

CHAPTER 1

INTRODUCTION

1.1 Background of Study

Pedestal crane is one of the offshore oil and gas production facilities that is used as a lifting machine to transfer offshore personnel, load equipments, tools and foods stuff from the supply boat to the fixed platform or vice versa. Proper design of the crane boom structure is important to make sure that the pedestal crane is properly operated at specific time when it is required. Basically there are two major considerations in the design of cranes [1]. The first is that the crane must be able to lift a load of a specified weight and the second is that the crane must remain stable and not topple over when the load is lifted and moved to another location [1].

A boom crane is distinguished from other cranes by its use of a single boom which pivots and rotates on a base at one end; the payload is hoisted from the other [2]. Both cranes in Figure 1.1 (a) and Figure 1.1 (b) are pedestal crane boom. Crane in Figure 1.1 (a) uses cables to pivot the boom up and down. That motion is called luffing. Instead of using a cable system to luff the boom, the pedestal crane in Figure 1.1 (b) uses a hydraulic cylinder. Both cranes use a motor to rotate the boom about a vertical axis. That rotation motion is called slewing.

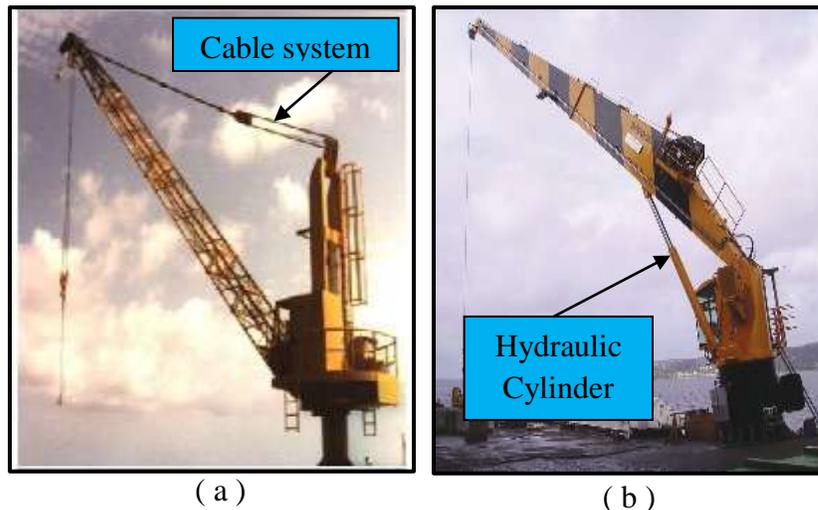


Figure 1.1: Pedestal crane boom

Fixed offshore platform is located at the middle of the sea which is exposing to adverse wind condition especially during monsoon season. During monsoon season, the operation of pedestal crane boom is not allowed because of safety issue. Wind and storms can influence the entire operation of cranes and can even destroy the whole crane [3]. Other than that, in the crane operation; there are also limitations in the hoisting speed (acceleration) for picking up or down the payload to avoid excessive dynamic loads or deformations in the crane system. Due to the safety issue; the analysis of the dynamic characteristics is therefore significant for both the design and operation of such crane.

1.2 Problem Statement

Pedestal crane is often used as one of the offshore oil and gas production facility for personnel transfer, loading of equipment, tools and foods stuff from the supply boat to the platform and vice versa. The pedestal crane components are the operator cabin which is mounted on a pedestal, the crane boom structure and the lifting block. In the crane operation, the hoisting speed (acceleration) for picking up or lowering down the payload provides sustained cyclic or dynamic loads to the crane system. Thus, the analysis of the dynamic characteristics of the offshore crane boom structure due to excitation by the payload is therefore significant for both the design and operation of such crane.

1.3 Objective

The objective of this project is to study the dynamic characteristic of the offshore crane boom structure due to excitation by the payload. Dynamic analysis will be conducted on the crane boom structure to get the value of natural frequencies, the mode shapes of crane boom structure and also harmonic response of the crane due to excitation by the payload.

1.4 Scope of study

This project is carried out to study dynamic characteristics of the offshore crane boom structure due to excitation by the payload; by solving using approximate solution and simulation in ANSYS Workbench software. This project will give more focus on the dynamic analysis of the crane boom structure assembly only; which consists of boom assembly and 16T main hook assembly. The crane boom structure will be modeled to a vibration model as a continuous system. The model is a solid tapered cantilever beam. The crane boom structure is fixed – free end and there is no slewing motion where the pedestal crane is fixed at one location without rotation of the base of the pedestal crane.

Mathematical model will be developed based on eigenvalue and eigenfunction problem and that mathematical model will be solved by approximate solution. Dynamic analysis will be done first on the crane boom structure without payload at 0° angle and act as a baseline model. Then, simulation of the model will be done using ANSYS Workbench software. The result obtained from the approximate solution will be compared with the result obtained from the simulation in ANSYS Workbench software.

The model then will be lifted to 55° angle lift. The hoisting speed for picking up or lowering down the payload provides sustained cyclic or dynamic loads to the crane system. The loading is only by the payload without any excitation by the wind load. The harmonic response analysis for crane boom structure at 55° angle will be done in ANSYS Workbench software. The amount of payload is varied; 4 tones, 8 tones, 12 tones, 16 tones, 20 tones to view the effect of loading on the crane boom structure. The material of the crane boom structure is steel. This project will be excluding the physical experiment.

CHAPTER 2

LITERATURE REVIEW / THEORY

2.1 Vibration Analysis

A vibration system is a dynamic system for which the response (output) depends on the excitations (inputs) and the characteristics of the system (e.g. mass, stiffness, and damping) in Figure 2.1 [4]. The excitation and response of the system are both time dependent. The response of the system sometimes can be viewed in frequency response. Vibration analysis of a given system involves determination of the response for the excitation specified. The analysis usually involves mathematical modeling, derivation of the governing equations of motion, solution of the equations of motion, and interpretation of the results [5].

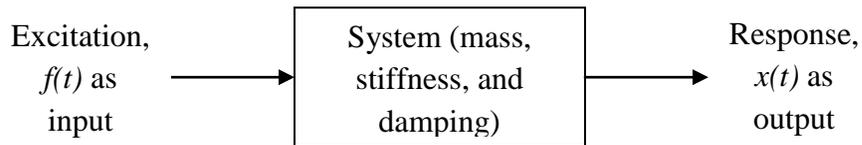


Figure 2.1: Input – output relationship of a vibratory system [4]

Different mathematical model requires different approaches (e.g. D'Alembert's principle, Newton's second law of motion, and Hamilton's principle) to be used in deriving the equations of motion of the system. Then, the equations of motion can be solved using variety of techniques to obtain analytical (closed-form) or numerical solutions, depending on the complexity of the equations involved. The solution of the equations of motion provides the displacement, velocity and acceleration responses of the system [6].

2.2 Vibration of Continuous System

In dealing with discrete system, mass, damping and elasticity were assumed to be present only at certain discrete points in the system. However, it is not possible to identify discrete masses, dampers or springs in dealing with distributed or continuous systems. Continuous systems such as beams, rods (bars), cables and strings where elasticity and mass are considered to be distributed parameters are distributed systems [6]. We consider the continuous distribution of elasticity, mass and damping and assume each of the infinite number of elements of the system can vibrate [6].

2.3 Eigenvalue Problem

The equations of motion of many continuous systems are in the form of non homogeneous linear partial differential equations of order two or higher subject to boundary and initial conditions. The boundary conditions may be homogeneous or non homogeneous. The initial conditions are usually stated in terms of the values of the field variable and its time derivative at time zero.

The solution procedure basically involves two steps. In the first step, the non homogeneous part of the equation of motion is neglected and the homogeneous equation is solved using the separation of variables technique. This leads to an eigenvalue problem whose solution yields an infinite set of eigenvalues and the corresponding eigenfunctions. The eigenfunction are orthogonal and form a complete set in the sense that any function $f(\vec{X})$ that satisfies the boundary conditions of the problem can be represented by a linear combination of the eigenfunctions.

The equation of motion of an undamped continuous system is in the form of a partial differential equation which can be expressed as [5]:

$$M(\vec{X}) \frac{\partial^2 w(\vec{X}, t)}{\partial t^2} + K[w(\vec{X}, t)] = f(\vec{X}, t) + \sum_{j=1}^s F_j(t) \delta(\vec{X} - \vec{X}_j), \quad \vec{X} \in V \quad (2.1)$$

Where:

\vec{X} is a typical point in the domain of the system (V),

$M(\vec{X})$ is the mass distribution,

$w(\vec{X}, t)$ is the field variable or displacement of the system that depends on spatial variable \vec{X} and (t),

$K[w(\vec{X}, t)]$ is the stiffness distribution of the system,

$f(\vec{X}, t)$ is the distributed force acting on the system,

$F_j(t)$ is the concentrated force acting at the point $\vec{X} = \vec{X}_j$ of the system,

s is the number of concentrated forces acting on the system, and

$\delta(\vec{X} - \vec{X}_j)$ is the Dirac delta function.

In the case of free vibration, f and all F_j will be zero and Eqn. (2.1) reduces to the homogeneous form

$$M(\vec{X}) \frac{\partial^2 w(\vec{X}, t)}{\partial t^2} + K[w(\vec{X}, t)] = 0, \quad \vec{X} \in V \quad (2.2)$$

For the natural frequencies of vibration, we assume the displacement $w(\vec{X}, t)$ to be a harmonic function as

$$w(\vec{X}, t) = W(\vec{X}) e^{i\omega t} \quad (2.3)$$

Where:

$W(\vec{X})$ denotes the mode shape (also called eigenfunction or normal mode), and

ω indicates the natural frequency of vibration.

Using Eqn. (2.3), Eqn. (2.2) can be represented as

$$K[W] = \lambda M[W] \quad (2.4)$$

Where:

$\lambda = \omega^2$ is also called the eigenvalue of the system.

Eqn. (2.4) along with the boundary condition defines the eigenvalue problem of the system. The solution of the eigenvalue problem yields an infinite number of eigenvalues $\lambda_1, \lambda_2, \dots, \lambda_n$ and the corresponding eigenfunction $W_1(\vec{X}), W_2(\vec{X}), \dots, W_n(\vec{X})$. The eigenvalue problem is said to be homogeneous, and the amplitudes of the eigenfunctions $W_i(\vec{X}), i = 1, 2, \dots$, are arbitrary. Thus, only the shapes of the eigenfunctions can be determined uniquely.

By solving eigenvalue problem, we can get an infinite number of eigenvalues which will be solved to get the natural frequencies of the system and also corresponding eigenfunctions which is the mode shapes of the systems.

2.4 Natural Frequency

Natural frequency is the frequency or frequencies at which a system will undergo free vibration [7] In the other words, natural frequency is the number of times where a system will oscillate (move back and forth) between its original position and its displaced position after an initial disturbance and left to vibrate on its own without any external forces. For example, we can consider a simple beam fixed at one end and having a mass attached to its free end, as shown in Figure 2.2. If the beam tip is pulled downward, then released, the beam will oscillate at its natural frequency.

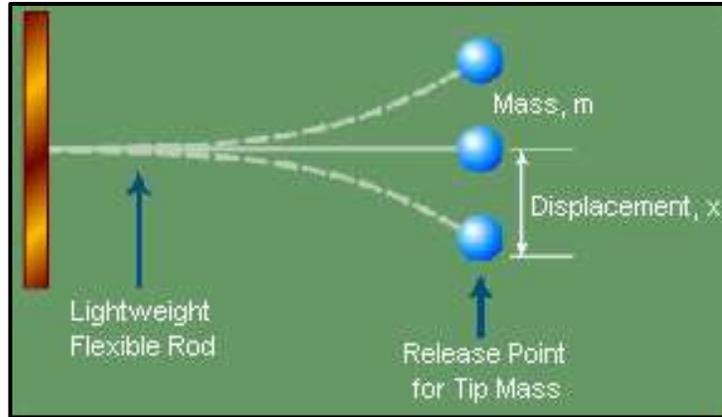


Figure 2.2: Natural frequency of simple fixed beam with free end [8]

For lateral vibration of the uniform beam, we can derive the equation of motion of the beam and then solve the equation of motion by considering the initial conditions and four boundary conditions for finding a total response of the beam. The natural frequency of the beam is

$$\omega = \beta^2 \sqrt{\frac{EI}{\rho A}} = (\beta l)^2 \sqrt{\frac{EI}{\rho A l^4}} \quad [9]$$

Where:

ω is natural frequency,

E is elasticity of the beam,

I is moment of inertia,

ρ is density of the beam,

A is cross section area,

l is length of the beam, and

β is constant value that can be determined from the boundary conditions of the beam as indicated in the *Appendix 1*.

For non uniform beam, cross section area $A(x)$ and moment of inertia $I(x)$ vary along x axis. Natural frequencies depend on the geometry, the boundary conditions (method of support or attachment), the masses of the components, and the strength of the restoring forces or moments.

2.5 Mode shape

Mode shape is the relationship between the amplitudes (one per DOF) of the independent motions of a system in free vibration [7]. There is one mode shape for each natural frequency, and it depends on the value of that natural frequency [7]. To determine the vibration of a system, the mode shape is multiplied by a function that varies with time, thus the mode shape always describes the curvature of vibration at all points in time, but the magnitude of the curvature will change. The mode shape is dependent on the shape of the surface as well as the boundary conditions of that surface [10]. The value of natural frequency and mode shape will be different for different type of structure, supports and boundary condition applied during the analysis.

2.6 Harmonic Response Analysis

The vibration analysis can be viewed as input/output relation where the force is the input while the vibration is the output. In a structural system, any sustained cyclic load will produce a sustained cyclic or harmonic response. Harmonic analysis results are used to determine the steady state response of a linear structure to loads that vary sinusoidally (harmonically) with time, thus enabling us to overcome resonance, fatigue and other harmful effects of forced vibrations. Harmonic response analysis is a linear analysis. Some nonlinearity, such as plasticity will be ignored, even if they are defined. All loads and displacements vary sinusoidally at the same known frequency (although not necessarily in phase) [11].

This analysis technique calculates only the steady-state, forced vibrations of a structure. The transient vibrations, which occur at the beginning of the excitation, are not encountered for in a harmonic response analysis. In this harmonic response analysis, all loads as well as the structure's response of the structure to cyclic loads over a frequency range and obtain a graph of some response quantity (usually displacements or accelerations) versus frequency. "Peak" responses are identified from the graphs of response versus frequency and stresses are the reviewed at those peak frequencies [11].

2.7 Resonance

Resonance is the buildup of large vibration amplitude that occurs when a structure or an object is excited at its natural frequency [8]. Resonance can be either desirable or undesirable [8]. Acoustic resonance, a desirable resonance, occurs in many different musical instruments. It also occurs in auditoriums. Undesirable mechanical resonance can cause bridges to collapse, aircraft wings to break, and machinery to break or malfunction [8]. The example of undesirable resonance is the collapse of Tacoma Narrows Bridge. Dynamic analysis of a crane boom structure purposely done to get the value of natural frequencies of the crane boom structure and its' mode shapes to assure that the design is not in resonance condition to avoid the collapse of crane boom structure. Harmonic response results will show the "peak" frequency where the maximum amplitude will occur.

CHAPTER 3

METHODOLOGY / PROJECT WORK

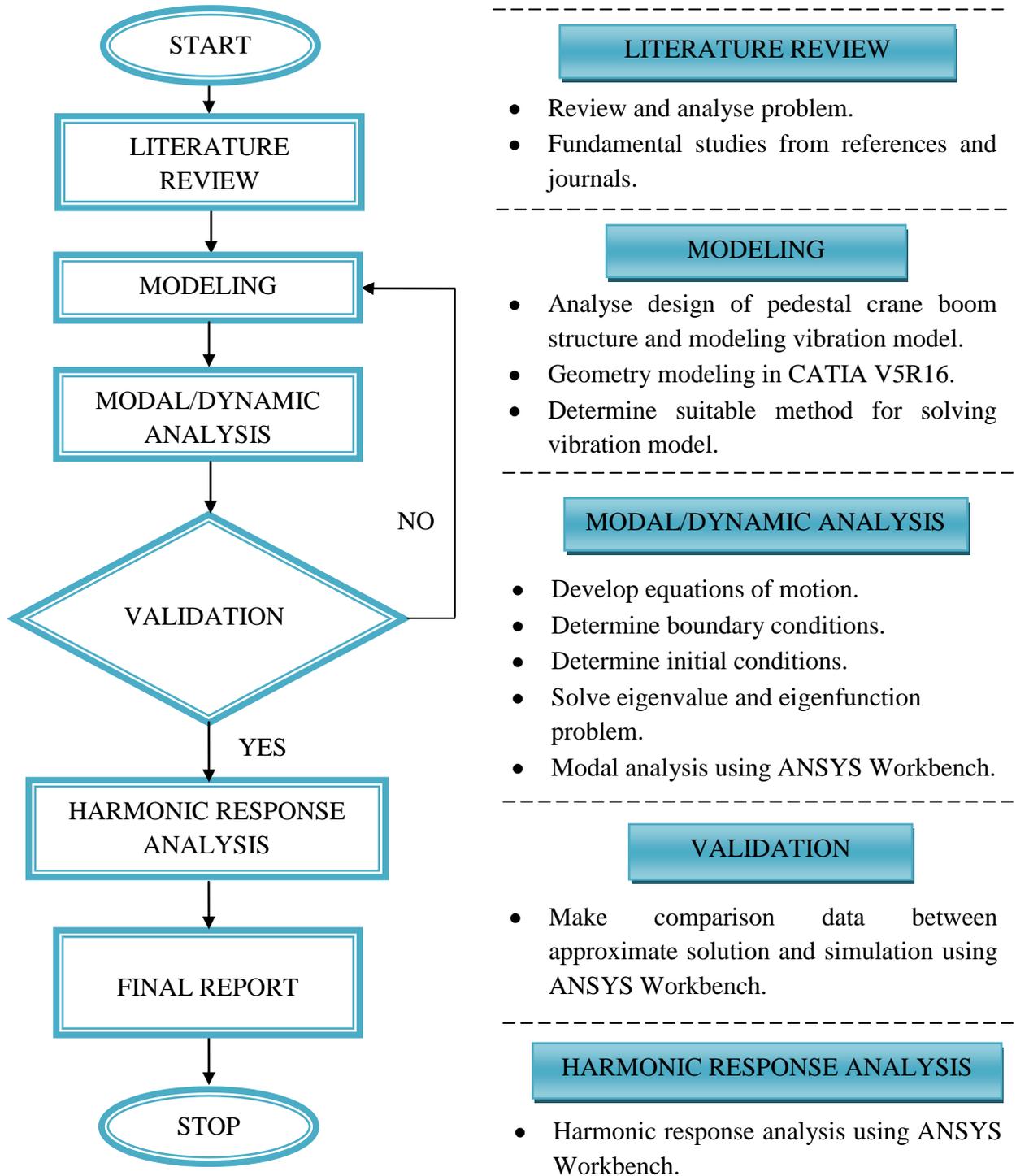


Figure 3.1: Methodology of the project

3.1 Literature Review

The literature review was firstly done by studying on the fundamental of vibration of continuous system. The study provides the information of behaviour and response of the continuous system (e.g. bar, beam, plate) under the transverse and longitudinal vibration forces. Besides, the study also can provide the method of finding the natural frequency, mode shape and total response of a continuous system.

Datta and Sill [12] have determined the natural frequencies of undamped transverse vibrations of cantilevered beams of constant width with linearly varying depth. Besides, Zhou and Cheung [13] have investigated the free vibration of a wide range of tapered Timoshenko beams. The Rayleigh Ritz method is used to derive the eigenfrequency equation of the beam. Other than that, Mohamed Hussein Taha and Samir Abohadima [14] also have developed mathematical model for free vibrations of non uniform flexural beams. The resulted equations are solved by transformation to the Bessel equations to obtain mode shapes and natural frequencies.

3.2 Modeling

The design of the crane boom structure was analysed. The crane boom structure was modeled to a vibration model as a continuous system. The model is a solid tapered cantilever beam. The crane boom structure is fixed – free end and there is no slewing motion where the pedestal crane is fixed at one location without rotation of the base of the pedestal crane. The solid tapered cantilever beam at rest (0° angle) is analyzed first, to be as a baseline model for further analysis. Then, the solid tapered cantilever beam is raised to 55° angle for harmonic response analysis.

3.2.1 Modeling vibration model

Variable cross section beams are widely used as structural elements in engineering. This is because they help designers to improve the strength characteristics and overcome weight and geometrical restrictions. Figure 3.2 below shows the schematic diagram of the system. The crane boom structure is modeled as a solid tapered cantilever beam with linearly varying width, b_0 and depth, h_0 .

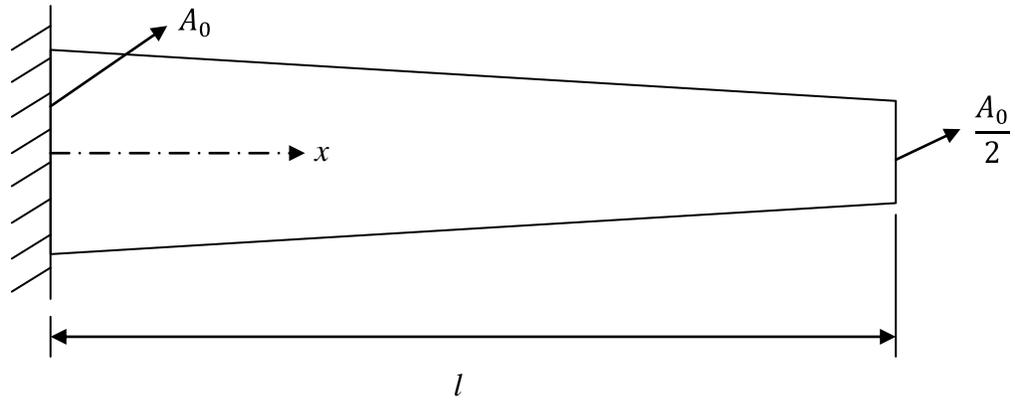


Figure 3.2: Front view of the schematic diagram of the system

Where:

l is the length of the beam,

A_0 is the cross section area of the beam,

b_0 is the width of the beam, and

h_0 is the thickness of the beam.

Variation of cross section area of the solid tapered cantilever beam is as follows:

$$A(x) = A_i + (A_j - A_i) \frac{x}{l} \quad (3.1)$$

$$A(x) = A_0 + \left(\frac{A_0}{2} - A_0 \right) \frac{x}{l} \quad (3.2)$$

$$A(x) = A_0 - \frac{A_0 x}{2l} \quad (3.3)$$

$$A(x) = A_0 \left(1 - \frac{x}{2l} \right) \quad (3.4)$$

Then, variation of moment of inertia of the solid tapered cantilever beam is as follows:

$$I(x) = I_i + (I_j - I_i) \frac{x}{l} \quad (3.5)$$

$$I(x) = \frac{b_0 h_0^3}{12} + \left(\frac{\frac{b_0 (h_0)}{2} \left(\frac{h_0}{2} \right)^3}{12} - \frac{b_0 h_0^3}{12} \right) \frac{x}{l} \quad (3.6)$$

$$I(x) = \frac{b_0 h_0^3}{12} - \left(\frac{5 b_0 h_0^3}{64} \right) \frac{x}{l} \quad (3.7)$$

$$I(x) = \frac{1}{4} b_0 h_0^3 \left(\frac{1}{3} - \frac{5x}{16l} \right) \quad (3.8)$$

$$I(x) = \frac{1}{4} (b_0 h_0) h_0^2 \left(\frac{1}{3} - \frac{5x}{16l} \right) \quad (3.9)$$

$$I(x) = \frac{(A_0) h_0^2}{4} \left(\frac{1}{3} - \frac{5x}{16l} \right) \quad (3.10)$$

3.2.2 Geometry modeling in CATIA V5R16

The solid tapered cantilever beam was modeled using CATIA V5R16 for simulation using ANSYS Workbench software. Figure 3.3 (a) shows the geometry model/structure of solid tapered cantilever beam at rest (0° angle) while figure 3.3 (b) shows the system at 55° angle. The properties of the geometry model are stated in Table 3.1.

Table 3.1: Mechanical properties of the geometry

PROPERTIES	DETAILS
Material type	Steel
Density	7850 kg/m ³
Modulus of Elasticity	200 GPa
Mass	11500 kg
Length	20 m

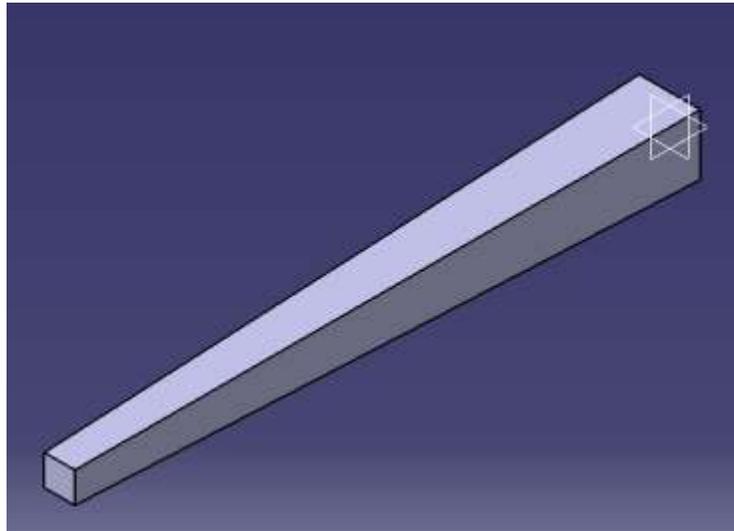


Figure 3.3 (a): Geometry model of the solid tapered cantilever beam at rest (0° angle)

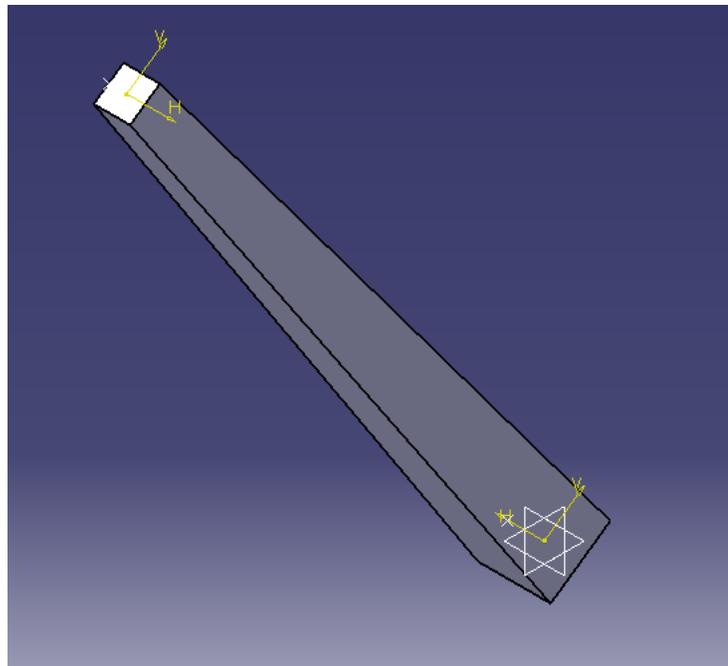


Figure 3.3 (b): Geometry model of the solid tapered cantilever beam at lift (55° angle)

3.2.3 Method used for Solving Continuous System

Several methods are used for solving the continuous system models which are exact solutions and approximate solutions. The exact solution consists of Newtonion method and energy method. Exact solutions are possible only in relatively few simple cases of continuous systems. The exact solutions are particularly difficult to find for two- and three-dimensional problems. Exact solutions are often desirable because they provide valuable insight into the behaviour of the system through ready access to the natural frequencies and mode shapes.

Most of the continuous system consider in several reference books have uniform mass and stiffness distribution and simple boundary conditions. However, some vibration problem may pose insurmountable difficulties either because the governing differential equation is difficult to solve or the boundary condition may be extremely difficult or impossible to satisfy. In such cases we may be satisfied with an approximate solution of the vibration problems.

The approximate methods can be classified into two categories. The first category is based on the expansion of the solution in the form of a finite series consisting of known functions multiplied by unknown function. The second category of methods is based on a simple lumping of system properties. All the approximate methods basically convert a problem described by partial differential equations into a problem described by a set of ordinary differential equations. There are two classes of methods that are based on series expansions: Rayleigh –Ritz methods and weighted residual methods.

3.2.4 Method used in this project

In this dynamic study, the crane boom structure is simplified to a solid tapered cantilever beam. Approximate solutions is more satisfied compared to exact solutions because of non uniform stiffness and mass distribution of the solid tapered cantilever beam. There are a lot of approximate methods that can be used. Rayleigh - Ritz method is chosen to be used in solving this dynamic analysis of the solid tapered cantilever beam.

Rayleigh – Ritz method is considered as an extension of Rayleigh’s method [7]. The method is based on the fact that Rayleigh’s quotient gives an upper bound for the first eigenvalue, $\lambda_1 = \omega_1^2$:

$$R(X(x)) \geq \lambda_1 \quad (3.11)$$

In the Rayleigh – Ritz method, the shape of deformation of the continuous system, $X(x)$, is approximated using trial family of admissible functions that satisfy the geometric boundary conditions of the problem:

$$(X(x)) = \sum_{i=1}^n c_i \phi_i(x) \quad (3.12)$$

Where c_1, c_2, \dots, c_n are unknown (constant) coefficients, also called Ritz coefficients, and $\phi_1(x), \phi_2(x), \dots, \phi_n(x)$ are the known trial family of admissible functions. The function $\phi_i(x)$ can be a set of assumed mode shapes, polynomials, or even eigenfunctions.

When eqn. (3.12) is substituted into the expression for Rayleigh’s quotient, R , Rayleigh’s quotient becomes a function of the unknown coefficients c_1, c_2, \dots, c_n :

$$R = R(c_1, c_2, \dots, c_n) \quad (3.13)$$

The coefficients c_1, c_2, \dots, c_n are selected to minimize Rayleigh’s quotient using necessary conditions:

$$\frac{\partial R}{\partial c_i} = 0, \quad i = 1, 2, \dots, n \quad (3.14)$$

Eqn. (3.14) denotes a system of n algebraic homogeneous linear equations in the unknown c_1, c_2, \dots, c_n . For the coefficients c_1, c_2, \dots, c_n to have a nontrivial solution, the determinant of the coefficient matrix is set equal to zero. This yields the frequency equation in the form of a polynomial in ω^2 of order n . The roots of the frequency equation provide the approximate natural frequencies of the system $\omega_1, \omega_2, \dots, \omega_n$. Using the approximate natural frequency ω_i in eqn. (3.4), corresponding approximate mode shape $c_1^{(i)}, c_2^{(i)}, \dots, c_n^{(i)}$ can be determined (for $i = 1, 2, \dots, n$).

3.3 Modal / Dynamic Analysis

From research and study of fundamental studies and journal, several sets of vibration analysis method were developed for solving the dynamic mathematical relations. During modeling process, the suitable method of finding all dynamic characteristic of harmonic response of the offshore crane boom structure was determined.

Approximate approach is chosen for solving this problem as the exact solution of eigenvalue problems of many continuous systems is difficult, sometimes impossible, either because of non uniform stiffness and mass distribution or because of complex boundary conditions. Rayleigh – Ritz method is used for solving dynamic analysis of the solid tapered cantilever beam. Solid tapered cantilever beam at rest (0° angle) is analyzed first, to be as a baseline model for further analysis.

3.3.1 Dynamic analysis of the tapered solid cantilevered beam at rest (0° angle) using Rayleigh Ritz Method

Variation of cross section area of the tapered solid cantilevered beam is:

$$A(x) = A_0 \left(1 - \frac{x}{2l}\right) \quad (3.15)$$

Variation of moment of inertia of the tapered solid cantilevered beam is:

$$I(x) = \frac{(A_0)h_0^2}{4} \left(\frac{1}{3} - \frac{5x}{16l}\right) \quad (3.16)$$

By using Rayleigh – Ritz method, following functions are used as trial functions:

$$\phi_1(x) = \sin \frac{\pi x}{2l}, \quad \phi_2(x) = \sin \frac{3\pi x}{2l}, \quad \phi_3(x) = \sin \frac{5\pi x}{2l}, \quad (3.17)$$

At free end beam, the boundary conditions are as follows:

$$EI \frac{\partial^2 w}{\partial x^2} = 0 \quad (3.18)$$

$$\frac{\partial}{\partial x} \left(EI \frac{\partial^2 w}{\partial x^2} \right) = 0 \quad (3.19)$$

The system is subjected to the initial conditions:

$$w(x, t = 0) = w_0(x) \quad (3.20)$$

$$\frac{\partial w}{\partial t}(x, t = 0) = \dot{w}_0(x) \quad (3.21)$$

The maximum kinetic energy can be found by assuming the transverse deflection function, $w(x, t)$ to be harmonic as:

$$w(x, t) = W(x) \cos \omega t \quad (3.22)$$

$$T_{max} = \frac{1}{2} \rho \omega^2 \int_0^l A(x) [W(x)]^2 dx \quad (3.23)$$

Since the maximum value of $w(x, t) = W(x)$, the maximum value of V is given by:

$$V_{max} = \frac{1}{2} E \int_0^l I(x) \left[\frac{d^2 W(x)}{dx^2} \right]^2 dx \quad (3.24)$$

By equating T_{max} to V_{max} , the Rayleigh quotient of the beam can be expressed as:

$$R(X(x)) = \lambda = \omega^2 = \frac{V_{max}}{T_{max}} \quad (3.25)$$

Therefore:

$$R = \omega^2 = \frac{V_{max}}{T_{max}} = \frac{\frac{1}{2} E \int_0^l I(x) \left[\frac{d^2 W(x)}{dx^2} \right]^2 dx}{\frac{1}{2} \rho \int_0^l A(x) [W(x)]^2 dx} \quad (3.26)$$

Using:

$$W(x) = \sum_{i=1}^3 c_i \phi_i(x) = c_1 \sin \frac{\pi x}{2l} + c_2 \sin \frac{3\pi x}{2l} + c_3 \sin \frac{5\pi x}{2l} \quad (3.27)$$

$$\frac{dW(x)}{dx} = \frac{c_1 \pi}{2l} \cos \frac{\pi x}{2l} + \frac{3c_2 \pi}{2l} \cos \frac{3\pi x}{2l} + \frac{5c_3 \pi}{2l} \cos \frac{5\pi x}{2l} \quad (3.28)$$

$$\frac{d^2 W(x)}{dx^2} = \frac{c_1 \pi^2}{4l^2} \cos \frac{\pi x}{2l} + \frac{9c_2 \pi^2}{4l^2} \cos \frac{3\pi x}{2l} + \frac{25c_3 \pi^2}{4l^2} \cos \frac{5\pi x}{2l} \quad (3.29)$$

Therefore:

$$V_{max} = \frac{1}{2} E \int_0^l I(x) \left[\frac{d^2 W(x)}{dx^2} \right]^2 dx \quad (3.30)$$

$$V_{max} = \frac{E}{2} \int_0^l \left[\frac{(A_0)h_0^2}{4} \left(\frac{1}{3} - \frac{5x}{16l} \right) \right] \left[\frac{c_1 \pi^2}{4l^2} \cos \frac{\pi x}{2l} + \frac{9c_2 \pi^2}{4l^2} \cos \frac{3\pi x}{2l} + \frac{25c_3 \pi^2}{4l^2} \cos \frac{5\pi x}{2l} \right]^2 dx \quad (3.31)$$

$$V_{max} = \frac{E(A_0)h_0^2}{1536l^3} [(17\pi^4 + 60\pi^2)c_1^2 + (1377\pi^4 + 270\pi^2)c_2^2 + (10625\pi^4 + 1500)c_3^2] \quad (3.32)$$

$$T_{max} = \frac{1}{2} \rho \int_0^l A(x) [W(x)]^2 dx \quad (3.33)$$

$$T_{max} = \frac{\rho}{2} \int_0^l A_0 \left(1 - \frac{x}{2l} \right) \left(c_1 \sin \frac{\pi x}{2l} + c_2 \sin \frac{3\pi x}{2l} + c_3 \sin \frac{5\pi x}{2l} \right)^2 dx \quad (3.34)$$

$$T_{max} = \frac{\rho A_0 l}{2} \left[c_1^2 \left(\frac{3}{8} - \frac{1}{2\pi^2} \right) + c_2^2 \left(\frac{3}{8} - \frac{1}{18\pi^2} \right) + c_3^2 \left(\frac{3}{8} - \frac{1}{50\pi^2} \right) + c_1 c_2 \left(\frac{3}{\pi^2} \right) - c_1 c_3 \left(\frac{3}{9\pi^2} \right) + c_2 c_3 \left(\frac{1}{\pi^2} \right) \right] \quad (3.35)$$

Rayleigh's quotient is given by:

$$R = \omega^2 = \frac{V_{max}(c_1, c_2, c_3)}{T_{max}(c_1, c_2, c_3)} \quad (3.36)$$

The necessary conditions for the minimization of R are given by:

$$\frac{\partial R}{\partial c_i} = \frac{T_{max} \frac{\partial V_{max}}{\partial c_i} - V_{max} \frac{\partial T_{max}}{\partial c_i}}{T_{max}^2} \quad (3.37)$$

Or

$$\frac{\partial V_{max}}{\partial c_i} - \frac{V_{max}}{T_{max}} \left(\frac{\partial T_{max}}{\partial c_i} \right) = \frac{\partial V_{max}}{\partial c_i} - \omega^2 \frac{\partial T_{max}}{\partial c_i} = 0, \quad i = 1, 2, 3 \quad (3.38)$$

Using:

$$\frac{\partial V_{max}}{\partial c_1} = (34 \pi^4 + 120 \pi^2)c_1 \quad (3.39)$$

$$\frac{\partial V_{max}}{\partial c_2} = (32754\pi^4 + 540 \pi^2)c_2 \quad (3.40)$$

$$\frac{\partial V_{max}}{\partial c_3} = (21250 \pi^4 + 3000 \pi^2)c_3 \quad (3.41)$$

$$\frac{\partial T_{max}}{\partial c_1} = \frac{\rho A_0 l}{2} \left[c_1 \left(\frac{3}{4} - \frac{1}{\pi^2} \right) + c_2 \left(\frac{1}{\pi^2} \right) - c_3 \left(\frac{1}{9\pi^2} \right) \right] \quad (3.42)$$

$$\frac{\partial T_{max}}{\partial c_2} = \frac{\rho A_0 l}{2} \left[c_2 \left(\frac{3}{4} - \frac{1}{9\pi^2} \right) + c_1 \left(\frac{1}{\pi^2} \right) + c_3 \left(\frac{1}{\pi^2} \right) \right] \quad (3.43)$$

$$\frac{\partial T_{max}}{\partial c_3} = \frac{\rho A_0 l}{2} \left[c_3 \left(\frac{3}{4} - \frac{1}{\pi^2} \right) - c_1 \left(\frac{1}{9\pi^2} \right) + c_2 \left(\frac{1}{\pi^2} \right) \right] \quad (3.44)$$

Eqn. 3.38 can be expressed as

$$\frac{E(A_0)h_0^2}{1536l^3} \begin{bmatrix} 34 \pi^4 + 120 \pi^2 & 0 & 0 \\ 0 & 32754\pi^4 + 540 \pi^2 & 0 \\ 0 & 0 & 21250 \pi^4 + 3000 \pi^2 \end{bmatrix} \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \end{Bmatrix} = \omega^2 \frac{\rho A_0 l}{2} \begin{bmatrix} \frac{3}{4} - \frac{1}{\pi^2} & \frac{1}{\pi^2} & -\frac{1}{9\pi^2} \\ \frac{1}{\pi^2} & \frac{3}{4} - \frac{1}{9\pi^2} & \frac{1}{\pi^2} \\ -\frac{1}{9\pi^2} & \frac{1}{\pi^2} & \frac{3}{4} - \frac{1}{\pi^2} \end{bmatrix} \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \end{Bmatrix} \quad (3.45)$$

Or

$$[k]\vec{c} = \lambda[m]\vec{c} \quad (3.46)$$

Where:

$$[k] = \begin{bmatrix} 34\pi^4 + 120\pi^2 & 0 & 0 \\ 0 & 32754\pi^4 + 540\pi^2 & 0 \\ 0 & 0 & 21250\pi^4 + 3000\pi^2 \end{bmatrix} \quad (3.47)$$

$$[m] = \begin{bmatrix} \frac{3}{4} - \frac{1}{\pi^2} & \frac{1}{\pi^2} & -\frac{1}{9\pi^2} \\ \frac{1}{\pi^2} & \frac{3}{4} - \frac{1}{9\pi^2} & \frac{1}{\pi^2} \\ -\frac{1}{9\pi^2} & \frac{1}{\pi^2} & \frac{3}{4} - \frac{1}{\pi^2} \end{bmatrix} \quad (3.48)$$

$$\lambda = \frac{768\omega^2\rho l^4}{E h_0^2} \quad (3.49)$$

Where:

$$\rho = 7850 \text{ kg/m}^3$$

$$l = 20\text{m}$$

$$E = 200 \text{ GPa}$$

$$h_0 = 2\text{m}$$

$$[[k] - \lambda_i^{(n)}[m]] \vec{c}^{(i)} = 0 \quad (3.50)$$

$$\lambda \begin{bmatrix} \left[\begin{array}{ccc} 34\pi^4 + 120\pi^2 & 0 & 0 \\ 0 & 32754\pi^4 + 540\pi^2 & 0 \\ 0 & 0 & 21250\pi^4 + 3000\pi^2 \end{array} \right] - \left[\begin{array}{ccc} \frac{3}{4} - \frac{1}{\pi^2} & \frac{1}{\pi^2} & -\frac{1}{9\pi^2} \\ \frac{1}{\pi^2} & \frac{3}{4} - \frac{1}{9\pi^2} & \frac{1}{\pi^2} \\ -\frac{1}{9\pi^2} & \frac{1}{\pi^2} & \frac{3}{4} - \frac{1}{\pi^2} \end{array} \right] \right] \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad (3.51)$$

Solving governing equation of motion above will get the value of eigenvalues, natural frequencies and Ritz coefficients as follow:

$$\vec{\lambda} = \begin{Bmatrix} 16.9 \\ 447.4 \\ 3326.3 \end{Bmatrix} \quad (3.52)$$

$$\vec{\omega} = \begin{Bmatrix} 3.7438 \\ 19.2627 \\ 52.5231 \end{Bmatrix} \quad (3.53)$$

$$\vec{c}^{(1)} = \begin{Bmatrix} -0.9875 \\ -0.1566 \\ 0.0168 \end{Bmatrix} \quad (3.54)$$

$$\vec{c}^{(2)} = \begin{Bmatrix} -0.0022 \\ 0.9867 \\ 0.1627 \end{Bmatrix} \quad (3.55)$$

$$\vec{c}^{(3)} = \begin{Bmatrix} 0.0001 \\ -0.0255 \\ 0.9997 \end{Bmatrix} \quad (3.56)$$

The transverse deflection function, $w(x, t)$ is to be harmonic as

$$w(x, t) = W(x) \cos \omega t \quad (3.57)$$

Where:

$$W(x) = \sum_{i=1}^3 c_i \phi_i(x) = c_1 \sin \frac{\pi x}{2l} + c_2 \sin \frac{3\pi x}{2l} + c_3 \sin \frac{5\pi x}{2l} \quad (3.58)$$

Therefore:

For $\omega_1 = 3.7438 \text{ Hz}$;

$$w_1(x, t) = -0.9875 \sin \frac{\pi x}{2l} - 0.0022 \sin \frac{3\pi x}{2l} + 0.0001 \sin \frac{5\pi x}{2l} (\cos 3.7438t) \quad (3.59)$$

For $\omega_2 = 19.2627 \text{ Hz}$;

$$w_2(x, t) = -0.1566 \sin \frac{\pi x}{2l} + 0.9867 \sin \frac{3\pi x}{2l} - 0.0255 \sin \frac{5\pi x}{2l} (\cos 19.2627t) \quad (3.60)$$

For $\omega_3 = 52.5231 \text{ Hz}$;

$$w_3(x, t) = 0.0168 \sin \frac{\pi x}{2l} + 0.1627 \sin \frac{3\pi x}{2l} + 0.9997 \sin \frac{5\pi x}{2l} (\cos 52.5231t) \quad (3.61)$$

3.3.2 Modal analysis of the solid tapered cantilever beam at rest (0° angle) using Simulation in ANSYS Workbench software

ANSYS is a general purpose finite element modeling package for numerically solving a wide variety of mechanical problems. The problems include: static or dynamic structural analysis (linear and non – linear), heat transfer and fluid problems, as well as acoustic and electromagnetic problems. It permits an evaluation or simulation of design without having to build and destroy multiple prototypes in testing. The ANSYS software is capable of simulating modal analysis and harmonic response analysis of the solid tapered cantilever beam. There are several stages involved during the process of simulating the structure. The stages and description are stated as below:

i. *Beginning of the Analysis*

The analysis was started by importing the geometry model/structure that was developed using CATIA V5R16 into ANSYS Workbench. Figure 3.4 shows the beginning interface of ANSYS workbench.

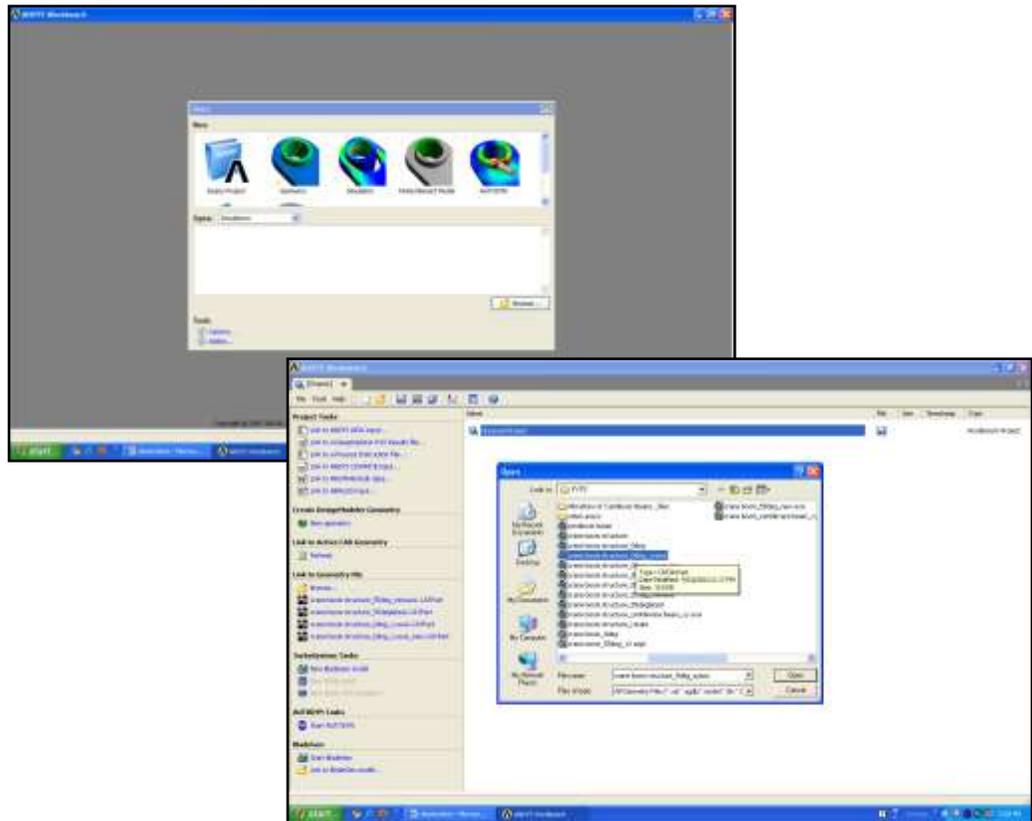


Figure 3.4: The beginning interface of ANSYS Workbench

ii. *Meshing the geometry*

The geometry of the solid tapered cantilevered beam was meshed first before proceeding to modal analysis. Meshing the structure allows the actual structure to be divided into several pieces elements, each of which is assumed to behave as continuous structural member called a finite element. Figure 3.5 shows the complete meshing of the geometry model.

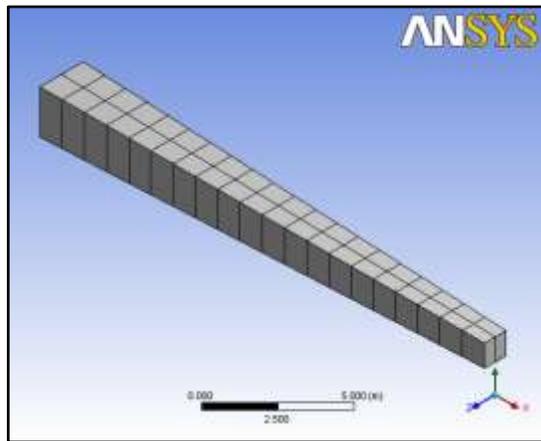


Figure 3.5: Geometry model with complete meshing

iii. *Modal Analysis*

After meshing process, the geometry model/structure was converted to simulation and modal analysis was done on the geometry/structure. There are several types of analysis in the ANSYS Workbench software; Static Structural, Flexible Dynamic, Rigid Dynamic, Harmonic Response, Modal, Linear Buckling, Random Vibration and Shape Optimization. Modal analysis was selected to be simulated to solve eigenvalue problem to get the natural frequencies and mode shapes of the model/structure. Boundary conditions and initial conditions were determined before solving the modal analysis. Result of natural frequencies and mode shapes obtained from the modal analysis will be compared with the result obtained from the approximate solution. This modal analysis was done on the solid tapered cantilevered beam at rest (0° angle) which will be act as baseline model for modal analysis and harmonic response analysis of the solid tapered cantilevered beam at lift (55° angle).

3.4 Validation

The modal/dynamic analysis of the solid tapered cantilevered beam at rest (0° angle) was done in two ways; approximate solution and simulation in ANSYS Workbench. The data and result of natural frequencies and mode shapes of the model/structure was evaluated and compared. Then, the solid tapered cantilevered beam at rest (0° angle) was act as a baseline model for further analysis and harmonic response analysis.

3.5 Harmonic Response Analysis

In a structural system, any sustained cyclic load will produce a sustained cyclic or harmonic response. During the real operation of the pedestal crane, the crane boom structure will be lifted to a certain angle to lift payloads from a boat to a fixed platform or vice versa. In this analysis, the crane boom structure is lifted to 55° angle lift and static at that angle lift without any further luffing motion (the crane boom move up and down) and slewing (rotation) motion. The hoisting speed for picking up or lowering down the payload provides sustained cyclic or dynamic loads to the crane system. The loading is only by the payload without any excitation by the wind load.

Modal analysis was simulated first on the solid tapered cantilever beam at 55° lift angle to find the natural frequencies and mode shape of the crane boom structure at certain lift angle. Then, harmonic response analysis was simulated to find the harmonic response of the crane boom structure and also to find maximum amplitude of deformation and acceleration of the crane boom structure when exposed to a harmonic load. The amount of payload is varied; 4 tones, 8 tones, 12 tones, 16 tones, 20 tones to view the effect of loading on the crane boom structure.

3.6 Final Report

Lastly, all result obtained from the approximate solution and simulation using ANSYS were evaluated. Discussion on the result will show the important of studying dynamic characteristics of the crane boom structure under excitation by the payload.

CHAPTER 4

RESULT & DISCUSSION

4.1 Natural Frequencies and Mode Shapes

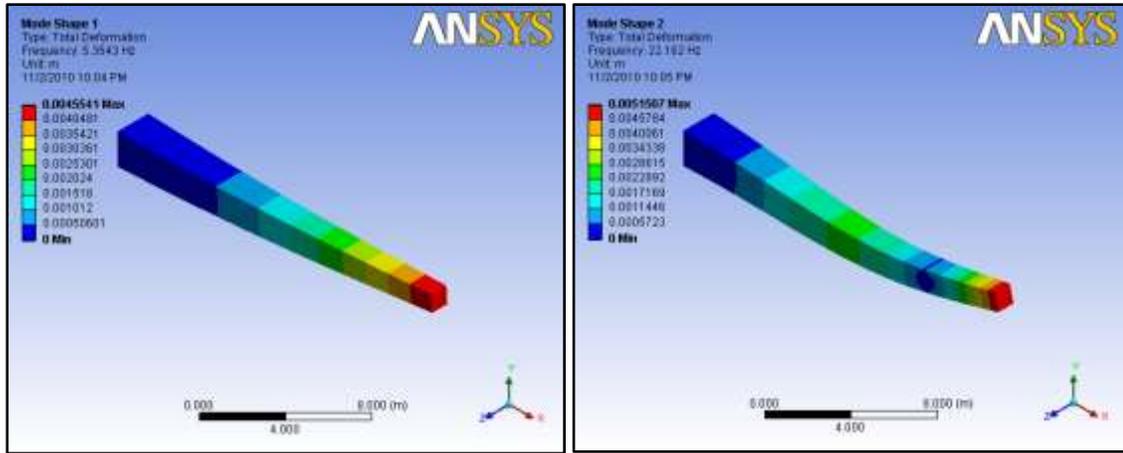
One of the important considerations for crane design and operation is the dynamic characteristics of the crane structure without payload which is the free vibration of the crane boom structure. Table 4.1 shows the natural frequencies for three different mode of the crane boom structure at rest (0° angle).

Table 4.1: Natural frequencies of the crane boom structure at rest (0° angle)

Mode	Frequency (Hz): Approximate solution	Frequency (Hz): Simulation in ANSYS Workbench	Error percentage (%)
1	3.7438	5.3543	30.1
2	19.2627	22.162	13.1
3	52.5231	53.232	1.3

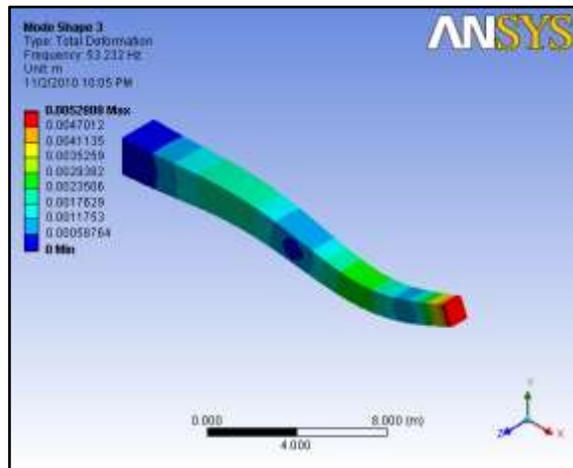
The natural frequency is different for associated mode. As mode, N is increased, the frequency will be increased. Figure 4.1 shows the mode shapes of the crane boom structure at rest (0° angle) using simulation in ANSYS Workbench. A mode shape is a specific pattern of vibration executed by a mechanical system at a specific frequency [15]. Mark H. Richardson stated that the conceptual conclusions that we can make regarding modes are [16]:

1. Modes are unique and inherent to any structure.
2. No external loads or forces are required to define modes.
3. Modes will only change if the mass, damping, or stiffness properties of the structure are changed. Changes in boundary are also reflected by changes in the mass matrix [M], damping matrix [C] and stiffness matrix, so modes will also change if the boundary conditions change.



(a)

(b)



(c)

Figure 4.1: Mode shape of the crane boom structure at rest (0° angle); (a) Mode shape 1, (b) Mode shape 2, (c) Mode Shape 3

The results of mode shapes are unique and there are no external loads or forces to define those modes. For solid tapered cantilever beam, cross section area $A(x)$ and moment of inertia $I(x)$ is varied along x axis. The mode shapes that resulted are different with the mode shapes for uniform cantilever beam. Basically, natural frequencies depend on the geometry, the boundary conditions (method of support or attachment), and the masses of the components and the strength of the restoring forces or moments.

Before simulating harmonic response analysis, modal analysis was done on the crane boom structure at certain angle lift (55° angle). Table 4.2 shows the difference between natural frequencies for crane boom structure at rest (0° angle) and at 55° angle lift.

Table 4.2: Difference between natural frequencies for crane boom structure at rest (0° angle) and at 55° angle lift

Mode	Natural Frequencies (Hz) at 0° angle	Natural Frequencies (Hz) at 55° angle
1	5.3543	5.3549
2	22.162	22.165
3	53.232	53.240

The result in Table 4.2 shows a little bit changes of natural frequencies of the offshore crane boom structure at different lift angle. From the point of view of dynamics, lift angle changes both the stiffness and inertia distribution of the whole structure and thus its effect on the dynamic properties mode-dependent.

Figure 4.2 shows the mode shapes of the crane boom structure at 55° lift angle using simulation in ANSYS Workbench.

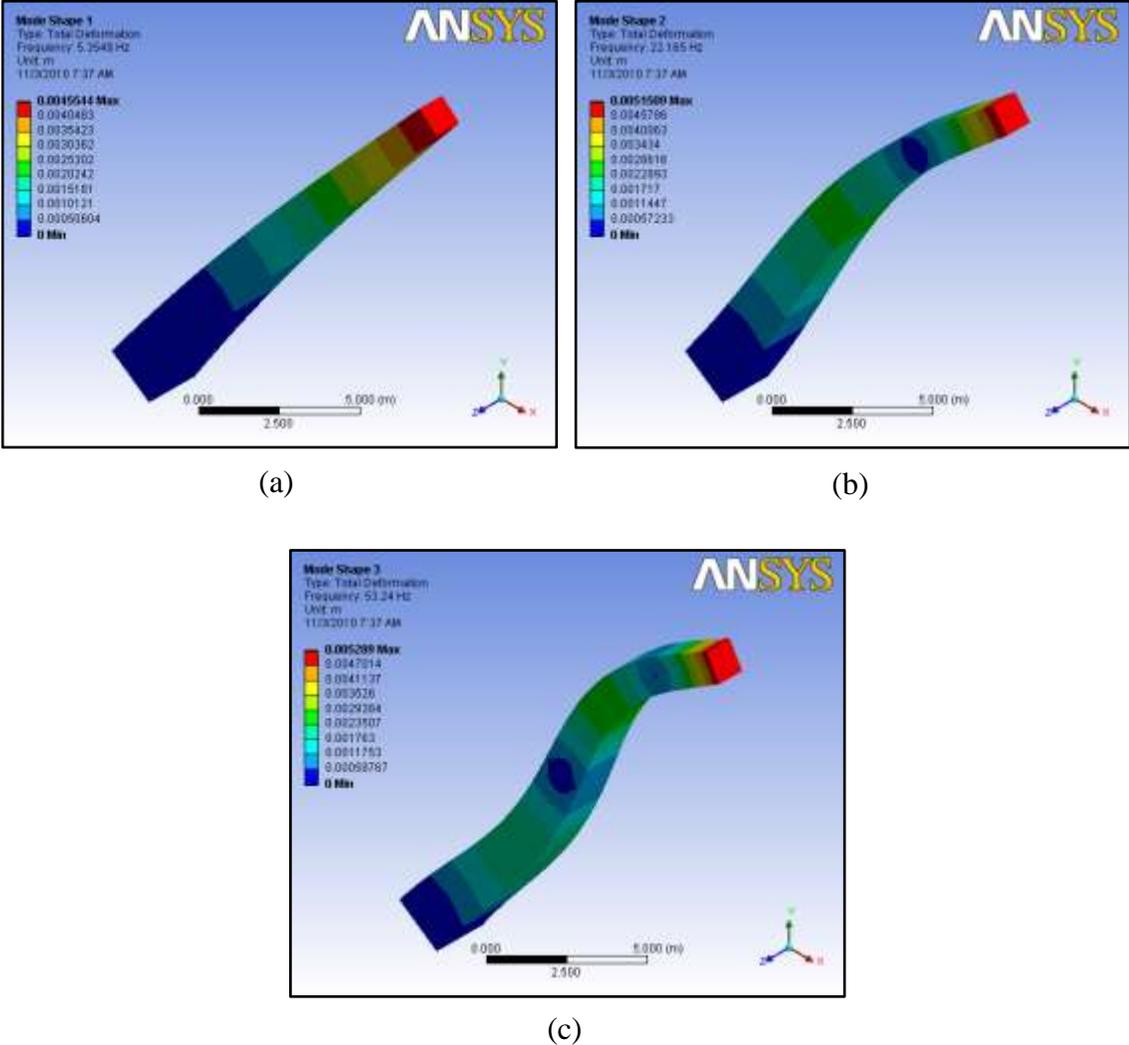


Figure 4.2: Mode shape of the crane boom structure at lift (55° angle); (a) Mode shape 1, (b) Mode shape 2, (c) Mode Shape 3

4.2 Harmonic Response

Any excitation or impulse of acceleration is fundamental important both in spectral analysis of the dynamic system and in dynamic responses in structures. In this harmonic response analysis using ANSYS Workbench, all loads as well as the structure's response varies sinusoidally at the same frequency. This typical harmonic analysis have calculated the response of the crane boom structure to cyclic loads over a frequency range and obtained a graph of response quantities (Displacement or deformation and acceleration) versus frequency.

4.2.1 Deformation of the Structure

In crane operations, the hoisting speed for picking up or lowering down the payload provides sustained cyclic or dynamic loads to the crane system. Figure 4.3 shows the frequency response of the amplitude of the crane deformation at different amount of payload. From the result, it shows that the maximum deformations are at three 'peak' frequencies which are same with the natural frequencies; 5.3549 Hz, 22.165 Hz and 53.240 Hz.

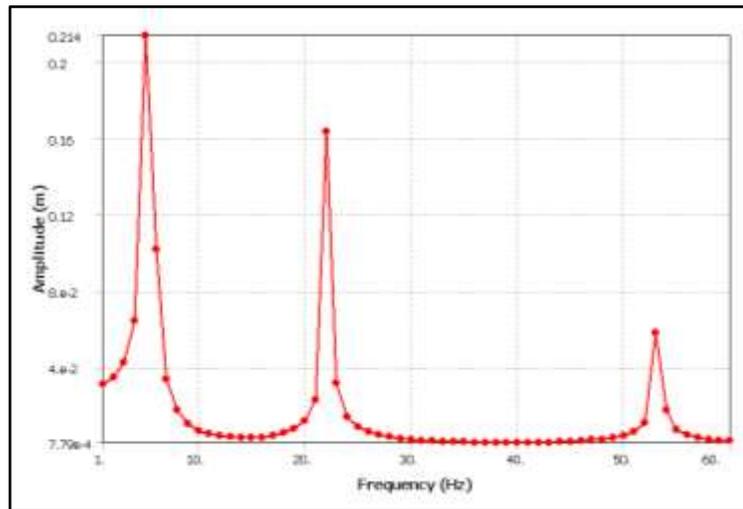


Figure 4.3 (a): Deformation amplitude of the structure excited by 4 Tones payload

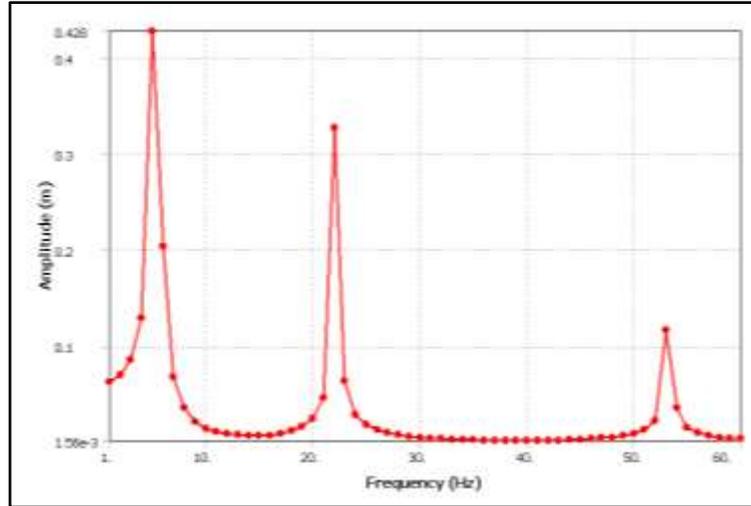


Figure 4.3 (b): Deformation amplitude of the structure excited by 8 Tones payload

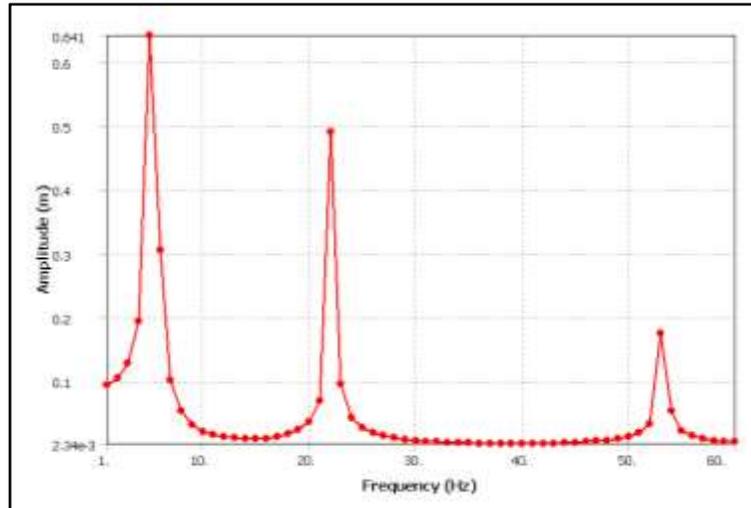


Figure 4.3 (c): Deformation amplitude of the structure excited by 12 Tones payload

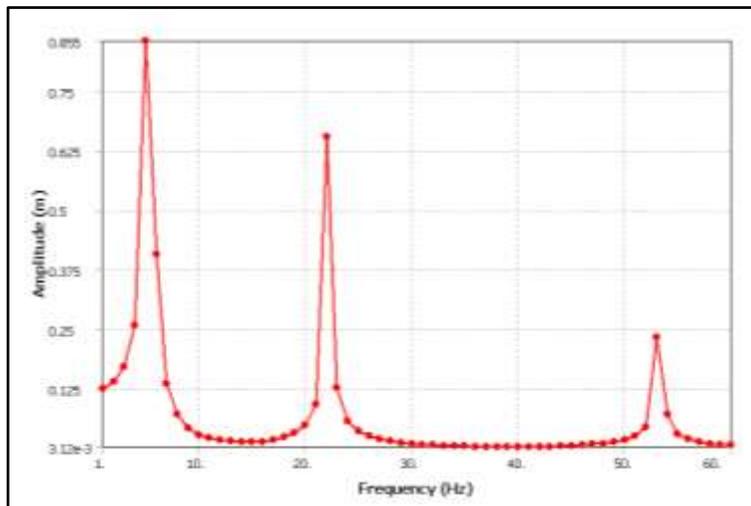


Figure 4.3 (d): Deformation amplitude of the structure excited by 16 Tones payload

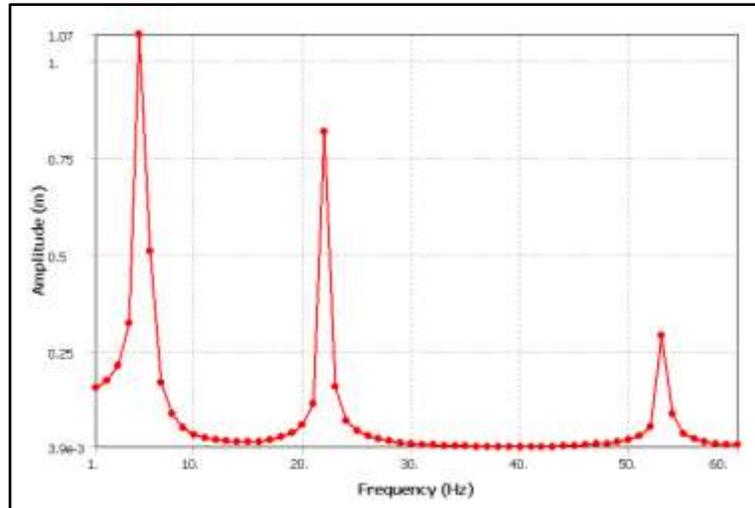


Figure 4.3 (e): Deformation amplitude of the structure excited by 20 Tones payload

The result shows the effect of payload on the dynamic characteristics of the crane boom structure. As the amount of payload increases, the maximum amplitude will also be increased. The harmonic load was expressed as

$$F \cos \omega t \quad (4.1)$$

Where:

F is the amount of payload,

ω is the frequency of hoisting speed, and

t is time.

Therefore, as amount of force increase, the amount of harmonic load will increase and it will affect the total frequency response of the crane boom structure. The graph of the amplitude of the structure's deformation represented the dynamic characteristics of the crane boom structure that was excited by the varying amounts of payload. The graph also represented the characteristics of the three first mode shapes. Three 'peak' frequencies represented the maximum amplitude of deformation at the natural frequencies. Based on resonance concept, resonance is the buildup of large vibration amplitude that occurs when a structure or an object is excited at its natural frequency [8]. The result satisfied the theory.

From the result, we can view that the amplitude of deformation at the first mode/frequency is higher compared to the amplitude of deformation at second and third mode. This can be explained by the description of the unique mode shapes. For the first mode shape, the deformation is based on one loop only while for second and third mode, the maximum amplitude is shared by 2 or more loop. That is why we can see that there are two or more amplitude of deflection (loop) for second and third mode. This can be viewed in figure 4.4 below:

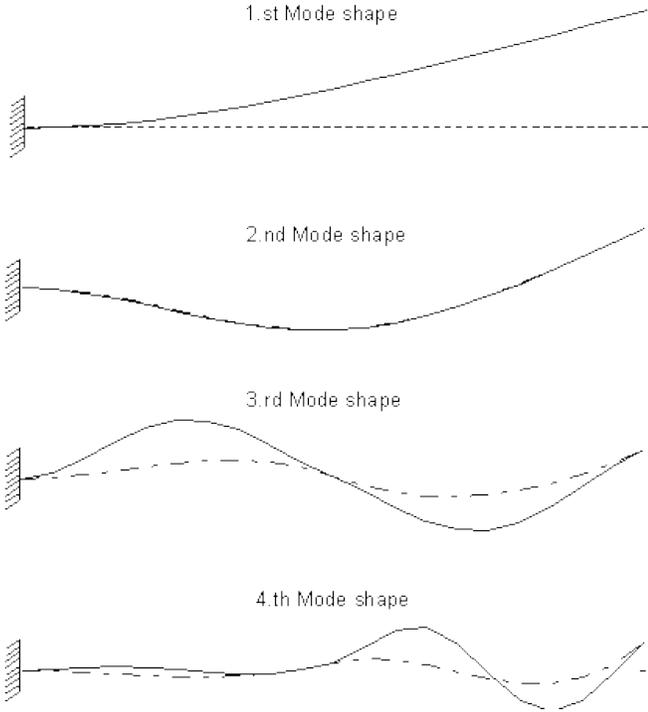


Figure 4.4: The mode shape for a cantilever beam

For the mode shape, when the time is varied; all particles along the solid tapered cantilever beam are oscillating in the same frequency and in phase (reaching the equilibrium point together) but each has different amplitude.

4.2.2 Acceleration of the Structure

Based on theory stated by Rao [17], by using complex number representation, a harmonic response can be represented as

$$\vec{X} = Ae^{i\omega t} \quad (4.2)$$

Where:

A is the amplitude,

ω is circular frequency (rad/sec), and

t is time.

The differentiation of the harmonic response with respect to time gives

$$\frac{d\vec{X}}{dt} = \frac{d}{dt}(Ae^{i\omega t}) = i\omega Ae^{i\omega t} = i\omega\vec{X} \quad (4.3)$$

$$\frac{d^2\vec{X}}{dt^2} = \frac{d}{dt}(i\omega Ae^{i\omega t}) = -i\omega^2 Ae^{i\omega t} = -\omega^2\vec{X} \quad (4.4)$$

From the equation 4.4, we can see that the acceleration is dependent on the quadratic value of frequency. As frequency increase by quadratic value, the acceleration of the structure will accelerate at high acceleration. The result of structure's acceleration shows that the 'peak' acceleration are at three 'peak' frequencies which are same with the natural frequencies; 5.3549 Hz, 22.165 Hz and 53.240 Hz. The results also satisfied the resonance concept.

Figure 4.5 shows the response quantities of acceleration at different amounts of payload. We can see that that three peak frequencies shows that the maximum acceleration of the crane boom structure at three different mode shape. The maximum amplitude of acceleration at third mode is higher that acceleration at first and second mode. This is because the acceleration is increasing by quadratic value of frequency. That satisfied the theory of equation 4.4.

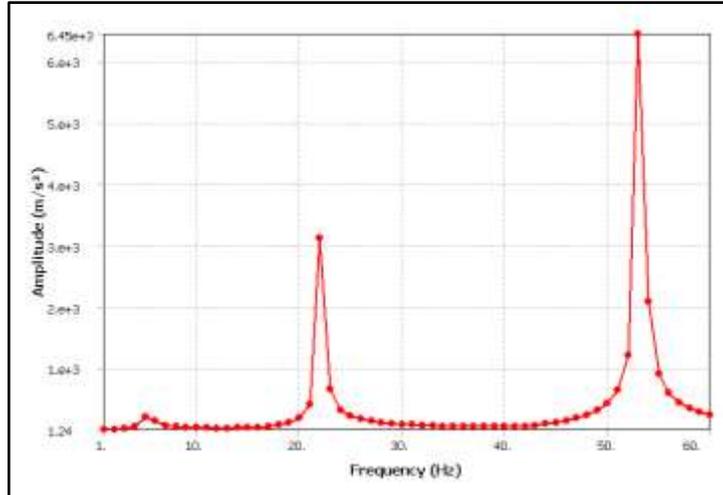


Figure 4.5 (a): Acceleration amplitude of the structure excited by 4 Tones payload

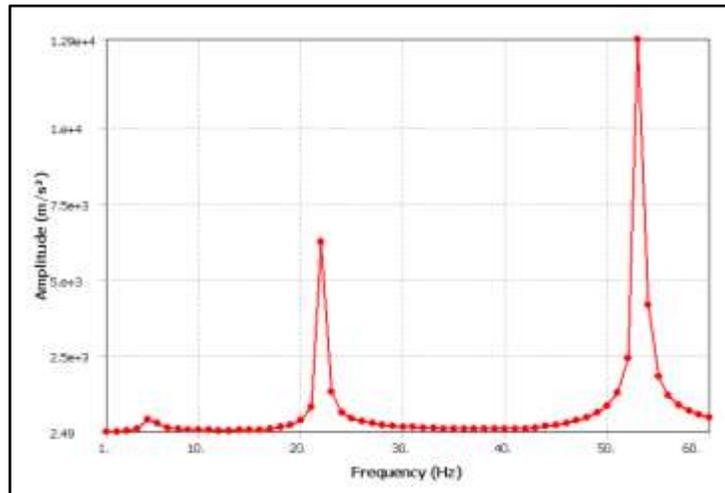


Figure 4.5 (b): Acceleration amplitude of the structure excited by 8 Tones payload

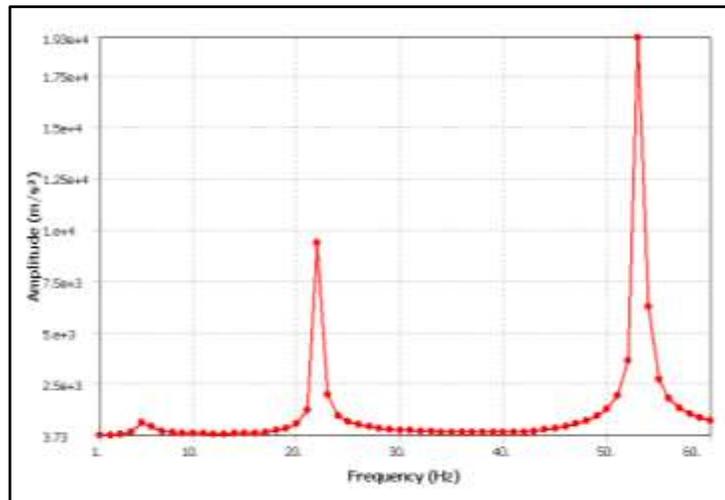


Figure 4.5 (c): Acceleration amplitude of the structure excited by 12 Tones payload

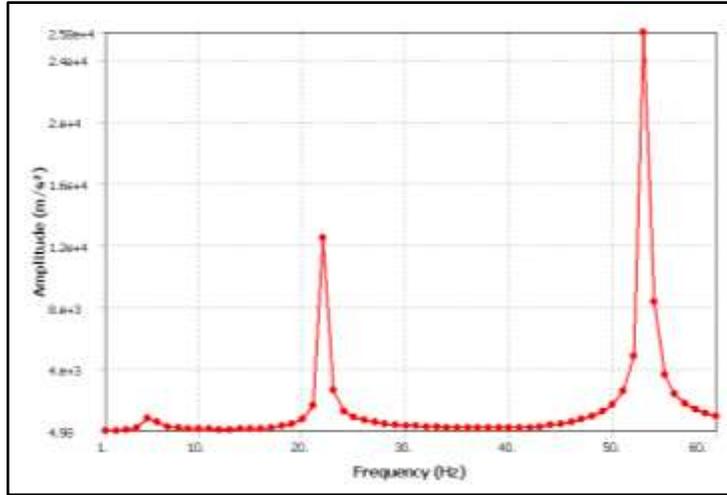


Figure 4.5 (d): Acceleration amplitude of the structure excited by 16 Tones payload

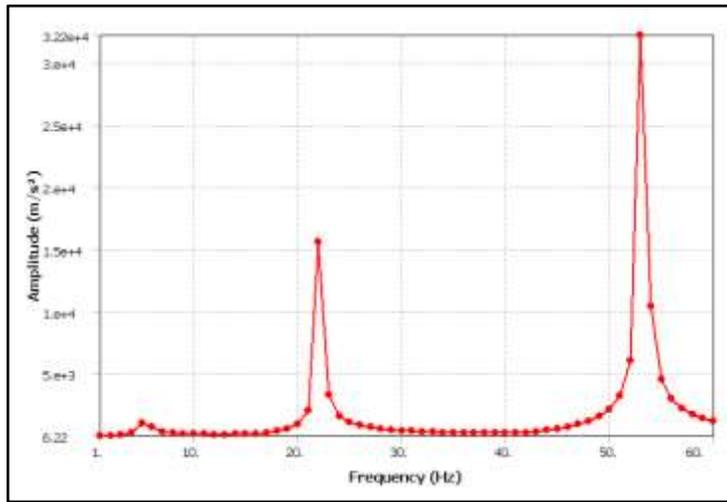


Figure 4.5 (e): Acceleration amplitude of the structure excited by 20 Tones payload

There are four main points regarding this study of harmonic response of the offshore crane boom structure:

- 1) The natural frequency is different for associated mode. As mode, N is increased, the frequency will be increased.
- 2) A little bit changes of natural frequencies of the offshore crane boom structure at different lift angle. This is because lift angle changes both the stiffness and inertia distribution of the whole structure and thus its effect on the dynamic properties mode-dependent.
- 3) The varying amount of payload has affected the dynamic characteristics of the crane boom structure. As the amount of payload increases, the maximum amplitude of structure's deformation and acceleration will also be increased.
- 4) The result of structure's deformation and acceleration in harmonic response analysis shows that the 'peak' deformation and acceleration are at three 'peak' frequencies which are same with the natural frequencies; 5.3549 Hz, 22.165 Hz and 53.240 Hz. The results satisfied the resonance concept.

CHAPTER 5

CONCLUSION AND RECOMMENDATION

In a conclusion, this project is a comprehensive research study about harmonic response of the offshore crane boom structure. The project is related to the study on the dynamic characteristics of the offshore crane boom structure upon excitation by the payload. The hoisting speed for picking up or lowering down the payload provides sustained cyclic or dynamic loads to the crane system. Harmonic analysis results are used to determine the steady state response of a structure to loads that vary sinusoidally (harmonically) with time, thus enabling us to overcome resonance, fatigue and other harmful effects of forced vibrations.

From the study, we could find the values of the natural frequency, the harmonic response on the crane boom and the mode shape under the influence of hoisting speed for picking up or lowering down the payload. The variation amounts of the payload have influenced the harmonic response of the crane boom structure. The dynamic analysis was done by approximate solution and simulation using ANSYS Workbench software. By using approximate solution, the mathematical model was developed based on simplified solid tapered cantilevered beam geometry using Rayleigh-Ritz method. The analysis was firstly done on free vibration of the structure at 0° angle where that analysis will be act as a baseline model. The result obtained was compared with the result obtained from the simulation using ANSYS Workbench software.

Then, the dynamic analysis was done on the structure at 55° angle to view the effect of angle lift to the dynamic characteristics of the crane boom structure. Harmonic response analysis was done to the structure with varying the amount of payload. The natural frequencies of the crane boom structure at 55° angle are 5.3549 Hz, 22.165 Hz and 53.240 Hz. Harmonic response result shows that the maximum amplitude of the deformation and acceleration of the crane boom structure are at the same frequencies with the natural frequencies. That conclusion satisfied the concept of resonance. This dynamic characteristic is really important in design and operation of the crane boom structure.

Recommendation

This research study can be expanded to investigate the dynamic characteristics of the crane boom structure under following condition:

- The swinging of the payload

Increasing the amounts of payload naturally will cause the swinging of the payload during picking up or lowering down the payload. The swinging of the payload will be critical if the crane operation is exposed to adverse wind condition. In this project, the amounts of payload were varied with assumption that the payload is not swinging and without any excitation by the wind load.

- Exposed to adverse wind condition

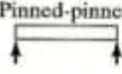
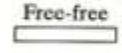
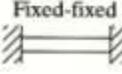
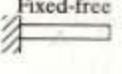
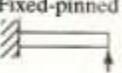
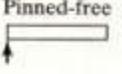
The crane operation naturally exposed to the adverse wind condition. This is because the location of the fixed platform at the middle of the sea. Wind and storms can influence the entire operation of cranes and can even destroy the whole crane. This show the important of study on harmonic response of the crane boom structure due to excitation by the wind load. In this project, the analysis was done with assumption that there are no wind load to excite the crane boom structure.

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APPENDIX 1

End Conditions of Beam	Frequency Equation	Mode Shape (Normal Function)	Value of $\beta_n l$
 Pinned-pinned	$\sin \beta_n l = 0$	$W_n(x) = C_n[\sin \beta_n x]$	$\beta_1 l = \pi$ $\beta_2 l = 2\pi$ $\beta_3 l = 3\pi$ $\beta_4 l = 4\pi$
 Free-free	$\cos \beta_n l \cdot \cosh \beta_n l = 1$	$W_n(x) = C_n[\sin \beta_n x + \sinh \beta_n x + \alpha_n (\cos \beta_n x + \cosh \beta_n x)]$ where $\alpha_n = \left(\frac{\sin \beta_n l - \sinh \beta_n l}{\cosh \beta_n l - \cos \beta_n l} \right)$	$\beta_1 l = 4.730041$ $\beta_2 l = 7.853205$ $\beta_3 l = 10.995608$ $\beta_4 l = 14.137165$ ($\beta l = 0$ for rigid body mode)
 Fixed-fixed	$\cos \beta_n l \cdot \cosh \beta_n l = 1$	$W_n(x) = C_n[\sinh \beta_n x - \sin \beta_n x + \alpha_n (\cosh \beta_n x - \cos \beta_n x)]$ where $\alpha_n = \left(\frac{\sinh \beta_n l - \sin \beta_n l}{\cos \beta_n l - \cosh \beta_n l} \right)$	$\beta_1 l = 4.730041$ $\beta_2 l = 7.853205$ $\beta_3 l = 10.995608$ $\beta_4 l = 14.137165$
 Fixed-free	$\cos \beta_n l \cdot \cosh \beta_n l = -1$	$W_n(x) = C_n[\sin \beta_n x - \sinh \beta_n x - \alpha_n (\cos \beta_n x - \cosh \beta_n x)]$ where $\alpha_n = \left(\frac{\sin \beta_n l + \sinh \beta_n l}{\cos \beta_n l + \cosh \beta_n l} \right)$	$\beta_1 l = 1.875104$ $\beta_2 l = 4.694091$ $\beta_3 l = 7.854757$ $\beta_4 l = 10.995541$
 Fixed-pinned	$\tan \beta_n l - \tanh \beta_n l = 0$	$W_n(x) = C_n[\sin \beta_n x - \sinh \beta_n x + \alpha_n (\cosh \beta_n x - \cos \beta_n x)]$ where $\alpha_n = \left(\frac{\sin \beta_n l - \sinh \beta_n l}{\cos \beta_n l - \cosh \beta_n l} \right)$	$\beta_1 l = 3.926602$ $\beta_2 l = 7.068583$ $\beta_3 l = 10.210176$ $\beta_4 l = 13.351768$
 Pinned-free	$\tan \beta_n l - \tanh \beta_n l = 0$	$W_n(x) = C_n[\sin \beta_n x + \alpha_n \sinh \beta_n x]$ where $\alpha_n = \left(\frac{\sin \beta_n l}{\sinh \beta_n l} \right)$	$\beta_1 l = 3.926602$ $\beta_2 l = 7.068583$ $\beta_3 l = 10.210176$ $\beta_4 l = 13.351768$ ($\beta l = 0$ for rigid body mode)