

**NUMERICAL MODELING OF HYDRAULIC FRACTURING IN  
SHALE OIL FORMATION**

BY

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Dissertation submitted in partial fulfillment of  
the requirements for the  
Bachelor of Engineering (Hons)  
(Petroleum Engineering)

September 2014

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# **CERTIFICATION OF APPROVAL**

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September 2014

## CERTIFICATION OF ORIGINALITY

This is to certify that I, **ELAMIN MOHAMED ELAMIN ABDALLA (15795), PETROLEUM ENGINEERING** is responsible for the work submitted in this project , that the work is original except specified in the references and acknowledgements, and herein have not undertaken or done by unspecified sources or persons.

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## **ABSTRACT**

Shale oil reservoir is one of the modern studies that oil and gas industries have started to concern about it. Although its permeability is very low, shale oil reservoirs can be produced by using many techniques such as hydraulic fracturing method. Since the shale oil reservoir is much complicated, it is challenging to study hydraulic fracturing in this reservoir. Among the literature review, basic theories of hydraulic fracturing technique, how the process is performed, equipment used, and some fracture geometry models, will be discussed.

There are several computer software programs that have been established to help petroleum engineers and planners to model hydraulic fracturing. These programs use numerical methods to model fracture propagation through the targeted formation. The user of this model will insert the necessary input parameters to that model. Eventually, the final output of these models will be the fracture geometry which is mainly the width and length of the fracture. The aim of this study is to analyze the two dimensional models which are Perkins-Kern-Nordgen, PKN and Geertsma de-Klerk, KGD fracture propagation models to obtain the geometry of the fracture based on the rock data as well as fracture treatment data.

The realization of this project will intensify the knowledge and it will help in the future researches of hydraulic fracturing for shale oil reservoir.

## ACKNOWLEDGEMENTS

Bissmillahirrahmanirrahim,

Alhamdulillah. Thanks to Allah SWT, Who, Alone, brings forgiveness and light and new life to those who call upon Him. It is he who creates from the very beginning, and he can restore life.

Deepest thanks and appreciation to my supervisor, **Mr. Mohammad Amin Shoushtari**, for the patient guidance, encouragement and advice he has provided to me to complete my project. Honestly, his knowledge and experience in the topic , and his willingness to share the knowledge he have, has granted me the chance to get effective knowledge, upgrade my learning scenario, and complete the project within the time allocated.

Besides, I would like to give my appreciation to **UNIVERSITI TEKNOLOGI PETRONAS** for giving me the platform to enhance my research skills and preparing me to have a background of the real world in the coming future with this project.

In addition, I would like to give thanks to my parents, for their faith in me and allowing me to be as ambitious as I wanted. I would like to thank my friends for their cooperation, encouragement, constructive suggestion and full of support for the project completion, from the beginning till the end.

It is my pleasure that I complete this project and got an effective knowledge that can help me in my coming future. I expect that this project will be a good research paper that will be offered to oil and gas industries especially the companies that working in Hydraulic fracturing.

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## ABBREVIATIONS AND NOMENCLATURES

API	American Petroleum Institute
$h_f$	Fracture height (ft)
T	Time (min)
E	Young's modulus of elasticity (kpa)
G	Shear modulus of rock formation
$h_R$	Reservoir interval height (m)
K	Cohesion of modulus ( $kpa \sqrt{cm}$ )
$K_c$	Critical stress intensity factor ( $kpa \sqrt{cm}$ )
L	Fracture length
$\Delta p$	$p - \sigma_H$ (Kpa)
$q_0$	Flow rate at fracture entrance $m^3/min$
$V_{sp}$	Spurt loss ( $m^3$ )
$\mu$	Fracturing fluid viscosity (Kpa.min)
$\nu$	Poisson's ratio of rock formation
W	Fracture width (m)
$\lambda$	Lame coefficient (psi)
$F_1$	$\int_0^1 \frac{\left(1 - \frac{2}{\pi} \sin^{-1} \lambda\right)^2}{(1 - \lambda^2)^{3/2}} d \lambda$

# CHAPTER 1

## INTRODUCTION

### 1.1 Background of study

Shale is known as one of the most problematical rock types in the majority of its related applications. Therefore it is estimated that the annual cost of this shale formation problems cost the oil industries billions of dollars each year[3]. Shale is defined as a type of sedimentary rock that is cemented-like sandstone- except in the case of shale the particles are much smaller and finer as clay or silt. Shale is typically deposited in a more quite environment like a lake or a shallow sea floor. The particles of shale are deposited in horizontal layers and cemented together and they tend to form very thin bed. Considering engineering properties, shale rock is characterized by a low compressive strength (less than 100 MPa” and high sensitivity to water. The properties of shale formation are very important that it can make the shale oil as one of the economical and reliable resources[7].

Shale oil – sometimes referred to as light tight oil- is a type of unconventional reservoir that has a very low permeability. Currently, shale oil is quickly rising as an economical unconventional resource in the oil and gas industries. There is a tendency for shale oil production to spread widely during the following decades. On the other hand, the production of shale oil is different and it will be done by using different procedures unlike the oil that is produced by using artificial lift methods[8].

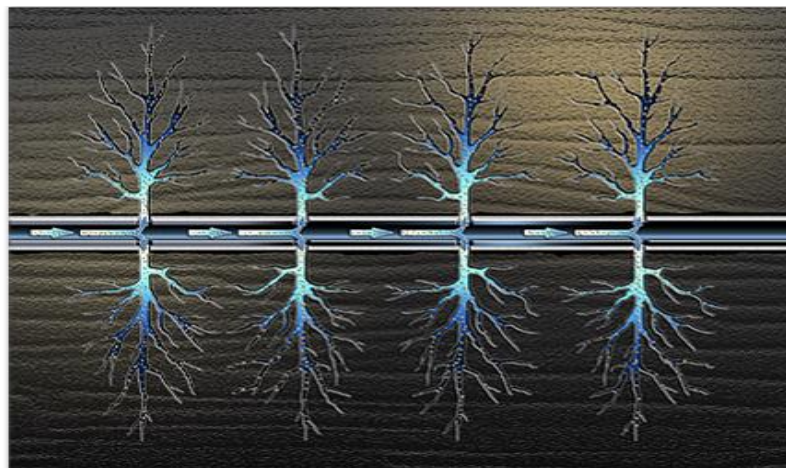
There are many techniques that is used to produce the shale oil. One of these techniques is by injecting carbon dioxide that can increase the pressure of the subsurface to let the oil lifts through the rock. Other method of producing shale oil is by waterflooding. The water that was placed underground will push the hydrocarbon into adjacent producing

well. This method is relatively cheap and it can cost about USD 15 per barrel. Other method used to produce shale oil is called hydraulic fracturing and in our study we will focus mainly on this method in details.

### 1.1.1 Hydraulic Fracturing

The process of hydraulic fracturing involves pumping numerous gallons of a high pressurized fluid to the well to create fractures in the rock. This fracture will later allow oil or gas to flow to the wellbore. In order to make the production potential of the well as large as possible, the shale formation will be fractured hydraulically[7].

Hydraulic fracturing is one of the methods that provide a safe extraction of hydrocarbon from underground shale formations. The initial wellbore is drilled using a drillpipe and bit. Drilling mud is pumped down through the drillpipe to cool and lubricate the bit, stabilize the wellbore, and remove cuttings and lift them up to the surface. The drilling continues until it reaches the target formation and doing all the processes of casing and cementing of every section individually. After that, a special tool of perforation is then lowered into the well creating holes in the shale layer which will allow the hydrocarbons to enter the well created channels. After removing the perforation tool, the fracturing fluid is pumped into the well and opens the tiny fractures in shale as we can see in the figure below:



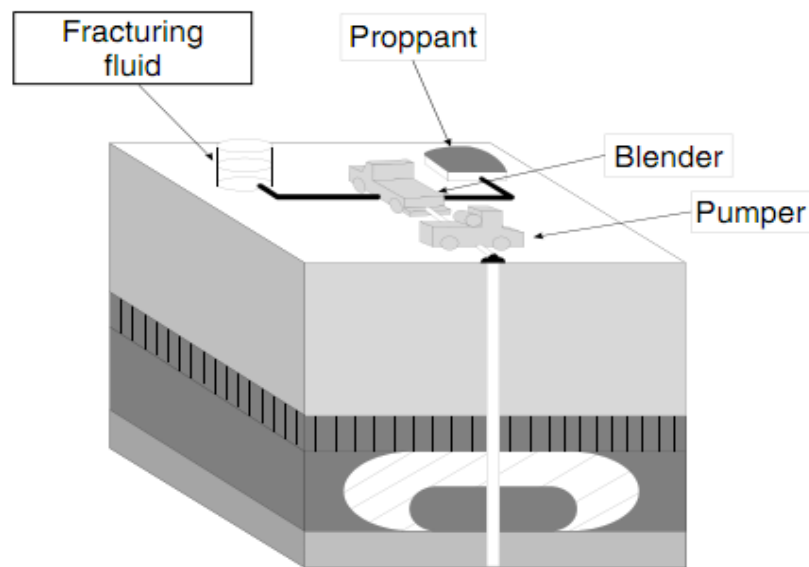
**Figure 1: Several Hydraulic Fractures in Formation[8]**

### 1.1.2 Fracturing equipment

The hydro-fracking process is done by using special equipment include: truck-mounted pumps, blenders, fluid tanks, and proppant tanks. Figure 2 illustrates the hydraulic fracturing equipment for oil and gas wells[7].

The hydraulic fracturing procedure can be divided into two stages. The first stage is called the pad stage where the fracturing fluid only is injected into the well to break the formation and generate a pad. The creation of this pad results from the injection rate of the fracturing fluid that must be higher than the flow rate at which the fluid will escape into the formation. The pad will enlarge to a required size and the slurry stage will start. In this stage, the fracturing fluid is mixed with sand and proppant by using the blender and the resulted mixture will be injected to the pad[8].

The fracture will be filled with sand and proppant and the pump will be shut down and the process of hydraulic fracturing is said to be completed.



**Figure 2: schematic to show the equipment layout in hydraulic fracturing treatments of oil and gas wells[8]**

## **1.2 Problem statement**

In the oil and gas industries, there are several softwares that are used for hydraulic fracturing. These softwares were made by the well-known companies such as Halliburton, Schlumberger, Baker Hughes...etc. However, these softwares are not free and the user must pay in order to get the license before setting them up. Moreover, those softwares must be used in a high performance system because it needs a high resolution and high contrast graphics.

Microsoft Excel is one of the useful programs that will be utilized in our study. This program is not only cheap but friendly for majority of its users because almost everybody worldwide who is using computer nowadays is familiar with how to use and generate the functions by using Microsoft Excel. Hence, with the aid of Microsoft Excel we will analyze some hydraulic fracturing propagation models and differentiate between them. These models will describe several aspects in hydraulic fracturing especially the width and length of the fracture. In addition, there is a lack of knowledge on applying these models in shale oil reservoirs.

## **1.3 Objectives and Scope of study**

### **1.3.1 Objectives**

The objectives of our study include:

- 1) Identify the appropriate models that will be used to:
  - Describe hydraulic fracturing in shale formation as well as to obtain closed-form solutions for a complex solid/fluid mechanics interaction problem.
  - Relate injection rate,  $q$ , time of treatment,  $t$ , and fluid leak off,  $q_l$ , with fracture width,  $w$ , and Length,  $L$ .
- 2) Analyze the effect of flow conditions like injection rate, fracturing fluid type and its viscosity, in the geometry of the fracture mainly the width and length and relate it with the breakdown pressure.

### **1.3.2 Scope of study**

The scope of our study is to

- Recognize and review the theoretical concepts of hydraulic fracturing, its history, terminology, and the present models used.
- Model the parameters required in hydraulic fracturing such as width and length by using two dimensional models mainly Perkins-Kern-Nordgren, PKN model, and Geertsma-deKlerk, GDK model.
- To analyze simulation results of our study to make a comparison between PKN and GDK models.
- The result of this work will be supported by the data extracted from the previous studies which considers the effect of the fracturing fluid flow characteristics in porous media.

## **CHAPTER 2**

### **LITERATURE REVIEW**

#### **2.1 The Idea of hydraulic fracturing**

The idea of hydraulic fracturing was begun during the last century, mainly in 1930s. a company called Dow Chemical has revealed that by injecting a high pressurized fluid downhole, the rock formation will be deformed and cracked. Currently, hydraulic fracturing is expensively used to develop the productivity of oil and gas wells. In 1950s, mainly in North America, 70% of gas wells, and 50% of oil wells was hydraulically fractured (Valko and Economides, 1995).

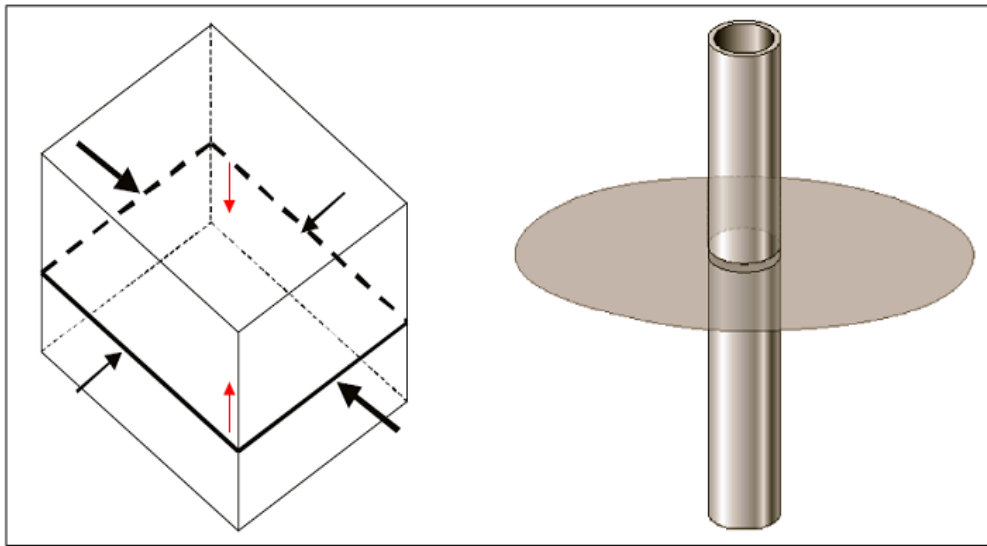
#### **2.2 Theory of hydraulic fracturing**

During the mid-century, hydraulic fracturing was projected as a key to increase the production of oil and gas from low pressure and low permeability reservoirs (Murphy& Carney, 1977). The theory of hydraulic fracturing method is that the pressure exerted by the fluid column should be greater than the formation stress and this can assist in the cracking process. The fluid will create the fracture and the fracture propagation will be increased by sing other chemicals and proppants to keep the fracture open. After the fracture was created, it will be the channel between the formation and the wellbore as a path. This path will be supported by a simulation design which consists of two principle stages. The first stage is referred to as the pad stage. In this stage, we will not use any proppant and the fluid is used to propagate the fracture and improve the fracture width. During the second stage which is referred to as slurry stage, the fluid and proppant will be mixed together. The main purpose of this stage is to assure that the proppant was placed in the fracture to have unlimited concentration at the fracture length when pumping finished[6].

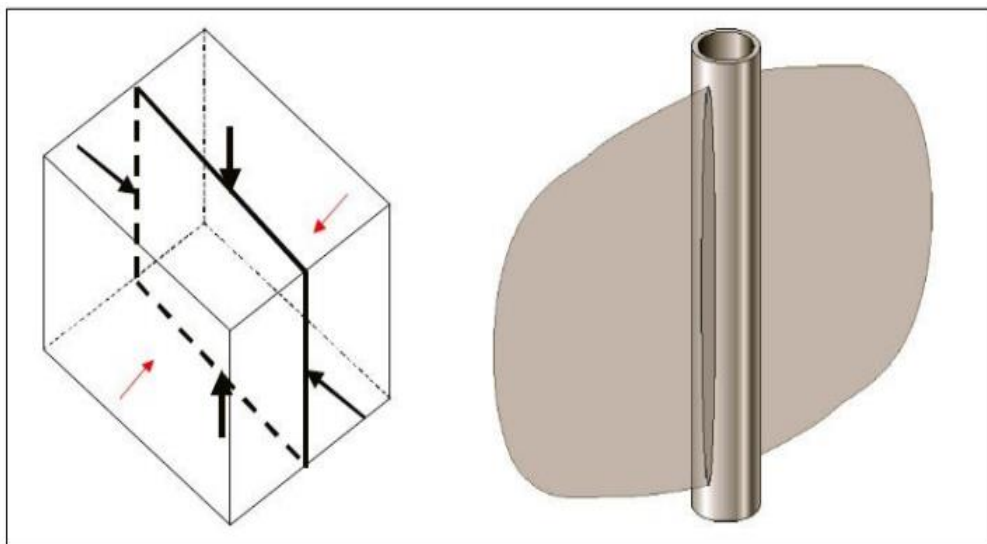


### 2.3 The extent of the fracture

According to Hydraulic fracturing Operations-Well Construction and Integrity Guideline (2009), the extent of the fracture is including both horizontal and vertical fracture. The horizontal fractures are made in the direction that is perpendicular to the least stress. Figures 3 &4 below demonstrate that the confining stress is exerted on the rock cube in three dimensions[5].



**Figure 3: Horizontal fracture perpendicular to the least stress[5]**



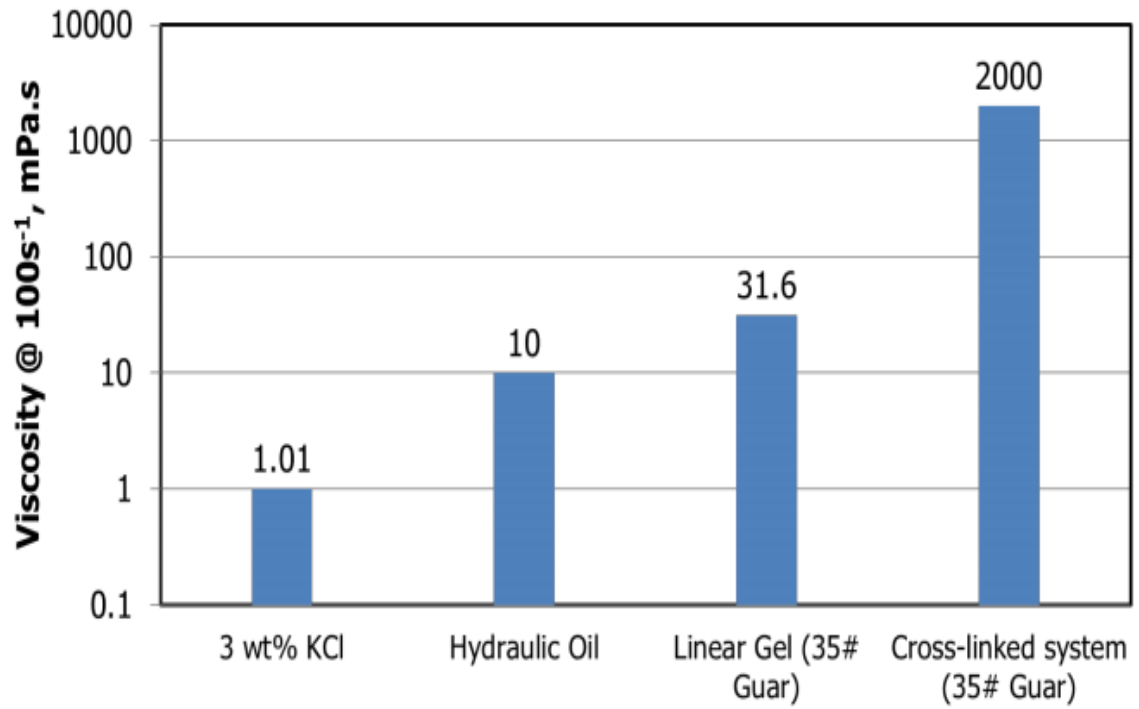
**Figure 4: Vertical fracture due to horizontal least stress[5]**

## 2.4 Viscosity and breakdown pressure

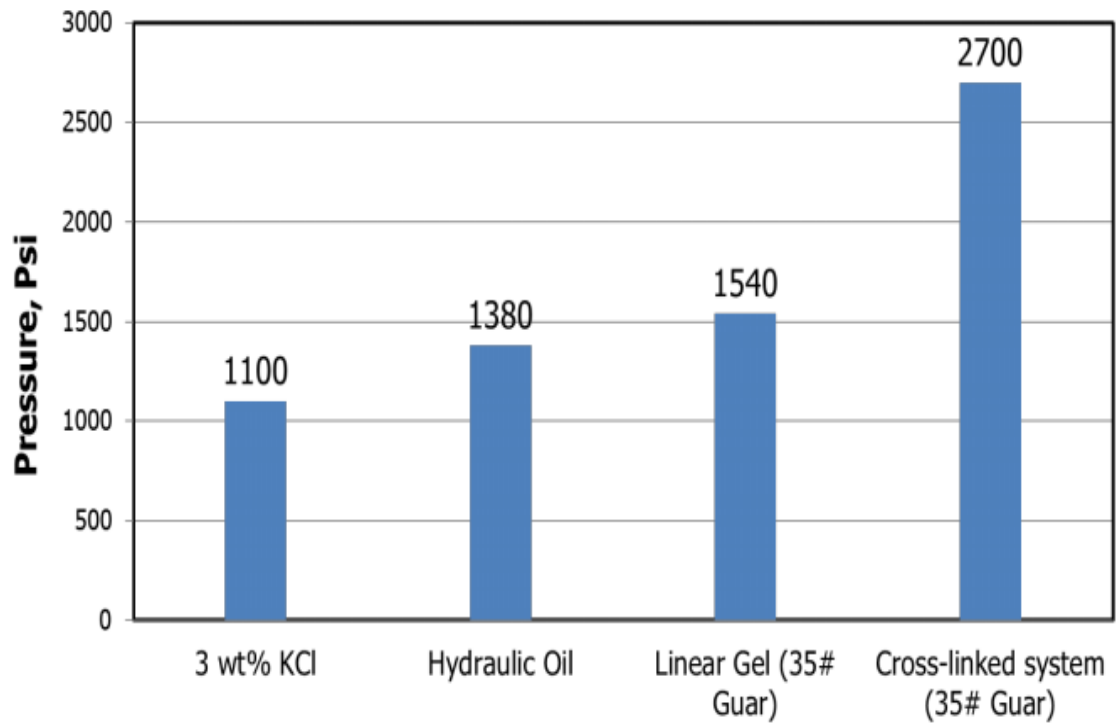
This study is mainly focusing in shale formation and the process of producing the shale oil from shale formation by using hydraulic fracturing. From the papers that were reviewed, the shale formation is a complicated formation that cannot be evaluated easily by using a traditional laboratory setting. The main characteristics of shale formation include very low permeability, the presence of micro-fractures and sensitivity to contacting fluids. The shale formation is a fine-grained rock that can be rich source of oil. Over the past decade, the combination of horizontal drilling and hydraulic fracturing has allowed access to large volumes of shale oil and shale gas that were previously uneconomical to produce, Seale (2007).

According to the experiments that were conducted, the main parameter in the design of the fracture is the fluid type selection and viscosity. In one of the experiments that was done, there are four different fluids that was injected which are 3wt% KCL solution, isobar oil, linear polymer gel (35 pptg guar) and crosslinked polymer gel (35 pptg guar). While each of the four fluids was injected into the core, the injection pressure was continuously increased until the shale broke at an indicated pressure referred to as (the breakdown pressure). Figure 5 demonstrates the viscosity of the four fluids, where figure 6 illustrating the breakdown pressure as a result of pumping those fluids.

From the experiment that was conducted, the results that was shown in the figures below show that the 3wt% KCL solution breakdown the shale formation at 1100 psi, while cross-linked gel breakdown the shale at 2700 psi the breakdown pressure in shale formation has a big relationship with the viscosity of the fracturing fluid. If the viscosity of the fluid is low, the required pressure that will break the shale formation is low. In contrast, if the fracture fluid loss has a high viscosity, a higher pressure is required to breakdown the shale formation



**Figure 5: Viscosity of four different fracture fluids that was injected[4]**

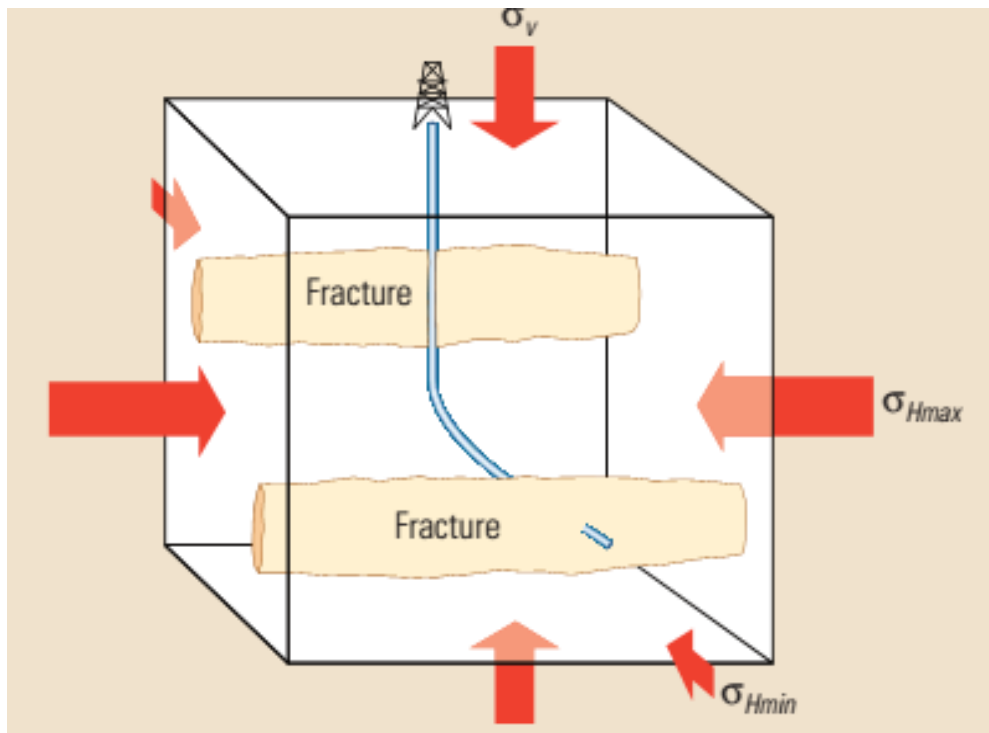


**Figure 6: Breakdown pressure increased with increasing fracture fluid viscosity[4]**

## 2.5 The physics of fracturing

The In-situ stress field will dictate the orientation, size, and the magnitude of the pressure needed to create it. The stress field may be defined by three principal compressive stresses which are oriented perpendicular to each other as illustrated in the figure below. The orientations and magnitudes of these stresses is determined by the regime tectonic in the region and by depth, pore pressure, and rock properties, which determine how stress is transmitted and distributed among formations. These three principal stresses increase with depth. The rate of this depth increasing will define the vertical gradient.

The in-situ stresses will control the orientation and propagation direction of hydraulic fractures. Hydraulic fractures are tensile fractures, and they open in the direction of least resistance. If the maximum principal compressive stress is the overburden stress, then the fractures are vertical propagation parallel to the maximum horizontal stress when the fracturing pressure exceeds the minimum horizontal stress.



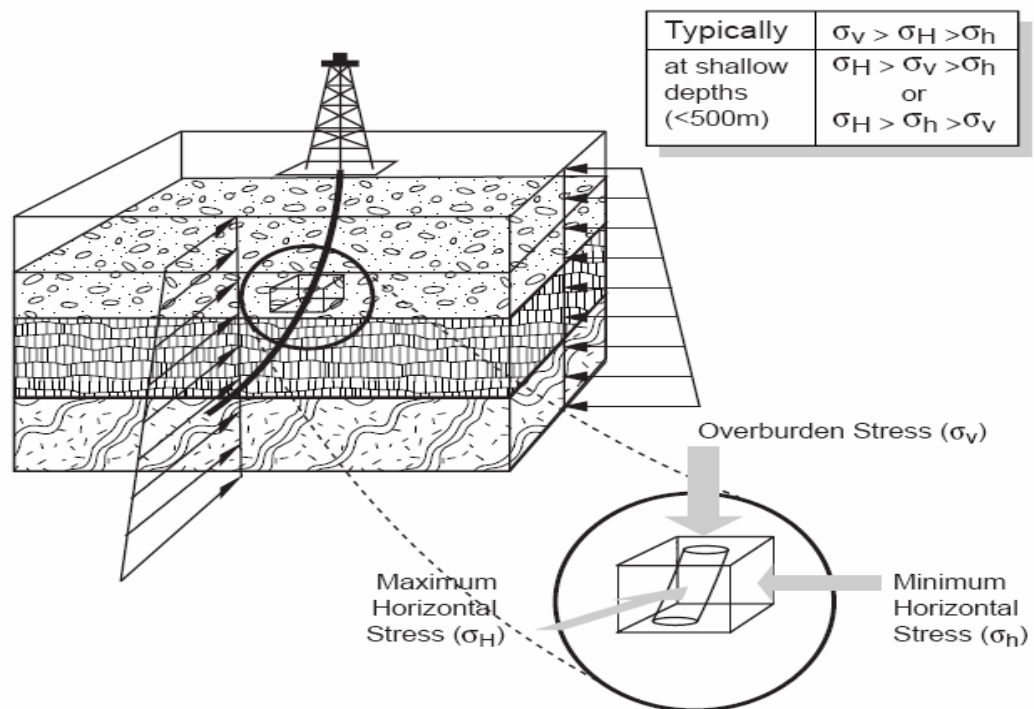
**Figure 7: In situ stresses and hydraulic fracture propagation[4]**

## 2.6 Rock mechanics and rock properties

Rock mechanics is one of the most significant factors that has to be considered properly in hydraulic fracturing. Some of the important properties of rock mechanics are briefly explained in the following section.

### 2.6.1 Insitu stress

There are three principle earth stresses oriented at right angles to one another as shown in the figure below:



**Figure 8: Insitu stresses in subsurface (Davies. 2007)**

The three principle stresses are:

$\sigma_v$  = Vertical stress (overburden stress)

$\sigma_H$  = Maximum horizontal stress

$\sigma_h$  = Minimum horizontal stress

Normally below 500m in a tectonically relaxed environment the vertical stress is the greatest. An average value of 1 to 1.1 psi/ft. is measured for wells at reasonable depth (Davies.2007).

Above 500 m,

$$\sigma_v > \sigma_H > \sigma_h$$

At shallow depth (< 500 m)

$$\sigma_H > \sigma_v > \sigma_h \text{ Or}$$

$$\sigma_H > \sigma_h > \sigma_v$$

The primary rock properties of interest for hydraulic fracturing calculations are the elastic properties, particularly the stiffness of the rock[2]. This usually defaults to the modulus of elasticity because most calculations are based on linear elasticity.

### **2.6.2 Linear Elasticity**

The assumption that rock behaves as a linear elastic material provides a major simplification of the theory of elasticity with the result that many fracturing problems are traceable and have analytic solutions. These solutions have been essential for the development of hydraulic fracturing theory. It should be remembered, however, that many rocks show considerable non-linear behavior over the stress-loading range of interest, and the effect of the non-linearity should be considered in certain instances. The basic assumption of the theory of linear elasticity is that the components of stress are linear functions of the components of strain[2].

### **2.6.3 Young's Modulus**

Young's modulus is the ratio of stress to strain. This can be as a result that the amount of strain that is caused by a given stress is a function of the material stiffness[2]. The Stiffness can be characterized by the slope of the stress-strain plot and it is referred to as Young's Modulus (E).

$$E = \frac{\text{Stress}}{\text{Strain}} \dots\dots\dots (1)$$

$$E = \frac{\sigma}{\epsilon} = Ib/in^2$$

For determining modulus for fracturing calculations, the confining pressure is normally set equal to the mean effective stress acting on the reservoir rock. The value of  $E$  can be determined from the resultant stress/strain value[2].

#### 2.6.4 Poisson's ratio

Poisson's ratio is the ratio of lateral expansion to longitudinal contraction for a rock under a uniaxial stress condition[2]. In other words, it is the ratio of strain perpendicular to the applied stress to strain along the axis applied stress. The poisson's ratio can be calculated from the following formula:

$$\nu = \frac{\lambda}{2(\lambda + G)} \dots\dots\dots (2)$$

Where

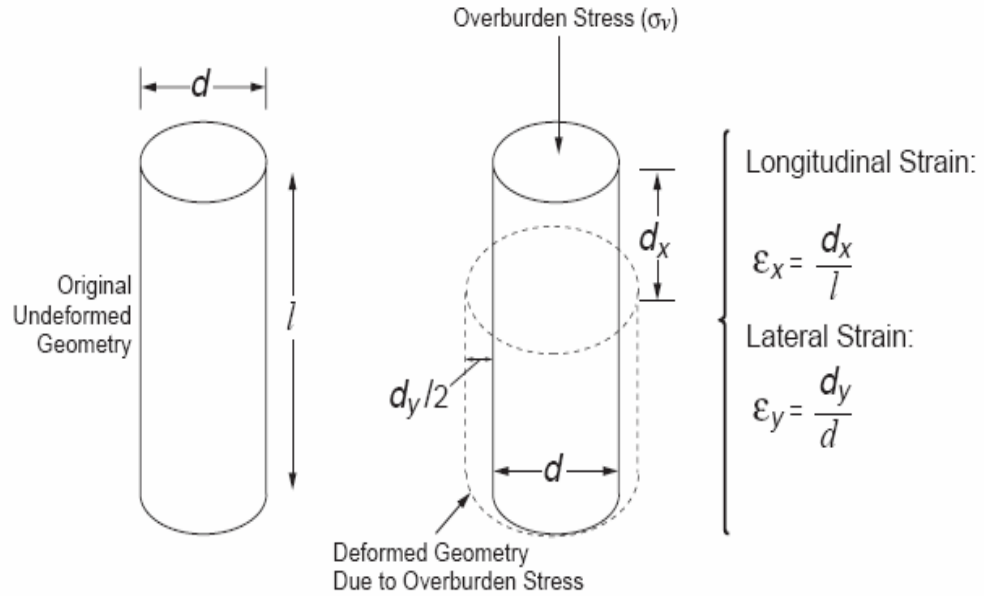
$\nu$  = Poisson's ratio

$\lambda$  = lame coefficient, psi

$G$  = Shear modulus, psi

Poisson's ratio is also a function of the stress or strain (nonlinear), and a value at a particular stress or over a range of stresses needs to be computed[2].

Compressive stress applied to a block of material along a particular axis causes it to shorten along that axis but also to expand in all directions perpendicular to that axis as shown in the figure below.



**Figure 9: Measurement of Poisson's ratio (Davies.2007)**

### 2.6.5 Shear and Bulk Moduli

One of the moduli that are useful for modeling the mechanics of the rock are the shear modulus,  $G$ , and the bulk modulus,  $K$ . Shear modulus,  $G$ , arises from linear elasticity and it can be computed by using the following formula:

$$G = \frac{E}{2(1+\nu)} \dots\dots\dots (3)$$

The bulk modulus,  $K$ , is the ratio of hydrostatic pressure to the volumetric strain it produces.  $K$  is can be calculated through the following formula

$$K = \frac{E}{3(1-2\nu)} \dots\dots\dots (4)$$

The bulk modulus,  $K$ , is also related to another important factor which is called critical stress intensity factor,  $K_C$ , by the following formula:



$$K_C = \frac{\sqrt{2}}{\pi} K \dots\dots\dots (5)$$

## 2.7 Fracture propagation models

The mathematical fracture propagation model is very important to relate the injection rate,  $q$ , time of treatment,  $t$ , and fluid leak off,  $q_l$ , with fracture dimensions such as width,  $w$ , and length,  $L$ . There are several models used to get the fracture geometry and these models will complete the calculation at a relatively less time as well as speeding up the calculation. One of these models is the two-dimensional fracture propagation models that involve four principles: the flow of the fluid, the mechanics of the rock, continuity, and the width of the fracture. These two-dimensional models are used in case that we have strong barriers[2]. In our study, we will focus on the variations between two models of fracturing which are PKN and GDK models.

There are two complementary models that were chosen to describe hydraulically induced fracture propagation in rocks. Both models include rectangular and a radial (circular) propagation mode. The two contrasting models are known as the PKN and Geertsma-de Klerk (GDK) models[2].

### 2.7.1 The PKN model

Perkins and Kern (1961) make a derivation that can solve for a fixed height vertical fracture as illustrated in the figure below. Nordgren (1972) added leak off and storage inside the fracture (because of width increasing) to the Perkins and Kern model[2].

According to Gidley (1989) the assumptions for PKN model are as follows:

- The fracture has a fixed height,  $h_f$ , independent of fracture length.
- The fracturing fluid pressure,  $p$ , is constant in vertical cross sections perpendicular to the propagation direction.
- The stiffness of the reservoir rock, its resistance to the deformation under the action of  $p$ , prevails in the vertical plane.

- In these cross sections the equation below will relate height,  $h_f$ , fluid pressure,  $p$ , and local fracture width.

$$W(x) = \frac{2(1-\nu)L \Delta p}{G} \sqrt{(1-x^2)} \dots\dots\dots (6)$$

The cross sections obtain an elliptic shape with maximum width in the center:

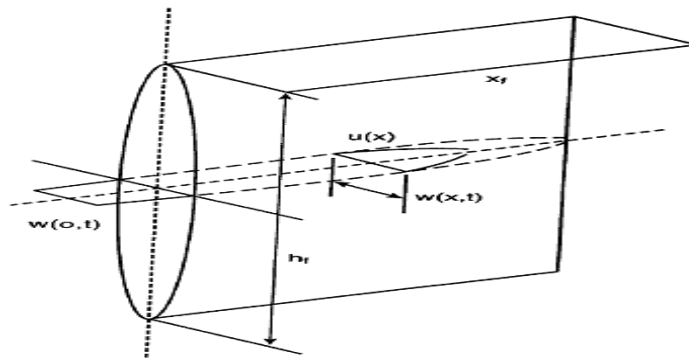
$$W(x, t) = \frac{(1-\nu)h_f}{G} (p - \sigma_H) \dots\dots\dots (7)$$

- The fluid pressure gradient in  $x$  direction is determined by the flow resistance in the elliptical flow channel.

The average width of the PKN fracture is expressed as:

$$\bar{W} = 0.3 \left[ \frac{q_i \mu (1-\nu)x_f}{G} \right]^{1/4} \left( \frac{\pi}{4} \gamma \right) \dots\dots\dots (8)$$

Where  $\gamma$  is 0.75. It is important to highlight that even in the contained fractures, the PKN is only effective when the length of the fracture is at least three times the height.



**Figure 10: The PKN fracture geometry [1]**

### 2.7.2 The KGD model

The KGD model assumes that the height of the fracture is constant and will not overcome the pay zone (i.e., the stresses in the layers above and below the pay zone are large enough to prevent fracture growth out of the pay zone).

According to Gidley (1989) the assumptions of GDK model are as follow:

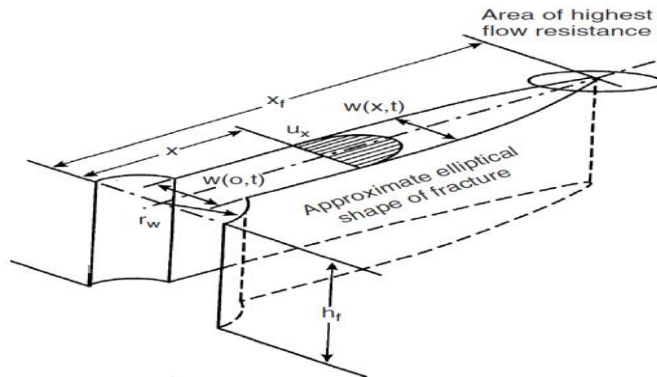
- Fixed fracture height,  $h_f$ .
- The stiffness of the rock is considered in the horizontal plane only
- Fracture width does not depend on fracture height and is constant in the vertical direction.
- The fluid pressure gradient is with respect to a narrow, rectangular slit of variable width,

$$P(0, t) - P(x, t) = P_f - P = \frac{12 \mu q_0}{h_f} \int_0^x \frac{dx}{w^3(x, t)} \dots \dots \dots (9)$$

- The shape of the fracture in the horizontal plane is elliptic with maximum width at the wellbore.

$$W(0, t) = \frac{2(1-\nu)L(p_f - \sigma_h)}{G} \dots \dots \dots (10)$$

A fracture model is shown in the figure below.



**Figure 11: The KGD model [1]**

The average width can be calculated by using the following formula

$$\bar{w} = 0.29 \left[ \frac{[\mu q_i(1-v)x_f^2]}{G h_f} \right]^{1/4} \left( \frac{\pi}{4} \right) \dots\dots\dots (11)$$

Where

$\bar{w}$  = average width, in

$q_i$  = pumping rate bpm

G and v represent the rock's elastic properties, i.e., shear modulus and Poisson's ratio, respectively where

$$E = 2(1 + v)G \dots\dots\dots (12)$$

The table below is showing the equations for fracture length and fracture width for constant injection rate

**Table 1: Equations for fracture length, width[4]**

<b>PKN Model</b>	
<b>Length (m)</b>	<b>Width (m)</b>
$C_1 \left[ \frac{G q_0^3}{(1-v)\mu h_f^4} \right]^{1/5} t^{4/5}$	$C_2 \left[ \frac{(1-v)q_0^2 \mu}{G h_f} \right]^{1/5} t^{1/5}$
<b>GDK Model</b>	
<b>Length(m)</b>	<b>Width (m)</b>
$C_4 \left[ \frac{G q_0^3}{(1-v)\mu h_f^3} \right]^{1/6} t^{2/3}$	$C_5 \left[ \frac{(1-v)q_0^3 \mu}{G h_f^3} \right]^{1/6} t^{1/3}$

The values of C are showed in the following table:

**Table 2: Values for C1 through C6 In table 1 [4]**

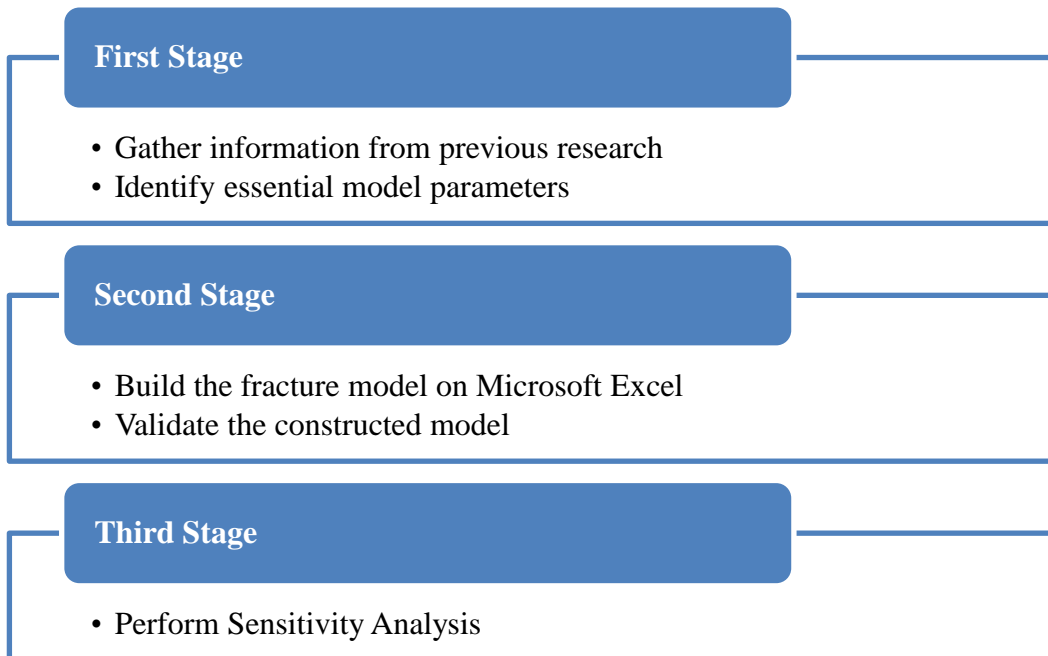
<b>Values for C1 through C6</b>		
	<b>One wing</b>	<b>Two wings</b>
<b>PKN</b>		
C1	0.68	0.45
C2	2.5	1.89
C3	2.75	2.31
<b>GDK</b>		
C4	0.68	0.48
C5	1.87	1.32
C6	2.27	1.19

## CHAPTER 3

### METHODOLOGY

#### 3.1 Research methodology

In this study which is a numerical modeling of hydraulic fracturing in shale oil formation, our objective will be achieved by several steps. Firstly, we have to identify the best model of all parameters mentioned earlier followed by developing a computer code by using Microsoft excel. The last step in our scope is that we will conduct sensitivity analysis that is the study of how the uncertainty in the output a mathematical model can be distributed to different sources of uncertainty in its inputs. Following diagram shows the planned steps to complete the study.



**Figure 12: General description of the workflow**

### 3.2 Gantt chart

#### 3.2.1 Final Year Project 1

No	Detail/ week	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
1	Topic selection															
2	Gathering important information from previous papers															
	• Shale oil reservoir															
	• Hydraulic fracturing theories															
	• Fracture propagation models															
3	Extended Proposal															
4	Proposal Defense															
5	Project work															
6	Interim report															
	• Reporting															
	• Submission															

### 3.2.2 Final Year Project 2

No	Detail/ week	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	Project work														
	• Studying of the propagation models														
	• Comparison of models														
	• Parametric studies														
2	Reporting , Documentation and Submission														
	• Progress report														
	• Pre-SEDEX														
	• Final Draft (Dissertation)														
	• Technical Paper														
	• Oral Presentation (Viva)														



## CHAPTER 4

### RESULTS AND DISCUSSION

#### 4.1 Prediction of fracture dimensions

In this section we will discuss our results that was obtained from the calculation of the fracture dimensions specifically length and width of the fracture. In our calculation, we did a comparison between PKN and GDK models. The comparison is mainly based on applying different values of injection rate as well as the time of injection. This comparison was conducted for both cases whether there is a leakoff or without leak off.

To become more familiar with the effect of shale oil rock, we took the critical stress intensity factor,  $K_C$ , of shale oil is about 8850.43 Kpa $\sqrt{cm}$  and the static shear modulus,  $G$ , of shale rock is about 20000 Kpa. The poisson's ratio of the shale formation was estimated as 0.3. By using Eq. 5 the cohesion modulus was found to be:

$$K = K_C \frac{\pi}{\sqrt{2}} = \frac{8850.43 \times \pi}{\sqrt{2}} = 19660.7 \text{ kPa} \sqrt{cm}$$

The following is the summary of assumed rock data:

**Table 3: summary of assumed rock data [2]**

Rock data	
$G$ (Kpa)	20000
$\nu$	0.3
$K_C$ (Kpa $\sqrt{cm}$ )	8859.43
$h_R$ (m)	30
$\sigma_H$ (Kpa)	$1.9 \times 10^4$
$\sigma_{Ha}$	$2.1 \times 10^4$

The fracture treatment data is illustrated in the following table

**Table 4: Fracture treatment data [2]**

Fracture treatment data	
$q_0$ ( $m^3/min$ )	2
$t$ (minutes)	200
$\mu$ (Kpa. min)	0.00000167
$K_l$ ( $m/m^{1/2}$ )	0.00046
$V_{sp}$ ( $L/m^2$ )	0.41
PKN $h_f$ (m)	39
GDK $h_f$ (m)	30
F1	0.75

#### 4.1.1 No fluid loss case

The equations that are illustrated in table 1 will be used to determine the length and the width of the fracture.

For the PKN model, the length of the fracture after 200 minutes ( $C_1 = 0.45$ ) is calculated as follow:

$$L = 0.45 \left[ \frac{20000 * 0.2^3}{(1-0.3) * 0.00000167 * 39^4} \right]^{1/5} (200^{4/5}) = 70.54169957 \text{ m}$$

And for the wellbore fracture width

$$W = 1.89 \left[ \frac{(1-0.3) * 0.2^2 * 0.00000167}{20000 * 39} \right]^{1/5} (200)^{1/5} = 0.012365845 \text{ m}$$

For GDK model where the fracture height is taken as 30 m, because it is a plane strain configuration with  $C_4 = 0.48$

$$L = 0.48 \left[ \frac{20000 * 0.2^3}{(1-0.3) * 0.00000167 * 30^3} \right]^{1/6} (200)^{2/3} = 68.03799689 \text{ m}$$

And for the wellbore fracture width ( $C_5 = 1.32$ )

$$W = 1.32 \left[ \frac{(1-0.3) * 0.2^3 * 0.00000167}{20000 * 30^3} \right]^{1/6} (200)^{1/3} = 0.012416591 \text{ m}$$

The comparison between these two models can be illustrated by changing the values of the injection rate  $q_0$  as well as the time of injection. We will repeat the previous calculations for both models by increasing the value of  $q_0$  and  $t$ .

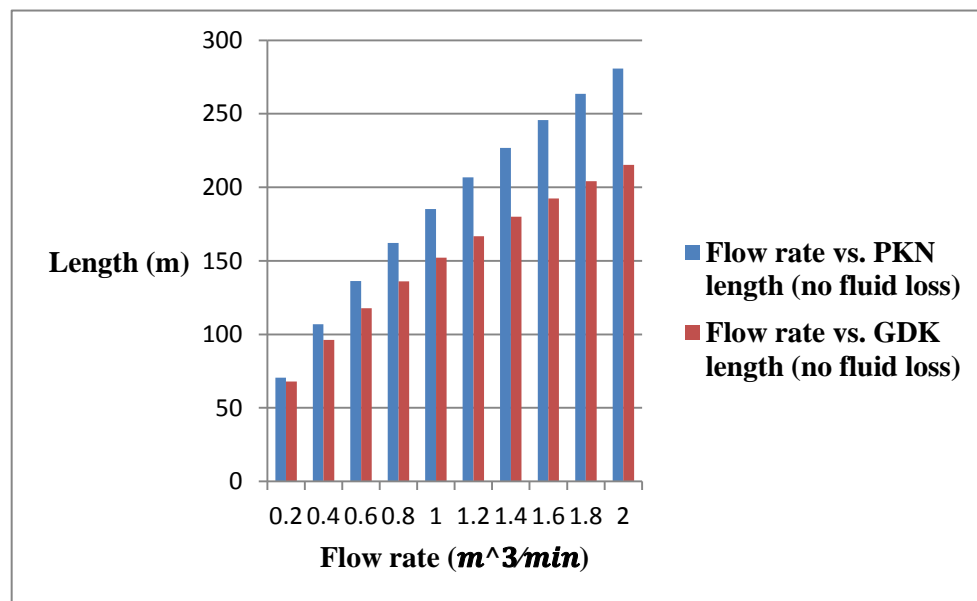
**Table 5: Result of the length and width for both models when the injection rate is 0.2 (No Fluid loss case)**

No fluid loss case		
	Length (m)	Width (m)
<b>PKN model</b>	280.8316	0.031062
<b>GDK model</b>	215.155	0.039265

Starting from an injection rate of 0.2, the results and graphs are shown as follow:

**Table 6: Length changes due to injection rate change (no fluid loss)**

No fluid loss case		
	Length (m)	
$q_0$ ( $m^3/min$ )	PKN	GDK
0.2	70.5417	68.0379969
0.4	106.9212	96.220258
0.6	136.3699	117.845267
0.8	162.0623	136.075994
1	185.2797	152.137586
1.2	206.6982	166.658376
1.4	226.7278	180.011619
1.6	245.6405	192.440516
1.8	263.6279	204.113991
2	280.8316	215.155038

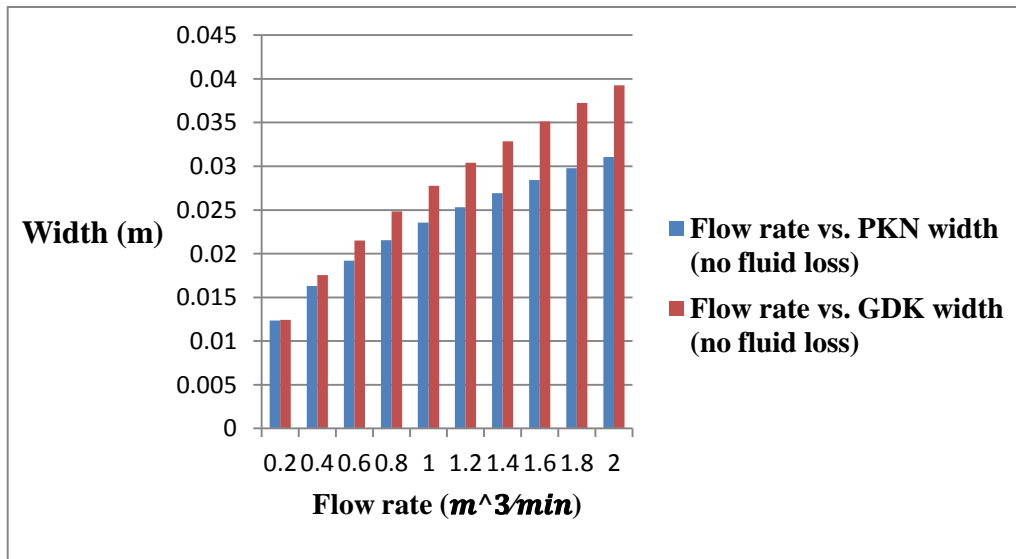


**Figure 13: Length vs flow rate for both models ( no fluid loss )**

From the graph and table shown above, we can observe that when we increase the flow rate, we observed that the length in PKN model will increase more than GDK length.

**Table 7: Width changes due to injection rate change (no fluid loss)**

No fluid loss case		
$q_0$ ( $m^3/min$ )	Width (m)	
	PKN	GDK
0.2	0.012366	0.01241659
0.4	0.016317	0.01755971
0.6	0.01919	0.02150617
0.8	0.02153	0.02483318
1	0.02354	0.02776434
1.2	0.025321	0.03041431
1.4	0.026932	0.03285121
1.6	0.028409	0.03511942
1.8	0.02978	0.03724977
2	0.031062	0.03926471

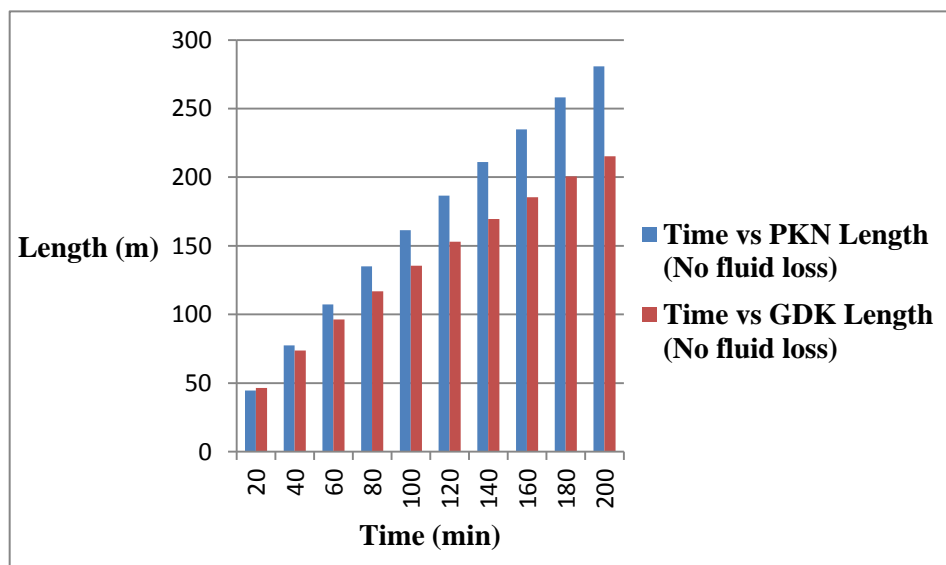


**Figure 14: Width vs flow rate for both models ( no fluid loss )**

On the other hand, we can observe that when we increase the flow rate, we observed that the width in GDK model will increase more than PKN length. When the time of injection increases, we will also observe the increasing the fracture length and width. Starting from a time injection of 20 minutes, the results are shown below.

**Table 8: Length changes due to injection time change (no fluid loss)**

No fluid loss case		
t (minutes)	Length (m)	
	PKN	GDK
20	44.5088	46.3537477
40	77.49433	73.5819878
60	107.1872	96.4196807
80	134.9255	116.804125
100	161.2954	135.53918
120	186.6238	153.056703
140	211.1176	169.622534
160	234.9189	185.414991
180	258.1309	200.561018
200	280.8316	215.155038

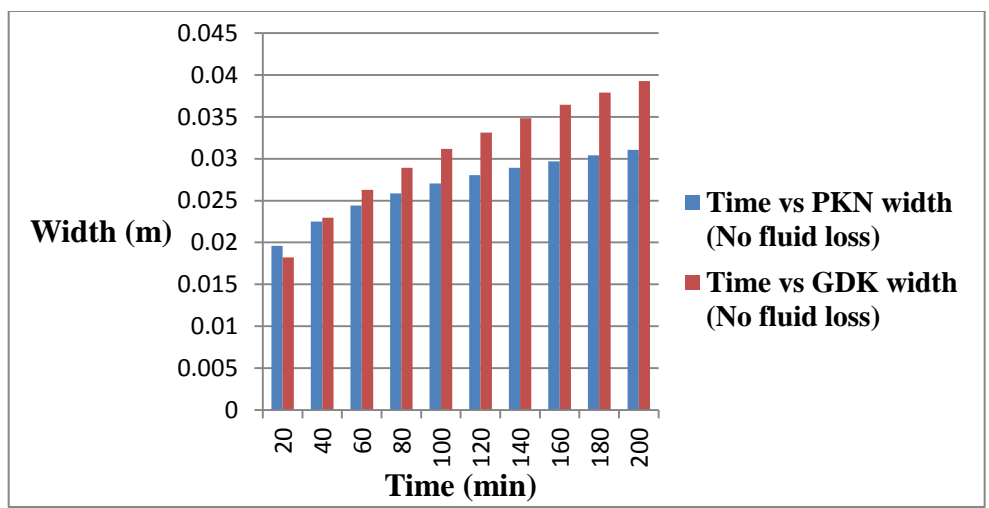


**Figure 15: Length vs time for both models ( no fluid loss )**

Here we can observe that, as time goes by, we will have more length for GDK model at the beginning. But after a certain time, the length of PKN model starts to increase and override the GDK length.

**Table 9: Width changes due to injection time change (no fluid loss)**

No fluid loss case		
t (minutes)	Width(m)	
	PKN	GDK
20	0.019599	0.01822506
40	0.022513	0.02296214
60	0.024415	0.02628509
80	0.02586	0.02893049
100	0.027041	0.03116442
120	0.028045	0.03311714
140	0.028923	0.03486329
160	0.029706	0.03645013
180	0.030414	0.03790966
200	0.031062	0.03926471



**Figure 16: Width vs time for both models ( no fluid loss )**

In contrast, we can observe that, at the beginning, we will have more width by PKN model. But as time goes by, the GDK model start to override PKN model.

#### 4.1.2 Incorporation of fluid loss

The second case that we are going to analyze it to have a comparison between the two models is when we have a leak. We will also use the same values of the injection rate and time of injection and the results are shown below.

**Table 10: Result of the length and width for both models when the injection rate is 0.2 (Fluid loss case)**

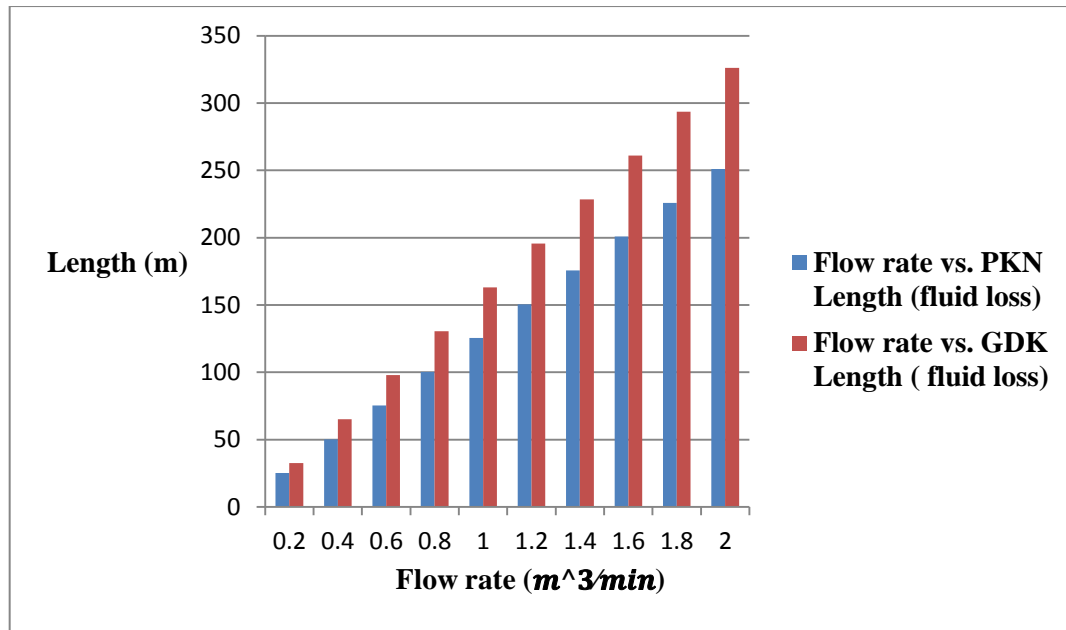
Incorporation of fluid loss		
	Length (m)	Width (m)
<b>PKN model</b>	250.9243	0.029534
<b>GDK model</b>	326.2016	0.039674

The following are the results that obtained in case that there is a fluid loss

**Table 11: Length changes due to injection rate changes (fluid loss)**

Fluid loss case		
	Length (m)	
$q_0$ ( $m^3/min$ )	PKN	GDK
0.2	25.09243	32.6201564
0.4	50.18486	65.2403128
0.6	75.27728	97.8604691
0.8	100.3697	130.480626
1	125.4621	163.100782
1.2	150.5546	195.720938
1.4	175.647	228.341095
1.6	200.7394	260.961251
1.8	225.8319	293.581407
2	250.9243	326.201564



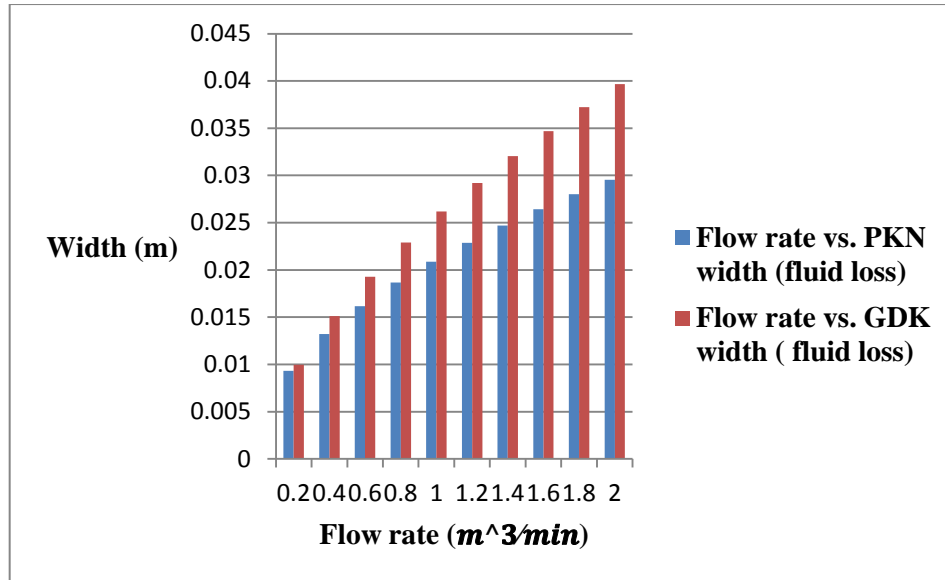


**Figure 17: Length vs. flow rate for both models (fluid loss)**

From the above graph and table, we can clearly observe that GDK model can give greater length as the injection rate is increasing.

**Table 12: Width changes due to injection rate changes (fluid loss)**

Fluid loss case		
$q_0$ ( $m^3/min$ )	Width (m)	
	PKN	GDK
0.2	0.00934	0.00996577
0.4	0.013208	0.01510528
0.6	0.016177	0.01926565
0.8	0.018679	0.02289533
1	0.020884	0.02617538
1.2	0.022877	0.02920127
1.4	0.02471	0.03203094
1.6	0.026416	0.03470283
1.8	0.028019	0.03724401
2	0.029534	0.03967445



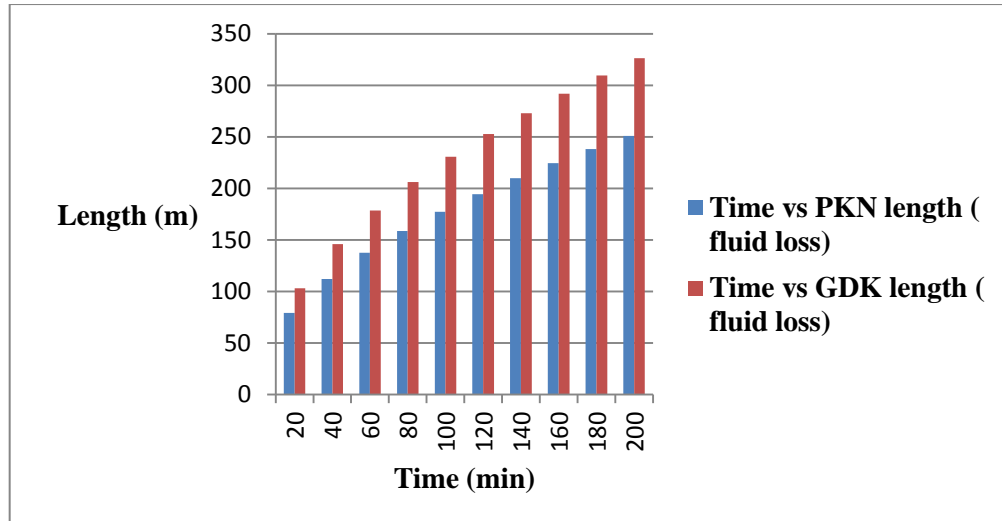
**Figure 18: Width vs. flow rate for both models (fluid loss)**

The graph and table above, illustrated that when we have fluid loss, we can have a greater width for GDK model than what we have in PKN model.

When we are dealing with the injection time, we will also observe the increasing of the fracture length and width from the graphs and tables. The results are illustrated below.

**Table 13: Length changes due to injection time changes (fluid loss)**

Fluid loss case		
t (minutes)	Length (m)	
	PKN	GDK
20	79.34922	103.153992
40	112.2167	145.881774
60	137.4369	178.667955
80	158.6984	206.307984
100	177.4303	230.659338
120	194.3651	252.674645
140	209.9383	272.919809
160	224.4335	291.763548
180	238.0477	309.461975
200	250.9243	326.201564

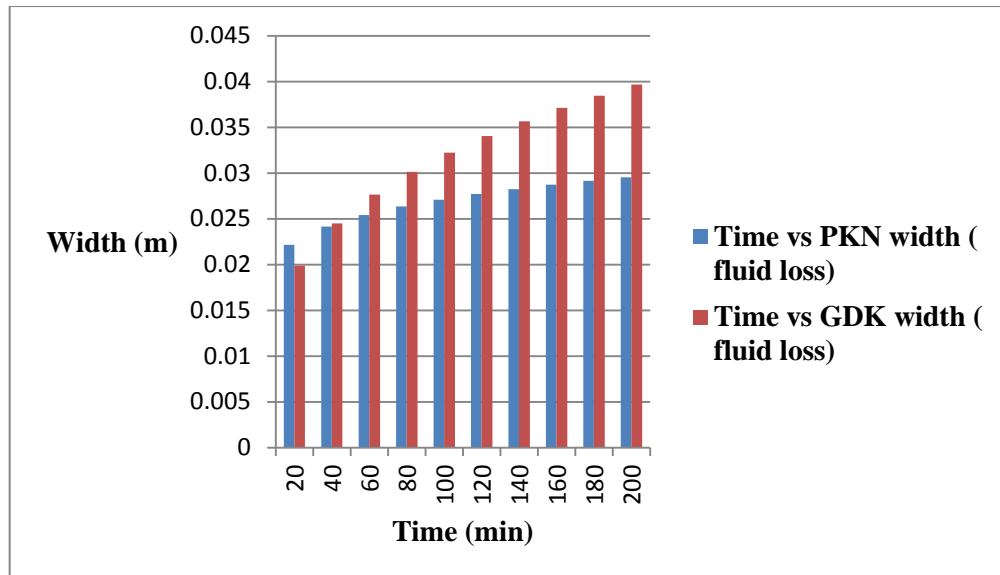


**Figure 19: Length vs. injection time changes for both models (fluid loss)**

As time increases, in case that we have fluid loss, we can observe from the table and graph above, GDK model can give more length than PKN model.

**Table 14: Width changes due to injection time changes (fluid loss)**

Fluid loss case		
t (minutes)	Width(m)	
	PKN	GDK
20	0.022147	0.01988433
40	0.024152	0.02448048
60	0.025408	0.02764695
80	0.026338	0.03013901
100	0.027083	0.03222567
120	0.027707	0.03403739
140	0.028246	0.03564842
160	0.028722	0.03710547
180	0.029148	0.03844003
200	0.029534	0.03967445



**Figure 20: Width vs. injection time changes for both models (fluid loss)**

At the beginning, the width of PKN model is greater than GDK model, but after 50 minutes, the width of the GDK model will override the width of the PKN model.

## **CHAPTER 5**

### **CONCLUSION AND RECOMMENDATIONS**

As a conclusion, shale oil currently embraces a reliable and economic resource as well as in the future. Therefore, the oil and gas industries worldwide must focus attention on the means and techniques of producing shale oil from shale rock especially the hydraulic fracturing method. Although hydraulic fracturing is a complex and requires a lot of tools and professional engineers, it is considered an effective method of shale oil production.

From the results obtained, in the case that there is no leak off, we observed that PKN model can give more length than GDK model. On the other hand, in the case that we have fluid loss, we also observed that GDK model can provide more length and width than PKN model.

Besides, the fracture propagation models that are used, mainly PKN and KGD models were found to be very useful model for the process of hydraulic fracturing. Also these models can facilitate the calculation of prediction of dimensions of the fracture and complete it in a relatively short time. In the future, scientists and engineers must invent new models that can represent the hydraulic fracturing and this will increase the effective production of shale oil reservoirs. In addition, engineers and scientists must conduct an experimental research in order to validate the success of using PKN and GDK model.

Below are some future recommendations:

- Experimental research has to be conducted to validate the success of using PKN and GDK models.

- Environmental impacts of hydraulic fracturing in shale oil reservoir, particularly on the amount of water consumed.
- Study on shale oil production.

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