Preprocessing of Seismic Raw Data by Ray Tracing Method

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By

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CERTIFICATION OF APPROVAL

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Approved by:

____

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Project Supervisor

UNIVERSITI TEKNOLOGI PETRONAS

TRONOH, PERAK

December 2014

CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

____

MUSFIRAH BINTI YUSOF

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ABSTRACT

Synthetic database for S-wave diffraction angle with source at different depth does not exist. Objective is to develop a synthetic velocity model for generating synthetic reflection and refraction data in standard Society of Exploration Geoscientist-Y (SEG-Y) format database. Velocity seismic profile database consisting of diffraction at the fractures and ray fan size are simulated. Nine layers velocity model is built by using finite difference method thus producing seismic wave propagation together with seismic ray tracing. Seismic ray tracing creates numerous number of rayfans. The angle of the reflection and refraction are calculated and tabled.

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CHAPTER 1

INTRODUCTION

1.0 INTRODUCTION

This chapter covers the project background, problem statement, objective, scope of study, and study feasibility of this project.

1.1 Project Background

Raytracing is a technique of generating image by tracing the path of sound or light. Seismic P-S waves are utilized by geophysicist to identify the fracture of earth. It is known that hydrocarbon is trapped in between fractures. P-S waves are shot from source and reflected/refracted before there are recorded by geophones [21]. Ray or the path of propagating waves will provide the characteristic of the targeted reservoir i.e. the fractures and anisotropy properties [6]. Filtering under the subject signal processing is important to remove the unnecessary noise [21]. Wave propagation in isotropic and anisotropic model is commonly used as a tools to measure the depth of the targeted reservoir [32]. In geophysics, Huygens principle, Snell’s law reflection and refraction has been utilized to build rayfan. Rayfan is a method to identify the fracture underneath us [6].

1.2 Problem Statement

Shear wave tends to split and diffract while it’s propagating through the fractures or anisotropic medium. Variation of the source of the depth will generate a different rayfan geometry due to the reflection and refraction of the wave of the fractures. Limitation of the reflection/refraction gradient due to the variation of the depth of the source is yet studied.

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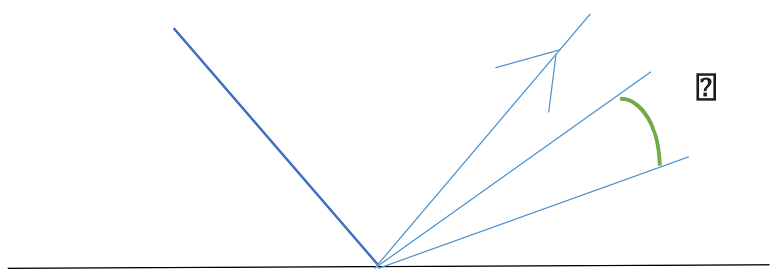


Figure 1: Ray fan and the angle of the reflected ray fan

1.3 Objective

The objective is:

- To develop a synthetic velocity model for generating synthetic reflection and refraction data in standard Society of Exploration Geoscientist-Y (SEG-Y) format database.

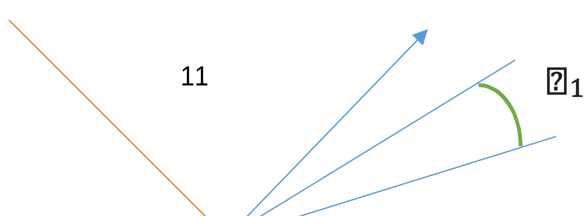


Figure 2: $\square_1$ is the reflected angle and $\square_2$ is the refracted angle of the ray fan

1.4 Scope of Study

The study is limited to designing an anti-cline velocity model that can generate seismic data in Society of Exploration Geophysicists-Y (SEGY) format. The anti-cline velocity model should have the following specifications:

- Limited to maximum ten layers earth model
- 30 Hz to 40 Hz.
- Depth 1000m
- Width 2000m
- Number of shot (1 shot, zero offset) within 0 – 500 meters

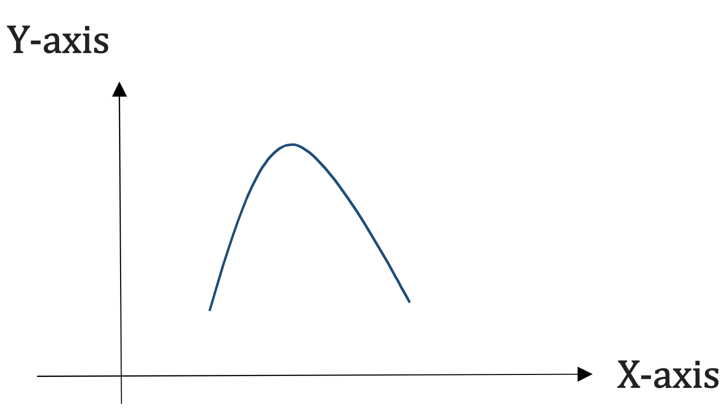


Figure 3: Anti-cline velocity model

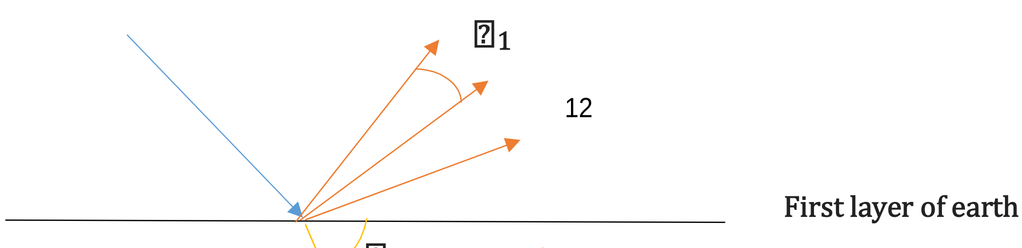


Figure 4: Two layers earth model for $\square_1$ and $\square_2$

1.5 Feasibility of Study

Build up a reference data base for geo scientist to interpret the recorded sample/dataset.

CHAPTER 2

LITERATURE REVIEW

2.0 Vertical Seismic Profile (VSP)

There are many advantages of zero-offset vertical seismic profile. One of it is to provide the most accurate velocity information in the vicinity of boreholes [2]. The first break times of direct down going S-wave is picked to estimate the velocity of the S-wave for 3-component VSP. But the direct down going S-wave arrival might be too weak to be picked precisely [9]. Figure 5 shows the zero-offset vertical seismic profile while Figure 6 shows the 3-component VSP method.

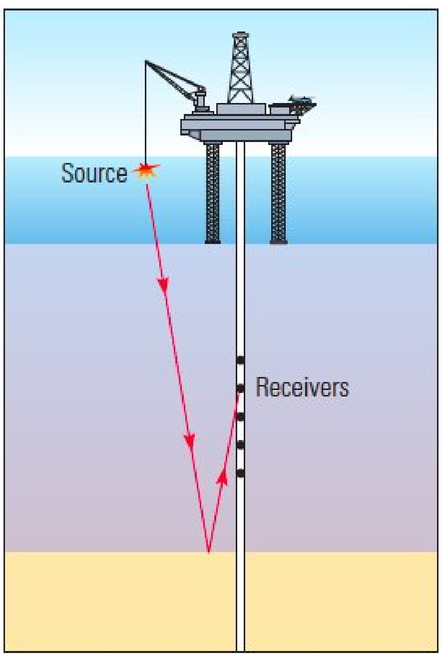


Figure 5: Zero offset VSP

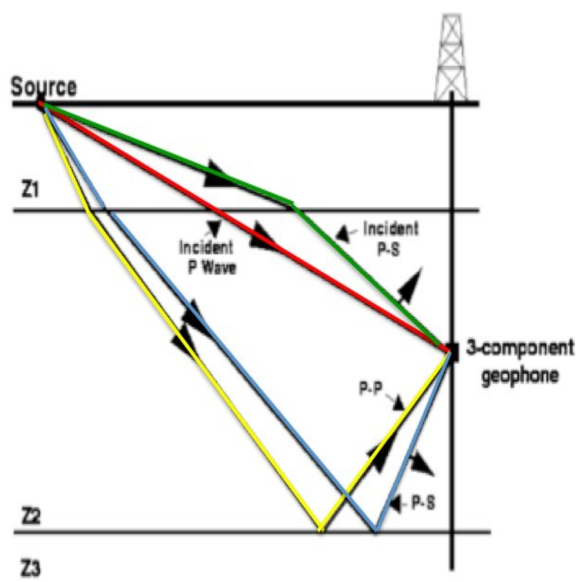


Figure 6: 3-Component VSP

2.1 Traveltime Tomography, t

The traveltime tomography starts when the forward modelling by ray tracing is done exactly when the initial guess is given. Later, the associated traveltimes is computed. The iteration of the tomography must be done to meet to the best approximation of the true model by reducing the variances between the experiential traveltimes and the modeled ones [14]. Traveltimes is computed iteratively by using traveltime tomography. And to update the velocity, the differences between modeled ones and observed ones are used. Hence the traveltime calculation is a critical step for traveltime tomography [15].

2.1.1 Velocity

The primary (P) and secondary (S) waves travel within the earth. The P-wave moves in the direction of propagation. The velocity of P-wave can be determined by using the equation (2.1) [4].

$$Vp = \sqrt{\frac{K+\frac{4}{3}\mu}{\rho}}$$

(2.1)

$$= \sqrt{\frac{\lambda + 2\mu}{\rho}}$$

where K is the bulk modulus, μ is the shear modulus and ρ is the density [4].

S-wave is perpendicular to the direction of wave propagation. The velocity of the S-wave can be calculated by using the equation (2.2) [5].

$$Vs = \sqrt{\frac{\mu}{\rho}} \qquad (2.2)$$

2.2 Ray Tracing

Inversion method is based on ray tracing. The shooting method is adopted for ray tracing. The shooting method fixes one end of the ray path (source point), takes initial incidence angle and initial azimuth, and starts the initial ray tracing [6]. The radius of target is analyzed using the bisection method, then the error is minimized and the ray path is linearly interpolated by the program. After setting a new ray parameter, the program shoots new rays and analyzes the radius of target again until the ray path achieves the end point (receiver point) [22].

Speed of sound in the ocean depth is changed due to changes in density and temperature, to achieve a local minimum near the depth of 800-1000 meters. Such local minima, called acoustic emission channel as a waveguide, the sound tends to bend towards it. Ray tracing a path through the ocean can be used to very large distances to calculate the sound, the impact is equipped with acoustic emission channels, as well as reflection and refraction off the surface and the bottom of the ocean [7]. Since then, the location of high and low signal intensity can be calculated, which is in the field of marine acoustics, acoustic communications, and acoustic temperature measurement is useful [6].

The formula used to calculate the ray tracing is shown in equation (2.3). Basically, ray tracing is calculated using Snell's law. Based on the equation, (2.3), b is the slope of the gradient, v is the velocity, and z is the depth [22]. In equation (2.4) and (2.5), u is the range of the variable, is the vertical slowness and p is the horizontal slowness [22].

$$b = \frac{v2 - v1}{z2 - z1} \tag{2.3}$$

$$x(p) = \left(\frac{}{bup}\right)\Bigg|_{u2}^{u1} \tag{2.4}$$

$$t(p) = \left(\frac{1}{b}\right)\left[ln\left(\frac{u+n}{p}\right) - \left(\frac{n}{u}\right)\right]\Bigg|_{u2}^{u1} \tag{2.5}$$

2.3 Snell's Law

Snell's law or usually known as law of refraction (sometimes known as Snell - Descartes law) refers to the time between light or other waves passing through the boundary between two different isotropic media, used to describe the incidence and refraction angles formula relationship between, for an example, water, glass and air [27].

Snell's law states that the ratio of the sine of the angle of incidence and refraction, and the ratio is equivalent to the reciprocal of the ratio of the phase velocity in the two media, or equivalent refractive index [27]

$$\frac{\sin\theta 1}{\cos\theta 2} = \left(\frac{V1}{V2}\right) = \left(\frac{n2}{n1}\right) \tag{2.6}$$

where $\theta 1$ *and* $\theta 2$ are the P-wave and S-wave angles of incidence and reflection, $V1$ and $V2$ are the corresponding P-wave velocity and S-wave wave and $n1$ and $n2$ are the refractive index of the respective medium [29].

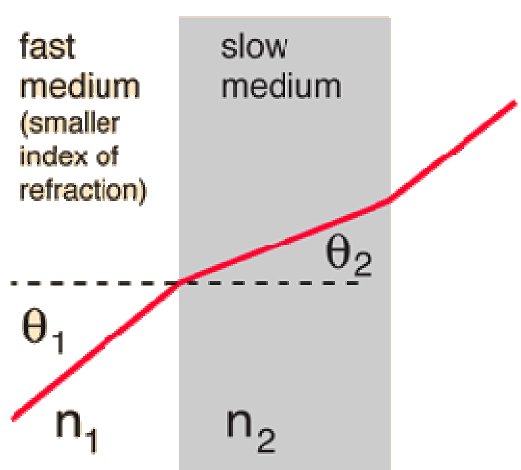


Figure 7: Refraction angle in two media to the direction of propagation.

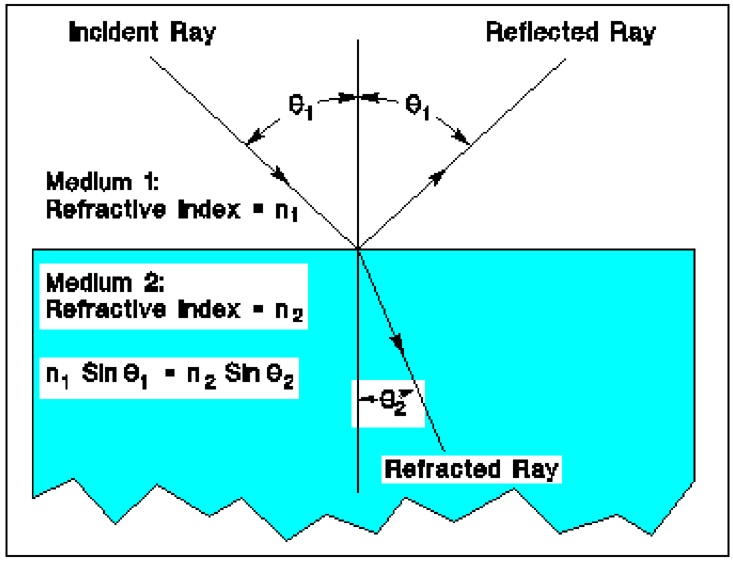


Figure 8: Reflection and refraction phenomena in two different media.

2.4 Huygens's Principle

The basic understanding in Huygens principle is that each point on the wavefront is the source of the wave. These waves spread out in the forward direction at the same speed of the source wave. Before, the new wave is a line tangent to all of the wavelets. This principle is used to explain refraction, interference and diffraction. Diffraction is curved around the edge of the obstacle waves [26].

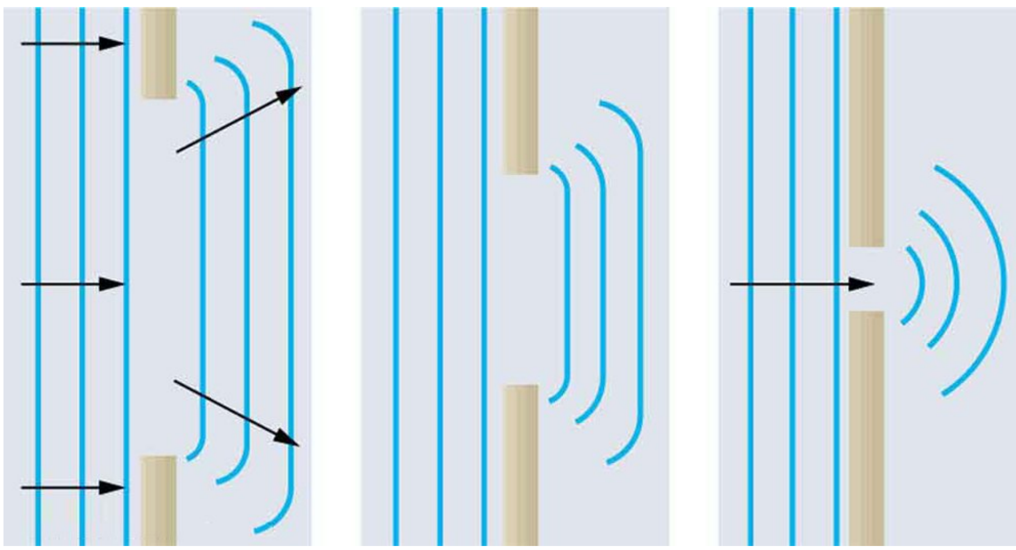


Figure 9: Diffraction phenomena for different slits

Huygens's Principle is applied to the front straight wave amazing opening. Curved wavefront after passing through the edge of the opening, a process called diffraction. More extreme bending amount of the small opening, the wave is the most remarkable characteristic of the fact that approximately the same size as the wavelength of the object coincides with the interaction [24].

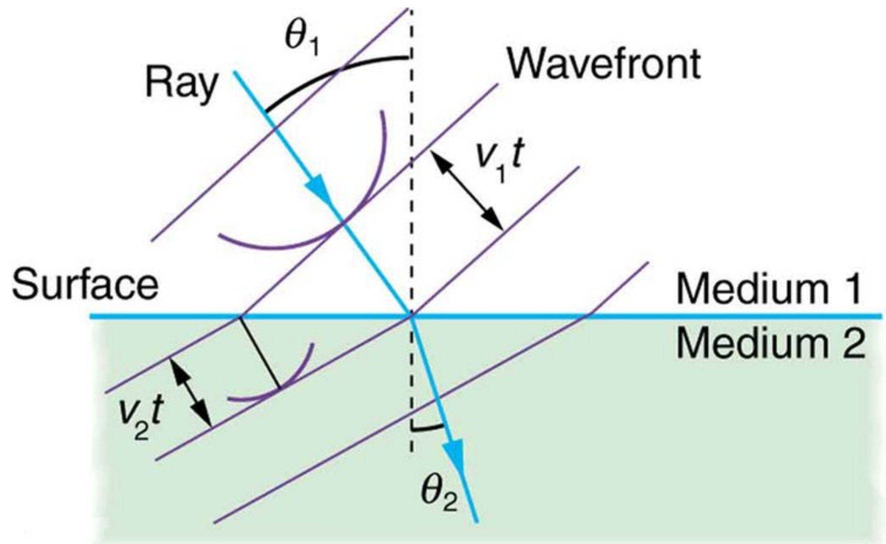


Figure 10: Application of Huygens's Principle to a straight line travelling in a different media with different speed.

Huygens's Principle can be used in explaining the law of refraction for a wavefront passing from one medium to another [29]. Each wavelet is issued when the wavefront crossing the interface between the media. Because the speed of light is smaller in the second medium, the wave does not travel in a given time, thus, the new wavefront changes direction. This explains why a ray changes direction, becoming closer to vertical when in the second medium [31].

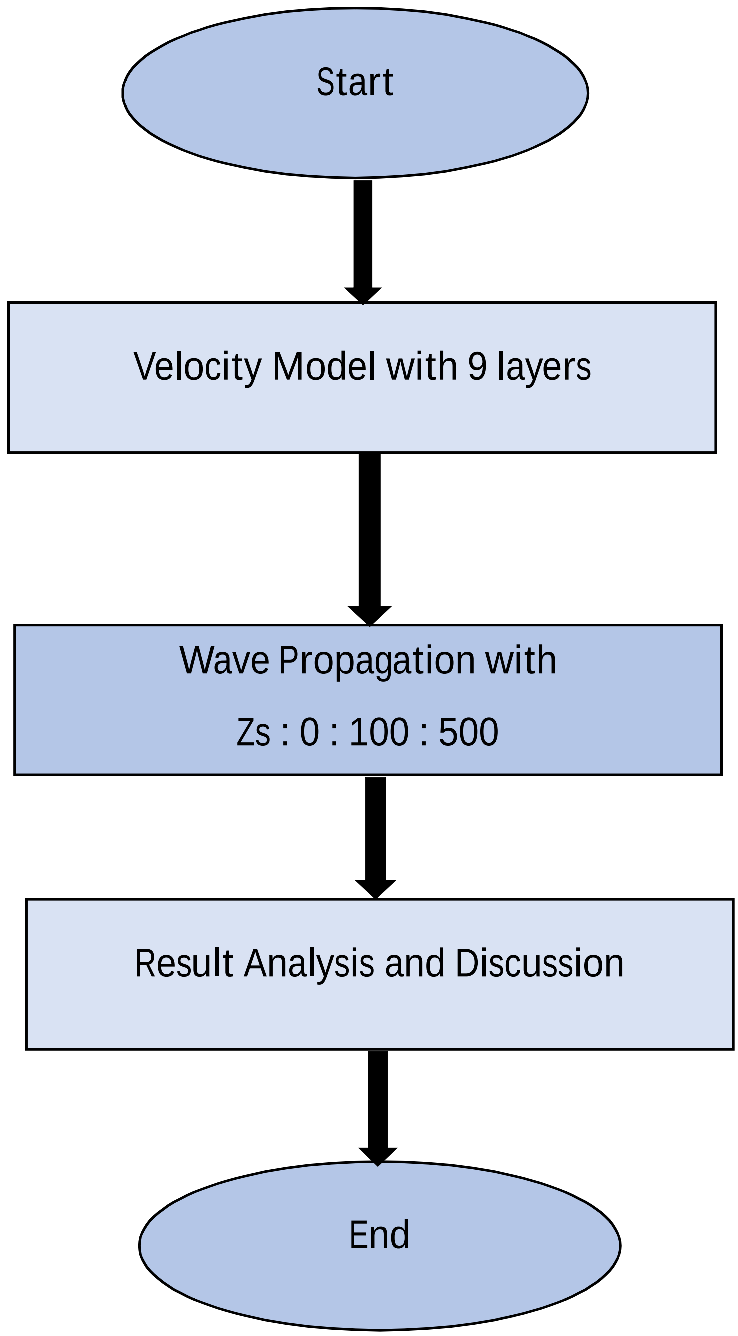
CHAPTER 3

METHODOLOGY

3.0 Introduction

This chapter will discuss the method that will be used in this research. In 3.1, the flow chart is presented. Under 3.2, nine layers velocity model is explained. In 3.2.1, Finite difference is introduced. In 3.3, seismic wave propagation is discussed. In 3.4, ray tracing is explained. In 3.4.1, Snell's law is proposed. In 3.4.2, Huygens's Principle is shared. In 3.4.3, gradient of reflection and refraction is explained and in 3.5, the Gantt chart is shown.

3.1 Flow Chart



3.2 Nine layers velocity model

Nine layers of velocity model are to be built by using Finite Difference as illustrated in Figure 11 below.

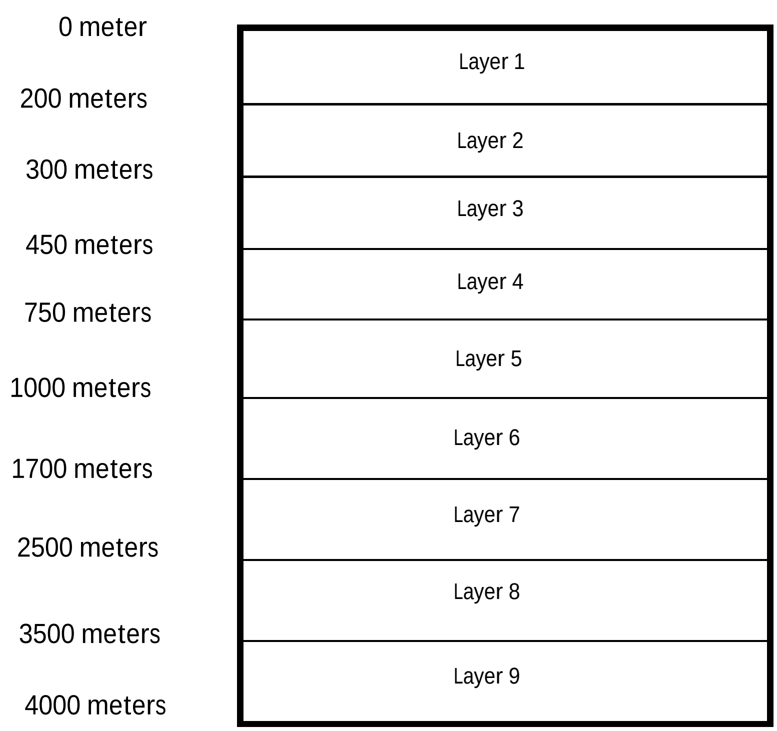


Figure 11: Nine layers velocity model.

3.2.1 Finite difference

To build the velocity model, the finite difference method is going to be used. Iteration starts from the nth receiver, and the S-wave velocity of the nth layer is inverted first. Based on the P-wave velocity, the initial S-wave velocity and the PS-wave ray path, we can calculate the PS-wave traveltime for the nth receiver [14].

Then, compare the calculated PS-wave traveltime with the picked one, and update the S-wave velocity using the traveltime difference [21].

When the difference between the calculated and picked PS-wave traveltimes is less than a given criterion, stop the iteration and save the S-wave velocity of the nth layer [21].

Next, go back to step 1, calculate the PS-wave traveltime of the (n − 1)th receiver using the calculated S-wave velocity of the nth receiver[21].

When traveltimes for all receivers are calculated, we can obtain the whole S- wave velocity all different receiver depths. Note that here upgoing converted PS-wave is assumed, such that the inversion starts from the last receiver point and iteratively goes up to the first receiver point. That is, the S-wave velocity of a single layer is updated at each iteration [21].

The inversion steps by using downgoing converted S-wave are similar to that of using upgoing converted S-wave. The only difference is that the inversion starts from the first downgoing converted wave receiver to the last downgoing converted wave receiver [14].

Finite difference is an approximate derivatives of partial differential equations by a linear combination of function values at the grid points. The function of fp of the forward finite difference can be defined as in (3.1) below

$$\Delta fp = f_{p+1} - f_p \tag{3.1}$$

The equation (3.2) shown below is called second forward finite difference.

$$f''(x_i) = \frac{f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)}{h^2} - O(h) \tag{3.2}$$

where

$$O(h) = f^{(3)}(x_i)h + \tfrac{7}{12} f^{(4)}(x_i)h^2 + ... \tag{3.3}$$

3.3 Seismic wave propagation

Seismic wave propagation is consisting of 2 types of waves which are P-wave and S-wave. Seismic wave propagation will produce a ray tracing as illustrated in Figure 12 below.

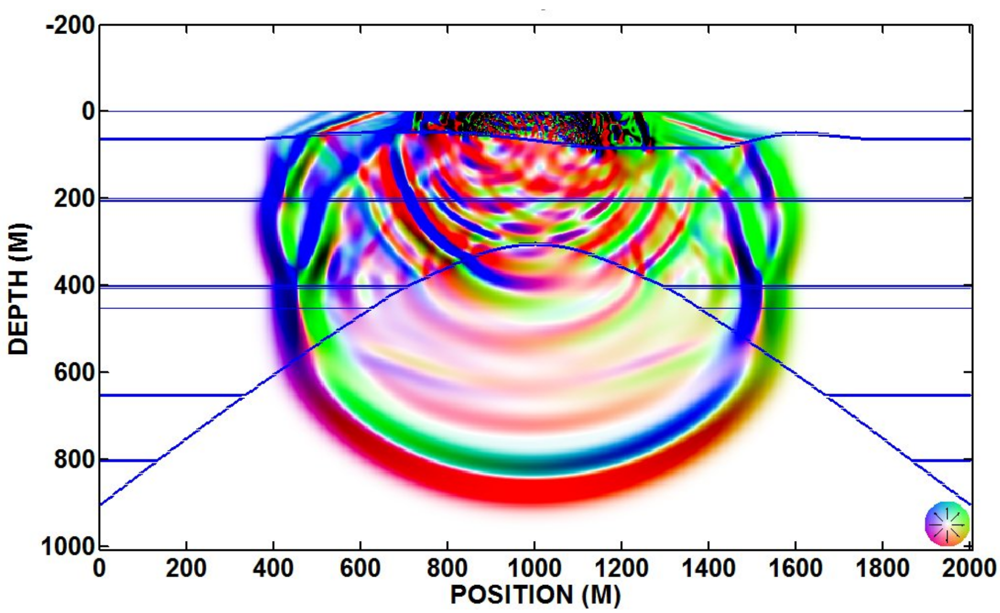


Figure 12: Seismic Wave Propagation using Finite Difference Method.

Seismic wave propagation can be calculated by using equation (3.4) shown below

$$(\Delta^2 + k^2)\, u = 0 \tag{3.4}$$

Based on equation (3.4), *k* is the wave number and *u* is the strain replacement.

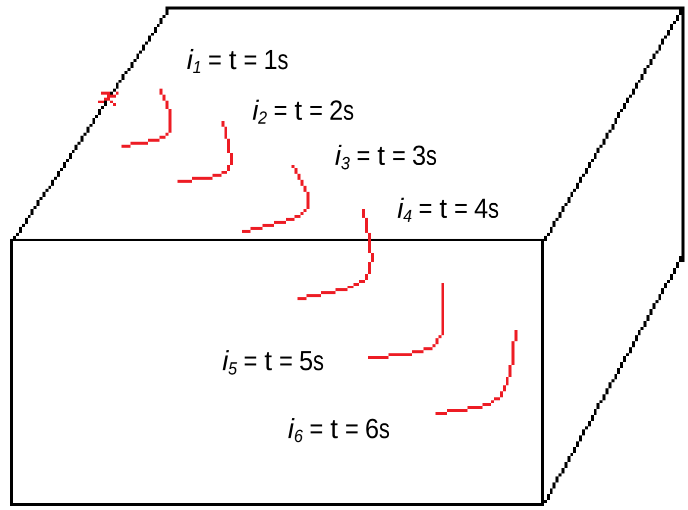


Figure 13: 3D model grid x grid

Figure 13 shows the 3D model grid by grid. To insert a motion or a wave inside the model, Finite Difference is used to let the wave propagate in the model.

(3.5)

Equation (3.5) is the equation of seismic sound wave where A is the amplitude, k is the wave number, c is the speed and t is the time.

3.4 Ray tracing

The ray tracing will be produced by using Snell's law method and Huygens's Principle as shown in Figure 14.

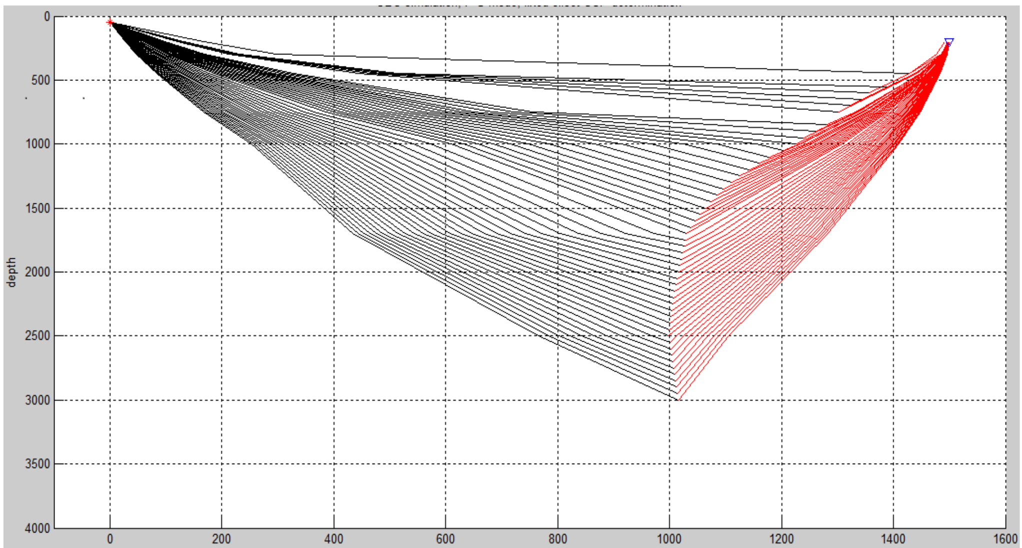


Figure 14: Ray tracing by using Snell's law method and Huygens's Principle.

3.4.1 Snell's Law

Snell's law equation that will be used in this research is as in (3.6)

$$\frac{\sin\theta 1}{\cos\theta 2} = \left(\frac{V1}{V2}\right) = \left(\frac{n2}{n1}\right) \tag{3.6}$$

where $\theta 1$ and $\theta 2$ are the P-wave and S-wave angles of incidence and reflection, $V1$ and $V2$ are the corresponding P-wave velocity and S-wave wave and $n1$ and $n2$ are the refractive index of the respective medium [29].

3.4.2 Huygens's Principle

Huygens principle is that each point on the wavefront is the source of the wave. These waves spread out in the forward direction at the same speed of the source wave. Before, the new wave is a line tangent to all of the wavelets. This principle is used to explain refraction, interference and diffraction. Diffraction is curved around the edge of the obstacle waves [26].

3.4.3 Gradient of reflection and refraction

The simulation performed on Matlab 2013a will produce a set of 10 rayfans in five different depth of source. The depth of the source will be varied from 100 meters to 500 meters. Based on these rayfan, the value of the refraction and reflection angle can be calculated.

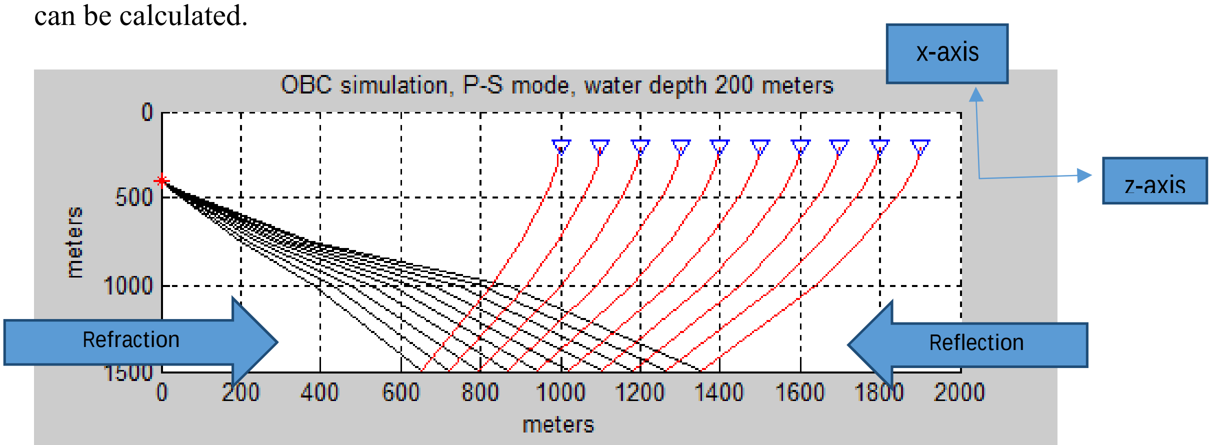


Figure 15: Rayfan produced from source of 400 meters.

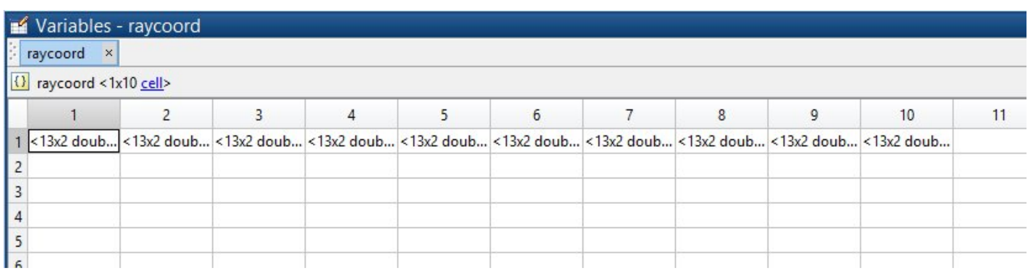


Figure 16: Details of each rayfan is tabulated under raycoord.

The details of each rayfan is then used to calculate the gradient of the reflection angle and the refraction angle. The gradient is to be calculated using equation (3.7) as shown below.

(3.7)

From the gradient calculated, the angle of the reflection and refraction can be calculated by converting the value of the gradient to degree by using the scientific calculator.

3.5 Gantt Chart

Table 1: Gantt chart.

ACTIVITY	WEEK NO													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Literature Review														
Mathematical Modelling										*				
Result/Analysis														

CHAPTER 4

RESULT AND DISCUSSION

4.0 Introduction

By using Matlab R2013a, ten set of rayfan are produced at different depth of source which varies from 100 meters to 500 meter.

The data that has been collected from the Matlab is the element for gradient. The gradient is then calculated by using equation (4.1) below

$$gradient = \frac{y2 - y1}{x2 - x1} \tag{4.1}$$

From the gradient calculated, the angle of the reflection can be calculated. The obtained value of gradient is converted to degree by using the scientific calculator.

4.1 Results

The results of the reflection and refraction angle obtained through the Matlab process are tabulated in Table 1 and Table 2. Table 1 is the reference table of the refraction angle of the rayfans for the depth of source from 100 meters to 500 meters while Table 2 is the reference table of the reflection angle of the rayfans for the depth of source from 100 meters to 500 meters.

The rayfan is then will be produced with different depth of source. It will be varies from 100 meters to 500 meters.

Table 1: The reference table of refraction angle of rayfan.

Depth(m) \\ $\square$	**Refraction 1 $\square_1$**	**Refraction 2 $\square_2$**	**Refraction 3 $\square_3$**	**Refraction 4 $\square_4$**	**Refraction 5 $\square_5$**	**Refraction 6 $\square_6$**	**Refraction 7 $\square_7$**	**Refraction 8 $\square_8$**	**Refraction 9 $\square_9$**	**Refraction 10 $\square_{10}$**

Depth(m) θ	Reflection 1 θ_1	Reflection 2 θ_2	Reflection 3 θ_3	Reflection 4 θ_4	Reflection 5 θ_5	Reflection 6 θ_6	Reflection 7 θ_7	Reflection 8 θ_8	Reflection 9 θ_9	Reflection 10 θ_{10}
100	Gradient = 0.502 Degree = 26.68°	Gradient = 0.556 Degree = 29.08°	Gradient = 0.610 Degree = 31.40°	Gradient = 0.666 Degree = 33.66°	Gradient = 0.722 Degree = 35.84°	Gradient = 0.78 Degree = 37.94°	Gradient = 0.838 Degree = 39.97°	Gradient = 0.898 Degree = 41.93°	Gradient = 0.959 Degree = 43.81°	Gradient = 1.021 Degree = 45.61°
200	Gradient = 0.534 Degree = 28.10°	Gradient = 0.591 Degree = 30.60°	Gradient = 0.650 Degree = 33.02°	Gradient = 0.709 Degree = 35.34°	Gradient = 0.770 Degree = 37.60°	Gradient = 0.832 Degree = 39.76°	Gradient = 0.896 Degree = 41.85°	Gradient = 0.960 Degree = 43.84°	Gradient = 1.027 Degree = 45.75°	Gradient = 1.094 Degree = 47.58°
300	Gradient = 0.566 Degree = 29.52°	Gradient = 0.628 Degree = 32.11°	Gradient = 0.691 Degree = 34.63°	Gradient = 0.755 Degree = 37.05°	Gradient = 0.821 Degree = 39.38°	Gradient = 0.888 Degree = 41.62°	Gradient = 0.958 Degree = 43.77°	Gradient = 1.029 Degree = 45.81°	Gradient = 1.102 Degree = 47.78°	Gradient = 1.177 Degree = 49.64°
400	Gradient = 0.597 Degree = 30.82°	Gradient = 0.662 Degree = 33.50°	Gradient = 0.729 Degree = 36.08°	Gradient = 0.797 Degree = 38.55°	Gradient = 0.867 Degree = 40.93°	Gradient = 0.939 Degree = 43.18°	Gradient = 1.012 Degree = 45.36°	Gradient = 1.088 Degree = 47.41°	Gradient = 1.165 Degree = 49.36°	Gradient = 1.245 Degree = 51.22°
500	Gradient = 0.642 Degree = 32.69°	Gradient = 0.714 Degree = 35.52°	Gradient = 0.788 Degree = 38.24°	Gradient = 0.865 Degree = 40.86°	Gradient = 0.945 Degree = 43.38°	Gradient = 1.028 Degree = 45.78°	Gradient = 1.114 Degree = 48.09°	Gradient = 1.203 Degree = 50.28°	Gradient = 1.298 Degree = 52.39°	Gradient = 1.395 Degree = 54.36°

Table 2: The reference table of reflection angle of rayfan.

100	Gradient = -0.242 Degree = 13.61°	Gradient = -0.263 Degree = 14.75°	Gradient = -0.28 Degree = 15.84°	Gradient = -0.303 Degree = 16.87°	Gradient = -0.322 Degree = 17.84°	Gradient = -0.339 Degree = 18.74°	Gradient = -0.356 Degree = 19.59°	Gradient = -0.371 Degree = 20.37°	Gradient = -0.386 Degree = 21.09°	Gradient = -0.399 Degree = 21.74°
200	Gradient = -0.249 Degree = 13.99°	Gradient = -0.271 Degree = 15.15°	Gradient = -0.291 Degree = 16.24°	Gradient = -0.311 Degree = 17.27°	Gradient = -0.329 Degree = 18.20°	Gradient = -0.347 Degree = 19.14°	Gradient = -0.363 Degree = 19.97°	Gradient = -0.379 Degree = 20.74°	Gradient = -0.393 Degree = 21.44°	Gradient = -0.405 Degree = 22.07°
300	Gradient = -0.257 Degree = 14.42°	Gradient = -0.279 Degree = 15.59°	Gradient = -0.300 Degree = 16.69°	Gradient = -0.320 Degree = 17.73°	Gradient = -0.34 Degree = 18.69°	Gradient = -0.356 Degree = 19.58°	Gradient = -0.372 Degree = 20.39°	Gradient = -0.387 Degree = 21.14°	Gradient = -0.400 Degree = 21.82°	Gradient = -0.41 Degree = 22.42°
400	Gradient = -0.268 Degree = 15.01°	Gradient = -0.291 Degree = 16.20°	Gradient = -0.312 Degree = 17.31°	Gradient = -0.331 Degree = 18.34°	Gradient = -0.35 Degree = 19.31°	Gradient = -0.368 Degree = 20.18°	Gradient = -0.383 Degree = 20.98°	Gradient = -0.398 Degree = 21.70°	Gradient = -0.411 Degree = 22.33°	Gradient = -0.422 Degree = 22.89°
500	Gradient = -0.281 Degree = 15.70°	Gradient = -0.304 Degree = 16.91°	Gradient = -0.326 Degree = 18.03°	Gradient = -0.346 Degree = 19.07°	Gradient = -0.364 Degree = 20.01°	Gradient = -0.381 Degree = 20.865°	Gradient = -0.396 Degree = 21.626°	Gradient = -0.410 Degree = 22.31°	Gradient = -0.422 Degree = 22.90°	Gradient = -0.433 Degree = 23.40°

Table 1 shows the reference table of the refraction angle of rayfan when the source is placed at the depth of 100 meters up to 500 meters.

4.2 Discussions

Based on Table 1, when the source is placed at the depth of 100 meters, the value of the refraction angle for $\square_1$ until $\square_{10}$ is increasing. From the table, the value of $\square_1$ is 26.677202106763° while the value of $\square_{10}$ is 45.609601707727°. The findings can be proven by Snell's law. This law establishes that

$$\text{refraction angle}, \sin i' = (\frac{d}{AB}) \tag{4.2}$$

Where d is the depth of the source, i' is the refraction angle and *AB* is the distance. Based on (4.2), as the value of d is increased, the value of i' will increase too. It proven the findings that as the ray parameters is increased, the refraction angle also will be increased.

When the source is placed at 500 meters, the value of $\square_1$ is 32.685015460608° and the value of $\square_{10}$ is 54.362451634773°. The values show that as the ray parameters is increasing, the value of the refraction angle also will be increased. This is proven by the formula (4.2) as the Snell's law mention that the refraction angle will increase when the ray parameters increased.

From Table 1, the value of $\square_1$ when the source is placed at 100 meters is 26.677202106763° while when the source is placed at 500 meters, the value of $\square_1$ is 32.685015460608°. From this findings, it can be said that the value of $\square_1$ will increase along with the value of the source. The value of $\square_1$ when the source is placed at 100 meter is smaller compared to $\square_1$ when the source is placed at 500 meter. This findings, follow the Snell's law in (4.2). As the depth of the source is small which is at 100 meters, the angle of the refraction will be small while when the depth of the source is big, which is at 500 meters, the angle of the refraction will be high.

Based on Table 1, the value of $\square_{10}$ when the source is placed at 100 meters is 45.609601707727° while when the source is at 500 meters, the value of $\square_{10}$ is

54.362451634773°. Based on the findings, it can be said that as the source is placed deeper, the refraction angle will be bigger.

This is proven by the Snell's law in (4.2) that says when the depth of the source is small which is placed at 100 meters, the refraction angle will be small while when the depth of the source is placed deeper which is at 500 meters, the refraction angle will be big.

Table 2 shows the reference table of reflection angle of rayfan when the source is placed at the depth of 100 meters up to 500 meters.

Based on Table 2, when the source is placed at the depth of 100 meters, it shows that the value of the reflection angle of each rayfan is increasing. From the findings, the value of $\square_{1}$ is 13.60950532258° while the value of $\square_{10}$ is 21.73678964194°. The findings can be proven by Snell's law which is

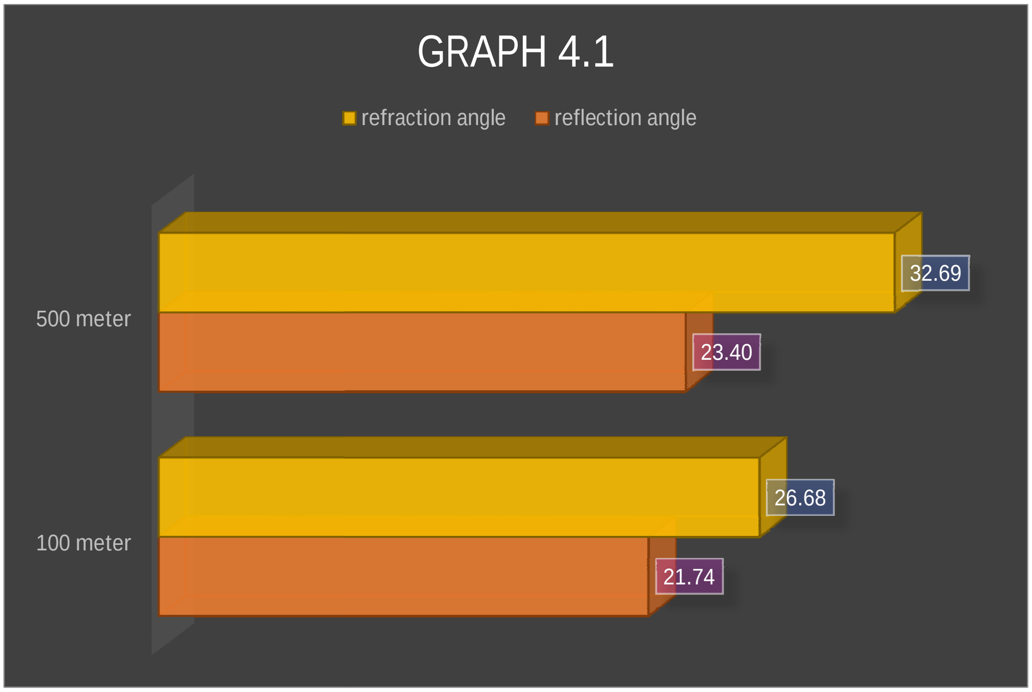
$$\text{reflection angle}, \sin i = \left(\frac{d}{AB}\right) \tag{4.3}$$

Where i is the reflection angle, d is the depth of the source and AB is the distance. Based on (4.3), the reflection angle is proportional to the depth of the source. This proves the findings that as the depth of the source is increased, the reflection angle also will be increased.

When the depth of the source is at 500 meters, based on the Table 2, the value of $\square_{1}$ is 15.69798951905° and the value of $\square_{10}$ is 23.39776391593°. The findings show that the value of the reflection angle is increased when the depth of the source is increased. This is proven by the Snell's law in (4.3) as the value of the reflection angle will increase when the depth is increased.

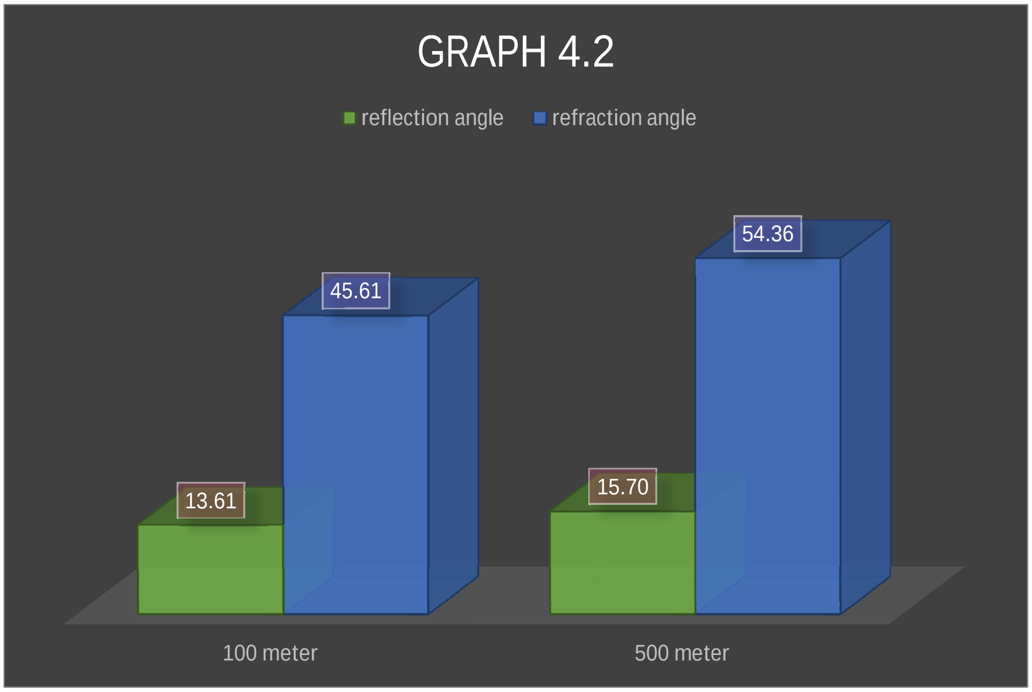
Based on Table 2, the value of reflection angle, $\square_{1}$ for the depth of source of 100 meters is 13.60950532258° while for the depth of 500 meters, the value of $\square_{1}$ is 15.69798951905°. Based on this findings, the value of $\square_{1}$ is increasing as the source is placed deeper. The reflection angle, $\square_{1}$ when the source is placed at 100 meter is smaller compared to $\square_{1}$ when the source is placed at 500 meter. This findings, follow the Snell's law in (4.3). As the value of d is small, which is when the source is placed at 100 meters, the angle of the reflection will be small while when the value of d is big, which is when the source is placed at 500 meters, the angle of the reflection will be big.

Based on Table 2, the value of $\square_{10}$ when the source is placed at 100 meters is 21.73678964194° while when the source is at 500 meters, the value of $\square_{10}$ is 23.39776391593°. Based on the findings, it can be said that as the source is placed deeper, the reflection angle will be bigger. This is proven by the Snell's law in (4.3) that says when the value of *d* is small, the reflection angle will be small while when the value of *d* is big, the reflection angle will be big.



Graph 4.1: Graph of reflection angle $\square_{10}$ and refraction angle $\square_{1}$ for 100 meters and 500 meters.

Graph 4.1 shows the reflection angle, $\square_{10}$ and refraction angle, $\square_{1}$ for the depth of the source at 100 meters and 500 meters. Based on this graph, for the depth of 100 meters, the value of the reflection angle, $\square_{10}$ is 21.73678964194° and refraction angle, $\square_{1}$ 26.677202106763° while for the depth of 500 meters, the value of the reflection angle, $\square_{10}$ is 23.39776391593° and refraction angle, $\square_{1}$ 32.685015460608°. From the value obtained, it can be said that the value of both $\square_{1}$ and $\square_{10}$ for the reflection angle and refraction angle are not the same for the depth of 100 meters and 500 meters. The percentage of the difference for $\square_{1}$ for both depth is 81.62% while the percentage of the difference for $\square_{10}$ for both depth is 92.9%.



Graph 4.2: Graph of reflection angle $\square_{1}$ and refraction angle $\square_{10}$ for 100 meters and 500 meters.

Graph 4.2 shows the reflection angle, $\square_{1}$ and refraction angle, $\square_{10}$ for the depth of the source at 100 meters and 500 meters. Based on this graph, for the depth of 100 meters, the value of the reflection angle, $\square_{1}$ is 13.60950532258° and refraction angle, $\square_{10}$ 45.609601707727° while for the depth of 500 meters, the value of the reflection angle, $\square_{1}$ is 15.69798951905° and refraction angle, $\square_{10}$ 54.362451634773° . From the value obtained, it can be said that the value of both $\square_{1}$ and $\square_{10}$ for the reflection angle and refraction angle are not the same for the depth of 100 meters and 500 meters.

Based on these finding, it can be said that these findings obey the Huygens' Principle. The principle is best in describing reflection and refraction. Huygens' Principle applied to the front straight wave from one medium to another, where its speed is less than traveling. The ray is bent towards the vertical, since the wavelet in the second medium has a lower speed.

CHAPTER 5

CONCLUSION

5.0 Conclusion

The fact that the synthetic database for s-wave diffraction angle with source at different depth have not exist has been repealed. Based on the findings obtained through the research, it can be concluded that the objective of this project which is to develop a synthetic velocity model for generating synthetic reflection and refraction data in standard Society of Exploration Geoscientist-Y (SEG-Y) format database is achieved. Nine layers velocity model have been built by using finite difference method thus producing seismic wave propagation together with seismic ray tracing. Seismic ray tracing created numerous number of rayfans. The reflected and refracted angle have been calculated and tabulated.

5.1 Recommendation

Upon achieving the main objective of this research with aforementioned details, one of the recommendation and suggestion for the future work is to improve the synthetic model. The limitation of the depth can be increased from 1000 meters to 5000 meters. The number of the shot can be increased to 2 to 5 slots so that the variations of the reflection and refraction angles can be seen.

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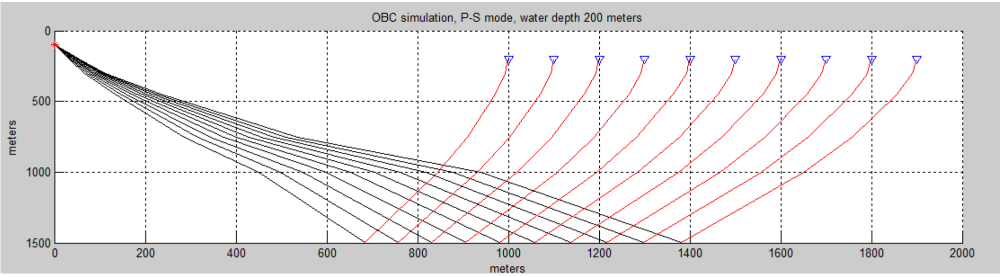
APPENDIX A

For depth 100 meters from sea level, the coding below is used.

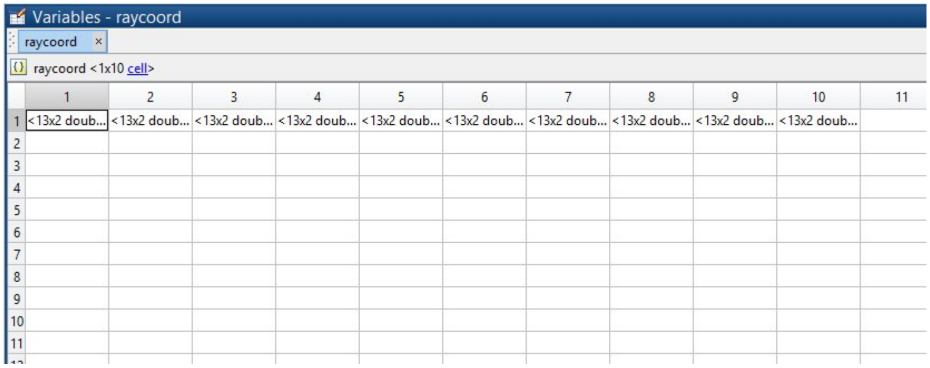
```
case {1}

%
% OBC primaries
```

```
%

%OBC recording
%make pwave model
zp=[0 200 300 450 750 1000 1700 2500 3500 4000];
vp=[1500 1600 2000 2250 2700 2100 3200 3750 4000 4200];
%make s-wave model
zs=zp;
vs=[0 300 700 1000 1250 1500 1800 1500 2100 1900];
zsrc=100;  `set the depth of the source.'
zrec=200;
zd=1500;
xoff=1000:100:1900;

%simulate source in the middle of the water layer and receiver on the
bottom
figure;subplot(2,1,1);flipy
%Trace P-P rays and plot in upper subplot
[t,p,L,raycoord]=traceray_pp(vp,zp,zsrc,zrec,zd,xoff,100,-1,10,1,1,2);
%put source and receiver markers
line(xoff,zrec*ones(size(xoff)),'color','b','linestyle','none','marker','v'
)
line(0,zsrc,'color','r','linestyle','none','marker','*')
%annotate plot
title('OBC simulation, P-P mode, water depth 200 meters')
xlabel('meters');ylabel('meters');grid
%plot traveltime versus offset in lower subplot
subplot(2,1,2);flipy;
plot(xoff,t);grid;xlabel('meters');ylabel('seconds')
xlim([0 max(xoff)])

figure;subplot(2,1,1);flipy
%Trace P-S rays and plot in upper subplot
[t,p,L,raycoord]=traceray_ps(vp,zp,vs,zs,zsrc,zrec,zd,xoff,10,-1,10,1,1,2);
%put source and receive markers
line(xoff,zrec*ones(size(xoff)),'color','b','linestyle','none','marker','v'
)
line(0,zsrc,'color','r','linestyle','none','marker','*')
%annotate plot
title('OBC simulation, P-S mode, water depth 200 meters')
grid;xlabel('meters');ylabel('meters');
subplot(2,1,2);flipy;
plot(xoff,t);grid;xlabel('meters');ylabel('seconds');
xlim([0 max(xoff)])

pos=get(gcf,'position');
set(gcf,'position',[1.1*pos(1) .9*pos(2) pos(3:4)])
```

After simulated the coding above, a set of rayfan are produced.



The data will be stored in raycoord as shown below.



Each column indicates the value of each rayfan.

Depth of 100 meters, the first ray fan,

Variables - raycoord{1, 1}

raycoord | raycoord{1, 1}

raycoord{1, 1} <13x2 double>

	1	2	3
1	0	100.0000	
2	31.9138	200	
3	66.1965	300	
4	132.7130	450	
5	286.4438	750	
6	449.9078	1000	
7	685.0977	1.5000e+03	
8	685.0977	1.5000e+03	
9	844.6666	1000	
10	910.1426	750	
11	972.2376	450	
12	993.7372	300	
13	999.8291	200.0000	
14			

Refraction 1, $\square_1$,

$$gradient = \frac{685.0977 - 31.9138}{1500 - 200}$$

$$gradient = 0.50244915384615$$

Convert 0.50244915384615 in degree and the value will be 26.677202106763°

Reflection 1, $\square_1$,

$$gradient = \frac{685.0977 - 999.8291}{1500 - 200}$$

$$gradient = -0.24210107692308$$

Convert -0.24210107692308 in degree, then 360 minus the value and the final value will be 13.60950532258°.

Depth of 100 meters, the second rayfan

Variables - raycoord{1, 2}

raycoord × raycoord{1, 2} ×

raycoord{1, 2} <13x2 double>

	1	2
1	0	100.0000
2	34.7895	200
3	72.2116	300
4	145.3164	450
5	315.2504	750
6	498.6207	1000
7	757.6604	1.5000e+03
8	757.6604	1.5000e+03
9	931.6077	1000
10	1.0028e+03	750
11	1.0701e+03	450
12	1.0934e+03	300
13	1.1000e+03	200.0000

Refraction 2, ⍰$_2$,

$$gradient = \frac{757.6604 - 34.7895}{1500 - 200}$$

$$gradient = 0.55605453846154$$

Convert 0.55605453846154 into degree and the value will be 29.076446252083°

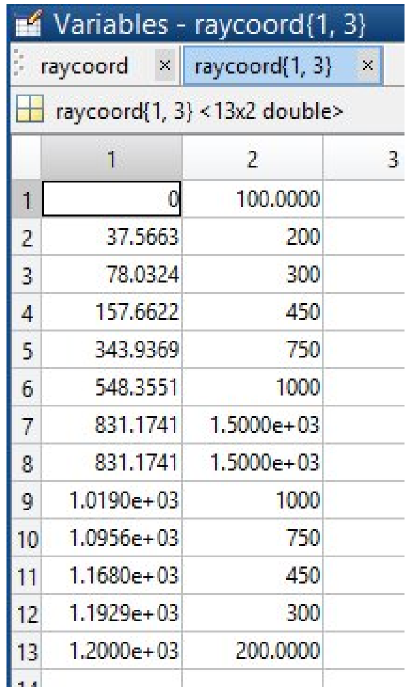
Reflection 2, ⍰$_2$,

$$gradient = \frac{757.6604 - 1100}{1500 - 200}$$

$$gradient = -0.26333815384615$$

Convert -0.26333815384615 in degree, then 360 minus the value and the final value will be 14.75322157656°.

Depth of 100 meters, the third rayfan



Variables - raycoord{1, 3}

raycoord | raycoord{1, 3}

raycoord{1, 3} <13x2 double>

	1	2	3
1	0	100.0000	
2	37.5663	200	
3	78.0324	300	
4	157.6622	450	
5	343.9369	750	
6	548.3551	1000	
7	831.1741	1.5000e+03	
8	831.1741	1.5000e+03	
9	1.0190e+03	1000	
10	1.0956e+03	750	
11	1.1680e+03	450	
12	1.1929e+03	300	
13	1.2000e+03	200.0000	

Refraction 3, ⍰$_3$,

$$gradient = \frac{831.1741 - 37.5663}{1500 - 200}$$

$$gradient = 0.61046753846154$$

Convert 0.61046753846154 into degree and the value will be 31.402710342635°

Reflection 3, ⍰$_3$,

$$gradient = \frac{831.1741 - 1200}{1500 - 200}$$

$$gradient = -0.28371223076923$$

Convert -0.28371223076923 in degree, then 360 minus the value and the final value will be 15.83928791437°.

Depth of 100 meters, the fourth rayfan

Variables - raycoord{1, 4}

raycoord × | raycoord{1, 4} ×

raycoord{1, 4} <13x2 double>

	1	2	3
1	0	100.0000	
2	40.2403	200	
3	83.6502	300	
4	169.7340	450	
5	372.4946	750	
6	599.3285	1000	
7	905.8399	1.5000e+03	
8	905.8399	1.5000e+03	
9	1.1070e+03	1000	
10	1.1889e+03	750	
11	1.2660e+03	450	
12	1.2925e+03	300	
13	1.3000e+03	200.0000	
14			

Refraction 4, ⍰$_4$,

$$gradient = \frac{905.8399 - 40.2403}{1500 - 200}$$

$$gradient = 0.66584584615385$$

Convert 0.66584584615385 into degree and the value will be 33.657496270177°

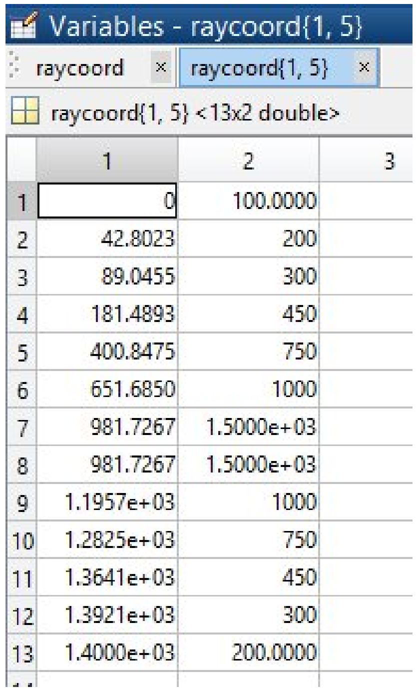
Reflection 4, ⍰$_4$,

$$gradient = \frac{905.8399 - 1300}{1500 - 200}$$

$$gradient = -0.30320007692308$$

Convert -0.30320007692308 in degree, then 360 minus the value and the final value will be 16.86730756544°

Depth of 100 meters, the fifth rayfan



Variables - raycoord{1, 5}

raycoord | raycoord{1, 5}

raycoord{1, 5} <13x2 double>

	1	2	3
1	0	100.0000	
2	42.8023	200	
3	89.0455	300	
4	181.4893	450	
5	400.8475	750	
6	651.6850	1000	
7	981.7267	1.5000e+03	
8	981.7267	1.5000e+03	
9	1.1957e+03	1000	
10	1.2825e+03	750	
11	1.3641e+03	450	
12	1.3921e+03	300	
13	1.4000e+03	200.0000	

Refraction 5, ▯$_5$,

$$gradient = \frac{981.7267 - 42.8023}{1500 - 200}$$

$$gradient = 0.72224953846154$$

Convert 0.72224953846154 into degree and the value will be 35.838681529366°

Reflection 5, ▯$_5$,

$$gradient = \frac{981.7267 - 1400}{1500 - 200}$$

$$gradient = -0.32174869230769$$

Convert -0.32174869230769 in degree, then 360 minus the value and the final value will be 17.83551140434°

Depth of 100 meters, the sixth rayfan

Variables - raycoord{1, 6}

raycoord × | raycoord{1, 6} ×

raycoord{1, 6} <13x2 double>

	1	2	3
1	0	100.0000	
2	45.2435	200	
3	94.1986	300	
4	192.8817	450	
5	428.8998	750	
6	705.5808	1000	
7	1.0589e+03	1.5000e+03	
8	1.0589e+03	1.5000e+03	
9	1.2851e+03	1000	
10	1.3766e+03	750	
11	1.4623e+03	450	
12	1.4917e+03	300	
13	1.5000e+03	200.0000	

Refraction 6, $\square_6$,

$$gradient = \frac{1058.9 - 45.2435}{1500 - 200}$$

$$gradient = 0.77973576923077$$

Convert 0.77973576923077 into degree and the value will be 37.944817017912°

Reflection 6, $\square_6$,

$$gradient = \frac{1058.9 - 1500}{1500 - 200}$$

$$gradient = -0.33930769230769$$

Convert -0.33930769230769 in degree, then 360 minus the value and the final value will be 18.74246969169°

Depth of 100 meters, the seventh rayfan

Variables - raycoord{1, 7}

raycoord | raycoord{1, 7}

raycoord{1, 7} <13x2 double>

	1	2	3
1	0	100.0000	
2	47.5554	200	
3	99.0904	300	
4	203.8608	450	
5	456.5337	750	
6	761.1854	1000	
7	1.1374e+03	1.5000e+03	
8	1.1374e+03	1.5000e+03	
9	1.3752e+03	1000	
10	1.4710e+03	750	
11	1.5607e+03	450	
12	1.5913e+03	300	
13	1.6000e+03	200.0000	

Refraction 7, $\square_7$,

$$gradient = \frac{1137.4 - 47.5554}{1500 - 200}$$

$$gradient = 0.838342$$

Convert 0.838342 into degree and the value will be 39.974517007685°

Reflection 7, $\square_7$,

$$gradient = \frac{1137.4 - 1600}{1500 - 200}$$

$$gradient = -0.35584615384615$$

Convert -0.35584615384615 in degree, then 360 minus the value and the final value will be 19.58790586959°

Depth of 100 meters, the eight rayfan

Variables - raycoord{1, 8}

raycoord × | raycoord{1, 8} ×

raycoord{1, 8} <13x2 double>

	1	2	3
1	0	100.0000	
2	49.7299	200	
3	103.7026	300	
4	214.3741	450	
5	483.6099	750	
6	818.6882	1000	
7	1.2173e+03	1.5000e+03	
8	1.2173e+03	1.5000e+03	
9	1.4660e+03	1000	
10	1.5659e+03	750	
11	1.6591e+03	450	
12	1.6910e+03	300	
13	1.6999e+03	200.0000	

Refraction 8, ⍰$_8$,

$$gradient = \frac{1217.3 - 49.7299}{1500 - 200}$$

$$gradient = 0.89813084615385$$

Convert 0.89813084615385 into degree and the value will be 41.927989171917°

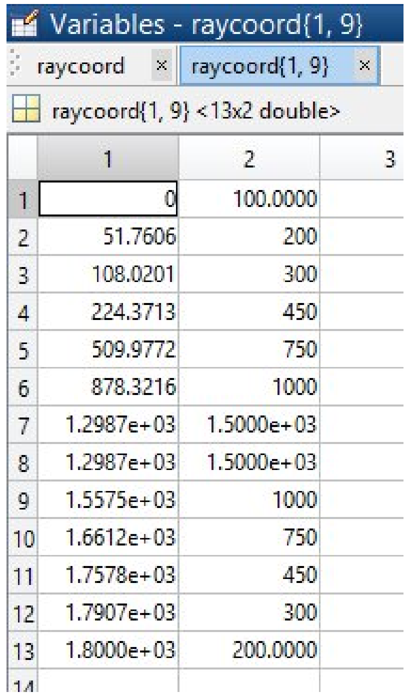
Reflection 8, ⍰$_8$,

$$gradient = \frac{1217.3 - 1699.9}{1500 - 200}$$

$$gradient = -0.37123076923077$$

Convert -0.37123076923077 in degree, then 360 minus the value and the final value will be 20.36647531114°

Depth of 100 meters, the ninth rayfan



Variables - raycoord{1, 9}

raycoord × | raycoord{1, 9} ×

raycoord{1, 9} <13x2 double>

	1	2	3
1	0	100.0000	
2	51.7606	200	
3	108.0201	300	
4	224.3713	450	
5	509.9772	750	
6	878.3216	1000	
7	1.2987e+03	1.5000e+03	
8	1.2987e+03	1.5000e+03	
9	1.5575e+03	1000	
10	1.6612e+03	750	
11	1.7578e+03	450	
12	1.7907e+03	300	
13	1.8000e+03	200.0000	
14			

Refraction 9, ⍰$_9$,

$$gradient = \frac{1298.7 - 51.7606}{1500 - 200}$$

$$gradient = 0.95918415384615$$

Convert 0.95918415384615 into degree and the value will be 43.806524910954°

Reflection 9, ⍰$_9$,

$$gradient = \frac{1298.7 - 1800}{1500 - 200}$$

$$gradient = -0.38561538461538$$

Convert -0.38561538461538 in degree, then 360 minus the value and the final value will be 21.08740659799°

Depth of 100 meters, the tenth rayfan

Variables - raycoord{1, 10}

raycoord | raycoord{1, 10}

raycoord{1, 10} <13x2 double>

	1	2	3
1	0	100.0000	
2	53.6386	200	
3	112.0219	300	
4	233.7843	450	
5	535.4156	750	
6	940.2256	1000	
7	1.3816e+03	1.5000e+03	
8	1.3816e+03	1.5000e+03	
9	1.6498e+03	1000	
10	1.7569e+03	750	
11	1.8565e+03	450	
12	1.8904e+03	300	
13	1.8999e+03	200.0000	
14			

Refraction 10, ⍰$_{10}$,

$$gradient = \frac{1381.6 - 53.6386}{1500 - 200}$$

$$gradient = 1.0215087692308$$

Convert 1.0215087692308 into degree and the value will be 45.609601707727°

Reflection 10, ⍰$_{10}$,

$$gradient = \frac{1381.6 - 1899.9}{1500 - 200}$$

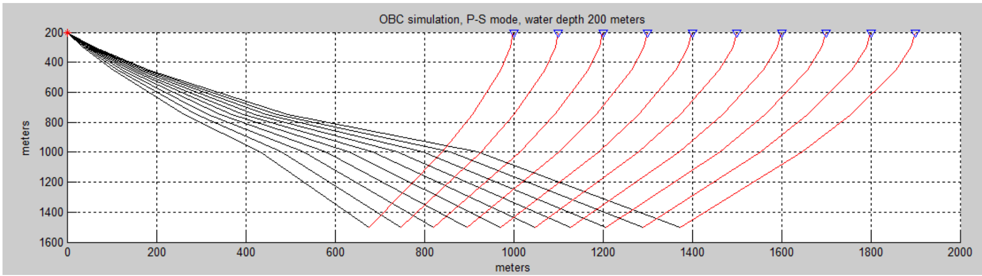
$$gradient = -0.39869230769231$$

Convert -0.39869230769231 in degree, then 360 minus the value and the final value will be 21.73678964194◦

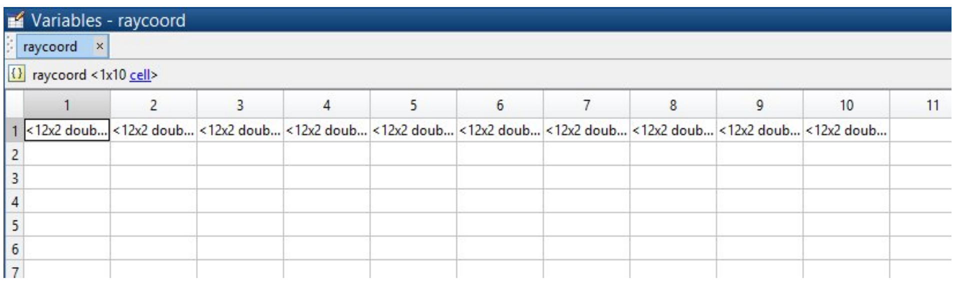
For depth of 200 meters from sea level, the coding below is used.

```
case {1}

%
% OBC primaries
%

%OBC recording
%make pwave model
zp=[0 200 300 450 750 1000 1700 2500 3500 4000];
vp=[1500 1600 2000 2250 2700 2100 3200 3750 4000 4200];
%make s-wave model
zs=zp;
vs=[0 300 700 1000 1250 1500 1800 1500 2100 1900];
zsrc=200;
zrec=200;
zd=1500;
xoff=1000:100:1900;

%simulate source in the middle of the water layer and receiver on the
bottom
figure;subplot(2,1,1);flipy
%Trace P-P rays and plot in upper subplot
[t,p,L,raycoord]=traceray_pp(vp,zp,zsrc,zrec,zd,xoff,100,-1,10,1,1,2);
%put source and receiver markers
line(xoff,zrec*ones(size(xoff)),'color','b','linestyle','none','marker','v'
)
line(0,zsrc,'color','r','linestyle','none','marker','*')
%annotate plot
title('OBC simulation, P-P mode, water depth 200 meters')
xlabel('meters');ylabel('meters');grid
%plot traveltime versus offset in lower subplot
subplot(2,1,2);flipy;
plot(xoff,t);grid;xlabel('meters');ylabel('seconds')
xlim([0 max(xoff)])

figure;subplot(2,1,1);flipy
%Trace P-S rays and plot in upper subplot
[t,p,L,raycoord]=traceray_ps(vp,zp,vs,zs,zsrc,zrec,zd,xoff,10,-1,10,1,1,2);
%put source and receive markers
line(xoff,zrec*ones(size(xoff)),'color','b','linestyle','none','marker','v'
)
line(0,zsrc,'color','r','linestyle','none','marker','*')
%annotate plot
title('OBC simulation, P-S mode, water depth 200 meters')
grid;xlabel('meters');ylabel('meters');
subplot(2,1,2);flipy;
plot(xoff,t);grid;xlabel('meters');ylabel('seconds');
xlim([0 max(xoff)])

pos=get(gcf,'position');
set(gcf,'position',[1.1*pos(1) .9*pos(2) pos(3:4)])
```

After simulated the coding above, a set of rayfan are produced.



The data will be stored in raycoord as shown below.



Each column indicates the value of each rayfan.

Depth of 200 meters, the first rayfan

Variables - raycoord{1, 1}

raycoord × | raycoord{1, 1} ×

raycoord{1, 1} <12x2 double>

	1	2	3
1	0	200.0000	
2	35.3275	300	
3	104.0218	450	
4	263.0778	750	
5	432.9881	1000	
6	676.0447	1.5000e+03	
7	676.0447	1.5000e+03	
8	840.4053	1000	
9	907.7855	750	
10	971.6410	450	
11	993.7366	300	
12	999.9944	200.0000	
13			

To calculate the angle of refrection 1, $\square_1$,

$$gradient = \frac{676.0447 - 35.3275}{1500 - 300}$$

$$gradient = 0.533931$$

Convert 0.533931 into degree and the value will be 28.099140699819°

Reflection 1, $\square_1$,

$$gradient = \frac{676.0447 - 999.9944}{1500 - 200}$$

$$gradient = -0.24919207692308$$

Convert -0.24919207692308 in degree, then 360 minus the value and the final value will be 13.99266758553°

<u>Depth of 200 meters, the second rayfan</u>

Variables - raycoord{1, 2}

raycoord | raycoord{1, 2}

raycoord{1, 2} <12x2 double>

	1	2	3
1	0	200.0000	
2	38.5226	300	
3	113.9700	450	
4	289.7353	750	
5	480.4956	1000	
6	748.0535	1.5000e+03	
7	748.0535	1.5000e+03	
8	927.0273	1000	
9	1.0002e+03	750	
10	1.0694e+03	450	
11	1.0932e+03	300	
12	1.1000e+03	200.0000	
13			

To calculate the angle of refraction 1, ▯$_2$,

$$gradient = \frac{y2 - y1}{x2 - x1}$$

$$gradient = \frac{748.0535 - 38.5226}{1500 - 300}$$

$$gradient = 0.59127575$$

Convert 0.59127575 into degree and the value will be 30.594795399417°

4.2.3 The third rayfan

Variables - raycoord{1, 3}

raycoord | raycoord{1, 3}

raycoord{1, 3} <12x2 double>

	1	2	3
1	0	200.0000	
2	41.6154	300	
3	123.7474	450	
4	316.3733	750	
5	529.2750	1000	
6	821.2581	1.5000e+03	
7	821.2581	1.5000e+03	
8	1.0143e+03	1000	
9	1.0930e+03	750	
10	1.1672e+03	450	
11	1.1928e+03	300	
12	1.2000e+03	200.0000	
13			

To calculate the angle of refraction 2, $\square_3$,

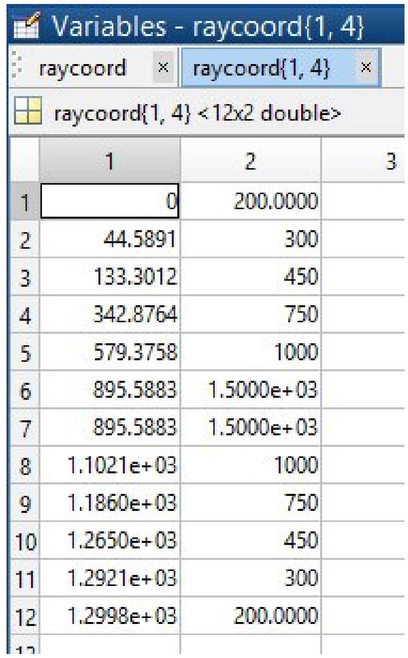
$$gradient = \frac{y2 - y1}{x2 - x1}$$

$$gradient = \frac{821.2581 - 41.6154}{1500 - 300}$$

$$gradient = 0.64970225$$

Convert 0.64970225 into degree and the value will be 33.011873081645°

4.2.4 The fourth rayfan



Variables - raycoord{1, 4}

raycoord | raycoord{1, 4}

raycoord{1, 4} <12x2 double>

	1	2	3
1	0	200.0000	
2	44.5891	300	
3	133.3012	450	
4	342.8764	750	
5	579.3758	1000	
6	895.5883	1.5000e+03	
7	895.5883	1.5000e+03	
8	1.1021e+03	1000	
9	1.1860e+03	750	
10	1.2650e+03	450	
11	1.2921e+03	300	
12	1.2998e+03	200.0000	

To calculate the angle of refraction 3, $\square_4$,

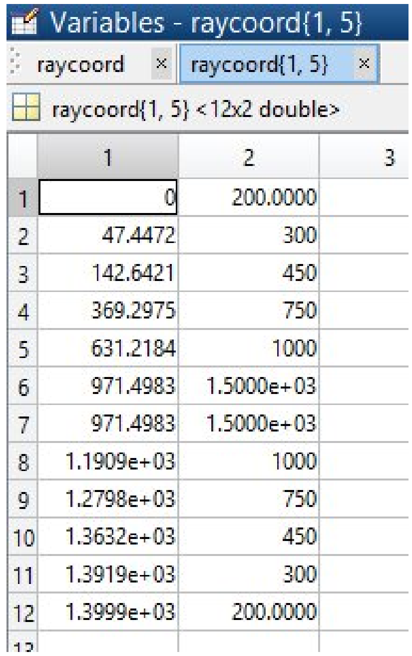
$$gradient = \frac{y2 - y1}{x2 - x1}$$

$$gradient = \frac{895.5883 - 44.5891}{1500 - 300}$$

$$gradient = 0.709166$$

Convert 0.709166 into degree and the value will be 35.342969718565°

4.2.5 The fifth rayfan



	1	2	3
1	0	200.0000	
2	47.4472	300	
3	142.6421	450	
4	369.2975	750	
5	631.2184	1000	
6	971.4983	1.5000e+03	
7	971.4983	1.5000e+03	
8	1.1909e+03	1000	
9	1.2798e+03	750	
10	1.3632e+03	450	
11	1.3919e+03	300	
12	1.3999e+03	200.0000	

To calculate the angle of refraction 4, ▯$_5$,

$$gradient = \frac{y2 - y1}{x2 - x1}$$

$$gradient = \frac{971.4983 - 47.4472}{1500 - 300}$$

$$gradient = 0.77004258333333$$

Convert 0.77004258333333 into degree and the value will be 37.597802815901°

4.2.6 The sixth rayfan

Variables - raycoord{1, 6}

raycoord × | raycoord{1, 6} ×

raycoord{1, 6} <12x2 double>

	1	2	3
1	0	200.0000	
2	50.1614	300	
3	151.6716	450	
4	395.3729	750	
5	684.6649	1000	
6	1.0486e+03	1.5000e+03	
7	1.0486e+03	1.5000e+03	
8	1.2802e+03	1000	
9	1.3737e+03	750	
10	1.4613e+03	450	
11	1.4913e+03	300	
12	1.4997e+03	200.0000	
13			

To calculate the angle of refraction 5, $\square_{6}$,

$$gradient = \frac{y2 - y1}{x2 - x1}$$

$$gradient = \frac{1048.6 - 50.1614}{1500 - 300}$$

$$gradient = 0.83203216666667$$

Convert 0.83203216666667 into degree and the value will be 39.761545405078°

4.2.7 The seventh rayfan

Variables - raycoord{1, 7}

raycoord × | raycoord{1, 7} ×

raycoord{1, 7} <12x2 double>

	1	2	3
1	0	200.0000	
2	52.7400	300	
3	160.4085	450	
4	421.1615	750	
5	740.2955	1000	
6	1.1275e+03	1.5000e+03	
7	1.1275e+03	1.5000e+03	
8	1.3707e+03	1000	
9	1.4685e+03	750	
10	1.5599e+03	450	
11	1.5912e+03	300	
12	1.6000e+03	200.0000	
13			

To calculate the angle of refraction 6, ▯$_7$,

$$gradient = \frac{y2 - y1}{x2 - x1}$$

$$gradient = \frac{1127.5 - 52.7400}{1500 - 300}$$

$$gradient = 0.89563333333333$$

Convert 0.89563333333333 into degree and the value will be 41.848684596547°

4.2.8 The eighth rayfan

Variables - raycoord{1, 8}

raycoord | raycoord{1, 8}

raycoord{1, 8} <12x2 double>

	1	2	3
1	0	200.0000	
2	55.1537	300	
3	168.7402	450	
4	446.3227	750	
5	797.9179	1000	
6	1.2077e+03	1.5000e+03	
7	1.2077e+03	1.5000e+03	
8	1.4616e+03	1000	
9	1.5634e+03	750	
10	1.6584e+03	450	
11	1.6909e+03	300	
12	1.7000e+03	200.0000	
13			

To calculate the angle of refraction 7, ⍰$_8$,

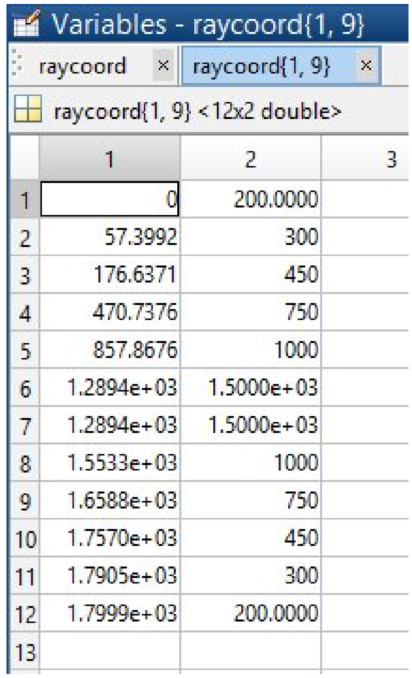
$$gradient = \frac{y2 - y1}{x2 - x1}$$

$$gradient = \frac{1207.7 - 55.1537}{1500 - 300}$$

$$gradient = 0.96045525$$

Convert 0.96045525 into degree and the value will be 43.844431640077°

4.2.9 The ninth rayfan


Variables - raycoord{1, 9}

raycoord{1, 9} <12x2 double>

	1	2	3
1	0	200.0000	
2	57.3992	300	
3	176.6371	450	
4	470.7376	750	
5	857.8676	1000	
6	1.2894e+03	1.5000e+03	
7	1.2894e+03	1.5000e+03	
8	1.5533e+03	1000	
9	1.6588e+03	750	
10	1.7570e+03	450	
11	1.7905e+03	300	
12	1.7999e+03	200.0000	
13			

To calculate the angle of refraction 8, ⍰$_{9}$,

$$gradient = \frac{y2 - y1}{x2 - x1}$$

$$gradient = \frac{1289.4 - 57.3992}{1500 - 300}$$

$$gradient = 1.0266673333333$$

Convert 1.0266673333333 into degree and the value will be 45.75386692915°

4.2.10 The tenth rayfan

Variables - raycoord{1, 10}

raycoord{1, 10} <12x2 double>

	1	2	3
1	0	200.0000	
2	59.4688	300	
3	184.0509	450	
4	494.2126	750	
5	920.3782	1000	
6	1.3727e+03	1.5000e+03	
7	1.3727e+03	1.5000e+03	
8	1.6457e+03	1000	
9	1.7546e+03	750	
10	1.8557e+03	450	
11	1.8901e+03	300	
12	1.8997e+03	200.0000	
13			

To calculate the angle of refraction 9, $\square_{10}$,

$$gradient = \frac{y2 - y1}{x2 - x1}$$

$$gradient = \frac{1372.7 - 59.4688}{1500 - 300}$$

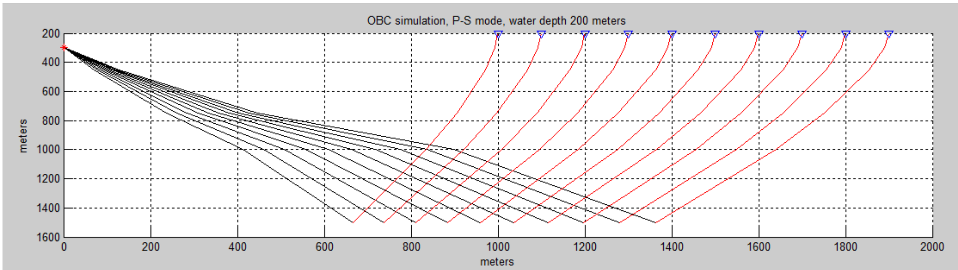
$$gradient = 1.0943593333333$$

Convert 1.0943593333333 into degree and the value will be 47.579661396831°

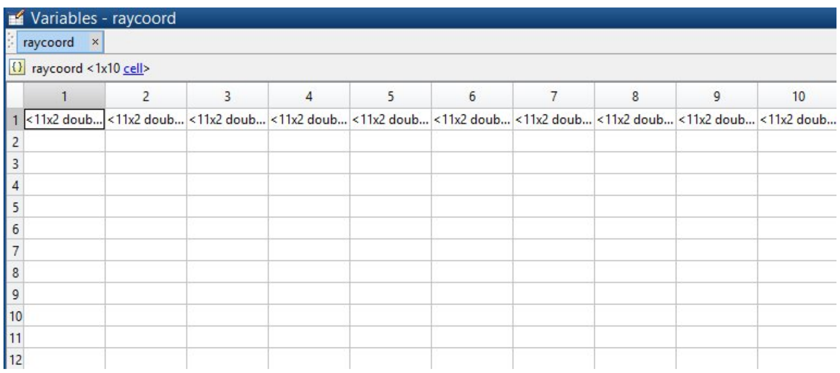
a. For depth of 300 meters from sea level, the coding below is used.

```
case {1}

%
% OBC primaries
%

%OBC recording
%make pwave model
zp=[0 200 300 450 750 1000 1700 2500 3500 4000];
vp=[1500 1600 2000 2250 2700 2100 3200 3750 4000 4200];
%make s-wave model
zs=zp;
vs=[0 300 700 1000 1250 1500 1800 1500 2100 1900];
zsrc=300;
zrec=200;
zd=1500;
xoff=1000:100:1900;

%simulate source in the middle of the water layer and receiver on the
bottom
figure;subplot(2,1,1);flipy
%Trace P-P rays and plot in upper subplot
[t,p,L,raycoord]=traceray_pp(vp,zp,zsrc,zrec,zd,xoff,100,-1,10,1,1,2);
%put source and receiver markers
line(xoff,zrec*ones(size(xoff)),'color','b','linestyle','none','marker','v'
)
line(0,zsrc,'color','r','linestyle','none','marker','*')
%annotate plot
title('OBC simulation, P-P mode, water depth 200 meters')
xlabel('meters');ylabel('meters');grid
%plot traveltime versus offset in lower subplot
subplot(2,1,2);flipy;
plot(xoff,t);grid;xlabel('meters');ylabel('seconds')
xlim([0 max(xoff)])

figure;subplot(2,1,1);flipy
%Trace P-S rays and plot in upper subplot
[t,p,L,raycoord]=traceray_ps(vp,zp,vs,zs,zsrc,zrec,zd,xoff,10,-1,10,1,1,2);
%put source and receive markers
line(xoff,zrec*ones(size(xoff)),'color','b','linestyle','none','marker','v'
)
line(0,zsrc,'color','r','linestyle','none','marker','*')
%annotate plot
title('OBC simulation, P-S mode, water depth 200 meters')
grid;xlabel('meters');ylabel('meters');
subplot(2,1,2);flipy;
plot(xoff,t);grid;xlabel('meters');ylabel('seconds');
xlim([0 max(xoff)])

pos=get(gcf,'position');
set(gcf,'position',[1.1*pos(1) .9*pos(2) pos(3:4)])
```

After simulated the coding above, a set of rayfan are produced.



The data will be stored in raycoord as shown below.



Each column indicates the value of each rayfan.

The first rayfan

Variables - raycoord{1, 1}

raycoord × raycoord{1, 1} ×

raycoord{1, 1} <11x2 double>

	1	2	3
1	0	300.0000	
2	71.1665	450	
3	236.3041	750	
4	413.6893	1000	
5	665.6962	1.5000e+03	
6	665.6962	1.5000e+03	
7	835.4501	1000	
8	904.9673	750	
9	970.7939	450	
10	993.5554	300	
11	999.9985	200.0000	
12			

To calculate the angle of reflection 1, ⍰$_1$,

$$gradient = \frac{y2 - y1}{x2 - x1}$$

$$gradient = \frac{665.6962 - 71.1665}{1500 - 450}$$

$$gradient = 0.56621876190476$$

Convert 0.56621876190476 into degree and the value will be 29.519353172458°

The second rayfan

Variables - raycoord{1, 2}

raycoord | raycoord{1, 2}

raycoord{1, 2} <11x2 double>

	1	2	3
1	0	300.0000	
2	78.1034	450	
3	260.5274	750	
4	459.8883	1000	
5	737.1294	1.5000e+03	
6	737.1294	1.5000e+03	
7	921.7456	1000	
8	997.1152	750	
9	1.0683e+03	450	
10	1.0929e+03	300	
11	1.0998e+03	200.0000	
12			

To calculate the angle of refraction 1, ⍰$_2$,

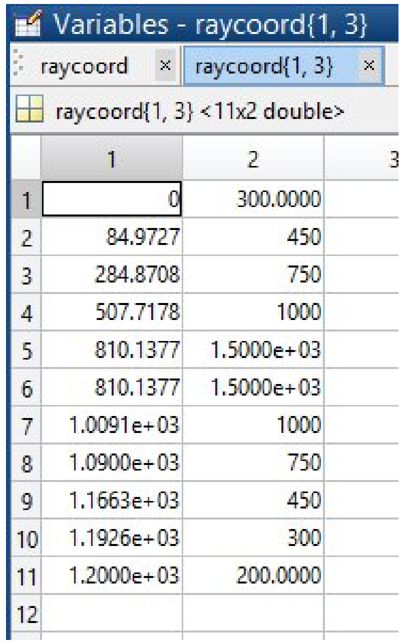
$$gradient = \frac{y2 - y1}{x2 - x1}$$

$$gradient = \frac{737.1294 - 78.1034}{1500 - 450}$$

$$gradient = 0.62764380952381$$

Convert 0.62764380952381 into degree and the value will be 32.114182609376°

<u>The third rayfan</u>



Variables - raycoord{1, 3}

raycoord × | raycoord{1, 3} ×

raycoord{1, 3} <11x2 double>

	1	2	3
1	0	300.0000	
2	84.9727	450	
3	284.8708	750	
4	507.7178	1000	
5	810.1377	1.5000e+03	
6	810.1377	1.5000e+03	
7	1.0091e+03	1000	
8	1.0900e+03	750	
9	1.1663e+03	450	
10	1.1926e+03	300	
11	1.2000e+03	200.0000	
12			

To calculate the angle of refraction 2, ▯$_3$,

$$gradient = \frac{y2 - y1}{x2 - x1}$$

$$gradient = \frac{810.1377 - 84.9727}{1500 - 450}$$

$$gradient = 0.69063333333333$$

Convert 0.69063333333333 into degree and the value will be 34.630251518783°

The fourth rayfan

Variables - raycoord{1, 4}

raycoord × | raycoord{1, 4} ×

raycoord{1, 4} <11x2 double>

	1	2	3
1	0	300.0000	
2	91.7136	450	
3	309.1471	750	
4	557.1181	1000	
5	884.4483	1.5000e+03	
6	884.4483	1.5000e+03	
7	1.0970e+03	1000	
8	1.1832e+03	750	
9	1.2643e+03	450	
10	1.2921e+03	300	
11	1.3000e+03	200.0000	
12			

To calculate the angle of refraction 3, ▯$_4$,

$$gradient = \frac{y2 - y1}{x2 - x1}$$

$$gradient = \frac{884.4483 - 91.7136}{1500 - 450}$$

$$gradient = 0.75498542857143$$

Convert 0.75498542857143 into degree and the value will be 37.052272774087°

The fifth rayfan

Variables - raycoord{1, 5}

raycoord × | raycoord{1, 5} ×

raycoord{1, 5} <11x2 double>

	1	2
1	0	300.0000
2	98.3057	450
3	333.3049	750
4	608.3482	1000
5	960.2517	1.5000e+03
6	960.2517	1.5000e+03
7	1.1857e+03	1000
8	1.2769e+03	750
9	1.3624e+03	450
10	1.3917e+03	300
11	1.4000e+03	200.0000
12		

To calculate the angle of refraction 4, $\square_5$,

$$gradient = \frac{y2 - y1}{x2 - x1}$$

$$gradient = \frac{960.2517 - 98.3057}{1500 - 450}$$

$$gradient = 0.82090095238095$$

Convert 0.82090095238095 into degree and the value will be 39.38260527553°

The sixth rayfan

The seventh rayfan

Variables - raycoord{1, 6}

raycoord × | raycoord{1, 6} ×

raycoord{1, 6} <11x2 double>

	1	2	3
1	0	300.0000	
2	104.7130	450	
3	357.2269	750	
4	661.6001	1000	
5	1.0376e+03	1.5000e+03	
6	1.0376e+03	1.5000e+03	
7	1.2753e+03	1000	
8	1.3711e+03	750	
9	1.4607e+03	450	
10	1.4913e+03	300	
11	1.5000e+03	200.0000	
12			

To calculate the angle of refraction 5, ⍰$_6$,

$$gradient = \frac{y2 - y1}{x2 - x1}$$

$$gradient = \frac{1037.6 - 104.7130}{1500 - 450}$$

$$gradient = 0.88846380952381$$

Convert 0.88846380952381 into degree and the value will be 41.619931115936°

The seventh rayfan

The eight rayfan

Variables - raycoord{1, 7}

raycoord | raycoord{1, 7}

raycoord{1, 7} <11x2 double>

	1	2	3
1	0	300.0000	
2	110.8957	450	
3	380.7696	750	
4	717.0801	1000	
5	1.1166e+03	1.5000e+03	
6	1.1166e+03	1.5000e+03	
7	1.3656e+03	1000	
8	1.4657e+03	750	
9	1.5591e+03	450	
10	1.5910e+03	300	
11	1.5999e+03	200.0000	
12			

To calculate the angle of refraction 6, ▯$_7$,

$$gradient = \frac{y2 - y1}{x2 - x1}$$

$$gradient = \frac{1116.6 - 110.8957}{1500 - 450}$$

$$gradient = 0.95781361904762$$

Convert 0.95781361904762 into degree and the value will be 43.76559874102°

The eight rayfan

Variables - raycoord{1, 8}

raycoord × raycoord{1, 8} ×

raycoord{1, 8} <11x2 double>

	1	2	3
1	0	300.0000	
2	116.7853	450	
3	403.6608	750	
4	774.7407	1000	
5	1.1968e+03	1.5000e+03	
6	1.1968e+03	1.5000e+03	
7	1.4564e+03	1000	
8	1.5603e+03	750	
9	1.6572e+03	450	
10	1.6902e+03	300	
11	1.6994e+03	200.0000	
12			

To calculate the angle of refraction 7, ⍰$_{8}$,

$$gradient = \frac{y2 - y1}{x2 - x1}$$

$$gradient = \frac{1196.8 - 116.7853}{1500 - 450}$$

$$gradient = 1.0285854285714$$

Convert 1.0285854285714 into degree and the value will be 45.807319227313°

The ninth rayfan

Variables - raycoord{1, 9}

raycoord × | raycoord{1, 9} ×

raycoord{1, 9} <11x2 double>

	1	2	3
1	0	300.0000	
2	122.4197	450	
3	426.0283	750	
4	835.6557	1000	
5	1.2795e+03	1.5000e+03	
6	1.2795e+03	1.5000e+03	
7	1.5489e+03	1000	
8	1.6564e+03	750	
9	1.7564e+03	450	
10	1.7904e+03	300	
11	1.7999e+03	200.0000	
12			

To calculate the angle of refraction 8, $\square_9$,

$$gradient = \frac{y2 - y1}{x2 - x1}$$

$$gradient = \frac{1279.5 - 122.4197}{1500 - 450}$$

$$gradient = 1.1019812380952$$

Convert 1.1019812380952 into degree and the value will be 47.777625346021°

The tenth rayfan

Variables - raycoord{1, 10}

raycoord × raycoord{1, 10} ×

raycoord{1, 10} <11x2 double>

	1	2	3
1	0	300.0000	
2	127.6547	450	
3	447.2615	750	
4	898.9795	1000	
5	1.3633e+03	1.5000e+03	
6	1.3633e+03	1.5000e+03	
7	1.6414e+03	1000	
8	1.7522e+03	750	
9	1.8550e+03	450	
10	1.8899e+03	300	
11	1.8997e+03	200.0000	
12			

To calculate the angle of refraction 9, ⍰$_{10}$,

$$gradient = \frac{y2 - y1}{x2 - x1}$$

$$gradient = \frac{1363.3 - 127.6547}{1500 - 450}$$

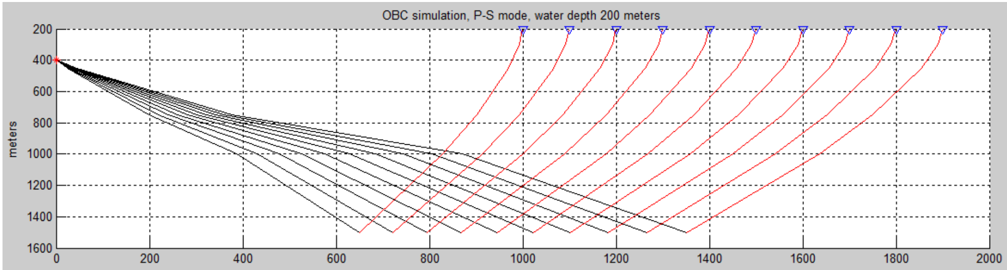
$$gradient = 1.176805047619$$

Convert 1.176805047619 into degree and the value will be 49.643500038196°

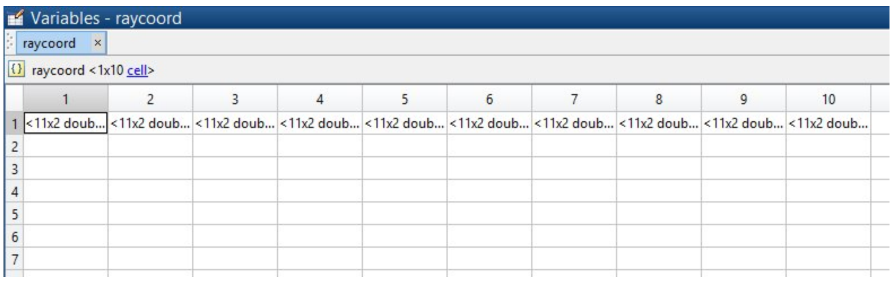
<u>For depth of 400 meters from sea level, the coding below is used.</u>

```
case {1}

%
% OBC primaries
%

%OBC recording
%make pwave model
zp=[0 200 300 450 750 1000 1700 2500 3500 4000];
vp=[1500 1600 2000 2250 2700 2100 3200 3750 4000 4200];
%make s-wave model
zs=zp;
vs=[0 300 700 1000 1250 1500 1800 1500 2100 1900];
zsrc=400;
zrec=200;
zd=1500;
xoff=1000:100:1900;

%simulate source in the middle of the water layer and receiver on the
bottom
figure;subplot(2,1,1);flipy
%Trace P-P rays and plot in upper subplot
[t,p,L,raycoord]=traceray_pp(vp,zp,zsrc,zrec,zd,xoff,100,-1,10,1,1,2);
%put source and receiver markers
line(xoff,zrec*ones(size(xoff)),'color','b','linestyle','none','marker','v'
)
line(0,zsrc,'color','r','linestyle','none','marker','*')
%annotate plot
title('OBC simulation, P-P mode, water depth 200 meters')
xlabel('meters');ylabel('meters');grid
%plot traveltime versus offset in lower subplot
subplot(2,1,2);flipy;
plot(xoff,t);grid;xlabel('meters');ylabel('seconds')
xlim([0 max(xoff)])

figure;subplot(2,1,1);flipy
%Trace P-S rays and plot in upper subplot
[t,p,L,raycoord]=traceray_ps(vp,zp,vs,zs,zsrc,zrec,zd,xoff,10,-1,10,1,1,2);
%put source and receive markers
line(xoff,zrec*ones(size(xoff)),'color','b','linestyle','none','marker','v'
)
line(0,zsrc,'color','r','linestyle','none','marker','*')
%annotate plot
title('OBC simulation, P-S mode, water depth 200 meters')
grid;xlabel('meters');ylabel('meters');
subplot(2,1,2);flipy;
plot(xoff,t);grid;xlabel('meters');ylabel('seconds');
xlim([0 max(xoff)])

pos=get(gcf,'position');
set(gcf,'position',[1.1*pos(1) .9*pos(2) pos(3:4)])
```

After simulated the coding above, a set of rayfan are produced.



The data will be stored in raycoord as shown below.



Each column indicates the value of each rayfan.

<u>The first rayfan</u>

Variables - raycoord{1, 1}

raycoord | raycoord{1, 1}

raycoord{1, 1} <11x2 double>

	1	2	3
1	0	400.0000	
2	24.8777	450	
3	198.6119	750	
4	386.7835	1000	
5	651.3784	1.5000e+03	
6	651.3784	1.5000e+03	
7	828.6104	1000	
8	901.0790	750	
9	969.6200	450	
10	993.2962	300	
11	999.9931	200.0000	
12			

To calculate the angle of reflection 1, $\square_1$,

$$gradient = \frac{y2 - y1}{x2 - x1}$$

$$gradient = \frac{651.3784 - 24.8777}{1500 - 450}$$

$$gradient = 0.59666733333333$$

Convert 0.59666733333333 into degree and the value will be 30.823147331081°

<u>The second rayfan</u>

Variables - raycoord{1, 2}

raycoord | raycoord{1, 2}

raycoord{1, 2} <11x2 double>

	1	2	3
1	0	400.0000	
2	27.2904	450	
3	219.2522	750	
4	431.2583	1000	
5	722.2852	1.5000e+03	
6	722.2852	1.5000e+03	
7	914.8017	1000	
8	993.2595	750	
9	1.0673e+03	450	
10	1.0928e+03	300	
11	1.1000e+03	200.0000	
12			

To calculate the angle of refraction 1, $\square_2$,

$$gradient = \frac{y2 - y1}{x2 - x1}$$

$$gradient = \frac{722.2852 - 27.2904}{1500 - 450}$$

$$gradient = 0.66189980952381$$

Convert 0.66189980952381 into degree and the value will be 33.500567671214°

The third rayfan

Variables - raycoord{1, 3}

raycoord | raycoord{1, 3}

raycoord{1, 3} <11x2 double>

	1	2	3
1	0	400.0000	
2	29.6613	450	
3	239.9448	750	
4	477.4635	1000	
5	794.6813	1.5000e+03	
6	794.6813	1.5000e+03	
7	1.0018e+03	1000	
8	1.0859e+03	750	
9	1.1650e+03	450	
10	1.1923e+03	300	
11	1.1999e+03	200.0000	
12			

To calculate the angle of refraction 2, ⍰$_3$,

$$gradient = \frac{y2 - y1}{x2 - x1}$$

$$gradient = \frac{794.6813 - 29.6613}{1500 - 450}$$

$$gradient = 0.72859047619048$$

Convert 0.72859047619048 into degree and the value will be 36.076724468995°

The fourth rayfan

Variables - raycoord{1, 4}

raycoord | raycoord{1, 4}

raycoord{1, 4} <11x2 double>

	1	2	3
1	0	400.0000	
2	31.9756	450	
3	260.5855	750	
4	525.5356	1000	
5	868.5461	1.5000e+03	
6	868.5461	1.5000e+03	
7	1.0894e+03	1000	
8	1.1788e+03	750	
9	1.2627e+03	450	
10	1.2915e+03	300	
11	1.2996e+03	200.0000	
12			

To calculate the angle of refraction 3, $\square_4$,

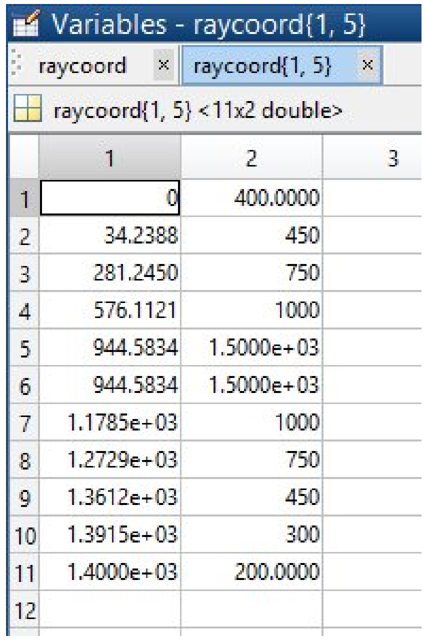
$$gradient = \frac{y2 - y1}{x2 - x1}$$

$$gradient = \frac{868.5461 - 31.9756}{1500 - 450}$$

$$gradient = 0.79673380952381$$

Convert 0.79673380952381 into degree and the value will be 38.545517206392°

<u>The fifth rayfan</u>



Variables - raycoord{1, 5}

raycoord{1, 5} <11x2 double>

	1	2	3
1	0	400.0000	
2	34.2388	450	
3	281.2450	750	
4	576.1121	1000	
5	944.5834	1.5000e+03	
6	944.5834	1.5000e+03	
7	1.1785e+03	1000	
8	1.2729e+03	750	
9	1.3612e+03	450	
10	1.3915e+03	300	
11	1.4000e+03	200.0000	
12			

To calculate the angle of refraction 4, ⍰$_5$,

$$gradient = \frac{y2 - y1}{x2 - x1}$$

$$gradient = \frac{944.5834 - 34.2388}{1500 - 450}$$

$$gradient = 0.86699485714286$$

Convert 0.86699485714286 into degree and the value will be 40.925119760391°

<u>The sixth rayfan</u>

Variables - raycoord{1, 6}

raycoord × | raycoord{1, 6} ×

raycoord{1, 6} <11x2 double>

	1	2	3
1	0	400.0000	
2	36.4078	450	
3	301.5388	750	
4	628.8016	1000	
5	1.0219e+03	1.5000e+03	
6	1.0219e+03	1.5000e+03	
7	1.2679e+03	1000	
8	1.3668e+03	750	
9	1.4592e+03	450	
10	1.4908e+03	300	
11	1.4997e+03	200.0000	
12			

To calculate the angle of refraction 5, $\square_6$,

$$gradient = \frac{y2 - y1}{x2 - x1}$$

$$gradient = \frac{1021.9 - 36.4078}{1500 - 450}$$

$$gradient = 0.938564$$

Convert 0.938564 into degree and the value will be 43.184818363871°

The seventh rayfan

Variables - raycoord{1, 7}

raycoord × | raycoord{1, 7} ×

raycoord{1, 7} <11x2 double>

	1	2	3
1	0	400.0000	
2	38.4937	450	
3	321.5629	750	
4	684.5132	1000	
5	1.1016e+03	1.5000e+03	
6	1.1016e+03	1.5000e+03	
7	1.3588e+03	1000	
8	1.4619e+03	750	
9	1.5580e+03	450	
10	1.5908e+03	300	
11	1.6000e+03	200.0000	
12			

To calculate the angle of refraction 6, ⍰$_7$,

$$gradient = \frac{y2 - y1}{x2 - x1}$$

$$gradient = \frac{1101.6 - 38.4937}{1500 - 450}$$

$$gradient = 1.0124821904762$$

Convert 1.0124821904762 into degree and the value will be 45.35536595857°

<u>The eight rayfan</u>

Variables - raycoord{1, 8}

raycoord | raycoord{1, 8}

raycoord{1, 8} <11x2 double>

	1	2	3
1	0	400.0000	
2	40.4579	450	
3	340.9234	750	
4	742.9280	1000	
5	1.1828e+03	1.5000e+03	
6	1.1828e+03	1.5000e+03	
7	1.4503e+03	1000	
8	1.5572e+03	750	
9	1.6566e+03	450	
10	1.6904e+03	300	
11	1.6999e+03	200.0000	
12			

To calculate the angle of refraction 7, θ_8,

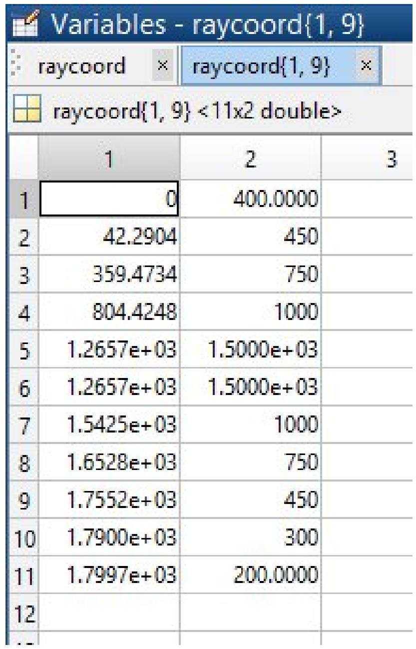
$$gradient = \frac{y2 - y1}{x2 - x1}$$

$$gradient = \frac{1182.8 - 40.4579}{1500 - 450}$$

$$gradient = 1.0879448571429$$

Convert 1.0879448571429 into degree and the value will be 47.411889587529°

The ninth rayfan



Variables - raycoord{1, 9}

raycoord | raycoord{1, 9}

raycoord{1, 9} <11x2 double>

	1	2	3
1	0	400.0000	
2	42.2904	450	
3	359.4734	750	
4	804.4248	1000	
5	1.2657e+03	1.5000e+03	
6	1.2657e+03	1.5000e+03	
7	1.5425e+03	1000	
8	1.6528e+03	750	
9	1.7552e+03	450	
10	1.7900e+03	300	
11	1.7997e+03	200.0000	
12			

To calculate the angle of refraction 8, θ_9,

$$gradient = \frac{y2 - y1}{x2 - x1}$$

$$gradient = \frac{1265.7 - 42.2904}{1500 - 450}$$

$$gradient = 1.165152$$

Convert 1.165152 into degree and the value will be 49.361922251113°

<u>The tenth rayfan</u>

Variables - raycoord{1, 10}

raycoord | raycoord{1, 10}

raycoord{1, 10} <11x2 double>

	1	2	3
1	0	400.0000	
2	43.9862	450	
3	377.1005	750	
4	869.5886	1000	
5	1.3509e+03	1.5000e+03	
6	1.3509e+03	1.5000e+03	
7	1.6361e+03	1000	
8	1.7494e+03	750	
9	1.8543e+03	450	
10	1.8900e+03	300	
11	1.8999e+03	200.0000	
12			

To calculate the angle of refraction 9, $⍰_{10}$,

$$gradient = \frac{y2 - y1}{x2 - x1}$$

$$gradient = \frac{1350.9 - 43.9862}{1500 - 450}$$

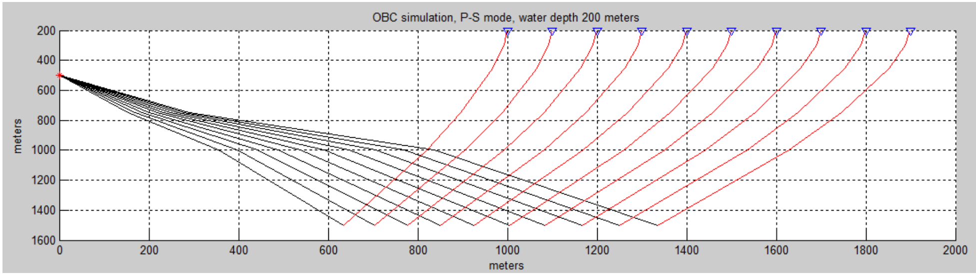
$$gradient = 1.2446798095238$$

Convert 1.2446798095238 into degree and the value will be 51.22092651179°

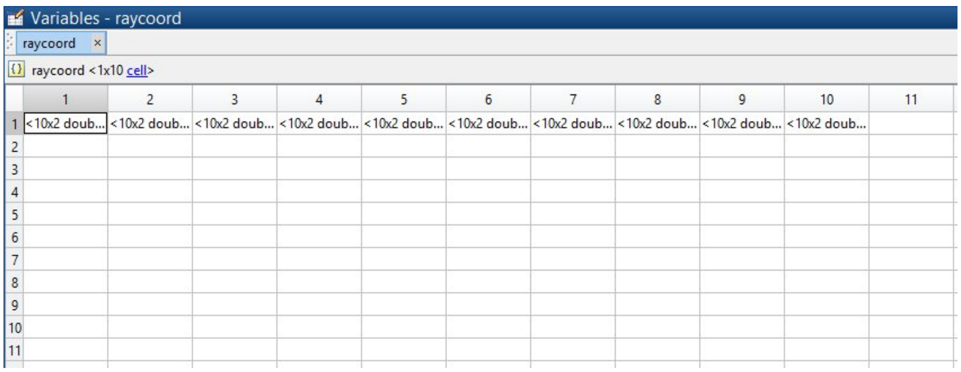
4.4 <u>For depth of 500 meters from sea level, the coding below is used.</u>

```
case {1}

%
% OBC primaries
%

%OBC recording
%make pwave model
zp=[0 200 300 450 750 1000 1700 2500 3500 4000];
vp=[1500 1600 2000 2250 2700 2100 3200 3750 4000 4200];
%make s-wave model
zs=zp;
vs=[0 300 700 1000 1250 1500 1800 1500 2100 1900];
zsrc=500;
zrec=200;
zd=1500;
xoff=1000:100:1900;

%simulate source in the middle of the water layer and receiver on the
bottom
figure;subplot(2,1,1);flipy
%Trace P-P rays and plot in upper subplot
[t,p,L,raycoord]=traceray_pp(vp,zp,zsrc,zrec,zd,xoff,100,-1,10,1,1,2);
%put source and receiver markers
line(xoff,zrec*ones(size(xoff)),'color','b','linestyle','none','marker','v'
)
line(0,zsrc,'color','r','linestyle','none','marker','*')
%annotate plot
title('OBC simulation, P-P mode, water depth 200 meters')
xlabel('meters');ylabel('meters');grid
%plot traveltime versus offset in lower subplot
subplot(2,1,2);flipy;
plot(xoff,t);grid;xlabel('meters');ylabel('seconds')
xlim([0 max(xoff)])

figure;subplot(2,1,1);flipy
%Trace P-S rays and plot in upper subplot
[t,p,L,raycoord]=traceray_ps(vp,zp,vs,zs,zsrc,zrec,zd,xoff,10,-1,10,1,1,2);
%put source and receive markers
line(xoff,zrec*ones(size(xoff)),'color','b','linestyle','none','marker','v'
)
line(0,zsrc,'color','r','linestyle','none','marker','*')
%annotate plot
title('OBC simulation, P-S mode, water depth 200 meters')
grid;xlabel('meters');ylabel('meters');
subplot(2,1,2);flipy;
plot(xoff,t);grid;xlabel('meters');ylabel('seconds');
xlim([0 max(xoff)])

pos=get(gcf,'position');
set(gcf,'position',[1.1*pos(1) .9*pos(2) pos(3:4)])
```

After simulated the coding above, a set of rayfan are produced.



The data will be stored in raycoord as shown below.



	1	2	3	4	5	6	7	8	9	10	11
1	<10x2 doub...	<10x2 doub...	<10x2 doub...	<10x2 doub...	<10x2 doub...	<10x2 doub...	<10x2 doub...	<10x2 doub...	<10x2 doub...	<10x2 doub...	
2											
3											
4											
5											
6											
7											
8											
9											
10											
11											

Each column indicates the value of each rayfan.

<u>The first rayfan</u>

Variables - raycoord{1, 1}

raycoord | raycoord{1, 1}

raycoord{1, 1} <10x2 double>

	1	2	3
1	0	500.0000	
2	153.4134	750	
3	354.9626	1000	
4	634.6279	1.5000e+03	
5	634.6279	1.5000e+03	
6	820.6446	1000	
7	896.5628	750	
8	968.2638	450	
9	993.0015	300	
10	999.9922	200.0000	
11			

To calculate the angle of reflection 1, $\square_1$,

$$gradient = \frac{y2 - y1}{x2 - x1}$$

$$gradient = \frac{634.6279 - 153.4134}{1500 - 750}$$

$$gradient = 0.64161933333333$$

Convert 0.64161933333333 into degree and the value will be 32.685015460608°

<u>The second rayfan</u>

Variables - raycoord{1, 2}

raycoord × raycoord{1, 2} ×

raycoord{1, 2} <10x2 double>

	1	2	3
1	0	500.0000	
2	169.5377	750	
3	397.3313	1000	
4	704.8197	1.5000e+03	
5	704.8197	1.5000e+03	
6	906.5623	1000	
7	988.6059	750	
8	1.0659e+03	450	
9	1.0925e+03	300	
10	1.1000e+03	200.0000	
11			

To calculate the angle of refraction 1, ⍰$_2$,

$$gradient = \frac{y2 - y1}{x2 - x1}$$

$$gradient = \frac{704.8197 - 169.5377}{1500 - 750}$$

$$gradient = 0.71370933333333$$

Convert 0.71370933333333 into degree and the value will be 35.515804457087°

<u>The third rayfan</u>

Variables - raycoord{1, 3}

raycoord | raycoord{1, 3}

raycoord{1, 3} <10x2 double>

	1	2	3
1	0	500.0000	
2	185.7221	750	
3	441.8445	1000	
4	776.8167	1.5000e+03	
5	776.8167	1.5000e+03	
6	993.4527	1000	
7	1.0812e+03	750	
8	1.1637e+03	450	
9	1.1920e+03	300	
10	1.2000e+03	200.0000	
11			

To calculate the angle of refraction 2, $\square_3$,

$$gradient = \frac{y2 - y1}{x2 - x1}$$

$$gradient = \frac{776.8167 - 185.7221}{1500 - 750}$$

$$gradient = 0.78812613333333$$

Convert 0.78812613333333 into degree and the value will be 38.242568117824°

<u>The fourth rayfan</u>

Variables - raycoord{1, 4}

raycoord{1, 4} <10x2 double>

	1	2	3
1	0	500.0000	
2	201.8832	750	
3	488.7727	1000	
4	850.7175	1.5000e+03	
5	850.7175	1.5000e+03	
6	1.0813e+03	1000	
7	1.1745e+03	750	
8	1.2617e+03	450	
9	1.2916e+03	300	
10	1.3000e+03	200.0000	
11			

To calculate the angle of refraction 3, $\square_4$,

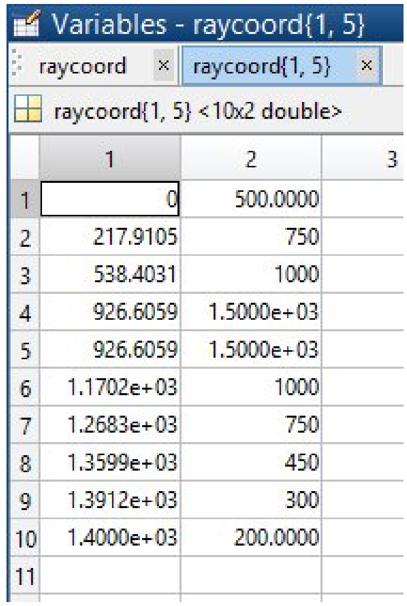
$$gradient = \frac{y2 - y1}{x2 - x1}$$

$$gradient = \frac{850.7175 - 201.8832}{1500 - 750}$$

$$gradient = 0.8651124$$

Convert 0.8651124 into degree and the value will be 40.863488989292°

<u>The fifth rayfan</u>



Variables - raycoord{1, 5}

raycoord | raycoord{1, 5}

raycoord{1, 5} <10x2 double>

	1	2	3
1	0	500.0000	
2	217.9105	750	
3	538.4031	1000	
4	926.6059	1.5000e+03	
5	926.6059	1.5000e+03	
6	1.1702e+03	1000	
7	1.2683e+03	750	
8	1.3599e+03	450	
9	1.3912e+03	300	
10	1.4000e+03	200.0000	
11			

To calculate the angle of refraction 4, $?_5$,

$$gradient = \frac{y2 - y1}{x2 - x1}$$

$$gradient = \frac{926.6059 - 217.9105}{1500 - 750}$$

$$gradient = 0.9449272$$

Convert 0.9449272 into degree and the value will be 43.378039000763°

<u>The sixth rayfan</u>

Variables - raycoord{1, 6}

raycoord × raycoord{1, 6} ×

raycoord{1, 6} <10x2 double>

	1	2	3
1	0	500.0000	
2	233.6625	750	
3	591.0266	1000	
4	1.0045e+03	1.5000e+03	
5	1.0045e+03	1.5000e+03	
6	1.2602e+03	1000	
7	1.3627e+03	750	
8	1.4582e+03	450	
9	1.4908e+03	300	
10	1.5000e+03	200.0000	
11			

To calculate the angle of refraction 5, □$_{6}$,

$$gradient = \frac{y2 - y1}{x2 - x1}$$

$$gradient = \frac{1004.5 - 233.6625}{1500 - 750}$$

$$gradient = 1.0277833333$$

Convert 1.0277833333 into degree and the value will be 45.784979397914°

The seventh rayfan

Variables - raycoord{1, 7}

raycoord × raycoord{1, 7} ×

raycoord{1, 7} <10x2 double>

	1	2	3
1	0	500.0000	
2	248.9648	750	
3	646.9067	1000	
4	1.0845e+03	1.5000e+03	
5	1.0845e+03	1.5000e+03	
6	1.3511e+03	1000	
7	1.4576e+03	750	
8	1.5567e+03	450	
9	1.5904e+03	300	
10	1.5999e+03	200.0000	
11			

To calculate the angle of refraction 6, $\square_7$,

$$gradient = \frac{y2 - y1}{x2 - x1}$$

$$gradient = \frac{1084.5 - 248.9648}{1500 - 750}$$

$$gradient = 1.11404693$$

Convert 1.11404693 into degree and the value will be 48.087954130692°

The eight rayfan

Variables - raycoord{1, 8}

raycoord × | raycoord{1, 8} ×

raycoord{1, 8} <10x2 double>

	1	2	3
1	0	500.0000	
2	263.6289	750	
3	706.3024	1000	
4	1.1665e+03	1.5000e+03	
5	1.1665e+03	1.5000e+03	
6	1.4429e+03	1000	
7	1.5530e+03	750	
8	1.6552e+03	450	
9	1.6900e+03	300	
10	1.6997e+03	200.0000	
11			

To calculate the angle of refraction 7, $\square_{8}$,

$$gradient = \frac{y2 - y1}{x2 - x1}$$

$$gradient = \frac{1166.5 - 263.6289}{1500 - 750}$$

$$gradient = 1.20382813$$

Convert 1.20382813 into degree and the value will be 50.284151669739°

The ninth rayfan

Variables - raycoord{1, 9}

raycoord × raycoord{1, 9} ×

raycoord{1, 9} <10x2 double>

	1	2	3
1	0	500.0000	
2	277.5287	750	
3	769.7590	1000	
4	1.2510e+03	1.5000e+03	
5	1.2510e+03	1.5000e+03	
6	1.5361e+03	1000	
7	1.6494e+03	750	
8	1.7543e+03	450	
9	1.7900e+03	300	
10	1.7999e+03	200.0000	
11			

To calculate the angle of refraction 8, ⍰$_9$,

$$gradient = \frac{y2 - y1}{x2 - x1}$$

$$gradient = \frac{1251.0 - 277.5287}{1500 - 750}$$

$$gradient = 1.29796173$$

Convert 1.29796173 into degree and the value will be 52.387951017624°

The tenth rayfan

Variables - raycoord{1, 10}

raycoord | raycoord{1, 10}

raycoord{1, 10} <10x2 double>

	1	2	3
1	0	500.0000	
2	290.2602	750	
3	836.3188	1000	
4	1.3364e+03	1.5000e+03	
5	1.3364e+03	1.5000e+03	
6	1.6291e+03	1000	
7	1.7451e+03	750	
8	1.8524e+03	450	
9	1.8888e+03	300	
10	1.8989e+03	200.0000	
11			

To calculate the angle of refraction 9, ⍰$_{\mathbf{10}}$,

$$gradient = \frac{y2 - y1}{x2 - x1}$$

$$gradient = \frac{1336.4 - 290.2602}{1500 - 750}$$

$$gradient = 1.3948530666667$$

Convert 1.3948530666667 into degree and the value will be 54.362451634773