Comparison of ARX and OBF-ARX models for Closed-loop Identification of Multiple Input Multiple Output (MIMO) System

by

Nazrin Syafiq Bin Roslan

13769

Dissertation submitted in partial fulfilment of
the requirements for the
Bachelor of Engineering (Hons)
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CERTIFICATION OF APPROVAL

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Approved by,

____

(Dr. Lemma Dendena Tufa)

UNIVERSITI TEKNOLOGI PETRONAS
TRONOH, PERAK
September 2014

CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

NAZRIN SYAFIQ BIN ROSLAN
910302-07-5765

ABSTRACT

ARX is simple and effective model structure for closed-loop system identification. OBF-ARX is also shown to be very effective and more advantageous for closed-loop identification of system involving time delays. However, these arguments are done in most literature in the context of SISO systems. This project will focus on the two systems which will be discussed and compared for identification of MIMO systems. The MIMO system used is the Wood & Berry Distillation Column. In this project, a mathematical model will be developed based on the distillation column with a closed loop system using the experimental data obtained from SIMULINK MATLAB. The Wood & Berry models will be used to compare the ARX and OBF-ARX model and to obtain the suitable and better model choice for a close loop system of MIMO system. The model structure with the highest average fitness value will be selected as the best model structure and the comparison plot of X_D and X_B of both condition of certain and uncertain time delays was plotted.

ACKNOWLEDGEMENTS

This report is done through the help and support from everyone, including my parents, supervisors, and my friend. Here, please allow me to dedicate my acknowledgement of gratitude towards them:

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Next, sincere thanks to my parents and my friends who never fail to support me. They will always be by my side whenever I am unhappy or feeling stress. I appreciate their time. Without them, I will not be able to complete the report will full courage.

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ABBREVIATIONS AND NOMENCLATURES

ARMAX - Autoregressive Moving Average Model

ARX - Autoregressive Model

MIMO - Multiple-Input Multiple-Output

OBF-ARX – Orthonormal Basis Filter of Autoregressive Model

PID - Proportional-Integral-Derivative

SISO - Single-Input Single-Output

CHAPTER 1:
INTRODUCTION

1.1 BACKGROUND OF STUDY

According to Chinedu and Deanbair (2008), system identification is the process of developing or improving the mathematical representation of a physical system using experimental data. In other word, it is the field of modelling dynamic systems from experimental data (i.e. input/output patterns). There are three types of identification techniques: Modal parameter identification, Structural-model parameter identification (primarily used in structural engineering) and Control-model identification (primarily used in mechanical and aerospace systems). This system identification helps to utilize both input and output data or can only include the output data.

A set of candidate models can be obtained by specifying within which collection of models that the user is going to choose for a suitable one. This model choosing is the most difficult part of system identification. During this stage, the user must equip with prior knowledge with engineering intuition and insight. Sometimes, a model set is obtained after careful modelling. Then, basic physical laws and other well-established relationships are constructed to know the physical parameters in a model. Meanwhile, a black box can be obtained when standard linear models are employed without referring to the physical background (Nagarajaiah, 2009).

The user can choose the best model from the set with the guidance from the data. This is known as identification method. The assessment of model quality is based on how the models perform when the models attempt to reproduce the measured data. After settling, the one that describes the data according to the chosen criterion best will be chosen as the particular model. Such test is known as model validation. Model validation involves various procedures to access how the model relates to observed data, to prior knowledge, and to its intended use. On the other hand, a model will be rejected if the numerical procedure fails to get the best model, the criterion is not well chosen, the model set was not appropriate, and the data set is not informative enough (Ljung, 1999).

AR(X) model is the simplest model incorporating the stimulus signal. The estimation of the ARX model is the most efficient of the polynomial estimations because it solves the linear regression equations in the analytic form. However, disturbances are part of the system dynamics. When the disturbances of the systems are not white noise, the coupling between the deterministic and stochastic dynamics can bias the estimation of the AR(X) model (Nagarajaiah, 2009). The parameters of the ARX model structure can be represented by

$$y(t) + ay(t-1) = B_1u(t-1) + Bu(t-2) + e(t) \qquad \text{Eq.(1)}$$

It can be estimated by least-squares method according to Instrument (2010)

Instrument (2010) found that Orthonormal Basis Filter (OBF-ARX) models have several advantages over the conventional linear models. They are consistent in parameters for most practical open-loop systems and the recently developed ARX-OBF and OBF-ARMAX structures lead to consistent parameters for closed loop identification also. They require relatively a fewer numbers of parameters to capture the dynamics of linear systems (parsimonious in parameters) and the model parameters can be easily estimated using linear least square method. MIMO systems can be easily handled using OBF and OBF based structures. In addition, recent works by Lemma and Ramasamy prove that OBF based structures show superior performance for multi-step ahead prediction of systems with uncertain time delays compared to most conventional model structures (Heuberger, 2005). The parameters of the OBF-ARX model structure can be represented by

$$y(k) = G(q)u(k) \qquad \text{Eq.(2)}$$

System identification can be divided into closed loop and open loop system identification. An open loop system identification is a process of developing the mathematical representation of a physical system without any feedback control. On the other hand, a closed loop system identification is a process of developing the mathematical representation of a physical system with feedback control. In a closed loop system identification process, the control valve is operating automatically while the control valve will operate manually in open loop system identification. In closed loop system identification, the input signal u(t) is correlated with e(t) while it is uncorrelated for open loop system identification. This research will only focus on closed loop system identification. Methods such as direct approach, indirect approach, and joint input-output approaches are used in closed loop system identification.

1.2 PROBLEM STATEMENT

ARX is simple and effective model structure for closed-loop system identification. OBF-ARX is also shown to be very effective and more advantageous for closed-loop identification of system involving time delays. However, these arguments are done in most literature in the context of SISO systems. In this project, the two systems will be compared for identification of MIMO systems.

1.3 OBJECTIVE

The objectives of comparing the ARX and OBF-ARX models for Closed-loop Identification of Multiple Input Multiple Output (MIMO) System are:

i. To generate identification data for a MIMO system (Wood and Berry distillation column).
ii. To develop ARX and OBF-ARX model using the data generated at (i) by using SIMULINK (MATLAB).
iii. To compare the accuracy and prediction capabilities of the ARX and OBF-ARX models.

1.4 SCOPE OF STUDY

The scope of study for this project will be:

i. Closed loop system and its properties
   Open loop system will not be included in this research

ii. Linear System
   Non-linear system is excluded in this project

iii. Auto Regressive (ARX) Model
   ARX model will be used to develop the mathematical model.

iv. Orthonormal Basis Filter (OBF) Model

OBF model will be used to develop mathematical model

v. Multiple-Input and Multiple Output (MIMO) System

Single-Input and Single-Output will not be considered in this project. Distillation. Column is an example of a MIMO system.

vi. Wood & Berry Distillation Column

This type of Distillation Column will be used as the standard.

CHAPTER 2:
LITERATURE REVIEW

Nelles (2001) suggest that models can be developed either from purely theoretical analysis or from experimental data or somewhere in between. The process of model development from experimental data is known as system identification. The identification test can be conducted either in open-loop (open-loop identification) or while the plant is under feedback control (closed-loop identification). Closed loop identification system, is well known which more aligned, relative to open loop identification, towards meeting the operating goals of the operation region. Another equally important aspect is that data generated under closed loop is more likely to contain control-relevant frequencies and so a controller based on the resulting model would be more suited to meet the performance specifications (Lemma & Ramasamy).

On the other hand, closed-loop conditions pose additional challenges for system identification. The fundamental problem is that of the correlation between the disturbances and the manipulated variables through the feedback. This correlation results in biased estimates of the model parameters when directly identifying the process dynamics from closed-loop input–output data. The awareness of these potential failings has motivated research efforts, which in turn have led to a better understanding of the properties of the existing methods when used with closed-loop data, as well as proposition of some remedies to circumvent the potential problems (Gevers & Ljung, 1986).

ARX model and OBF-ARX model was chosen in this project due to its consistency of model parameters and the number of parameters required to describe the system within acceptable degree of accuracy. These two models consistency was relate with the bias and optimality of the model parameters. ARX models, in fact is a suitable model class for linear control implementations. The parameter estimation problem is convex and easily handed for both SISO and MIMO system in contrast to ARMAX or State Space model. ARX model structure provides a much simpler estimation problem of multivariable system than the ARMAX model.

Aside, Orthonormal Basis Filter (OBF-ARX) models have several advantages over the conventional linear models. They are consistent in parameters for most practical open-loop systems and the recently developed ARX-OBF and OBF-ARMAX structures lead to consistent parameters for closed loop identification also. They require relatively a fewer numbers of parameters to capture the dynamics of linear systems (parsimonious in parameters) and the model parameters can be easily estimated using linear least square method [6]. MIMO systems can be easily handled using OBF and OBF based structures.

CHAPTER 3:
METHODOLOGY

3.1 Objective (i)

MATLAB is the main software used in this project. Wood and Berry Distillation. Column will be used in this project. The mathematical model is expressed in Equation 3. The model set-up is shown in Figure 1 below. The model is defined as:

$$\begin{bmatrix} x_D(s) \\ x_B(s) \end{bmatrix} = \begin{bmatrix} \frac{12.8e^{-s}}{16.7s+1} & \frac{-18.9e^{-3s}}{21s+1} \\ \frac{6.6e^{-7s}}{10.9s+1} & \frac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix} \begin{bmatrix} R(s) \\ S(s) \end{bmatrix} \qquad \text{Eq.(3)}$$

The following steps are taken to accomplish the objectives:

1. Introduce excitation signal on the distillation column
2. Collect the input-output data
3. Develop the ARX models

The Model set-up

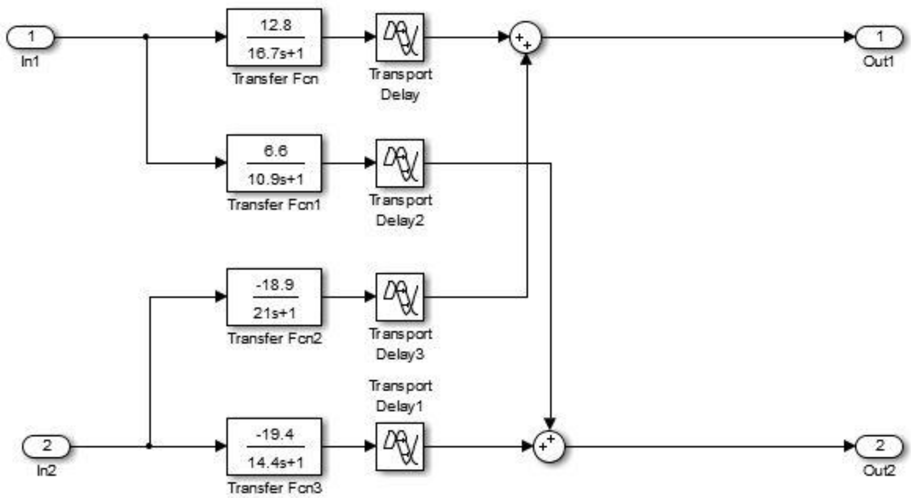


***Figure 1**: The subsystem of Wood and Berry Distillation Column*

The first section of the test concentrate on the effect of closed loop system identification when both loops are closed. The test was run with both integral controllers I1 and I2 are set as zero. Second test on the effect closed loop system identification when both Integral controllers, I1 and I2 are set at 16.37 and 14.46 respectively. The third section test studies on the effect of the Integral controller, I on the closed loop system identification under closed loops conditions. In this case, both proportional controller, Kc1 and kc2 are set at 0.604 and -0.127 respectively.

3.2 Objective (ii)

A simulation is performed by using MATLAB Model Predictive Control Toolbox. The data generated at (i) will be analyse and the selection of the appropriate ARX model and its size was performed prior to that. The appropriate model structure will be develop and for each simulation, sampling period is gathered, collected and tabulated.

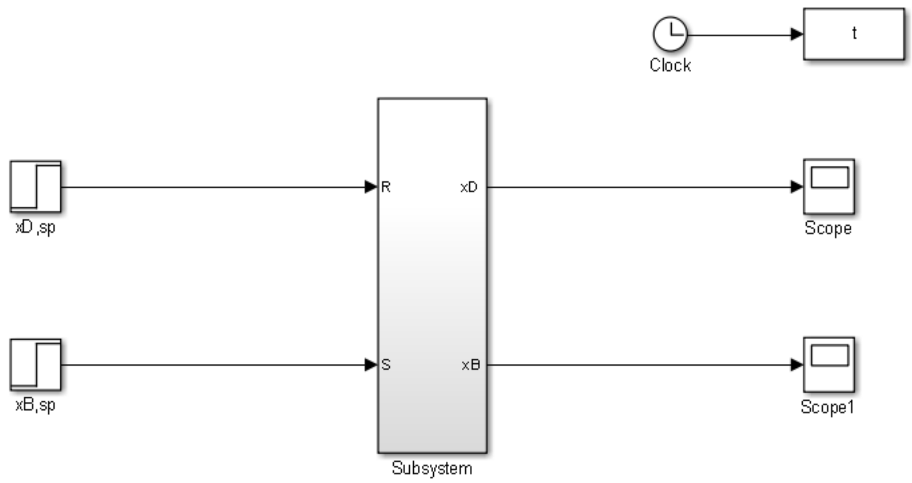


***Figure 2**: The Distillation Column or Predictive Control System*

3.3 Objective (iii)

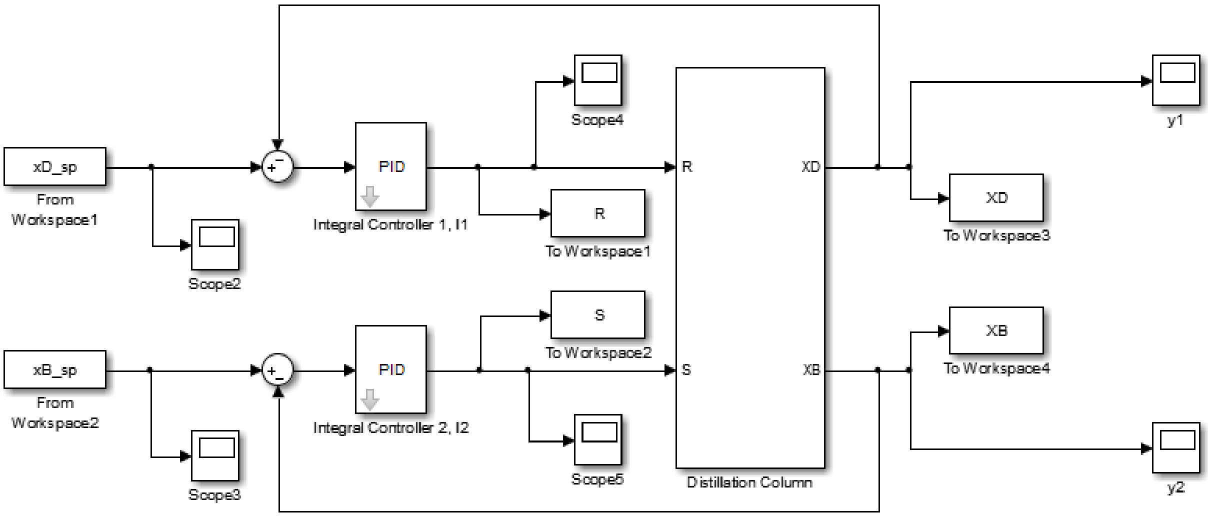


Figure 3: SIMULINK: PID Controller with Wood and Berry Distillation Column

This objective can be achieved by comparing the performance using residual error analysis. However it is more to day to day monitoring action and it is done in the actual plant condition after preliminary method of comparing is done. The preliminary method of comparing the models are by using fitness method of comparing. Using the model developed in (ii), the models can be compared using time delay which we included during modelling processes by taking the differences between actual data and predicted data and compare with the mean error. To ensure the result is accurate, the total number of parameters in both model is made into equal.

CHAPTER 4:
RESULT AND DISCUSSION

4.1 Certain Time Delays Condition

4.1.1 ARX Model development from SIMULINK MATLAB

The model developed from the MATLAB is recorded. The model can be represented by Eq(1) in Chapter 1. Three (3) ARX models were developed and used, which are [2*ones(2,2),2*ones(2,2), [1 3; 7 3]] named as Structure 1, [3*ones(2,2), 2*ones(2,2), [1 3; 7 3]] named as Structure 2 and [5*ones(2,2), 2*ones(2,2), [1 3; 7 3]] named as Structure 3. Generally Structure 1, Structure 2 and Structure 3 are combination of different polynomial orders [n_A n_B n_K] of [2 2 1], [3 2 1] and [5 2 1]. The structures contain combination of different polynomial orders as summarized in the table below. The first column of the model represents A(q), the second column represents B(q), while the last column represents the time delay.

Table 1: Table of different ARX model structure under certain time delays condition

ARX Structure	n_A	n_B	n_K
1	$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$	$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$	$\begin{bmatrix} 1 & 3 \\ 7 & 3 \end{bmatrix}$
2	$\begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix}$	$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$	$\begin{bmatrix} 1 & 3 \\ 7 & 3 \end{bmatrix}$
3	$\begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix}$	$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$	$\begin{bmatrix} 1 & 3 \\ 7 & 3 \end{bmatrix}$

Based on the three structures, a model parameter was obtained from the MATLAB. The model was developed using the determined parameter using Equation 1. The summary of the ARX parameter for both X_D and X_B for three different structures was shown in the table below:

Table 2: ARX parameter for three structures

	Structure 1	Structure 2	Structure 3
X_D	$A(z) = 1 - 0.1124\, z^{-1} - 0.6399\, z^{-2}$	$A(z) = 1 - 0.4983\, z^{-1} + 0.0539\, z^{-2} - 0.417\, z^{-3}$	$A(z) = 1 - 0.6127\, z^{-1} + 0.1842\, z^{-2} - 0.5343\, z^{-3} + 0.3666\, z^{-4} - 0.3958\, z^{-5}$
	$A_2(z) = -0.274\, z^{-1} + 0.1408\, z^{-2}$	$A_2(z) = 0.2169\, z^{-1} - 0.3045\, z^{-2} + 0.08961\, z^{-3}$	$A_2(z) = 0.4073\, z^{-1} - 0.3632\, z^{-2} + 0.4342\, z^{-3} - 0.64\, z^{-4} + 0.3253\, z^{-5}$
	$B_1(z) = 0.04153\, z^{-1} + 1.302\, z^{-2}$	$B_1(z) = 0.0239\, z^{-1} + 0.9749\, z^{-2}$	$B_1(z) = 0.01996\, z^{-1} + 0.9819\, z^{-2}$
	$B_2(z) = -1.078\, z^{-3} - 1.245\, z^{-4}$	$B_2(z) = -2.231\, z^{-3} - 0.8298\, z^{-4}$	$B_2(z) = 0.3144\, z^{-3} - 3.848\, z^{-4}$
X_B	$A(z) = 1 - 0.8834\, z^{-1} - 0.06006\, z^{-2}$	$A(z) = 1 - 0.4502\, z^{-1} - 0.4933\, z^{-2} + 0.09545\, z^{-3}$	$A(z) = 1 - 0.3285\, z^{-1} - 0.3871\, z^{-2} + 0.3368\, z^{-3} - 0.6201\, z^{-4} + 0.404\, z^{-5}$
	$A_1(z) = 0.4347\, z^{-1} - 0.3409\, z^{-2}$	$A_1(z) = 0.1768\, z^{-1} + 0.3123\, z^{-2} - 0.4675\, z^{-3}$	$A_1(z) = 0.1326\, z^{-1} + 0.2793\, z^{-2} - 0.5016\, z^{-3} + 0.3656\, z^{-4} - 0.4784\, z^{-5}$
	$B_1(z) = -0.04441\, z^{-7} + 0.5737\, z^{-8}$	$B_1(z) = -0.07994\, z^{-7} + 0.6444\, z^{-8}$	$B_1(z) = -0.1147\, z^{-7} + 0.4734\, z^{-8}$
	$B_2(z) = -1.878\, z^{-3} - 1.226\, z^{-4}$	$B_2(z) = -2.663\, z^{-3} - 1.074\, z^{-4}$	$B_2(z) = -0.5838\, z^{-3} - 3.509\, z^{-4}$

(Take note: the example below shows the first row of the values obtained only when n_K is set to
[1 3;7 3].

Which is simplified in terms of X_D and X_B, yield the equation below

$$X_D = \frac{B_1}{A} R + \frac{B_2}{A_2} S + \frac{1}{A} e(k) \qquad \text{Eq. (4)}$$

$$X_B = \frac{B_1}{A_1} R + \frac{B_2}{A} S + \frac{1}{A_1} e(k) \qquad \text{Eq. (5)}$$

When substitute into Eq. (3) and Eq. (4)

Structure 1:

$$X_D = \frac{0.04153\, z^{-1} + 1.302\, z^{-2}}{1 - 0.1124\, z^{-1} - 0.6399\, z^{-2}} R + \frac{-1.078\, z^{-3} - 1.245\, z^{-4}}{-0.274\, z^{-1} + 0.1408\, z^{-2}} S + \frac{1}{1 - 0.1124\, z^{-1} - 0.6399\, z^{-2}} e(k)$$

$$X_B = \frac{-0.04441\,z^{-7} + 0.5737\,z^{-8}}{0.4347\,z^{-1} - 0.3409\,z^{-2}}\,R + \frac{-1.878\,z^{-3} - 1.226\,z^{-4}}{1 - 0.8834\,z^{-1} - 0.06006\,z^{-2}}\,S + \frac{1}{0.4347\,z^{-1} - 0.3409\,z^{-2}}\,e(k)$$

Structure 2:

$$X_D = \frac{0.0239\,z^{-1} + 0.9749\,z^{-2}}{1 - 0.4983\,z^{-1} + 0.0539\,z^{-2} - 0.417\,z^{-3}}\,R + \frac{-2.231\,z^{-3} - 0.8298\,z^{-4}}{0.2169\,z^{-1} - 0.3045\,z^{-2} + 0.08961\,z^{-3}}\,S + \frac{1}{1 - 0.4983\,z^{-1} + 0.0539\,z^{-2} - 0.417\,z^{-3}}\,e(k)$$

$$X_B = \frac{-0.07994\,z^{-7} + 0.6444\,z^{-8}}{0.1768\,z^{-1} + 0.3123\,z^{-2} - 0.4675\,z^{-3}}\,R + \frac{-2.663\,z^{-3} - 1.074\,z^{-4}}{1 - 0.4502\,z^{-1} - 0.4933\,z^{-2} + 0.09545\,z^{-3}}\,S + \frac{1}{0.1768\,z^{-1} + 0.3123\,z^{-2} - 0.4675\,z^{-3}}\,e(k)$$

Structure 3:

$$X_D = \frac{0.01996\,z^{-1} + 0.9819\,z^{-2}}{1 - 0.6127\,z^{-1} + 0.1842\,z^{-2} - 0.5343\,z^{-3} + 0.3666\,z^{-4} - 0.3958\,z^{-5}}\,R + \frac{0.3144\,z^{-3} - 3.848\,z^{-4}}{0.4073\,z^{-1} - 0.3632\,z^{-2} + 0.4342\,z^{-3} - 0.64\,z^{-4} + 0.3253\,z^{-5}}\,S + \frac{1}{1 - 0.6127\,z^{-1} + 0.1842\,z^{-2} - 0.5343\,z^{-3} + 0.3666\,z^{-4} - 0.3958\,z^{-5}}\,e(k)$$

$$X_B = \frac{-0.1147\,z^{-7} + 0.4734\,z^{-8}}{0.1326\,z^{-1} + 0.2793\,z^{-2} - 0.5016\,z^{-3} + 0.3656\,z^{-4} - 0.4784\,z^{-5}}\,R + \frac{-0.5838\,z^{-3} - 3.509\,z^{-4}}{1 - 0.3285\,z^{-1} - 0.3871\,z^{-2} + 0.3368\,z^{-3} - 0.6201\,z^{-4} + 0.404\,z^{-5}}\,S + \frac{1}{0.1326\,z^{-1} + 0.2793\,z^{-2} - 0.5016\,z^{-3} + 0.3656\,z^{-4} - 0.4784\,z^{-5}}\,e(k)$$

For ARX model under both certain and uncertain time delay, the n_B of the ARX was set constatnt to [2 2;2 2] according to the discussion with supervisor. After the model was run and and the results was compared with the actual data, the given result was obtained.

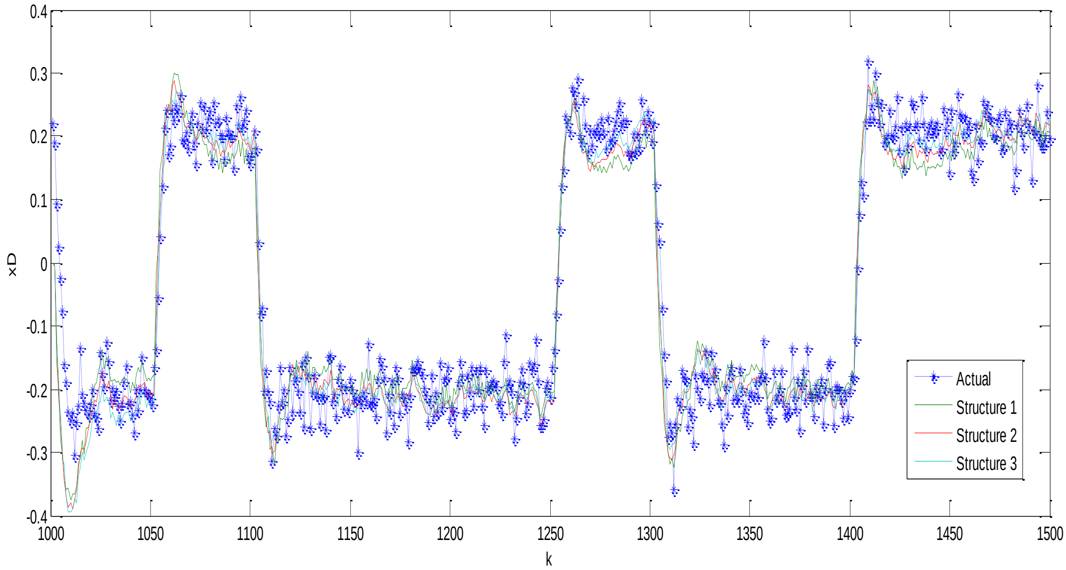


Figure 4: X_D plot of ARX model of different structure compared to Actual Data

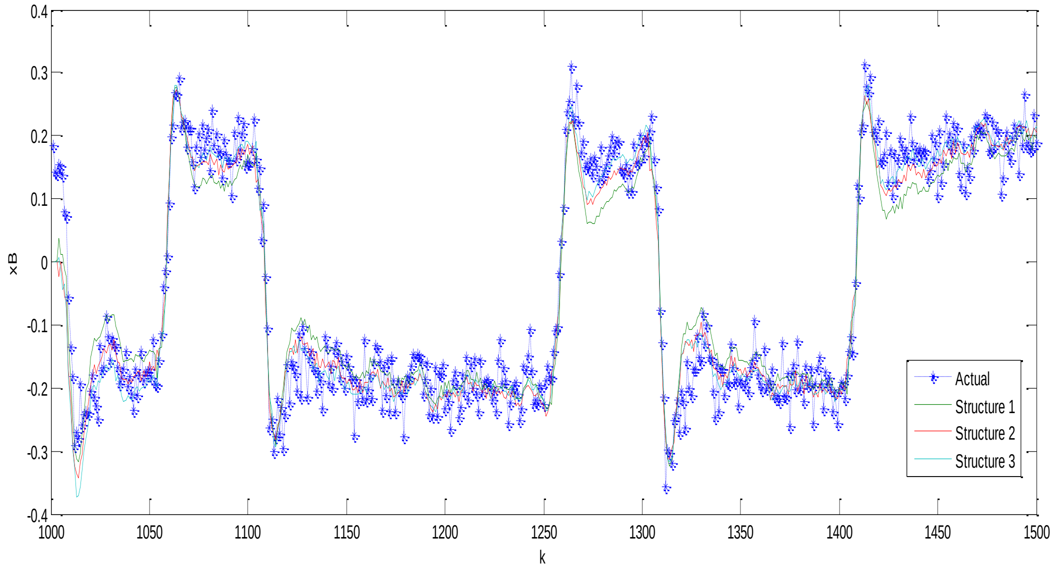


Figure 5: X_B plot of ARX model of different structure compared to Actual Data

From the graph shown above, it is hard to see the difference and deviation between these three structures because there are very close to each other in terms of best fit. However to distinguish between the best model and the actual data, these three structures was examined and compared using fitness table. The result is shown below:

Table 3: Table of fitness for ARX model structure with certain time delays.

	Structure 1	**Structure 2**	**Structure 3**
X_D	73.42	76.36	77.15
X_B	72.95	76.98	77.41
Average	73.19	76.67	77.28

Based on the table of fitness shown above, it can be conclude that structure 3 has the highest average value of fitness compared to Structure 1 and Structure 2. In fact, the value of fitness for X_D and X_B alone was higher compared to the Structure 1 and Structure 2. The plot of best fit of structure 3 compared to the actual data (validation data) was plotted in the graph show below:

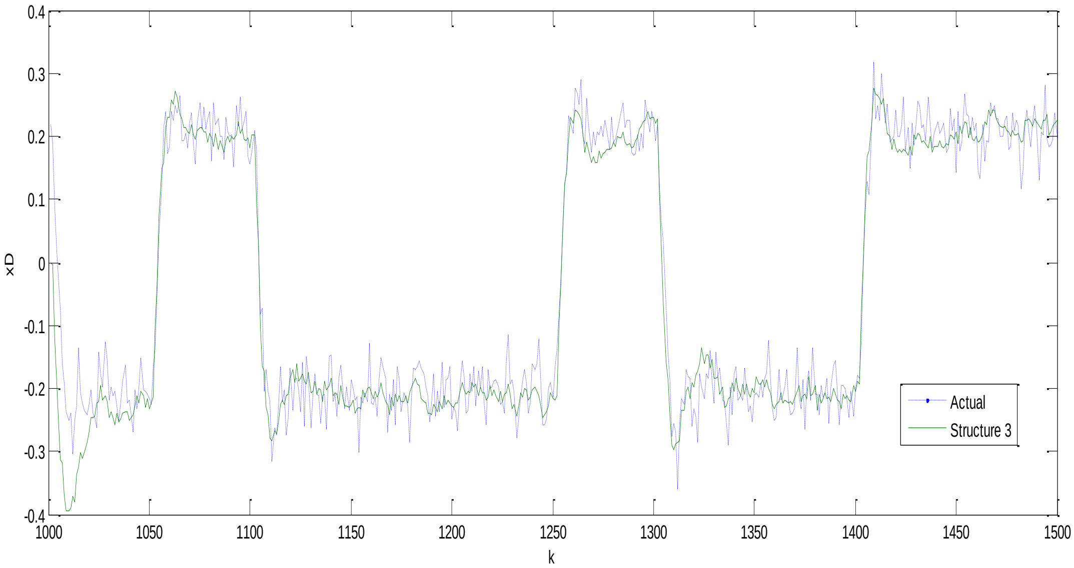


Figure 6: Best of X_D plotted against actual data for Structure 3

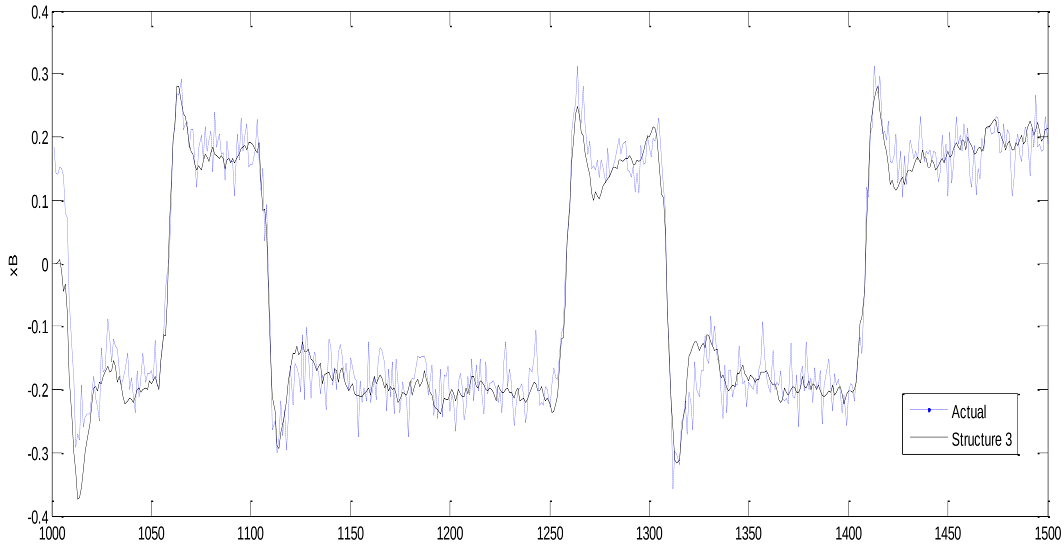


Figure 7: Best X_B plotted against actual data for Structure 3

4.1.2 OBF-ARX Model development from SIMULINK MATLAB

For OBF-ARX model structure, there are also 3 cases (structure) studied which is Structure 1, Structure 2 and Structure 3. The orders of n_A is kept constant by using order of $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$, however the n_{OBF} manipulated with different value. The structeres contain combination of different polynomial orders as stated in the table below:

Table 4: Table of different ARX model structure under certain time delays condition

OBF-ARX Structure	n_A	N_{OBF}	n_K
1	$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$	$\begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}$	$\begin{bmatrix} 1 & 3 \\ 7 & 3 \end{bmatrix}$
2	$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$	$\begin{bmatrix} 6 & 6 \\ 6 & 6 \end{bmatrix}$	$\begin{bmatrix} 1 & 3 \\ 7 & 3 \end{bmatrix}$
3	$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$	$\begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix}$	$\begin{bmatrix} 1 & 3 \\ 7 & 3 \end{bmatrix}$

For OBF-ARX model under both certain and uncertain time delay, the n_A of the OBF-ARX was set constant to [2 2;2 2] according to the discussion with supervisor. The n_A value would not affecting the result too much, thus it remains suitable of [2 2;2 2] After the model was run and and the results was compared with the actual data, the given result was obtained.

Result of Structure 1:

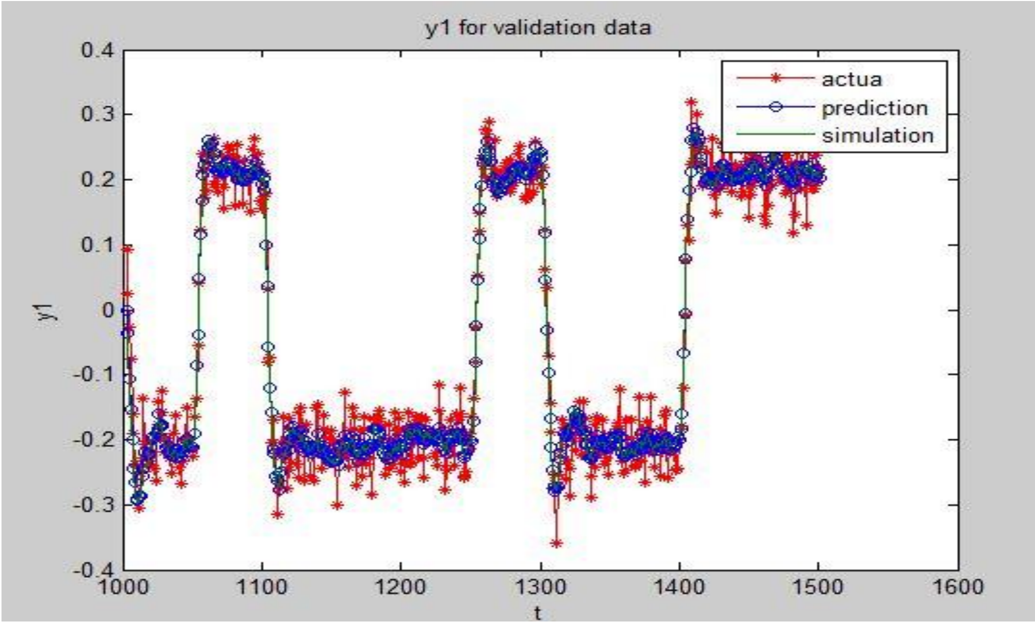


Figure 8: OBF-ARX Structure 1 for X_D

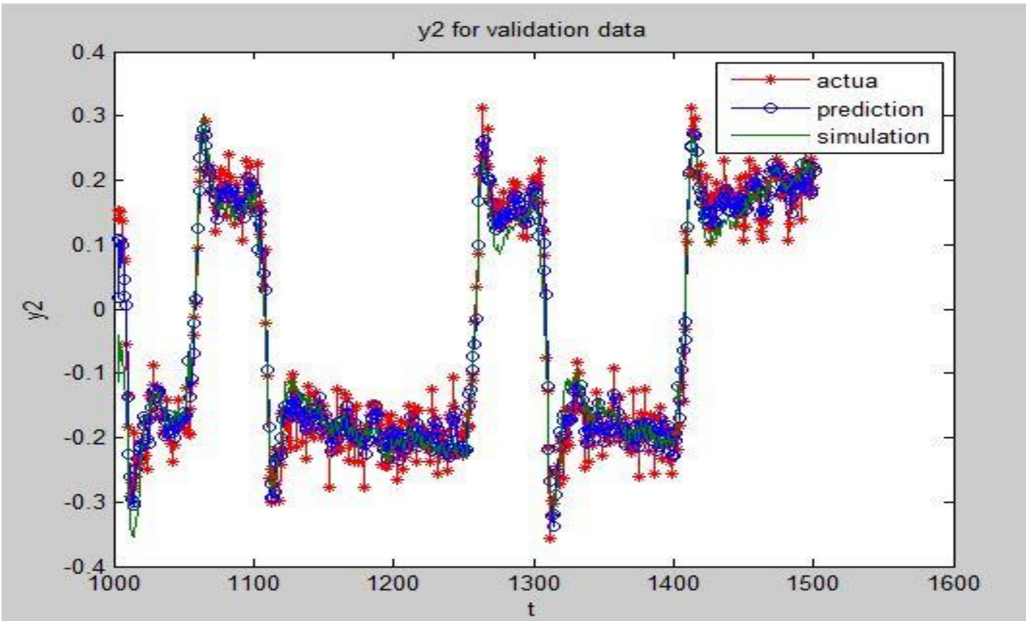


Figure 9: OBF-ARX Structure 1 for X_B

Result of Structure 2:

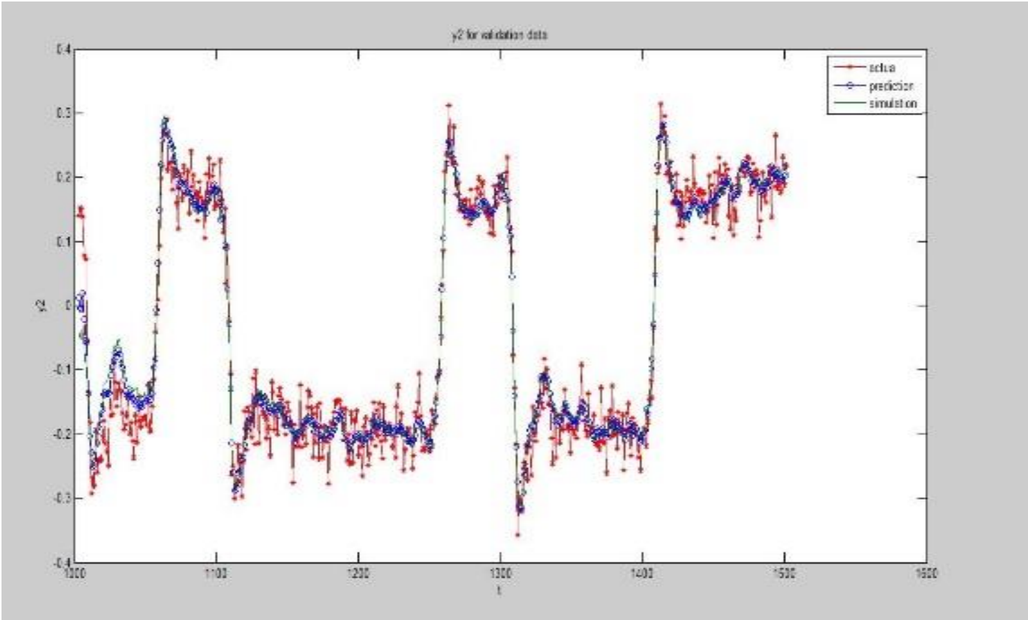


Figure 10: OBF-ARX Structure 2 for X_D

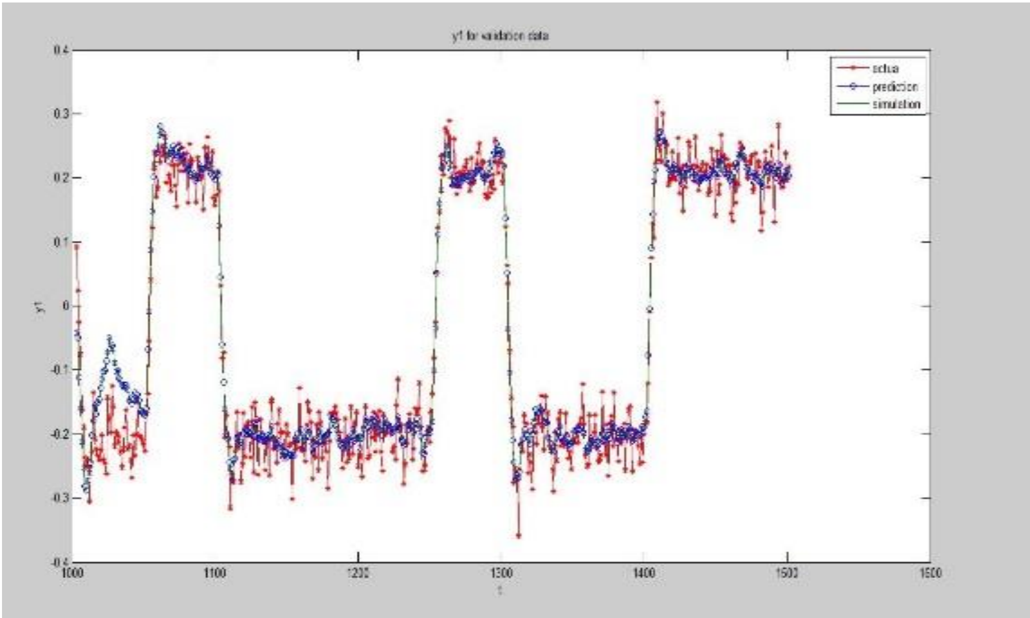


Figure 11: OBF-ARX Structure 2 for X_B

Result of Structure 3:

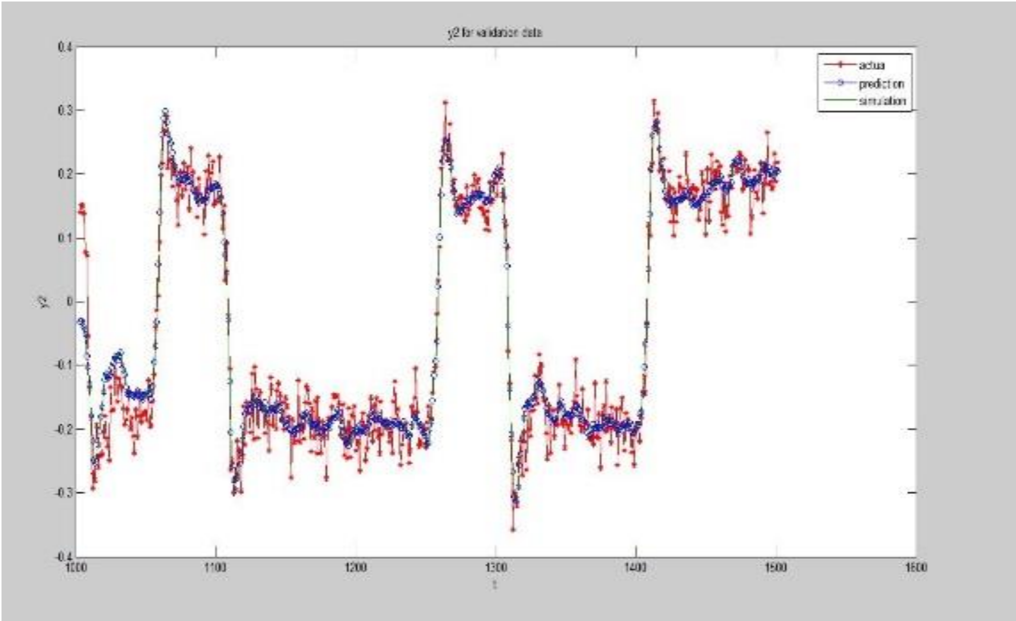


Figure 12: ARX Structure 3 for X_D

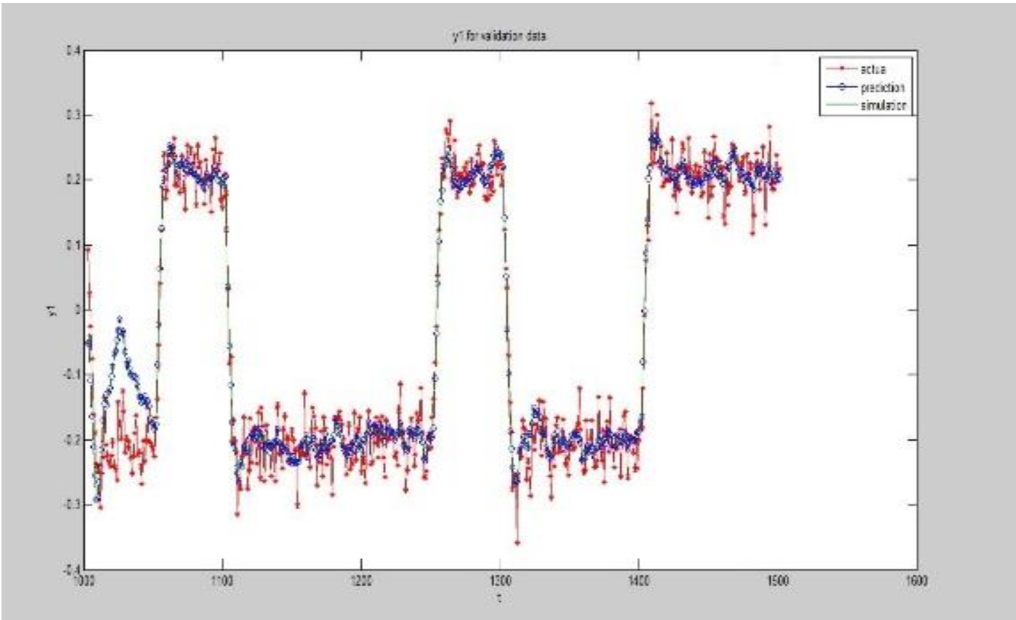


Figure 13: OBF-ARX Structure 3 for X_B

To distinguish between best model and the actual data, these three (3) structures was compared using fitness table mentioned earlier. The summary of the result data is shown below:

Table 5: Table of fitness for OBF-ARX model structure with certain time delays.

	Structure 1	**Structure 2**	**Structure 3**
x_D	83.97	79.56	77.63
x_B	79.19	78.61	78.27
Average	81.58	79.09	77.95

From the tabulated data of OBF-ARX model, it seems that under certain time delays, Structure 1 have the highest average of fitness which is 81.58% compared to rest two model which have only slight different with each other. However, more clear result will be obtain by further studies under uncertain time delay condition for three structures.

4.2 Uncertain Time Delays

4.2.1 ARX Model development from SIMULINK MATLAB

Under thie condition, there are also thrre structeres contain combination of different polynomial orders as stated in the table below. However, for time delays, randomly we use $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$, as if the time delays was not known just like in the actual plant.

Table 6: Table of different ARX model structure under uncertain time delays condition

ARX Structure	n_A	n_B	n_K
1	$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$	$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$	$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$
2	$\begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix}$	$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$	$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$
3	$\begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix}$	$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$	$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

Based on the three structures, a model parameter was obtained for uncertain time delays from the MATLAB. The model was developed using the determined parameter using Equation 1. The summary of the ARX parameter for both X_D and X_B for three different structures was shown in the table below:

Table 7: The summary of the ARX parameter for both X_D and X_B for three different structures

	Structure 1	Structure 2	Structure 3
X_D	$A(z) = 1 - 0.3012\ z^{-1} - 0.5467\ z^{-2}$	$A(z) = 1 - 0.4716\ z^{-1} - 215.6\ z^{-2} + 201.7\ z^{-3}$	$A(z) = 1 - 0.2509\ z^{-1} - 295.7\ z^{-2} + 276.5\ z^{-3} + 0.2041\ z^{-4} - 0.1995\ z^{-5}$
	$A_2(z) = -0.06294\ z^{-1} + 0.03122\ z^{-2}$	$A_2(z) = 0.3087\ z^{-1} + 215.5\ z^{-2} - 202\ z^{-3}$	$A_2(z) = 0.1796\ z^{-1} + 295.7\ z^{-2} - 276.5\ z^{-3} - 0.466\ z^{-4} + 0.1591\ z^{-5}$
	$B_1(z) = 1.304\ z^{-2} - 0.4022\ z^{-3}$	$B_1(z) = 692.8\ z^{-2} - 687.9\ z^{-3}$	$B_1(z) = 948.6\ z^{-2} - 942.3\ z^{-3}$
	$B_2(z) = -0.6752\ z^{-2} - 1.932\ z^{-3}$	$B_2(z) = 3291\ z^{-2} - 3300\ z^{-3}$	$B_2(z) = 4508\ z^{-2} - 4519\ z^{-3}$
X_B	$A(z) = 1 - 1.23\ z^{-1} + 0.4184\ z^{-2}$	$A(z) = 1 - 0.8572\ z^{-1} + 216.7\ z^{-2} - 202.7\ z^{-3}$	$A(z) = 1 - 0.7799\ z^{-1} + 268.3\ z^{-2} - 250.7\ z^{-3} - 0.5373\ z^{-4} + 0.3701\ z^{-5}$
	$A1(z) = 0.8744\ z^{-1} - 0.9323\ z^{-2}$	$A_1(z) = 0.6999\ z^{-1} - 216.8\ z^{-2} + 202.5\ z^{-3}$	$A_1(z) = 0.7123\ z^{-1} - 268.3\ z^{-2} + 250.7\ z^{-3} + 0.276\ z^{-4} - 0.4309\ z^{-5}$
	$B_1(z) = 1.037\ z^{-2} - 0.6904\ z^{-3}$	$B_1(z) = 695.2\ z^{-2} - 690.9\ z^{-3}$	$B1(z) = 859.2\ z^{-2} - 854.2\ z^{-3}$
	$B_2(z) = 1.671\ z^{-2} - 4.425\ z^{-3}$	$B_2(z) = 3306\ z^{-2} - 3315\ z^{-3}$	$B2(z) = 4086\ z^{-2} - 4097\ z^{-3}$

(Take note: the example below shows the first row of the values obtained only when n_K is set to

[2 2;2 2]. These parameters was substitute into Eq.(3) and Eq.(4) which is simplified in terms of X_D and X_B, yield the equation below:

Structure 1:

$$X_D = \frac{1.304\ z^{-2} - 0.4022\ z^{-3}}{1 - 0.3012\ z^{-1} - 0.5467\ z^{-2}} R + \frac{-0.6752\ z^{-2} - 1.932\ z^{-3}}{-0.06294\ z^{-1} + 0.03122\ z^{-2}} S + \frac{1}{1 - 0.3012\ z^{-1} - 0.5467\ z^{-2}} e(k)$$

$$X_B = \frac{1.037\ z^{-2} - 0.6904\ z^{-3}}{0.8744\ z^{-1} - 0.9323\ z^{-2}} R + \frac{1.671\ z^{-2} - 4.425\ z^{-3}}{1 - 1.23\ z^{-1} + 0.4184\ z^{-2}} S + \frac{1}{0.8744\ z^{-1} - 0.9323\ z^{-2}} e(k)$$

Structure 2:

$$X_D = \frac{692.8\,z^{-2} - 687.9\,z^{-3}}{1 - 0.4716\,z^{-1} - 215.6\,z^{-2} + 201.7\,z^{-3}}\,R + \frac{3291\,z^{-2} - 3300\,z^{-3}}{0.3087\,z^{-1} + 215.5\,z^{-2} - 202\,z^{-3}}\,S + \frac{1}{1 - 0.4716\,z^{-1} - 215.6\,z^{-2} + 201.7\,z^{-3}}\,e(k)$$

$$X_B = \frac{695.2\,z^{-2} - 690.9\,z^{-3}}{0.6999\,z^{-1} - 216.8\,z^{-2} + 202.5\,z^{-3}}\,R + \frac{3306\,z^{-2} - 3315\,z^{-3}}{1 - 0.8572\,z^{-1} + 216.7\,z^{-2} - 202.7\,z^{-3}}\,S + \frac{1}{0.6999\,z^{-1} - 216.8\,z^{-2} + 202.5\,z^{-3}}\,e(k)$$

Structure 3:

$$X_D = \frac{948.6\,z^{-2} - 942.3\,z^{-3}}{1 - 0.2509\,z^{-1} - 295.7\,z^{-2} + 276.5\,z^{-3} + 0.2041\,z^{-4} - 0.1995\,z^{-5}}\,R + \frac{4508\,z^{-2} - 4519\,z^{-3}}{0.1796\,z^{-1} - 295.7\,z^{-2} - 276.5\,z^{-3} - 0.466\,z^{-4} + 0.1591\,z^{-5}}\,S + \frac{1}{1 - 0.2509\,z^{-1} - 295.7\,z^{-2} + 276.5\,z^{-3} + 0.2041\,z^{-4} - 0.1995\,z^{-5}}\,e(k)$$

$$X_B = \frac{859.2\,z^{-2} - 854.2\,z^{-3}}{0.7123\,z^{-1} - 268.3\,z^{-2} + 250.7\,z^{-3} + 0.276\,z^{-4} - 0.4309\,z^{-5}}\,R + \frac{4086\,z^{-2} - 4097\,z^{-3}}{1 - 0.7799\,z^{-1} + 268.3\,z^{-2} - 250.7\,z^{-3} - 0.5373\,z^{-4} + 0.3701\,z^{-5}}\,S + \frac{1}{0.7123\,z^{-1} - 268.3\,z^{-2} + 250.7\,z^{-3} + 0.276\,z^{-4} - 0.4309\,z^{-5}}\,e(k)$$

After run the model and comparing with the actual data, the given result was obtained.

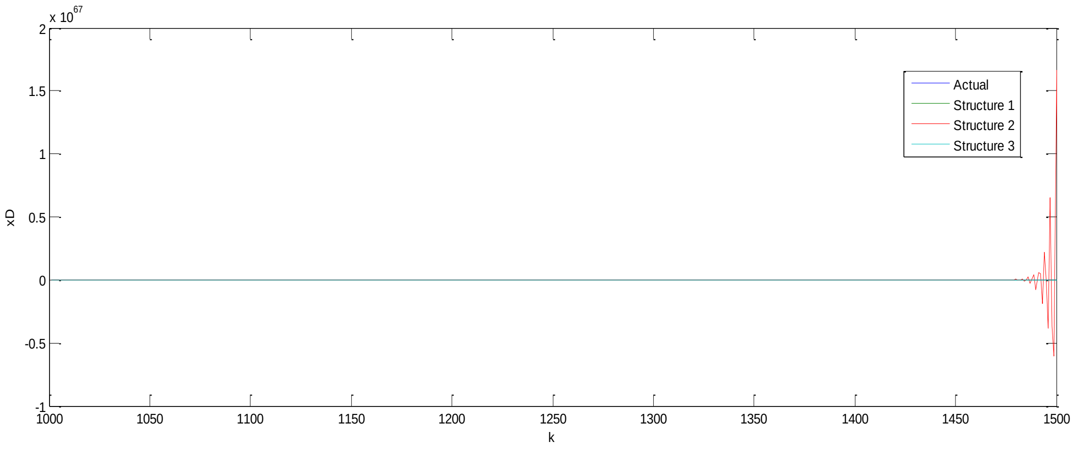


Figure 14: X_D plot of ARX model of different structure compared to Actual Data

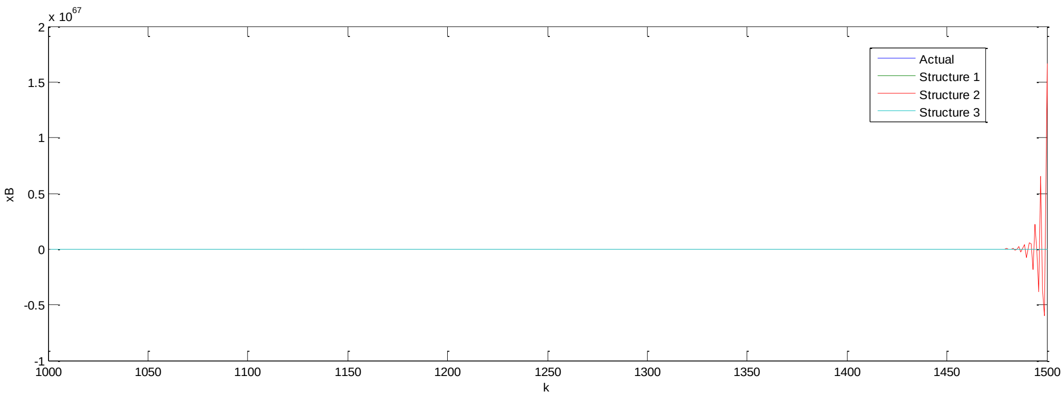


Figure 15: X_B plot of ARX model of different structure compared to Actual Data

From the graph show above, it is clearly seen the some of the parameters cannot be estimated using ARX model under uncertain time delays. This shows that ARX cannot perform best under uncertain time delays. However to distinguish between best model and the actual data, these three structures was compared using fitness table. The data is shown below:

Table 8: Table of fitness for ARX model structure with uncertain time delays.

	Structure 1	Structure 2	Structure 3
x_D	74.39	-1.9E+30	Can't Predict
x_B	73.37	-2.11+30	Can't Predict
Average	73.88	-2.00+30	-

Based on the table of fitness shown above, it can be conclude that structure 1 has the highest average value of fitness for both X_D and X_B. The plot of best fit of structure 1 compared to the actual data was plotted in the graph shown below:

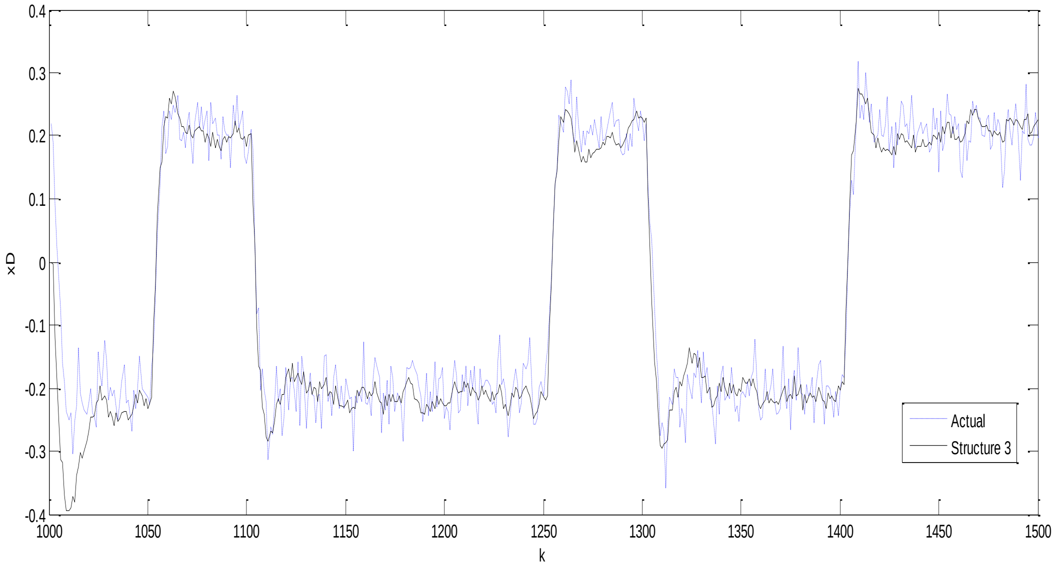


Figure 16: Best X_D plotted against Actual Data for Structure 1

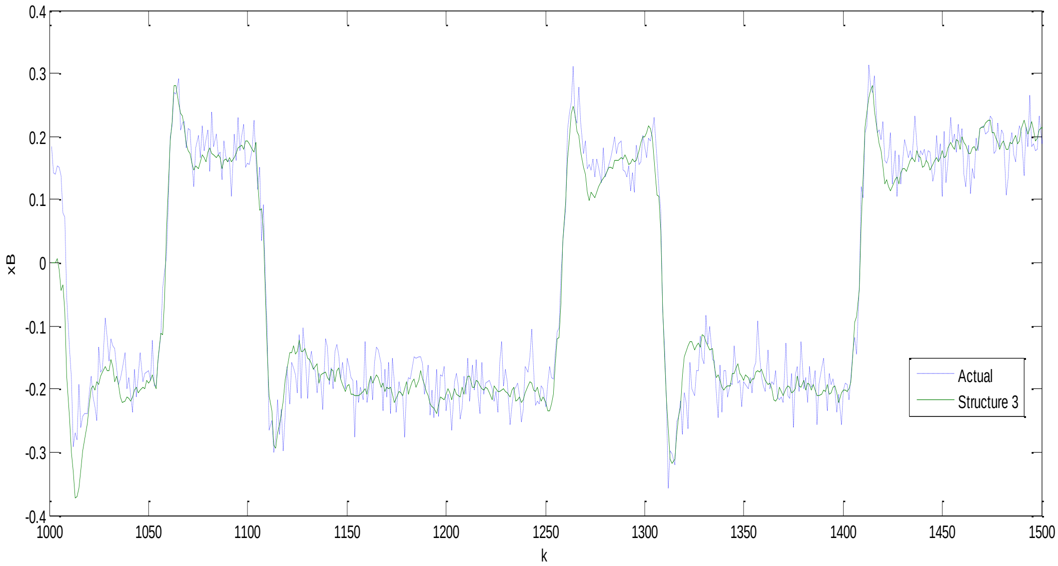


Figure 17: Best X_B plotted against Actual Data for Structure 1

4.2.2 OBF-ARX Model development from SIMULINK MATLAB

The orders of n_A is kept constant by using order of $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$, and also the n_K value also is set to $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$ because for $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, the model would not show much different and some of the strcuture cannot be predicted. However the n_{OBF} is manipulated with different value as in the certain time delays condition model structure. The structures contain combination of different polynomial orders as stated in the table below:

Table 9: Table of different ARX model structure under certain time delays condition

OBF-ARX Structure	$\mathbf{n_A}$	$\mathbf{n_{OBF}}$	$\mathbf{n_K}$
1	$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$	$\begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}$	$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$
2	$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$	$\begin{bmatrix} 6 & 6 \\ 6 & 6 \end{bmatrix}$	$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$
3	$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$	$\begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix}$	$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

For OBF-ARX model under uncertain time delay, the n_A of the OBF-ARX was set constant to [2 2;2 2] according to the discussion with supervisor. The n_A value would not affecting the result too much, thus it remains suitable of [2 2;2 2] After the model was run and and the results was compared with the actual data, the given result was obtained.

Result of Structure 1:

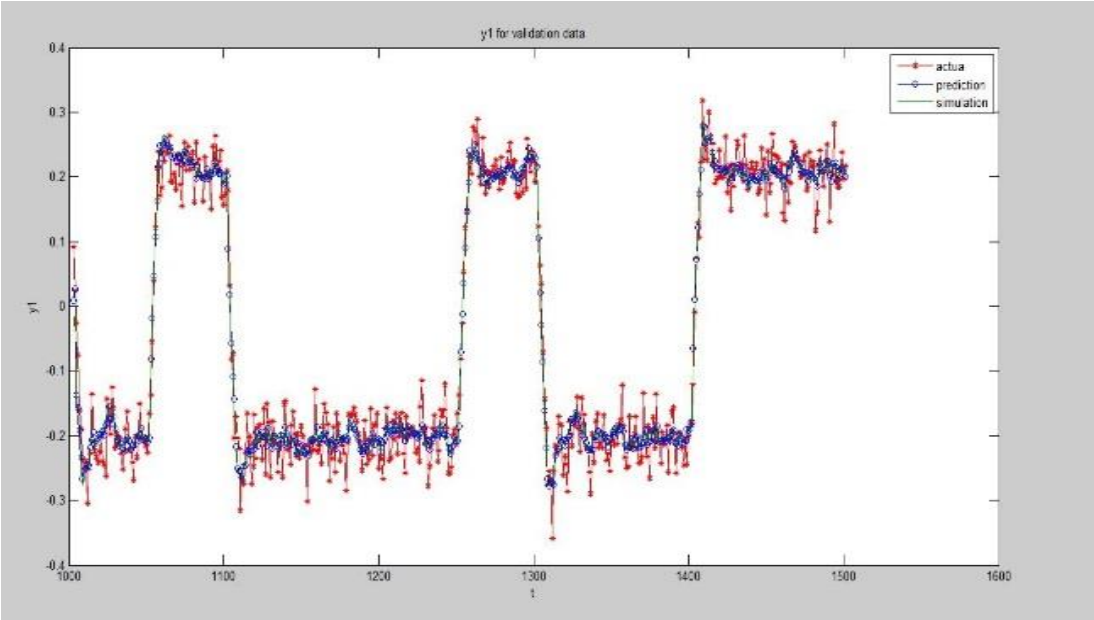


Figure 18: OBF-ARX Structure 1 for X_D

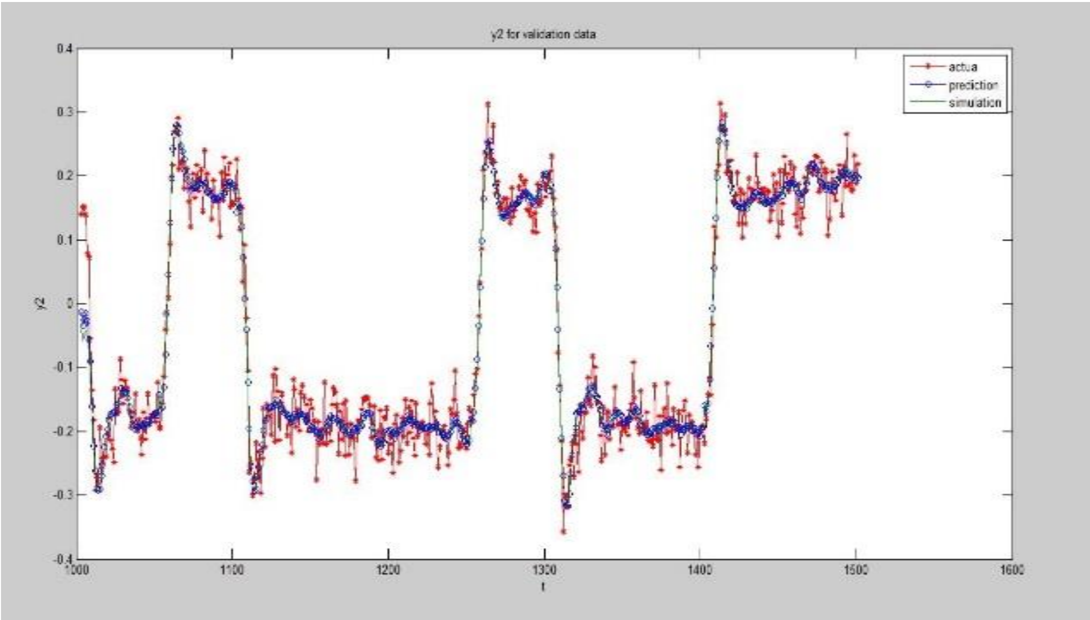


Figure 19: OBF-ARX Structure 1 for X_B

Result of Structure 2:

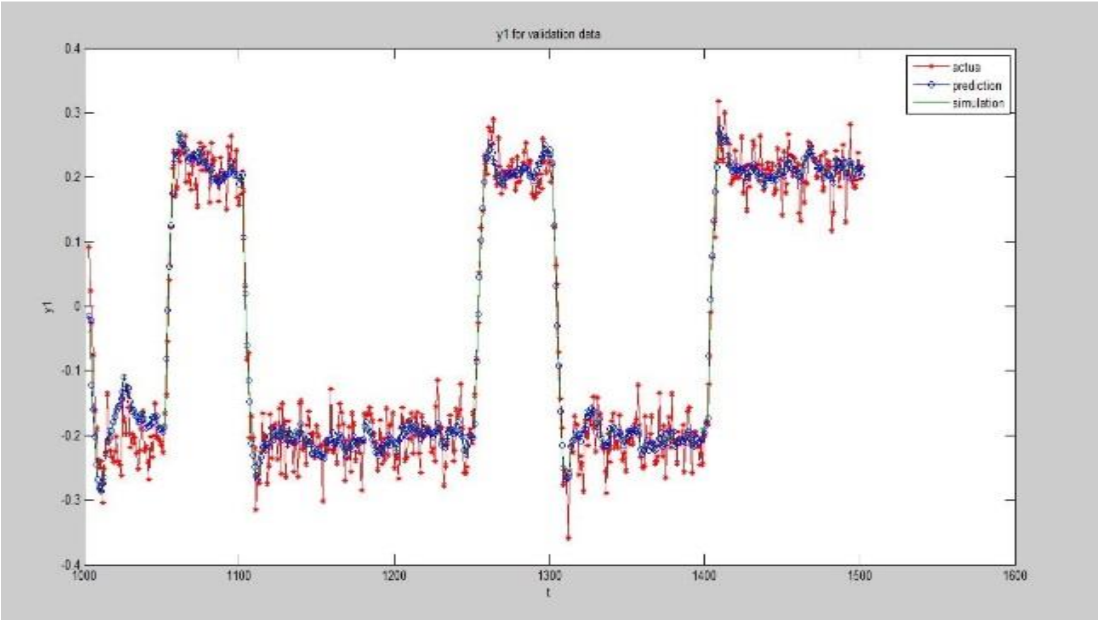


Figure 20: OBF-ARX Structure 2 for X_D

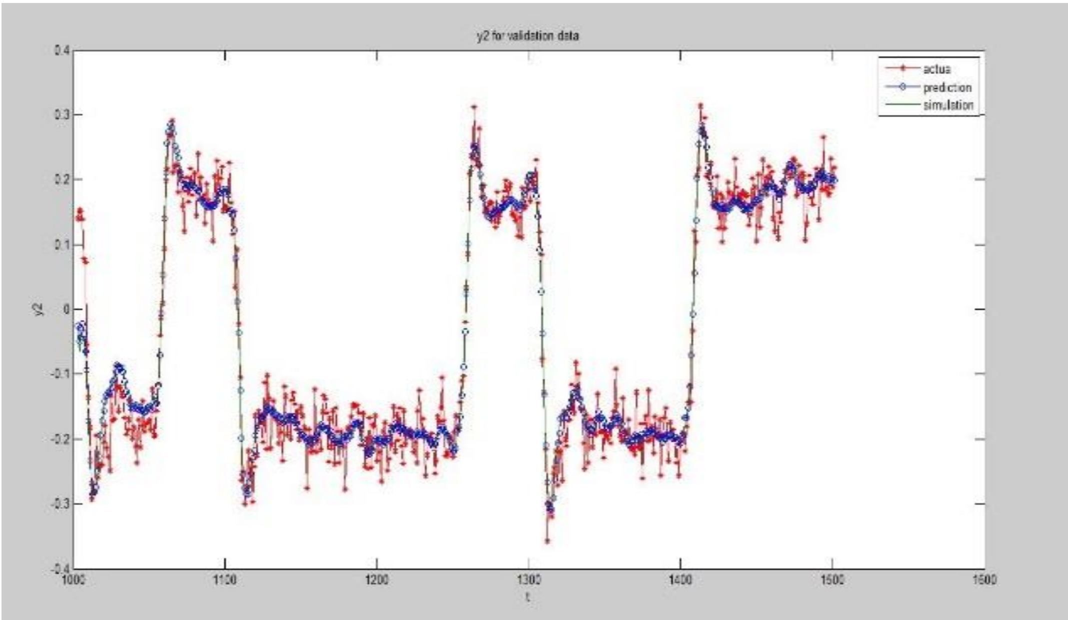


Figure 21: OBF-ARX Structure 2 for X_B

Result of Structure 3:

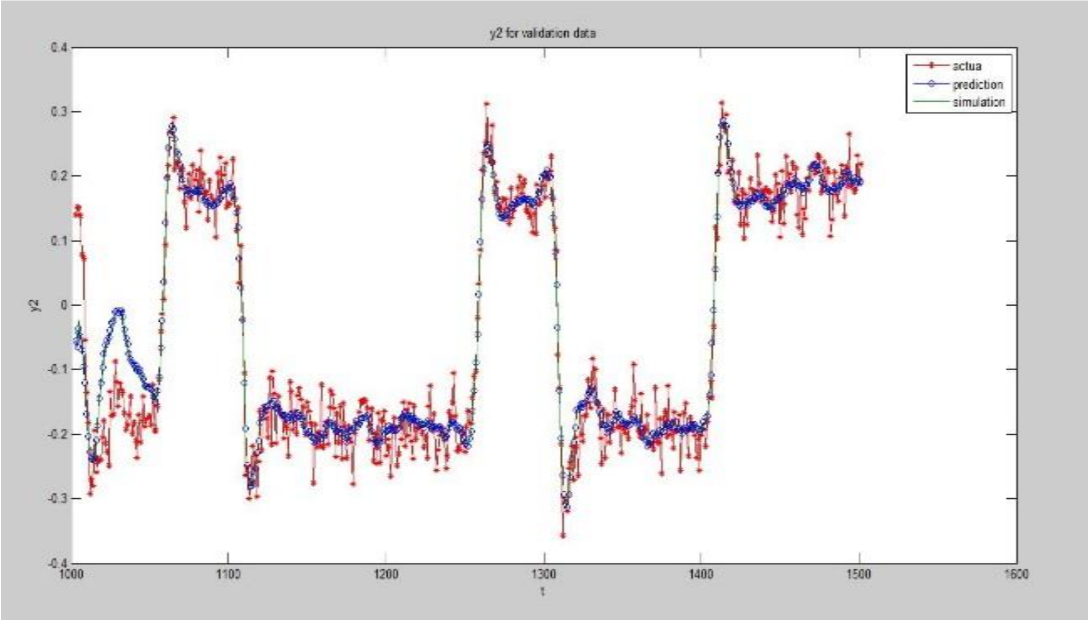


Figure 22: OBF-ARX Structure 3 for X_D

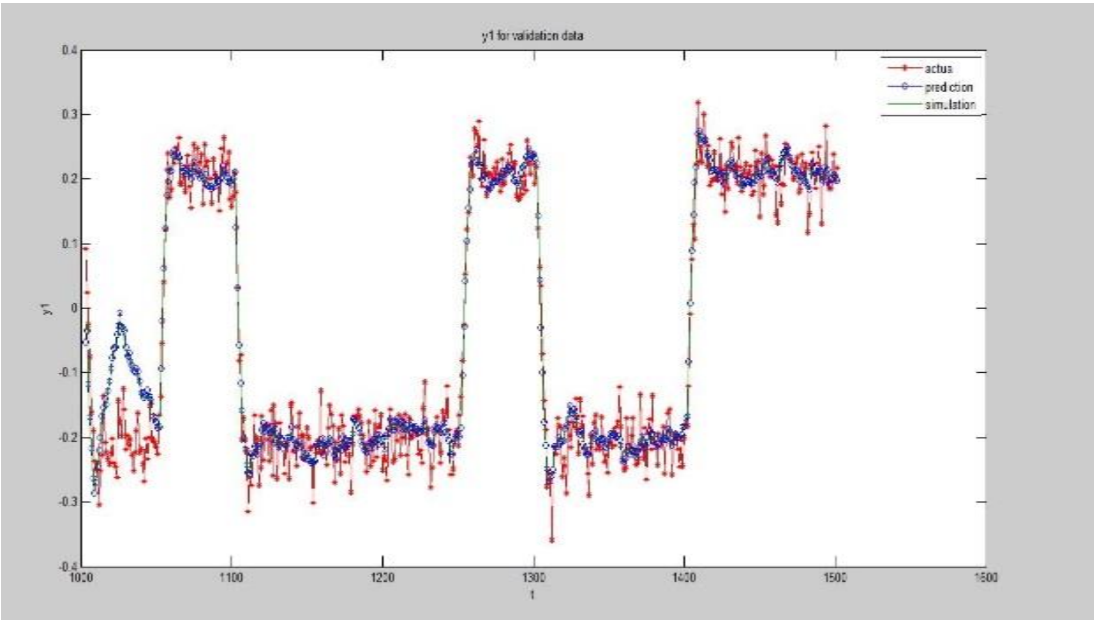


Figure 23: OBF-ARX Structure 3 for X_B

To distinguish between best model and the actual data, these three structures was compared using fitness table. The data is shown below:

Table 10: Table of fitness for OBF-ARX model structure with uncertain time delays.

	Structure 1	**Structure 2**	**Structure 3**
x_D	83.87	82.88	76.90
x_B	80.15	78.72	73.16
Average	82.01	80.80	75.03

For OBF-ARX model with uncertain time delays, it is found that structure 1 give the best fit of the data. A comparison data was tabulated between ARX and OBF-ARX time delays to see the different more clearer.

4.3 Comparing the best ARX and OBF-ARX Structure

4.3.1 Certain time delays condition

This part will further discuss about the comparison of the best structure of ARX and OBF-ARX obtained under certain time delays condition. For the ARX model under certain time delays, Structure 3 is proven to be the best structure as it has the highest average value of fitness. For OBF-ARX, Structure 1 seems the best structure model under this condition. These two best fit graph are the plot together against the actual data (validation data).

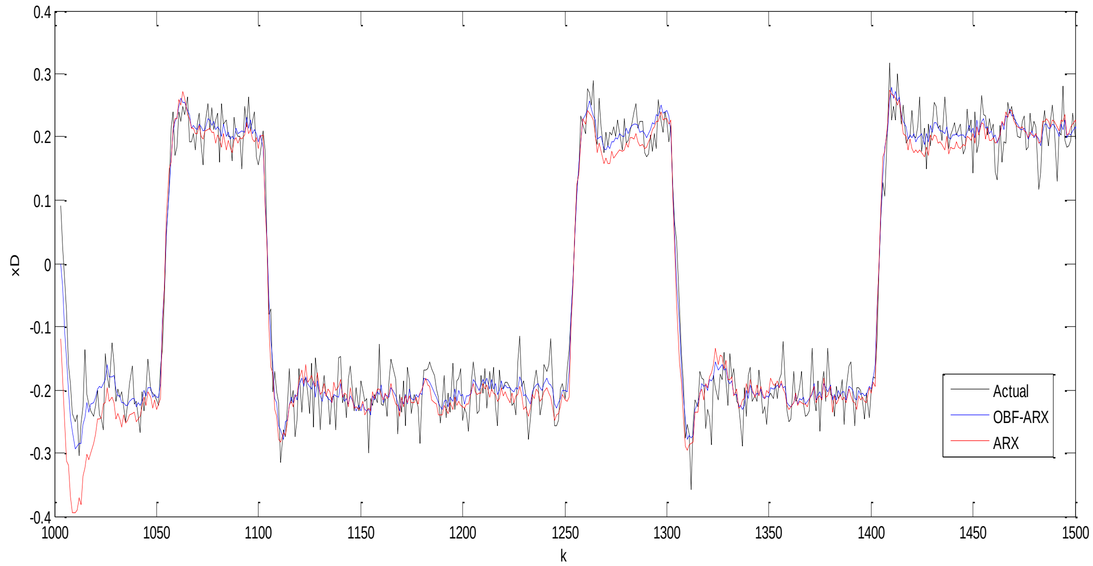


Figure 24: Best plot of X_D OBF-ARX with X_D ARX of Certain Time Delays

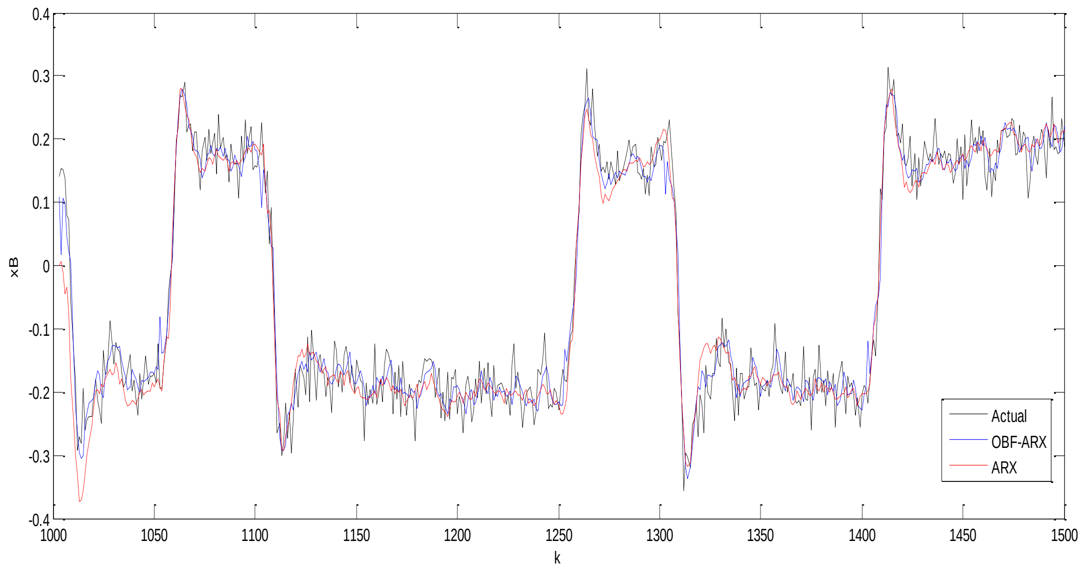


Figure 25: Best plot of X_B OBF-ARX with X_B ARX of Certain Time Delays

As we can see for certain time delays, ARX and OBF-ARX gives about similar approximation towards the actual data. From the two graph of X_D and X_B, these two best fit graph is very close to the actual data and it can be concluded under certain time delays, not much comparison can be made as the two model ARX and OBF-ARX shows about the same curve.

4.3.2 Uncertain time delays condition

In order to differentiate the structure of ARX and OBF-ARX, it need to be compared under uncertain time delays. It is because in actual condition of the plant, this such condition of unknown time delay will happen as it is impossible to shut down the whole operation of the plant to examine and extract the data from the distillation column. In actual condition, the distillation column also might work with uncertainties such as variety of feed composition, time, and different desired output.

Thus this steps is comparing the two best model for X_D and X_B for both model under uncertain time delay to know which one showing more promising characteristic and more reliable. The comparison of the best plot of ARX and OBF-ARX against actual data was shown as follows:

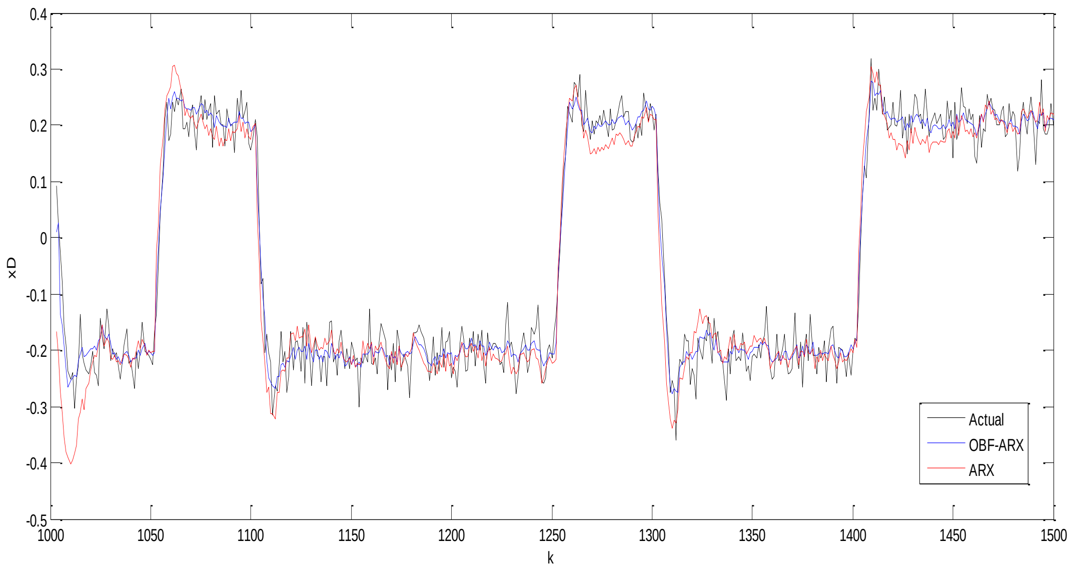


Figure 26: Best plot of X_D OBF-ARX with X_D ARX of Uncertain Time Delays

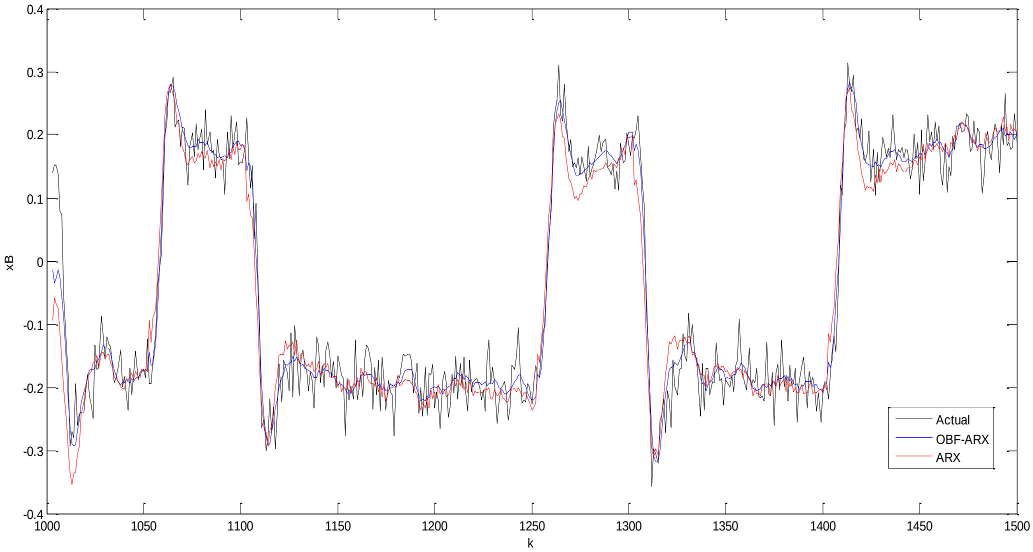


Figure 27: Best plot of X_B OBF-ARX with X_B ARX of Uncertain Time Delays

From the graph above, the graphs was examine in terms of fit in fitness table to give a clear picture of the deviation for each model structure against the actual data. The result is shown as follows:

Table 11: Comparison of fitness for X_D and X_B plot of OBF-ARX and ARX for both condition of time delays

	Certain Time Delay (%)		**Uncertain Time Delay (%)**	
	ARX (Structure 3)	**OBF-ARX** (Structure 1)	**ARX** (Structure 1)	**OBF-ARX (Structure 1)**
X_D	77.9	84.04	75.11	84.09
X_B	78.06	79.88	73.91	80.52

Thus from the fitness table above, it can be conclude that OBF-ARX has the best fits against all compared to the actual data under uncertain time delays.

CHAPTER 5:
CONCLUSION AND RECOMMENDATION

Distillation Column plays a very important role in the industries today. It is crucial to identify the effect of the controller parameters on the closed loop system identification, especially for a Multiple-Input Multiple-Output (MIMO) system. In this project, it is observed that different parameters give different effects on the closed loop system identification. And as the parameter of the certain input, varies and becomes bigger, some output will experience and awkward behaviour. For OBF-ARX under certain time delay and uncertain time delay if we compared, it does not depend on the condition of the time delay because the value of the fitness is almost the same whether the time delay is known or not known. However for ARX, for some cases it cannot be predicted and when the number of parameter is changed to certain value, it shows that the predicted fitness has a significance between both time delay conditions. It can can be conclude that OBF-ARX models have a promising characteristic which make them very reliable for control relevant system identification compared to conventional ARX models. A test was run with 1000 modelling data and 500 validation data shows that OBF-ARX has better fitness for certain time delay condition and even better in uncertain time delay condition. Thus it is an effective model for modelling system with uncertain time delays and also can be used in both open-loop and closed-loop identifications.

For future improvement, this comparison can be developed and examine further using residual error analysis. In the residual error analysis, the error of the model can be plotted to see the deviation of the error whether it increase, decrease or remain the same. Any changes in the error plot indicates that a certain process need to increase more bias or to reduce it. This method is more effective for comparing the models in actual plant condition and it is done by day-to-day monitoring activities. For early development of comparing the models, fitness table can be done however this techniques need to be comprehend with residual error analysis by plotting the error and scatter plot to see the regression is good or bad before the model being taken and implemented in the plant. This is to ensure to consistency of the models and to reduce the risk of operating the plant.

REFERENCES

Chinedu Agbalaka and Deanbair Oliver. Application of the EnKF and localization to automatic history matching of facies distribution and production data. Mathematical Geosciences, 40(4):353{374, May 2008. doi: 10.1007/s11004-008-9155-7.

Nagarajaiah, S. *System identification*. 2009, July 7. Retrieved February 22, 2014, from: http://sstl.cee.illinois.edu/apss/files/9aSNUIUCShortCourse1.pdf

Ljung, L., *System identification theory for the user*. 2nd ed. 1999, New Jersey: Prentice Hall. 609.

Nagarajaiah, S. *System identification*. 2009, July 7. Retrieved February 22, 2014, from: http://sstl.cee.illinois.edu/apss/files/9aSNUIUCShortCourse1.pdf

Instruments, N. *Selecting a model structure in the system identification process*. 2010 Retrieved February 22, 2014, from http://www.ni.com/white-paper/4028/en/.*arx*.

[Heuberger, P. S. C., Van den Hof, P. M. J. and Wahlberg, B. (2005). Modelling and Identification with Rational Orthogonal Basis Functions, Springer.

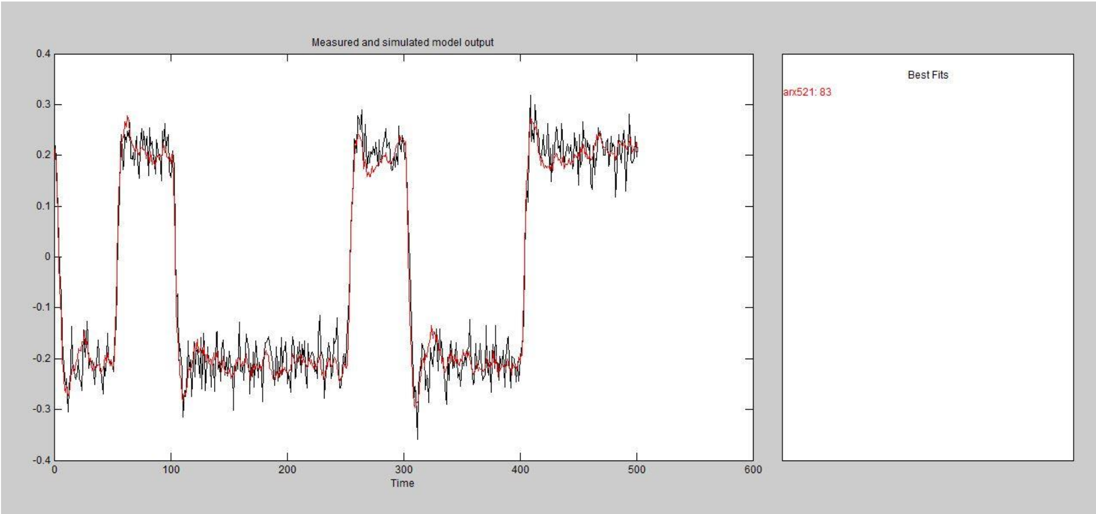
ARX. (n.d.) Retrieved August 20, 2014, from http://dali.feld.cvut.cz/ucebna/matlab/toolbox/ident/arx.html.

Nelles, O., *Nonlinear System Identification*. 2001, New York: Springer. 785.

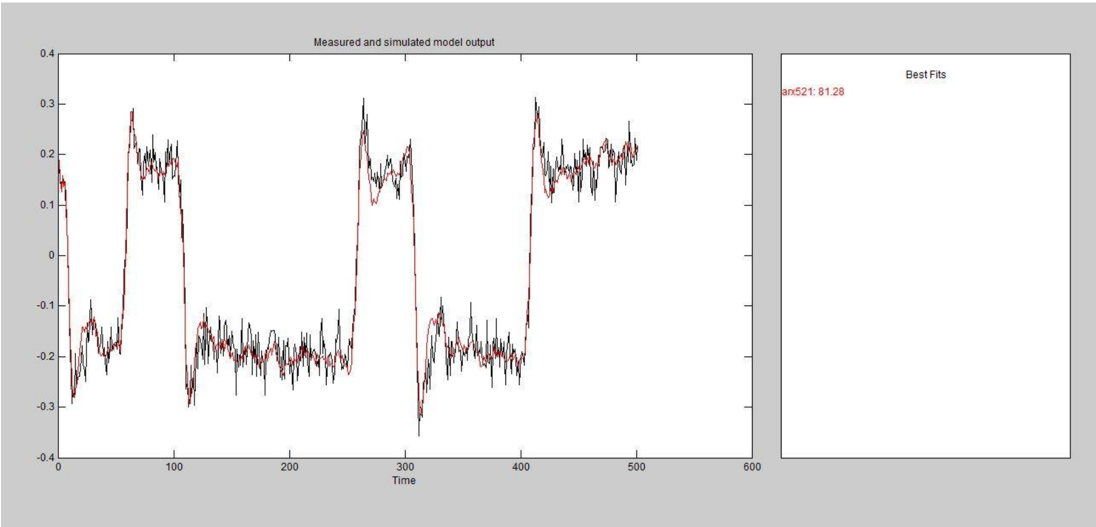
Lemma D. Tufa and M. Ramasamy, System Identification Using Orthonormal Basis Filter. Retrieved September 2, 2014, from http://www.intechopen.com/download/pdf/37953

M. Gevers, L. Ljung, Optimal experiment designs with respect to the intended model application Automatica, 22 (5) (1986), pp. 543–554

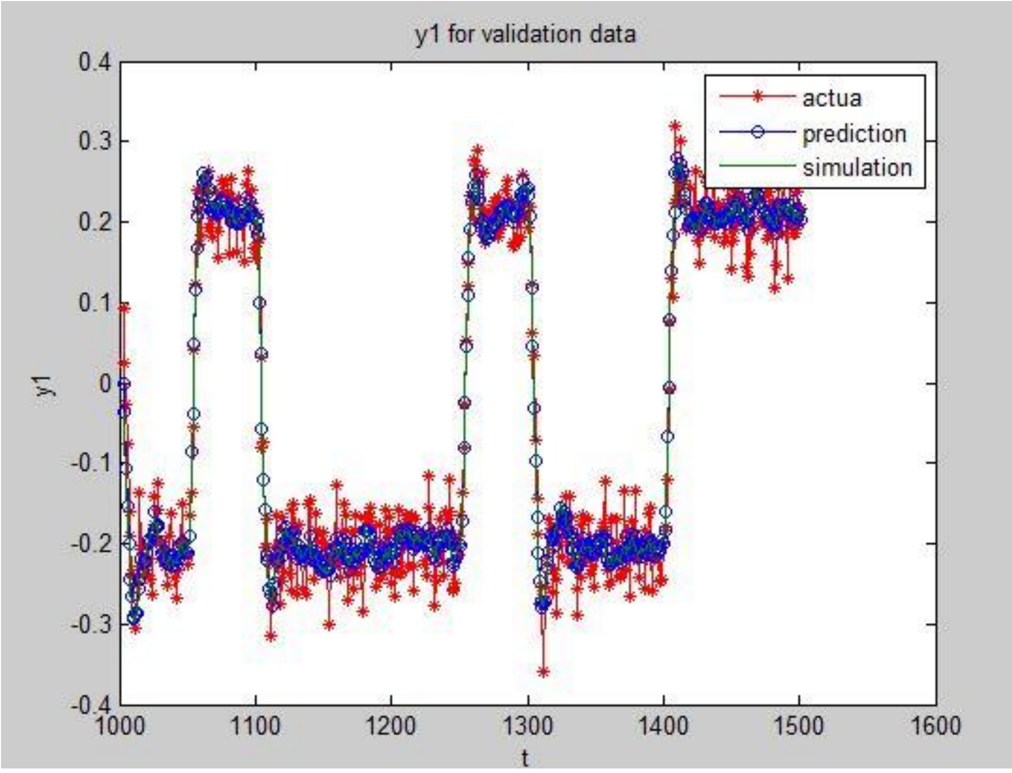
APPENDICES



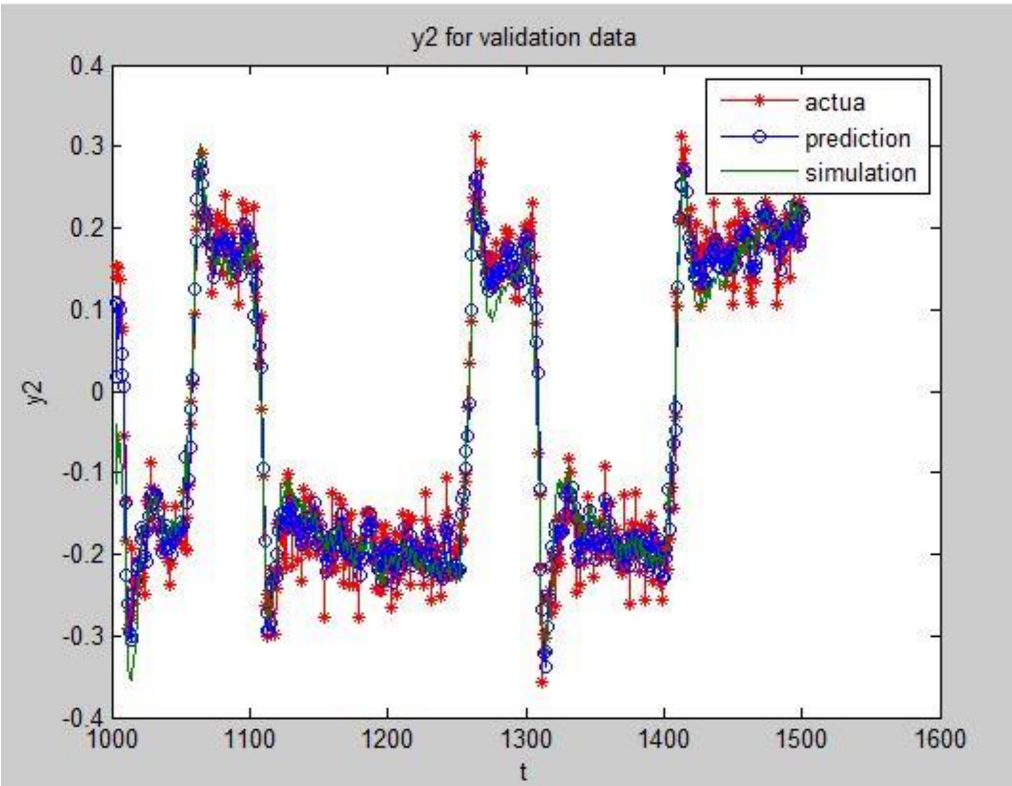
Appendix 1: Structure 3 - The best Structure of ARX for X_D under certain time delay



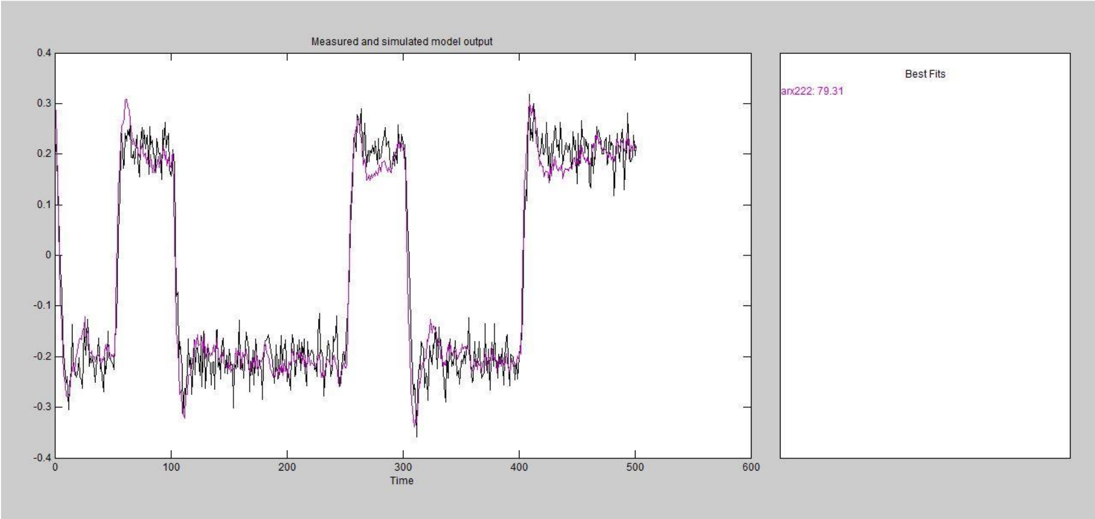
Appendix 2: Structure 3 - The best Structure of ARX for X_B under certain time delay



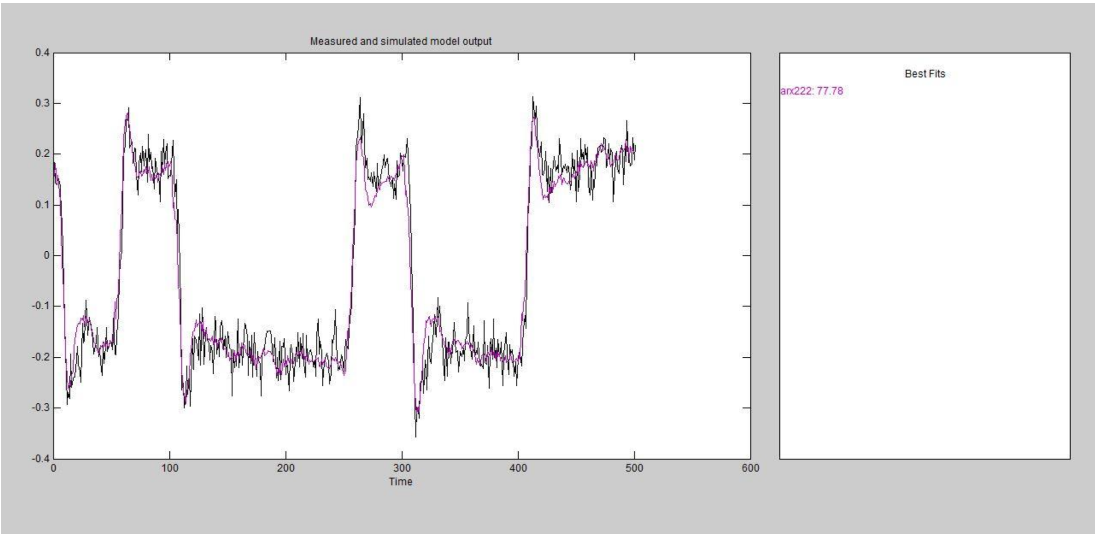
Appendix 3: Structure 1 - The best Structure of OBF-ARX for X_D under certain time delay



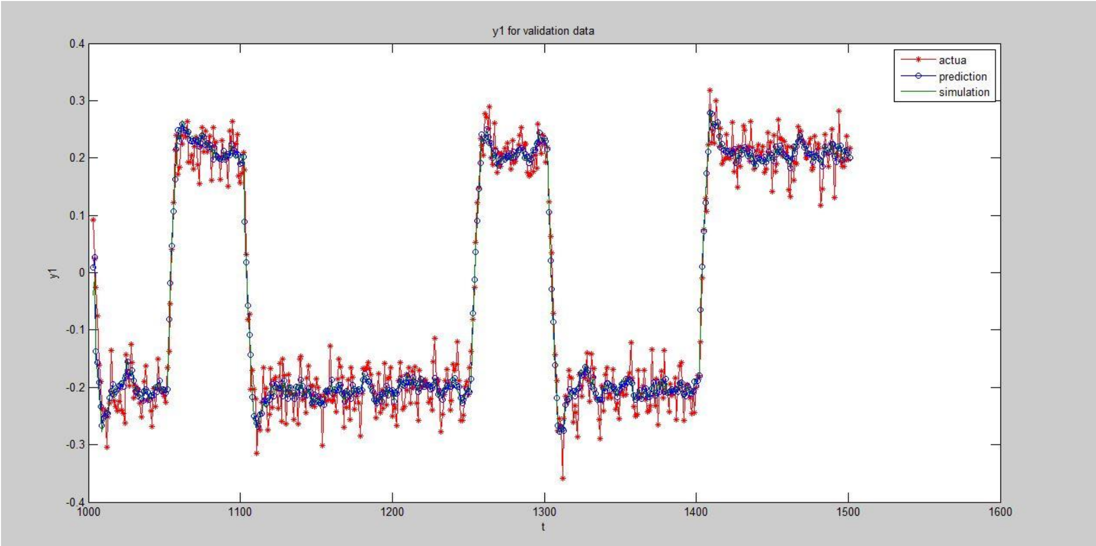
Appendix 4: Structure 1 - The best Structure of OBF-ARX for X_B under certain time delay



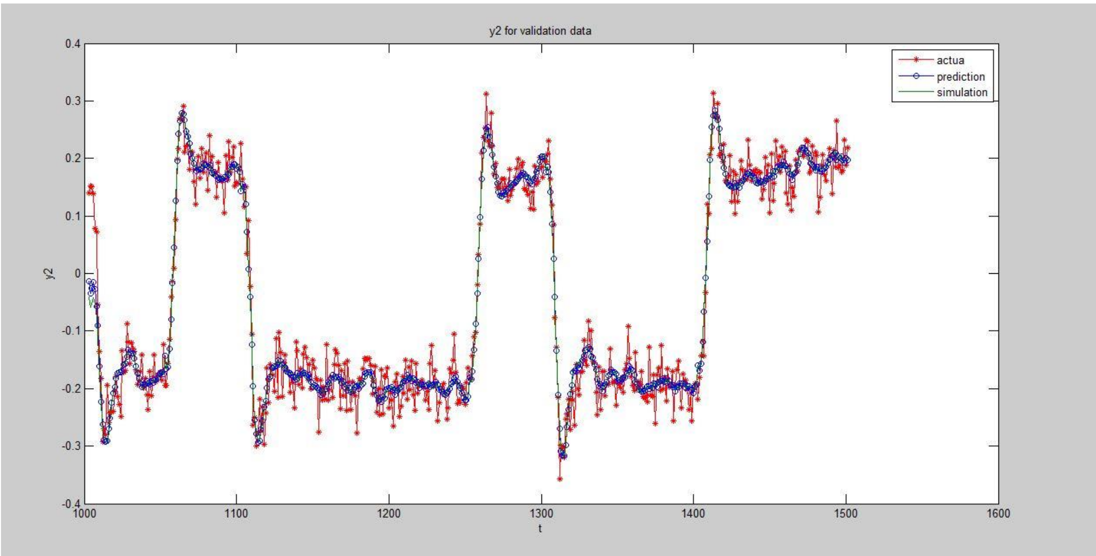
Appendix 5: Structure 1 - The best Structure of ARX for X_D under uncertain time delay



Appendix 6: Structure 1 - The best Structure of ARX for X_B under uncertain time delay



Appendix 7: Structure 1 - The best Structure of OBF-ARX for X_D under uncertain time delay



Appendix 8: Structure 1 - The best Structure of ARX for X_B under uncertain time delay