Effect of a Moving Load to A Jetty Gangway

by

Tengku Shahril Nizam Bin Tengku Mohd Yusof

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the requirements for the
Bachelor of Engineering (Hons)
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CERTIFICATION OF APPROVAL

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Approved by,

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(Ir. Idris Bin Ibrahim)

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TRONOH, PERAK
DECEMBER 2010

CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

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TENGKU SHAHRIL NIZAM

ABSTRACT

This thesis discuss of the study that have been done on the **Effect of a Moving Load to A Jetty Gangway.** The objective of the project is to study the dynamic characteristic of the jetty gangway structure due to moving load, which in this case is a personnel walking on the jetty gangway. The dynamics analysis of jetty gangway is essential for determining the structure integrity of a jetty gangway.

The structure of the telescopic jetty gangway is modeled as an I-beam structure that are attached; where the boundary condition at the extensional part is treated to be fixed; therefore continuous system can be applied in the analytical method. The structure is treated as a single structure, with different value of cross sectional areas at the beginning, middle and far end of the jetty gangway. The dynamical relations of the jetty gangway to the effect of a moving load will be developed. The first part of the project will only discuss about the free vibration of the structure. In the second part, the response and simulation of the structure under the forced vibration is being done and validated using results from the analytical solution. The response of the moving load is modeled using Fourier Series.

Further in this report, a list of results of free vibration and forced vibration consist of natural frequency, mode shapes and response will be shown and discussed. The thesis had discussed application moving load into the system and shown the response of the jetty gangway structure. The values of displacement compared using analytical approach and simulation approach are showing similarity. Throughout this thesis, the author concluded that the results obtained are valid as it was verified with the references.

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CHAPTER 1

INTRODUCTION

1.1 Background of Study

Telescopic jetty gangway is an alternative of the conventional jetty gangway structure designed to be used by the operation and maintenance personnel to board the ship from port or vice versa. The study on the static load of the structure has already been developed in the previous projects. The study emphasized on the deflection, bending moment, shear force and slope of the gangway structure. The study recommended that the dynamics analysis should be developed in order to design the jetty gangway based on its structural integrity studies. Therefore, further studies focusing on the dynamic part of the jetty gangway structure is undertaken to investigate its dynamic characteristics while at the same time fulfill the purpose of structural design of the jetty gangway. The Figure 1.1 below represents an example of telescopic jetty gangway:

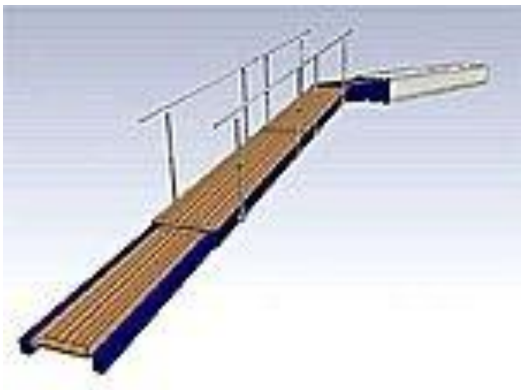


Figure 1.1: Telescopic jetty gangway

1.2 Problem Statement

The project is a continuation study from the previous Final Year Project. From the previous project, the study of dynamic load under the influence of the sea wave to the structure has been performed. The outcome of the study provided an insight for the design parameters for jetty gangway. However, the dynamic study can be further undertaken to study the effect of one of the major load applied to the jetty gangway structure, which is the moving load. The personnel walking on the jetty gangway excites the dynamic motion of its structure. This excitation provided to the structure is not localized or limited to just one point as it is based on the movement of the personnel. The dynamic load due to the moving load would create a harmonic/periodic motion of the jetty gangway which could affect the integrity of the structure. Thus, it is important to investigate the dynamic characteristics of the jetty gangway upon excitation by the moving load. The output of this study can be used to assist in improving the design of the gangway.

1.3 Objectives and Scopes of Study

The objective of this project is to study the dynamic characteristic of the jetty gangway structure due to moving load. The project aims to collect sufficient data of the mode shape, the value of natural frequency and the value of response of the structure which depends on the frequencies of the moving load.

The scope of study for this project concerns more on the dynamics analysis which means it will only discuss about the mathematical relations of the parameters of the jetty gangway. The study consists on finding the relation of a moving load (frequency and amplitude) with mode shape and response (displacement) to the gangway structure. Integrity of the structure can be further defined when the study of it under the influence of moving load completed. The values and mathematical relations will be validated using simulation software such as ANSYS during the second part of the project. No prototype will be built as only simulation stage involved.

CHAPTER 2

LITERATURE REVIEW

The project is based on earlier study of the telescopic jetty gangway structure which was done in previous final year projects. There will be generally three stages applied to this project which are dynamic analysis, modeling and simulation. For the dynamic analysis, a study through sources of reference is compulsory in order to solve the project. It is understood that the previous project adopted the continuous system approach which the fundamental of vibration on the continuous system is examined in order to study the behavior of the gangway under the influence of transverse and longitudinal vibrational force. The methods of finding the natural frequencies and the mode shape of a continuous shape were studied and compared in order to investigate the natural frequencies and mode shape of the gangway structure. Instead of studying the theoretical parts in the references, an analytical study on the journal and papers regarding the dynamic analysis on the structure was done in order to see the flow and reliable methods to be used in the related research project.

The moving load problem has been the subject of numerous research efforts in the last century. The importance of this problem is manifested in numerous applications in the field of transportation especially bridges, guideways, overhead cranes, cableways, rails, roadways, runways, tunnels, launchers and pipelines are example of structural elements to be designed to support moving loads.

According to B. Mehri et. Al [1], the literature concerning the forced vibration analysis of structures with moving bodies is broad. The most used method for determining these vibrations is the expansion of the applied loads and the dynamic responses in terms of the eigenfunctions of the undamped beams. This method leads to solutions presented as infinite series, which will be truncated after a number of terms and approximate solutions are then obtained.

Fryba et al. [2] used the Fourier sine (finite) integral transformation and the Laplace–Carson integral transformation to determine the dynamic response of beams due to moving loads and obtained this response in the form of series solutions. Ting et al. [3] formulated and solved the problem using the influence coefficients (static Green function). The distributed inertial effects of the beam were considered as applied external forces. Correspondingly, at each position of the mass, numerical integration had to be performed over the length of the beam.

Hamada [4] solved the response problem of a simply supported and damped Euler–Bernoulli uniform beam of finite length traversed by a constant force moving at a uniform speed by applying the double Laplace transformation with respect to both time and the length coordinate along the beam. He obtained in closed form an exact solution for the dynamic deflection of the considered beam. Yoshimura et al. [5] presented the analysis of dynamic deflections of a beam, including the effects of geometric non-linearity, subjected to moving vehicle loads. With the loads moving on the beam from one end to the other, the dynamic deflections of the beam and loads were computed by using the Galerkin method. Lee [6] presented a numerical solution based on integration programs using the Runge-Kutta method for integrating the response of clamped-clamped beam acted upon by a moving mass.

Esmailzadeh et al. [7] have studied the forced vibration of a Timoshenko beam with a moving mass. Gbadeyan and Oni [8] presented a technique based on modified generalized finite integral transforms and the modified Struble's method to analyze the dynamic response of finite elastic structures (Rayleigh beams and plates) having arbitrary end supports and under an arbitrary number of moving masses. Foda and Abduljabbar [9] used a Green function approach to determine the dynamic deflection of an undamped simply supported Euler–Bernoulli beam of finite length subject to a moving mass traversing its span at constant speed. Visweswara Rao [10] studied the dynamic response of an Euler-Bernoulli beam under moving loads by mode superposition. The time-dependent equations of motion in modal space were solved by the method of multiple scales and instability regions of parametric resonance were identified.

Wu et al. [11] presented a technique using combined finite element and analytical methods for determining the dynamic responses of structures to moving bodies and applied this technique to a clamped–clamped beam subjected to a single mass moving along the beam. Sun [12] obtained Green's function of the beam on an elastic foundation by means of Fourier transform. The theory of linear partial differential equation was used to represent the displacement of the beam in terms of convolution of the Green's function. Next he employed the theory of complex function to seek the poles of the integrand of the generalized integral. Theorem of residue was then utilized to represent the generalized integral using contour integral in the complex plane. Abu-Hilal [13] used a Green functions method for determining the dynamic response of Euler–Bernoulli beams subjected to distributed and concentrated loads. He used this method to solve single and multi-span beams, single and multi-loaded beams, and statically determinate and indeterminate beams. Several theories are applied to this project which will be discussed as the following;

2.1 Continuous system

According to Singiresu S. Rao, continuous system is a system of infinite degree of freedom in which it is not possible to identify discrete masses, damping and spring. It is necessary to consider continuous distribution of the mass, damping and elasticity and assume that each of the infinite number of point of the system can vibrate. When a system is modeled as a discrete system, the governing equations are ordinary differential equations which are easy to solve and if the system in modeled as a continuous system, the governing equation are partial differential equation which are more difficult.

2.2 Method for solving continuous system

There are several methods used for solving the continuous system model which are general method, Rayleigh method, finite deferential method and finite element method. All of the methods having their significant and the approach in solving the dynamics analysis are differ from respective methods. For the purpose of this project, the author just discussed the dynamic analysis in solving the force vibrational problem on a beam in axial and lateral directions. Detail of each method is elaborated below.

2.3 General method

The equation of motion of the element of beam developed by considering the free body diagram of the bending beam (lateral direction) as already shown by the Figure 1.2 below:

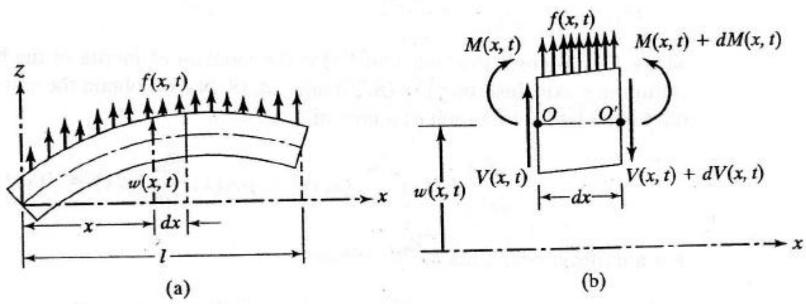


Figure 1.2: The bending beam with the respective variables

Based on the free body diagram yield two equations which are

- Force equation of motion in the z direction
- The moment equation of motion about point y-axis.

By applying the partial differential method on above equation, both equations can be joined together to form the equation of motion:

$$-\frac{\partial^2 M}{\partial X^2}(x,t) + f(x,t) = \rho A(x)\frac{\partial^2 w}{\partial t^2}(x,t) \ldots\ldots\ldots\ldots \quad (2.3.1)$$

Then , this equation been elaborated further by using Euler-Bernoulli or the elementary theory of bending of a beam which participated the relation of the bending moment and defection , yield the equation of motion for forced lateral vibration of a non-uniform and uniform beam. For our purpose, the equation of motion for uniform beam (equation 2.5.1) is applied.

$$EL\frac{\partial^2 w}{\partial X^4}\ (x,t) + \rho A\frac{\partial^2 w}{\partial t^2}\ (x,t) = f(x,t) \qquad (2.3.2)$$

As the equation of motion above involving the second –order derivative with respect to time and fourth-order derivative with respect to x, two initial conditions and four boundary conditions are needed for finding a unique solution for w(x,t).

The natural frequency of the beam given by:

$$w = B^2\sqrt{\frac{EL}{\rho A}} = (Bl)^2\sqrt{\frac{EL}{\rho A l^4}} \qquad (2.3.3)$$

The mode shape equation for formal function for the Fixed –free beam is

$$w_n(x) = c_n[sinB_n x - sinhB_n - a_n(cosB_n x - coshB_n x)] \qquad (2.3.4)$$

The forced vibration solution of a beam can be determined using the mode superposition principle. The deflection of the beam been represented by the equation below:

$$w(x,t) = \sum_{n=1}^{\infty} w_n(x)\, q_n(t) \qquad (2.3.5)$$

2.4 Lagrange method

Lagrange's formulation is an entirely scalar procedure, starting from the scalar quantities of kinetic energy, potential energy, and work expressed in terms of generalized coordinates.

We can derive the equation of motion of a multidegree of freedom system in matrix form Lagrange`s equation [Singiresu S. Rao, 2004].

$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_i}\right) - \frac{\partial T}{\partial x_i} + \frac{\partial V}{\partial x_i} = F_i, i = 1,2, \dots . n$……..….(2.4.1)

Where $F_i$is the non conservative generalized force corresponds or the external force.

The kinetic and potential energies of multidegree of freedom system can be express in matrix form as indicated below:

$T = \frac{1}{2}\dot{\vec{x}}^T[m]\dot{\vec{x}}$…………………………….…..(2.4.2)

$V = \frac{1}{2}\vec{x}^T[k]\vec{x}$…………………………….…..(2.4.3)

Based on the matrix,

$\frac{\partial T}{\partial \dot{x}_i} = [\vec{m}_i^T]\dot{\vec{x}}$…………………………......……(2.4.4)

By conducting the differential and several substations of equation, the desired equation of motion in matrix form can be obtained

$[m]\ddot{\vec{x}} + [k]\vec{x} = \vec{F}$……………………….....….. (2.4.5)

2.4.1-Advantages

The vibration analysis of continuous system required the solution of partial differential equation, which is quite difficult. In fact, analytical solutions do not exist for many partial differential equations. The analysis of multi degree of freedom, on the other hand, required the solution of set of ordinary differential equations, which is relatively simple. Hence, for simplicity of analysis, continuous system is often approximated as multi degree of freedom systems.

2.4.2- Disadvantages

However, as the number of degree of freedom increased, the solution of the characteristic equation becomes more complex. The mode shapes exhibit a property known as orthogonality , which often enables to simplify the analysis of multi-degree of freedom system.

2.5 Rayleigh method

The Rayleigh methods is based on Rayleigh principle, which can be stated as followed:

The frequency of vibration of conservative system vibrating about an equilibrium position has a stationary value in the neighborhood of natural mode. This stationary values, if fact, is a minimum value in the neighborhood of the fundamental natural mode.

Rayleigh method can be applied to find the fundamental natural frequency of continuous systems. This method is much simpler than exact analysis for system with varying distribution of mass and stiffness. In order to apply Rayleigh`s method, we need to derive expression for the maximum kinetic (equation 2.5.1), potential energies (equation 2.5.2) and Rayleigh quotient (equation 2.5.3).

$$U_{\max} = \frac{1}{2}\int \frac{M^2}{EI}\,dx = \frac{1}{2}\int EI\left(\frac{d^2y}{dx^2}\right)^2 dx \qquad \text{……………. (2.5.1)}$$

$$T_{\max} = \frac{1}{2}\int \dot{y}^2\,dm = \frac{1}{2}\omega^2\int y^2\,dm \qquad \text{……………………...(2.5.2)}$$

$$\omega^2 = \frac{\int EI\left(\frac{d^2y}{dx^2}\right)^2 dx}{\int y^2\,dm} \qquad \text{………………………………... (2.5.3)}$$

$R(\omega)$ or ω^2 is the Rayleigh`s quotient for the continuous system. If $\omega(x)$ is the mode shapes, then is equal to the square of the natural frequency of that mode. If ω is not a mode shape, the $R(\omega)$ is the scalar function for ω. For a discrete systems, $R(\omega)$ is a minimum when ω is a mode shape. Therefore, Rayleigh`s quotient can be used to approximate the lowest natural frequency for continuous system [Singiresu S. Rao, 2004].

2.5.1- Advantages

The usage of Rayleigh method is highly appreciated in continuous system. This is due to the significant of fundamental frequency in continuous system as the forced responses in many cases of continuous system are in large magnitudes. The Rayleigh method can be used to determine the fundamental frequency of a beam or shaft represented by a series of lumped masses.

2.5.2- Comparison between Rayleigh method and other (hysteretic) model

Based on the study conducted by Mr Sigurudur Erlingsson on the *live load induced vibration in Ullevi Stadium – dynamics soil analysis*, the respond of the structure can be found by two methods which are hysteretic model and Rayleigh model. The calculated responses from both models are differs but show the similarity in the overall behavior. The observation includes in comparison to the horizontal as well as the vertical components of both models. The result of the amplification peak in horizontal direction of the Rayleigh method clearly indicated but the result for hysteretic model only indicates a broad amplification with no predominant peak. This due to, the hysteretic model permits longer computational time step than the Rayleigh method as the time steps should be chosen small enough to ensure a stable solution.

2.6 Rayleigh- Ritz method

2.6.1 Introduction

The Rayleigh-Ritz method can be considered as an extension of Rayleigh method. It is based on the premise that the closer approximation to the exact natural mode can be obtained by superimposing a number of assumed functions than by using a single assumed function, as in Rayleigh method. The (equation 2.6.1.1) below stated the deflection equation when *n* functions are chosen.

$$W(x) = c_1 w_1(x) + c_2 w_2(x) + \ldots + c_n w_n(x) \ldots\ldots \quad (2.6.1.1)$$

If the assumed functions are suitably chosen, this method provided not only the approximate value of the fundamental frequency but also the approximate values of the higher natural frequencies and the mode shapes.

2.6.2- Advantages

It is usual to approach the problem in vibration problem by using energy principles either with a Rayleigh-Ritz Method (continuous series) or with the finite element method. A number of studies have been made which employed different versions of the latter technique and these have dealt with a variety of boundary conditions. However, not all possible boundary conditions have been considered and there has also been the problem of obtaining convergence of the solution when the number of terms in the series solution is increased. The Rayleigh-Ritz approaches are suitable for the symmetric balanced and unbalanced cases. It is demonstrated that the solution by using Rayleigh-Ritz method possesses good numerical characteristics and convergence properties.

In Rayleigh Ritz method, an arbitrary number of functions can be used in order to get the accurate result as the number of frequencies can be obtained is equal to the number of functions used. However, the amount of computation required becomes prohibitive for the asymmetric case where the finite element technique should be used and implemented.

2.7 Finite element method

2.7.1- Introduction

The basic idea in the finite element method is to find the solution of a complicated problem by replacing it with a simpler one. Since the actual problem is replaced by a simpler one in finding the solution, we will be able to find only an approximate solution rather than the exact solution. The existing mathematical tools will not be sufficient to find the exact solution (and sometimes, even an approximate solution) of most of the practical problems. Thus, in the absence of any other convenient method to find even the approximate solution of a given problem, the finite element method is considered as preferable. Moreover, in the finite element method, it will be possible to improve or refine the approximate solution by spending more computational effort.

2.7.2- Technical background

The finite element method consists of dividing the dynamic system into a series of elements by imaginary lines, and connecting the elements only at the intersections of these lines. These intersections are called nodes. The stresses and strains in each element are then defined in terms of the displacements and forces at the nodes, and the mass of the elements is lumped at the nodes. The Figure 2.1 below represents the finite element idealization of the beam:

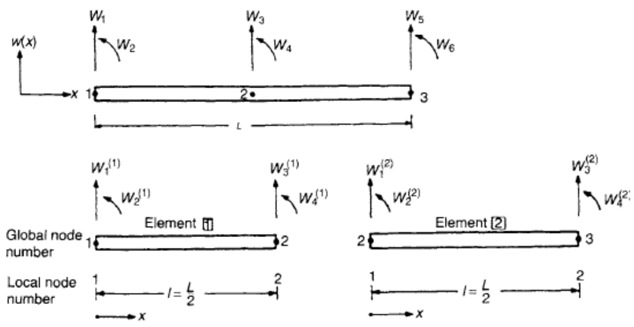


Figure 2.1: The finite idealization of the beam

The necessary equations for the approximate solution values can be formulated based on the divided structure of the beam element. The related equation is written below:

$$\omega(x) = W_1^{(e)}.\frac{1}{I^3}(2x^2 - 3Ix^2 + I^3) + W_3^{(e)}.\frac{1}{I^3}(3Ix^2 - 2x^3) + W_2^{(e)}.\frac{1}{I^2}(x^3 - 2Ix^2 + I^2x) + W_4^{(e)}.\frac{1}{I^2}(x^3 - Ix^2) \text{ ....(2.7.2.1)}$$

A series of equations is thus produced for the displacement of the nodes and hence the system. The formula of eigenvalue equation is stated below:

$$[K]\vec{W} = \lambda[M]\vec{W} \ldots\ldots\ldots\ldots\ldots (2.7.2.2)$$

where $\vec{W}$ is the eigenvector and λ is the eigenvalue. By solving these equations, the stresses, strains, natural frequencies and mode shapes of the system can be determined.

The accuracy of the finite element method is greatest in the lower modes, and increases as the number of elements in the model increases.

2.7.3- Advantages

The principal advantage of the finite element method is its generality; it can be used to calculate the natural frequencies and mode shapes of any linear elastic system. However, it is a numerical technique that requires a fairly large computer, and care has to be taken over the sensitivity of the computer output to small changes in input. For beam type systems the finite element method is similar to the lumped mass method, because the system is considered to be a number of rigid mass elements of finite size connected by massless springs. The infinite number of degrees of freedom associated with a continuous system can thereby be reduced to a finite number of degrees of freedom, which can be examined individually.

2.7.4- Equivalence of finite element and variational (Rayleigh Ritz) methods

If we compare the basic steps of the finite element method with the Rayleigh-Ritz method, we find that both are essentially equivalent. In both methods, a set of trial functions are used for obtaining an approximate solution. Both methods seek a linear combination of trial functions that optimizes (or makes stationary) a given functional. The main difference between the methods is that in the finite element method, the assumed trial functions are not defined over the entire solution domain and they need not satisfy any boundary conditions. Since the trial functions have to be defined over the entire solution domain, the 1Rayleigh-Ritz method can be used only for domains of simple geometric shape. Since elements with simple geometric shape can be assembled to approximate even complex domains, the finite element method proves to be a more versatile technique than the Rayleigh-Ritz method. The only limitation of the finite element method is that the trial functions have to satisfy the convergence (continuity and completeness) conditions [Singiresu S. Rao, 2004].

2.8 Finite differential method

2.8.1- Introduction

The main idea in the finite differential methods is to use approximations to derives solution of a complicated problem by replacing it by a simpler one. Thus the governing differential equation of motion and the associated boundary conditions, if applicable, are replaced by the corresponding finite difference equation. Three type of formulas, forward, backward, and central difference formulas, can be used to derived the finite difference equation.

2.8.2- Technical

In finite difference method, the solution domain replaced with a finite number of points, referred to as mesh or grid points, and seeks to determine the values of the desired solution at these points. The grid points are usually considered to be equally spaced along each of the independent coordinates.

By using Taylor`s series expansion, x_{i+1} and x_{i+2} can be expressed about the grid point *i* as

$$x_{i+1} = x_i + hx_1 \dot{+} \frac{h^2}{2} \ddot{x}_i + \frac{h^2}{6} \dddot{x}_i + \cdots \text{ .... (2.8.2.1)}$$

$$x_{i-1} = x_i - hx_1 \dot{+} \frac{h^2}{2} \ddot{x}_i - \frac{h^2}{6} \dddot{x}_i + \cdots \text{ ....(2.8.2.2)}$$

where $x_i = x(t = t_i)$ and $h = t_{i+1} - t_i = \Delta t$. By taking two terms only and subtracting both equation above, the central difference approximation to the first derivate of $t = t_i$

$$\dot{x}_i = \frac{dy}{dt}|ti = \frac{1}{2h}(x_{i+1} - x_{i-1}) \text{..............................(2.8.2.3)}$$

By taking terms up to the second derivative and adding the x_{i+1} and x_{i-1} , the central difference formula for the second derivative:

$$\ddot{x}_i = \frac{d^2y}{dt^2}|ti = \frac{1}{h^2}(x_{i+1} - 2x_i + x_{i-1}) \text{.....................(2.8.2.4)}$$

2.8.3-Advantages

A disadvantage of the finite difference method is that it is conditionally stable that is the time step A_t has to be smaller than a critical time step (A_t). If the time step A_t is larger than *(At)c, i,* the integration is unstable in the sense that any errors resulting from the numerical integration or round-off in the computations grow and makes the calculation of X meaningless in most cases.

2.9 Approach of Solving Moving Load

There are several approaches from the earlier review, stated that options to solve moving load. After screening the suitable methods to apply for the project, the author extract the definitions as below;

2.9.1 Dirac Delta Function

From http://en.wikipedia.org/wiki/Dirac_delta_function, the Dirac delta or Dirac's delta is a mathematical construct introduced by theoretical physicist Paul Dirac. Informally, it is a generalized function representing an infinitely sharp peak bounding unit area: a 'function' δ(*x*) that has the value zero everywhere except at *x* = 0 where its value is infinitely large in such a way that its total integral is 1. In the context of signal processing it is often referred to as the unit impulse function. In applied mathematics, the delta function is often manipulated as a kind of limit (a weak limit) of a sequence of functions, each member of which has a tall spike at the origin: for example, a sequence of Gaussian distributions centered at the origin with variance tending to zero.

2.9.2 Green Function

The source may be physically located within the region of interest, it may be simulated by certain boundary conditions on the surface of that region, or it may consist of both possibilities. A Green's function is a solution to the relevant partial differential equation for the particular case of a point source of unit strength in the interior of the region and some designated boundary condition on the surface of the region. Solutions to the partial differential equation for a general source function and appropriate boundary condition can then be written in terms of certain volume and surface integrals of the Green's function.

2.9.3 Fourier Series

An infinite series whose terms are constants multiplied by sine and cosine functions and that can, if uniformly convergent, approximate a wide variety of functions. In mathematics, an infinite series used to solve special types of differential equations. It consists of an infinite sum of sines and cosines, and because it is periodic (i.e., its values repeat over fixed intervals), it is a useful tool in analyzing periodic functions.

2.10 Study of Non-Linear Beam Analysis By Mesut Simsek

From the study by Mesut Simsek, researcher from Yildiz Technical University, Turkey,in his journal " Non-linear Vibration Analysis of a Functionally graded Timeshenko beam under action of a moving load" , he specified the technique used for study the non-linear beam structure. In this reference, the effect of large deflection, material distribution, velocity of the moving load and excitation frequency on the beam displacement, bending moments and stresses have been examined in detail.

In this journal, the result gain from the study was compared with the existing result of a linear beam which was gain from the previous study. This is conducted in order to verify the result gain from his study where the result gain from linear beam will be treated as references values. Below are one of the results gain from the Mesut`s study which shows the transverse displacement of the beam with respect to the time for different value of the load velocity (20m/s) and the excitation of frequency (20 rad/s). Both lines of the graph represent the displacement of a linear and non-linear beam.

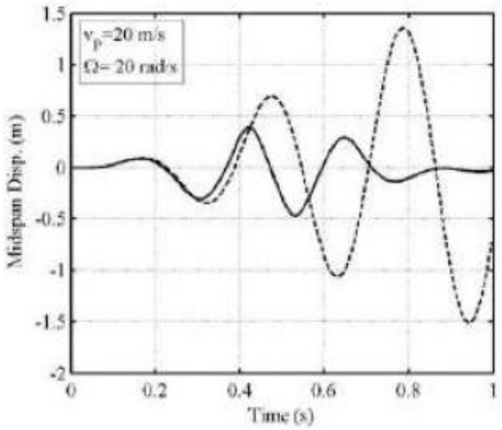


Figure 2.2: The Displacement of the Beam With Respect of Time for Linear (-) and Non-Linear (---) [Mesut Simsek,2010]

The comparison of linear and non-linear beam which have been adopted in the reference can also be implemented in this project.

CHAPTER 3

METHODOLOGY

The project adopts a methodology based on the flowchart below:

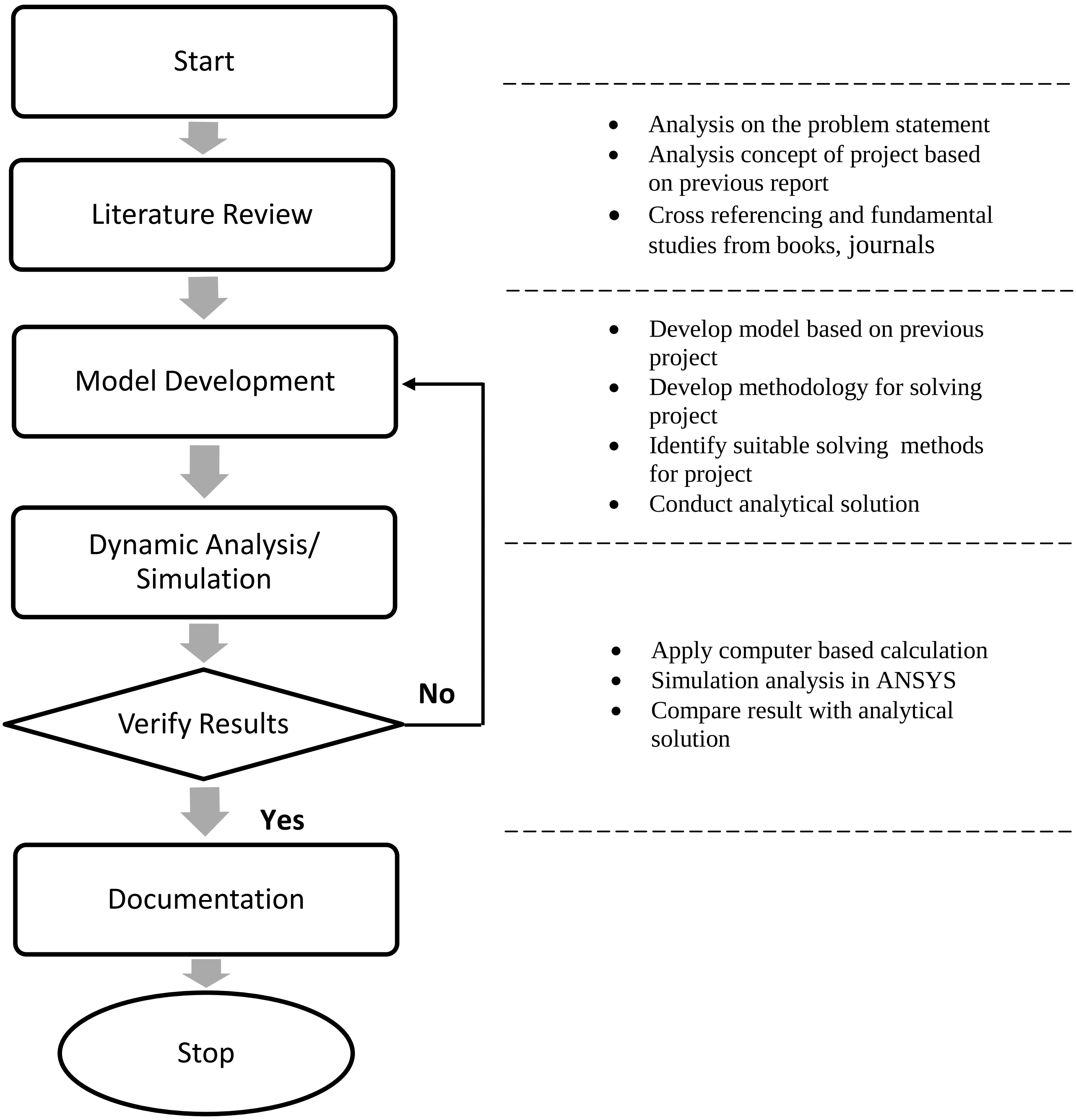


Figure 3.1: The flow of the project

The project will begin with finding of literature review from sources of reference such as journals and books for the fundamental understanding about the project. A complete design of a jetty gangway with its specifications will be developed along with the dimension referring from the actual standards. It is assumed that the dynamic analysis will be more precise and more reliable by using the actual dimension of a jetty gangway.

For the modeling stage, the structural design is taken directly from the previous project which will be applied as the model in this project. Figure 3.2 below represents the general mode for the jetty gangway.

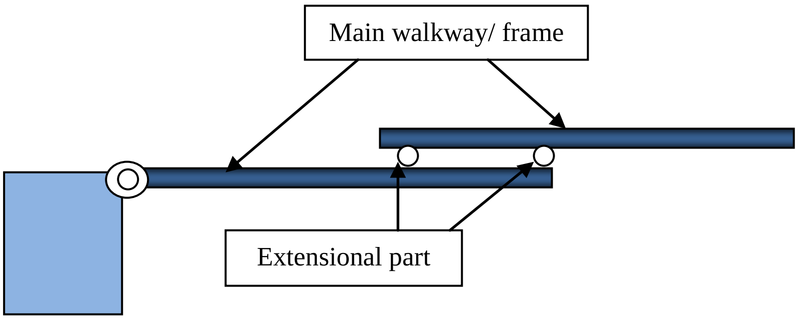


Figure 3.2: The main part of the telescopic jetty gangway

Based on the Jetty gangway structure above, several parameters will be used in the calculation and analysis that need to be investigated. The parameters are:

- Length of the upper and lower walkway
- Width of the upper and lower walkway
- Thickness of the upper and lower walkway
- Material type (affect density, Modulus of Elasticity, Poisson ratio and mass)

The parameter values above will be treated as a constant during the investigation. The standard data of the gangway will be used in order to specify the exact values of the mentioned parameters. The only parameters been manipulated are the value of the moving load amplitude and its frequency. However, during the analytical solution, the model is being considered as one (1) whole body which has an overlap part. This part is located at the extensional part of the jetty gangway. This assumption is taken in order to simplify the dynamical analysis of the model.

During the simulation stage, the simulation software will be used to compare the result with the earlier analytical solution. Approximation method will be applied during this stage which is using ANSYS to validate the result and simulate the structure with the effect of a moving load.

The next step is to study on the basic fundamental of continuous system. The study will include finding information regarding method to define behavior of the jetty gangway with effect of moving load. This will comprise of method on finding the natural frequency and mode shape of the structure under the transverse and longitudinal forces which is considered of a continuous system. Basically, during this activity there will be theoretical understanding involved which mostly referred through the books and journals. This activity is mostly done in order to seek for ideas and method to be used in the project.

After completed with literature review on dynamic analysis, the step to find methodology for solving calculation on dynamics mathematical relation will be developed. This will comprise of method on finding natural frequency and mode shape of the structure in continuous system.

The calculation step generated will be used to perform analytical analysis, in order to determine the equations and its values of the natural frequency, displacement and mode shape of the structure under the influence of a personnel moving on it. The project is then furthered with verification of the data and the graphs.

The project will then be continued with the verification result from the analytical analysis by using simulation software, ANSYS. The obtained result will be proven either if the pattern obtained is similar to the result from the ANSYS. The calculation step will be conducted again if the data are not coherent.

At the end of this project, the outcome can help to determine the behavior of the jetty gangway under influence of a moving load which will then able to provide insightful design parameters for development of the detail design of a telescopic jetty gangway.

Table 1: Gantt chart showing the planner for the first part

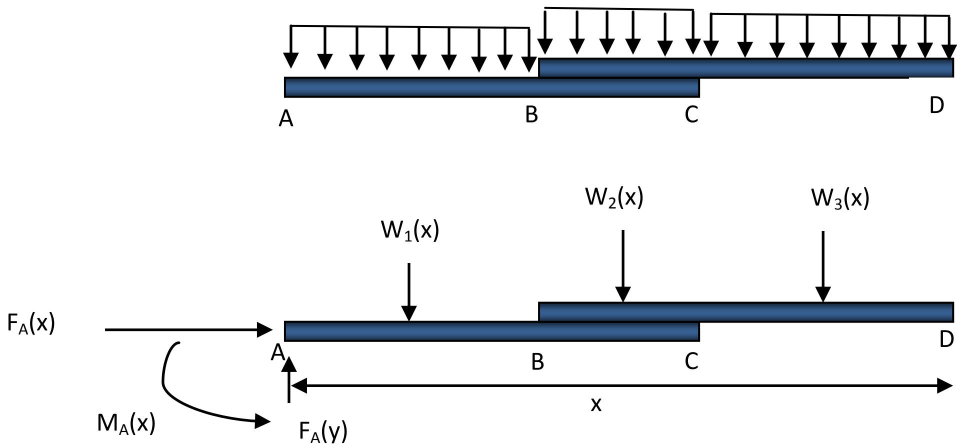
No	Activity	Subject	1	2	3	4	5	6	7	Mid- semester break	8	9	10	11	12	13	14
1	Literature Review	Topic Selection	■														
2	Literature Review	Understanding the basic concept of Jetty gangway		■	■												
3	Literature Review	Fundamental of dynamic study on Jetty gangway			■	■											
4	Documentation	Submission of Preliminary report					●										
5	Literature Review	Method on finding natural frequency					■	■									
6	Literature Review	Method on finding mode shape						■	■								
7	Documentation	Seminar									●						
8	Documentation	Submission of Progress report									●						
9	Literature Review	Generate method for dynamic analysis									■	■	■				
10	Develop methodology	Analysis of the solution											■				
11	Literature Review	Analytical solution for Continuous System												■			
12	Analytical analysis	Analysis of Analytical solution													■	■	
13	Documentation	Submission of Interim report final draft															●
14	Documentation	Oral presentation															●

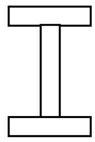
Table 2: Gantt chart showing the planner for the second part

No	Activity	Subject	1	2	3	4	5	6		7	8	9	10	11	12	13	14
1	Review	Continuation on Project Work (Planning)	■						Mid- semester break								
2	Develop methodology	Method on solving response		■	■	■											
4	Documentation	Submission of Progress Report 1				●											
5	Analytical analysis	Method on solving with simulation software				■	■	■		■							
6	Analytical analysis	Analysis of the solution									■	■					
7	Documentation	Seminar									●						
8	Documentation	Submission of Progress Report 2									●						
9	Analytical analysis	Wrap up of project data											■	■			
11	Documentation	Poster Exhibition												●			
12	Analytical analysis	Overall project review													■	■	
13	Documentation	Submission of dissertation final draft															●
14	Documentation	Oral presentation															●
15	Documentation	Submission of Dissertation (hard bound)															

3.1 Approach to Finding Free Forced Vibrations

Several approaches have been studied in order to solve the structure problem. The free forced vibration is discussed in this sub-section.





The structure is considered as a simplified I-beam model

Figure 3.3: Load distribution and free body diagram of the gangway

The data of the beam are as the following;

E, Modulus of elasticity: 69 *Gpa* (aluminium)

m, Mass of beam: 106.173*kg* for A-B and C-D, 106.173kg for B-C

Length between point B and point C: 1, point A-B or C-D: 2m

L, Total length: 5.0 *m*

H , height: 611mm

A, Cross Sectional Area: point A-B & C-D: 0.0198 m^2, point B-C: 0.0396 m^2

I, Moment of Inertia: point A-B & C-D: 0.000108 m^4,

point B-C: 0.000216 m^4

ρ, Density: 2700 kg/m^3

The governing equation of a beam subject to a concentrated force, shown in Fig. 3.3 above, can be given by *w(x,t);*

$$EI\frac{\partial^4 y}{\partial x^4}+\rho A\frac{\partial^4 y}{\partial t^4}=0 \qquad (3.1.1)$$

where w*(x,t)*, represents the deflection of the beam, *x* represents the traveling direction of the moving load, and *t* represents time. Also, *EI* is the rigidity of the beam, *E* is Young's modulus of elasticity, *I* is the cross sectional moment of inertia of the beam, ρ is the density per unit length of the beam and *A* is the cross-sectional area of the beam. The beam length is *l*.

Its particular solution can be sought in the following form;

$$y(x,t)= Y(x)g(t) \qquad (3.1.2)$$

Y(x) comprising function of x while *g(t)* comprising function of *t*.

Separating (1) as per (2) yields;

$$\frac{EI}{\rho A y(x)}.\frac{\partial^4 y(x)}{\partial x^4} = -\frac{\ddot{g}(t)}{g(t)} \qquad (3.1.3)$$

Let the φ^2= separation constant, and be applied to both sides of equation (3.1.3).

Apply constant to left side of equation (3.1.3):

$$\frac{EI}{\rho A y(x)}.\frac{\partial^4 Y(x)}{\partial x^4} = \varphi^2$$

rearranging yields;

$$\frac{\partial^4 Y(x)}{\partial x^4} - \frac{\rho A y(x)\varphi^2}{EI} = 0 \qquad (3.1.4)$$

Equation (3.1.4) can be rewritten to;

$$\frac{\partial^4 Y(x)}{\partial x^4} - \frac{\lambda^4 y(x)}{EI} = 0 \qquad (3.1.5)$$

Where

$$\lambda^4 = \frac{\rho A \varphi^2}{EI} \qquad (3.1.6)$$

Apply constant to right side of equation (3.1.3):

$$-\frac{\ddot{g}(t)}{g(t)} = \varphi^2$$

rearranging yields;

$$\ddot{g}(t) + \varphi^2 g(t) = 0 \qquad (3.1.7)$$

Applying sinusoidal force to equation g(t);

$g(t) = F_D \sin \omega_n t$, and substitute into equation (3.1.5) yields;

$$-\omega_n^2 F_D \sin \omega_n t, + \varphi^2 F_D \sin \omega_n t = 0 \qquad (3.1.8)$$

Simplify equation (3.1.6) equals;

$$\omega_n^2 = \varphi^2 \qquad (3.1.9)$$

For the relation of equation (3.1.6) can be replaced by (3.1.9) to become;

$$\lambda^4 = \frac{\rho A \omega_n^2}{EI} \qquad (3.1.10)$$

The formal homogenous function for the Fixed –free beam is:

$$W(x) = A_n \sinh(\lambda_n x) + B_n \cosh(\lambda_n x) + C_n \sin(\lambda_n x) + D_n \cos(\lambda_n x) + \ldots \qquad (3.1.11)$$

The boundary conditions and the initial conditions for the simply supported beam are:

At point (x=0)

Deflection: Y (x=0) =0 (3.1.12)

Slope: $\frac{dY(x)}{dx} = 0$ (3.1.13)

At point (x=L), where L=5

Bending moment: $\frac{d^2Y(x=L)}{dx^2} = 0$ (3.1.14)

Shear force: $-\frac{EI\, d^3Y(x=L)}{dx^3} = 0$ (3.1.15)

The derivative of equation (4.1.11) are as the following:

$$W^I(x) = A_n\lambda_n \cosh(\lambda_n x) + B_n\, \lambda_n \sinh\,(\lambda_n x) + C_n\, \lambda_n \cos(\lambda_n x) - D_n\lambda_n \sin(\lambda_n x)$$

(3.1.16)

$$W^{II}(x) = A_n{\lambda_n}^2 \sinh(\lambda_n x) + B_n\, {\lambda_n}^2 \cosh(\lambda_n x) - C_n{\lambda_n}^2 \sin(\lambda_n x) - D_n$$

(3.1.17)

$$W^{III}(x) = A_n{\lambda_n}^3 \cosh(\lambda_n x) + B_n\, {\lambda_n}^3 \sinh(\lambda_n x) - C_n\, {\lambda_n}^3 \cos(\lambda_n x) + D_n{\lambda_n}^3\, s\,in(\lambda_n x)$$

(3.1.18)

Then substitute the equation (3.1.11) and it derivatives into respective boundary condition equation

From equation (3.1.11) and (3.1.12)

$$B_n + D_n = 0 \quad (3.1.19)$$

From equation (3.1.16) and (3.1.13)

$$A_n + C_n = 0 \tag{3.1.20}$$

From equation (3.1.17) and (3.1.14)

$$A_n \sinh(\lambda_n L) + B_n \cosh(\lambda_n L) - C_n \sin(\lambda_n L) - D_n \cos(\lambda_n L) = 0 \tag{3.1.21}$$

From equation (3.1.18) and (3.1.15)

$$A_n \lambda_n{}^3 \cosh(\lambda_n L) + B_n \lambda_n{}^3 \sinh(\lambda_n L2) - C_n \lambda_n{}^3 \cos(\lambda_n L) + D_n \lambda_n{}^3 \, s\,in(\lambda_n L) = 0 \tag{3.1.22}$$

Then, rearrange the equation (3.1.19), (3.1.20), (3.1.21) & (3.1.22) into the matrix form,

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ \sinh(\lambda_n L) & \cosh(\lambda_n L) & -\sin(\lambda_n L) & -\cos(\lambda_n L) \\ \lambda_n{}^3 \cosh(\lambda_n L) & \lambda_n{}^3 \sinh(\lambda_n L) & -\lambda_n{}^3 \cos(\lambda_n L) & \lambda_n{}^3 \, s\,in(\lambda_n L) \end{bmatrix} \begin{bmatrix} A_n \\ B_n \\ C_n \\ D_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{3.1.23}$$

The non-zero solution of this set of equations exists if and only if its characteristic determinant is equal to zero

$$\left\| \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ \sinh(\lambda_n L) & \cosh(\lambda_n L) & -\sin(\lambda_n L) & -\cos(\lambda_n L) \\ \lambda_n{}^3 \cosh(\lambda_n L) & \lambda_n{}^3 \sinh(\lambda_n L) & -\lambda_n{}^3 \cos(\lambda_n L) & \lambda_n{}^3 \, s\,in(\lambda_n L) \end{bmatrix} \right\| = 0$$

The determinants need to be simplified in order to plot a graph for finding the λ_n (root value). The full calculation steps for the determinants simplification is attached in the appendix for references.

The simplification equation is:

$$cos(\lambda_n L)\ cosh(\lambda_n L) + 1 = 0 \qquad (3.1.24)$$

where L = 5m

A graph is being plotted using Microsoft Excel based on equation (3.1.23) for finding the value of λ_n.

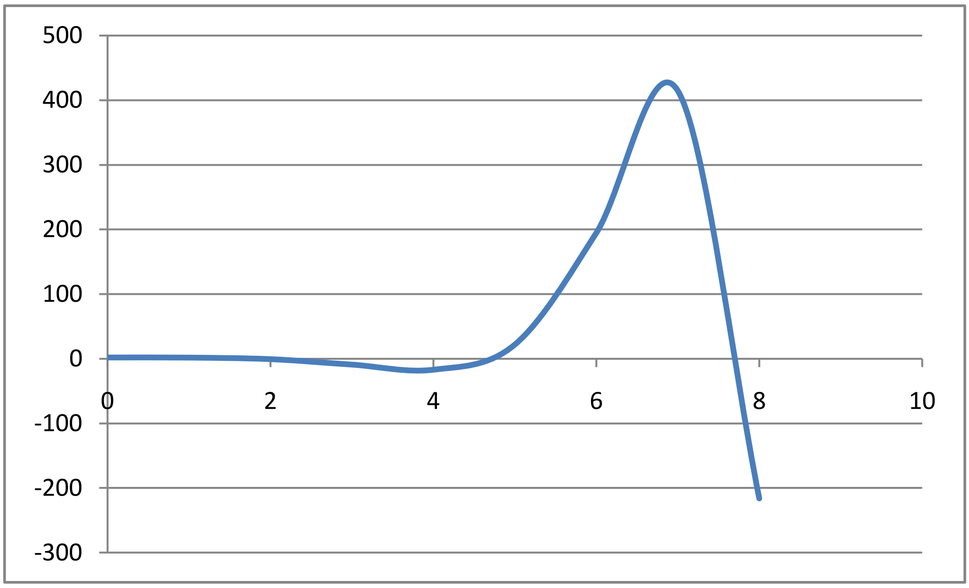


Figure 3.4: Graph plot of $y((\lambda_n L) = \ cos(\lambda_n L)\ cosh(\lambda_n L) + 1$

Assuming $x = (\lambda_n L)$

From this diagram, the first three roots are

$x_1 = 1.764259$ (interception 1)

$x_2 = 4.433154$ (interception 2)

$x_3 = 7.657489$ (interception 3)

Substitute L = 5m

$$\lambda_1 = 0.3528518$$

$$\lambda_2 = 0.8866508$$

$$\lambda_3 = 1.53094978$$

The relationship (10)

$$\lambda_n^4 = \frac{\rho A {\omega_n}^2}{EI}$$

$$\omega_n = \sqrt{\frac{EI\lambda^4}{\rho A}}$$

offers values for the wanted natural frequencies

$$\omega_1 = \sqrt{\frac{EI\lambda_1^4}{\rho A}} = 46.486\ s^{-1}$$

$$\omega_2 = \sqrt{\frac{EI\lambda_2^4}{\rho A}} = 293.525\ s^{-1}$$

$$\omega_3 = \sqrt{\frac{EI\lambda_3^4}{\rho A}} = 875.107\ s^{-1}$$

Finding the mode shape of the beam

For each of these roots the set of equation (3.1.23) becomes linearly dependant. The equations allow the constants A_n , $B_{n,}$ $C_{n,}$ and D_n to be computed.

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ \sinh(\lambda_n L) & \cosh(\lambda_n L) & -\sin(\lambda_n L) & -\cos(\lambda_n L) \\ \lambda_n{}^3\cosh(\lambda_n L) & \lambda_n{}^3\sinh(\lambda_n L) & -\lambda_n{}^3\cos(\lambda_n L) & \lambda_n{}^3\, s\,in(\lambda_n L) \end{bmatrix} \begin{bmatrix} A_n \\ B_n \\ C_n \\ D_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{F_D}{EI} \end{bmatrix}$$

From the matrix form equation above, the values of constant A_n , B_n, C_n & D_n can be found since values of λ_n already been determined in calculation above.

The matrix been calculated by using MATLAB, and the calculation steps is attached in appendix for references.

Result of calculating

$$\begin{bmatrix} A_1 \\ B_1 \\ C_1 \\ D_1 \end{bmatrix} = \begin{bmatrix} -0.0102 \\ 0.0138 \\ 0.0102 \\ 0.0138 \end{bmatrix} \qquad \begin{bmatrix} A_2 \\ B_2 \\ C_2 \\ D_2 \end{bmatrix} = \begin{bmatrix} 0.3798 \\ -0.3735 \\ -0.3798 \\ 0.3735 \end{bmatrix} \qquad \begin{bmatrix} A_3 \\ B_3 \\ C_3 \\ D_3 \end{bmatrix} = \begin{bmatrix} -0.9405 \\ 0.9412 \\ 0.9405 \\ -0.9412 \end{bmatrix}$$

After getting all of the values above, the mode shape of the lower beam can be determined.

Introducing the values of λ_n , ω_n,A_n, B_n, C_n & D_n according to equation (3.1.11),

$$\therefore\ W_1(x) = -0.0102\sinh(\lambda_n x) + 0.0138\cosh(\lambda_n x) + 0.0102\sin(\lambda_n x) - 0.0138\cos(\lambda_n x) \qquad (3.1.25)$$

$$\therefore\ W_2(x) = 0.3798\sinh(\lambda_n x) - 0.3735\cosh(\lambda_n x) - 0.3798\sin(\lambda_n x) + 0.3735\cos(\lambda_n x) \quad (3.1.26)$$

$$\therefore\ W_3(x) = -0.9405\sinh(\lambda_n x) + 0.9412\cosh(\lambda_n x) + 0.9405\sin(\lambda_n x) - -0.9412\cos(\lambda_n x) \quad (3.1.27)$$

3.2 Approach to Finding Forced Vibrations

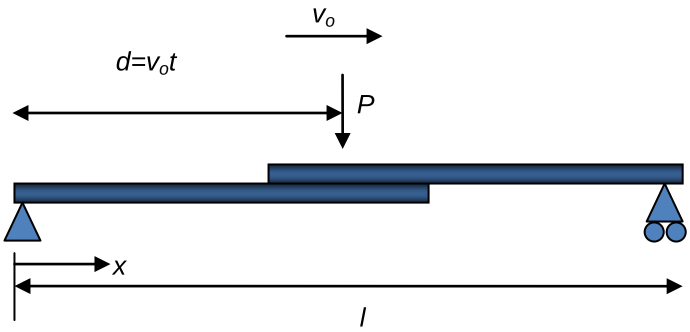


Figure 3.5: Jetty gangway subjected to a moving concentrated load

Consider the gangway as a beam subjected to a concentrated load *P* that moves at a constant speed v_o along the beam as shown in the figure above. The boundary conditions of the fix-simply supported beam are given by:

$$w(0,t) = 0 \quad (3.2.1)$$

$$\frac{d^2w}{dx^2}(0,t) = 0 \quad (3.2.2)$$

$$w(l,t) = 0 \quad (3.2.3)$$

$$\frac{d^2w}{dx^2}(l,t) = 0 \quad (3.2.4)$$

Assume that the beam to be at rest initially, so that initial conditions:

$$w(x,0) = 0 \quad (3.2.5)$$

$$\frac{d^2w}{dt^2}(x,0) = 0 \quad (3.2.6)$$

The approach is represented using a Fourier Series. For this, the concentrated load *P* acting at $x=d$ is assumed to be distributed uniformly over an elemental length $2\Delta x$ centred at $x=d$ as shown in the Figure 3.6. Now the distributed force, *f*(*x*), can be defined as

$$f(x) = \begin{cases} 0 & for\ 0 < x < d - \Delta x \\ \frac{P}{2\Delta x} & for\ d - \Delta x \le x \le d + \Delta x \\ 0 & for\ d + \Delta x < x < l \end{cases}$$

(3.2.7)

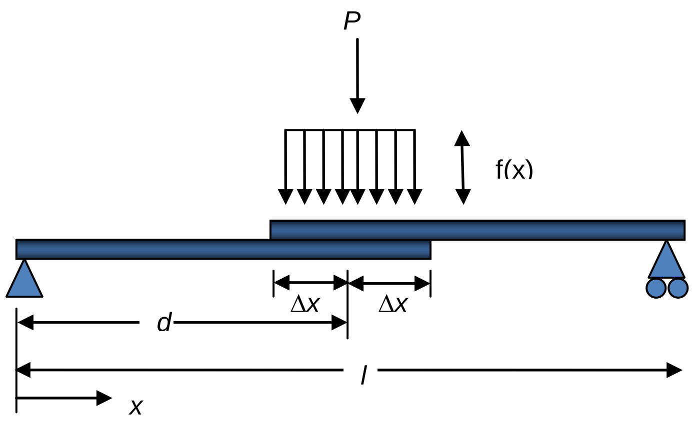


Figure 3.6: Concentrated load assumed to be uniformly distributed over a length $2\Delta x$

From Fourier Series analysis, it is known that if a function *f*(x) is defined only over a finite interval (e.g., from x_o to x_o+L), the definition of the function *f*(*x*) can be extended for all values of *x* and can be considered to be periodic with period *L*. The Fourier series expansion of the extended periodic function converges to the function *f*(*x*) in the original interval from x_o+L. As a specific case, if the *f*(*x*) is defined over the interval 0 to *l*, its Fourier series expansion in terms of only sine terms is given by

$$f(x) = \sum_{n=1}^{\infty} f_n \sin\frac{n\pi x}{l}$$

(3.2.8)

Where the coefficients f_n are given by

$$f_n = \frac{2}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}\,dx$$

(3.2.9)

The Fourier coefficients f_n can be computed, using Eq. (4.2.7) for $f(x)$, as

$$f_n = \frac{2}{l}\left[\int_0^{d-\Delta x}(0)\left(\sin\frac{n\pi x}{l}\right)dx + \int_{d-\Delta x}^{d+\Delta x}\frac{P}{2\Delta x}\left(\sin\frac{n\pi x}{l}\right)dx + \int_{d+\Delta x}^{l}(0)\left(\sin\frac{n\pi x}{l}\right)dx +\right]$$

$$= \frac{P}{l\Delta x}\int_{d-\Delta x}^{d+\Delta x}\sin\frac{n\pi x}{l}\,dx = \frac{2P}{l}\sin\frac{n\pi d}{l}\frac{\sin(n\pi\Delta x/l)}{n\pi\Delta x/l}$$

(3.2.10)

Since P is actually a concentrated load acting at x=d, we let $\Delta x \to 0$ in Eq. (3.2.10) with

$$\lim_{\Delta x\to 0}\frac{\sin(n\pi\Delta x/l)}{(n\pi\Delta x/l)} = 1$$

(3.2.11)

to obtain the coefficients

$$f_n = \frac{2P}{l}\sin\frac{n\pi d}{l}$$

(3.2.12)

Thus, the Fourier sine series expansion of the concentrated load acting at x=d can be expressed as

$$f(x) = \frac{2P}{l}\sum_{n=1}^{\infty}\sin\frac{n\pi d}{l}\sin\frac{n\pi x}{l}$$

(3.2.13)

Using d=v_ot in Eq. (4.2.7), the load distribution can be represented in terms of x and t as

$$f(x,t) = \frac{2P}{l}\left(\sin\frac{\pi x}{l}sin\frac{\pi v_o t}{l} + \sin\frac{2\pi x}{l}sin\frac{2\pi v_o t}{l} + \sin\frac{3\pi x}{l}sin\frac{3\pi v_o t}{l} + \cdots\right)$$

(3.2.14)

The response of the beam under the *n*th component of the load represented by Eq.(3.2.14), can be obtained using the below equation, which is the response solution of the beam:

$$w(x,t) = \frac{f_0 l^4}{EI(n\pi)^4[1-(\omega/\omega_n)^2]} \sin\frac{n\pi x}{l}\left(sin\omega t - \frac{\omega}{\omega_n} sin\omega_n t\right) \tag{3.2.15}$$

Therefore yields,

$$w(x,t) = \frac{2Pl^3}{EI(n\pi)^4[1-(2\pi v_0/l\omega_n)^2]} \sin\frac{n\pi x}{l}\left(sin\frac{2\pi v_0}{l} t - \frac{2\pi v_0}{\omega_n l} sin\omega_n t\right) \tag{3.2.16}$$

3.3 Simulation using ANSYS

Simulation software is being used to verify the vibrational response of the structure due to moving load. The models have been constructed in ANSYS, and analysis of moving load is being performed. 25 nodes are being placed at the beam and the load is being applied subject to velocity and time. The boundary conditions for the model is being set that to have fixed point at the near end of the beam and simply supported at the far end of the beam. The properties of the beam are set as in Chaper 3.1. The middle part (B-C) is set to have different properties as from the side parts (A-B and C-D).

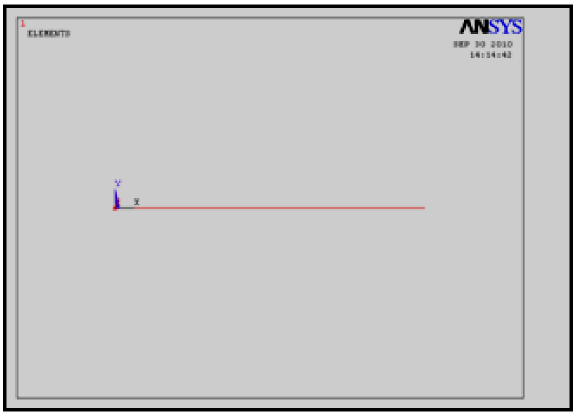


Figure 3.7: Model of the beam

CHAPTER 4

RESULTS AND DISCUSSION

4.1 Free Forced Vibration

4.1.1 Results

The graphs below show the mode shapes of n=1, n=2 and n=3, which are plotted based on equations (3.1.25), (3.1.26) and (3.1.27) as below;

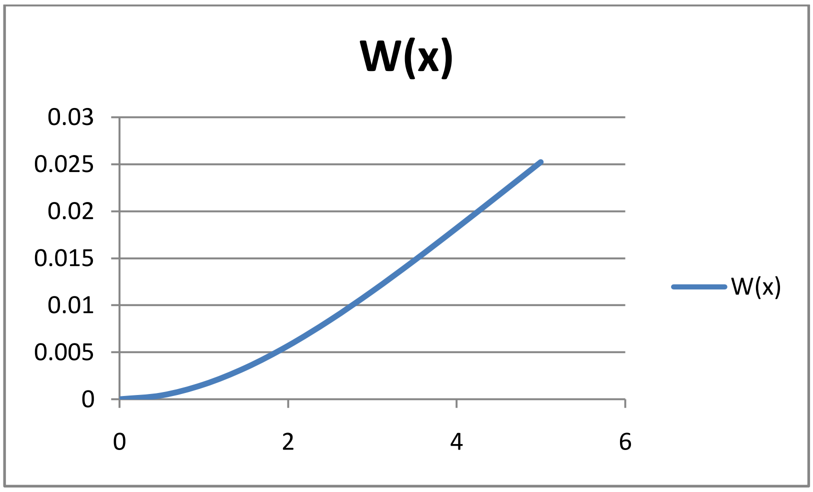


Figure 4.1: The mode shape n= 1 of the beam

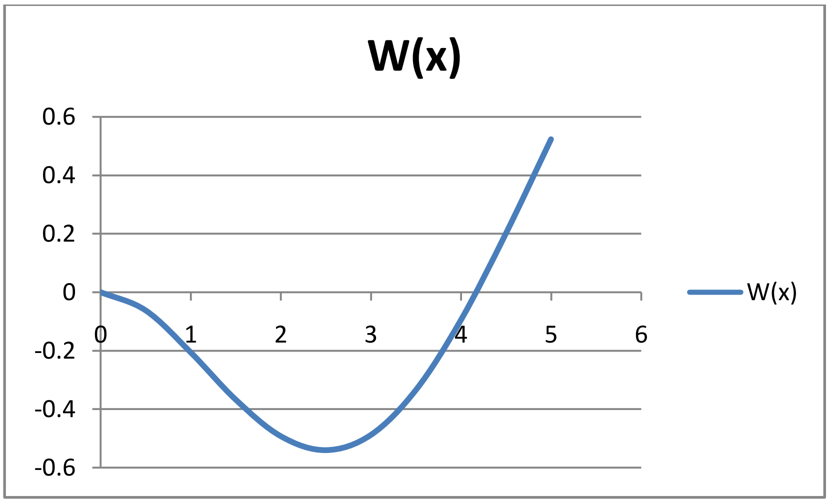


Figure 4.2: The mode shape n = 2 of the beam

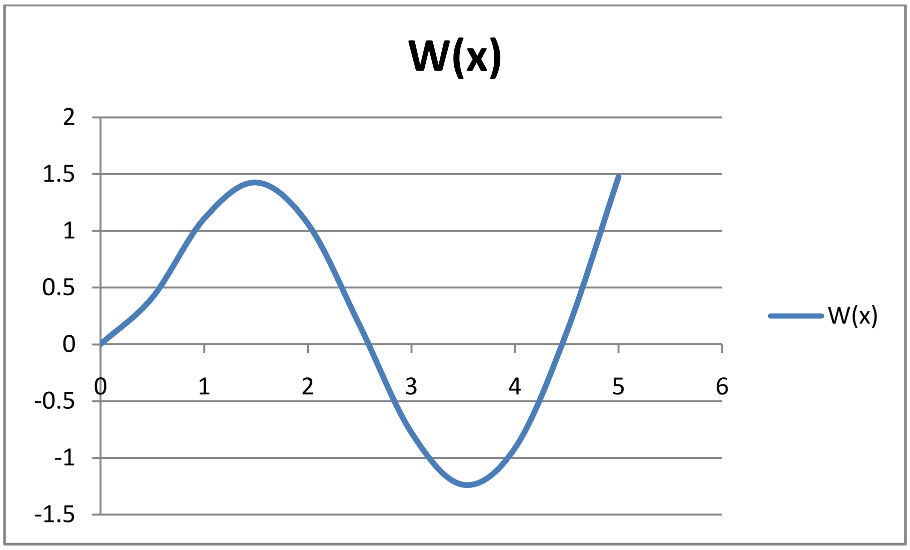


Figure 4.3: The mode shape n = 3 of the beam

4.1.2 Discussion

In the result section, a set of value for natural frequency is shown. The values of natural frequency change with the different mode or n value. The value of λ also play role in affecting the value of natural frequency. The modes are chosen to be limit until three.

From the mode shapes graphs, we can study the pattern and compare it with the general shape. It can be seen that the mode shape graphs for n=1, n=2, and n=3 of a beam with respective frequency are roughly similar to the general natural modes. This shows that the mode shape obtained from the calculation results are valid since the pattern is almost similar with the expected mode shape pattern. However, the mode shape obtained was not really what is as expected from the beginning. This is because the graphs are smooth eventhough there are overlap part at the beam. The overlap part is containing double of bulk mass, therefore it supposed to show the difference in the mode shape when the graphs pass the point B-C at the beam.

The problem is expected to occur because of insufficient boundary conditions applied to the beam. Therefore, the beam should be segmented into 3 parts and for every part, several boundary conditions and properties will be applied into equations during the analytical steps.

4.2. Forced Vibration

4.2.1 Results

The graphs showing the response of the beam according certain criteria are plotted based on equation (4.2.15);

Load, P	1000N
Length, L	5m
Modulus of elasticity,E	69GPa
Moment of Inertia, I	$0.000108m^4$
Moment of Inertia, I (b-c)	$0.000216m^4$
Nat. Freq.	$46.486s^{-1}$

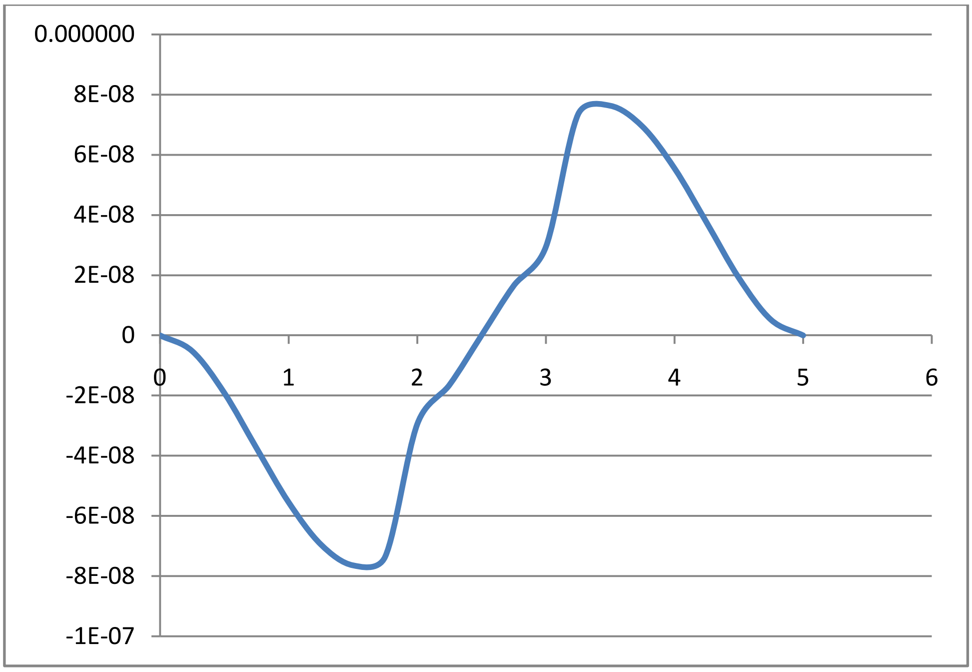


Figure 4.4: The response of the beam when n = 1, v_o=1m/s

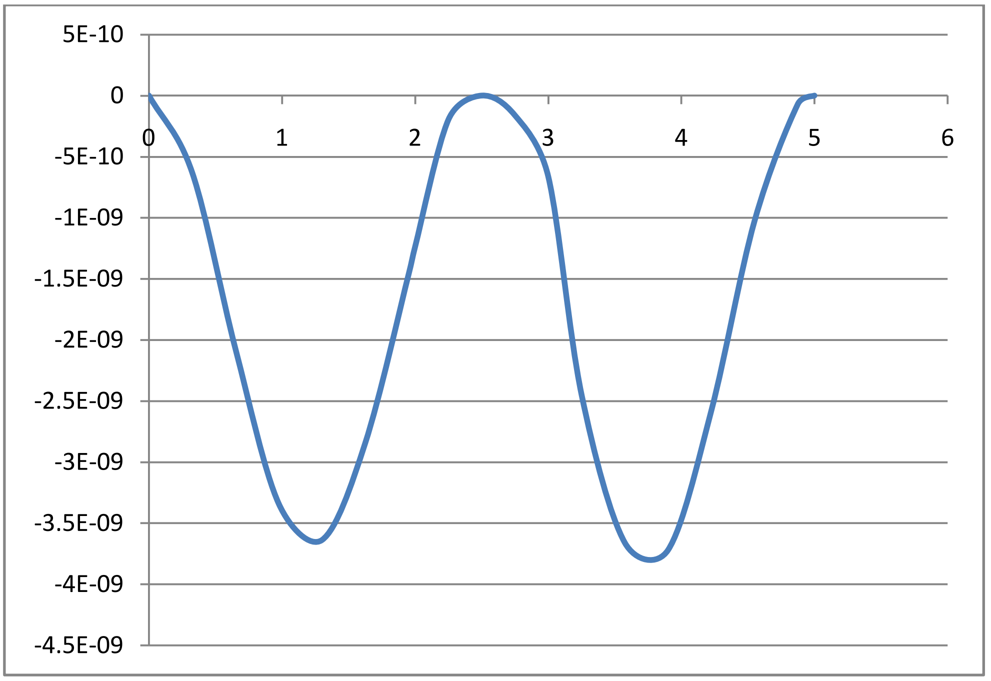


Figure 4.5: The response of the beam when n = 2, v_o=1.3m/s

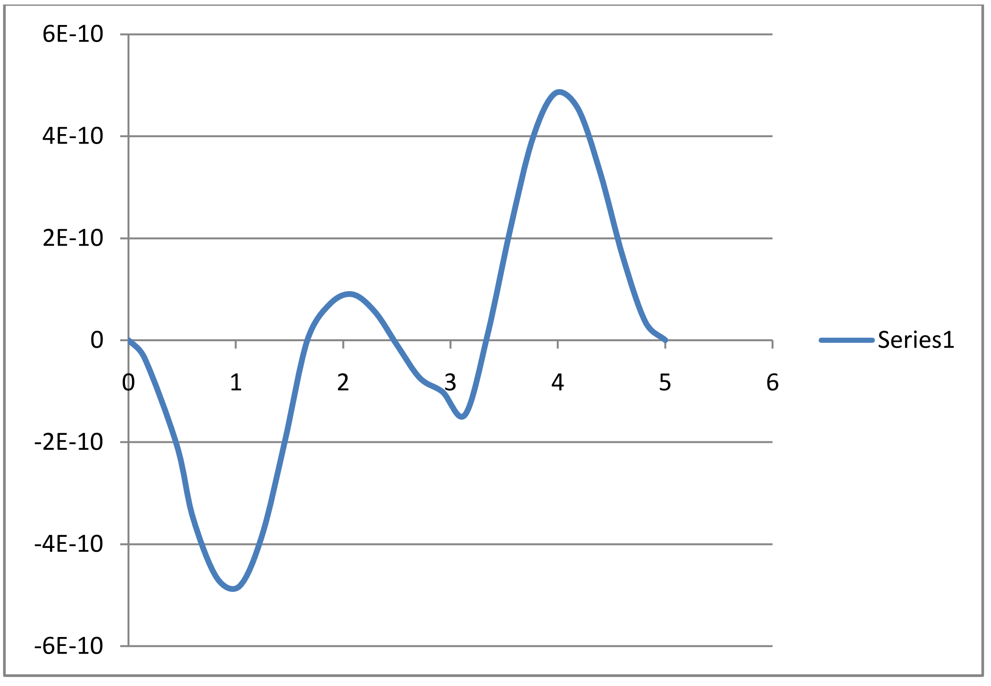


Figure 4.6: The response of the beam when n = 3, v_o=1.5m/s

4.2.2 Discussion

Figure 4.4 to Figure 4.6 refer to response of the beam when applied to certain frequency of moving load. The moving load moves in different constant velocity for each model, varies from $1ms^{-1}$, $1.3ms^{-1}$ to $1.5ms^{-1}$. The mode shape for the structure, in general, is following the typical mode shape pattern without any abnormal graphs shown. From the figures, the responses of the beam will show clear sinusoidal characteristics as the value of the moving load frequencies increased. It can be visibly observed since the number of peak and dip of beam displacement increased as the value of frequencies increased.

It can also visibly seen that a smooth transition in the displacement at overlap part of the beam (from 2m to 3m) when the moving load past the part. The displacement transformed to be slower at the overlap part. This is due to the transition in the stiffness of the beam.

4.3 ANSYS Simulation

4.3.1 Results of Mode Shapes:

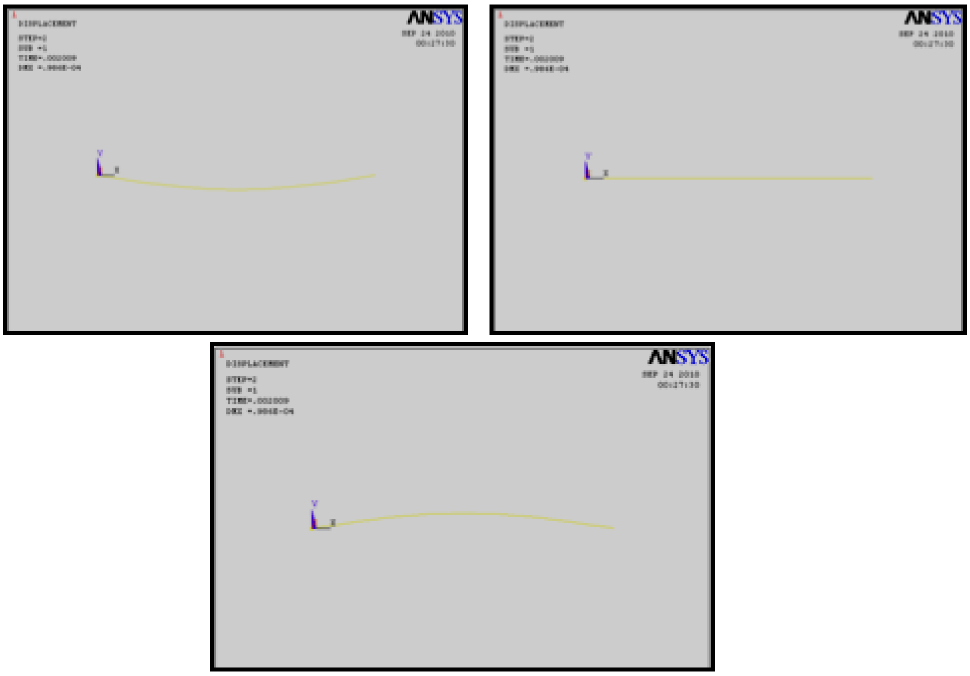


Figure 4.7: Animation of mode 1

The above figures show the animation of first mode using ANSYS. The velocity of the moving load is set to be at v= $1ms^{-1}$. The displacement of the beam subject to y axis is shown along the length beam.

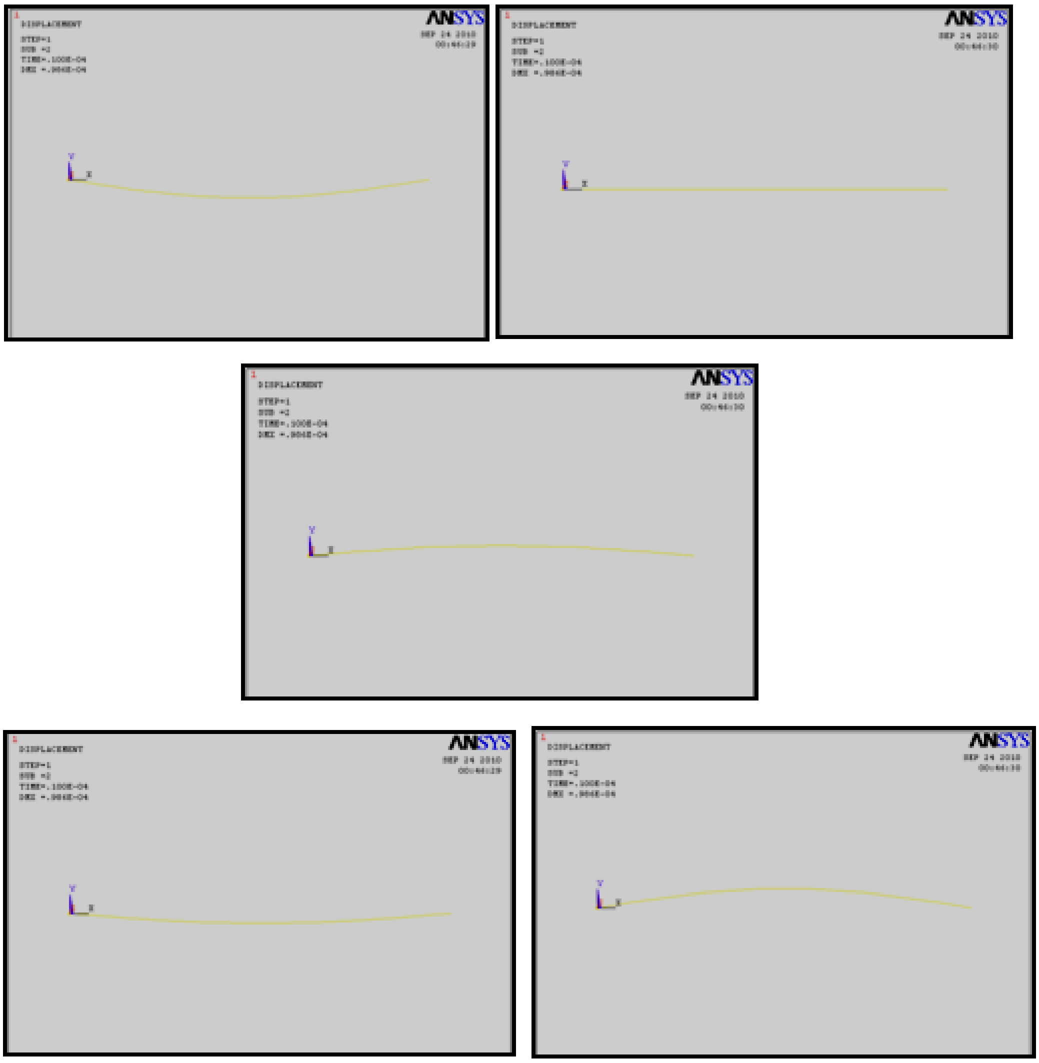
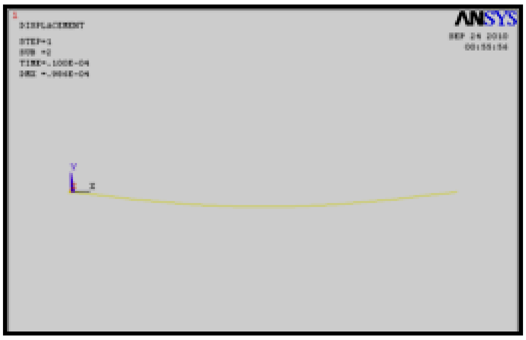
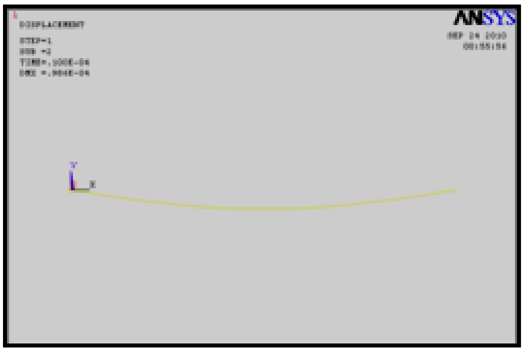
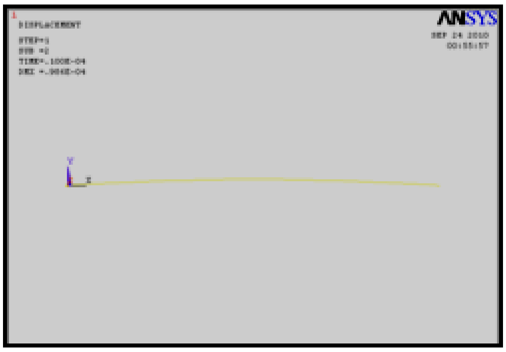
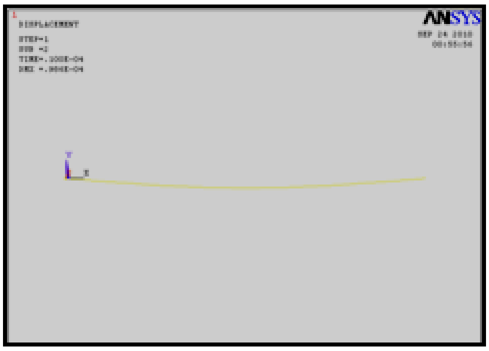
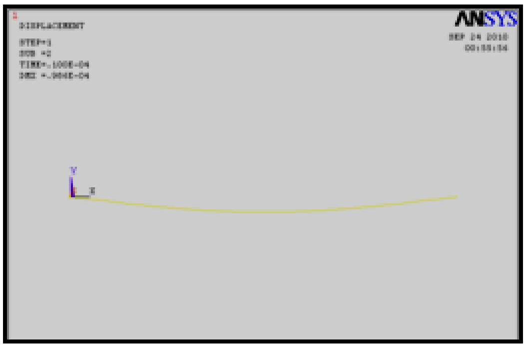
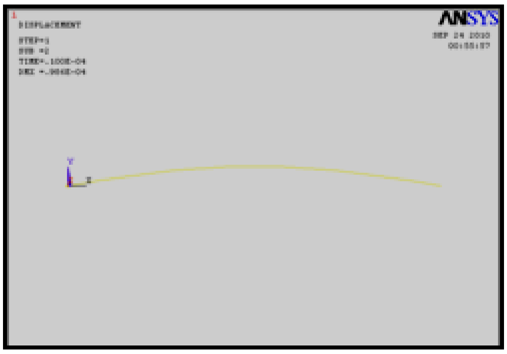
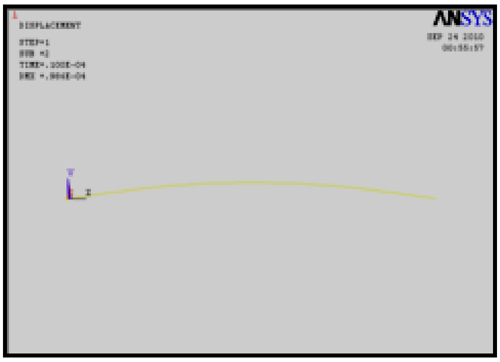


Figure 4.8: Animation of mode 2

The above figures show the animation of second mode using ANSYS. The velocity of the moving load is set to be at v= $1.3ms^{-1}$. The displacement of the beam subject to y axis is shown along the length beam.

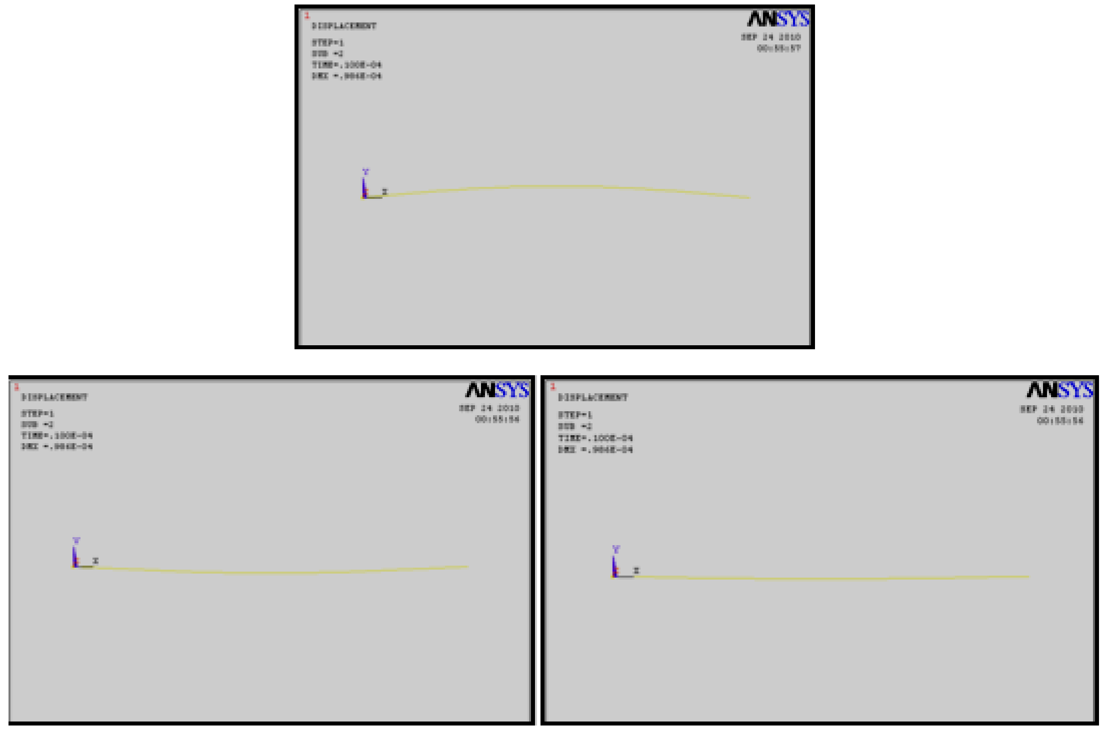


Figure 4.9: Animation of mode 3

The above figures show the animation of the third mode using ANSYS. The velocity of the moving load is set to be at v= 1.5ms^{-1}. The displacement of the beam subject to y axis is shown along the length beam.

4.3.2 Discussion

The mode shapes from Figure 4.7 to Figure 4.9 represent the mode shape of the beam from Mode 1 to Mode 3. The mode shapes behave like a usual mode shape and we can say the result of the simulation is valid. The mode shapes also represent the results obtained from the numerical analysis in the Forced Vibration.

4.3.3 Results of Deformed Shapes:

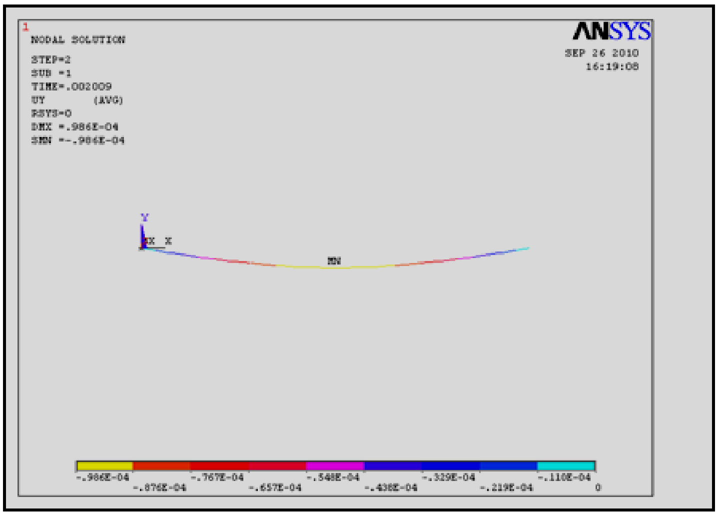


Figure 4.10: Deformed shape of mode 1

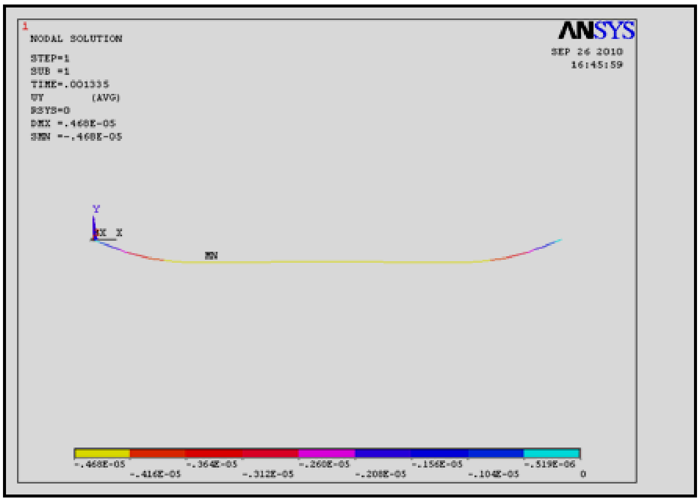


Figure 4.11: Deformed shape of mode 3

4.3.4 Discussion

The deformed shape of mode 1 as shown in the Figure 4.10 shows that the stress load is more concentrated at the area where the beam is supported. It can also visibly seen that the middle part of the beam experience less stress. This is because the middle beam part is overlapped and has difference element properties. Therefore, the middle beam is stiffer compared to the part approximating the supports.

For the higher velocity of moving load as in Figure 4.11, the beam displaced more frequently making the stress to move further to the support of the beam. The length of the flat zone is longer compared to Mode 1. The potential of the beam to experience defect could first be caused from the support area of the beam.

4.3.5 Results of Displacement versus Time:

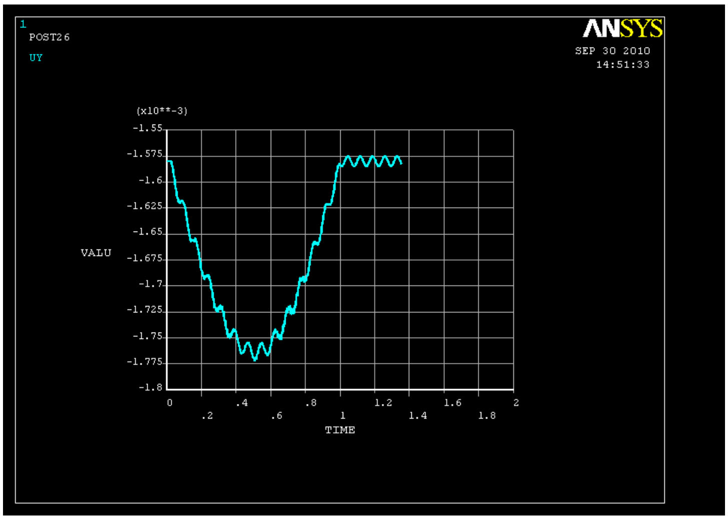


Figure 4.12: The displacement versus time for velocity= 1ms^{-1}

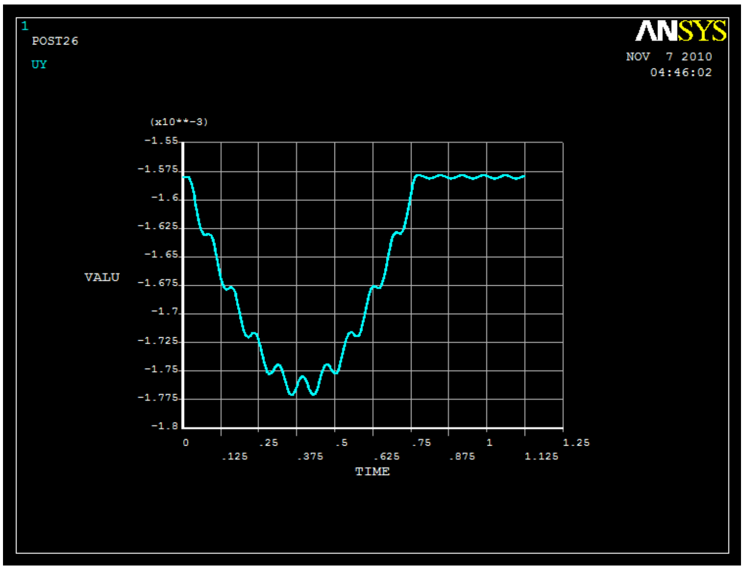


Figure 4.13: The displacement versus time for velocity= 1.3ms^{-1}

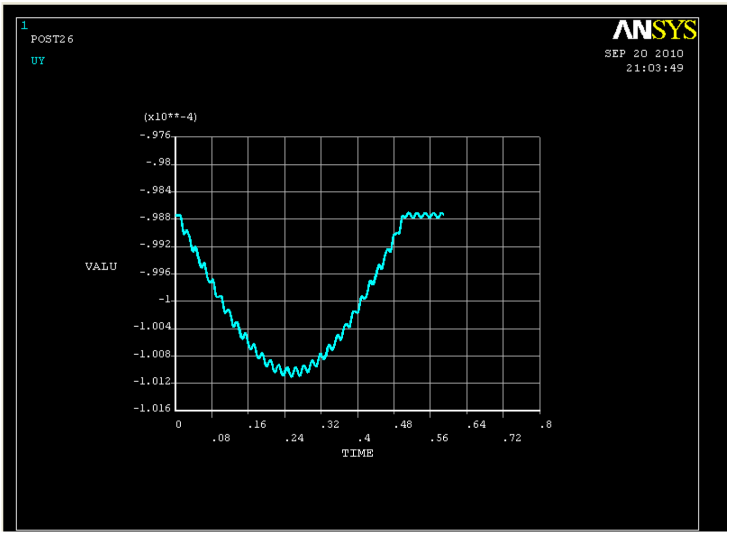


Figure 4.14: The displacement versus time for velocity= 1.5ms^{-1}

4.3.6 Discussion

The above graphs are plotted as the displacement of the beam subjected to time based on the moving load velocity. The shape of the both figures are almost the same, only that in Figure 4.14 the displacement is more frequent compared from Figure 4.12 and Figure 4.13. This could be affected by the higher velocity of the speed applied to the beam.

CHAPTER 5

CONCLUSION AND RECOMMENDATION

5.1 Conclusion

The project was related to study the dynamic characteristics of the jetty gangway structure under the influence of a moving load. The approach of solving response is developed by using Fourier Series and the response obtained are shown in the graphs. The jetty gangway structure is modeled as a single structure with different cross sectional areas at different sections of the beam, where the near end and middle extension part are considered fix, while the far end is simply supported. The results concluded that the approach adopted in this report has been approved as similarity is achieved when comparison with the reference is made.

There are three main relations can be found thoroughly in this project which are the natural frequency, the mode shape and the response of the structure under the influence of a moving load. The pattern of the mode shape under the influence of moving load shows transition pattern when approximating the middle part of the model. This is due to the complexity of the middle beam profile.

The project had discussed application moving load into the system and shown the response of the beam. The values of displacement compared using analytical approach and simulation approach are showing similarity.

5.2 Recommendation

Other than using Fourier Series to solve the moving load problem, there are several more options to be used as an alternative. The approach should be taken and compared for a further verification. The other alternatives are Green Function and Dirac Delta Function.

The complexity of the structure may also be solved more accurately using non-linear approaches such as Finite Element Method and Extended Hamilton Principle. The Finite Element Method can be used to calculate natural frequencies and mode shapes of any linear elastic system that may require more computational effort in order to get the values better, however the solutions might be tedious and lengthy.

The study should be extended by more powerful operating software than ANSYS or MATLAB. The software such as BEDAS can simulate more accurate Dynamic Analysis solution for beams and bridges under the influence of moving load. The simulation data are required to show the real behavior of the structure under a moving load. Therefore, an extension for parametric study for a variation under several parameters of the structure can be done to improve the integrity of the telescopic jetty gangway. The manipulation of the parameters will generate different results of natural frequency, mode shape and response.

CHAPTER 6

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Appendix

Determinant simplification:

$$\left\|\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ \sinh(\lambda_n L) & \cosh(\lambda_n L) & -\sin(\lambda_n L) & -\cos(\lambda_n L) \\ {\lambda_n}^3\cosh(\lambda_n L) & {\lambda_n}^3\sinh(\lambda_n L) & -{\lambda_n}^3\cos(\lambda_n L) & {\lambda_n}^3 s\,in(\lambda_n L) \end{bmatrix}\right\| = 0$$

$$= -1\begin{bmatrix} 1 & 1 & 0 \\ \sinh(\lambda_n L) & -\sin(\lambda_n L) & -\cos(\lambda_n L) \\ {\lambda_n}^3\cosh(\lambda_n L) & -{\lambda_n}^3\cos(\lambda_n L) & {\lambda_n}^3 s\,in(\lambda_n L) \end{bmatrix} - 1\begin{bmatrix} 1 & 1 & 0 \\ \sinh(\lambda_n L) & -\sin(\lambda_n L) & -\cos(\lambda_n L) \\ {\lambda_n}^3\cosh(\lambda_n L) & -{\lambda_n}^3\cos(\lambda_n L) & {\lambda_n}^3 s\,in(\lambda_n L) \end{bmatrix}$$

$$= -1\left[1\begin{bmatrix} -\sin(\lambda_n L) & -\cos(\lambda_n L) \\ -{\lambda_n}^3\cos(\lambda_n L) & {\lambda_n}^3 s\,in(\lambda_n L) \end{bmatrix} - 1\begin{bmatrix} \sinh(\lambda_n L) & -\cos(\lambda_n L) \\ {\lambda_n}^3\cosh(\lambda_n L) & {\lambda_n}^3 s\,in(\lambda_n L) \end{bmatrix}\right] - 1\left[1\begin{bmatrix} \cosh(\lambda_n L) & -\sin(\lambda_n L) \\ {\lambda_n}^3\sinh(\lambda_n L) & -{\lambda_n}^3\cos(\lambda_n L) \end{bmatrix} + \right.$$

$$\left. 1\begin{bmatrix} \sin(\lambda_n L) & \cos(\lambda_n L) \\ {\lambda_n}^3\cosh(\lambda_n L) & {\lambda_n}^3 s\,inh(\lambda_n L) \end{bmatrix}\right] = \mathbf{0}$$

$$= {\lambda_n}^3(sin^2\lambda l + cos^2\lambda l + sinh\lambda l sin\lambda l + cosh\lambda l cos\lambda l + cosh\lambda l cos\lambda l - sinh\lambda l sin\lambda l - sinh^2\lambda l + cosh^2\lambda l) = 0$$

$$= 1 + 2\, cosh\lambda l cosh\lambda l + 1 = 0$$

$$= cosh\lambda l cosh\lambda l + 1 = 0$$

Matlab calculation of the matrix function page 30:

To get started, select MATLAB Help or Demos from the Help menu.

```
>> E=6.9000e+10

I=1.0800e-4


E =

  6.9000e+010

I =

  1.0800e-004

>> L=5

L =

     5

>> Fd=1000

Fd =

       1000


>> X=0.3528518
```

When n=1;

```
X =


    0.3529
```

```
>> A=[0 1 0 1;1 0 1 0; sinh(X*L) cosh(X*L) -sin(X*L) -cos(X*L); (X^3)*cosh(X*L) (X^3)*sinh(X*L) -(X^3)*cos(X*L) (X^3)*sin(X*L)]

A =

         0    1.0000         0    1.0000

    1.0000         0    1.0000         0

    2.8330    3.0043   -0.9813    0.1923

    0.1320    0.1245    0.0084    0.0431


>> B=[0;0;0;(-Fd/(E*I))]

B =

  1.0e-003 *

         0

         0

         0

   -0.1342


>> c=inv(A)*B


c =


   -0.0102

    0.0138

    0.0102

   -0.0138
```

When n=2;

```
>> X=0.8866508

X =

    0.8867

>> A=[0 1 0 1;1 0 1 0; sinh(X*L) cosh(X*L) -sin(X*L) -cos(X*L); (X^3)*cosh(X*L) (X^3)*sinh(X*L) -(X^3)*cos(X*L) (X^3)*sin(X*L)]

A =

         0    1.0000         0    1.0000
    1.0000         0    1.0000         0
   42.0965   42.1084    0.9613    0.2755
   29.3513   29.3430    0.1921   -0.6701

>> B=[0;0;0;(-Fd/(E*I))]

B =

  1.0e-003 *

         0
         0
         0
   -0.1342
```

```
>> c=inv(A)*B

c =

  1.0e-003 *

    0.3798
   -0.3735
   -0.3798
    0.3735
```

When n=3;

```
X =

    1.5309

>> A=[0 1 0 1;1 0 1 0; sinh(X*L) cosh(X*L) -sin(X*L) -cos(X*L); (X^3)*cosh(X*L) (X^3)*sinh(X*L) -(X^3)*cos(X*L) (X^3)*sin(X*L)]
```

```
A =

  1.0e+003 *

     0         0.0010         0    0.0010
    0.0010         0      0.0010         0
    1.0553    1.0553   -0.0010   -0.0002
    3.7868    3.7868   -0.0007    0.0035

>> B=[0;0;0;(-Fd/(E*I))]
B =
  1.0e-003 *
         0
         0
         0
   -0.1342

>> c=inv(A)*B
c =
  1.0e-004 *
   -0.9405
    0.9412
    0.9405
   -0.9412
```