Preventive Maintenance & Replacement Scheduling Model for Repairable and Maintainable Systems

by

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14183
Dissertation submitted in partial fulfillment of
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CERTIFICATION OF APPROVAL

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CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

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(Jamshid Lutfullaev)

ABSTRACT

Maintenance plan is required when there is a need to decide on production rate and preventive maintenance (PM) scheduling for complex production systems in industry. The more complex the system is, the more components that may fail and cause the entire piece of equipment to be taken out of service, that in turn could affect production rate.

Usually, most of equipment has its PM schedule set during plant design phase and generally follow recommendation from equipment manufacturer. After years of operation (operation and maintenance phase), the PM schedule might not be optimised due to various reasons – lack of full information on failure rate, repair rate, maintenance cost, etc. Thus, the objectives of this project are to develop an appropriate model for preventive maintenance and replacement scheduling using Mixed Integer Non – Linear Programming (MINLP), and also to analyze the results and effectiveness of the optimized model through trade – offs curves, that are one of the project deliverables. The project is initiated by identifying problem and objectives, study on literature review regarding various types of preventive maintenance and replacement scheduling model, and then come out with a model concept. It was found out that various parametric change effects were observed in component age graphs and scheduling tables in two developed models. In Model 1, three different trends were seen for each three cases such as cost increment was observed as required reliability increased where no changes happened in required reliability when shutdown cost varied and required reliability increased when time period was decreased respectively. In Model 2, it can be also seen that as time period decreased required reliability increased where higher amount of given budget can also increase required reliability of components. After all, these studies should be useful for maintenance planners and engineers tasked with the problem of developing maintenance plans for complex systems of components.

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CHAPTER 1: INTRODUCTION

1.1 Project Background

In contrast with the past years, global economy today is not in a good state as there is economic recession going on and we can see its influence almost in each aspect of our life including in industry. One of the cost influences in industry can be experienced when production stops because of equipment breakdown or failures occur. Mexican Gulf Oil Spill accident was an example of such costs where due to a failure of a small valve that caused more than 14 billion US dollars for cleaning-up and other activities as declared by the British Petroleum [1].

Therefore, as equipment age there is a need to conduct a checking whether they are operating as expected, and for how long they could be reliable. Thus, PM works are conducted in order to sustain ongoing successful business and avoid equipment failure. Alternatively, replacement activities may take place in case if equipment is found non-repairable. The process of scheduling therefore is crucial since it serves to assess production tasks by relying on the available resources and predefined objectives.

There were several studies conducted regarding PM optimization and replacement plans for different type of cases to address scheduling problems. However, it is found that there were only few studies done in the integration of preventive maintenance optimization and equipment replacement problem, especially for multi-component systems. The contribution of this project work is that this approach considers a realistic dependency between components that affects maintenance and replacement optimal decisions, and shows how to develop time-based patterns of maintenance and replacement actions that minimize the total cost of those actions including the cost of unexpected failures and maximize the overall reliability of the system.

Hence, it is hereby proposed to formulate a simplified but reliable model for optimizing the scheduling of preventive maintenance for repairable and maintainable systems. Once the model is formulated, the effect of such factors as available budget, minimum required reliability, and shut-down cost to minimum overall maintenance cost and reliability could be investigated. In order to conduct this research project, existing problems will be addressed and tackled accordingly.

1.2 Problem Statement

In business market, customer satisfaction determines its continuous growth and sustainability where demand and supply principle apply as one of the main driving mechanism and measurement of it. For instance, in industry, customer satisfaction can be evaluated through production rate decrement or increment. However, there are two different production influencing factors to be considered such as production planning and maintenance works.

Production planning is concerned with allocating limited resources to a set of jobs along with certain objective functions to be optimized. But, sometimes equipment unavailability periods occur due to unexpected failures or just because of performing scheduled preventive maintenance activities.

Maintenance works are basically concentrated on ensuring continuous operation and growth of industrial processes. In this regard, preventive maintenance (PM) schedules are considered as an integral part of the maintenance process, since optimal PM schedule facilitates the minimization of maintenance costs and ensures permanent production.

However, the interdependent relationship between the industrial operations and maintenance works presumes separate planning and implementation in real life cases. At the same time, this interrelationship could eventually result in a conflict of interests, since interruptions resulting from PM works may potentially cause unsatisfied production demand.

Considering above, one can realize that it is very important to have an optimized scheduling plan that helps to avoid such cases. In other words, failing to have a proper PM scheduling plan might negatively influence the production which consequently affects customer demand satisfaction. Therefore this work will aim to address the problem of ensuring continuous industrial operations during PM works by creating the optimal preventive maintenance plan for complex industrial processes.

Thus, this project work focuses on to develop a Bi-objective Optimization Model taking into consideration two criteria of equipment, namely (1) reliability for preventive maintenance aspect and (2) total operational costs for both preventive maintenance and production planning decisions that allows decision maker to achieve compromise solution meeting at best for the above two criteria using General Algebraic Modelling System (GAMS) programming language.

1.3 Research Objectives

The objectives of this study are as follows:

- To develop preventive maintenance and replacement scheduling models using Mixed Integer Non-Linear Programming (MINLP) method and,
- To investigate the effects of such factors as available budget, minimum required reliability, and shut-down cost to minimum overall maintenance cost and reliability.

1.4 Scope of Study

The scope of this research work is limited to:

- systems connected in series,
- systems whose failure distribution is governed by Non-homogeneous Poisson Process (NHPP), and
- A system with maximum number of sub-systems or workstations up to 10.

All the models are to be solved using GAMS.

CHAPTER 2: LITERATURE REVIEW

2.1 Preventive Maintenance and Its Importance

Preventive maintenance (PM) is an action carried out at prescribed intervals which intends to reduce the failure rate or the performance of degradation of equipment [2]. Preventive maintenance is practiced in order to avoid any types of unexpected shutdowns and equipment damaging cases which would lead to corrective or repair activities. This type of management approach is mostly based on time scheduling or in other word cyclic tasks, such as inspection, adjustments and lubrications which are designed to keep the equipment in the levels of reliability and availability [3]. Couzens and Hiroshige [5] defines that, Preventive Maintenance could be based on two categories: *Time - based* or *Run – based* that involves periodically inspecting, servicing, cleaning, or replacing parts to prevent sudden failure where *Predictive* stands for on – line monitoring of equipment in order to use important/expensive parts to the limit of their serviceable life.

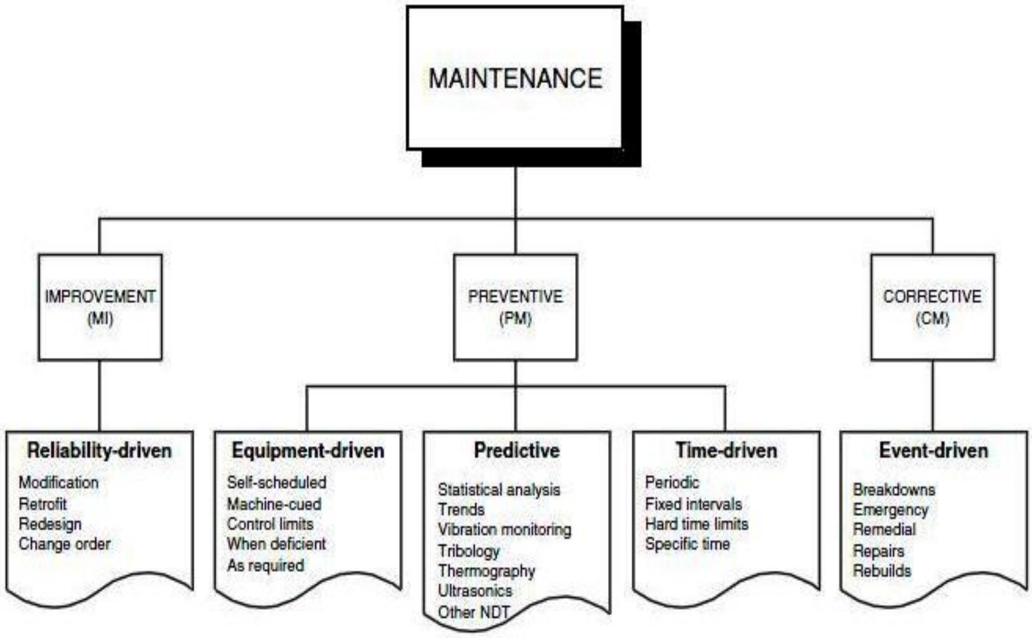


Figure 2.1.1: Structure of maintenance [3]

PM is planned maintenance conducted while equipment is functioning normally to avoid future failures. This type of maintenance aims to reduce risk of failures which may consist of inspection, adjustments, lubrication, parts replacement, calibration, and repair of equipment that are beginning to wear out. PM is generally performed on a regular basis, regardless of whether or not functionality or performance is degraded [2]. Classification of preventive maintenance tasks are divided into the below categories [2]:

- *Age-based maintenance.* In this case PM tasks are conducted at a specified age of the equipment. The age is measured by time concepts such as number of take – offs of an airplane and etc.
- *Clock-based maintenance.* In this case PM tasks are carried out at specified calendar times. This type of maintenance is easier to manage than age - based maintenance, since the maintenance tasks can be scheduled to predefine times. Figure 2.1.2 represents an example for this type of maintenance:

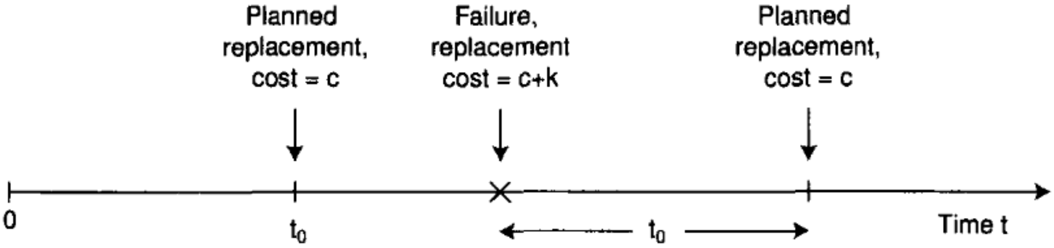


Figure 2.1.2: Age replacement policy and costs [3]

As it is shown in figure 2.1.2, when the equipment has reached the age t_0, the cost of preventive replacement is *c*, and the cost of replacing failed equipment (before age t_0) is $c+k$. The cost *c* covers the hardware and man – hour costs, while *k* is the extra cost incurred by the unplanned replacement, such as production loss [3].

- *Condition-based maintenance.* In this case PM tasks are done according to one or more condition variables of the equipment. Maintenance is conducted when a condition variable comes close or passes a critical value. For instance, vibration, temperature, and number of particles in the lube oil can be considered as examples of condition variables. The condition variables may be monitored continuously or at regular intervals. Another name for condition-based maintenance is *predictive maintenance.*
- *Opportunity maintenance.* This type is normally applied for multi- equipment systems, where maintenance tasks on other equipment or a system shutdown

intervention provides an opportunity for carrying out maintenance on equipment that are not the cause of the opportunity.

It is obvious that nothing lasts forever; this term clarifies the importance of prevention of any type of break downs or unexpected interruptions when machines or equipments run. Whenever production demand is made, equipment must be ready to operate and not break down during the operating time. If equipment fail during an operational cycle, there will be delays in producing the product and delivering it to customers. In these days of intense competitiveness, delays in delivery can result in losing customers. Preventive maintenance is required so that equipment is reliable enough to develop a production schedule that, in turn, is dependable enough to give a customer firm delivery dates [3].

In most of the cases, companies will purchase another identical piece of equipment when equipment is not reliable enough to schedule to the capacity. Then, if the first one breaks down on a critical order, they have a back up. With the price of equipment today, however, this back-up can be an expensive solution to a common problem. Unexpected equipment failures can be reduced, if not almost eliminated, by a good preventive maintenance program. With equipment availability at its highest possible level, redundant equipment will not be required [6].

2.2 Scheduling

Scheduling is considered one of the advantages to doing preventive maintenance rather than waiting until equipment fails and then doing emergency repairs. Schedule planning for inspections and preventive activities can be done in advance to make sure that the most convenient time for production is chosen, that maintenance parts and materials are available, and that the maintenance workload is relatively uniform [4].

2.3 Repairable Systems

A repairable system is a system which, after failing to perform one or more of its functions satisfactorily, can be restored to fully satisfactory performance by any method, other than replacement of the entire system [7].

2.4 Maintainable Systems

According to ReliaSoft [8], maintainable or maintainability is defined as the probability of performing a successful repair action within a given time. In other words, maintainability measures the ease and speed with which a system can be restored to operational status after a failure occurs. This is similar to system reliability analysis except that the random variable of interest in maintainability analysis is time-to-repair rather than time-to-failure.

2.5 Reliability

Dr. Walter A. Shewart [9] states that reliability is defined as the probability that a device will perform its intended function during a specified period of time under stated conditions. Reliability may also be defined in the following ways:

- The idea that an item is fit for a purpose with respect to time
- The capacity of a designed, produced, or maintained item to perform as required over time
- The capacity of a population of designed, produced or maintained items to perform as required over specified time
- The resistance to failure of an item over time
- The probability of an item to perform a required function under stated conditions for a specified period of time
- The durability of an object.

2.6 Mixed Integer Nonlinear Programming

In order to simplify decision making process in industry, several tools, software, mathematical models have been developed. One of them is MINLP programming targeting to overcome optimization problems such as maximizing or minimizing.

Michael R.B [10] claims that Mixed Integer Nonlinear Programming (MINLP) refers to mathematical programming with continuous and discrete variables and nonlinearities in the objective function and constraints. The use of MINLP is a natural approach of formulating problems where it is necessary to simultaneously optimize the system structure (discrete) and parameters (continuous). MINLPs have been used in various applications, including the process industry and the financial, engineering, management science and operations research sectors. MINLP solver technology is for particularly algorithms handling large-scale, highly combinatorial and highly nonlinear problems.

2.7 Review of Earlier Research on Scheduling

The effectiveness of the preventive maintenance scheduling under different conditions such as shop load, job sequencing rule, maintenance capacity and strategy was studied in several earlier studies. For instance, a multi- criteria approach to find optimal preventive maintenance intervals of components in a paper factory production line with total expected costs and reliability as the objective functions was proposed by Sortrakul et al [11] could find an approximation for optimal control policies and values of input factors by combining analytical formulation with simulation-based statistical tools such as experimental design and response surface methodology in a production and preventive maintenance planning problem. Moreover, integrated preventive maintenance and job shop scheduling problem for a single-machine was tackled in Pan et al [12]. In these studies, minimization of total weighted expected completion time is considered as the objective function. As a comparison, the obtained computational results of integrated model were compared with the results achieved from solving preventive maintenance scheduling and job scheduling problems independently.

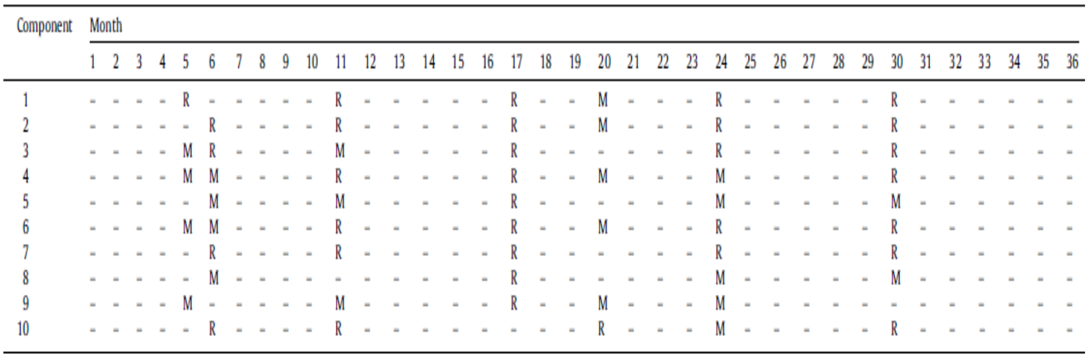
In another study, five objectives functions of maintenance cost, makespan and weighted completion time of jobs, total weighted tardiness, and machine availability were considered simultaneously in a multi-objective integrated production and maintenance planning problem solved by a multi-objective genetic algorithm [13]. Apart from that, Bi-objective integrated production and maintenance scheduling models have been presented to determine the Pareto-optimal front of the assignment of production tasks to machines along with preventive maintenance activities in a production system [14] whereby these studies developed and tested series of genetic algorithms to solve the problem.

Moghaddam [15] found out that none of the reviewed research studies considered the simultaneous combination of total operational costs, system reliability and overall availability of the production system in their modelling approach. The contribution of the research work was solving multi – objective problems using goal programming method.

As a continuation of the research work [15], a hybrid dynamic programming/branch-and-bound algorithm was implemented to find the global optimal solutions where each component was analyzed separately. Defining a general configuration for multi-component systems and developing mathematical models to determine optimal PM and replacement scheduling were the main contributions of the research [16].

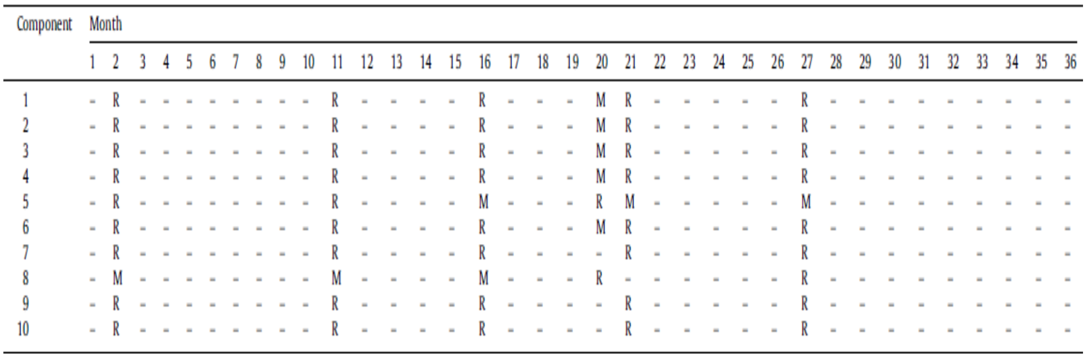
As noticed in the case study [16], the minimum effective age of each component is equal to zero at the beginning of planning horizon where they were shown from the second month on. For instance, component 1 does not receive any maintenance action at the first four periods, but it is replaced at the 5th period, maintained at 10th period and so on. This causes the effective age drops to zero and the component 1 works as a new one at the beginning of the next period. As a final outcome of the research work, the following two schedules were presented:

Table 2.7.1: Optimal maintenance and replacement schedule for Case-1



Component	Month																																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
1	-	-	-	-	R	-	-	-	-	-	R	-	-	-	-	-	R	-	-	M	-	-	-	R	-	-	-	-	-	R	-	-	-	-	-	-
2	-	-	-	-	-	R	-	-	-	-	R	-	-	-	-	-	R	-	-	M	-	-	-	R	-	-	-	-	-	R	-	-	-	-	-	-
3	-	-	-	-	M	R	-	-	-	-	M	-	-	-	-	-	R	-	-	-	-	-	-	R	-	-	-	-	-	R	-	-	-	-	-	-
4	-	-	-	-	M	M	-	-	-	-	R	-	-	-	-	-	R	-	-	M	-	-	-	M	-	-	-	-	-	R	-	-	-	-	-	-
5	-	-	-	-	-	M	-	-	-	-	M	-	-	-	-	-	R	-	-	-	-	-	-	M	-	-	-	-	-	M	-	-	-	-	-	-
6	-	-	-	-	M	M	-	-	-	-	R	-	-	-	-	-	R	-	-	M	-	-	-	R	-	-	-	-	-	R	-	-	-	-	-	-
7	-	-	-	-	-	R	-	-	-	-	R	-	-	-	-	-	R	-	-	-	-	-	-	R	-	-	-	-	-	R	-	-	-	-	-	-
8	-	-	-	-	-	M	-	-	-	-	-	-	-	-	-	-	R	-	-	-	-	-	-	M	-	-	-	-	-	M	-	-	-	-	-	-
9	-	-	-	-	M	-	-	-	-	-	M	-	-	-	-	-	R	-	-	M	-	-	-	M	-	-	-	-	-	-	-	-	-	-	-	-
10	-	-	-	-	-	R	-	-	-	-	R	-	-	-	-	-	-	-	-	R	-	-	-	M	-	-	-	-	-	R	-	-	-	-	-	-

Table 2.7.2: Optimal maintenance and replacement schedule for Case-2



Component	Month																																		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
1	-	R	-	-	-	-	-	-	-	-	R	-	-	-	-	R	-	-	-	M	R	-	-	-	-	-	R	-	-	-	-	-	-	-	-	-
2	-	R	-	-	-	-	-	-	-	-	R	-	-	-	-	R	-	-	-	M	R	-	-	-	-	-	R	-	-	-	-	-	-	-	-	-
3	-	R	-	-	-	-	-	-	-	-	R	-	-	-	-	R	-	-	-	M	R	-	-	-	-	-	R	-	-	-	-	-	-	-	-	-
4	-	R	-	-	-	-	-	-	-	-	R	-	-	-	-	R	-	-	-	M	R	-	-	-	-	-	R	-	-	-	-	-	-	-	-	-
5	-	R	-	-	-	-	-	-	-	-	R	-	-	-	-	M	-	-	-	R	M	-	-	-	-	-	M	-	-	-	-	-	-	-	-	-
6	-	R	-	-	-	-	-	-	-	-	R	-	-	-	-	R	-	-	-	M	R	-	-	-	-	-	R	-	-	-	-	-	-	-	-	-
7	-	R	-	-	-	-	-	-	-	-	R	-	-	-	-	R	-	-	-	-	R	-	-	-	-	-	R	-	-	-	-	-	-	-	-	-
8	-	M	-	-	-	-	-	-	-	-	M	-	-	-	-	M	-	-	-	R	-	-	-	-	-	-	R	-	-	-	-	-	-	-	-	-
9	-	R	-	-	-	-	-	-	-	-	R	-	-	-	-	R	-	-	-	-	R	-	-	-	-	-	R	-	-	-	-	-	-	-	-	-
10	-	R	-	-	-	-	-	-	-	-	R	-	-	-	-	R	-	-	-	-	R	-	-	-	-	-	R	-	-	-	-	-	-	-	-	-

For Case-1, it was concluded that if all components had the same reliability parameters, the structure and the frequency of activities in the optimal schedule would be affected by just ratio of the maintenance and replacement costs. In the second case, less reliable components with higher deterioration rate were replaced more frequently than the more reliable components with lower deterioration rate. It was also found that the value of the improvement factor does not affect the structure of optimal schedule.

2.8 Summary of Research Gaps

It is found that none of the reviewed research studies considered parametric variation effects on multi – objective function combinations in their modelling approach. In addition, most of these efforts try to model single-component or single-machine production systems using difficult way of solving approach where complex programming tools or languages were used instead. Hence the first contribution of this research is to develop and simplify comprehensive mathematical model to be able to capture broader aspects of the production and maintenance scheduling problems in multi-component manufacturing systems without any predefined user preferences and much effort to formulate problem for the system. Therefore, the contribution of this research work is to develop and test a novel solution procedure to achieve exact Pareto-optimal solutions using combination of simulation and optimization methods.

CHAPTER 3: METHODOLOGY

3.1 Introduction

The first chapter has outlined the problem statement, main objectives, and scopes of the project. Through chapter 2, it was shown that there are indeed gaps in the modelling of preventive maintenance scheduling for repairable systems. In the current chapter, we present the methods adopted to address the project objectives.

Figure 3.1 demonstrates the flow chart of the methodology adopted in the current project. As expected, it starts with a detailed literature review and concludes with result analysis and documentations. The intermediate stages, however, include model development, simulation for a particular case and refinement of the model by changing model setting and solver types. Once the model is confirmed, the work concentrates on investigation of the effect of major factors to the schedules of part of a system or the whole system. The Gantt chart and the corresponding milestones for the project are shown in Appendix A.

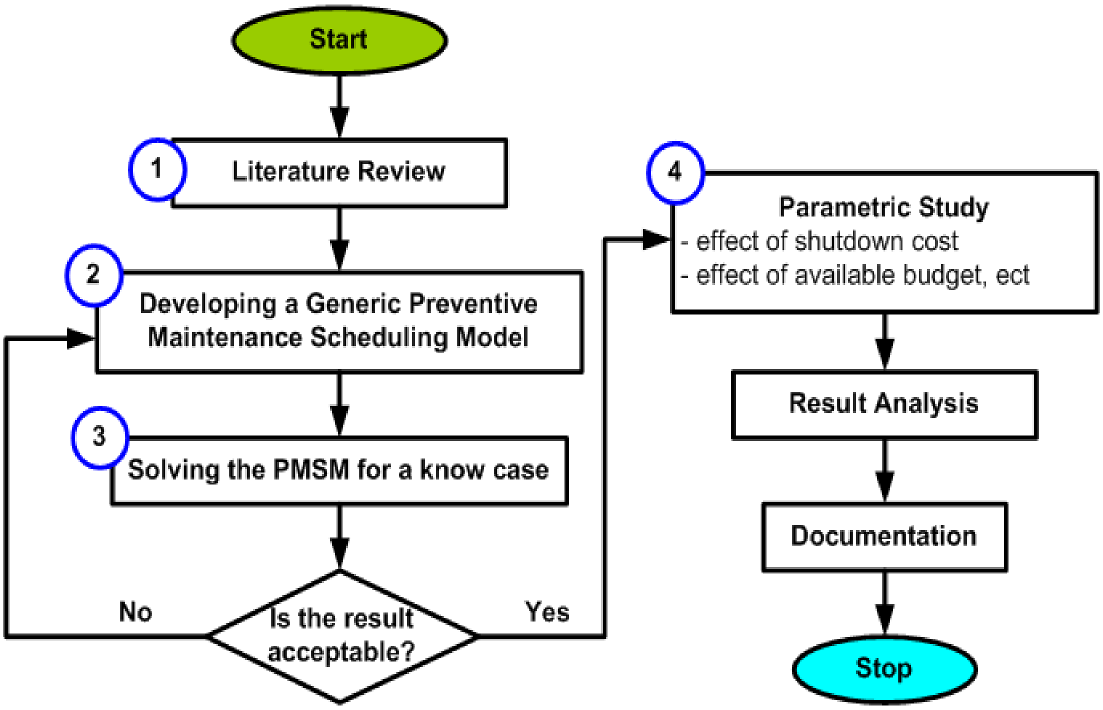


Figure 3.1.1: Process Flow Chart

3.1.1 Literature Review

In order to understand optimization problems and analyze the gap among preventive maintenance and replacement scheduling related researches, several literature reviews were done through searching for Online Scientific Journals, books and magazines.

3.1.2 Generic Model for Preventive Maintenance Scheduling

To develop the model, case study data from a research work was adopted. In the model, two objectives are aimed to achieve such as Minimizing total cost subject to reliability constraint and Maximizing system reliability subject to a budget constraints. Apart from these, the objective functions are subjected to several constraints, including time period and number of components to consider.

3.1.3 Solving the PMS model

According to the nature of this optimization problem, different types of solvers were considered to solve the model. Referring to the previous research work [16], it was found out that MINLP solving method was utilized. Thus, the strategy adopted to solve the model was General Algebraic Modeling System (GAMS) which has many solver tools that suit any types of optimization problems. Besides that, assumptions were made to simplify the solving process.

3.1.4 Result Analysis and Reporting

Having solved the model, analysis of the solution and reporting will be made based on comparison of schedules and graphs generated due to their varying effects on total cost, replacement or maintaining actions including reliability performances of the components.

3.2 Preventive Maintenance Scheduling Problem

This project work initiated by identifying the current problem and determining the objectives. Once done, the author carried out an extensive study on the project by gathering required data and information from available journals, articles, books and references. This enables the author to understand more on the project to be carried out and able to correlate the project with other previous researches done by researchers.

Experimental procedures are developed where preventive maintenance schemes or schedules data are gathered from series of components with an increasing rate of occurrence of failure (ROCOF). The data is taken from a research paper, produced by Kamran Moghaddam entitled ‘Preventive maintenance and replacement scheduling for repairable and maintainable system using dynamic programming’.

A model is then developed based on the data provided by the research work in order to assess the existing PM schedules by inputting all the data obtained and propose ways to further to optimize. General Algebraic Modelling System (GAMS) is utilized as a main tool to develop the PM optimization model.

Two different models are considered to achieve goals such as Minimizing total cost subject to a reliability constraint and Maximizing system reliability subject to a budget constraint under the following problem formulations and settings [16]:

Parameters of the Equations:

N: number of components

T: length of the planning horizon

J: Number of intervals

Weibull - Reliability: $\begin{cases} \lambda\text{: scale parameter(characteristic life)of component } i \\ \beta\text{: shape parameter of component } i \end{cases}$

α_i: improvement factor of component i

F_i: unexpected failure cost of component i

M_i: maintenance cost of component i

R_i: replacement cost of component i

RR_{series}: required reliability of the series system of components

GB: given budget in the series system of components

Z: Fixed cost of the system.

Decision Variables:

$$\begin{cases} Xi,j : \text{ effective age of component } i \text{ at the start of period } j \\ X'i,j : \text{ effective age of component } i \text{ at the end of period } j \end{cases}$$

Note that $m_{i,j}$ and $r_{i.j}$ are binary variables of maintenance and replacement actions for component i in period t and they cannot be equal to one simultaneously:

$$m_{i,j} = \begin{cases} 1 & if\ component\ i\ at\ period\ t\ is\ maintained \\ 0 & otherwise \end{cases}$$

$$r_{i,j} = \begin{cases} 1 & if\ component\ i\ at\ period\ t\ is\ replaced \\ 0 & otherwise \end{cases}$$

3.2.1 Total Cost Minimization Problem

Objective Function for Case–1: *Minimizing total cost subject to a reliability constraint:*

$$\text{Min}\quad f_1=\sum_{i=1}^{N}\sum_{j=1}^{T}[F_i\cdot\lambda_i((X'_{i,j})^{\beta_i}-(X_{i,j})^{\beta_i})+M_i\cdot m_{i,j}+R_i\cdot r_{i,j}]+\sum_{j=1}^{T}[Z(1-\prod_{i=1}^{N}(1-(m_{i,j}+r_{i,j})))]$$

3.2.2 Overall Reliability Maximization Problem

Objective Function for Case–2: *Maximizing system reliability subject to a budget constraint:*

$$\text{Max}\ f_2=\prod_{i=1}^{N}\prod_{j=1}^{T}\exp[-\lambda_i((X'_{i,j})^{\beta_i}-(X_{i,j})^{\beta_i})]$$

Subject to (Constraints):

1) The initial age for each component is set to zero indicating that all components are brand new ones at the beginning:

   $X_{i,1} = 0$

   $i = 1, \ldots\ldots, N$

2) The following set calculates the effective age of components depending on which action was taken in the previous period as follows:

   $X_{i,j} = (1-m_{i,j-1})(1-r_{i,j-1})X'_{i,j-1}+m_{i,j-1}(\alpha_i X'_{i,j-1})$

$i = 1, \ldots\ldots, N; j = 2, \ldots.., J$

3) The age of component *i* at the <u>end</u> of period *j*:

   $X'_{i,j} = X_{i,j} + (\frac{T}{J})$

   $i = 1, \ldots.., N; j = 1, \ldots\ldots, J$

4) In the above cost function, the condition of $m_{i,j} + r_{i,j} \leq 1$ should be held for all components over the intervals of the planning horizon stating that either a maintenance or a replacement action should be carried out for each workstation, hence:

   $m_{i,j} + r_{i,j} \leq 1$

   $i = 1, \ldots.., N; j = 1, \ldots.., J$

These below two constraints 5) and 6) function in the following manner:

If a component was replaced in the previous period then $r_{i,j-1}=1, m_{i,j-1}=0$, so that its effective age drops down to $X_{i,j} = 0$, if a component is minimally repaired then: $r_{i,j-1}=0$, $m_{i,j-1}=1$ and its effective age becomes: $X_{i,j}\,(\alpha_i X'_{i,j-1})$. Finally if no action was taken, $r_{i,j-1}=0, m_{i,j-1}=0$, the component continues its normal aging as $X_{i,j} = X'_{i,j-1}$, thus:

5) $m_{i,j}, r_{i,j} = 0$ or 1

   $i = 1, \ldots.., N; j = 1, \ldots.., J$

6) $X_{i,j}, X'_{i,j} \geq 0$

   $i = 1, \ldots.., N; j = 1, \ldots.., J$

All the above listed constraints apply for both cases including these two followings exclusively for:

7) Constraint for Case – 1:

   $\prod_{i=1}^{N} \prod_{j=1}^{T} \exp[-\lambda_i((X'_{i,j})^{\beta_i} - (X_{i,j})^{\beta_i})] \geq RR_{series}$

8) Constraint for Case – 2:

   $\sum_{i=1}^{N} \sum_{j=1}^{T} \left[F_i \cdot \lambda_i(\left(X'_{i,j}\right)^{\beta_i} - \left(X_{i,j}\right)^{\beta_i}\right) + M_i \cdot m_{i,j} + R_i \cdot r_{i,j}\right] \leq GB$

3.3 Solution Methods

In the current project, all the optimization problems are solved using GAMS developed by General Algebraic Modelling System Development Corporation [17]. As reported by the company, GAMS is specifically designed for modelling linear, nonlinear and mixed integer optimization problems. The system is especially useful with large, complex problems. GAMS is especially useful for handling large, complex, one-of-a-kind problems which may require many revisions to establish an accurate model. The system models problems in a highly compact and natural way. The user can change the formulation quickly and easily, can change from one solver to another, and can even convert from linear to nonlinear with little trouble.

CHAPTER 4: RESULTS AND DISCUSSION

4.1 Introduction

An example developed by Moghaddam [16] was adopted as an extension for the study to develop the models. The models are developed based on parametric enhancement of the data given in the research paper using GAMS. A repairable and maintainable series system of components with an increasing rate of occurrence (ROCOF) is chosen which is subjected to deterioration over a discrete number of periods. In each period (which could be defined a week, a month, etc.) it is assumed that one of the three distinct actions can be taken for each component in the system:

1) *Do nothing* - In this case, no action is to be taken on the component. This is often referred to as leaving a component in a state of ‘‘bad-as-old’’, where the component of interest continues to age normally.
2) *Maintenance* - the component is maintained, which places it into a state somewhere between ‘‘good as- new’’ and ‘‘bad-as-old’’. In this paper, the maintenance action reduces the effective age of the component by a stated percentage of its actual age. Because components subject to wear out experience an increasing ROCOF, this reduction in effective age results in a reduction in ROCOF as well.
3) *Replacement* - In this case, the component is to be replaced, immediately placing it in a state of ‘‘good-as-new’’, i.e.., its age is effectively returned to time zero.

4.2 Solving the Preventive Maintenance Scheduling Model for a Known Case

Model-1 from the case study [3] has been solved under the same settings stated in the case study [3] using GAMS software without any changes in the formulations to see and compare for validating purposes. The following case study data from Table 4.2.1 was utilized to solve the problem.

Table 4.2.1: Case study Data [3]

Component	λ	β	α	Failure Cost ($)	Maintenance Cost ($)	Replacement Cost ($)
1	0.00022	2.20	0.62	250	35	200
2	0.00035	2.00	0.58	240	32	210
3	0.00038	2.05	0.55	270	65	245
4	0.00034	1.90	0.50	210	42	180
5	0.00032	1.75	0.48	220	50	205
6	0.00028	2.10	0.65	280	38	235
7	0.00015	2.25	0.75	200	45	175
8	0.00012	1.80	0.68	225	30	215
9	0.00025	1.85	0.52	215	48	210
10	0.00020	2.15	0.67	255	55	250

According to the results obtained, replacement and maintenance activities take place starting from the 4th month until the month of 29 that can be seen in the attached optimal replacement and maintenance schedule table in the appendices section of this research work. The difference in the value obtained from minimizing total cost in the research paper [3] is comparatively lower (13797.10$) than 20071.586$ which has been obtained using GAMS programming. Moreover, required reliability was found to be 50% as expected while minimizing the total cost in the GAMS programming solution case. It was also found out that the main reason to vary the values of minimized total costs is selection of a proper solver options in GAMS programming.

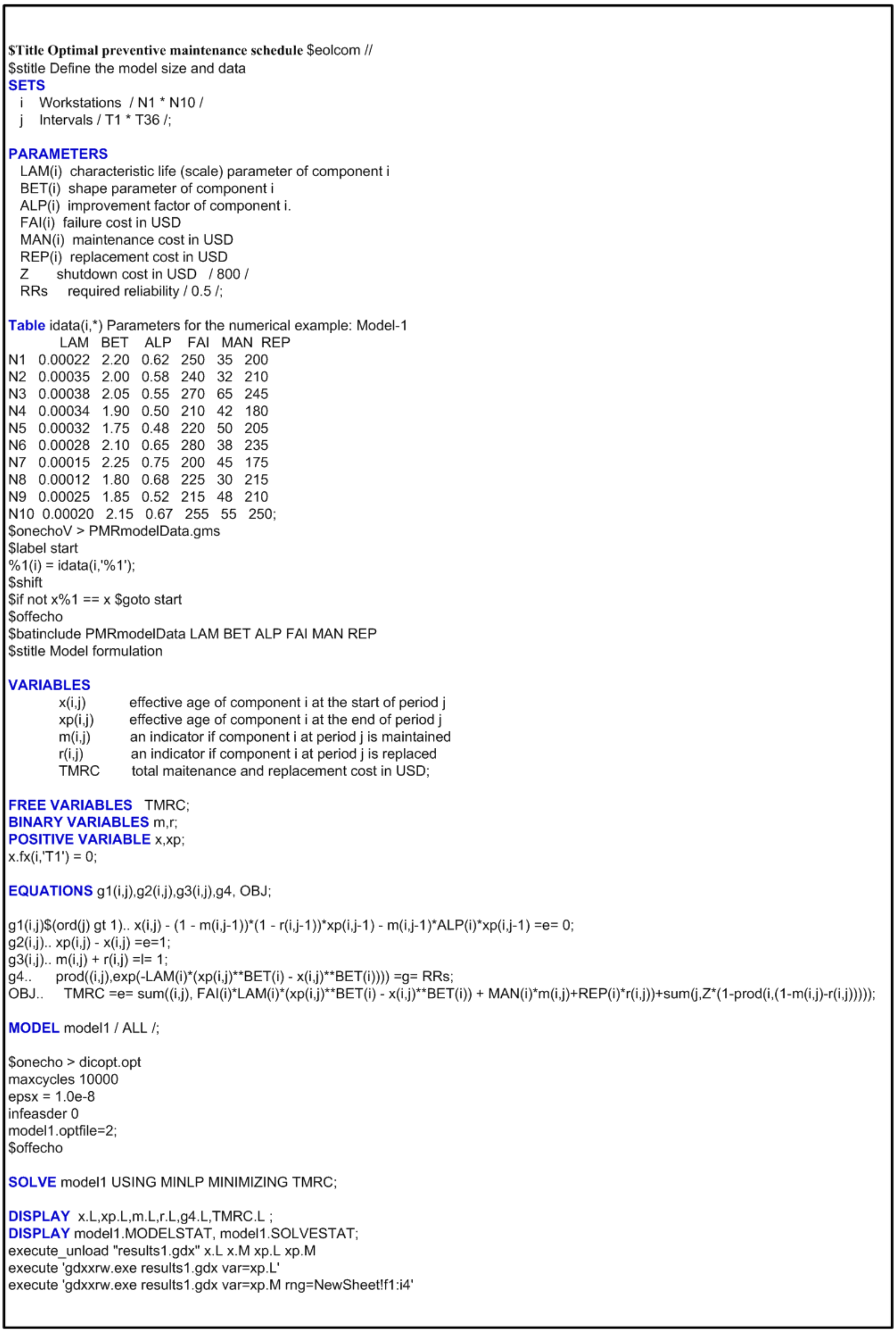
Investigated parametric studies based on the research paper Moghaddam [16] were done under the following settings:

For Model 1: Reliability (RR), Shutdown cost (Z) and Time variations (T) and;
For Model 2: Given budget (GB) and Time variations (T).

Under the scenarios, results obtained are demonstrated in these following sequences:

Model 1:

1) RR variations of: 99% , 100% and 80% and the effects on Total Cost
2) Z variations of: 900 $, 700$ and 1000$ and the effects on Reliability
3) T variations of: 12 months and 24 months and the effects on Reliability



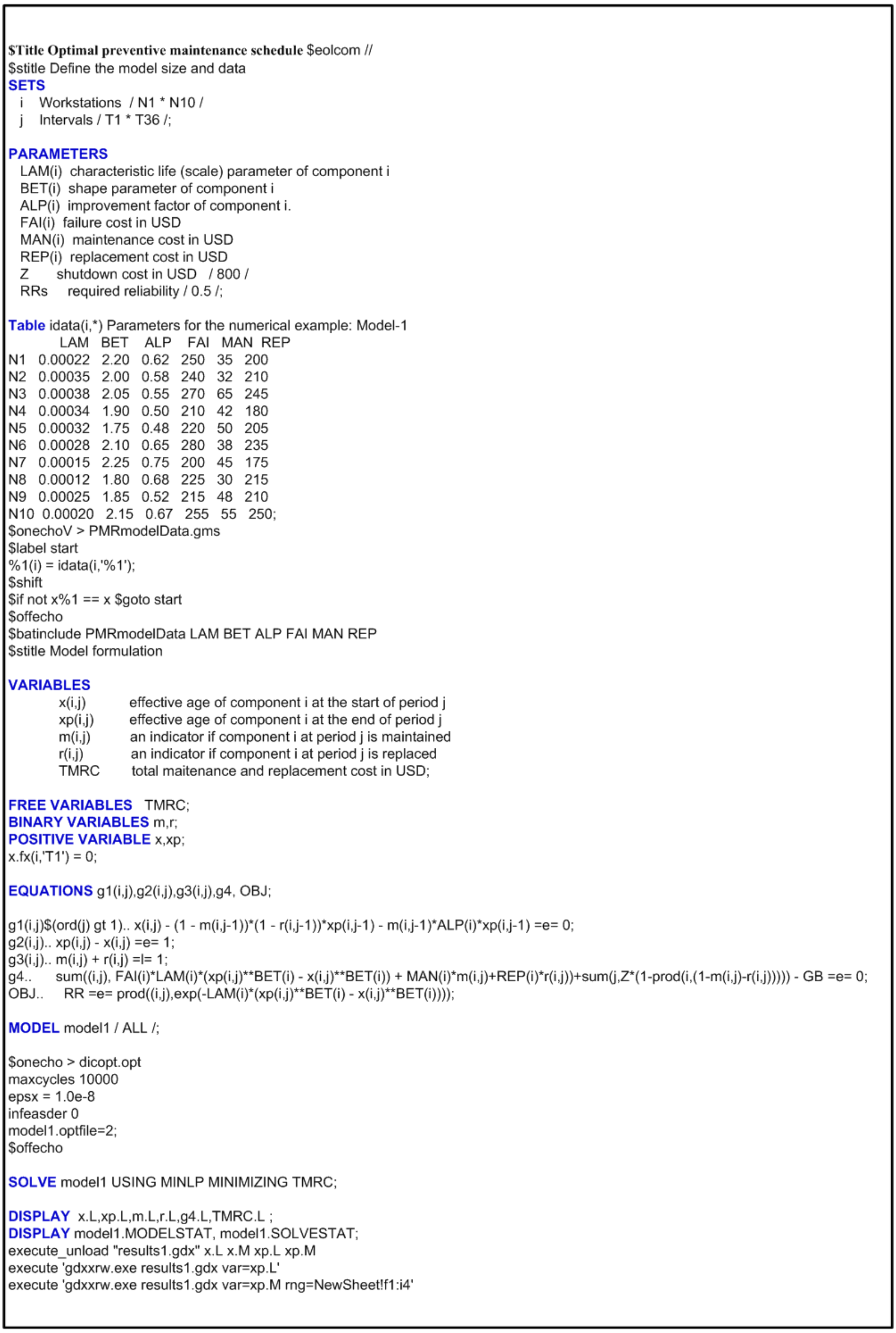
```
$Title Optimal preventive maintenance schedule $eolcom //
$stitle Define the model size and data
SETS
  i   Workstations  / N1 * N10 /
  j   Intervals / T1 * T36 /;

PARAMETERS
  LAM(i)  characteristic life (scale) parameter of component i
  BET(i)  shape parameter of component i
  ALP(i)  improvement factor of component i.
  FAI(i)  failure cost in USD
  MAN(i)  maintenance cost in USD
  REP(i)  replacement cost in USD
  Z       shutdown cost in USD   / 800 /
  RRs     required reliability / 0.5 /;

Table idata(i,*) Parameters for the numerical example: Model-1
        LAM   BET   ALP   FAI  MAN  REP
N1  0.00022  2.20  0.62  250  35  200
N2  0.00035  2.00  0.58  240  32  210
N3  0.00038  2.05  0.55  270  65  245
N4  0.00034  1.90  0.50  210  42  180
N5  0.00032  1.75  0.48  220  50  205
N6  0.00028  2.10  0.65  280  38  235
N7  0.00015  2.25  0.75  200  45  175
N8  0.00012  1.80  0.68  225  30  215
N9  0.00025  1.85  0.52  215  48  210
N10 0.00020  2.15  0.67  255  55  250;
$onechoV > PMRmodelData.gms
$label start
%1(i) = idata(i,'%1');
$shift
$if not x%1 == x $goto start
$offecho
$batinclude PMRmodelData LAM BET ALP FAI MAN REP
$stitle Model formulation

VARIABLES
          x(i,j)         effective age of component i at the start of period j
          xp(i,j)        effective age of component i at the end of period j
          m(i,j)         an indicator if component i at period j is maintained
          r(i,j)         an indicator if component i at period j is replaced
          TMRC           total maitenance and replacement cost in USD;

FREE VARIABLES   TMRC;
BINARY VARIABLES m,r;
POSITIVE VARIABLE x,xp;
x.fx(i,'T1') = 0;

EQUATIONS g1(i,j),g2(i,j),g3(i,j),g4, OBJ;

g1(i,j)$(ord(j) gt 1).. x(i,j) - (1 - m(i,j-1))*(1 - r(i,j-1))*xp(i,j-1) - m(i,j-1)*ALP(i)*xp(i,j-1) =e= 0;
g2(i,j).. xp(i,j) - x(i,j) =e=1;
g3(i,j).. m(i,j) + r(i,j) =l= 1;
g4..     prod((i,j),exp(-LAM(i)*(xp(i,j)**BET(i) - x(i,j)**BET(i)))) =g= RRs;
OBJ..     TMRC =e= sum((i,j), FAI(i)*LAM(i)*(xp(i,j)**BET(i) - x(i,j)**BET(i)) + MAN(i)*m(i,j)+REP(i)*r(i,j))+sum(j,Z*(1-prod(i,(1-m(i,j)-r(i,j)))));

MODEL model1 / ALL /;

$onecho > dicopt.opt
maxcycles 10000
epsx = 1.0e-8
infeasder 0
model1.optfile=2;
$offecho

SOLVE model1 USING MINLP MINIMIZING TMRC;

DISPLAY  x.L,xp.L,m.L,r.L,g4.L,TMRC.L ;
DISPLAY model1.MODELSTAT, model1.SOLVESTAT;
execute_unload "results1.gdx" x.L x.M xp.L xp.M
execute 'gdxxrw.exe results1.gdx var=xp.L'
execute 'gdxxrw.exe results1.gdx var=xp.M rng=NewSheet!f1:i4'
```

Figure 4.2.1: GAMS Program for Model-1

Model 2:

1) Time variations of: 12 months and 48 months and the effects on Reliability
2) GB variations of: 10000$ and 30000$ and the effects on Reliability

To solve the Models, parameters used in the coding remains the same except the studied ones.



```
$Title Optimal preventive maintenance schedule $eolcom //
$stitle Define the model size and data
SETS
   i    Workstations  / N1 * N10 /
   j    Intervals / T1 * T36 /;

PARAMETERS
   LAM(i)  characteristic life (scale) parameter of component i
   BET(i)  shape parameter of component i
   ALP(i)  improvement factor of component i.
   FAI(i)  failure cost in USD
   MAN(i)  maintenance cost in USD
   REP(i)  replacement cost in USD
   Z       shutdown cost in USD   / 800 /
   RRs     required reliability / 0.5 /;

Table idata(i,*) Parameters for the numerical example: Model-1
         LAM   BET    ALP    FAI   MAN  REP
N1   0.00022  2.20   0.62   250   35   200
N2   0.00035  2.00   0.58   240   32   210
N3   0.00038  2.05   0.55   270   65   245
N4   0.00034  1.90   0.50   210   42   180
N5   0.00032  1.75   0.48   220   50   205
N6   0.00028  2.10   0.65   280   38   235
N7   0.00015  2.25   0.75   200   45   175
N8   0.00012  1.80   0.68   225   30   215
N9   0.00025  1.85   0.52   215   48   210
N10  0.00020  2.15   0.67   255   55   250;
$onechoV > PMRmodelData.gms
$label start
%1(i) = idata(i,'%1');
$shift
$if not x%1 == x $goto start
$offecho
$batinclude PMRmodelData LAM BET ALP FAI MAN REP
$stitle Model formulation

VARIABLES
          x(i,j)         effective age of component i at the start of period j
          xp(i,j)        effective age of component i at the end of period j
          m(i,j)         an indicator if component i at period j is maintained
          r(i,j)         an indicator if component i at period j is replaced
          TMRC           total maitenance and replacement cost in USD;

FREE VARIABLES   TMRC;
BINARY VARIABLES m,r;
POSITIVE VARIABLE x,xp;
x.fx(i,'T1') = 0;

EQUATIONS g1(i,j),g2(i,j),g3(i,j),g4, OBJ;

g1(i,j)$(ord(j) gt 1).. x(i,j) - (1 - m(i,j-1))*(1 - r(i,j-1))*xp(i,j-1) - m(i,j-1)*ALP(i)*xp(i,j-1) =e= 0;
g2(i,j).. xp(i,j) - x(i,j) =e= 1;
g3(i,j).. m(i,j) + r(i,j) =l= 1;
g4..      sum((i,j), FAI(i)*LAM(i)*(xp(i,j)**BET(i) - x(i,j)**BET(i)) + MAN(i)*m(i,j)+REP(i)*r(i,j))+sum(j,Z*(1-prod(i,(1-m(i,j)-r(i,j))))) - GB =e= 0;
OBJ..     RR =e= prod((i,j),exp(-LAM(i)*(xp(i,j)**BET(i) - x(i,j)**BET(i))));

MODEL model1 / ALL /;

$onecho > dicopt.opt
maxcycles 10000
epsx = 1.0e-8
infeasder 0
model1.optfile=2;
$offecho

SOLVE model1 USING MINLP MINIMIZING TMRC;

DISPLAY  x.L,xp.L,m.L,r.L,g4.L,TMRC.L ;
DISPLAY model1.MODELSTAT, model1.SOLVESTAT;
execute_unload "results1.gdx" x.L x.M xp.L xp.M
execute 'gdxxrw.exe results1.gdx var=xp.L'
execute 'gdxxrw.exe results1.gdx var=xp.M rng=NewSheet!f1:i4'
```

Figure 4.2.2: GAMS Program for Model-2

4.3 Solution for Benchmark Case

Two objectives were considered to achieve by solving the benchmark case such as Minimizing total cost and Maximizing overall reliability. Having solved for each of the objectives, results varied due to the constraints and assumptions accordingly.

4.3.1 Minimizing Total Cost for Single Objective Problem

In this case, it was attempted to minimize total cost subject to the constraint that some minimum level of system reliability over the planning horizon is achieved and it was assumed that components are arranged in series. It is important to note that other system configurations (parallel, k-out-of-n, complex, etc.) could be handled just by modifying and adapting the fixed cost section and the reliability function based on the configuration of the parallel, k-out-of-n, or other complex systems, but for the sake of simplicity, only series systems were considered in this paper.

4.3.2 Maximizing Overall Reliability for Single Objective Problem

Maximizing overall reliability was addressed through modifying the formulation and introducing a budgetary constraint, GB. The objective of this model was to maximize the system reliability, depending on user’s choice of maintenance and replace decisions, such that budgeted total cost is not exceeded.

4.4 Parametric Study

To study the influences of changing variables on various criteria such as effects on shutdown cost, available budget and required reliability, parametric studies have been conducted. Interestingly, some studied parameters contributed positively in decreasing shutdown cost and increasing required reliability while negative effects were also observed.

4.4.1 Effect of Shutdown Cost

Three different values of shutdown cost were taken into consideration such as 900 $, 700 $ and 1000 $. After each run of the program, required reliabilities due to these values were found to be the same which did not have any effect on them. Moreover, in these three cases, components were subjected to either maintain or replace starting fifth month to twenty eighth.

4.4.2 Effect of Available Budget

It was found out that available budget played an important role in increasing the required reliability for components. For instance, 10000$ and 30000$ were assigned for each run and reliability outcomes became 39 % and 69 % due to respective investment amounts. Thus, it can be concluded that the more budget made available the more reliable components became.

4.4.3 Effect of minimum required reliability

Minimum required reliability variables of 99%, 100% and 80% were set to see the influences on total cost. The results yielded were determined as 1023987.447$ for 99% and 100% and 52877.626$ for 80% reliability requirements respectively. Only replacing actions can be seen in schedule from the month 1 till 35 for the first two requirements where both maintenance and replacement works took place in the third case from the month of 2 to 33.

4.5 Summary

Based on the results discussed above, most influential trends can be seen in availability of budget and required reliability changes whereby their variation effects reflect on total cost accordingly. In addition, the component overhauling actions due to the shutdown cost remained in the same time interval. Detailed information on the results is fully attached in the appendices section of this research work.

CHAPTER 5: CONCLUSION AND RECOMMENDATION

5.1 Conclusion

In two models discussed in chapter 4, it was found out that various parametric change effects can be observed in component age graphs and scheduling tables. In Model 1, three different trends were seen for each three cases such as cost increment occurs as required reliability increases, no changes happened in required reliability when shutdown cost varied and required reliability increased when time period was decreased respectively. In Model 2, it can be also seen that as time period decreases required reliability increases while higher amount of given budget can also increase required reliability of components in the second case. In this paper, there were presented two non-linear mixed-integer optimization models for preventive maintenance and replacement scheduling of multi-component systems. These models seek to minimize the total cost subject to achieving some minimal reliability and maximize the total reliability of the system subject to a budgetary constraint. Such models should be useful to maintenance planners as they try to develop effective plans according to their analyzing period, availability of given budgets, reliability requirements and observe changes in their systems. Complexity of the models precludes solution of extremely large problems with hundreds of components and/or periods.

In this project work, only few types of GAMS solvers were used to analyze the problem namely (1) BARON 14.4, (2) CONOPT, (3) MINOS and (4) SNOPT and (5) DICOPT. But among these solvers, the results obtained using DICOPT was satisfying the given constraints and objectives in the problems addressed. Moreover, solution for such problems can be found accurately using GAMS software which has many types of solver functions and allows user to deal with gigantic problems widely.

5.2 Recommendation

Extension for further study could be application of different solver types which use various algorithms of GAMS. Moreover, another potential continuation of this project work could be collecting real data from any plant for more components or workstations and conduct parametric studies by decreasing and increasing the number of components and investigate their effects on component ages, total costs and etc. Additionally, prospective researchers can examine the models through goal programming method and adding more objective functions to optimize using GAMS software and apply its suitable solver tool that might change the behaviour of parameters in the problem.

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APPENDICES

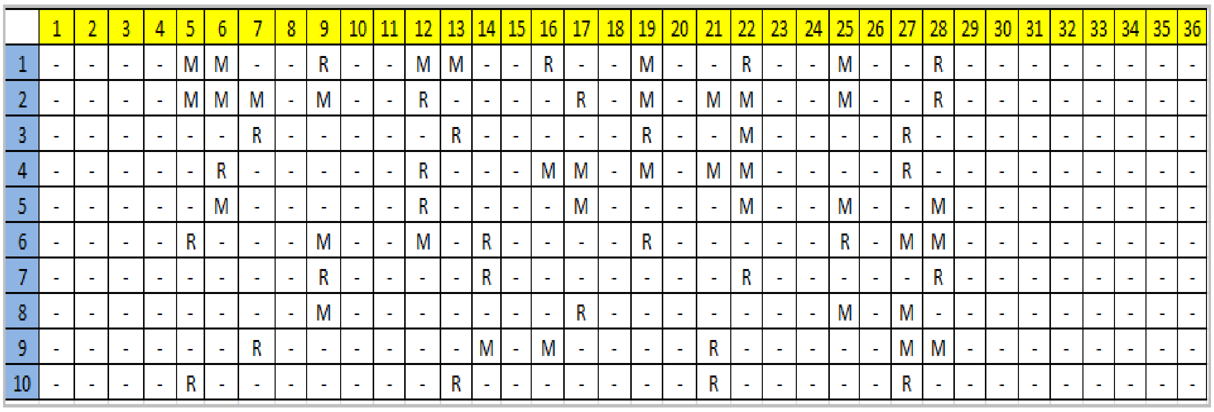
A. Project Gantt Chart and Milestones

№	Detail/Week	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	Review Literatures on Optimization Models (Maintainability, Repairability, Cost Minimizations and etc.)	🟦													
2	Synthesizing & Assessment of Benchmark Problems on Optimization		🟦	🟦	🟦	🟦	🟦								
3	Submission of Progress Report							🟦							
4	Synthesizing of Generic Scheduling Problem								🟦						
5	Selecting or Integration of Alternative Solvers								🟦						
6	Interpreting the Obtained Results									🟦					
7	Result Analyzing & Documentation										🟦	🟦			
8	Submission of Draft Report												🟦		
9	Submission of Dissertation													🟦	
10	Submission of Technical Paper													🟦	
11	Oral Presentation													🟦	
12	Submission of Project Dissertation (Hard Copy)														🟦

Remarks:

🟦	Work Progress

Result Obtained for a Known Case

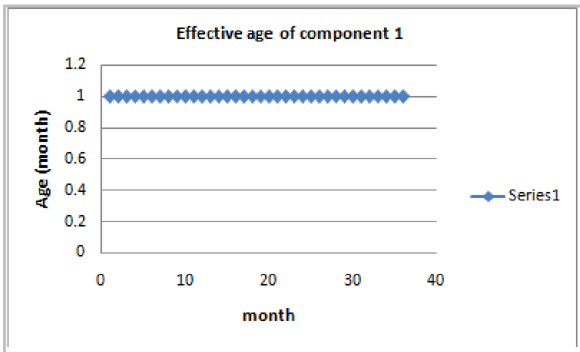


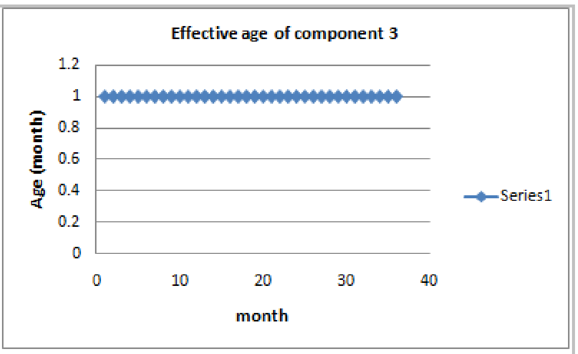
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
1	-	-	-	-	M	M	-	-	R	-	-	M	M	-	-	R	-	-	M	-	-	R	-	-	M	-	-	R	-	-	-	-	-	-	-	-
2	-	-	-	-	M	M	M	-	M	-	-	R	-	-	-	-	R	-	M	-	M	M	-	-	M	-	-	R	-	-	-	-	-	-	-	-
3	-	-	-	-	-	-	R	-	-	-	-	-	R	-	-	-	-	-	R	-	-	M	-	-	-	-	R	-	-	-	-	-	-	-	-	-
4	-	-	-	-	-	R	-	-	-	-	-	R	-	-	-	M	M	-	M	-	M	M	-	-	-	-	R	-	-	-	-	-	-	-	-	-
5	-	-	-	-	-	M	-	-	-	-	-	R	-	-	-	-	M	-	-	-	-	M	-	-	M	-	-	M	-	-	-	-	-	-	-	-
6	-	-	-	-	R	-	-	-	M	-	-	M	-	R	-	-	-	-	R	-	-	-	-	-	R	-	M	M	-	-	-	-	-	-	-	-
7	-	-	-	-	-	-	-	-	R	-	-	-	-	R	-	-	-	-	-	-	-	R	-	-	-	-	-	R	-	-	-	-	-	-	-	-
8	-	-	-	-	-	-	-	-	M	-	-	-	-	-	-	-	R	-	-	-	-	-	-	-	M	-	M	-	-	-	-	-	-	-	-	-
9	-	-	-	-	-	-	R	-	-	-	-	-	-	M	-	M	-	-	-	-	R	-	-	-	-	-	M	M	-	-	-	-	-	-	-	-
10	-	-	-	-	R	-	-	-	-	-	-	-	R	-	-	-	-	-	-	-	R	-	-	-	-	-	R	-	-	-	-	-	-	-	-	-

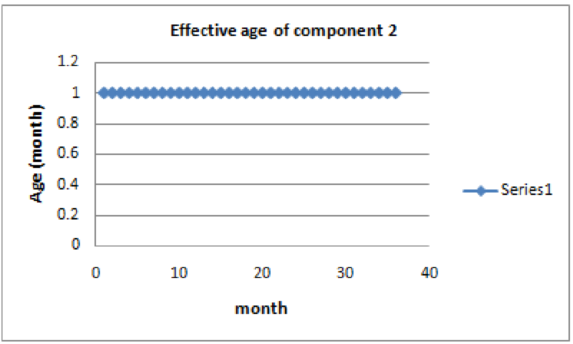
Table 1: Optimal maintenance and replacement schedule based on

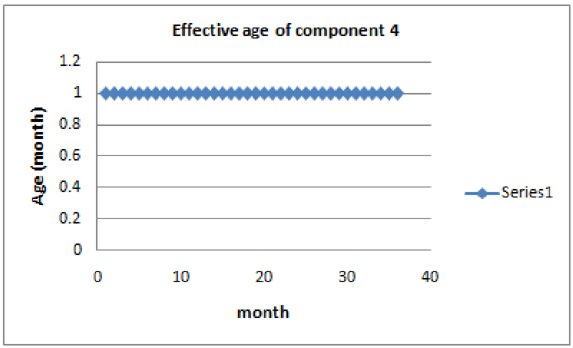
Total cost = 20071.586, RR=50%

Model -1 Result Graphs and Tables









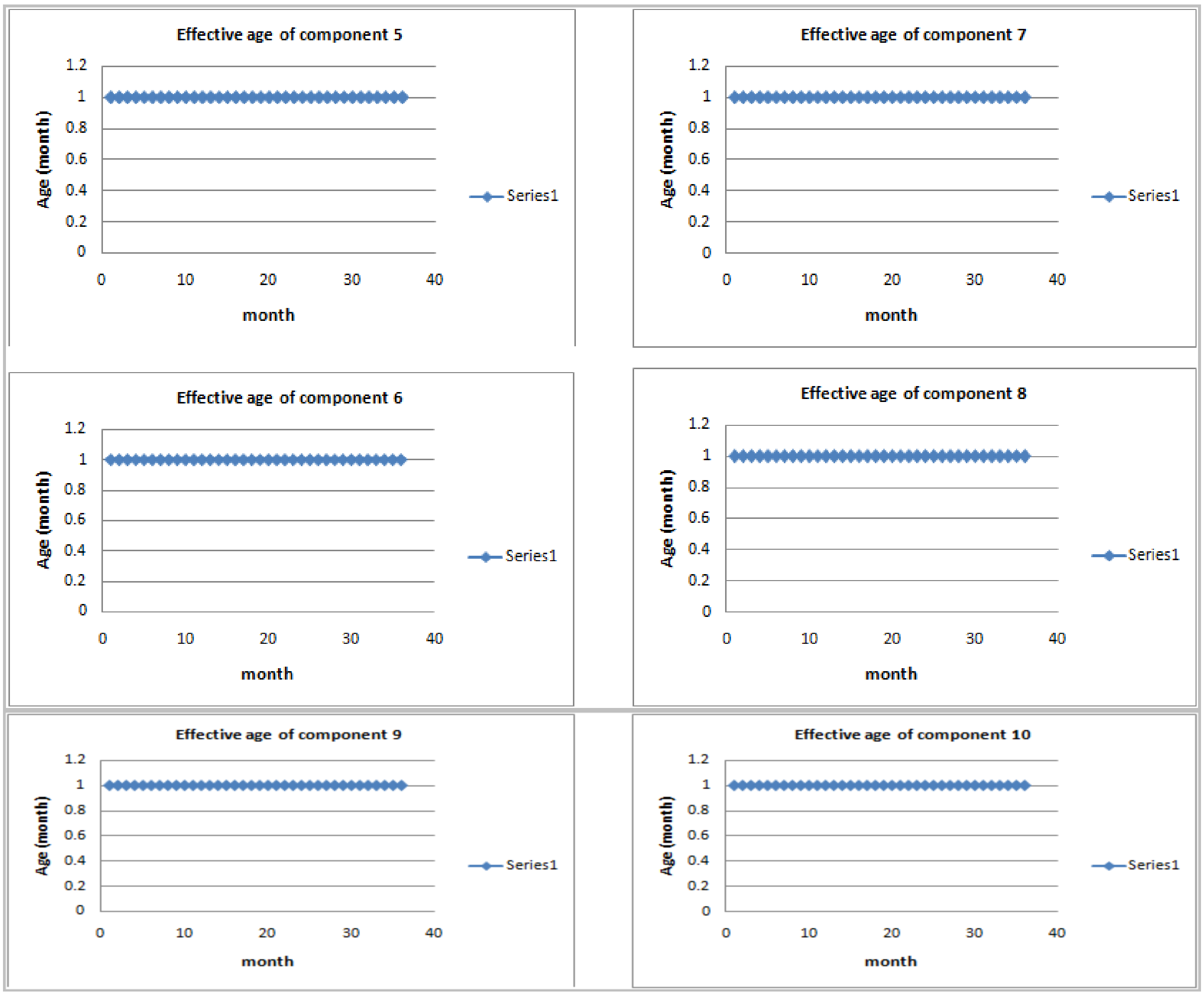
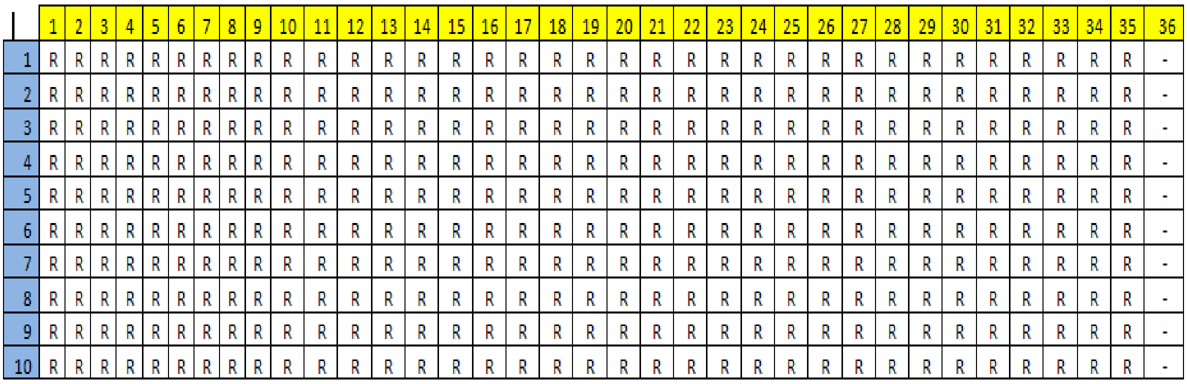


Figure 1: Effective age of components at period T=36, (Total cost =102397.447, RR=99%) for Model 1.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
1	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	-
2	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	-
3	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	-
4	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	-
5	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	-
6	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	-
7	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	-
8	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	-
9	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	-
10	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	-

Table 2: Optimal maintenance and replacement schedule based on

Total cost =102397.447, RR=99%

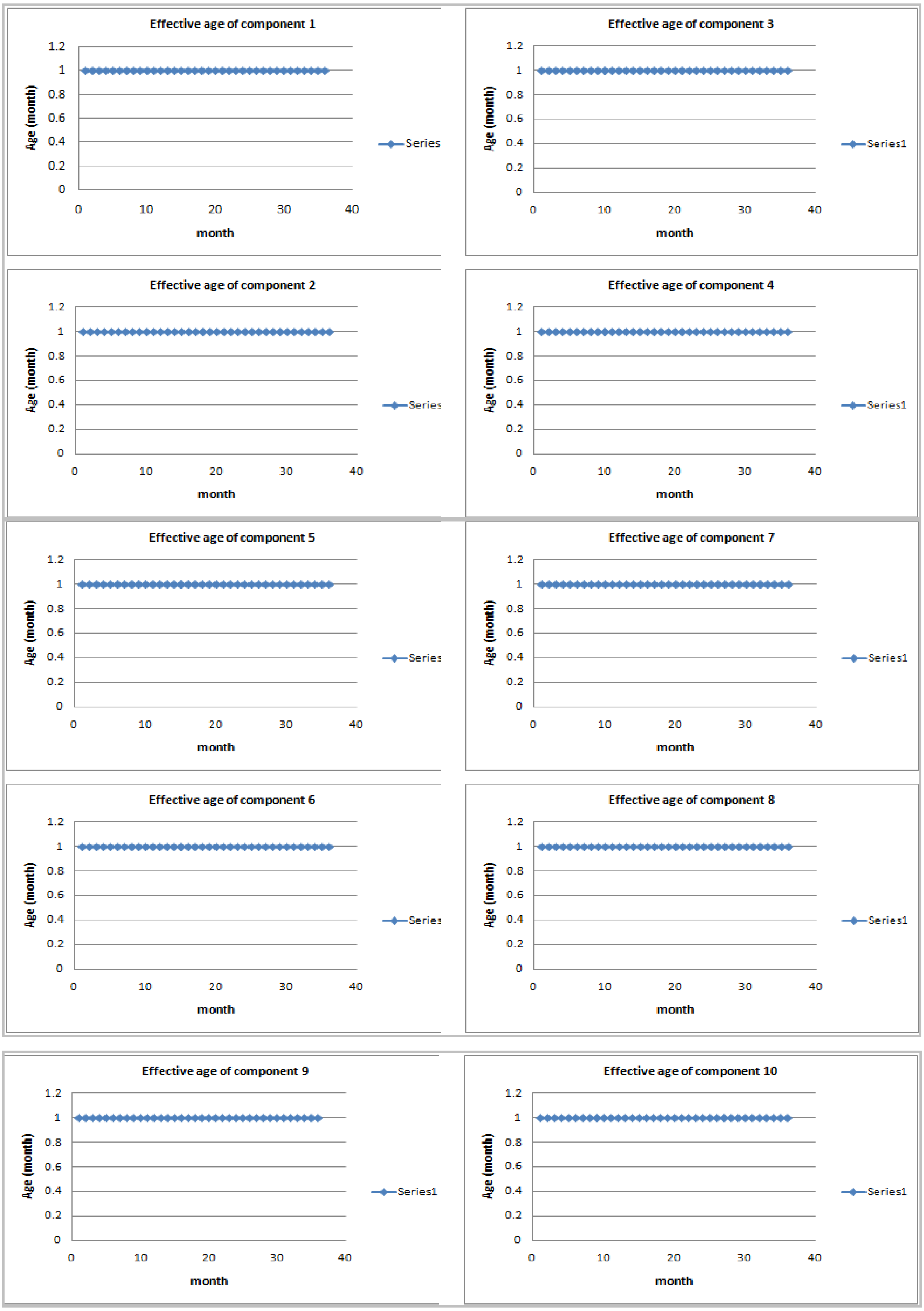
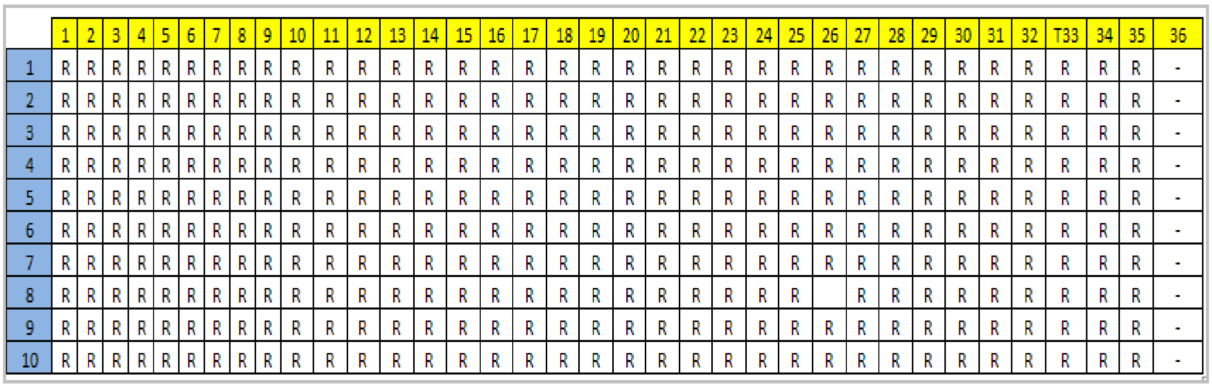


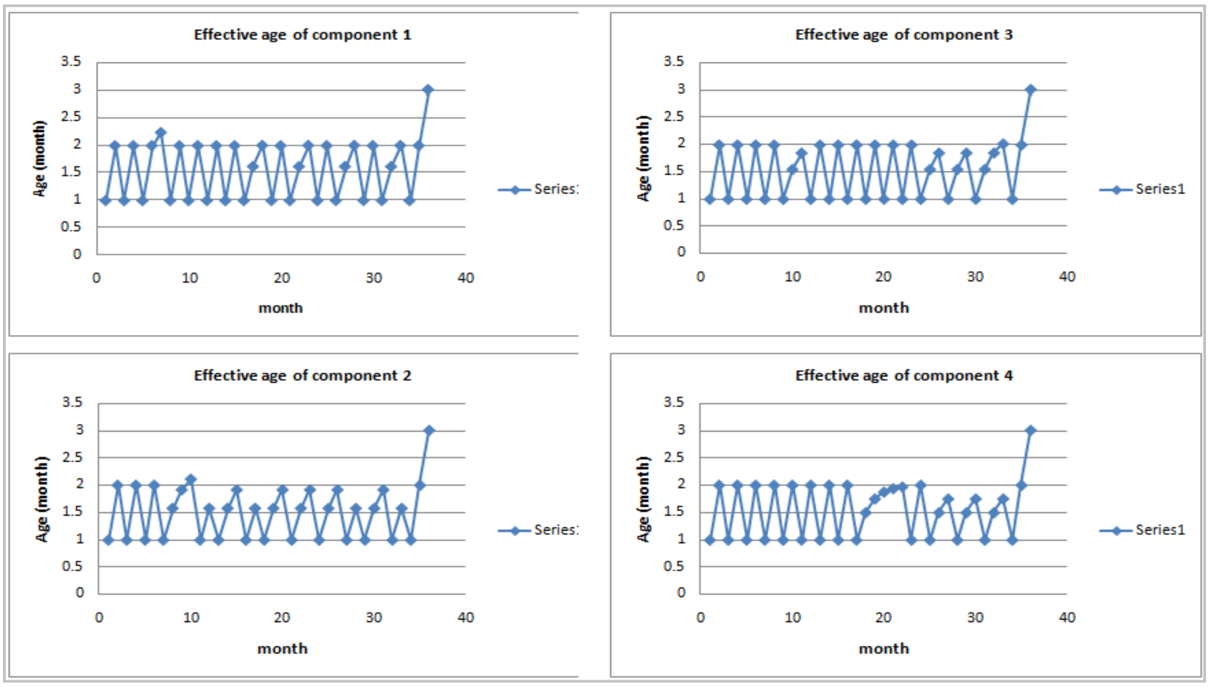
Figure 2: Effective age of components at period T=36, (Total cost =102397.447, RR=100%) for Model 1.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	T33	34	35	36
1	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	-
2	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	-
3	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	-
4	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	-
5	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	-
6	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	-
7	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	-
8	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R		R	R	R	R	R	R	R	R	R	-
9	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	-
10	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	-

Table 3: Optimal maintenance and replacement schedule based on

Total cost =102397.447, RR=100%



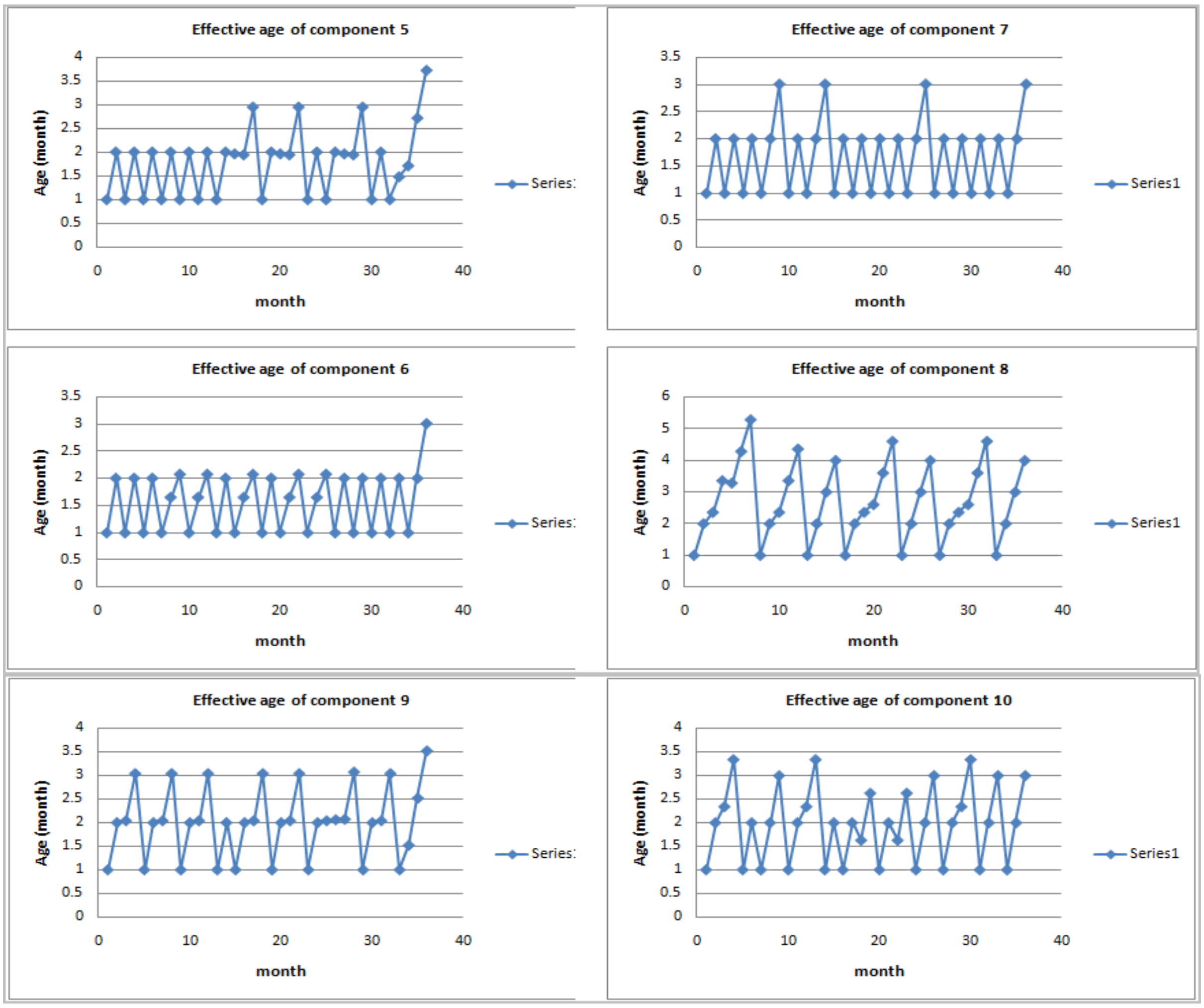


Figure 3: Effective age of components at period T=36, (Total cost = 52877.626, RR=80%) for Model 1.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
1	-	R	-	R	-	M	R	-	R	-	R	-	R	-	R	M	M	R	-	R	M	M	R	-	R	M	M	R	-	R	M	M	R	-	-	-
2	-	R	-	R	-	R	M	M	M	R	M	R	M	M	R	M	R	M	M	R	M	M	R	M	M	R	M	R	M	M	R	M	R	-	-	-
3	-	R	-	R	-	R	-	R	M	M	R	-	R	-	R	-	R	-	R	-	R	-	R	M	M	R	M	M	R	M	M	M	R	-	-	-
4	-	R	-	R	-	R	-	R	-	R	-	R	-	R	-	R	M	M	M	M	M	R	-	R	M	M	R	M	M	R	M	-	R	-	-	-
5	-	R	-	R	-	R	-	R	-	R	-	R	-	M	M	-	R	-	M	M	-	R	-	R	-	M	M	-	R	-	R	M	M	-	-	-
6	-	R	-	R	-	R	M	M	R	M	M	R	-	R	M	M	R	-	-	M	M	R	M	M	R	-	R	-	R	-	R	-	R	-	-	-
7	-	R	-	R	-	R	-	-	R	-	R	-	-	R	-	R	-	R	-	R	-	R	-	-	R	-	R	-	R	-	R	-	R	-	-	-
8	-	M	-	M	-	-	R	-	M	-	-	R	-	-	-	R	-	M	M	-	-	R	-	-	-	R	-	M	M	-	-	R	-	-	-	-
9	-	M	-	R	-	M	-	R	-	-	-	R	-	R	-	M	-	R	-	M	-	R	-	M	M	M	-	R	-	M	-	R	M	-	-	-
10	-	M	-	R	-	R	-	-	R	-	M	-	R	-	R	-	R	-	R	-	R	-	R	-	-	R	-	M	-	R	-	-	R	-	-	-

Table 4: Optimal maintenance and replacement schedule based on

Total cost =52877.626, RR=80%

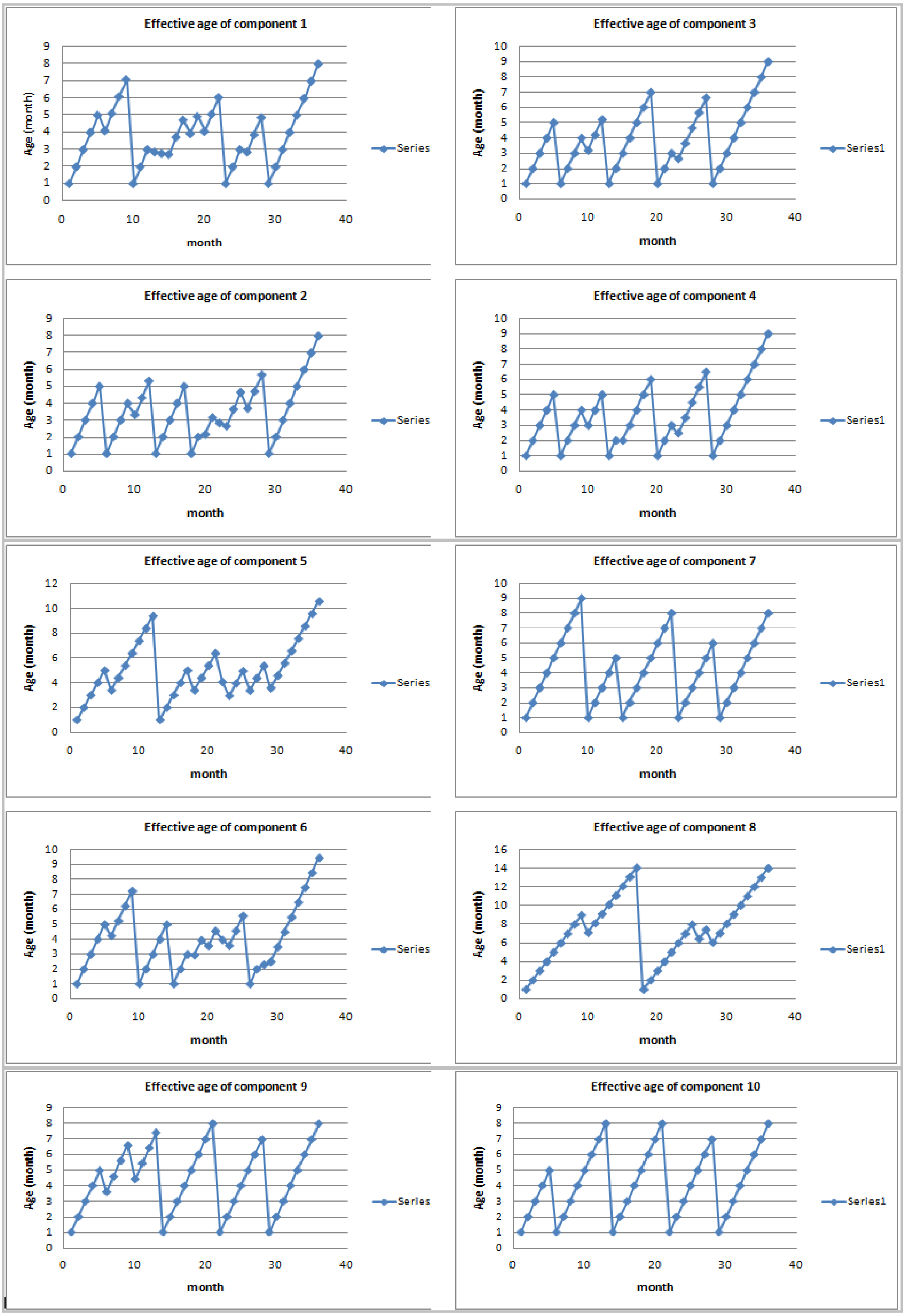
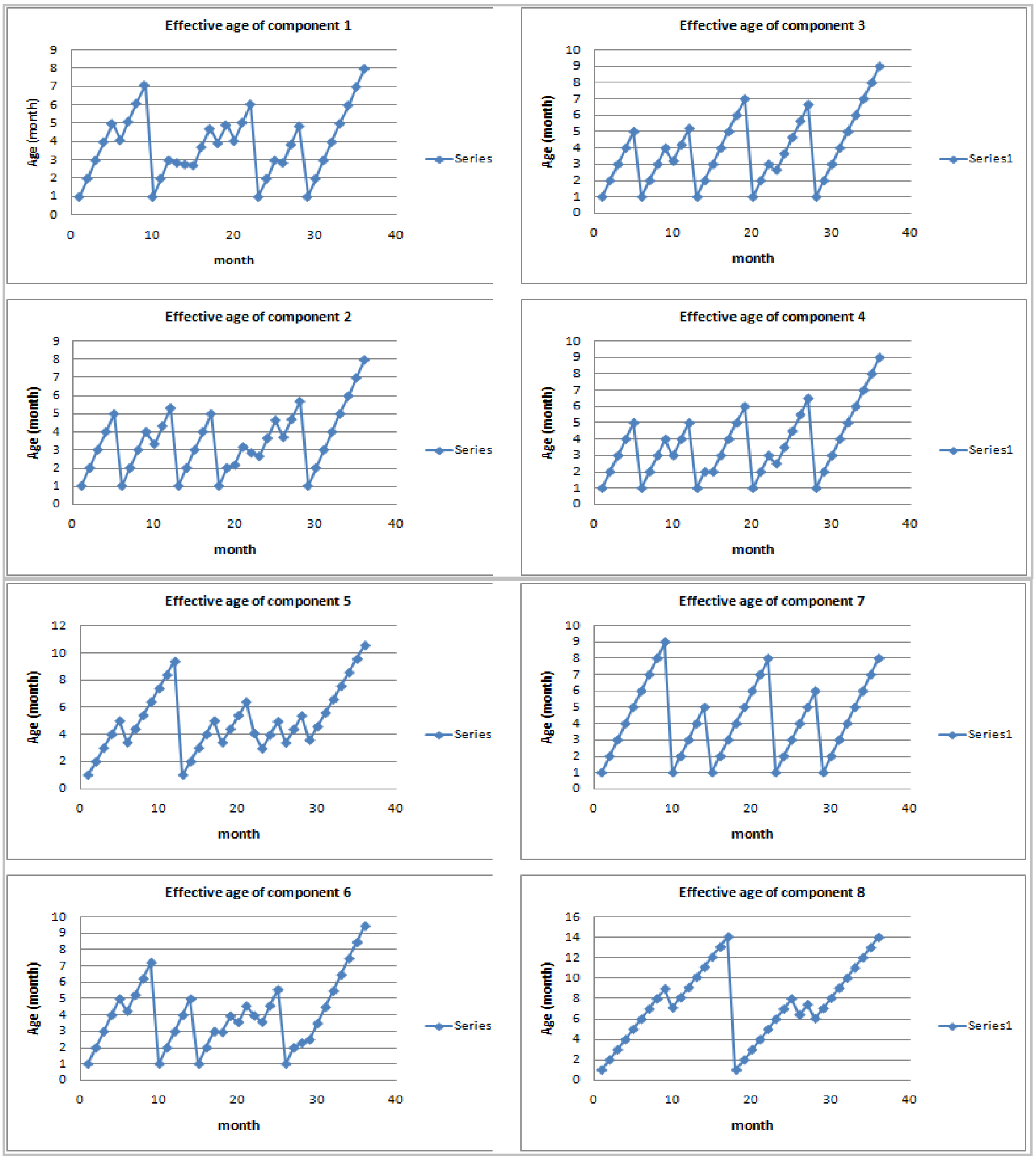


Figure 4: Effective age of components at period T=36, (Z = 900$, RR=50%) for Model 1.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
1	-	-	-	-	M	-	-	-	R	-	-	M	M	M	-	-	M	-	M	-	-	R	-	-	M	-	-	R	-	-	-	-	-	-	-	-
2	-	-	-	-	R	-	-	-	M	-	-	R	-	-	-	-	R	-	M	-	M	M	-	-	M	-	-	R	-	-	-	-	-	-	-	-
3	-	-	-	-	R	-	-	-	M	-	-	R	-	-	-	-	-	-	R	-	-	M	-	-	-	-	R	-	-	-	-	-	-	-	-	-
4	-	-	-	-	R	-	-	-	M	-	-	R	-	M	-	-	-	-	R	-	-	M	-	-	-	-	R	-	-	-	-	-	-	-	-	-
5	-	-	-	-	M	-	-	-	-	-	-	R	-	-	-	-	M	-	-	-	M	M	-	-	M	-	-	M	-	-	-	-	-	-	-	-
6	-	-	-	-	M	-	-	-	R	-	-	-	-	R	-	-	M	-	M	-	M	M	-	-	R	-	M	M	-	-	-	-	-	-	-	-
7	-	-	-	-	-	-	-	-	R	-	-	-	-	R	-	-	-	-	-	-	-	R	-	-	-	-	-	R	-	-	-	-	-	-	-	-
8	-	-	-	-	-	-	-	-	M	-	-	-	-	-	-	-	R	-	-	-	-	-	-	-	M	-	M	-	-	-	-	-	-	-	-	-
9	-	-	-	-	M	-	-	-	M	-	-	-	R	-	-	-	-	-	-	-	R	-	-	-	-	-	-	R	-	-	-	-	-	-	-	-
10	-	-	-	-	R	-	-	-	-	-	-	-	R	-	-	-	-	-	-	-	R	-	-	-	-	-	-	R	-	-	-	-	-	-	-	-

Table 5: Optimal maintenance and replacement schedule based on Z =900$, RR=50%



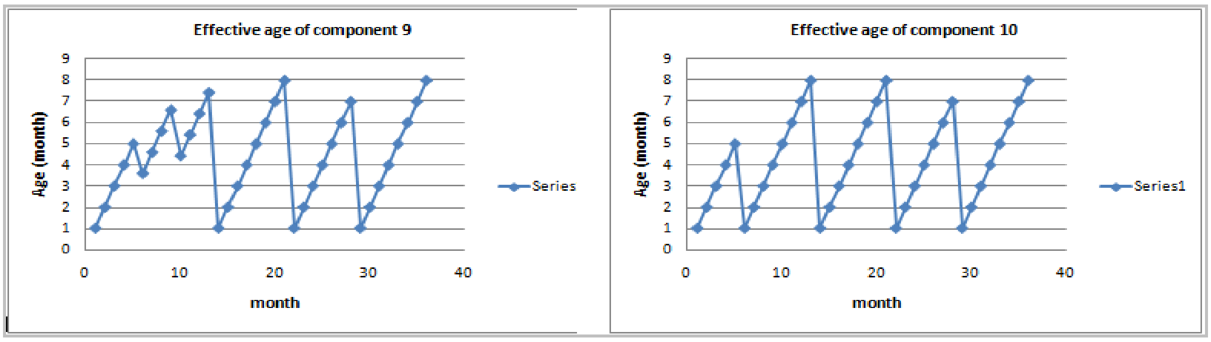
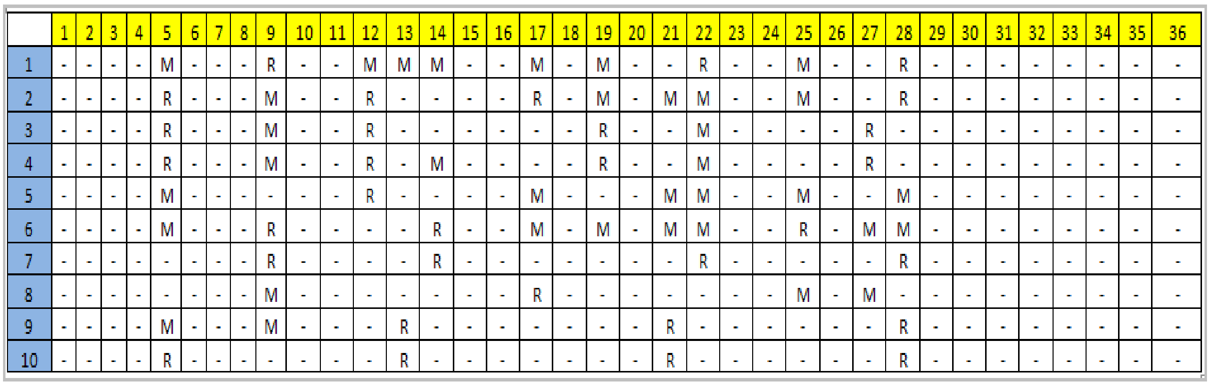
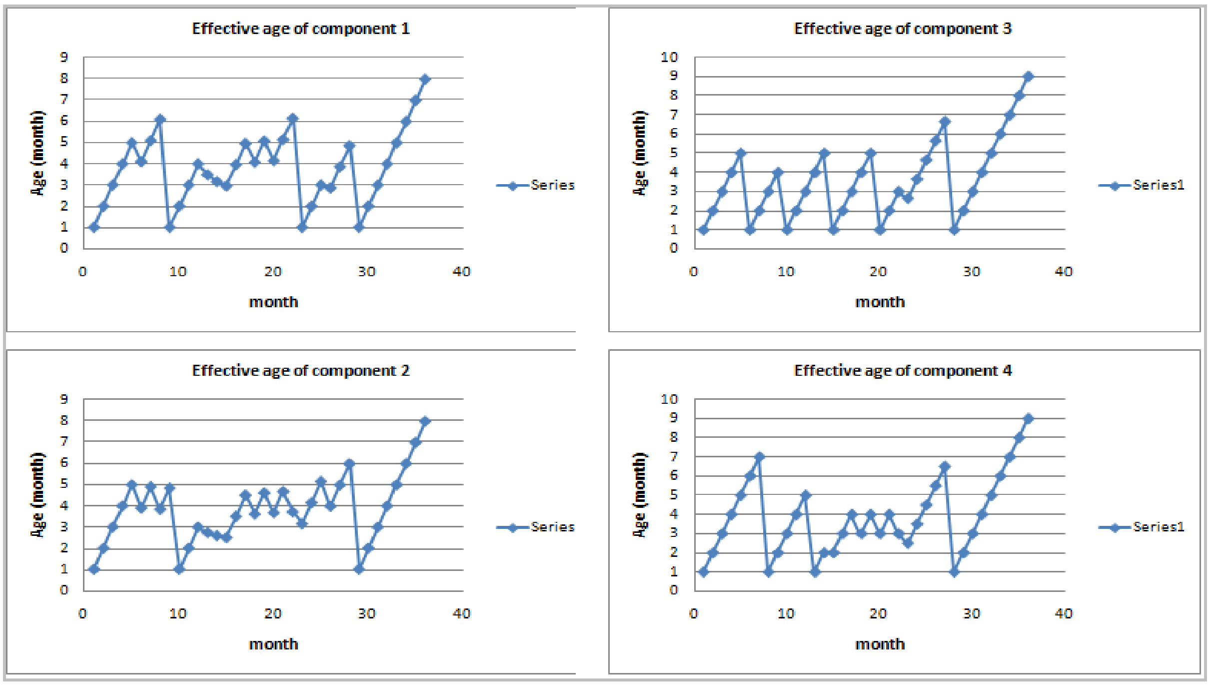


Figure 5: Effective age of components at period T=36, (Z = 700$, RR=50%) for Model 1.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
1	-	-	-	-	M	-	-	-	R	-	-	M	M	M	-	-	M	-	M	-	-	R	-	-	M	-	-	R	-	-	-	-	-	-	-	-
2	-	-	-	-	R	-	-	-	M	-	-	R	-	-	-	-	R	-	M	-	M	M	-	-	M	-	-	R	-	-	-	-	-	-	-	-
3	-	-	-	-	R	-	-	-	M	-	-	R	-	-	-	-	-	-	R	-	-	M	-	-	-	-	R	-	-	-	-	-	-	-	-	-
4	-	-	-	-	R	-	-	-	M	-	-	R	-	M	-	-	-	-	R	-	-	M	-	-	-	-	R	-	-	-	-	-	-	-	-	-
5	-	-	-	-	M	-	-	-	-	-	-	R	-	-	-	-	M	-	-	-	M	M	-	-	M	-	-	M	-	-	-	-	-	-	-	-
6	-	-	-	-	M	-	-	-	R	-	-	-	-	R	-	-	M	-	M	-	M	M	-	-	R	-	M	M	-	-	-	-	-	-	-	-
7	-	-	-	-	-	-	-	-	R	-	-	-	-	R	-	-	-	-	-	-	-	R	-	-	-	-	-	R	-	-	-	-	-	-	-	-
8	-	-	-	-	-	-	-	-	M	-	-	-	-	-	-	-	R	-	-	-	-	-	-	-	M	-	M	-	-	-	-	-	-	-	-	-
9	-	-	-	-	M	-	-	-	M	-	-	-	R	-	-	-	-	-	-	-	R	-	-	-	-	-	-	R	-	-	-	-	-	-	-	-
10	-	-	-	-	R	-	-	-	-	-	-	-	R	-	-	-	-	-	-	-	R	-	-	-	-	-	-	R	-	-	-	-	-	-	-	-

Table 6: Optimal maintenance and replacement schedule based on Z =700$, RR=50%



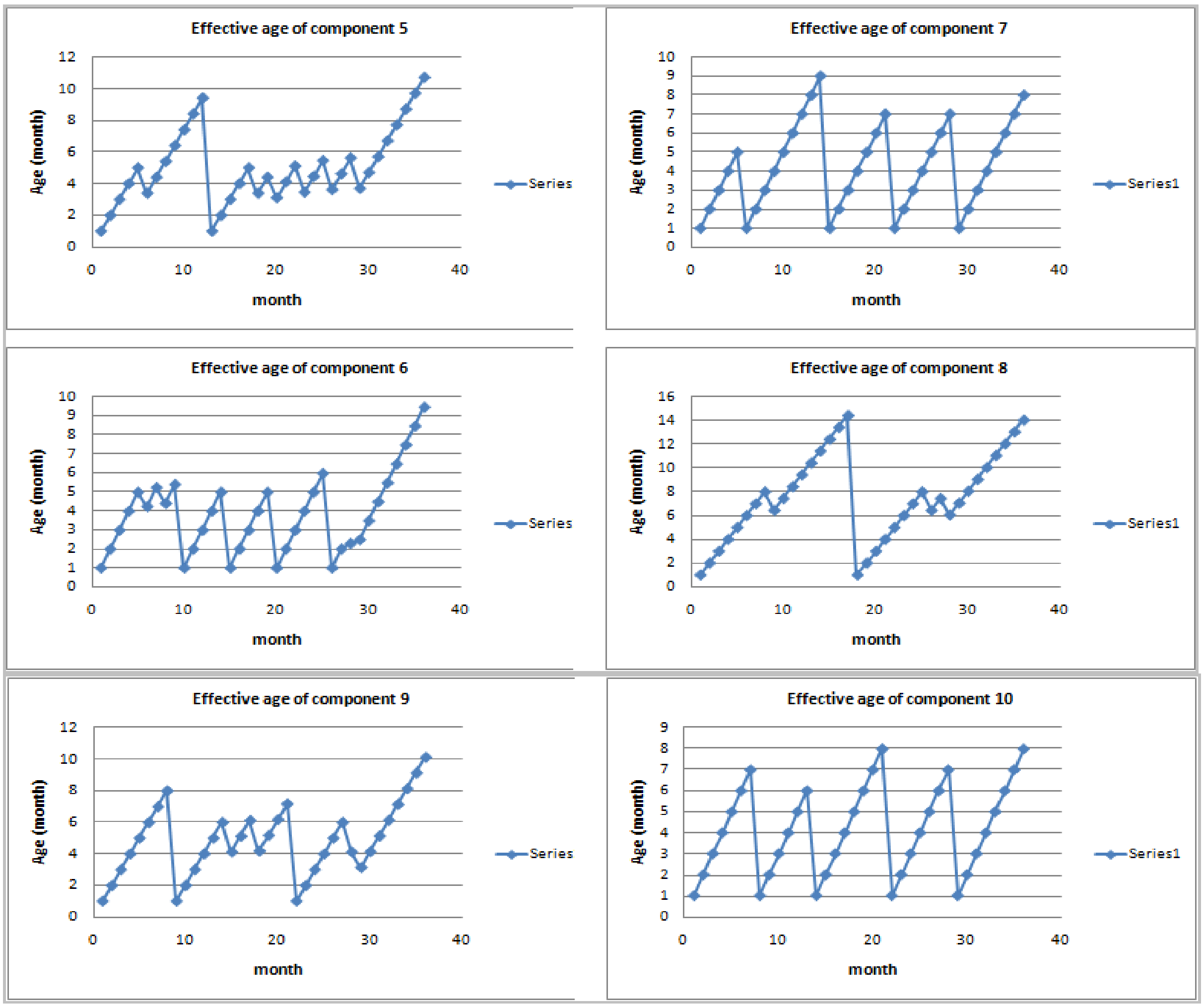
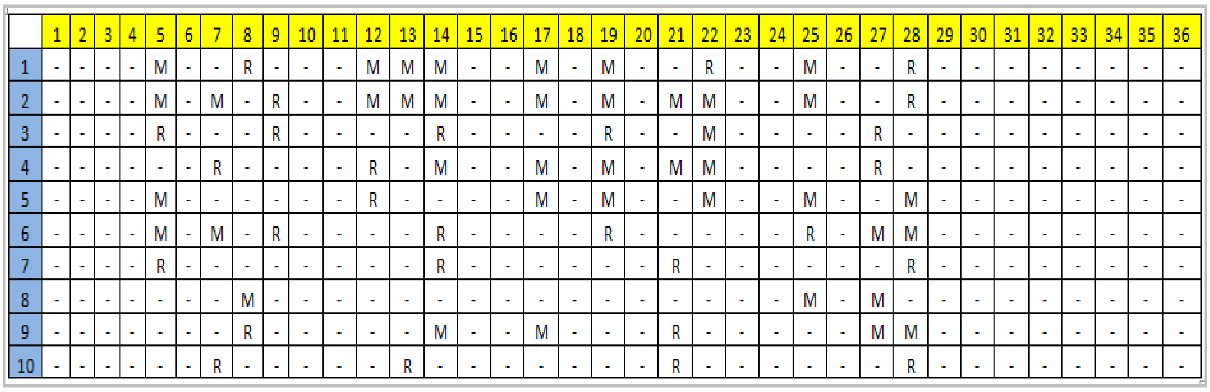


Figure 6: Effective age of components at period T=36, (Z = 1000$, RR=50%) for Model 1.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
1	-	-	-	-	M	-	-	R	-	-	-	M	M	M	-	-	M	-	M	-	-	R	-	-	M	-	-	R	-	-	-	-	-	-	-	-
2	-	-	-	-	M	-	M	-	R	-	-	M	M	M	-	-	M	-	M	-	M	M	-	-	M	-	-	R	-	-	-	-	-	-	-	-
3	-	-	-	-	R	-	-	-	R	-	-	-	-	R	-	-	-	-	R	-	-	M	-	-	-	-	R	-	-	-	-	-	-	-	-	-
4	-	-	-	-	-	-	R	-	-	-	-	R	-	M	-	-	M	-	M	-	M	M	-	-	-	-	R	-	-	-	-	-	-	-	-	-
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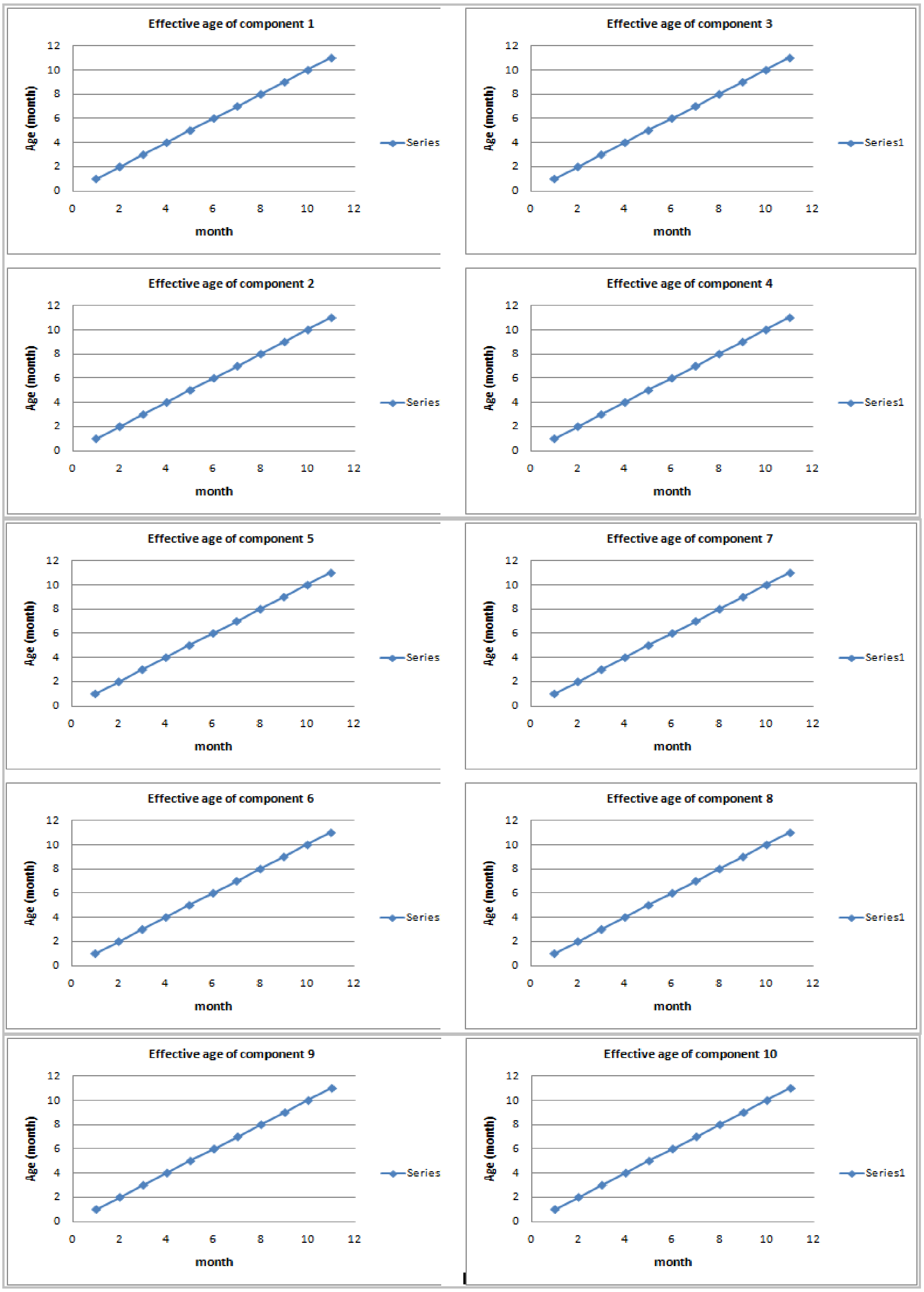
6	-	-	-	-	M	-	M	-	R	-	-	-	-	R	-	-	-	-	R	-	-	-	-	-	R	-	M	M	-	-	-	-	-	-	-	-
7	-	-	-	-	R	-	-	-	-	-	-	-	-	R	-	-	-	-	-	-	R	-	-	-	-	-	-	R	-	-	-	-	-	-	-	-
8	-	-	-	-	-	-	-	M	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	M	-	M	-	-	-	-	-	-	-	-	-
9	-	-	-	-	-	-	-	R	-	-	-	-	-	M	-	-	M	-	-	-	R	-	-	-	-	-	M	M	-	-	-	-	-	-	-	-
10	-	-	-	-	-	-	R	-	-	-	-	-	R	-	-	-	-	-	-	-	R	-	-	-	-	-	-	R	-	-	-	-	-	-	-	-

Table 7: Optimal maintenance and replacement schedule based on Z =1000$, RR=50%



Effective age of component 1

Effective age of component 3

Effective age of component 2

Effective age of component 4

Effective age of component 5

Effective age of component 7

Effective age of component 6

Effective age of component 8

Effective age of component 9

Effective age of component 10

Figure 7: Effective age of components at period T=12, (RR=67.3%) for Model 1.

As there was done no maintenance or replacement actions for above T=12 graphs, there was no scheduling table required as components kept on aging.

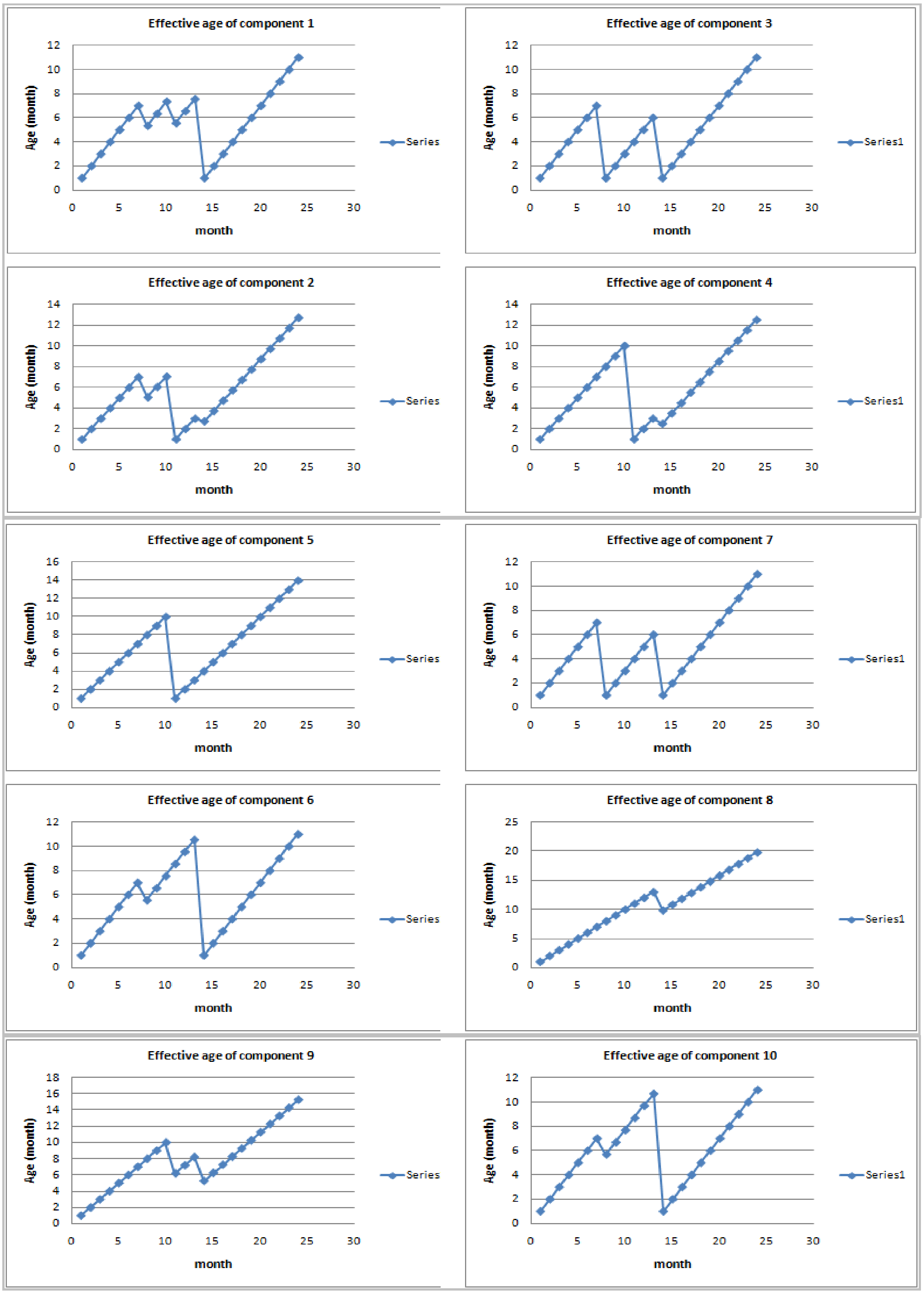
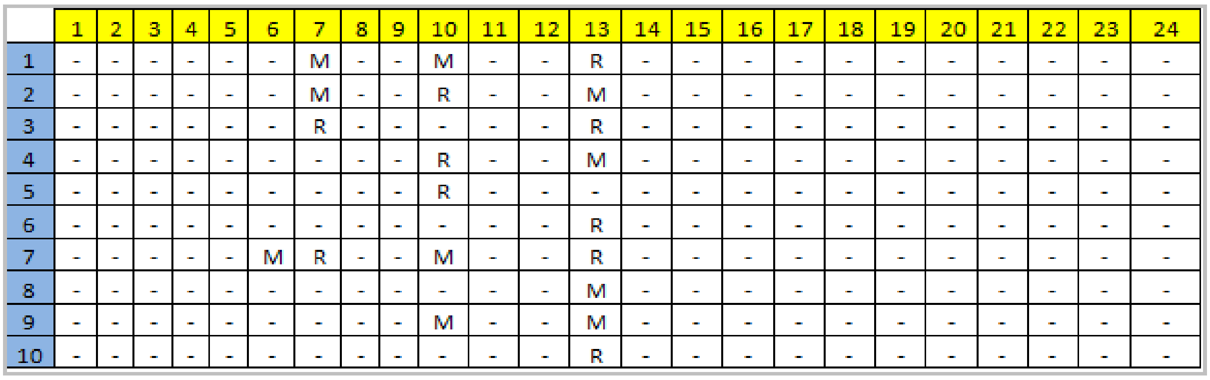


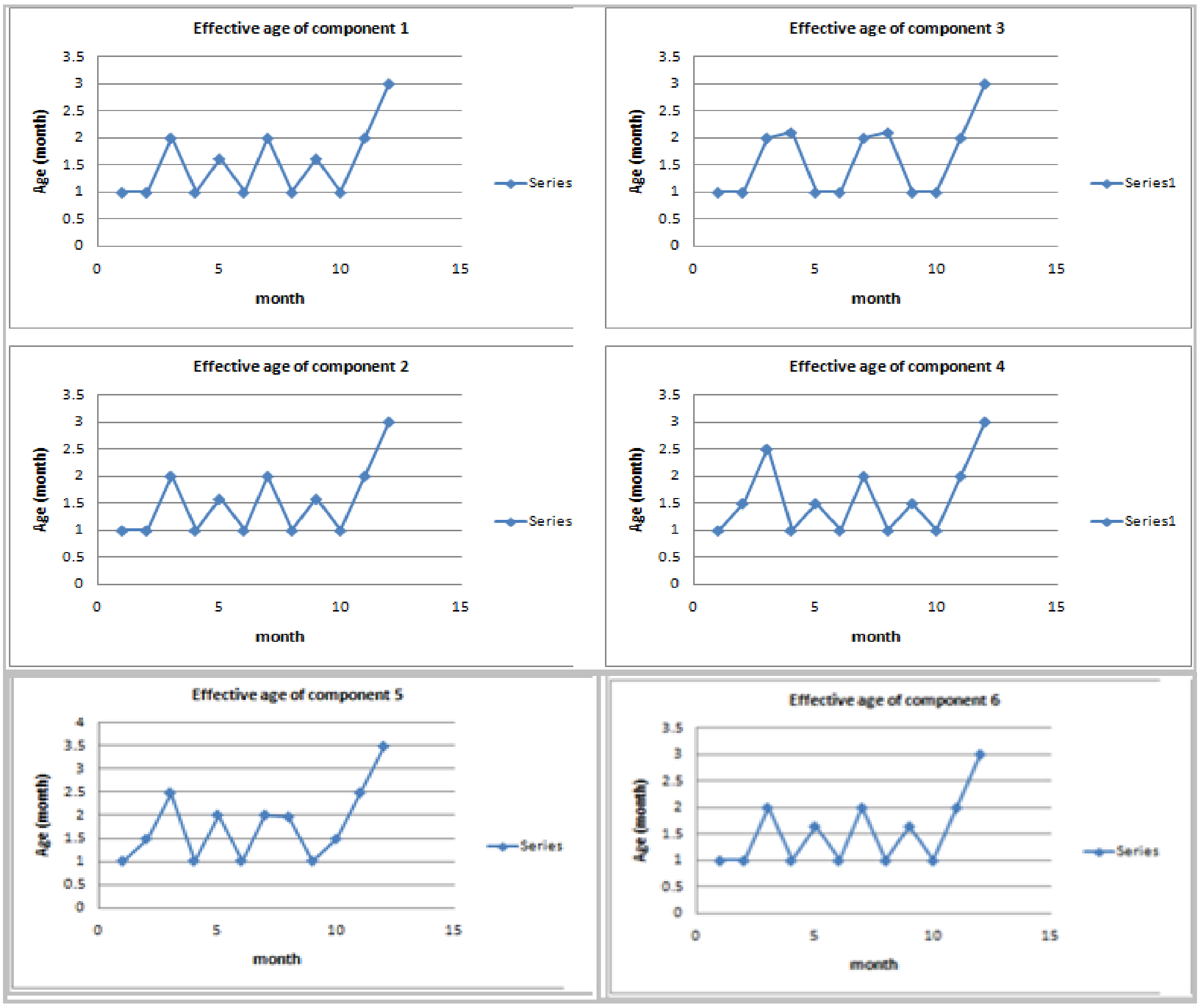
Figure 8: Effective age of components at period T=24, (RR=50.1%) for Model 1.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	-	-	-	-	-	-	M	-	-	M	-	-	R	-	-	-	-	-	-	-	-	-	-	-
2	-	-	-	-	-	-	M	-	-	R	-	-	M	-	-	-	-	-	-	-	-	-	-	-
3	-	-	-	-	-	-	R	-	-	-	-	-	R	-	-	-	-	-	-	-	-	-	-	-
4	-	-	-	-	-	-	-	-	-	R	-	-	M	-	-	-	-	-	-	-	-	-	-	-
5	-	-	-	-	-	-	-	-	-	R	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6	-	-	-	-	-	-	-	-	-	-	-	-	R	-	-	-	-	-	-	-	-	-	-	-
7	-	-	-	-	-	M	R	-	-	M	-	-	R	-	-	-	-	-	-	-	-	-	-	-
8	-	-	-	-	-	-	-	-	-	-	-	-	M	-	-	-	-	-	-	-	-	-	-	-
9	-	-	-	-	-	-	-	-	-	M	-	-	M	-	-	-	-	-	-	-	-	-	-	-
10	-	-	-	-	-	-	-	-	-	-	-	-	R	-	-	-	-	-	-	-	-	-	-	-

Table 8: Optimal maintenance and replacement schedule based on T =24, RR=50.1%

Model - 2 Result Graphs and Tables



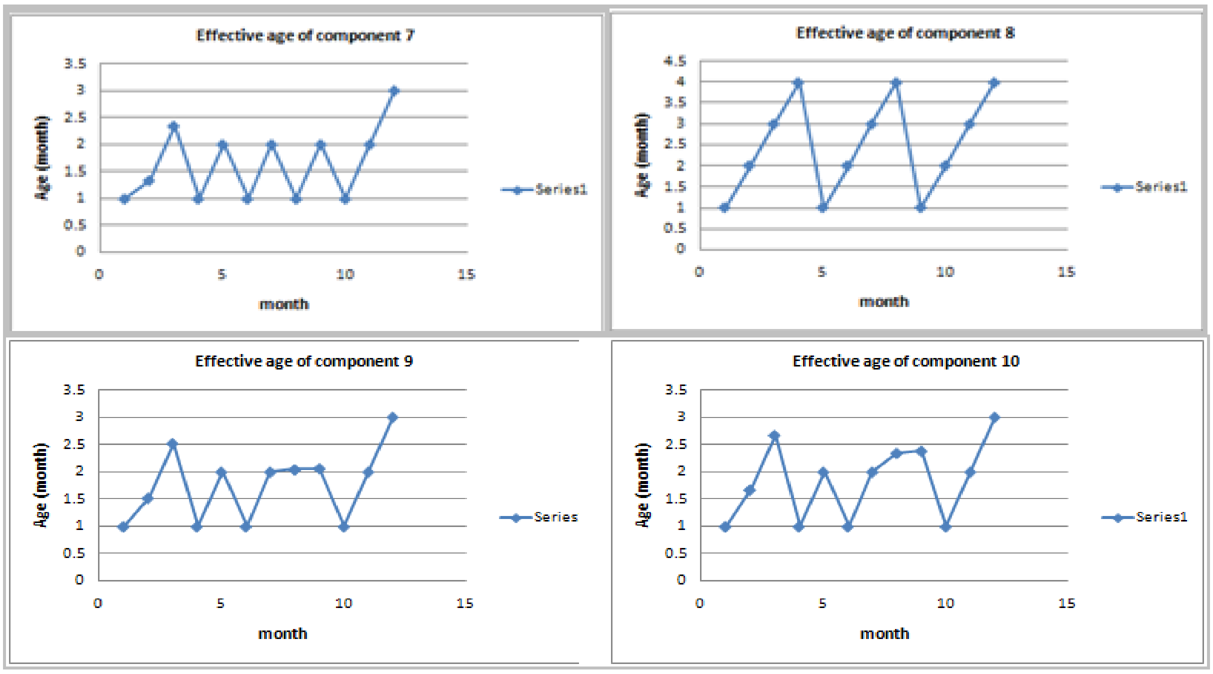
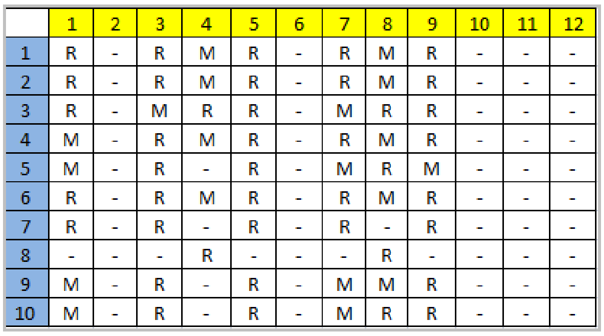


Figure 9: Effective age of components at period T=12, (RR=93%) for Model 2.



	1	2	3	4	5	6	7	8	9	10	11	12
1	R	-	R	M	R	-	R	M	R	-	-	-
2	R	-	R	M	R	-	R	M	R	-	-	-
3	R	-	M	R	R	-	M	R	R	-	-	-
4	M	-	R	M	R	-	R	M	R	-	-	-
5	M	-	R	-	R	-	M	R	M	-	-	-
6	R	-	R	M	R	-	R	M	R	-	-	-
7	R	-	R	-	R	-	R	-	R	-	-	-
8	-	-	-	R	-	-	-	R	-	-	-	-
9	M	-	R	-	R	-	M	M	R	-	-	-
10	M	-	R	-	R	-	M	R	R	-	-	-

Table 9: Optimal maintenance and replacement schedule based on T =12, RR=93%

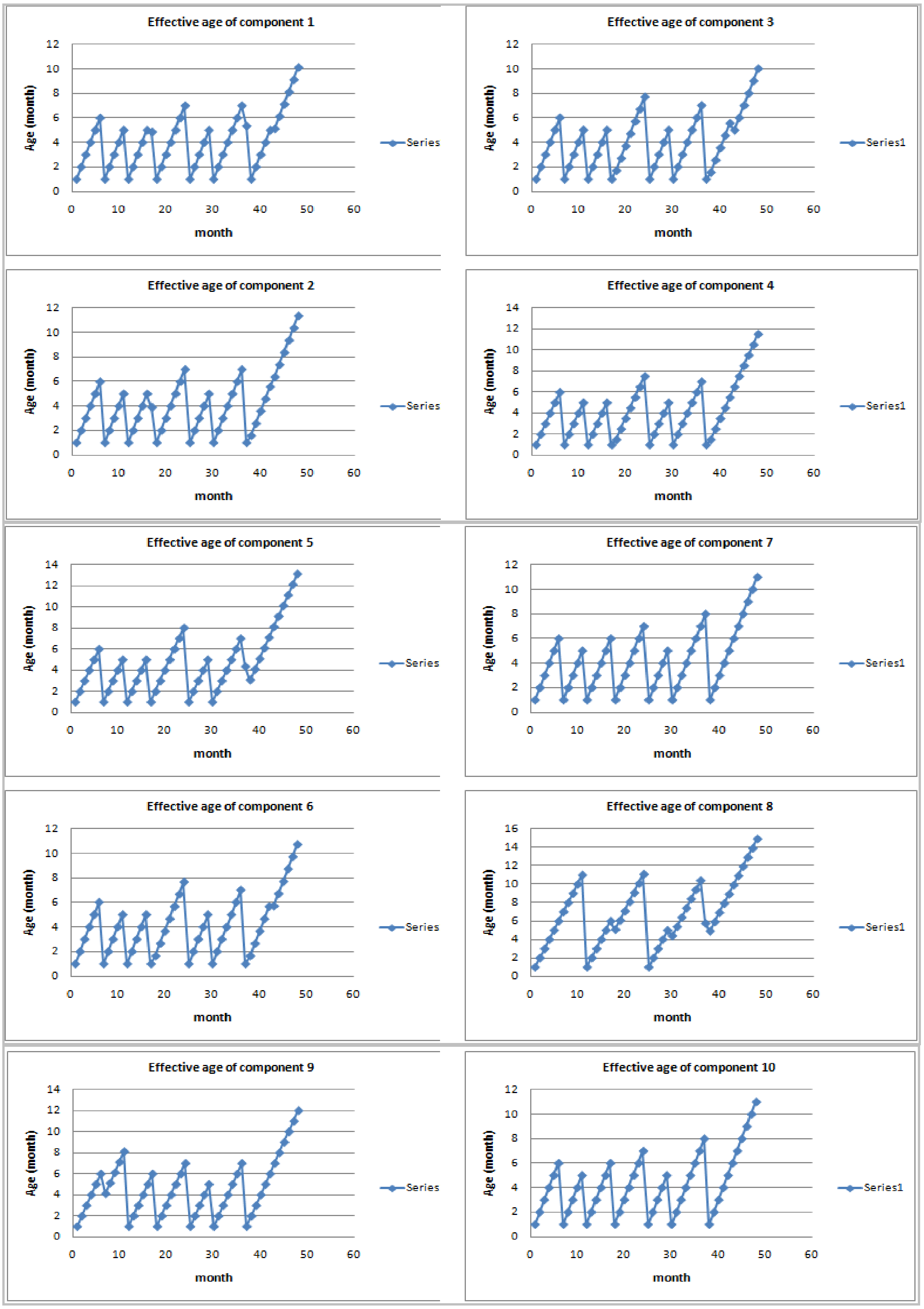
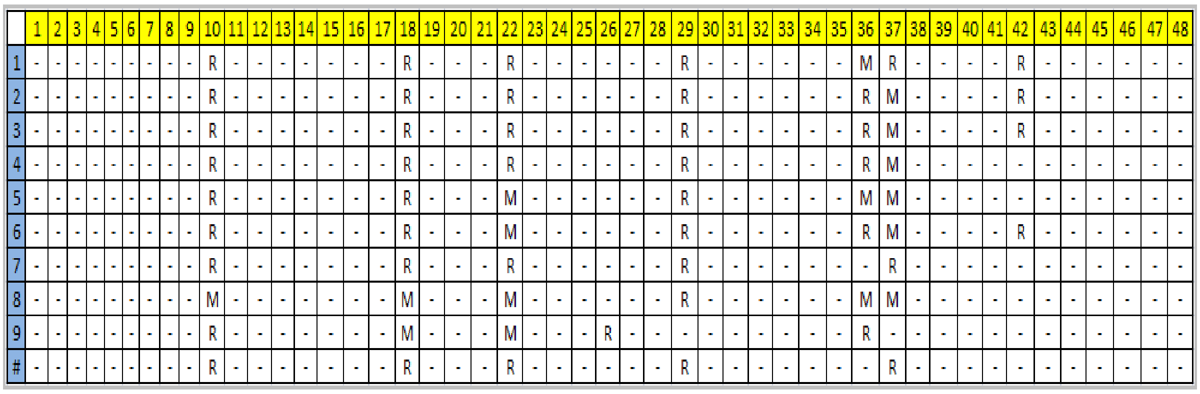
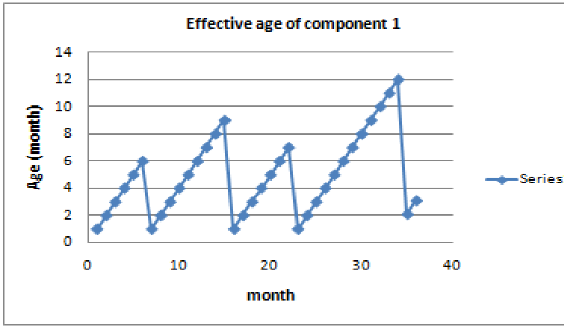


Figure 10: Effective age of components at period T=48, (RR=31.2%) for Model 2.

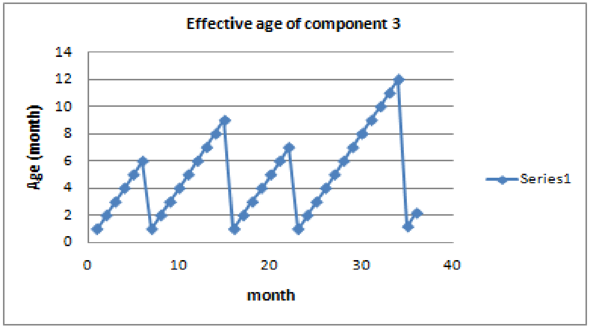


	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48
1	-	-	-	-	-	-	-	-	-	R	-	-	-	-	-	-	-	R	-	-	-	R	-	-	-	-	-	-	R	-	-	-	-	-	-	M	R	-	-	-	-	R	-	-	-	-	-	-
2	-	-	-	-	-	-	-	-	-	R	-	-	-	-	-	-	-	R	-	-	-	R	-	-	-	-	-	-	R	-	-	-	-	-	-	R	M	-	-	-	-	R	-	-	-	-	-	-
3	-	-	-	-	-	-	-	-	-	R	-	-	-	-	-	-	-	R	-	-	-	R	-	-	-	-	-	-	R	-	-	-	-	-	-	R	M	-	-	-	-	R	-	-	-	-	-	-
4	-	-	-	-	-	-	-	-	-	R	-	-	-	-	-	-	-	R	-	-	-	R	-	-	-	-	-	-	R	-	-	-	-	-	-	R	M	-	-	-	-	-	-	-	-	-	-	-
5	-	-	-	-	-	-	-	-	-	R	-	-	-	-	-	-	-	R	-	-	-	M	-	-	-	-	-	-	R	-	-	-	-	-	-	M	M	-	-	-	-	-	-	-	-	-	-	-
6	-	-	-	-	-	-	-	-	-	R	-	-	-	-	-	-	-	R	-	-	-	M	-	-	-	-	-	-	R	-	-	-	-	-	-	R	M	-	-	-	-	R	-	-	-	-	-	-
7	-	-	-	-	-	-	-	-	-	R	-	-	-	-	-	-	-	R	-	-	-	R	-	-	-	-	-	-	R	-	-	-	-	-	-	-	R	-	-	-	-	-	-	-	-	-	-	-
8	-	-	-	-	-	-	-	-	-	M	-	-	-	-	-	-	-	M	-	-	-	M	-	-	-	-	-	-	R	-	-	-	-	-	-	M	M	-	-	-	-	-	-	-	-	-	-	-
9	-	-	-	-	-	-	-	-	-	R	-	-	-	-	-	-	-	M	-	-	-	M	-	-	-	R	-	-	-	-	-	-	-	-	-	R	-	-	-	-	-	-	-	-	-	-	-	-
#	-	-	-	-	-	-	-	-	-	R	-	-	-	-	-	-	-	R	-	-	-	R	-	-	-	-	-	-	R	-	-	-	-	-	-	-	R	-	-	-	-	-	-	-	-	-	-	-

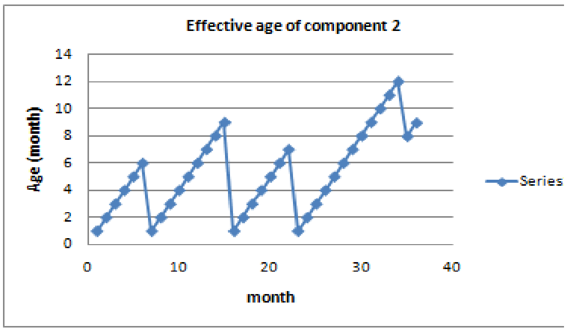
Table 10: Optimal maintenance and replacement schedule based on T =48, RR=31.2%



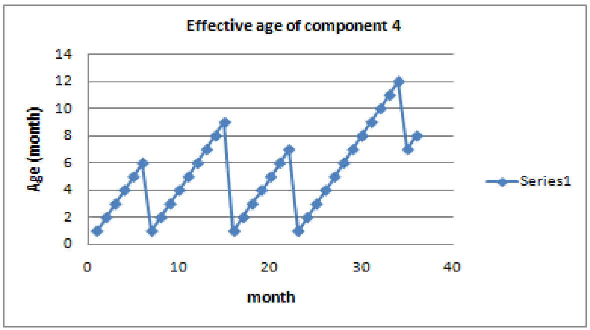
Effective age of component 1



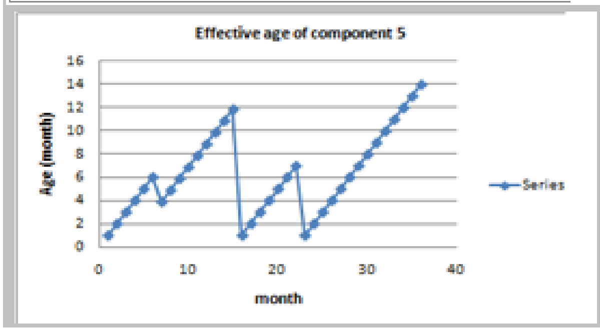
Effective age of component 3



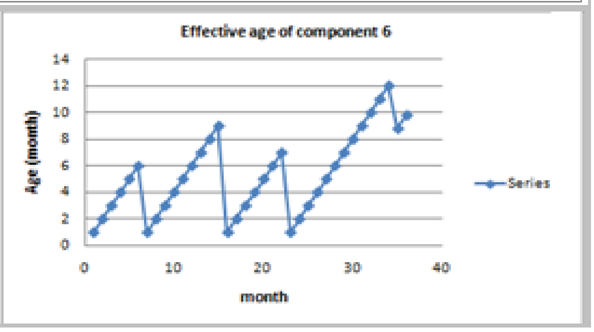
Effective age of component 2



Effective age of component 4



Effective age of component 5



Effective age of component 6

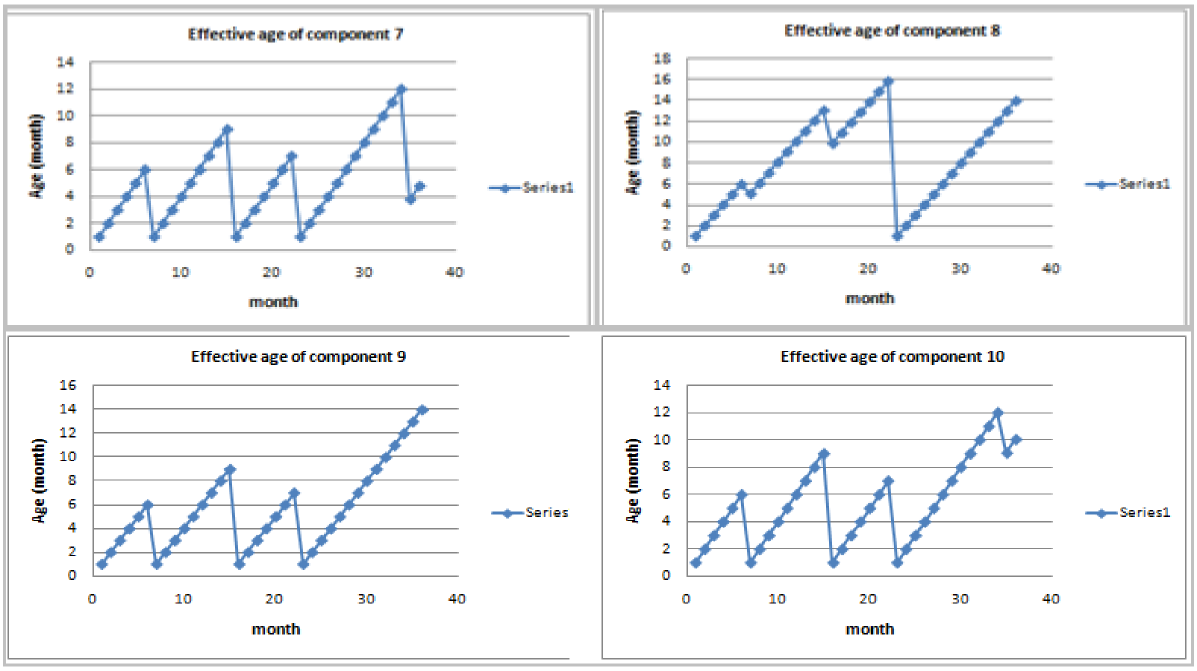
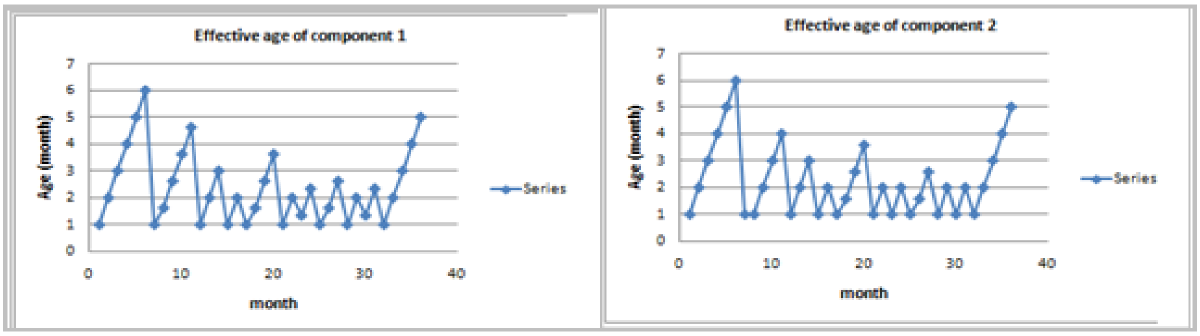


Figure 11: Effective age of components at period T=36, (GB=10000$, RR=39.3%) for Model 2.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
1	-	-	-	-	-	R	-	-	-	-	-	-	-	-	R	-	-	-	-	-	-	R	-	-	-	-	-	-	-	-	-	-	-	R	-	-
2	-	-	-	-	-	R	-	-	-	-	-	-	-	-	R	-	-	-	-	-	-	R	-	-	-	-	-	-	-	-	-	-	-	M	-	-
3	-	-	-	-	-	R	-	-	-	-	-	-	-	-	R	-	-	-	-	-	-	R	-	-	-	-	-	-	-	-	-	-	-	R	-	-
4	-	-	-	-	-	R	-	-	-	-	-	-	-	-	R	-	-	-	-	-	-	R	-	-	-	-	-	-	-	-	-	-	-	M	-	-
5	-	-	-	-	-	M	-	-	-	-	-	-	-	-	R	-	-	-	-	-	-	R	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6	-	-	-	-	-	R	-	-	-	-	-	-	-	-	R	-	-	-	-	-	-	R	-	-	-	-	-	-	-	-	-	-	-	R	-	-
7	-	-	-	-	-	R	-	-	-	-	-	-	-	-	R	-	-	-	-	-	-	R	-	-	-	-	-	-	-	-	-	-	-	R	-	-
8	-	-	-	-	-	M	-	-	-	-	-	-	-	-	M	-	-	-	-	-	-	R	-	-	-	-	-	-	-	-	-	-	-	-	-	-
9	-	-	-	-	-	R	-	-	-	-	-	-	-	-	R	-	-	-	-	-	-	R	-	-	-	-	-	-	-	-	-	-	-	-	-	-
10	-	-	-	-	-	R	-	-	-	-	-	-	-	-	R	-	-	-	-	-	-	R	-	-	-	-	-	-	-	-	-	-	-	M	-	-

Table 11: Optimal maintenance and replacement schedule based on T =36, GB=10000$, RR=39.3%



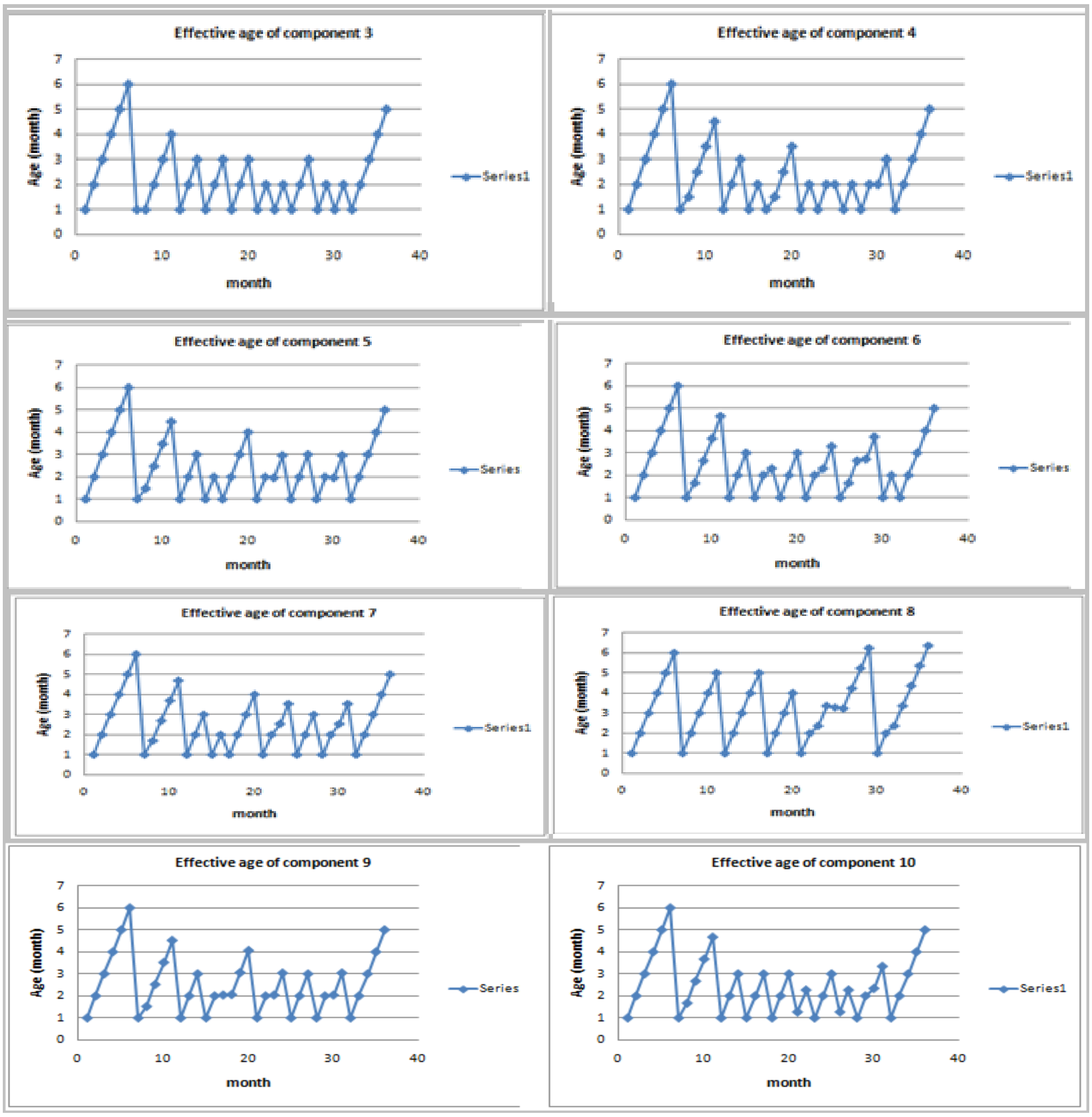


Figure 12: Effective age of components at period T=36, (GB=30000$, RR=69%) for Model 2.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
1	-	-	-	-	-	R	M	-	-	-	R	-	-	R	-	R	M	-	-	R	-	R	-	R	M	-	R	-	R	-	R	-	-	-	-	-
2	-	-	-	-	-	R	R	-	-	-	R	-	-	R	-	R	M	-	-	R	-	R	-	R	M	-	R	-	R	-	R	-	-	-	-	-
3	-	-	-	-	-	R	R	-	-	-	R	-	-	R	-	-	R	-	-	R	-	R	-	R	-	-	R	-	R	-	R	-	-	-	-	-
4	-	-	-	-	-	R	M	-	-	-	R	-	-	R	-	R	M	-	-	R	-	R	-	M	R	-	R	-	M	-	R	-	-	-	-	-
5	-	-	-	-	-	R	M	-	-	-	R	-	-	R	-	R	-	-	-	R	-	M	-	R	-	-	R	-	M	-	R	-	-	-	-	-
6	-	-	-	-	-	R	M	-	-	-	R	-	-	R	-	M	R	-	-	R	-	M	-	R	M	-	M	-	R	-	R	-	-	-	-	-
7	-	-	-	-	-	R	R	-	-	-	R	-	-	R	-	R		-	-	R	-	R	-	R	-	-	R	-	R	-	R	-	-	-	-	-
8	-	-	-	-	-	R	-	-	-	-	R	-	-	-	-	R	-	-	-	R	-	M	-	M	M	-	-	-	R	-	M	-	-	-	-	-
9	-	-	-	-	-	R	M	-	-	-	R	-	-	R	-	M	M	-	-	R	-	M	-	R	-	-	R	-	M	-	R	-	-	-	-	-
10	-	-	-	-	-	R	M	-	-	-	R	-	-	R	-	-	R	-	-	R	-	R	-	-	R	-	R	-	M	-	R	-	-	-	-	-

Table 12: Optimal maintenance and replacement schedule based on T =36, GB=30000$, RR=69%

Remarks:

🟨	Number of Months
🟦	Number of components