Analysis of slot flow approximations for annular flow of yield-power-law fluids

by

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(Dissertation Final Draft)

September 2014

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CERTIFICATION

CERTIFICATION OF APPROVAL

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This is a dissertation submitted to the
Petroleum Engineering Department of
Universiti Teknologi PETRONAS
as partial requirement for fulfillment of
BACHELOR OF ENGINEERING (Hons)
(PETROLEUM)

Approved by,

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September 2014

CERTIFICATION OF ORIGINALITY

This is a certification that the work submitted in this project is of my responsibility and that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

ARMENIO OSVALDO MANUEL NHANALE

Abstract

It is the concern of both service companies and operators to understand the rheological behavior of drilling fluids under drilling conditions. In the past few year major advances have been done in the study of Yield Power Law fluids due to its capacity to study both power-law fluids at low shear and Bingham plastic fluids.

Understanding pressure drop-flow rate behavior in annulus is necessary to improve drilling operations. This thesis presents mathematical relationships between mud circulating rate and the respective pressure drop for Yield-power Law in a concentric annulus, with no drillpipe rotation. Based on this mathematical model pressure drop-flow rate relationships are obtained.

These results will help define the minimum pump power rate to be used during mud circulation into the well during drilling operations; it will also determine the pressure drop variation throughout the drillstring when circulating mud at a certain rate.

Acknowledgements

I am using this opportunity to express my sincere and profound gratitude to everyone who supported me throughout the course of this Final Year Project especially to my Supervisor Dr. Narahari. I am thankful for their aspiring guidance, invaluably constructive criticism and friendly advice during the project work.

I express my warm thanks to my parents for supporting me throughout this journey and for always believing in my potential. You both have always been my inspiration and I am grateful to have you both in my life and I hope to still make you proud.

To my "sisters" and my "my brother", thank you for all the prayers and the love that you have shown to me from the beginning and I know that I can always count on all of you to be there when I need it.

To my closest friends, they know who they are, thank you for always being there through thick and thin, and also for supporting me in all aspects, I sincerely hope that we manage to achieve all the great things that we have planned.

And thanks to me for being able to stay strong and positive even in the harsh moments.

Thank you,

Armenio Osvaldo Manuel Nhanale

Contents

List of Figures

List of Tables

Nomenclatures

$dp/_{dL}$ - Pressure drop per unit length

h – Gap of annulus in the slot

K – Fluid consistency index

n - Fluid behavior index

p – Pressure

r – Radius

r_1 - Radius of outer pipe of annulus

r_2 – Radius of inner pipe of annulus

q – Flow rate

u – Fluid velocity

y_a - Distance of top of inner layer from bottom plate

y_b - Distance of bottom of top layer from bottom plate

τ - Shear Stress

γ - Shear Rate

τ_y - Yield Stress

τ_0 - Integer constant

w – Width of the slot

1. Introduction

1.1. Background

Drilling fluids are used while drilling oil and natural gas wells to maintain primary control of the well being drilled. Its functions include:

- Circulate the cuttings;
- Suspend cutting during trip;
- Set of cuttings on surface system;
- Provide hydrostatic to prevent formation fluids from invading the well and delivering hydraulic energy to the formation under the bit.

Successful drilling operation is dependent on mud weight, yield point, gel strength, plastic viscosity and thixotropy. The behavior of a drilling fluid is determined by the flow regime, which is determining factor on the ability of that fluid to perform its basic functions.

The flow can be under two different regimes, either laminar or turbulent, depending on the fluid velocity, size and shape of the flow channel, fluid density, and viscosity. If the pump rate is low enough for the flow to be laminar the fluid model can be employed to develop the mathematical relation between flow rate and frictional pressure, therefore throughout this dissertation the flow will be considered to be laminar.

The rheological behavior of drilling fluids during drilling operation is a major concern for both service companies and operators. Characterization of fluid flow during drilling operation is usually analyzed using rheological models. These model alone cannot make a full description of the fluid flow over their entire shear range, therefore understanding of rheological models needs to be supported by practical experience to make a fully description of the fluid behavior.

Frictional pressures, swab and surge pressures, and slip velocity of cutting in fluids are estimated using these models. The commonly used models in the drilling industry are Newtonian, Bingham Plastic, Power Law and Yield Power Law. Understanding these models will improve drilling operations when using any of these fluids.

Fluids that do not exhibit a direct proportionality between shear stress and shear rate are classified as non- Newtonian (Bourgoyne Jr, et al. 1986).

A number of models have been developed by researchers to describe the drilling mud behavior, power law and Bingham plastic models are the most used since predictions can be made easily using this model (Kelessidis, et al. 2006). Power law model is a useful correction of the Newtonian Fluid Model but it might yield errors when fluid has yield stress.

The yield power law model gives better rheological data and a wider range of drilling fluid compared to the Bingham Plastic Model, but not until the last decade it started being used due to the complexity of the model development (Kelessidis, et al. 2006). This dissertation presents mathematical relations between mud circulating rate and the associated pressure drop for Yield Power Law fluids flowing through annulus which is defined as slot instead of considering it to be radial.

1.2. Problem Statement

Avoiding complicated mathematical models that yield analytically intractable solution is crucial when predicting drilling mud behavior in annulus. Several mathematical models have been used to describe the mud behavior in pipes and annulus using slot-flow approximation but yield has been proved to be model that covers a wider range of drilling fluids.

1.3. Objectives and Scope of study

1.3.1. Objectives

- Develop an approach that can be used to estimate pressure drop – flow rate relationships in an annulus
- Develop a solution that can be used to select drilling mud to be used during drilling operations
- Ensure that all the steps involved in the estimation of pressure drop – flow rate relationship can be analytically traced

2. Literature Review and/or Theory

2.1. Yield Power Law Fluids Overview

Yield Power Law fluids are described by a three-parameter rheological model mathematically commonly written as:

$\boldsymbol{\tau = \tau_y + K\gamma^n}$ (Skalle 2011)……………..………………….. (1)

Where:

τ – Shear Stress

τ_y - Yield Stress

K - Fluid Consistency Index

γ - Shear Rate

n - Fluid Behavior Index

The Yield Power Law (YPL) model is preferred over the Bingham plastic and power law models because its solution provides more accurate models of rheological behavior since the stress that is experienced by the fluid is characterized using three parameters namely yield stress, fluid consistency index and fluid behavior index unlike the other models that use one or two parameters (Newtonian, Bingham and Power Law models). It also can be used to predict Newtonian fluids ($\tau_y = 0; n = 1$), power-law fluids ($\tau_y = 0$) and Bingham plastic fluids ($n = 1$) when there is adequate experimental data and they must be generated using non linear regression of the viscometer data.

Like Bingham Plastic fluids, YPL fluid models require a minimum stress to initiate flow but decreasing stress with increasing shear. According to (Omosebi & Adenunga, 2012) this model is preferred over the other for the following reasons:

- It can be used to describe most of drilling fluid flow
- Incorporates a yield stress value which is necessary for several hydraulics, and
- It represent the Bingham and Power Law models as special cases

2.2. Slot Flow Approximation

For the purpose of simplified integration of the velocity profile and the pressure drop flow rate relationship the annulus is represented as slot and an equivalent relation between the shear stress and pressure gradient is found. Integration using this method is less complicated than if we were to consider the annulus to be radial. For accurate results the annulus must obey the following condition $r_1/r_2 > 0.3$.

$A = \pi(r_2^2 - r_1^2)$ - 2.1

$h = r_2 - r_1$ - 2.2

Based on Newton's first law of motion if the fluid is in a steady state its velocity is constant resulting resultant of forces that are exerted on the fluid element to be equal to zero

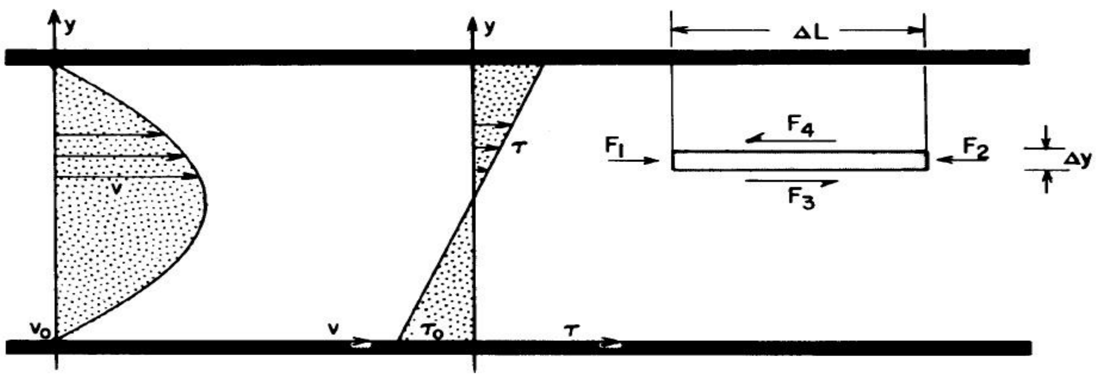

Figure 1. Free body diagram for fluid element in a narrow slot (Bourgoyne Jr, et al. 1986)

Fig.1 one represents a fluid element in a narrow slot, and we can observe that there are four forces acting around the fluid. Forces one and two act perpendicularly to the area of the fluid element therefore the force is represented by the pressure when deriving the slot flow equations while forces three and four are parallel to the cross sectional area of the fluid therefore the force is related to the shear stress. By considering that the fluid element in figure 1 to width "W" and thickness "Δy", the force applied in point can be said to be:

$$F_1 = pW\Delta y$$

The same can be applied for the subsequent for in the different points

$$F_2 = p_2 W\Delta y = \left(p - \frac{dp_f}{dL}\Delta L\right) W\Delta y$$

$$F_3 = \tau W\Delta L$$

$$F_4 = \tau_{y+\Delta y} W\Delta L = \left(\tau + \frac{d\tau}{dy}\Delta y\right) W\Delta L$$

By assuming a steady flow the sum of the forces acting on the fluid element are equal to zero

$$F_1 - F_2 + F_3 - F_4 = 0$$

$$pW\Delta y - \left(p - \frac{dp_f}{dL}\Delta L\right) W\Delta y + \tau W\Delta L - \left(\tau + \frac{d\tau}{dy}\Delta y\right) W\Delta L = 0$$

$$\frac{dp_f}{dL} - \frac{d\tau}{dy} = 0$$

$$\boldsymbol{\tau = y\frac{dp_f}{dL} + \tau_0}$$

Equation one is later used in this paper to find the pressure flow rate relationship for yield power law based on the approach used by (Bourgoyne Jr, et al. 1986).

2.3. Pipe eccentricity

Traditionally investigators have assumed that the drillstring and the hole/casing are concentric. That is a wrong assumption to make since it is almost physically impossible to have a fully concentric annulus specially when during directional drilling when the pipe weight causes a strong tendency for the pipe to lie against the hole as it is illustrated in case 2 and 3 of the fig. 3.

According to (Iyoho & Azar, 1981) manipulation of equations describing non-Newtonian flow through parallel plate is generally easier than conventional annular-flow equations, in Figure 2 and 3 the eccentric annulus is represented by a nonrectangular slot.

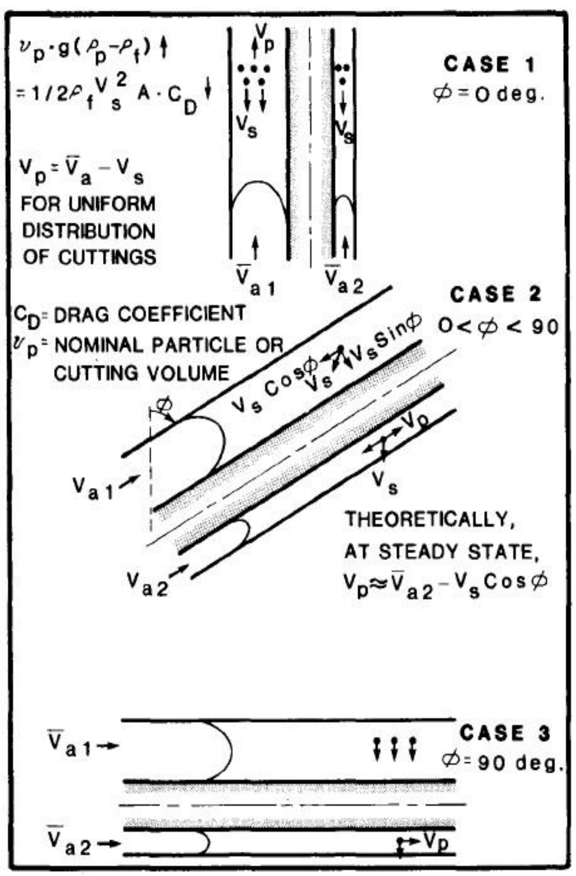

Figure 3. Fluid and Particle dynamics in eccentric annuliof variable inclination (Iyoho & Azar, 1981)

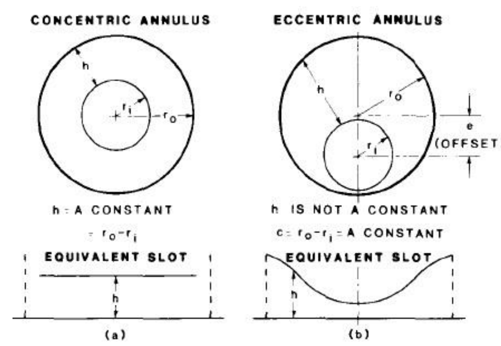

Figure 2. Slot equivalents of concentric and eccentric annuli (Iyoho & Azar, 1981)

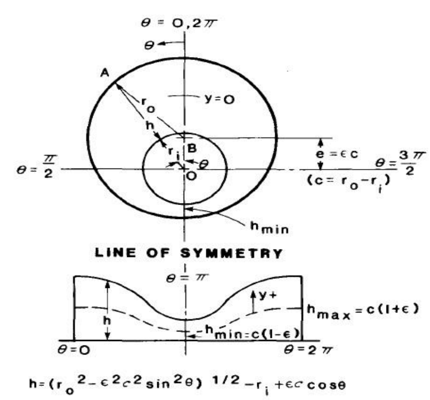

Figure 4. Nomenclature for eccentric annulus and equivalent slot (Iyoho & Azar, 1981)

The derivation of equation for variable slot height h can be found in appendix A.

2.4. Drillstring Rotation Effects

The changes in frictional pressure losses are attributed to different flow phenomena such as (Ahmed, 2006):

- **Shear Thinning**: Tendency that Non-Newtonian flow has to reduce its frictional pressure losses, usually caused by the coupling of axial and rotational flow through apparent viscosity function is dependent on shear rate.
- **Inertial/Acceleration Effects:** Usually attributed to the geometric irregularities and eccentricity of the annulus. If the changes in annular geometry are constant this results in an increase in frictional pressure losses.

  Irregular flow patterns can be observed in highly eccentric annulus, which can result in substantial variation of the velocity (both magnitude and direction) of a fluid element (inertial effect) along the streamline and an in increase in pressure loss.
- **Secondary Flow:** Centrifugal and shear-instabilities are the main reasons for secondary flow. Secondary flow patterns can be formed in annular flows and increase the friction pressure loss.

Based on (Ahmed, 2006) in fully eccentric annulus with relatively small pipe diameter the frictional pressure losses decreases under laminar flow (low rotational speed) and increases when the flow regime change to turbulent (high rotational speed). Under the same circumstances it was observed that in intermediate flow regimes the effects of drillstring rotation are minimal.

(Ahmed, 2006), also observed that in concentric annulus with the shear thinning effect, dominant in concentric and slightly eccentric annulus, causes the pressure loss to decrease when rotational speed is increased. Thus, in highly eccentric annulus, the shear thinning is overpowered by the inertial effect.

The inertial effects due eccentric and annular irregularities overcome the shear thinning, resulting in an increase in the friction pressure loss. This behavior is experienced most of the times during high flow rates.

Test were also conducted using the same fluid in a higher pipe diameter and it was observed that at the lowest flow rate a slight decrease in pressure loss can observed as the drillstring rotation increases. At intermediate flow rates, the annular pressure loss increases with the increase in drillstring rotation. At high flow rates, there is a 15% reduction in pressure loss when the pipe is rotating at 50 rpm. Geometric irregularities are the main reason for this effect. A 35% error margin discrepancy is observed at different flow rates (6 gpm and 14 gpm) when calculations are made without taking into consideration the drillstring rotation.

3. Methodology/Project Work

3.1. Methodology

In this project we will conduct an empirical research in a qualitative form. Meaning that question will be clearly defined and answered with evidence collected.

The empirical research follows A.D. Groot cycle which is illustrated in the figure bellow:

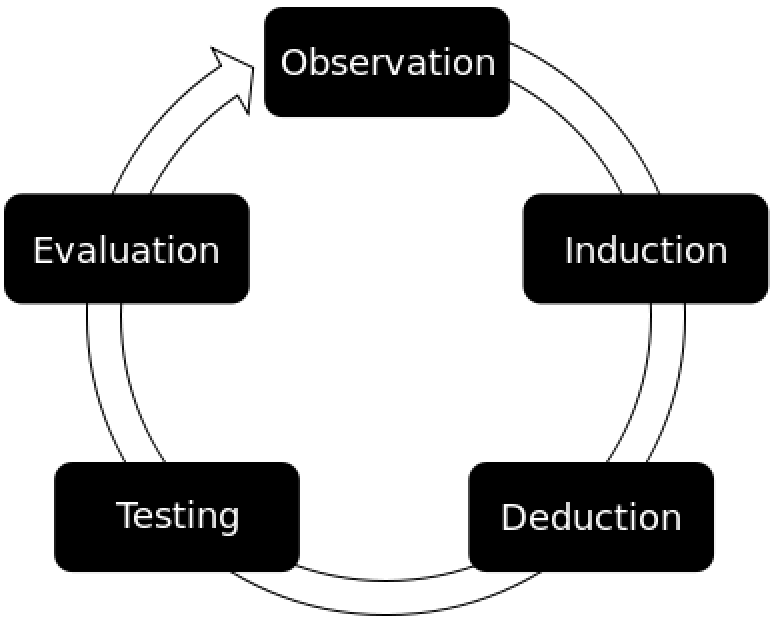

Figure 5. A.D. de Groot Empirical cycle

1. Observation – This is the stage where all the research material is gathered and organized, most of the material will be collected from well known Petroleum Journal such as: SPE, Journal of Petroleum Science and Engineering. Relevant Petroleum Engineering books will also be used as an auxiliary source for general concepts.
2. Induction – The stage where a hypothesis is formulated. The hypnoses that will be formulated from the knowledge acquired in the material mentioned above in the Observation will become our problem statement.

3. Deduction – Based on the literature, we will deduct its consequences with the newly gained empirical data.
4. Testing – Test the hypothesis with experimental data
5. Evaluation – At the last stage an evaluation of outcome of test will be performed

3.2. Objectives of the Empirical Research

- Do more than just reporting observations
- Provides a structure for improved understanding
- Combine extensive research with detailed case study
- Prove how relevant the theory is based in a real world environment (context)

3.3. Advantages of the Method

- Appropriate method for understanding and responding to dynamics of different situations
- Takes into consideration contextual differences
- Helps using what is available to build something new
- Helps meeting standards of professional research

3.4. Project Timelines

Table 1. Project Timeline

No.	Detail/ Week	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	Selection of Project Topic	■	■												
2	Preliminary Research Work		■	■	■	■									
3	Submission of Extended Proposal						●								
4	Proposal Defence								■	■					
5	Project work continues										■	■	■		
6	Submission of Interim Draft Report													●	
7	Submission of Interim Report														●

● Suggested milestone

■ Process

3.5. Summary of methodology

In the first stage of this research we intend to revise previous research related to the topic of this study, after a thorough understanding of the topic and its developments in the industry we should move forward to deeply analyze the slot-flow models for non-Newtonian, Bingham Plastic Model and Power-Law Model, fluid flow through annulus. Subsequently we should combine the knowledge acquired in both models to develop a slot-flow model for yield-power law fluids.

After developing the slot-flow model for yield-power law fluid we should gather rheological data from different samples using a viscometer and analyze the data using the Yield Power Law models, the results shall be expressed graphically and an adequate evaluation of the results is made.

4. Results and Discussion

The following mathematical model is developed under the assumption that:

- The flow is laminar
- The drillstring is placed concentrically
- Circular open hole of known diameter
- Incompressible drilling fluid
- Isothermal Flow
- No drillstring rotation

These assumptions do not constitute the reality of what goes on in the field therefore they won't describe precisely the flow of drilling mud in the well.

4.1. Yield Power Law Solution for flow in annulus

The equation for flow rate per unit width of the slot,

$$\frac{q}{w} = \int_0^h u dy = \int_0^{y_a} u dy + u_p \int_{y_a}^{y_b} dy + \int_{y_b}^h u dy$$

Equation 1. Velocity equations (Derivation in Appendix)

$$u = -\frac{\left(dp/dL \big/ K\right)^m}{m+1}\left[-y_a{}^{m+1} + (y_a - y)^{m-1}\right] \ldots.0 \le y \le y_a$$

$$u = \frac{y_a^{m+1}}{m+1}\left(\frac{dp/dL}{K}\right)^m \ldots\ldots y_a \le y \le y_b$$

$$u = \left(\frac{dp/dL}{K}\right)^m \frac{1}{m+1}\left[(h - y_b)^{m+1} - (y - y_b)^{m+1}\right] \ldots\ldots.. y_b \le y \le h$$

$$m = 1/n$$

$$\frac{q}{w} = uy\Big|_0^h - \int_0^h y\frac{du}{dy}dy = 0 - \int_0^h y\left(\frac{du}{dy}\right)dy$$

$$= -\int_0^{y_a} y\left(\frac{du}{dy}\right)dy - \int_{y_b}^h y\left(\frac{du}{dy}\right)dy = -I_1 - I_2$$

By manipulating the equation we find the follow,

$$I_1 = \left(\frac{dp/dL}{K}\right)^m \left[\frac{y_a^{\,m+2}}{(m+1)(m+2)}\right]$$

And

$$I_2 = -\left(\frac{dp/dL}{K}\right)^m \left[\frac{(h-y_b)^{m+2}}{m+2} + \frac{y_b(h-y_b)^{m+1}}{m+1}\right]$$

By solving the integral we have:

Equation 2

$$\frac{q}{w} = \left(\frac{dp/dL}{K}\right)^m \frac{1}{(m+1)(m+2)} \times \left[-y_a^{\,m+2} + (m+1)(h-y_b)^{m+2} + y_b(m+2)(h-y_b)^{m+1}\right]$$

By replacing y_a, y_b and defining $\tau_0 = \frac{h}{2}\frac{dp}{dL}$ we can represent the flow rate using τ_y and ${}^{dp}/_{dL}$.

Equation 3

$$q = \left(\frac{dp/dL}{K}\right)^m \left(\frac{w}{(m+1)(m+2)}\right) \times \left[-\left(-\frac{\tau_y+\tau_0}{dp/dL}\right)^{m+2} + (m+1)\left(h - \frac{\tau_y-\tau_0}{dp/dL}\right)^{m+2} + (m+2)\left(\frac{\tau_y-\tau_0}{dp/dL}\right)\left(h-\frac{\tau_y-\tau_0}{dp/dL}\right)^{m+1}\right]$$

Equation 4

$$q=\left[\frac{\pi\left(r_2^2-r_1^2\right)\left(r_2-r_1\right)^{1+\frac{1}{n}}\left(\frac{1}{K}\frac{dp}{dL}\right)^{\frac{1}{n}}}{2^{\frac{1}{n}}\left(\frac{1}{n}+1\right)\left(\frac{2}{n}+1\right)}\right]\times\left(1-\frac{\tau_y}{\left[\left(r_2-r_1\right)/2\right]\left(dp/dL\right)}\right)^{1+\frac{1}{n}}\times\left[\frac{\frac{\tau_y}{\left[\left(r_2-r_1\right)\right]\left(dp/\ dL\right)}+\frac{1}{n}+1}{2^{\frac{1}{n}}\left(\frac{1}{n}+1\right)\left(\frac{2}{n}+4\right)}\right]$$

Equation 3 was found based on the assumption made earlier the solution for the flow rate pressure drop relationship for yield power law fluids, using the same approach it was used by (Bourgoyne Jr, Millhein, Chenevert, & Young Jr, 1986) for the Bingham Plastic Model.

4.2. Rheological Data

The samples contain water and bentonite at 5% w/v, hydrated for 24h at room temperature. The data was collected using a Fann VG 35 Viscometer, R1-B1 Rotor Bob Configuration and F1 Torsion Spring.

Table 2. Rheological data

Shear Rate (1/s)	**Shear Stress (Pa)**		
Sample	S1 (5% Bentonite)	S2 (2% Bentonite)	S3 (0.3% Bentonite)
1021	18.17	12.83	4.50
851	16.67	9.75	4.17
681	14.75	8.58	3.58
510	12.75	8.00	3.08
340	9.92	7.33	2.42
170	6.50	5.58	1.75
136	6.17	5.42	1.50
102	5.75	5.08	1.42
51	4.00	4.00	1.08
34	4.17	4.17	1.00
17	3.33	3.67	0.67
10	3.25	3.67	0.58
5	2.80	3.25	0.50

The rheograms presented in fig.6, 7 and 8 and from table 2 clearly show a yield-pseudoplastic fluid behavior and yield power law models will give a more adequate solution for the rheological behavior.

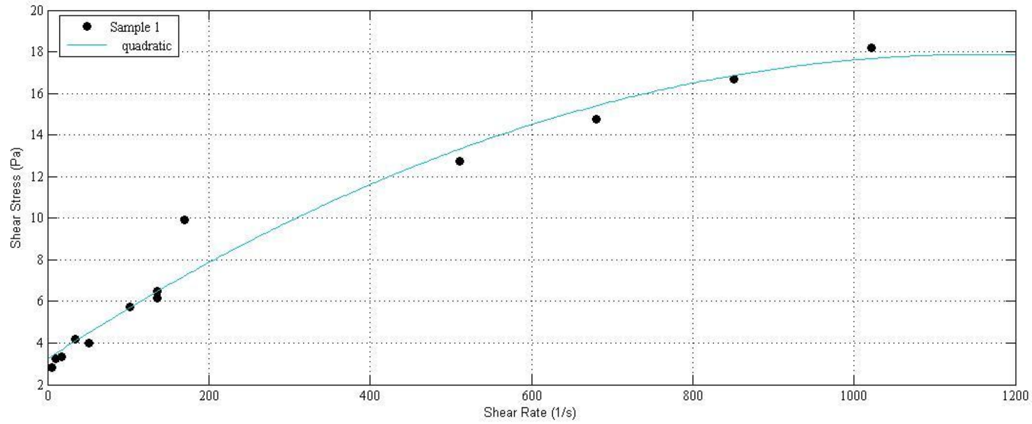

Figure 6. Rheogram of the original data for Sample 1

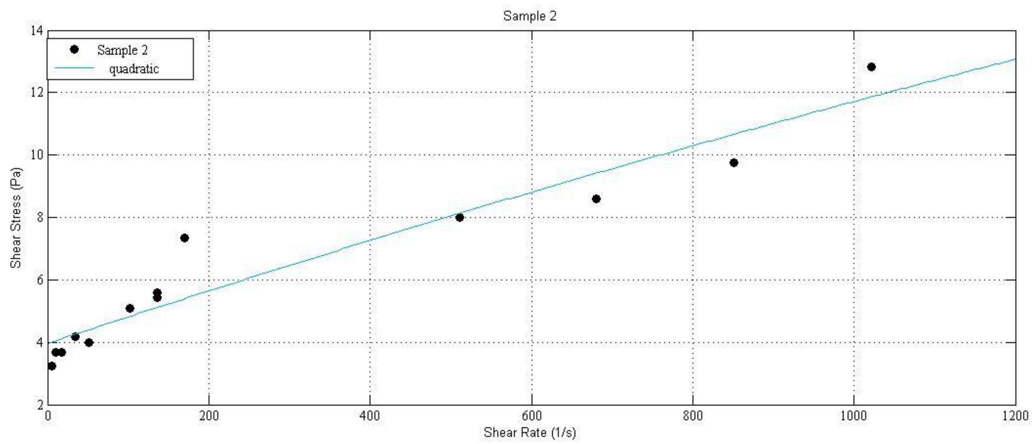

Figure 7. Rheogram of the original data for Sample 2

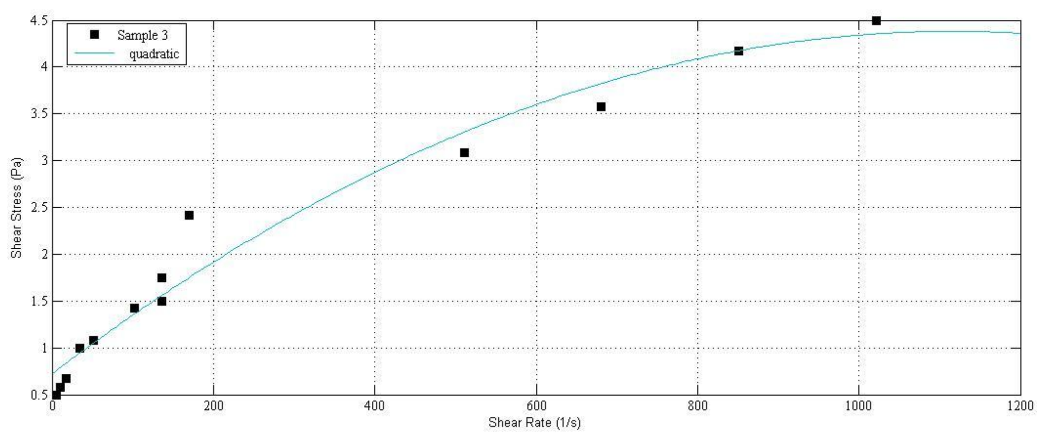

Figure 8. Rheogram of the original data for Sample 3

Values of yield Stress were obtained using the plot shown in fig, 9. The method is based on plotting the experimental rheological data using semi plot of $[\tau - \log \gamma]$. From the plot we can observe that for the lower shear rate values, if extrapolated so that $(\gamma \rightarrow 0)$, follow a straight line. From the straight we collect that value at which it crosses the y-axis, and that will be our estimation of yield stress.

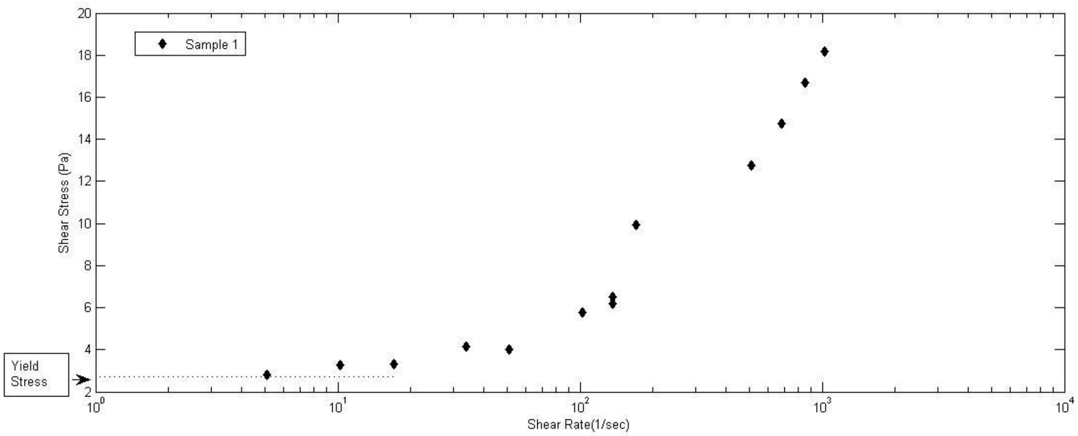

Figure 9. Determination of yield stress using graphical plot

4.3. Non-Linear Regression Results

Table 3. Rheological Parameters derived from non-linear regression

Sample	τ_y (Pa)	K (Pas^n)	n
S1	2.4095	0.1251	0.7012
S2	3.4701	0.0313	0.6436
S3	0.3793	0.0567	0.6196

This method is used to reduce the value of the squared sum (SS) of the difference between data and fit. This is a cyclical process where a first estimate of the yield stress needs to be defined, in this specific project we used the $[\tau - \log \gamma]$ (See fig. 9) to determine the first estimate of the yield stress.

The regression was done using EXCEL where the first iteration the SS is computed based on the initial parameter values (Experimental values of Shear Stress and Shear Rate). Excel repeats this step in order to find results that give the lowest possible value of SS.

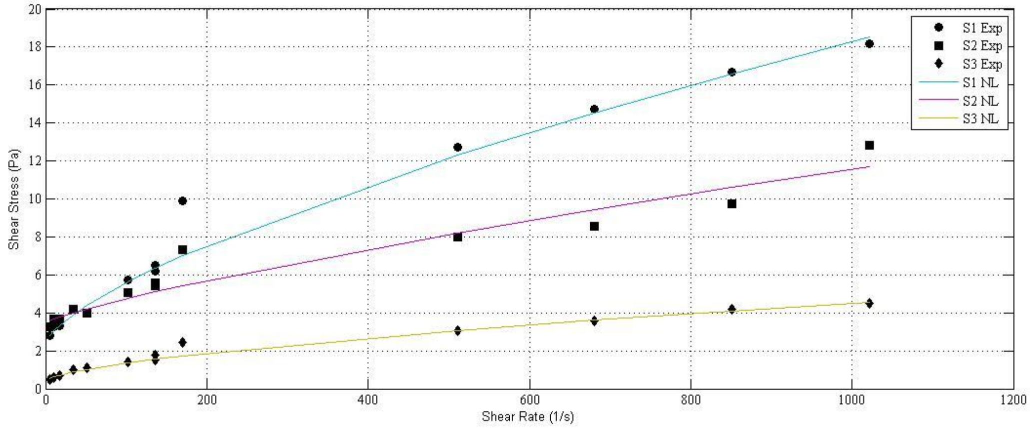

Figure 10. Comparison of Rheograms original data with results for the model

4.4. Pressure Drop Flow Rate Relationship

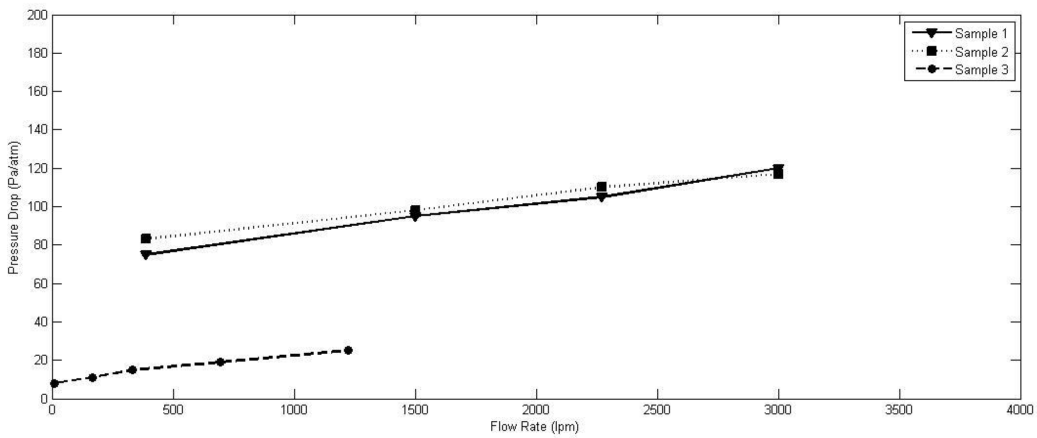

Figure 11. Pressure drop- flow rate profile for S1, S2, S3 in a concentric annulus 0.4445 m by 0.127 m

Pressure drop in the annulus affect the performance of a drilling program therefore when analyzing the pressure drop – flow rate relation of any given fluid this must be taken into consideration. From fig. 11 it is clear that sample 3 gives better performance in relation to the other two since it has the lower pressure drop range.

Pressure losses in the annulus can greatly affect the rate at which cuttings are transported and also require more energy to be pumped in order to fulfill drilling requirements.

Samples 1 and 2 are good for very high pump rates as it is shown in the graph, there is an increment in pressure drop as flow rate increase but the slope of the curve is very small which indicates that the losses happen at a very low rate. Samples 1 and 2 are recommended to be used when drilling deep wells where the mud pump rate needs to be high.

5. Conclusion and Recommendation

It is clear that the rheological parameters of drilling fluids have a major effect on the drilling program and they must be effectively determined. In this particular case Non-Linear regression was used but there is a wide range of approaches that could be used to determine the rheological parameters. These parameters are then used to compute the pressure profile of the three samples.

The pressure profiles were obtained using the yield power law model for drilling fluid using slot flow approximation. The approach used was based on the approach used to develop the Bingham Plastic Model by (Bourgoyne Jr, et al. 1986).

The solution for pressure drop versus flow rate relationship presented in this study can be used to derive pressure drop versus flow rate relationship that consider the eccentricity of the annulus, since it is almost impossible to have a fully concentric annulus, this will further improve the accuracy of the estimations made in relation to the fluid behavior in the annulus. The approach used in this paper was proved to be accurate by graphically comparing the experimental data with the model data. All graphs that relate experimental data and model data follow the same trend and have a very small difference in terms of values.

Appendix

1. Deviation of Equation for Variable Slot Height h

Referring to figures 2 and 3 the following definitions are applicable

$c = r_0 - r_i$ (A-1)

Where c is the constant radial clearance for concentric annulus, e is the offset between centers

$\epsilon = fractional\ eccentricity$

$\epsilon = \frac{e}{c}$ (A-2)

Where r_0 and r_i are outer and inner radii, respectively, h is the variable annular clearance.

Applying the cosine rule to triangle AOB

$r_0^2 = (h + r_i)^2 + e^2 - 2e(h + r_i)\cos\theta$

(A-3)

$= h^2 + r_i^2 + 2hr_i + e^2 - 2eh\cos\theta - 2er_i\cos\theta$

(A-4)

Rearranging,

$h^2 + 2h(r_i - e\cos\theta) + (e^2 - r_i^2 + r_o^2 - 2e\cos\theta) = 0$

(A-5)

Solving for h,

$h = \frac{1}{2}\left\{2(e\cos\theta - r_i) \pm [4(r_i - e\cos\theta)^2 - 4(e^2 - r_o^2 + r_i - 2er_i\cos\theta]^{1/2}\right\}$

(A-6)

Simplifying and ignoring the negative h value

$h = e\cos\theta - r_i + [r_o^2 - e^2(1 - cos^2\theta)]^{1/2}$ (A-7)

$= (r_0^2 - e^2 sin^2\theta)^{1/2} - r_i + e\cos\theta$ (A-8)

$Since\ e = \epsilon c$

$h = (r_o^2 - \epsilon^2 c^2 sin^2\theta)^{1/2} - r_i + \epsilon c\cos\theta$ (A-9)

If ϵ and/or c is small, $\epsilon^2 c^2$ is even smaller and Eq. A-9 degenerates the following: $[\boldsymbol{h = c(1+\epsilon\cos\theta) = c + e\cos\theta}]$

2. Derivation of velocity profile in concentric annulus

As mentioned in the literature review, integration of the force balance performed on a fluid element gives

$$\tau = y\frac{dp}{dL} + \tau_0$$

With τ_0 to be determined. y_a and y_b are the distances of the lower sheared and upper sheared surfaces from the bottom plate respectively. At y_a the shear stress τ_a must equal $(-\tau_y)$.

$$\tau_a = -\tau_y = \tau_0 + y_a\frac{dp}{dL}$$

Which gives,

$$y_a = -\frac{\tau_y + \tau_0}{dp/dL}$$

For the outer plug region shear stress is obtained at y_b

$$\tau_b = \tau_y = \tau_0 + y_b\frac{dp}{dL}$$

The shear stress in the fluid layer enclosed by the inner layer of the plug is (u =0 at y =0) and m=1/n:

$$\tau = -\tau_y - K\left(\frac{du}{dy}\right)^n = \tau_0 + y\frac{dp}{dL}$$

$$u = \frac{-K}{(m+1)\,dp/dL}\left\{-\left(-\frac{\tau_y+\tau_0}{K}\right)^{m+1} + \left(-\frac{\tau_y+\tau_0}{K} - \frac{dp/dL}{K}y\right)^{m+1}\right\};0 \le y \le y_a$$

In terms of y_a the equation becomes

$$u = -\frac{\left(\dfrac{dp/dL}{K}\right)^m}{m+1}\left\{-\left(y_a\right)^{m+1} + \left(y_a - y\right)^{m+1}\right\}; 0 \le y \le y_a$$

The plug velocity (u_p) is the velocity at $y = y_a$ given by,

$$u_p = \frac{y_a^{m+1}}{m+1}\left(\frac{dp/dL}{K}\right)^m ; y_a \le y \le y_b$$

The shear stress for the fluid region around the plug and the upper plate is (u =0 at y =h)

$$\tau = \tau_y + K\left(-\frac{du}{dy}\right)^n = \tau_0 + y\frac{dp}{dL}$$

In terms of y_b the previous equation becomes,

$$u = \frac{1}{dp/dL\, K^m (m+1)}\left[\left(\tau_0 - \tau_y + h\frac{dp}{dL}\right)^{m+1} - \left(\tau_0 - \tau_y + y\frac{dp}{dL}\right)^{m+1}\right]; y_b \le y \le h$$

$$u = \left(\frac{dp/dL}{K}\right)^m \frac{1}{m+1}\left[\left(h - y_b\right)^{m+1} - \left(y - y_b\right)^{m+1}\right]; y_b \le y \le h$$

The velocity of the plug is given when $y = y_b$, therefore the plug velocity equation becomes,

$$u_p = \left(\frac{dp/dL}{K}\right)^m \frac{\left(h - y_b\right)^{m+1}}{\left(m+1\right)}; y_a \le y \le y_b$$

$$y_a^{m+1} = \left(h - y_b\right)^{m+1}$$

By taking the (m+1)th root and only keeping the positive value we have

$$y_a = h - y_b \rightarrow y_a + y_b = h$$

Substituting the values of y_a and y_b given in the previous equation τ_0 is found to be,

$$\tau_0 = -\frac{h}{2}\frac{dp}{dL}$$

$$\tau_w = -\tau_0 = \frac{h}{2}\frac{dp}{dL}$$

For there to be fluid flow,

$$\frac{dp}{dL} > \frac{\tau_y}{h/2}$$

Then

$$y_a = \frac{h}{2} - \frac{\tau_y}{dp/dL}$$

$$y_b = \frac{h}{2} + \frac{\tau_y}{dp/dL}$$

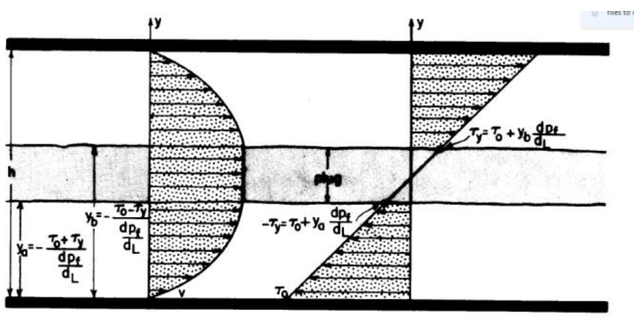

Figure 12. Laminar Flow of fluid in Slot (Bourgoyne Jr, et al. 1986)

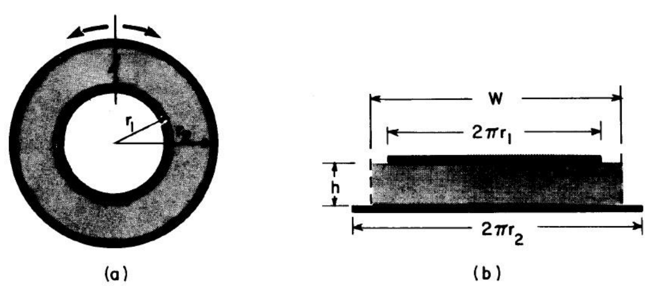

Figure 13. Represtention of fluid as a slot (Bourgoyne Jr, et al. 1986)

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