Modelling of Pseudo Hydrostatic Force in Two – Phase Flow with Different Layers

by

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Dissertation submitted in partial fulfilment of
the requirements for the
Bachelor of Engineering (Hons)
(Mechanical Engineering)

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CERTIFICATION OF APPROVAL

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A project dissertation submitted to the
Mechanical Engineering Programme
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Approved by,

____

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January 2010

CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources of persons

IYLIA ELENA BINTI ABDUL JAMIL

ABSTRACT

In the study of solid-in-liquid flow, shear stress is important in determining the force that is acting on the pipe wall. In case of homogenous suspension solid-in-liquid flow, the properties can be considered as mixture properties with constant concentration profile across the flow area. In the moving bed of particles with variable concentration, the shear estimation is not directly predictable and there is no existing clear mathematical formula to achieve this objective. In the present work, the method of finding the force acted on the pipe wall by the particles in the layer, which is termed the dry force will be presented using a method called the "pseudo hydrostatic pressure" method. To attain the equation for the dry force, a mathematical approach is taken with the assumptions that the flow is a horizontal, two-phase pipe flow (solid-liquid), incompressible and it is at steady-state. For initial study, only Newtonian fluid is to be considered in the case. The two-layer approach is taken whereby the flow will consist of one upper suspended layer of particles in the fluid, and the bottom layer which is the moving bed of particles. Thus, the developed mathematical model can be applicable in solving for the shear force in horizontal two-phase flows.

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CHAPTER 1

INTRODUCTION

1.1 Background

Liquid-solid flow is classified as one of the multiphase flows. It consists of flows in which particles are carried by the liquid and are also referred to as slurry flows. Slurry flows covers wide applications from the transport of coals and oil to the flow of mud. The flow of particles in fluids has a wide application in industrial processes. An example is the efficient combustion of coal particles in a furnace depends on the interaction of particles with air.

These flows are classified as homogeneous, heterogeneous, moving bed, or stationary bed. In horizontal flows, the homogeneous layer is the one where particles are suspended by the turbulence of the fluid. Heterogeneous layer contain coarse particles that tend to settle at the bottom of the pipe. The moving bed regime occurs when the particles settle on the bottom of the pipe and move along as a bed. The liquid-solid flows are complex, and due to this, the suspended layer is usually treated as a single-phase fluid with modified properties which depends on the solids concentration. [1]

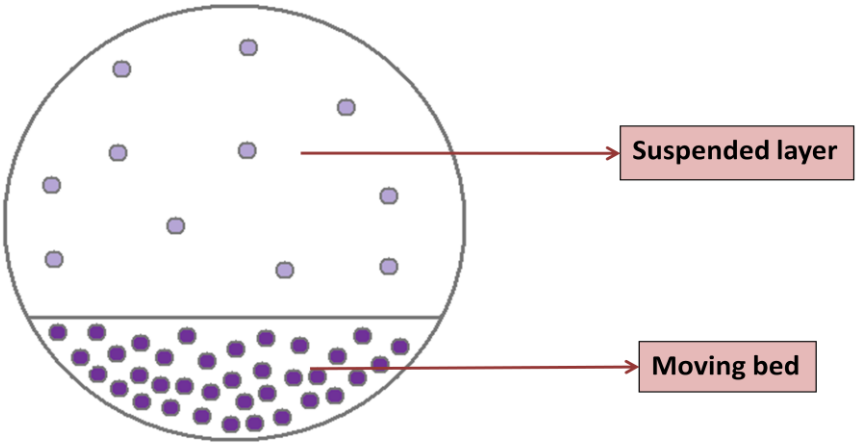


Figure 1 Cross sectional area of solid-liquid flow in circular pipe

The pseudo hydrostatic pressure is a method not unlike the hydrostatic pressure in stationary fluids. A clear understanding about the hydrostatic pressure is first required in this study.

A fluid is at an equilibrium state when the pressure on every side of a body of fluid is equal. At this state, the fluid is not moving, as all the shear stresses present are balanced by the normal pressure exerted by the fluid to its contact surface. Hydrostatic pressure is the pressure exerted by a fluid at equilibrium due to the force of gravity. Since pressure is defined as the force exerted on an area, and the only force acting on any such small cube of fluid is the weight of the fluid column above it, hydrostatic pressure can be calculated according to the following formula:

$$P = \rho g h$$

$\rho = density\ of\ fluid$

$g = gravitational\ force$

$h = height\ of\ water\ column\ above\ area\ of\ interest$

Because this is a study of flow in a pipe, it involves fluid at movement. To adopt the hydrostatic pressure concept here, the term ‘pseudo’ needs to be added as a prefix to the term hydrostatic. The word ‘pseudo’ describes something that is not genuine, but having the appearance of a certain characteristic. By ‘pseudo hydrostatic pressure’, it has the meaning of pressure of fluid that is assumed to be in stationary form, and is applied instead to a moving body of fluids.

1.2 Problem Statement

In the study of solid-in-liquid flow, there is no clear mathematical formula to determine the shear forces between the moving bed of particles and boundaries in the moving fluid. In the application of the transport of oil, this shear force is important in determining the force that is acting on the pipe wall by the moving bed, which contributes to determining the power required to pump the mixture of fluid and solid in the pipe to the surface or collection sump.

1.3 Objectives

The objectives of this project include:

- To establish a mathematical model to estimate the shear forces of the solid-liquid flow by applying the pseudo hydrostatic pressure method
- To solve and justify the model using real data

1.4 Scope of study

The scope of study includes extending the current search on the ways to determine the shear forces in solid-liquid flow in pipes. The search is intended to overcome the limitation and target to develop clear and general formula to be applied in such horizontal pipes. Also, the information gathered should result in a general mathematical formulation that can be used in any application involving the two-phase flow in pipes. Then the general formula is tested using field data, to justify its validity. Comparisons are made to any existing methods of calculating the shear stress using the same concept of pseudo hydrostatic pressure.

1.5 Significance of the work

The study on solid-liquid flow is important in its significant application in drilling oil from well in cleaning operations, and in the transport of sand in water in sedimentation. Creating a clear method to help in finding the required pumping power is very advantageous for the drilling process in the oil and gas industry, also in other applications. Apart from that, this study can be further used to provide a general equation to find the wall shear stress in solid-liquid Newtonian flows in horizontal pipe, for such applications.

CHAPTER 2

LITERATURE SURVEY

2.1 Multiphase Flow & Solid Transport

According to Kelessidis and Bandelis (2005) [2], the flow patterns created when two phase solid–liquid mixtures flow in conduits depend on several parameters like flow rates, conduit shape and size, fluid and solid properties and conduit inclination. Proper identification of the particular flow patterns leads to better estimation of the main parameters of interest, pressure drop and heat and mass transfer rates. The main parameters affecting the transition to the particular flow pattern are presented and the conditions for transition are discussed in their research paper.

The flow of solid–liquid mixtures in conduits is encountered in several situations of industrial significance like ore transportation with long pipelines, oil well and geothermal drilling, mineral and waste water processing. The flow geometry may be pipe or annulus in vertical, inclined or horizontal orientation. While the issues dealing with vertical configurations have been solved after many years of research, there are several problems and questions to be answered for the flow of two phase solid–liquid mixtures in horizontal and inclined conduits.

During the flow of solid-liquid mixtures in horizontal pipes or annuli, the liquid and solid phases may distribute in a number of geometrical configurations or flow patterns. The main parameters determining the particular flow pattern are the liquid velocity, the solids loading, the properties of liquid and solids (rheology and density of liquid, density, diameter and sphericity of solids), the inclination from vertical and conduit shape and size. A detailed description of the flow patterns has been given and these patterns are depicted in Fig. 1, in the direction from high (Figure 2a) to low liquid flow rates (Figure 2d). They are classified as suspended symmetric flow pattern (Figure 2d), suspended asymmetric flow pattern (Figure 2b), moving bed flow pattern (two layers) (Figure 2c) and finally stationary/moving bed flow pattern (three layers) (Figure 2a). At even lower liquid velocities the solids may pile up in the pipe (or annulus) and full blockage may occur.

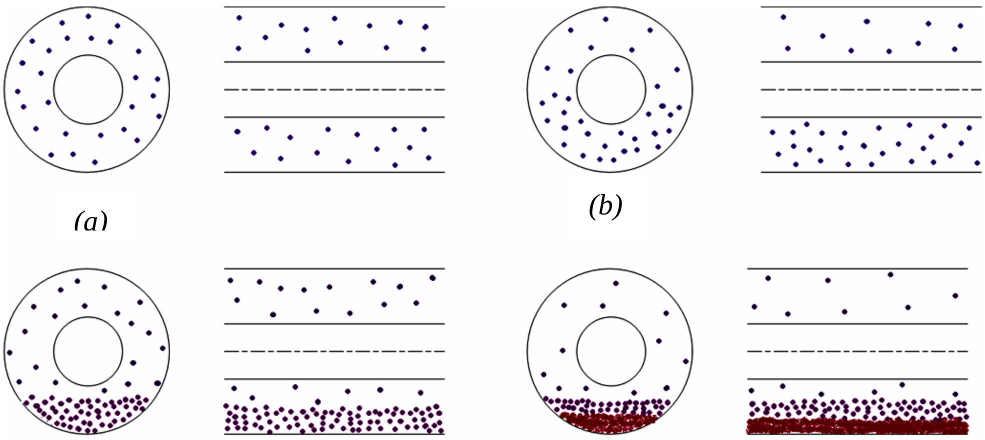

(a) *(b)* *(c)* *(d)*

Figure 2 (a) Suspended Symmetric, (b) Suspended Asymmetric,
(c) Suspended / Moving Beds and (d) Suspended / Moving / Stationary Bed [2]

The Wilson model (Van Riet et al, 1996) [3] is a widely used model for the hydraulic transport of solids in pipelines. A theoretical background of the model has been published piece by piece in a number of articles over the years. A variety of information provided in these publications makes the model difficult to reconstruct. A good understanding of the model structure is inevitable for the user who wants to extend or adapt the model to specific slurry flow conditions. An aim of this article is to summarise the model theory and submit the results of the numerical analysis carried out on the various model configurations. The numerical results show some differences when compared with the nomographs presented in the literature as the graphical presentations of the generalised model outputs. Model outputs are sensitive on a number of input parameters and on a model configuration used. A reconstruction of the nomographs from the computational model outputs is a subject to discussion. A schematic cross section of a pipe is illustrated in Figure 3 as it is defined in the two-layer model for the fully stratified flow and for the heterogeneous flow.

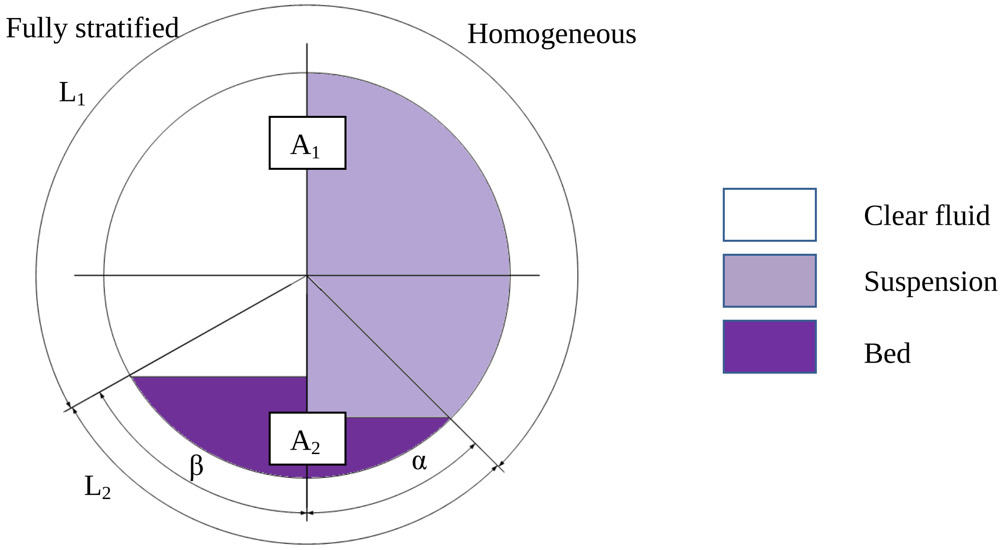


Figure 3 Schematic cross-section for two-layer model [3]

In the article, the geometry of the pipe cross section is defined by equations for the cross-sectional perimeters, the cross-sectional areas, and the equivalent hydraulic diameter of the non-circular waterway section above the bed. The shear stresses on the flow boundaries are determined using Nikuradse friction equation for turbulent flow in a hydraulically-rough pipe. The driving shear force on the bed surface is calculated using the Nikuradse equation multiplied by an empirical constant for shear stress on the bed surface.

The resisting mechanical friction force between bed and pipe wall is determined; this is the normal force exerted by the bed against the pipe wall multiplied by the mechanical friction coefficient μ. Then the viscous friction force between the bed and the pipe wall, and the force balance is calculated. However, it is said in the article that an implementation of this method is not appropriate for the two-layer model. The proposed method provides higher viscous shear stress between bed and pipe wall than is that for fluid.

Poirier (2000) [4], in his study to determine the requirements for transferring insoluble solids from the evaporator pot to the High Level Waste Tank Farm found that the primary parameters influencing flow regimes in horizontal pipelines are velocity and particle size.

The transition between a heterogeneous suspension and a heterogeneous suspension with a sliding bed is often called the deposition velocity or re-suspension velocity, depending on whether the velocity is decreasing or increasing [5]. The axial velocity in a transfer line should be greater than the deposition velocity or re-suspension velocity. Slurry transfers should occur as heterogeneous suspensions [6] [7].

The following are the properties assumed for the author to perform his analysis:

- Particle density is 3930 kg/m3. If the particle density is less, the minimum transport velocity will be less than determined in this analysis.
- Fluid density (water) is 1000 kg/m3. If the fluid density is higher, the minimum transport velocity will be less than determined in this analysis.
- Particle diameter is between 0.1 mm and 4.0 mm. Larger particle sizes would lead to larger minimum transport velocities.
- The fluid viscosity (water) is 1 cp. If the fluid viscosity is greater, the minimum transport velocity will be less than determined in this analysis.
- The pipe diameter is 2 inches.

By analyzing several papers on the determination of minimum transport velocity, the author estimated his required minimum transport velocity based on different correlations and reviewing of graphs. The result is as follows:

Table 1 Calculated Minimum Transport Velocity in Horizontal Pipeline [4]

Reference	u_t (0.1 mm particle)	u_t (4.0 mm particle)
Durand [6][8][9]	2.56032 m/s	2.56032 m/s
Wasp[8]	0.9144 m/s	1.6764 m/s
Newitt et. al. [5]	1.00584 m/s	5.1816 m/s
Turian and Yuan [10]	1.49352 m/s	3.6576 m/s
Average	1.49352 m/s	3.26136 m/s

Two methods could be used to determine the minimum transport value based on the values in Table 1:

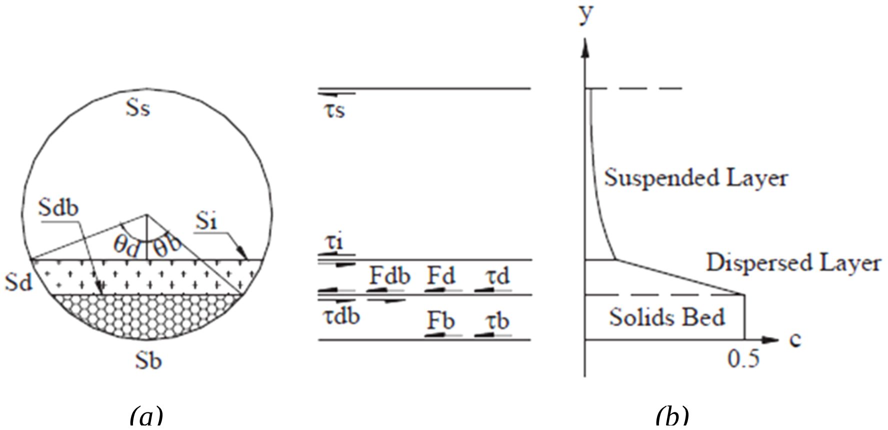
Method 1 is to select the maximum value (17 ft/sec). Method 2 is to calculate the average of the four values (10.7 ft/sec) and add 25% conservatism (13.4 ft/sec).[7] With the information available, the recommended minimum transport velocity would be estimated to be 13 – 17 ft/sec for a heterogeneous suspension. If the transport velocity is between 9 ft/sec and 17 ft/sec, the slurry could be transported as a heterogeneous suspension with a sliding bed or a heterogeneous suspension.

The properties of particles and transporting fluid can be used in this current study, in order to be able to apply the values of minimum transport velocity to determine the shear forces acting on the channel by the flow.

2.2 Pseudo Hydrostatic Pressure

Ramadan et al (2005) [11] used the pseudo hydrostatic pressure method in the three-layer model presented in their study. The purpose of their study is to overcome the limitations of any existing flow models, which are used to predict cutting transport in inclined and horizontal wells. According to a set of assumptions in their research, the model predicts the pressure loss and transport rate of solids in Newtonian and power-law fluid suspensions by assuming stratified flow conditions. Sets of stationary sand bed transport rate tests were performed to verify the predictions of the model. The average transport rates of the beds were predicted using the model.

The concept of dispersive layer has been employed by Doron and Barnea (1993) [14] to extend the two-layer modelling to a three-later scheme. Their model considered the existence of a dispersive layer, which is sandwiched between the suspended layer and the bed as shown in Figure 5a. The dispersed layer was considered to have a higher concentration gradient compared to the suspended layer (Figure 4b).



(a) (b)

Figure 4 (a) Schematic representation of shear stresses acting in the three-layer mechanics model; and (b) assumed concentration profiles in three-layer modelling scheme [11]

Material balance equations of the two phases and momentum equations of the three layers are combined to develop the model. Additional equations are introduced to estimate the average concentration of the suspended layer, and thickness and velocity of the dispersed layer. The thickness of the dispersed layer is modelled using the pseudo hydrostatic pressure gradient concept and assuming linearly varying particle concentration in the dispersed layer.

They used the pseudo hydrostatic pressure concept in wide range to calculate the force between moving bed and boundary and also in order to find the thickness of the dispersed layer, assuming linearly varying concentration in the moving bed layer. The authors had completed their missed part to get the force acting on the boundary of the moving bed layer. However, their approximated model lack of reference of how they found the final form of the force formulas.

In another application of the pseudo hydrostatic pressure, Lade and Inel (1997) [12] used the method in the study of rotating yield and plastic potential surfaces in their paper entitled Rotational Kinematic Hardening Model for Sand. Their experimental study stated that “since the hydrostatic axis uniquely defines the orientation of a surface, a pseudo hydrostatic axis may be employed to keep track of a rotating surface. The only variable required to describe this rotation in the triaxial plane is the angle between the original and the pseudo hydrostatic axes".

This mathematical study involves a surface, of which they are interested to study its pattern of rotation. However, it is not clearly stated how they come about with deciding the orientation of the pseudo hydrostatic axis. And the subject in study is not related to the study of flows; therefore, a further understanding needs to be acquired in order to comprehend this application.

Mingjun et al (1996) [13] also applied the concept of pseudo hydrostatic pressure in their report entitled Electrical Properties of Pyrolyzed Polypyrrolone Film Under Pressure. The experiment was to investigate the "temperature dependence of conductivity of polypyrrolone film pyrolyzed at different pyrolytic temperature. The result was measured as a function of pressure". To conduct their experiment, they placed their film samples in a pressure cap in Teflon cell, filled with an oil. The Teflon cell was then loaded, producing a pseudo hydrostatic pressure on the specimen.

The Teflon cell is used "as a container in a conventional piston-cylinder device". It is a technique that "has been evolved to generate hydrostatic and uniaxial stress regimes". From this, it may be assumed that the 'pseudo hydrostatic pressure' that is produced in the Teflon cell is just a pressure that is generated for the purpose of experiment, instead of being naturally existing. If this be the case, there is no relation of this application to the study of two-phase flow. However, to confirm this assumption, information regarding the Teflon cell pressure generation method can be searched.

Overall, the term 'pseudo hydrostatic' may be applied in different application, due to its wide meaning. There is no specific idea given on how the pseudo hydrostatic pressure method is applied in each different study.

CHAPTER 3

METHODOLOGY

3.1 Technique of Analysis

To find the shear stresses of the solid-liquid flow in a pipe, the technique to be used is mathematical modelling, using the pseudo hydrostatic pressure concept. It is actually based on using the imaginary pressure exerted by the two layers and in this case, preliminary assumptions will be handled to generalize the model:

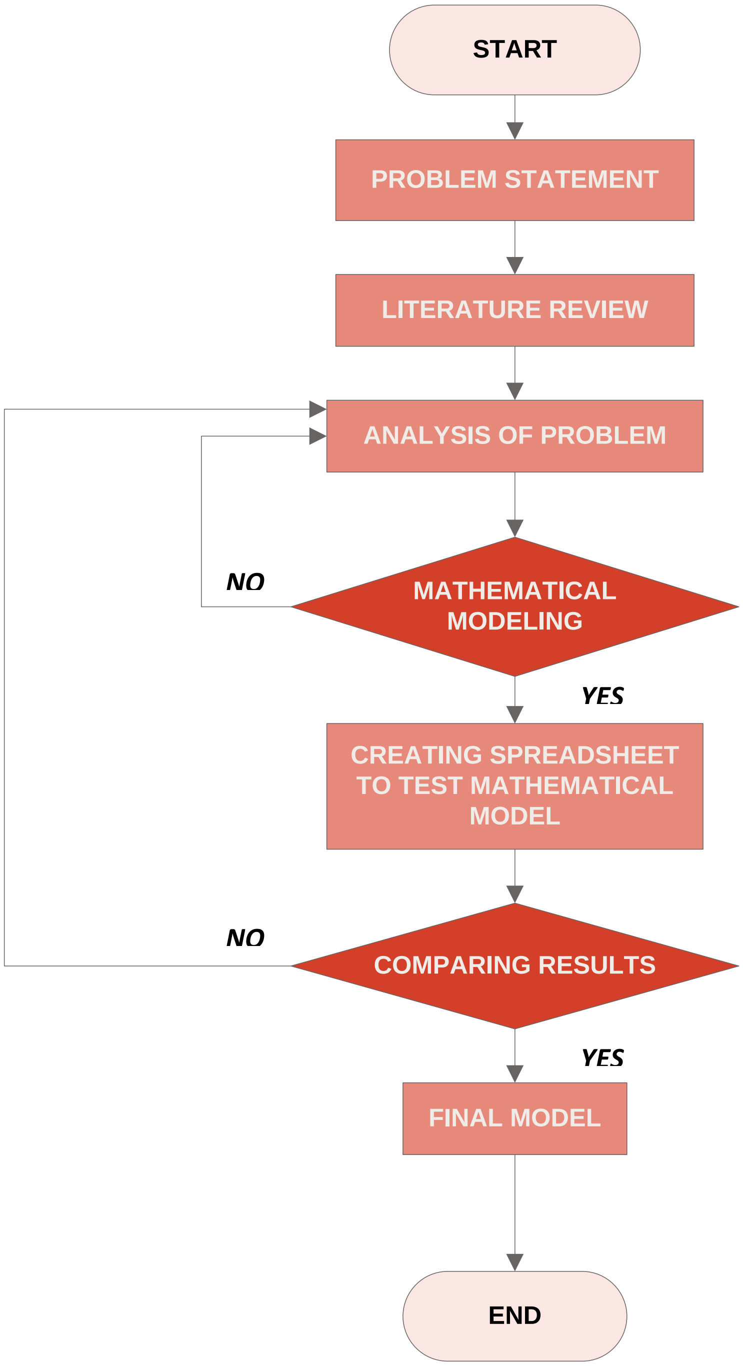
i. The flow is a two-phase pipe flow (solid-liquid)
ii. The pipe is horizontal
iii. Two layers in the flow:
    a. Upper layer is the homogeneous suspended layer
    b. Lower layer is the moving bed layer with linear concentration profile
iv. The flow is incompressible and at steady state, with Newtonian liquid phase

The model will be converted into computer program and comparisons and testing will be conducted against available model [11] using site data.

3.2 Gantt Chart & Milestone

No	Item / Week	1	2	3	4	5	6	7	8	9		10	11	12	13	14
1	**Commencement of project work**: • Creating Excel Spreadsheet • Studying various characteristics of particles in two-phase flows • Studying concentration of two-phase flow	■	■													
2	**Literature Search:** • Obtaining data for model testing	■	■	■			■	■								
3	**Improvement on mathematical equation**		■	■	■	■										
4	**Submission of Progress Report 1**					**x**										
5	**Project Work:** • Testing model using application data • Analyzing results with graphical methods					■	■	■								
6	**Submission of Progress Report 2**								**x**							
7	**Seminar**								**x**							
8	**Project work:** • Further testing against other available models • Finalizing model based on test results								■	■	■	■	■	■	■	
9	**Poster Exhibition**												**x**			
10	**Submission of Dissertation Final Draft**															**x**
12	**Oral Presentation**											**During Study Week**				
13	**Submission of Dissertation**										**7 Days After Oral Presentation**					

3.3 Execution Flow Chart



3.4 Required Equipment / Software

The software used in this project is Microsoft Excel.

CHAPTER 4

THEORY & MATHEMATICAL MODELLING

4.1 The Two-Layer Model

The flow of solid-in-liquid in pipes can be divided into two layers which are:

i. The upper layer: Homogeneous Suspended Layer
ii. The lower layer: Moving Bed Layer

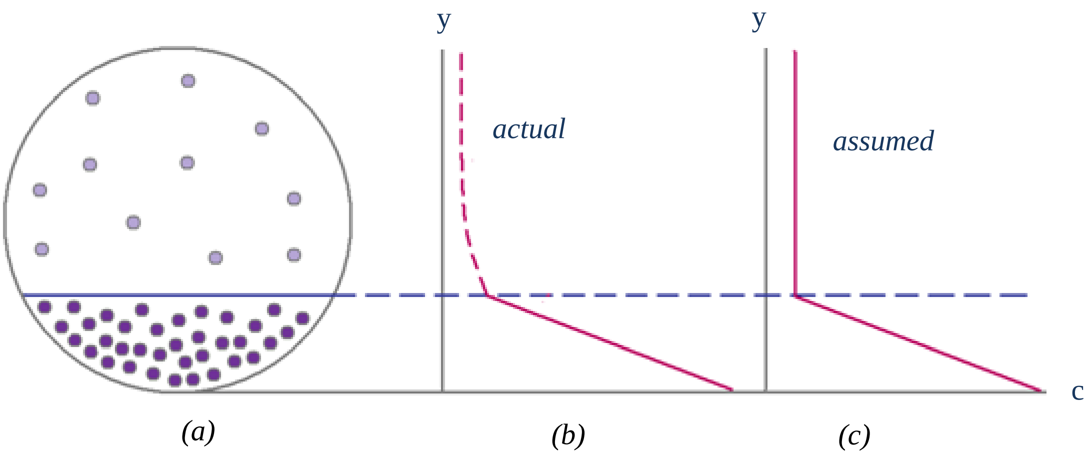


Figure 5 (a) The two-layer approach with the suspended region and the moving bed, (b) the concentration profile for suspended layer shown in dashed line and (c) concentration profile of suspended layer assumed to be homogeneous while concentration profile of the moving bed is linear

In the top layer or the suspended layer, the concentration profile is considered as homogeneous, having a constant concentration profile. This is because; there is only a small variation in its concentration (Figure 5(b)), which could be neglected and the profile of the suspended layer concentration, c_s is constant $(\frac{dC_s}{dy} = 0)$ while the moving bed has a linearly increasing concentration profile.

In a three-layer model, there is an additional layer at the bottom of the flow. This layer which is called dead bed or stationary bed has a maximum concentration of 0.5 [11]. Therefore in this two-layer model, the maximum concentration of the moving bed $c_{m,max}$ is taken as 0.5.

4.2 Assumptions

To mathematically model the process, these assumptions regarding the flow need to be made:

i. The flow is a two-phase pipe flow (solid-liquid)
ii. The flow is in horizontal pipe
iii. The fluid is taken as Newtonian fluid
iv. Two-layer approach is applied
    a. Upper layer is the homogeneous suspended layer
    b. Lower layer is the moving bed layer with linear concentration profile
v. No-slip condition between the two layers which neglects the interstitial shear force between the two layers
vi. The flow is incompressible and at steady state
vii. Analysis is made per unit length basis (flow properties is constant in the horizontal direction)

4.3 Derivation of Mathematical Model

Forces

The total force acting on the pipe wall boundaries is the summation of the forces acting on the wall of the upper suspended layer and the wall of the lower moving bed. It can be given by:

$$F_w = F_{sw} + F_{mw} \qquad (1)$$

The average suspended layer particle concentration c_s is very small compared to the average concentration of the particles in the moving bed layer ($c_s \ll c_m$) [5].

Therefore, the force acting on the upper wall only comes from the shear between the fluid (of mixed density) and the pipe wall:

$$F_{sw} = \tau_{sw} A_s \tag{2}$$

The moving bed layer has a higher concentration of particles which will exert additional force. This force is the dry friction force that is acted by the particles in the moving bed layer upon the bottom wall boundaries. The force between the moving bed and wall will become:

$$F_{mw} = (\tau_{mw} A_m + F_d) \tag{3}$$

This frictional force between the moving bed layer and the wall F_d will be determined using the pseudo hydrostatic pressure distribution on the wall and will be analysed per unit length basis.

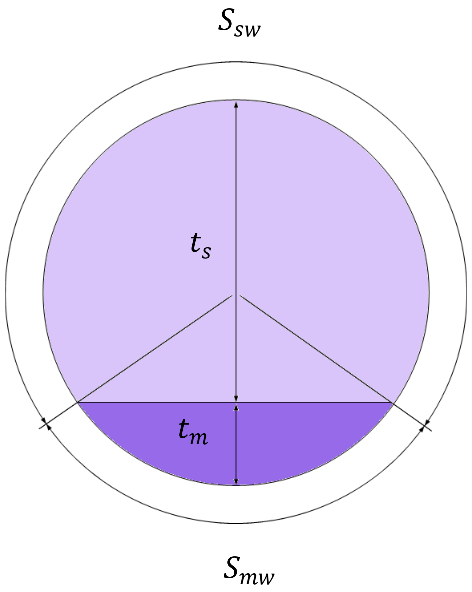


Figure 6 Thickness and Perimeter of each layer in determining the pseudo hydrostatic pressure

Area

By this simplification of the unit length basis, the area between each layer and the wall around becomes the wetted perimeter between them:

$$A_s = S_{sw} \; x \; 1 \; unit \; length \tag{4}$$

$$A_m = S_{mw} \; x \; 1 \; unit \; length \tag{5}$$

Density

According to the two phase flow assumption, the density of each of the two layers will be the mixed densities between the fluid and solids phases according to the solid concentrations in each layer. The density of the fluid phase ρ_f depends solely on the properties of fluid used. Meanwhile, the density of particles depends on both particle properties ρ_p and particles volumetric concentration c_i in the layer. This can be expressed by the following relation:

$$\rho_s = c_s \, \rho_p + (1 - c_s) \, \rho_f \tag{6}$$

$$\rho_m = c_m \, \rho_p + (1 - c_m) \, \rho_f \tag{7}$$

\

Dry Friction Force

To get the dry friction force F_d on moving bed wall, the pseudo hydrostatic pressure approach shall be used. Following the simple definition of the pseudo hydrostatic pressure distribution on the moving bed boundary, the pressure can be estimated as total force acting on that boundary per the area of wall in contact with the moving bed region for one unit length:

$$P = \frac{F}{A}$$

The dynamic friction coefficient between particles and channel wall is μ_d. Then the dry friction force will be written as:

$$F_d = \mu_d \, P_{Pseudo} \, A_m \tag{8}$$

$$P_{Pseudo} = P_m \tag{9}$$

The Pseudo Hydrostatic Pressure

Based on the pseudo hydrostatic pressure concept, the hydrostatic pressure distribution along the moving bed wall can be defined as:

$$P_{Pseudo} = \int_0^{t_m} \left[\rho_m \, c_m + \rho_f \, (1 - c_m)\right] g \, t$$

$$P_{Pseudo} = \int_0^{t_m} \rho_P \, c_m \, g \, dt + \int_0^{t_m} \rho_f \, (1 - c_m) \, g \, dt \tag{10}$$

Concentration

The relation of concentration in the two layers is given by:

The average particles volumetric concentration in the suspended layer is very small compared with the moving-bed layer. Thus we assume that the concentration profile is constant. Wilson (1987) and Hanes and Bowen (1985) assumed the vertical distribution of sediment in the moving bed layer, *c* to vary linearly as

$$c = c_o - \frac{z}{\delta_s}(c_o - c_\delta) \tag{11}$$

In the equation, c_δ is concentration at the top of the sheet layer and c_o is the maximum concentration. In our case, $c_\delta = c_s$, where at the interface of the suspended and moving layers, the concentration is equal. The maximum concentration is taken as the concentration at the bottom of the moving bed layer, therefore $c_o = c_{m,max}$. Hence we obtain the following relation

$$c = c_{m,max} - \frac{t}{t_m}\left(c_{m,max} - c_s\right) \tag{12}$$

where *t* is the height of the moving bed, and t_m is the maximum height.

Substituting Equation (12) into Equation (10), and integrating to find P_m:

$$P_m = \int_0^{t_m} \rho_p g \left[c_{m,max} - \frac{t}{t_m}\left(c_{m,max} - c_s\right)\right] dt + \int_0^{t_m} \rho_f g \left(1 - \left[c_{m,max} - \frac{t}{t_m}\left(c_{m,max} - c_s\right)\right]\right) g \, dt$$

$$P_m = \rho_p g \left[t c_{m,max} - \frac{t^2}{2t_m}\left(c_{m,max} - c_s\right)\right]_0^{t_m} + \rho_f g \left[t - t c_{m,max} + \frac{t^2}{2t_m}\left(c_{m,max} - c_s\right)\right]_0^{t_m}$$

$$P_m = \rho_p g \left(t_m c_{m,max} - \frac{t_m}{2}\left(c_{m,max} - c_s\right)\right) + \rho_f g \left(t_m - t_m c_{m,max} + \frac{t_m}{2}\left(c_{m,max} - c_s\right)\right)$$

$$P_m = \rho_p g t_m \left(c_{m,max} - \frac{\left(c_{m,max} - c_s\right)}{2}\right) + \rho_f g t_m \left(1 - c_{m,max} + \frac{\left(c_{m,max} - c_s\right)}{2}\right)$$

$$P_m = \rho_p g t_m \left(\frac{\left(c_{m,max} + c_s\right)}{2}\right) + \rho_f g t_m \left(1 - \frac{\left(c_{m,max} + c_s\right)}{2}\right) \tag{13}$$

We know from Equation *(8)*:

$$F_d = \mu_p \, P_{Pseudo} \; A_m$$

Combining with Equation *(13)*, the moving layer dry friction force per unit length becomes:

$$\boldsymbol{F_d = \mu_d \left[\rho_p \left(\frac{(c_{m,max} + c_s)}{2} \right) + \rho_f \left(1 - \frac{(c_{m,max} + c_s)}{2} \right) \right] g t_m \, S_{mw}} \tag{14}$$

CHAPTER 5

DEVELOPMENT OF CALCULATION PROGRAM

5.1 The Program

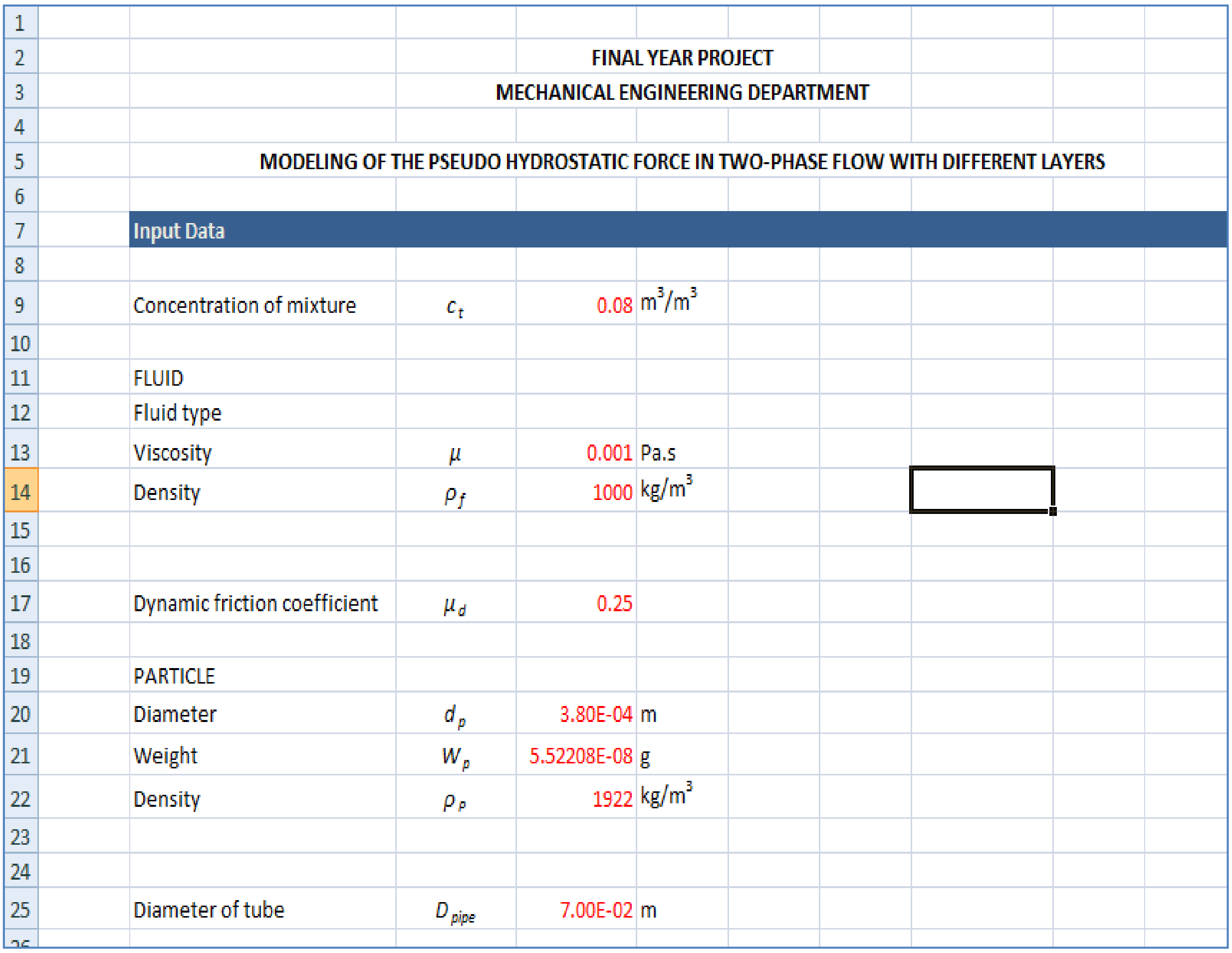
To test the developed model's validity, a calculation program is created using Microsoft Excel, including all inputs and desired outputs to be calculated. From the previous chapter, the governing equation to be solved is:

$$F_d = \mu_d \left[\rho_p \left(\frac{(c_{m,max} + c_s)}{2} \right) + \rho_f \left(1 - \frac{(c_{m,max} + c_s)}{2} \right) \right] g t_m S_{mw}$$

(14)

The purpose is to calculate the unknowns of the problem and obtain the value of the parameter of interest, F_d.

The next page shows screen shots of the program that is being developed.



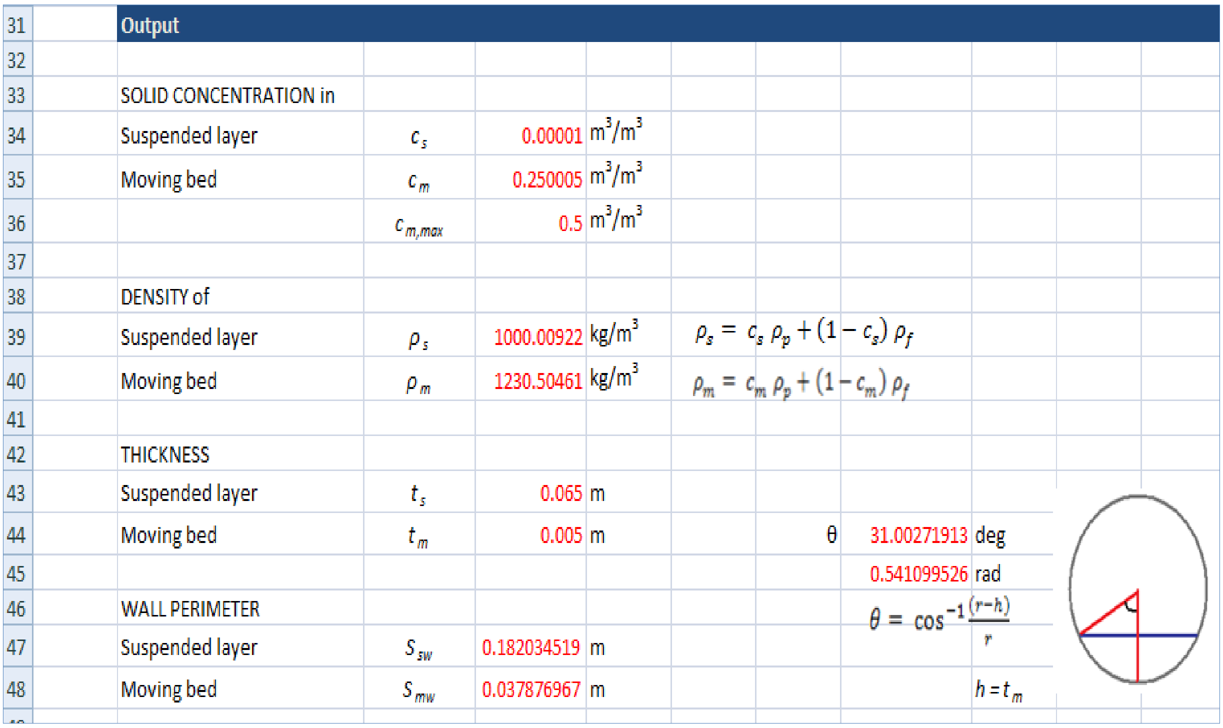
FINAL YEAR PROJECT
MECHANICAL ENGINEERING DEPARTMENT

MODELING OF THE PSEUDO HYDROSTATIC FORCE IN TWO-PHASE FLOW WITH DIFFERENT LAYERS

Input Data

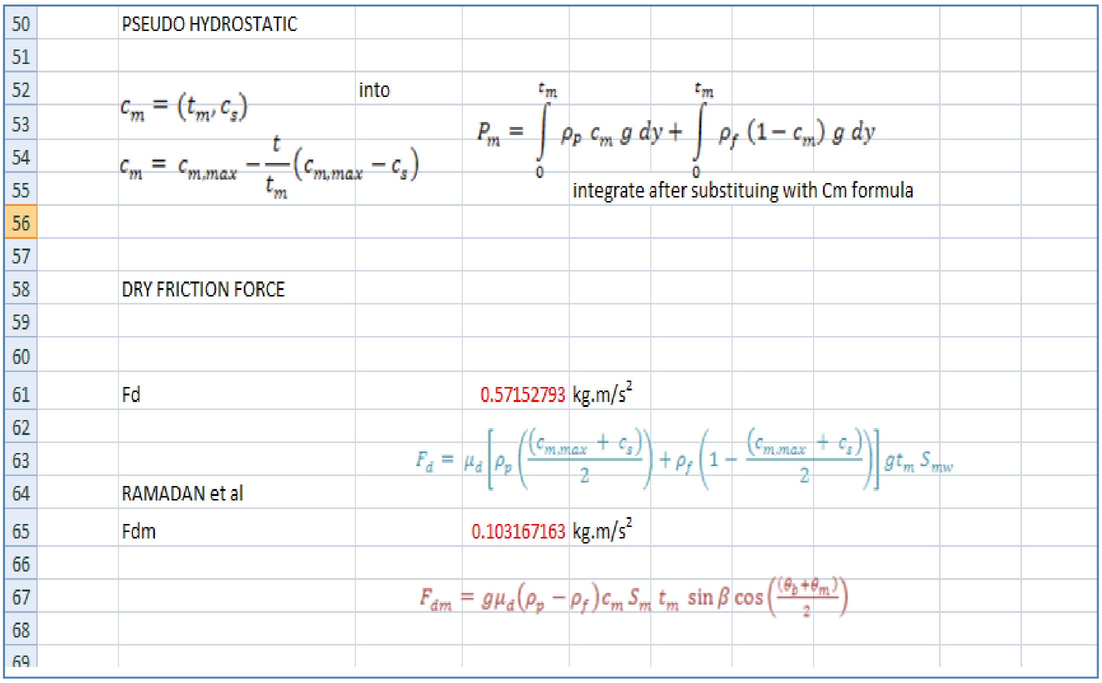
Concentration of mixture	c_t	0.08	m^3/m^3
FLUID			
Fluid type			
Viscosity	μ	0.001	Pa.s
Density	ρ_f	1000	kg/m^3
Dynamic friction coefficient	μ_d	0.25	
PARTICLE			
Diameter	d_p	3.80E-04	m
Weight	W_p	5.52208E-08	g
Density	ρ_p	1922	kg/m^3
Diameter of tube	D_{pipe}	7.00E-02	m

Figure 7 Screenshot of Input section in calculation program



31		Output								
32										
33		SOLID CONCENTRATION in								
34		Suspended layer	c_s	0.00001	m^3/m^3					
35		Moving bed	c_m	0.250005	m^3/m^3					
36			$c_{m,max}$	0.5	m^3/m^3					
37										
38		DENSITY of								
39		Suspended layer	ρ_s	1000.00922	kg/m^3	$\rho_s = c_s \rho_p + (1 - c_s) \rho_f$				
40		Moving bed	ρ_m	1230.50461	kg/m^3	$\rho_m = c_m \rho_p + (1 - c_m) \rho_f$				
41										
42		THICKNESS								
43		Suspended layer	t_s	0.065	m					
44		Moving bed	t_m	0.005	m			θ	31.00271913	deg
45									0.541099526	rad
46		WALL PERIMETER							$\theta = \cos^{-1}\frac{(r-h)}{r}$	
47		Suspended layer	S_{sw}	0.182034519	m					
48		Moving bed	S_{mw}	0.037876967	m					$h = t_m$

Figure 8 Screenshot of Output section in calculation program



PSEUDO HYDROSTATIC

$c_m = (t_m, c_s)$ into $P_m = \int_0^{t_m} \rho_p\, c_m\, g\, dy + \int_0^{t_m} \rho_f\, (1 - c_m)\, g\, dy$

$c_m = c_{m,max} - \frac{t}{t_m}(c_{m,max} - c_s)$

integrate after substituing with Cm formula

DRY FRICTION FORCE

Fd 0.57152793 kg.m/s²

$$F_d = \mu_d \left[\rho_p \left(\frac{(c_{m,max} + c_s)}{2}\right) + \rho_f \left(1 - \frac{(c_{m,max} + c_s)}{2}\right)\right] g t_m\, S_{mw}$$

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Fdm 0.103167163 kg.m/s²

$$F_{dm} = g\mu_d(\rho_p - \rho_f)c_m\, S_m\, t_m\, \sin\beta \cos\left(\frac{(\theta_b + \theta_m)}{2}\right)$$

Figure 9 Calculating the pseudo hydrostatic pressure

5.2 Testing

This model (Equation *(14)*) shall be tested for its accuracy against one existing model that also applies the pseudo hydrostatic pressure method in finding the dry friction force between the wall and the moving bed. Following is the equation derived by Ramadan et al [11]:

$$F_d = g\mu_d(\rho_p - \rho_f)c_d S_d\, t_d\, \sin\beta \cos\left(\frac{(\theta_b + \theta_d)}{2}\right)$$

(15)

Equation *(24)* is based on a three-layer approach, where there is another layer beneath the moving bed which is the stationary solids bed (Figure 10). The dry friction force, F_d is acting on the boundaries of S_d. This force is estimated using a pseudo hydrostatic pressure distribution, where μ_d is the dry dynamic friction coefficient between the particles and the wall of the channel and g is the gravitational acceleration. The thickness of the dispersed layer is t_d. The angular bed thicknesses (Figure 10) of the bed and dispersed layer are θ_b and θ_d, respectively.

To compare this equation with the one that has been modelled in this study, the parameters in Equation *(15)* has to be applied accordingly. Taking the two-layer approach, Equation *(15)* becomes

$$\boldsymbol{F_d = g\mu_d(\rho_p - \rho_f)c_m S_m t_m \sin\beta \cos\left(\frac{(\theta_m)}{2}\right)} \tag{16}$$

where the notation *m* represents the moving bed.

For an application in a horizontal pipe, $\sin\beta = 1$ because the inclination angle from the vertical axis is 90. However, this model depends only on the height and concentration of the moving bed, and also the angular distance that it makes from the bottom of the pipe (Figure 6). The relation between the angular distances of the layers is unclear. Apart from that, the author is simply taking the difference in the density values of the fluid and the particles.

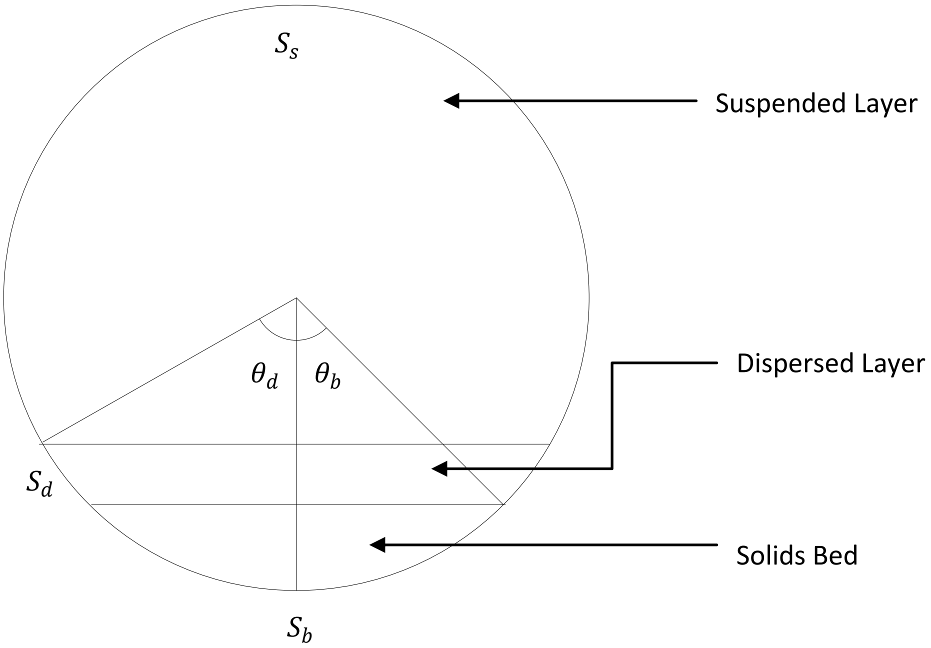

Figure 10 Geometrics of the three layer model [11]

5.3 Data

The data required in order to test the developed model will is taken from other research papers. This is so that a comparison can be made, and a more accurate result (in terms of percentage difference) will be obtained.

Values for the parameters of interest can be taken from Ramadan et al [11] where he uses the following:

Table 2 Constant computational data for the model prediction [11]

Density of water	1000 kg/m^3
Viscosity of water	0.001 Pa.s
Channel diameter	70 mm
Dynamic friction factor	0.25

To standardize calculations, initial conditions and values are made as follow:

i. Concentration for suspended layer, c_s is assumed to be relatively small (= 0.00001).

ii. Two values of particle density are used in the iteration, 2600 kg/m^3 and 1922 kg/m$^{3.}$

iii. Thickness of the moving bed layer is the variable pre-set parameter. The value will be from 0.005 m to 0.020 m in solving for F_d for each iteration.

CHAPTER 6

RESULTS & DISCUSSION

6.1 Iteration Results

Set 1: Diameter of Pipe = 0.007 m

Concentration of Suspended Layer = 0.00001

Particle density = 2600 kg/m^3

Table 3 Set 1: Dry Friction Force

	Modelled equation	Reference Equation
t_m	F_d	F_d
(m)	(N)	(N)
0.0050	0.6503	0.1790
0.0075	1.2022	0.3246
0.0100	1.8631	0.4928
0.0125	2.6215	0.6788
0.0150	3.4702	0.8789
0.0175	4.4045	1.0899
0.0200	5.4217	1.3092

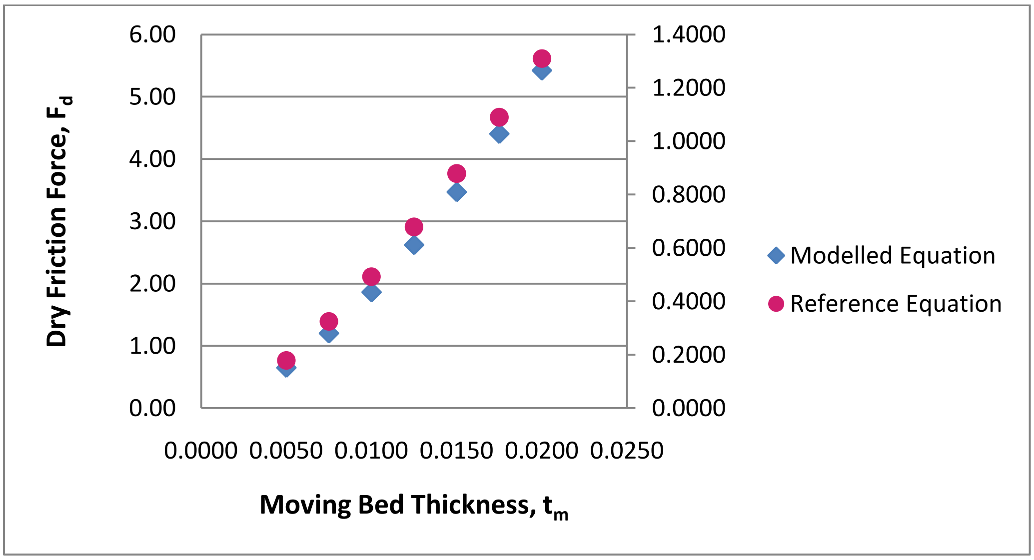

Figure 11 Set 1: Moving Bed Thickness vs. Dry Friction Factor

Set 2: Diameter of Pipe = 0.007 m

Concentration of Suspended Layer = 0.00001

Particle density = 1922 kg/m^3

Table 4 Set 2: Dry Friction Force

	Modelled equation	Reference Equation
t_m	F_d	F_d
(m)	(N)	(N)
0.0050	0.5715	0.1032
0.0075	1.0567	0.1870
0.0100	1.6375	0.2840
0.0125	2.3041	0.3912
0.0150	3.0500	0.5064
0.0175	3.8713	0.6280
0.0200	4.7652	0.7544

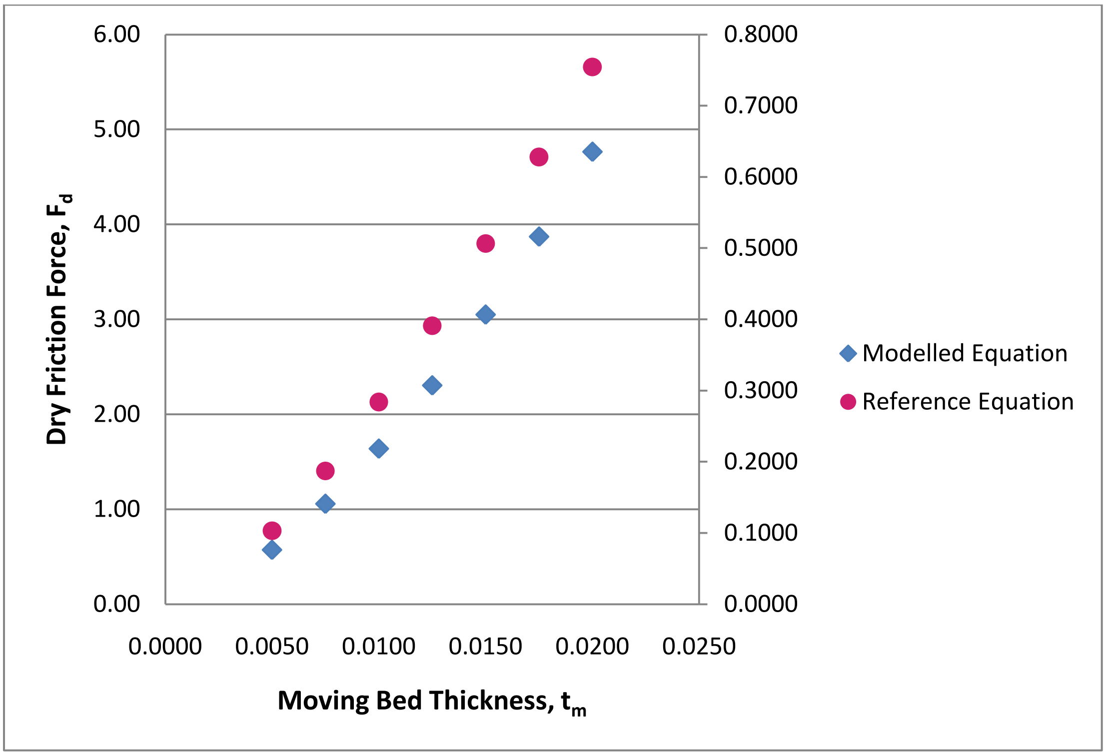

Figure 12 Set 2: Moving Bed Thickness vs. Dry Friction Factor

Set 3: Diameter of Pipe = 0.007 m

Concentration of Suspended Layer = 0.001

Particle density = 2600 kg/m^3

Table 5 Set 3: Dry Friction Force

	Modelled equation	Reference Equation
t_m	F_d	F_d
(m)	(N)	(N)
0.0050	0.6505	0.1794
0.0075	1.2029	0.3252
0.0100	1.8642	0.4938
0.0125	2.6230	0.6802
0.0150	3.4721	0.8806
0.0175	4.4071	1.0920
0.0200	5.4247	1.3118

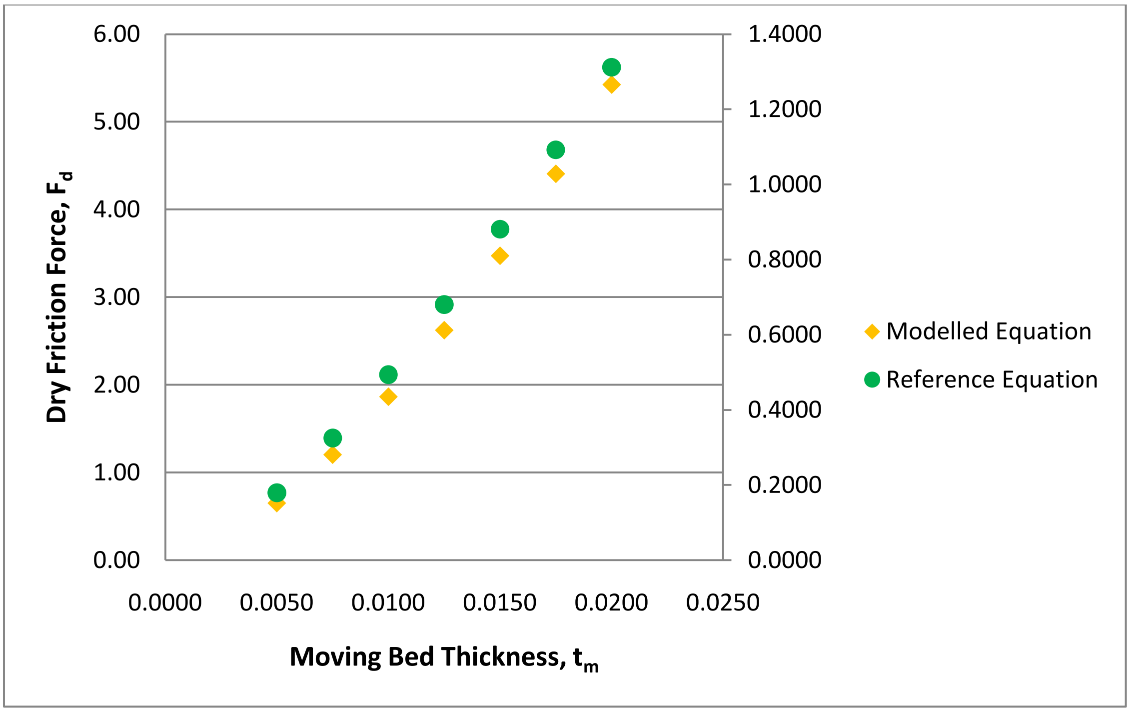

Figure 13 Set 3: Moving Bed Thickness vs. Dry Friction Factor

Set 4: Diameter of Pipe = 0.007 m

Concentration of Suspended Layer = 0.001

Particle density = 1922 kg/m^3

Table 6 Set 4: Dry Friction Force

	Modelled equation	Reference Equation
t_m	F_d	F_d
(m)	(N)	(N)
0.0050	0.5717	0.1034
0.0075	1.0571	0.1874
0.0100	1.6382	0.2846
0.0125	2.3051	0.3920
0.0150	3.0512	0.5074
0.0175	3.8727	0.6293
0.0200	4.7670	0.7559

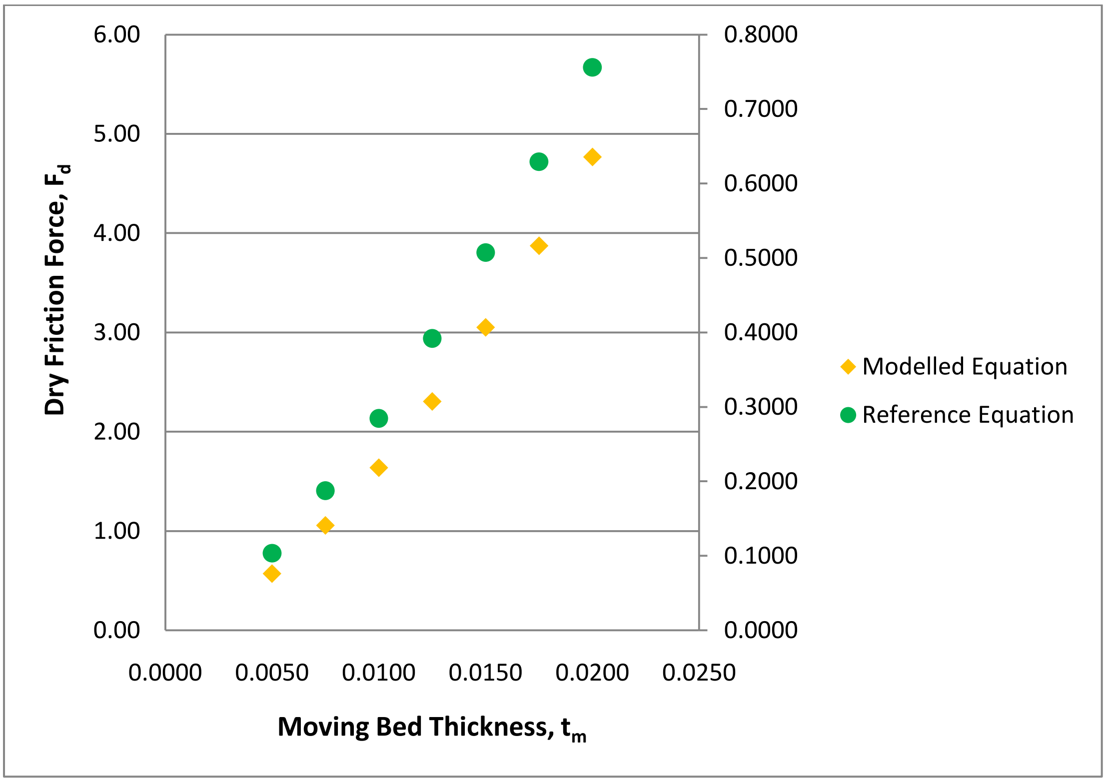

Figure 14 Set 4: Moving Bed Thickness vs. Dry Friction Factor

Set 5: Diameter of Pipe = 0.071 m

Concentration of Suspended Layer = 0.001

Particle density = 1922 kg/m^3

Table 7 Set 4: Dry Friction Force

	Modelled equation	Reference Equation
t_m	F_d	F_d
(m)	(N)	(N)
0.0500	65.4769	18.0372
0.0750	121.0460	32.7075
0.1000	187.5710	49.7218
0.1250	264.0960	68.5058
0.1500	349.5500	88.7176
0.1750	443.6090	110.0540
0.2000	545.9650	132.2520

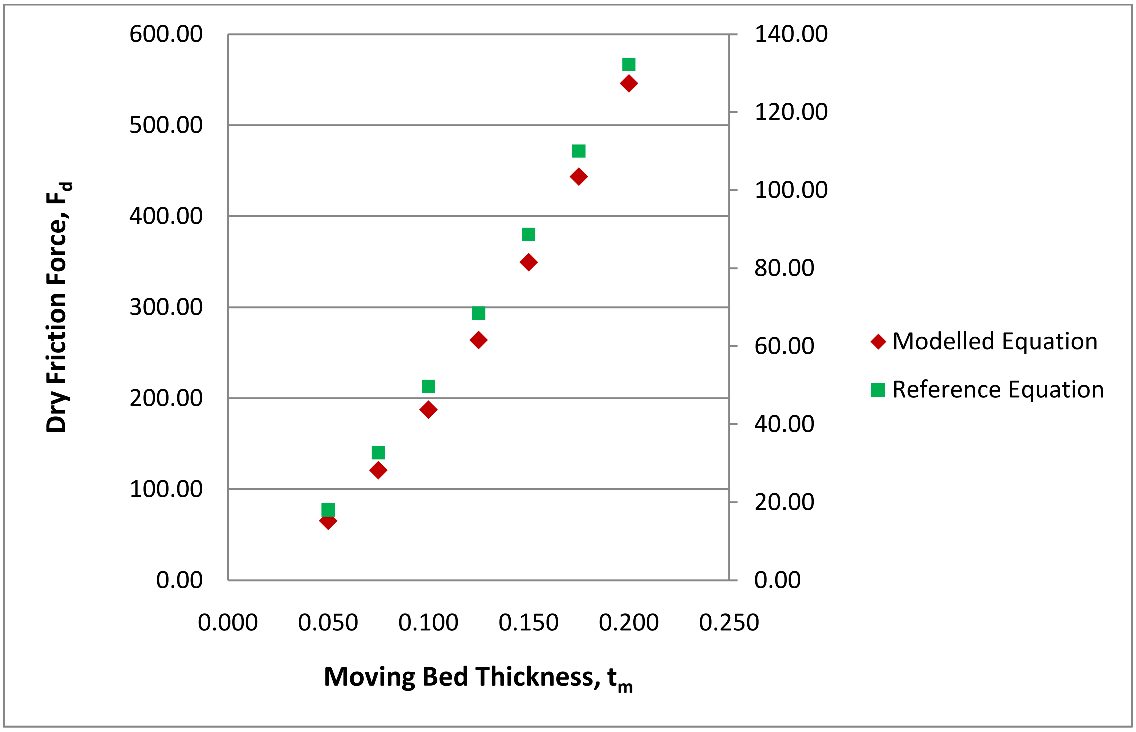

Figure 15 Set 5: Moving Bed Thickness vs. Dry Friction Factor

6.2 Comparison of Different Concentration

Set 6: Particle density = 2600 kg/m^3

Table 8 Set 6: Dry Friction Force at different concentrations for the modelled equation

	Modelled equation	
c_s	0.00001	0.001
t_m	F_d	F_d
(m)	(N)	(N)
0.0050	0.6503	0.6505
0.0075	1.2022	1.2029
0.0100	1.8631	1.8642
0.0125	2.6215	2.623
0.0150	3.4702	3.4721
0.0175	4.4045	4.4071
0.0200	5.4217	5.4247

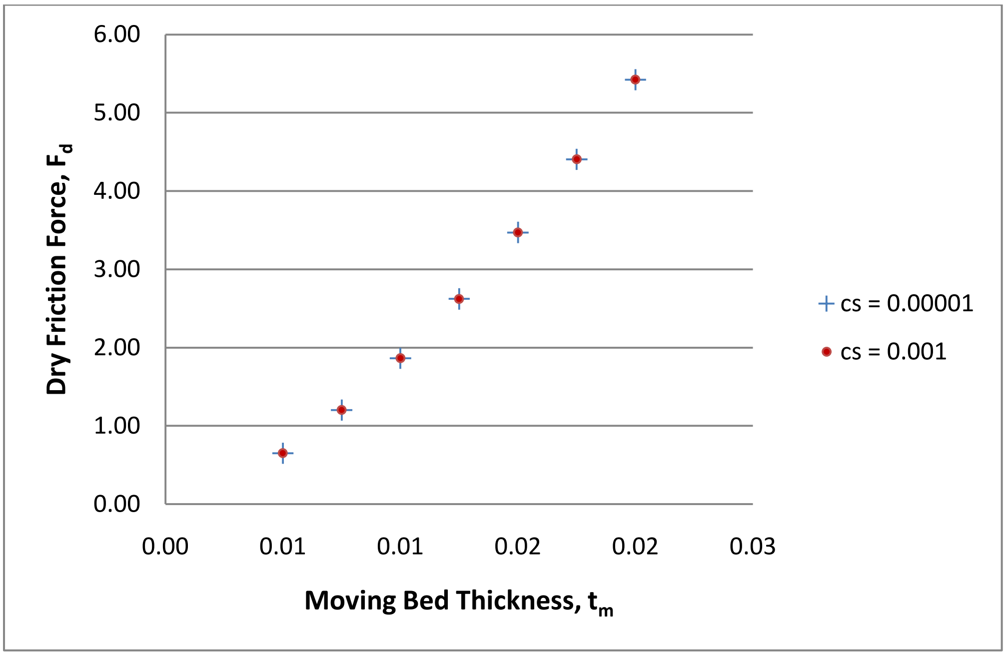

Figure 16 Set 6: Moving Bed Thickness vs. Dry Friction Factor for modelled equation

Table 9 Set 6: Dry Friction Force at different concentrations for the reference equation

	Reference Equation	
c_s	0.00001	0.001
t_m	F_d	F_d
(m)	(N)	(N)
0.0050	0.1790	0.1794
0.0075	0.3246	0.3252
0.0100	0.4928	0.4938
0.0125	0.6788	0.6802
0.0150	0.8789	0.8806
0.0175	1.0899	1.092
0.0200	1.3092	1.3118

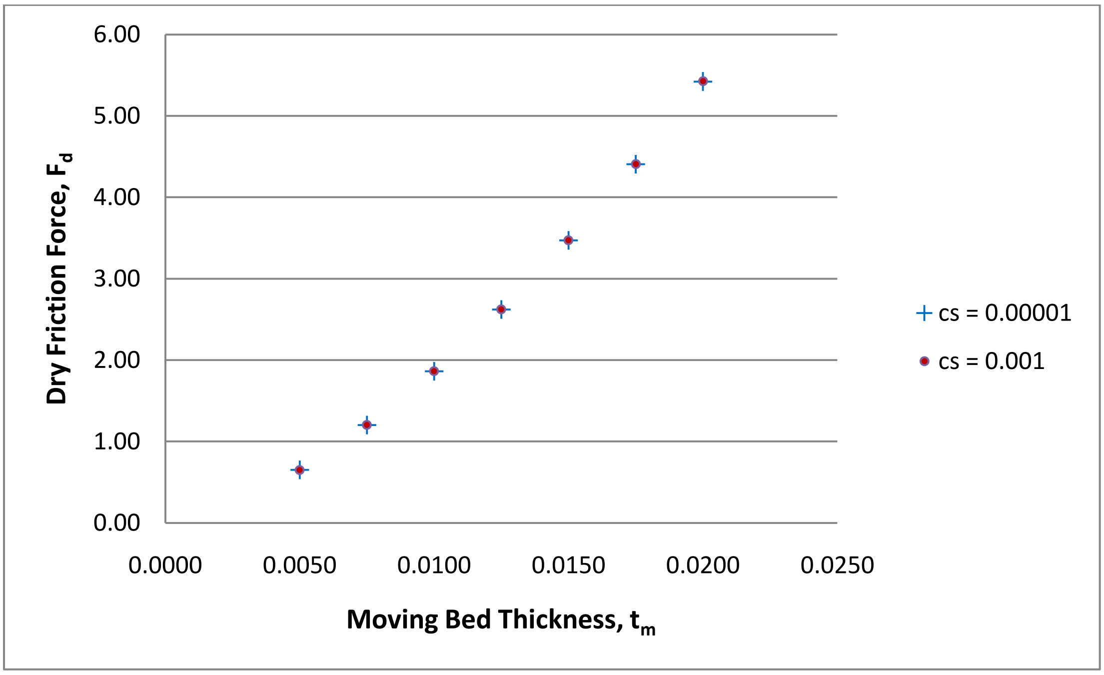

Figure 17 Set 6: Moving Bed Thickness vs. Dry Friction Factor for reference equation

6.3 Interpretation of Results

From the calculation results in section 6.1, Set 1 – Set 4, we can see that for both modelled equation and reference equation, the dry friction force is increasing with increasing moving bed thickness. However, the reference equation gives a much smaller value for each iteration. Taking example from the first row of Table 3, the percentage of error can be calculated as follows:

	Modelled equation	Reference Equation
t_m	F_d	F_d
(m)	(N)	(N)
0.0050	0.6503	0.1790

$$Error = \frac{|modelled - reference|}{reference}$$

$$= \frac{|0.6503 - 0.1790|}{0.1790}$$

$$= 2.633$$

The value of error is highly significant; however, it can be justified by the following explanations:

i. The reference equation is built for a three-layer application [11]. The assumptions made in developing the equation may only be suited to three-layer flows.

ii. In the original reference equation (Equation *(15)*), the author is taking the average angular distance between the dispersed layer and the solids bed (Figure 10).

$$F_d = g\mu_d(\rho_p - \rho_f)c_d S_d\, t_d\, \sin\beta \cos\left(\frac{(\theta_b + \theta_d)}{2}\right)$$

This average value might be insignificant when the equation is applied to a two-layer model, where the value of θ_b will be zero.

iii. The modelled equation finds the dry friction force acting on the pipe wall by the *layer of particles*. In the actual case, only particles in contact with the wall would exert dry friction force. This could mean that only a percentage of the pseudo hydrostatic force contributes to the dry friction force on the pipe wall in contact with the moving bed. For this, we can assume that if the contact between particles and lower layer pipe wall is 25% of total contact area between moving bed (fluid and particles) and pipe wall, the dry friction force could also be reduced to 25%, which could give an excellent agreement with the reference equation.

In section 6.2, comparison of the dry friction force value is being made by increasing the value of suspended layer concentration $c_{s.}$ Figure 15 and 16 show that there is not much difference in F_d while changing the concentration from 0.00001 to 0.001. This is because the concentration of the suspended layer is always much smaller than that of the moving bed. Therefore, any change in its value, provided still agreeing with the relation $(c_s \ll c_m)$, does not contribute to a high increment in the dry friction force.

CHAPTER 7

CONCLUSION &

RECOMMENDATIONS

7.1 Conclusion

At the end of this project, a general clear mathematical formula has been developed to find the dry friction force of a horizontal solid-liquid flow using the two-layer approach. The generalized model can be modified to match such application to serve in solving the complexity of calculating the boundary-moving bed force in different types of two phase flow with layers.

The basics of the calculation program have been made in Microsoft Excel. The developed mathematical model is tested against one available model that also applies the pseudo hydrostatic pressure method, using similar data [11].

Based on the calculated results, there is lack of agreement between the modelled equation and the reference equation. This difference is justified by several factors which include assumptions made for mathematical modelling and dissimilar application for different flow models (two-phase or three-phase).

7.2 Recommendations

Following this study, there are many improvements that can be made in order to achieve more excellent results and to expand the current search to wider applications:

i. The pseudo hydrostatic pressure method can be improved by applying the effect of differently shaped of bodies. For a curved surface, the total hydrostatic force on the whole surface area is the resultant force of its vertical and horizontal component.
ii. The effect of particle size and channel diameter can be included in future investigations.
iii. The current search can be extended to find the shear stress acting on the pipe wall. Following this, the effect of flow rate on the moving bed height can be included in the study.
iv. An experiment could be conducted to compare the modelled equation with experimental values.
v. The search can be extended to Non-Newtonian fluids.
vi. The application of the pseudo hydrostatic pressure can be considered on other flow models [2] and flow in inclined channels.

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