	0%	46	0	65
5.1	Prepare Technical		7/6/14	7/19/14		0%	10	0	0
5.2	Oral Presentation		7/22/14	7/22/14	1	0%	1	0	1
5.3	Submit Final report		9/1/14	9/8/14	7	0%	6	0	7

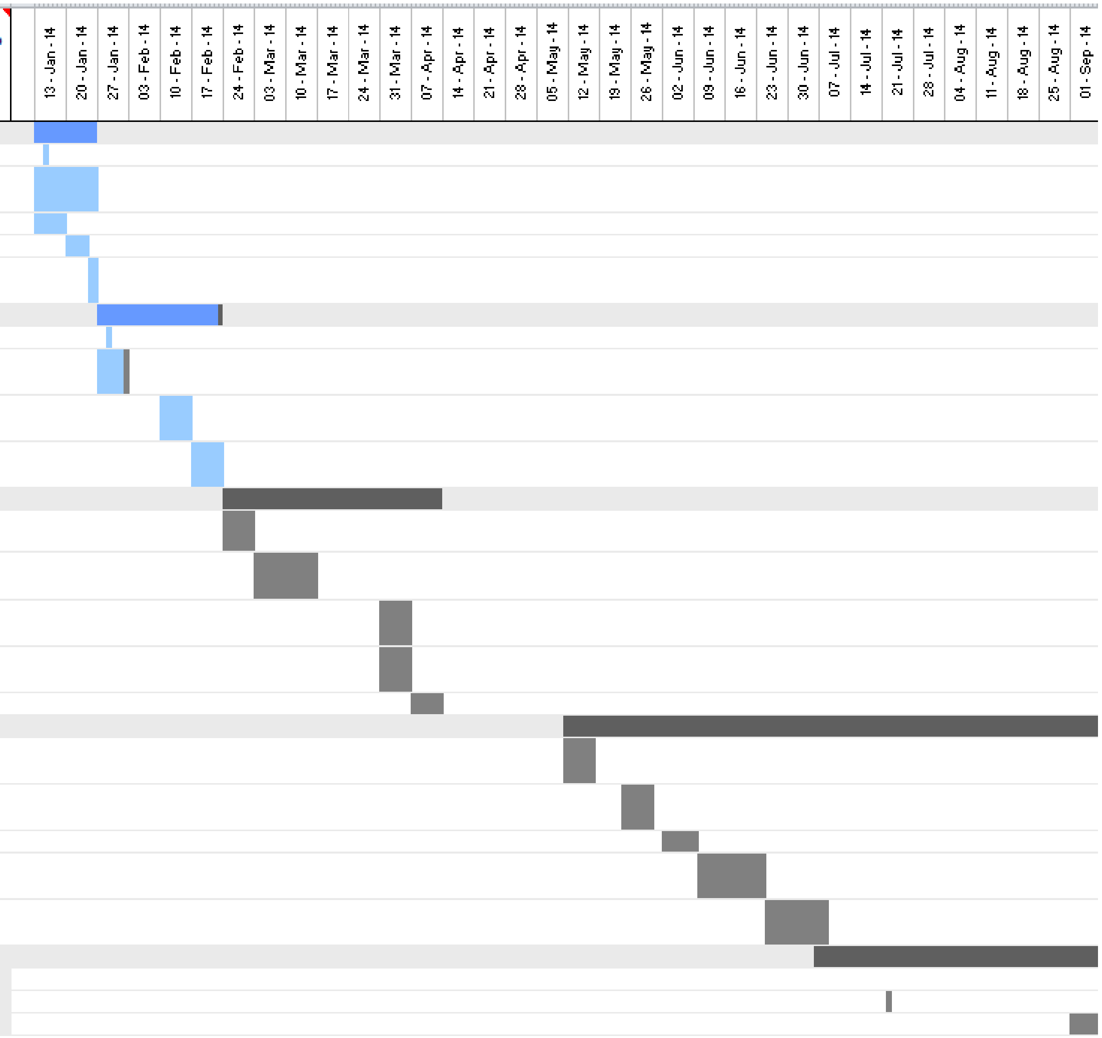


Figure 2: Gantt Chart of project

3. DESIGNING MONTE CARLO

From the research paper studied proposed that there had been several steps shall be abide in order to design and performing a Monte Carlo solution regardless what case study it shall be used for. It can be summarized into 9 main steps of which shown in the following process flow (al, 2001).

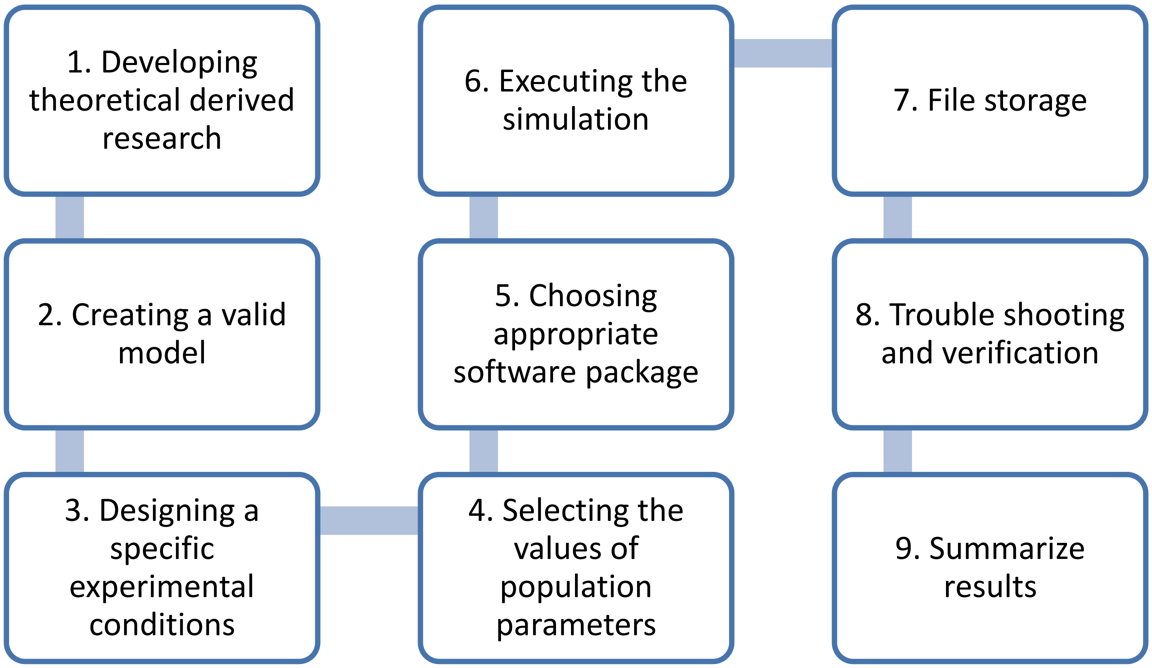


Figure 3: Designing Monte Carlo solution flow diagram (al, 2001)

A key of criticisms of Monte Carlo is lack of strong theory guiding the design analysis (al, 2001). In order to ensure the design analysis of Monte Carlo solution does not stray from its objective the researchers need to outline a clear research question at the beginning. For this project the main research interest is on the time to failure of the repairable system. So, for this project the theoretical population model will be using a Lifetime distribution models where a random sample size will be collected form this population to observe the collection of failure times from time =0 until time= infinity as a lifetime distribution model can be any ***probability density function*** (PDF) as the *f(t)* only give the range of time. While on the other hand, the corresponding ***Cumulative***

Distribution Function (CDF) where *F*(t) is a very useful function, as it gives the probability that a randomly selected unit will fail by time(t). (Monte Carlo Simulation) (Ebeling & Munson, 1997)

Second major criticism of Monte Carlo solution is lack of external validity (al, 2001). It suggested reviewing structural equation model applications across research though numerous journal papers. In this particular project objective, the structural equation to be used shall be the equation related to the four type distribution which aiming to identify the time to failure of repairable system. Another concern in designing process is regarding the size and the complexity of the model to be design. (al, 2001) (Monte Carlo Simulation)

As for choosing the software, the simulation of Monte Carlo solution shall only be design by using the Macro Excel and comparison analysis shall be perform with simulation produced using Weibull ++ software. A macro can be defined as the recording of a series of tasks. Where it is a form of automation of series of activities show a software program the steps to get something done, and the software will follow along. Using macros to automate the simple and repetitive activities could help user to save hours give if it had been used correctly. Marcos in Excel are written in Excel Visual Basic for Applications (VBA). This is a version of Visual Basic which is a prominent Microsoft programming language and had been developed specifically for use in Office-like applications.

4. GENERATING TIME TO FAILURE

The random number shall be generated using the Excel's pseudo random number generator [=RAND()] for different types of distribution. The methods are as follows:

As here in this project, we shall we using directly the $f(x)$ as the PDF and $F(x_o)$ as its CDF. Thus, we shall come out with;

$$F(x_o) = \int_0^{x_o} f(x)dx$$

Let $F(x)$ be a uniform random variable in the interval (0,1).

If $F(x)$ is Cumulative Density Function (CDF) for any type of distribution with an inverse $x = F(x)^{-1}$, then by substituting the random number to $F(x)$ and solving the value of x. Then, it shall produce random numbers of x having that PDF = $f(x)$.

Table 3: Excel expression for different distributions (O'Connor, 2012)

Distribution	PDF or f(t)	Excel Expression for time to failure
Exponential	$\lambda \exp(-\lambda t)$	-LN(1-RAND())/ λ λ = failure rate
Weibull	$\frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1} \exp\left\{-\left(\frac{t}{\alpha}\right)^{\beta}\right\}$	α {-LN(1- RAND()}^(1/ β) α = scale parameter, β = shape parameter
Normal	$\frac{1}{\sigma\sqrt{2\pi}}\exp\left\{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2\right\}$	NORMINV(RAND(), μ,σ) $\mu = mean$, $\sigma = variance$
Uniform	1/T	T x RAND() T = max time
Lognormal	$\frac{1}{\sigma t\sqrt{2\pi}}\exp\left\{-\frac{1}{2}\left(\frac{\ln t-\mu}{\sigma}\right)^2\right\}$	LOGINV(RAND(), μ,σ) $\mu = mean$, $\sigma = variance$

Table 3 is showing the summarized of the various Excel expression used in this project and its PDF equation used in developing the Excel spread sheet.

CHAPTER 4

RESULTS & DISCUSSION

As for the result of the project the four type distributions namely weibull distribution, Normal distribution, Lognormal distribution had been modelled using Monte Carlo method in the Excel spread sheet with different parameters each; as the parameters used was assumed by the used as the standard distribution parameters. In reality later, the parameter shall be acquired from the user upon the usage of the developed Excel spread sheet. Where the Excel itself contains a pseudo random number generator where the function is invoked using the Excel function =RAND() (Gedam & Beaudet, 2000) .

The simulation generated was compared with the simulation generated using the existing Weibull++ software. For each type of the distribution, the function equation had been used, derived and also arranged in unit of time as the main objective of this Monte Carlo simulation is to discover the prediction of time of failure of the repairable system. In order to simulate a distribution, random numbers need to be generated for a specified situation. Where, the random numbers shall be used to produce failure times from a specific distribution. To simulate any out of the four selected distribution, a special property of the probability density function (PDF) is being used.

Particularly in engineering applications the degree of repair on a current cycle and its duration are dependent on the history of the repair process. Thus, there shall be different level or type of repair that shall produce and require different type reliability distributions.

For all type of distributions, it has been defined that number of iterations of the Monte Carlo solution is 1000 which mean that the generating of random number that shall solve for time to failure will be done for thousand times and assuming that the system requires to simulate number of failure in 20 years of operation.

a. NORMAL DISTRIBUTION

Normal distribution also known as the bell curve distribution is the most commonly used for reliability and life data analysis.

$$F(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{t} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\, dx$$

Equation 1: Equation of function for Normal Distribution (Andrew & Moss, 2002)

From the equation of function for Normal distribution, the equation was arranged in order to obtain the time to failure of the system. Before obtaining the time to failure value that randomly generated, the parameters to be used in the equations are being determined.

There are two main parameters of Normal distribution, which the first is mean (μ) or the mean time to failure (MTTF). The mean can be assumed as value from negative infinity until positive infinity (-∞ to +∞.) Is also shall be known as location parameter of the *pdf*, as it locates the *pdf* along the abscissa. Second parameter is standard derivation (σ), a scale parameter of the Normal distribution *pdf.*

For the developed Excel spread sheet, the user is require to determine the mean and standard derivation of the Normal distribution upon generating time to failure of the distribution with the Monte Carlo simulation. Thus, without any specific data, it is assumed to use the standard Normal distribution's parameters value which is assuming that the mean (μ) value is 0 and the standard derivation (σ) value is 2. This is because; in this distribution assuming that the distribution shall have confidence interval of 95% where about 95% of the observations will fall within

2 standard deviations of the mean, which is the interval (-2,2) for the standard normal .It is mentioned in the “68-95-99.7 Rule” for Normal Distributions (Dubi, 2000).

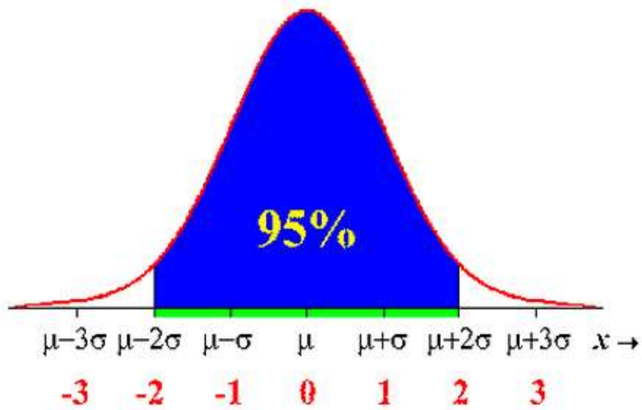


Figure 4: Normal Distribution curve with 95% confidence interval

The figure above shows; Normal distribution curve that approximately 95% of the observations fall within 2 standard deviations of the mean. The value of mean and standard derivation shall vary on how we want the data fall within our own confidence interval value. Although it may appear as if a Normal distribution does not include any values beyond a certain interval, the density is actually positive for all values.

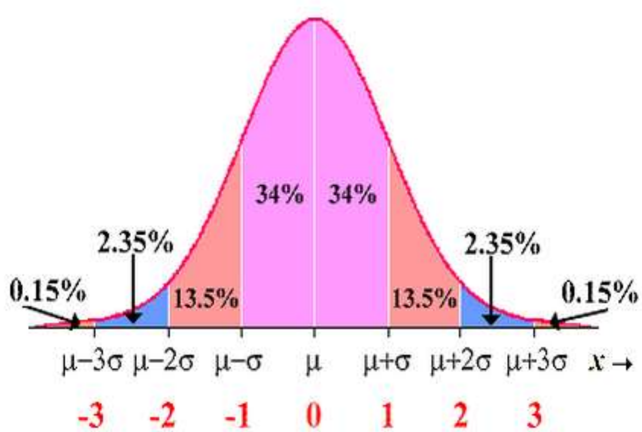


Figure 5: Normal Distributions 68-95-99.7 rule. (Dubi, 2000)

Then from the defined parameters, then both parameters will be call in an Excel function of Normal distributions; NORMINV(probability, μ, σ) where here the Monte Carlo solution will make generate random number for the probability. Therefore, the new Excel function used is NORMINV(RAND(), μ, σ) and it shall solve for random sample time to failure for this type distribution.

However, there are some who argue that the Normal distribution is inappropriate for modelling lifetime data because the left-hand limit of the distribution extends to negative infinity. Thus, it shall produce in negative values for results. Negative values for time are not accepted in most of the components of Weibull++, nor are they implemented. Certain components of the application reserve negative values for suspensions, or will not return negative results. Regardless, provided that the distribution in question has a relatively high mean and a relatively small standard deviation, the issue of negative failure times would not be a problem.

b. LOGNORMAL DISTRIBUTION

The Lognormal distribution is important in the description of natural phenomena as the natural growth rate is independent of size which is known as Gibrat's Law (Birolini, 2004) .A Lognormal distribution results if the variable is the product of a large number of independent, identically-distributed variables in the same way that a Normal distribution results if the variable is the sum of a large number of independent, identically-distributed variables. The random variable is restricted to positive values. Depending on the parameters, the distribution rapidly rises to its mode, and then declines slowly to become asymptotic to zero.

The Lognormal and the Normal distributions are closely related. When some random variable X has a Lognormal distribution, the variable Z for instance, has a Normal distribution when Z = ln(X). Alternatively, when the random variable Z has a standard Normal distribution, the random variable X has a Lognormal distribution when X = exp(Z).

$$f(x) = \frac{1}{\beta\sqrt{2\pi}} \exp\left[-\frac{(\ln x - \alpha)^2}{2\beta^2}\right]$$

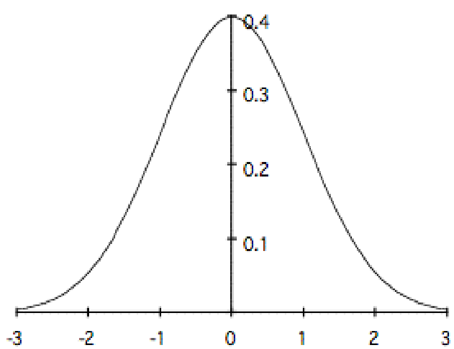
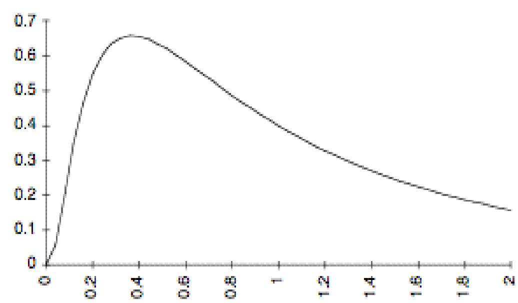
Equation 2: Equation of function for Lognormal Distribution (Andrew & Moss, 2002)

The Lognormal has two parameters that are the moments of the related Normal distribution. Where;

- $\alpha = \mu$ of Normal distribution
- $\beta = \sigma$ of Normal distribution

The figure below, the Normal distribution with mean 0 and standard deviation 1 is at the left. The corresponding Lognormal distribution is at the right.

Table 4: Relationship of Normal & Lognormal distribution (Birolini, 2004)

<table>
<tr><th>Random variable Z of Normal distribution</th><th>Random variable X of Lognormal distribution</th></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>$\mu_z = 0$ and $\sigma_z^2 = 1$</td><td>$\alpha = 0$, $\beta = 1$ where;
the $\mu_x = 1.6487$ and $\sigma_z^2 = 4.6708$</td></tr>
</table>

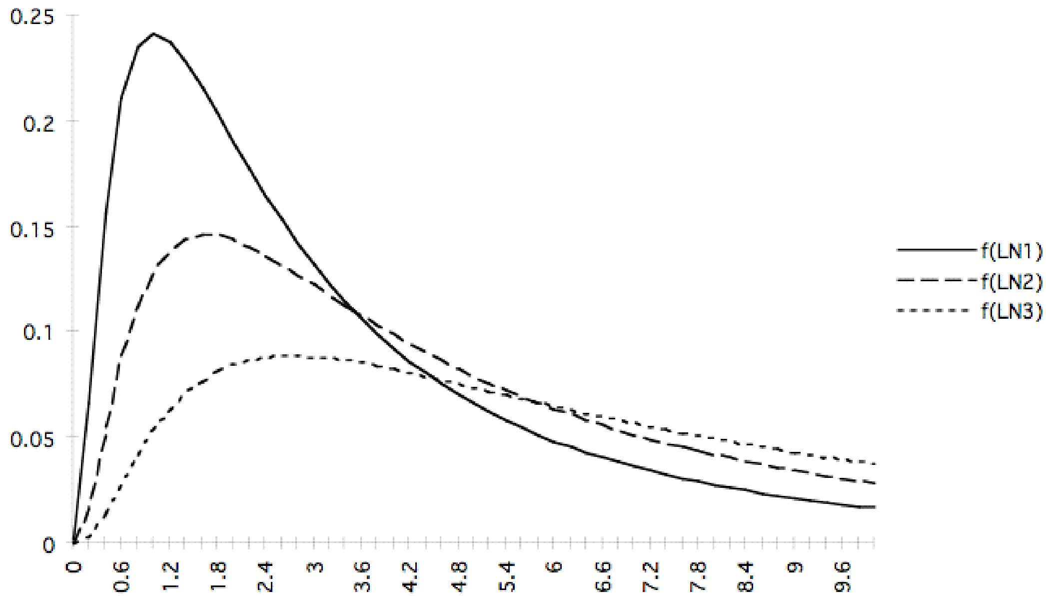


Figure 6: Lognormal distribution curve with different parameters value (Jensen, 2004)

In some cases, the mean and variance of the random variable X are the only parameters available to find the corresponding parameters of the distributions. Therefore, the statistical estimates of the mean and variance are used to compute the parameters of the distribution. Solving for the parameters in terms of the moments gives the expressions at the left.

c. WEIBULL DISTRIBUTION

Weibull distribution could be used for the repaired system that is not perfect which the "quality" of the repaired system is worse than the system's "quality" at the beginning of the previous cycle. (Ševcík, 2007) This condition could be considered as *degrading system*. However, since Weibull distribution could also suitable for general repair denotes a repair which lies between two well-known boundary cases, minimal repair and perfect repair. (Ševcík, 2007)

As for Weibull distribution the equation of function; CDF is dealing with two types of parameters as shown below.

$$F(t) = 1 - e^{\lambda t^{\beta}}$$

Equation 3:Equation of function for Weibull Distribution (Andrew & Moss, 2002)

F(t) is the fraction falling in the population by time t; as F(t) is also the probability of failure by time t. Then, the equation of function was arranged in manner where it is easier to calculate the time of failure using Weibull distribution function.

$$t = F^{-1} = \eta[-\ln(1 - F)]^{1/\beta}$$

$$t = [-\frac{\ln(1 - \xi)}{\lambda}]^{1/\beta}$$

For simulating Weibull Times to Failure (TTF) , the unit uniform random variable U had been substituted in the inverse expression to get:

$$t = F^{-1}U = \eta[-\ln(1 - F)]^{1/\beta}$$

Since both 1-U and U are a uniformly distributed variable in the interval of (0,1), the inverse expression is simplify as:

$$t = \eta\ [-\ln(1 - U)]^{\frac{1}{\beta}}$$

So, each U value shall produce a time to failure (TTF) from a Weibull distribution.

<u>Weibull Simulation in Excel</u>

The main parameters of this type of distribution are:

- β (beta)= slope
- η (eta)= the characteristic of life

The developed Excel spread sheet just requires the user to key in the value of the two parameters Eta and Beta. Once the user had entered the parameter, then the parameter will be used in the *pdf* for Weibull distribution of which as stated in Equation 2. Thus, as for Weibull distribution parameters, the value η is assigned to be 1 as it is assuming that 63.2% of the units will fail that time. Thus, it shall makes the β value is 2.0. (Andrew & Moss, 2002)

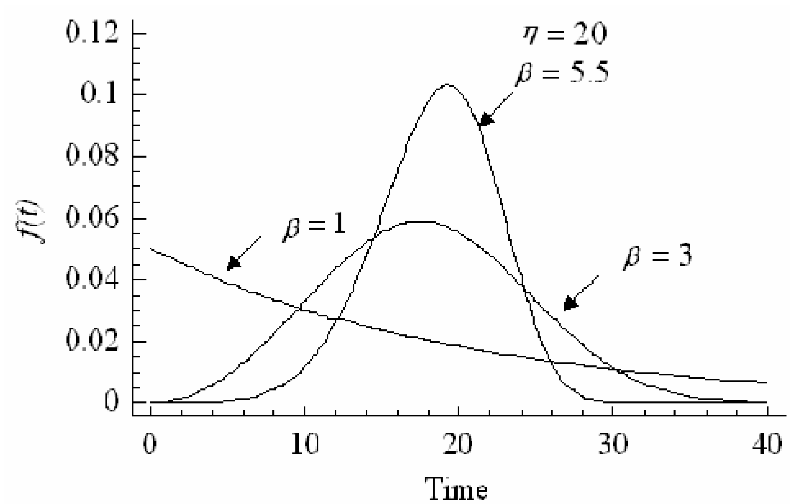


Figure 7: Example of Weibull Distribution curve (Barringer)

Throughout this Monte Carlo solution designing process, the sensitivity analysis design is also part of the design process. It had been studied on the purpose and method used by the previous researchers which had used the sensitivity analysis in their Monte Carlo simulation design.

Then, to investigate the suitability of the following simple procedure for estimating the parameters is used.

Then, the Excel spread sheet shall produce random samples for time to failure where at the end of table is a round up value of prediction where it shall state which year it will fail. The prediction of random number shall be done in repetitive manner by Monte Carlo Function as recorded earlier and number of iteration or repetition is depending on the user input. A given iteration of the subsystem is simulated by testing the state of the first component of the subsystem by generating a random number in the range [0,1].

From the generated data, the user can randomly produce a random set of data repeatedly with defined shortcut key earlier when recording the Monte Carlo solution in the Macros, The developed system actually checking if the first component does not fail, then the state of the next component is tested and this process is repeating including the series of subsystem. Then, if the subsystem fails the parallel component to it will also be tested until it also fails. If then, the system itself fails. After a number of these system simulations have been performed, the failure intensity of the system is calculated by dividing the number of observed failures by the total number of simulations.

d. EXPONENTIAL DISTRIBUTION

$$F(t) = 1 - e^{-\lambda t}$$

Equation 4: Equation of function for Exponential Distribution (Andrew & Moss, 2002)

The exponential distribution is a very commonly used distribution in reliability engineering. Due to its simplicity, it has been widely employed. It is a special case of the Weibull distribution where ß=1. The exponential distribution is used to describe units that have a constant failure rate λ.

The function of time of the exponential equation is shown as the following equation with ξ as the random numbers.

$$t = -\frac{\ln(1 - \xi)}{\lambda}$$

The general formula for the probability density function (pdf) of the exponential distribution is as below.

$$f(x) = \frac{1}{ß} e^{-(x-\mu)ß}$$

x ≥ μ : ß > 0

where μ is the location parameter and β is the scale parameter (the scale parameter is often referred to as λ = 1/β). The case where μ = 0 and β = 1 is called the standard exponential distribution. The equation for the standard exponential distribution is written below.

$$f(x) = e^{-x}$$

for x ≥ 0

The exponential distribution is shown as inversely proportional of the function equation to the time as shown in the graph below.

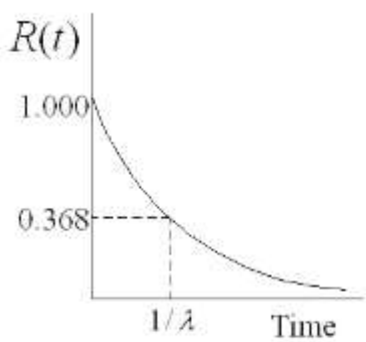


Figure 8: Example of Exponential Distribution curve

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CHAPTER 5

CONCLUSSION & RECOMMENDATION

1. CONCLUSSION

In conclusion, performing reliability analysis of different types of distribution using Excel spread sheet is possible with the suitable repair conditions and using the right equations. It is also part of reducing cost in any reliability study to be done and it would be user friendly as an Excel spread sheet is considered as general software that is used widely. However, in producing the Monte Carlo solution with four types of distribution; more studies and researches need to be done. The selection of which type of distribution to be used is based on what kind of repair needed or what level is the return state of the repaired system. The progress of the spread sheet development is quite time consuming as the validation of the spread sheet needs longer time than provided. A clearer instruction is also necessary in the Excel developed as to make it more user-friendly.

2. RECOMMENDATION

Some assumptions should be made regarding the data used, interface problems, dependence between elements, environmental influences and quality of manufacturers. Besides, operating conditions should be specified with care as well since it has important influences on the reliability. This project could also be improved later with cost and system effectiveness in considerations with the reliability. It should allow the end users to know the best possible usefulness to life-cycle cost ratio and at the same time knowing the ability of an item to meet service demand.

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APPENDIX

a. NORMAL DISTRIBUTION

	A	B	C	D
1	Failure Distribution	Parameters		
2	Normal	**μ (years)**	**σ**	
3		**0**	**2.0**	
4				
5	**Iteration**	1000		
6				
7				

Figure 9: Normal distribution_ defining parameters

Random Samples for Time to Failure			
Sample No	Time to Failure	System Age	Year of failure
1	1.12	1.12	2
2	1.60	2.71	3
3	0.88	3.59	4
4	4.71	8.31	9
5	1.16	9.46	10
6	2.22	11.68	12
7	0.44	12.12	13
8	2.68	14.80	15
9	1.70	16.50	17
10	0.97	17.46	18
11	1.94	19.41	20
12	3.15	22.56	23
13	0.90	23.46	24
14	2.63	26.09	27
15	1.42	27.51	28
16	1.87	29.38	30
17	2.31	31.68	32
18	0.29	31.97	32
19	0.01	31.98	32
20	1.33	33.31	34

RESULT	505	582	624	610	607	643	631	622	642	612	611	650	643	643	614	625	638	637	645	630

Simulated number of failure (in 20 years) **12.414**

No of repair recorded in each year

Years	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
# of repair	1	0	0	0	2	0	1	1	0	1	0	1	0	1	0	0	1	2	0	0

Simulation Run: No of repair recorded in each year

Iteration	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	2	1	0	0	1	3	0	1	1	0	0	1	0	1	0	0	1	1	0	1
2	0	1	1	1	0	0	1	0	2	0	1	0	1	1	1	0	0	0	1	0
3	1	0	0	0	0	1	0	0	2	0	0	1	1	0	1	0	0	3	0	0
4	0	0	0	1	0	0	1	0	1	1	0	1	0	0	1	1	0	1	0	0
5	0	0	1	0	1	2	0	0	1	0	0	1	0	0	1	0	1	0	3	1
6	1	0	0	2	1	1	3	0	1	1	1	0	0	0	0	1	1	0	0	0
7	0	0	0	2	0	2	2	0	2	1	1	0	1	1	1	1	0	2	4	0
8	0	1	1	2	0	0	0	2	4	0	0	0	0	0	2	0	1	0	0	0
9	1	0	1	0	0	0	0	1	1	0	1	1	1	1	0	0	1	0	2	1
10	1	1	0	0	1	2	2	0	0	0	3	0	1	2	0	1	0	1	1	1
11	2	0	1	0	1	0	1	1	0	2	0	1	0	1	1	1	0	1	0	0
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110	1	0	2	0	0	1	0	0	0	0	2	1	0	0	2	0	1	0	1	2
111	0	0	1	2	1	0	2	0	0	1	1	0	3	0	0	1	0	1	1	0
112	0	2	0	0	0	2	0	1	0	0	1	0	1	0	1	0	1	0	1	0
113	0	1	1	1	1	1	0	0	1	0	0	1	0	1	0	2	1	0	0	0
114	0	0	0	2	1	0	1	0	1	0	0	0	1	1	1	2	0	1	0	0
115	0	0	0	1	0	0	1	0	0	1	0	1	0	1	0	0	0	1	1	1

116	0	1	1	1	1	1	1	2	0	0	1	0	1	2	1	0	1	1	2	2
117	0	1	0	1	2	0	2	1	3	1	0	0	1	1	0	0	2	0	0	2
118	0	0	4	0	1	1	1	0	1	0	0	2	0	0	2	0	0	0	0	1
119	0	2	0	0	1	1	1	1	1	0	0	3	1	0	1	0	1	0	1	0
120	0	0	0	0	1	0	0	0	2	0	1	1	0	0	0	2	0	0	1	0
121	1	0	2	2	0	1	0	1	2	1	0	2	2	0	0	0	1	0	1	1
122	1	0	1	1	1	0	1	0	0	0	1	1	0	1	2	0	0	0	1	1
123	0	1	1	0	0	1	3	1	0	0	1	1	1	0	1	0	0	2	2	0
124	0	0	0	2	0	0	1	0	1	1	0	3	1	1	0	0	2	1	0	1
125	0	1	0	1	1	2	1	0	0	0	0	0	1	0	1	2	0	2	1	0
126	1	0	0	2	0	1	0	1	1	0	0	0	0	1	1	2	0	2	0	0
127	0	1	1	1	1	0	1	1	2	0	1	0	0	0	2	0	1	0	0	0
128	0	0	1	0	0	1	0	1	0	1	1	1	1	0	0	1	0	0	1	1
129	1	0	0	1	1	0	1	0	1	1	0	1	1	0	1	0	2	1	1	1
130	0	2	1	1	1	2	0	2	1	0	1	1	1	1	0	1	2	1	1	0
131	0	2	1	0	0	1	3	0	1	3	1	0	1	0	0	1	0	0	1	1
132	1	1	0	2	0	0	1	0	0	0	0	0	1	0	0	1	0	1	1	0
133	1	0	1	1	2	1	0	0	1	0	0	1	0	0	2	0	0	1	0	1
134	0	1	0	2	0	0	0	0	2	1	0	1	0	0	1	0	1	1	1	1
135	0	0	0	0	1	0	1	0	0	0	0	1	0	1	0	1	1	0	1	1
136	0	1	1	0	1	0	0	0	1	0	0	1	0	1	0	0	0	1	0	0
137	0	0	2	1	0	1	0	1	0	1	2	1	0	1	1	0	1	1	0	0
138	0	1	1	1	0	1	1	0	0	3	1	1	0	1	0	0	0	1	0	0
139	1	0	1	2	2	1	0	0	3	0	1	1	1	1	0	1	0	0	1	0
140	0	0	0	0	1	1	0	1	1	1	0	0	1	1	1	0	1	0	0	0
141	2	0	0	1	0	1	1	1	2	0	1	0	1	4	0	0	1	0	1	0
142	0	0	0	1	1	1	1	0	1	0	0	0	2	0	2	0	0	1	1	1
143	0	0	3	0	2	1	0	0	0	0	0	2	1	0	0	0	2	1	1	1
144	0	0	1	1	1	0	0	1	0	0	2	0	1	0	0	0	1	0	0	0
145	1	1	0	2	1	1	0	0	1	0	0	1	0	1	1	0	1	2	0	0
146	1	1	2	1	0	1	1	0	1	1	0	0	0	2	0	1	0	0	2	0
147	1	0	0	1	4	0	0	1	0	1	1	0	1	0	0	1	0	0	1	0
148	1	0	1	0	0	1	0	0	1	1	0	1	1	0	1	1	0	0	1	1
149	0	2	0	0	1	0	1	1	0	0	2	2	1	1	0	0	1	1	2	0
150	0	0	1	0	2	0	0	1	0	1	0	0	0	2	0	0	0	1	0	1
151	1	0	2	0	0	0	0	2	0	1	0	0	3	1	0	1	0	1	1	0
152	0	1	0	0	1	0	0	1	2	0	1	0	0	1	1	1	1	1	1	2
153	0	1	1	1	0	1	0	3	0	1	0	0	1	1	0	1	0	0	1	0
154	1	0	3	0	1	1	1	0	0	0	1	1	0	0	0	2	0	1	1	0
155	0	2	0	2	0	1	0	0	1	0	0	0	2	1	0	1	1	0	2	1
156	2	3	1	0	2	1	0	0	1	0	2	1	0	1	0	1	1	0	0	1
157	0	1	1	0	1	0	1	1	2	0	1	1	0	1	0	0	1	0	0	0

*The iteration was done for 1000 times for all of the distributions.

b. LOGNORMAL DISTRIBUTION

Failure Distribution	Parameters	
Normal	μ	σ
	0	2.0
Iteration	1000	

Figure 10: Lognormal distribution_ defining parameters

c. WEIBULL DISTRIBUTION

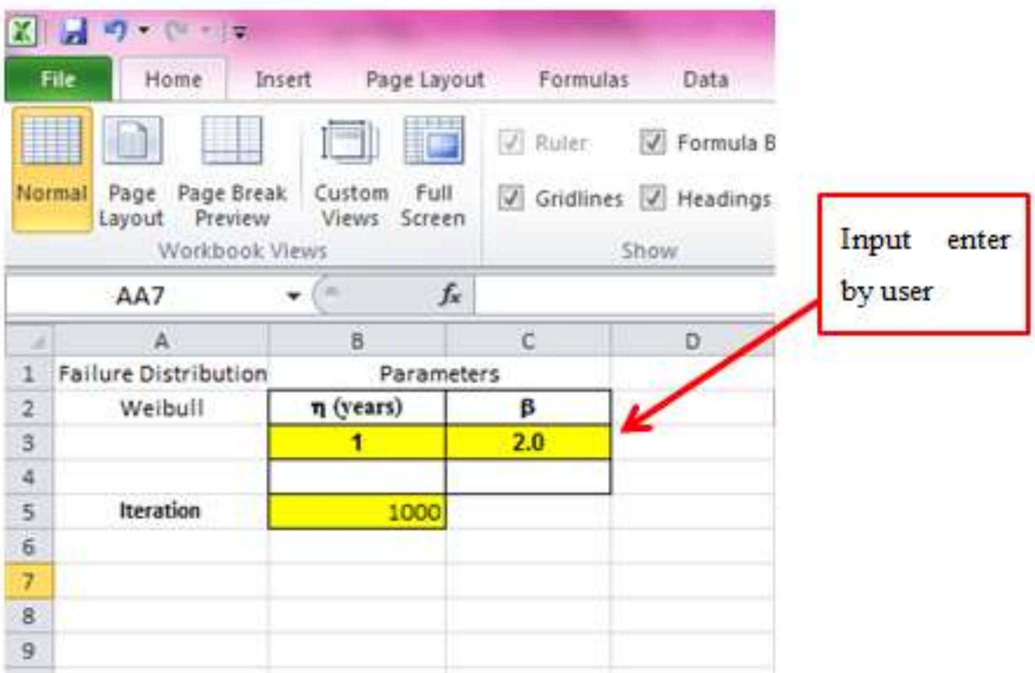

Figure 11: Weibull distribution_ defining parameters