

CERTIFICATION OF APPROVAL

Studies and Simulation of Five Link Suspension Systems

By

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Approved by,

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CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

SYAHRUL EHWAN BIN LAMAT

ABSTRACT

This report discuss of a project to study and simulate five link suspension that now many car manufacturer will implement it for their car. In this report there will be four chapter and followed by conclusion, references, and appendices.

In the first chapter, the author provides an introduction about the project background and the problem statement. The author also includes the objective and scope of study for the project in introduction part. After the introduction, in chapter two the author explains the literature review. It is about brief introduction on suspension system that has various kind of this system. In addition the author also explains about five link suspension system on it history, mechanism and others. Besides, there also will be a brief introduction on MATLAB Simulink software that will be used for the simulation.

For chapter three, the author will explain on the methodology of this project. The author provides the process flow for project planning for FYP1 and FYP2 and also the Gantt chart that include the milestone of both respectively. In this chapter, the author will discuss the detail step taken by the author to complete this project.

Now comes to the fourth chapter, there will be the result and discussion of the progress of the project. In this chapter, the author will write about findings on five link suspensions, kinematic analysis, the base model of the simulation which is for quarter car model, simulation of different design, and last the advantages and disadvantages of five link suspension system

Lastly, there will be the conclusion and reference that the author use in doing this project.

ACKNOWLEDGEMENT

Praise to Almighty God, because of giving me this opportunity to complete this project. Then, to my supervisor, Dr. Vu TrieuMinh, thanks a lot for the advice, help, comment, and useful tools to me along the progress of the project until the project completed.

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CHAPTER 1

INTRODUCTION

1. PROJECT BACKGROUND

Nowadays lots of people around the world demand for a high quality car. One of the specifications that we need for a high quality car is the suspension system that will give a good driving quality. Besides comfortable, a good suspension also can give us excellent steer handling. In order to produce a car with this specification, a lot of automotive engineers and vibrations engineers work hard doing research on car suspensions. And now they almost get the answer to that problem which is using five link suspension systems.

Throughout this project the author will study and simulate on this five link suspensions type to know further about this technology and how it works. Furthermore, the author also ready to improve this five link suspension systems on the structural positions of the link of the suspensions. MATLAB is one of the software chosen by the author to do this study and simulations project.

2. PROBLEM STATEMENT

Modeling the suspension of an automobile is of interest for many automotive and vibrations engineers. Of importance for these engineers are the ride qualities of the vehicle traversing over broken roads and control of body motion. When traveling over rough terrain, the vehicle exhibits bounce (up and down), pitch (rotation about the center of gravity along the vehicle's length) and roll (rotation about the center of gravity along the vehicle's width) motions. For this project, the bounce and pitch motion of the car over rough roads are of interest and will be analyzed in this report.

3. OBJECTIVE

3.1. Final Year Project Objective

The purpose of the project is to develop a framework, which will enhance students' skills in the process of applying knowledge, expanding thoughts, solving problems independently and presenting findings through minimum guidance and supervision.

3.2. Project Objective

There are five objectives that the author aim in this project and they are listed as below:

- a. To study the concept and mechanism of five link suspensions.
- b. To fomulate a mathematical model of five link suspensions
- c. To simulate five link suspension simuation using MATLAB
- d. To prove a critical problem of positining in five link suspension
- e. To list all the advantages and disadvantages of five link suspension

4. Scope of Study

In this project, the author has interest in studying the orientation and position of the linkages, road surface, velocity, and acelerations. The author also will doing research on fomulate the equations or mathematical model for five link suspension system and proceed on using MATLAB to produce the simulations. Throughout this project the author will have some assumptions for the calculation and simulations which as below.

- 1. Material of the strut bar are constant
- 2. The model of car is a quater car model
- 3. The link lenght are vary.
- 4. The weight of the strut bar are constant

CHAPTER 2

LITERATURE REVIEW

Suspension is the term given to the system of springs, shock absorbers and linkages that connects a vehicle to its wheels. Suspension systems serve a dual purpose – contributing to the car's roadholding/handling and braking for good active safety and driving pleasure, and keeping vehicle occupants comfortable and reasonably well isolated from road noise, bumps, and vibrations, etc. These goals are generally at odds, so the tuning of suspensions involves finding the right compromise. It is important for the suspension to keep the road wheel in contact with the road surface as much as possible, because all the forces acting on the vehicle do so through the contact patches of the tyres. The suspension also protects the vehicle itself and any cargo or luggage from damage and wear. The design of front and rear suspension of a car may be different.

1. SUSPENSION SYSTEMS

Suspension system used in an automobile is a system mediating the interface between the vehicle andthe road, and their functions are related to a wide range of drivability such as handing ability, stability, comfortability and so forth (Dixon, 1996). Since total optimization of such contents requires much of design freedom, a multi-link suspension system, that is rincipally a parallel six-bar universal linkage, is getting installed to passenger cars, mainly to high-grade cars. On the other hands, such design freedom makes the design process for determining link geometry, etc. more complicated, and it is not so easy to design the suspension system with promising insights. This leadsthe necessity of a new generation of design methodology that can realize a potential of the complicated system toward total optimality.

2. FIVE LINK / MULTI LINK SUSPENSION

The five-link suspension mechanism was first introduced by Deimler-Benz on their W201 andW124 series under the name "multilink suspension" (Fig. 1). Ever since hasbeen successfullyimplemented both in independent suspension systems and in rear axle guiding mechanisms bymany automobile manufacturers. Due to the larger number of design parameters, it has the capability of fulfilling better the complex kinematic and dynamic requirements imposed on suspensionsystems of today's automobiles. It is however much more difficult to synthesize than anyther suspension mechanism, due to its general spatial configuration. In case of multilink front suspension, the design problem is even more complex due to the fact that the kingpin is a virtual one corresponding to the momentary screw axis of the wheel carrier performing the steering motion relative to the chassis.



Figure 1: Five Link Suspension

The rear independent wheel or axle guiding mechanism(s) are, in the sense of mechanism theory, spatial motion generators (also known as rigid body guidance mechanisms). Research on motion generators synthesis and analysis has been carried out on both abstract and applied mechanisms by many researchers in the past. A general formulation of the mechanism synthesis problem for path, function and rigid body guidance based on optimization techniques is proposed by Aviles et al.

According to the authors, a global error function to be minimized is defined as a weighted-sum of some local error functions, previously minimized with respect to the Cartesian coordinates of the "basic points" of the mechanism. The so-called basic points are the centers of the joints and the points of the links required to generate certain paths throughout the working range of the mechanism. Although the method is general in its formulation, the main disadvantage lays in the large number of variables required to define the objective function, as well as in not including the ground joint coordinates among the design parameters.

3. MATLAB SIMULINK

MATLAB is s software package for high performance numerical computation and visualization. It provides an interactive environtment with hundreds of built-in functions for technical computation, graphics, and animation. Also, it provides easy extensibility with its own high-level programming language. The name MATLAB stands for MATrix LABarotary. MATLAB has a number of add-on software module, calles toolboxes, that perform more specialized computations. Toolboxes deal with applications such as Symbolic computation, Statistics, Financial analysis, Image and Signal Processing, Control System design, Fuzzy logic, Neutral Networks, Wavelets, Simulink, and others.

As aforementioned, Simulink is one of add-on for MATLAB. Simulink, developed by The MathWorks, is a tool for modeling, simulating and analyzing multidomain dynamic systems. Its primary interface is a graphical block diagramming tool and a customizable set of block libraries. It offers tight integration with the rest of the MATLAB environment and can either drive MATLAB or be scripted from it. Simulink is widely used in control theory and digital signal processing for multidomain simulation and design.

4. V-REALMS BUILDING

V-Realm Builder is a flexible, graphically oriented tool for 3-D editing and is available for Windows operating systems only. It is a native VRML authoring tool that provides a convenient interface to the VRML syntax. Its primary file format is VRML. Its graphical interface (GUI) offers not only the graphical representation of a 3-D scene and tools for interactive creation of graphical elements, but also a hierarchical tree-style view (tree viewer) of all the elements present in the virtual world

CHAPTER 3

METHODOLOGY

1. PROJECT PLANNING

1.1.Flowchart



1.2. Activity Detail

1.2.1. Title Selection

Students are required to select a title during the first two weeks on final year first semester. Some of the titles are already purposed by the lecturers. After selecting the title and the lecturer agreed to supervise the project, student required to write a proposal for the project to be approve by FYP Committee. After that, student can proceed with the project selected.

1.2.2. Data Gathering

The second step after selecting the title is data gathering part. In this part, all information, literature review, and data related to the project are gathered through books, journal, research paper, and internet. This will be included in preliminary report.

1.2.3. Conceptual Understanding

This part is related to the first objective of the project which is to study and understand the concept and mechanism of five link suspension systems. This will be the first result of the project.

1.2.4. Develop mathematical Model

From equation of motion and other equation related, all them being combined and simplified or derived to form desire equation. In this part, the author encountered lot of problem to formulate the equation. The author referred the journal [1] for the mathematical model part.

1.2.5. Develop Quarter Car Model Simulation

For simulation part, the author use MATLAB / Simulink as the simulation software.

1.2.6. Analyze Result

After obtaining the simulation of quarter car model, the result graph of time versus bouncing amplitude is obtained. Back to the simulation, the author will change the parameters of the linkage to different value to get the best orientation and position of the linkage. The result also will be compared to the result from simulation of other type of suspension system.

1.2.7. Conclusion

Last part of the project is to conclude the project which is based on the objective of the project.

1.3.Software Used

- 1.3.1. MATLAB / Simulink
- 1.3.2. V-Realms Building

2. MILESTONE

2.1.Final Year Project 1

	Task Name	Start	Finish	Jan 24, '10) Jan	31, '10	Feb 7, '10	Feb 14, '10) Fe	eb 21, '10	Feb	28, 10	Mar 7, '10	Mar	r 14, '10	Mar 2	1,'10	Mar 28, '10	Apr 4, '1	0 4	\pr 11, "	10 A	pr 18,	40	Apr 25, '1	10 M	lay 2, '1	4 0	vlay 9, '	10 /	May
				STT	SN	1 W F	STT	SMW	FS	TT	SM	1 W F	STT	S N	M W F	ST	TS	MWF	ST	TS	M W	FS	T	TS	MW	FS	; T :	TS	M V	/ F	S
1	Selection of Project Title	Mon 1/25/10	Fri 2/5/10	-															2												
2	Proposal Submission	Mon 1/25/10	Tue 2/2/10			∎ ₁										2															
3	Confirmation of Title	Wed 2/3/10	Fri 2/5/10			Į.										8															
4	🖃 Data Gathering	Wed 2/3/10	Fri 2/19/10			<u> </u>			•							2															
5	Suspension System	Wed 2/3/10	Mon 2/8/10																												
6	Five Link Suspension System	Tue 2/9/10	Wed 2/17/10													8															
7	Software Used	Thu 2/18/10	Fri 2/19/10					l 🎽	-							8															
8	Preliminary Report	Mon 2/22/10	Fri 2/26/10							_	,					8															
9	Working On Report	Mon 2/22/10	Thu 2/25/10							h																					
10	Submission of Report	Fri 2/26/10	Fri 2/26/10						-	- Ť	.					8															
11	Conceptual Understanding	Mon 3/1/10	Fri 3/12/10								1 -					8															
12	Concept	Mon 3/1/10	Fri 3/5/10									<u></u>	-84	•		8															
13	Mechanism	Mon 3/8/10	Fri 3/12/10										+	<u> </u>																	
14	Midsemester Break	Mon 3/15/10	Fri 3/19/10												<u> </u>																
15	- Progress Report	Mon 3/22/10	Fri 3/26/10													1	_														
16	Working on Report	Mon 3/22/10	Thu 3/25/10													-	_														
17	Submission of Report	Fri 3/26/10	Fri 3/26/10																												
18	Seminar	Mon 3/29/10	Fri 4/2/10													8		+	-												
19	Develop Mathematical Model	Mon 4/5/10	Fri 4/30/10													8			+								1				
20	- Interim Report	Mon 5/3/10	Eri 5/7/10																			-			-	- 1					
21	Working on Report	Mon 5/3/10	Thu 5/6/10													8												L			
22	Submission of Report	Eri 57/10	Fri 50/10													8												<u> </u>			
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2.2.Final Year Project 2

Task Name	Duration	Start	Finish	August	2010		September 2	2010		October 2)10		Noven	ber 201	0	
				22 25 28 31 3 6	9 12 15	18 21 24 27 3	0 2 5 8	11 14 17 20 2	3 26 2	2 5 8	11 14 17	7 20 23 26 29	1 4	7 10	13 16 19	22 25 2
Project work continues	17 days?	Mon 26/7/10	Tue 17/8/10			,										
Building Model	6 days?	Mon 26/7/10	Mon 2/8/10													
Search for simulations	11 days?	Tue 3/8/10	Tue 17/8/10	l i												
Submission Progress Report 1	1 day?	Mon 23/8/10	Mon 23/8/10													
Project work continues	22 days?	Mon 23/8/10	Tue 21/9/10							8 8 9 9 9 9 9 9 8 8 8 8 8 8 8 8 8 8 8 8						
Obtaining initial simulation result	11 days?	Mon 23/8/10	Mon 6/9/10													
Analysis result	11 days?	Tue 7/9/10	Tue 21/9/10													
Submission Progress Report 2	1 day?	Fri 24/9/10	Fri 24/9/10					1								
Seminar	1 day?	Mon 27/9/10	Mon 27/9/10													
Project work continues	33 days?	Mon 27/9/10	Wed 10/11/10						-					-		
Half car model simulations	16 days?	Mon 27/9/10	Mon 18/10/10									h i				
Simulate with different parameters	11 days?	Tue 19/10/10	Tue 2/11/10									*	i i			
Analysis result	6 days?	Wed 3/11/10	Wed 10/11/10										L È			
Poster Exibitions	1 day?	Mon 18/10/10	Mon 18/10/10									l.				
Oral presentation	6 days?	Mon 15/11/10	Mon 22/11/10													

CHAPTER 4

RESULT AND DISCUSSION

1. FINDING ON FIVE LINK SUSPENSION SYSTEM

Based on my further research there are many types of five link suspension system inplemented by various car manufacturer such as Audi, Mercedes Benz, BMW, and Honda. They design the five link suspension system based on how to arrange the link at the wheel and connect it to the body (chasis). It is because to put five link at the wheel will be crowded and it need a lots of space. So it is important to design the five linkage in perfect position.



Figure 2: Multi-link suspension on BMW 1 Series Coupé – Picture Credited to BMW

My second fiding is from Wikipedia that five link suspension system quite unique where it only allow the wheel move up and down and control the movement to right and left side during steering. Which means that there will be less movement to side of the wheel. Furthermore, the interesting part of five link suspension that as the spindle turns for steering, it alters the geometry of the suspension by apply torque to all four suspension arms. They have complex pivot systems designed to allow this to happen. Car manufacturers claim that this system gives even better road-holding properties, because all the various joints make the suspension almost infinitely adjustable.





Figure 3: Front Views – Movement Up and Down

Figure 4: Top View – Movement to side

To compare with three link suspension and four link suspension, five link suspension system is more stable in term of handling and steering the car. Below are the comparison of movement of the wheel between this three system.



Figure 5: Comparison of wheel movement – Picture credited to Honda

From my research, the main idea of my project is to study on the best arrangement of the linkage of five link suspension so that it will be very compact interm of space and good handling in term of comfortablility.

2. KINEMATIC ANALYSIS



Figure 6: Five Link Suspension

2.1. Position Problem

The five-link suspension mechanism has six degrees-of-freedom, of which five are trivial rotations of the connecting links around their own axes. Correspondingly, the position of the wheelcarrier can be specified using only one independent parameter viz the coordinate z_N of the origin of the Nx'y'z' reference frame relative to the central reference frame Oxyz. The remaining fiveparameters: coordinates x_N , y_N and angles α , β and γ that define the position and orientation of thewheel carrier can be determined by solving the following equations of constraint:

$$(x_{Ai} - x_{Bi})^{2} + (y_{Ai} - y_{Bi})^{2} + (z_{Ai} - z_{Bi})^{2} = l_{i}^{2} \quad (i = 1...5)$$

5

Describe the condition of the distance between joints A_i and B_i to remain constant during the working range of the mechanism. In the above Eq. (5), the coordinates x_{Bi} , y_{Bi} and z_{Bi} are determined by applying the following transformation to the Nx'y'z' reference frame:

$$\begin{bmatrix} x_{\mathrm{B}i} \\ y_{\mathrm{B}i} \\ z_{\mathrm{B}i} \end{bmatrix}_{Oxyz} = \begin{bmatrix} R_{\beta\alpha\gamma} \end{bmatrix} \begin{bmatrix} x'_{\mathrm{B}i} \\ y'_{\mathrm{B}i} \\ z'_{\mathrm{B}i} \end{bmatrix}_{Nx'y'z'} + \begin{bmatrix} x_{\mathrm{N}} \\ y_{\mathrm{N}} \\ z_{\mathrm{N}} \end{bmatrix}_{Oxyz}$$

Where $[R_{\beta\alpha\gamma}]$ is the transformation matrix that expresses the successive rotation of the wheel carrierrelative to 0xyz by the pitch angle β , yawangle α and roll angle γ :

$$[R_{\beta\alpha\gamma}] = [R_{\gamma,x}][R_{\alpha,z}][R_{\beta,y}] = \begin{bmatrix} c\alpha \cdot c\beta & -s\alpha & c\alpha \cdot s\beta \\ s\alpha \cdot c\beta \cdot c\gamma + s\beta \cdot s\gamma & c\alpha \cdot c\gamma & s\alpha \cdot s\beta \cdot c\gamma + c\beta \cdot s\gamma \\ s\alpha \cdot c\beta \cdot s\gamma + s\beta \cdot c\gamma & c\alpha \cdot s\gamma & s\alpha \cdot s\beta \cdot s\gamma + c\beta \cdot c\gamma \end{bmatrix}$$

In the above equation $[R_{\alpha,z}]$, $[R_{\beta,y}]$ and $[R_{\gamma,x}]$ are the basic rotation matrices while $c\alpha = cos\alpha, s\alpha = sin \alpha$ and so forth.

For a given value of the independent parameter z_N , the system of Eq. (5) in the unknowns α , β , γ , x_N and y_N can be very conveniently solved by minimizing the following objective function:

$$F_0(\alpha, \beta, \gamma, x_{\rm N}, y_{\rm N}) = \sum_{i=1}^{5} [(x_{{\rm A}i} - x_{{\rm B}i})^2 + (y_{{\rm A}i} - y_{{\rm B}i})^2 + (z_{{\rm A}i} - z_{{\rm B}i})^2]$$

In order to facilitate convergence, the starting point when minimizing F_0 can be taken the position of the wheel carrier (the same x_N , y_N and orientation angles α , β , and γ) imposed during synthesis for the same z_{Nj} . Once the displacement problem of the wheel carrier is solved, the diagram of thewheel track, recessional wheel motion, camber, and toe angle alteration can be generated. 6

2.2. Linear Velocity and Acceleration Analysis

The velocities of points B_i on the wheel carrier can be determined by differentiating once with respect to time the equations of constraint (5). The number of unknowns thus emerging is 15, and therefore 10 more equations must be added, like the time derivatives of following equations:

$$(x_{Bj} - x_{Bk})^2 + (y_{Bj} - y_{Bk})^2 + (z_{Bj} - z_{Bk})^2 = \text{const} \quad (j = 1 \dots 4 \text{ and } k = j + 1 \dots 5)$$

9

10

And,

$$(x_{\mathrm{B}i} - x_{\mathrm{N}})^{2} + (y_{\mathrm{B}i} - y_{\mathrm{N}})^{2} + (z_{\mathrm{B}i} - z_{\mathrm{N}})^{2} = \mathrm{const} \quad (i = 1 \dots 5)$$

Describe the condition that the wheel carrier is a rigid body. By differentiation these equationsonce with respect to time, a new independent parameter \dot{z}_N will emerge, which, the same as z_N must be specified as input during the numerical analysis.

The coefficients of the system of linear equations in the 17 unknowns \dot{x}_{Bi} , \dot{y}_{Bi} , \dot{z}_{Bi} (i= 1 . . . 5), \dot{x}_N and \dot{y}_N used for velocity analysis are summarized in Table 1 (see appendix 1).

By differentiating with respect to time the equations used to solve the velocity problem, asecond system of linear equations in the unknowns \ddot{x}_{Bi} , \ddot{y}_{Bi} , \ddot{z}_{Bi} (*i*= 1 . . . 5), \ddot{x}_{N} and \ddot{y}_{N} will befurther obtained, the coefficients of which are given in Table 2 (see appendix 2). In this case z_{N} , \dot{z}_{N} and \ddot{z}_{N} will bethe independent parameters that must be specified as inputs.

2.3. Angular Velocity and Acceleration Analysis

The components of the angular-velocity vector $(\omega_x, \omega_y, \omega_z)$ relative to the fixed reference frame 0_{xyz} can be determined using the following equation known from the rigid body kinematics:

$$\begin{bmatrix} \dot{\mathbf{x}}_{\mathrm{B}i} \\ \dot{\mathbf{y}}_{\mathrm{B}i} \\ \dot{\mathbf{z}}_{\mathrm{B}i} \end{bmatrix} = \begin{bmatrix} \dot{\mathbf{x}}_{\mathrm{N}} \\ \dot{\mathbf{y}}_{\mathrm{N}} \\ \dot{\mathbf{z}}_{\mathrm{N}} \end{bmatrix} + \begin{bmatrix} \omega_{x} \\ \omega_{y} \\ \omega_{z} \end{bmatrix} \times \begin{bmatrix} x_{\mathrm{B}i} - x_{\mathrm{N}} \\ y_{\mathrm{B}i} - y_{\mathrm{N}} \\ z_{\mathrm{B}i} - z_{\mathrm{N}} \end{bmatrix}$$

Written for any two different points of the wheel carrier for which the linear velocity are known. The expressions of ω_x , ω_y , and ω_z as derived from relation (11) are given in Eq.(A.1) in Appendix 3.

The components of the angular-acceleration vector (ε_x , ε_y , ε_z) can be determined writing the following equation, the same for two different points of the wheel carrier:

$$\begin{bmatrix} \ddot{x}_{Bi} \\ \ddot{y}_{Bi} \\ \ddot{z}_{Bi} \end{bmatrix} = \begin{bmatrix} \ddot{x}_{N} \\ \ddot{y}_{N} \\ \ddot{z}_{N} \end{bmatrix} + \begin{bmatrix} \varepsilon_{x} \\ \varepsilon_{x} \\ \varepsilon_{x} \end{bmatrix} \times \begin{bmatrix} x_{Bi} - x_{N} \\ y_{Bi} - y_{N} \\ z_{Bi} - z_{N} \end{bmatrix} + \begin{bmatrix} \omega_{x} \\ \omega_{x} \\ \omega_{x} \end{bmatrix} \times \begin{pmatrix} \begin{bmatrix} \omega_{x} \\ \omega_{x} \\ \omega_{x} \end{bmatrix} \times \begin{bmatrix} x_{Bi} - x_{N} \\ y_{Bi} - y_{N} \\ z_{Bi} - z_{N} \end{bmatrix} \end{pmatrix}$$

The expressions of ε_x , ε_y , and ε_z derived through analytical manipulations of relation (12) are summarized in Eq.(A.2) in Appendix 3. Alternatively, the components of the angular acceleration and be determined by differentiating once with respect to time the components of the angular velocity:

$$\varepsilon_x = \dot{\omega}_x, \varepsilon_y = \dot{\omega}_y$$
 and $\varepsilon_z = \dot{\omega}_z$

11

13

3. QUARTER CAR SIMULATION

From research, the author have obtained the simulation for five link suspension system using MATLAB. This simulation is for quater car model and the parameter have not been changed by the author.

Below are the simulation details shown in picture.



Figure 7: Overall simulation model

This is the overall model of the simulation. It consist of five connection to wheel carrier and one wheel. There are also the road surface selector, vehicle speed, shock graph, and force graph. This model taken from Reinhold von Schwerin "Multibode System Simulation".

3.1.Detail of the block



Figure 8: Control Arm 1 Block

This is the first control arm block where the control arm is jointed to the wheel carrier by normal joint shown by **Spherical 1** block. **Universal 1** is where the connection of the control arm with body frame, **Frame 1**.



Figure 9: Shock Absorber and Vehicle Speed block

For control arm 5, beside connected to body frame and wheel carrier, it also connected to the shock absorber. The other joint are the same as Control arm 1. There is also vehicle speed selector where we can change the speed of the car to see its effect to the car suspension.



Figure 10: Wheel Carrier with Wheel connected through revolute joint

Wheel carrier connected to wheel where in the middle of the connection is **Revolute**where it shows that the wheel are spinning during the test.



Figure 11: Road surface selector

There are three type of road surface can be select which are Bumpy, Flat, and SinWave. There is also Force on Road block for the graph.



Figure 12: Shock absorber block detail

Above is the shock absorber block detail where we can see the shock force block which is where we can get the shock on road graph.



3.2.Initial Result Obtained

Figure 13: Initial result for Sine Wave road surface



Figure 14: Initial result of Bumpy road surface



Figure 15: Initial result of Flat road surface

For flat road surface, we can see that the setting time before the flat line achieve is approximately 1.25 sec. It is a very short time, and this show how five link suspensions provides the comfort to the riders.

4. SIMULATE WITH DIFFERENT LINK PARAMETERS

In this simulation, we are using coordinate system (CS) as out tools to design the orientation of the linkage. The author will change the parameter by changing the CS of the linkage to make new design (orientation) of the suspension system.

Below are the details to change the parameters.



Figure 16: Control Arm 1 with Universal and Spherical joint

For example, the author will change the coordinate of Frame 1 so that the length of control arm 1 will change with different orientation. Detail of block Frame 1 is as below:

Ground		
Grounds one side	of a Joint to a fixed location in the World (coordinate system.
Parameters		
Location [x,y,z]:	[-64 413 327]	mm 💟
Show Machine	Environment port	



In these block parameters of Frame 1, the coordinate of the joint is [-64 413 327] which is [x y z] respectively. We change the coordinate to [-94 433 337] that also will changer the length and position of the first arm.

After change the block parameters for Frame 1, and then we need to change the parameters of Universal1 Block as shown below:

tepreseni he base (12 attach iensor an letermine	ts two rotationa (B) Body around ed to follower, L id actuator ports sign of forward	l degrees of freedom. The follower (F) two primitive revolute axes (R1, R2). I isted order of primitives is order of mot s can be added. Base-follower sequenc motion. This joint becomes singular if t	rotates with respect to R1 attached to base. ion during simulation. re and axes directions both revolutes align.
Connectio	on parameters		
Current b	base:	GND@Frame1	
Current f	ollower:	CS1@Rod#1	
Paramete Axes	Advanced		
Paramete Axes	Advanced	Axis of Action 1x y z1	Reference CS
Paramete Axes Name R1	Advanced Primitive revolute	Axis of Action I x v z l (null([-64 636 345]-[-64 413 327])*	Reference CS World
Axes Axes R1 R2	Advanced Primitive revolute revolute	Axis of Action 1x v z1 (null([-64 636 345]-[-64 413 327])) (null([-64 636 345]-[-64 413 327]))	Reference CS World

Figure 18: Universal 1 block parameters

In this block, it shows that the base of the joint is connected to Frame1 and the follower is connected to Rod#1 which is Arm1. The rotation of the joint is constraint to the coordinate of frame and wheel carrier where bot connected to Arm1. In addition, spherical joint are as follow:



Figure 19: Universal joint

The Universal block represents a composite joint with two rotational degrees of freedom (DOF) as two revolute primitives. A Joint block represents the relative degrees of freedom between two bodies, not the bodies themselves. For this joint the base and follower can be either.

Now we can proceed to next step as follows.



Figure 20: Detail of ControlArm1 block

Figure 19 show the block diagram inside ControlArm1 block. Here we can found the Rod#1 block where it actually connected to the Universal1 block through Conn1 block. Through Conn2 block, the rod connected to Spherical1 block which is another joint that connecting the rod to the wheel carrier. The coordinate that we need to change is inside the Rod#1 block parameters shown below:

Body Represe axes fo	ents a user-c r center of g	lefined ri ravity (C	gid body. Body defined G) and other user-spe	d by mas: cified Bor	s m, i dy cc	inertia tensor I, an ordinate systems.	nd coor This d	dinate origins an ialog sets Body ir	d nitial
position provide	and orienta s optional se	tion, unle ttings fo	ess Body and/or conne r customized body geo	cted Join metry an	its ar	e actuated separa lor.	itely. T	his dialog also	
Mass pi	roperties								
Mass:	2							kg	
Inertia:	[14597 -1	16; -1 1	091 1133; 16 1133 15	;037]				g*cm^2	
Positio	n Orienta	ation	Visualization						
Show Port	Port Side	Name	Origin Position Vector [x y z]	Unil	ts	Translated from Origin of	m	Component Axes of	冒
	Loft IT	CG	[-64 524.5 336]	mm		World		World	
	Len	100000000	F 64 412 2271	mm	~	World	~	World	×
-	Left	CS1	[-04 415 527]		the second second				
- - -	Left V Right V	CS1 CS2	[-64 636 345]	mm	~	World	~	World	1
- - - - - -	Left V Right V Right V	CS1 CS2 CS3	[-64 636 345] [0 0 0]	mm m	~	World World	~	World World	
- - - - - - -	Left V Right V Right V	CS1 CS2 CS3	[-64 636 345] [0 0 0]	mm	~	World World	~	World World	•



Parameter that needs to change is CS1 value and it will be the same as coordinate that we put in Frame1 block. For spherical joint, we do not need to change anything because the coordinate are connected to the other side of the rode. In addition spherical joint are as follows:



Figure 22: Spherical Motion of Follower (blue) Relative to Base (red)

The Spherical block represents three rotational degrees of freedom at a single pivot point, a "ball-in-socket" joint. Two rotational DOF specify a directional axis, and a third rotational DOF specifies rotation about that directional axis. The sense of each rotational DOF is defined by the right-hand rule, and the three rotations together form a right-handed system. A spherical joint is one of the SimMechanics primitive joints, along with prismatic and revolute.

After complete changing the entire necessary coordinate, and then we can run the simulation. Below is the result after changing the coordinate. For this result, the author use force graph as the result. In order to define it to displacement graph, we can use the analogy of relation between force and distance. The higher the force, it will result in higher in distance. For example for equation F=ma, where mass is constant and we measure the distance that can be achieve by the acceleration by five second then it make the time constant. So the higher the force, the higher the acceleration will be then this make distance higher because the time constant. Below are the result.



Figure 23: Result of bumpy road surface with different design

From the result above, we can see that the maximum amplitude is lesser than the previous design and this give us a better conform. But it is different in sine wave road surface condition which is as follows.





From the result above, the maximum amplitude is higher than the precious design which shows that it is lesser comfort on sine wave road condition. Below is the result of flat road surface condition.



Figure 25: Result of Flat road surface with different design

From the result of the shock force graph, we can see that the settling time is increased to approximately 1.65 second rather than the first one which is 1.25 second. In addition, the magnitude of the bouncing force is much too higher than previous design that means this design is not acceptable because of the wheel already touch the car body.

So this proves that the position problem of five link suspension is difficult to solve where we have a good result in bumpy road surface condition but it is worse when come to sine wave road surface and flat road surface condition. So only a few designs are acceptable to be used where we can have a good result in all three road surface.

5. ADVANTAGE AND DISADVANTAGE OF FIVE LINK SUSPENSION SYSTEM

5.1.Advantages

- **5.1.1.** Multi-link suspension allows the auto designer the ability to incorporate both good ride quality and good car handling in the same vehicle.
- **5.1.2.** A multi-link suspension will allow the vehicle to flex more; this means simply that the suspension will be able to move more easily to conform to the varying angles of off road.
- **5.1.3.** Multi-link equipped vehicles are ideally suited for sports such as rock crawling and desert racing
- **5.1.4.** In its simplest form the multi-link suspension is orthogonal that is, it is possible to alter one parameter in the suspension at a time, without affecting anything else

5.2. Disadvantages

- **5.2.1.** Multilink suspension is costly and complex
- **5.2.2.** It is also difficult to tune the geometry without a full 3D computer aided design analysis
- **5.2.3.** Need bigger space between body and wheel.

5.2.4.

CHAPTER 5

CONCLUSION

1. CONCLUSION

Through the whole project period, the author has gone through the entire step in the project. All the result has been obtained at the end of this project.

As a conclusion, it is proved that five link suspension systems is one of the best suspension systems which has very small settling time. It also proved that the position problem of five link suspension systems make five link suspension expensive in term of its design and it is very crucial and difficult to solve by simple calculation without a 3D simulation solutions. This show by putting a different value of the position will give a very different result.

Lastly, the author would like to state that MATLAB / Simulink are very powerful software to do the simulation for this project.

2. RECOMMENDATION

For future improvement and expand of this project, the author would like to give few recommendations as below:

- 1. Produce more design of the linkage position.
- 2. Compare the result with other suspension type with same condition
- 3. Improve the simulations by make it easier to understand

CHAPTER 6

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CHAPTER 7

APPENDICES

Appendix 1

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Appendix 2

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Appendix 3

Considering two points B_j and B_k $(j \neq k)$, the components of the angular velocity of the wheel carrier are $\omega_x = \frac{P1 \cdot \Delta x_j \cdot \Delta x_k + P2 \cdot \Delta y_j \cdot \Delta x_k + P3 \cdot \Delta x_j \cdot \Delta z_j}{\Delta x_j \cdot \Delta z_j \cdot \Delta x_k}$ $\omega_y = (\Delta y_k \cdot \omega_x - P3)/\Delta x_k$ $\omega_z = (\Delta z_j \cdot \omega_y - P1)/\Delta y_j$ while the components of the angular acceleration are $\varepsilon_x = \frac{Q1 \cdot \Delta x_j \cdot \Delta x_k + Q2 \cdot \Delta y_j \cdot \Delta x_k + Q3 \cdot \Delta x_j \cdot \Delta z_j}{\Delta x_j \cdot \Delta z_j \cdot \Delta y_k - \Delta y_j \cdot \Delta z_j \cdot \Delta x_k}$ $\varepsilon_y = (\Delta y_k \cdot \varepsilon_x - Q3)/\Delta x_k$ $\varepsilon_z = (\Delta z_j \cdot \varepsilon_y - Q1)/\Delta y_j$ (A.2)

Appendix 4

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