Energy Harvesting Using Mechanical Absorber

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CERTIFICATION OF APPROVAL

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A project dissertation submitted to the
Mechanical Engineering Programme
Universiti Teknologi PETRONAS
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(MECHANICAL)

Approved by,

____

(DR TAMIRU ALEMU LEMMA)

UNIVERSITI TEKNOLOGI PETRONAS
TRONOH, PERAK
May 2015

CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified I the references and acknowledgement, and that the original work contained herein have not been undertaken or done by unspecified sources or person.

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RAZIN AKMAL BIN RUSLAN

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ABSTRACT

In recent years, vibration energy harvesting is receiving a considerable amount of interest among the researchers. With the global concern on energy issues, this lead to an increase in the study of vibration based energy harvesting and use it as an alternative source of energy. This paper presents an idea on harvesting energy using the mass spring absorber system attached with a transducer. The aim is to investigate the effect of damping, mass, stiffness and bandwidth towards energy harvesting system performance. The stiffness, mass, damping and bandwidth will be the parameter to be studied. This will enable author to investigate the behavior of each parameter towards the system and how its effect the power output. Simple mechanical absorber system will be modeled based on the undamped, damped and forced vibration second degree of freedom equation of motion. Based on the mathematical equation derived, the stiffness, mass, damping and bandwidth is simulated and how its affect the system is observed. The results shows that amplitude will increase as damping is decreases, while amplitude will be decrease as damping higher. The optimum mass ratio (M2/M1) of the system is between 5%-10%. The acceptable range of bandwidth ratio is from 5% to 30%, as higher ratio will lead to less power output from the system.

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CHAPTER 1 : INTRODUCTION

1.1 Background Study

Vibration energy harvesting is the process which wasted vibration is harvested and converted to useful electrical energy. With the increasing demand of energy and environment concern, many technologies in energy harvesting are developed such as solar, geothermal and wind energy harvester. Since vibration exists everywhere, such as from building, vehicle system, human motion, machines and ocean waves, it can become a good alternative energy source if we are able to harvest and utilize it.

A vibration energy harvesting system usually consists of a mechanical system with external excitation, a transducer that converts the vibration energy into electric energy, mechanisms for motion transmission and magnification, power electronics and energy storage elements, and energy management and control strategies [1]. Even though researcher successfully to harvest energy using the system, but the output energy of the traditional vibration energy harvesting absorber is low. Majority of the researcher able to harvest energy less than 100 mW power, which has only limited applications in low power electronics. In addition, researcher previous work are only focused on harvesting energy at only particular frequency region.

Ocean waves and vehicle system are example of sources that produced large vibration. For example, Zuo cited in his article that the world ocean wave is producing approximately 8000-80000 TWh every year and total wave energy along the U.S coast is 2640 TWh/yr. Harvesting huge amount of vibration energy from this source can be useful because it can be used to power up high energy applications device. Besides relying on larger vibration sources, researcher also changing the design of the absorber system itself in order to maximize the output of the harvested energy.

Manipulating the absorber system parameter such as mass, damping, stiffness and bandwidth also will affect the output of the energy harvested. As for the energy harvesting device, transducer will be used. Many type of transducer are available to be selected. Conventional type transducer may not able to give out the desired output. So researcher keep manipulating and optimizing the conventional transducer so that it can maximize the energy harvested from it.

1.2 Problem Statement

Energy harvesting is relatively a topic aiming at discovering new sources of clean electric energy to avoid environment pollution from the classical oil based techniques. Researcher are looking at the efficiency of energy and started to considerate all form of energy to be utilized rather than letting the energy wasted.Vibration exists in almost all dynamic systems, including human motion. Recently, only researchers are interested to exploit vibration as energy source. The challenge however, is that there are trade-offs to make since maximization of the energy harvesting could mean just shifting the critical speeds and causing instead higher vibration. Under this research, we intend to find answers to the following research questions:

i. How does the bandwidth affect the maximum power that can be harvested from the dual mass vibration absorber?
ii. What would be the effect of absorber mass ratio to main mass vibration amplitude and maximum energy harvested?
iii. How does an absorber parameter (damping, stiffness, mass, bandwidth) affect the maximum power that can be harvested from the system?

1.3 Objectives

The objectives of this project are:

1. To develop a mathematical model for two mass. (Absorber and main mass system).
2. To find the optimum design parameter recommendation for the absorber system.(Damping, mass, stiffness and bandwidth)

1.4 Scope of Study

This project work is limited to:

i. Viscous damped type vibration absorber.
ii. An absorber connected to a piezoelectric device.
iii. A study based on a benchmark problem to be selected in the process.
iv. A study on the effect of bandwidth, damping, mass and stiffness towards the dual mass absorber system.

CHAPTER 2 : LITERATURE REVIEW

2.1 Dynamic Vibration Absorber

The vibration absorber, also called dynamic vibration absorber, is a mechanical devices used to reduce or eliminate unwanted vibration on the primary system. It consists of another mass and stiffness attached to the main or original mass that needs to be protected from vibration. [1]. The aim of using dynamic vibration absorber is to control the vibration of the host structure and to produce maximum vibration from the absorber mass so that energy can be harvest from it [2][4]. Dynamic vibration absorber also able to transfer vibration from primary structure to the secondary structure.

Typically, dynamic vibration absorber is modeled by a spring-mass-damper system with base excitation and the mechanical to electrical energy conversion occurs when a conversion mechanism is applied [3]. Vibration absorber consist of a comparatively small vibratory system k, m attached to the main mass M (Figure 1). The natural frequency $\sqrt{k/m}$ of the attached absorber is chosen to be equal to the frequency of ω of the disturbing force. This will let main mass M does not vibrate, and the small k and m vibrates in such way that its spring force is at all instant equal and opposite to $P_o\ sin\,\omega t$. Thus there is no net for acting on *M* and therefore that mass does not vibrate [5].

To obtain maximum energy conversion, the motion between the base and the oscillating mass should be maximized. It can be done when the oscillating mass is tuned to the dominant frequency of the base excitation force. Tuning an energy harvester assumes the ambient vibration is characterized by a single dominant frequency. If the forcing frequency is away from the tuned system’s resonant frequency, the resulting oscillation amplitude is small and corresponds to a smaller amount of harvested energy [7].

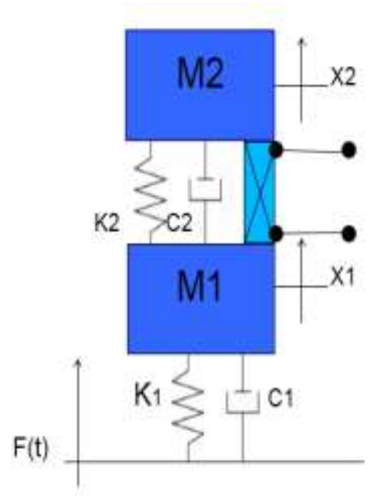


Figure 2.1: Free Body Diagram of Two Degree Freedom Vibration

2.1.1 Absorber System Design

This section shows the design of the absorber system from previous researcher. Researcher keep changing the design of the absorber system in order to maximize the output of the energy harvested from the system, while minimizing the vibration of the system. Hassan (2014) using the conventional absorber system without damping tune the absorber system to cancel the vibration while producing the maximum absorber mass to act as an energy harvester. Ali and Adhikari (2013) propose new design which is the aim is to control the primary structure vibration and to harvest energy from dynamic vibration absorber using fixed theory point method. Zuo et. al (2011) using the beam system replacing the old conventional design. The design comprises a multi-mode intermediate beam with a tip mass, called a 'dynamic magnifier', and an 'energy harvesting beam' with a tip mass. Liu and Coppola (2010) in his research compare old design with their purpose design which is the absorber mass is directly connected toward the base of the main structure. Wong and cui (2007) design is placing the primary mass above the secondary mass while attach it to the main structure. The design and summary are shown below.

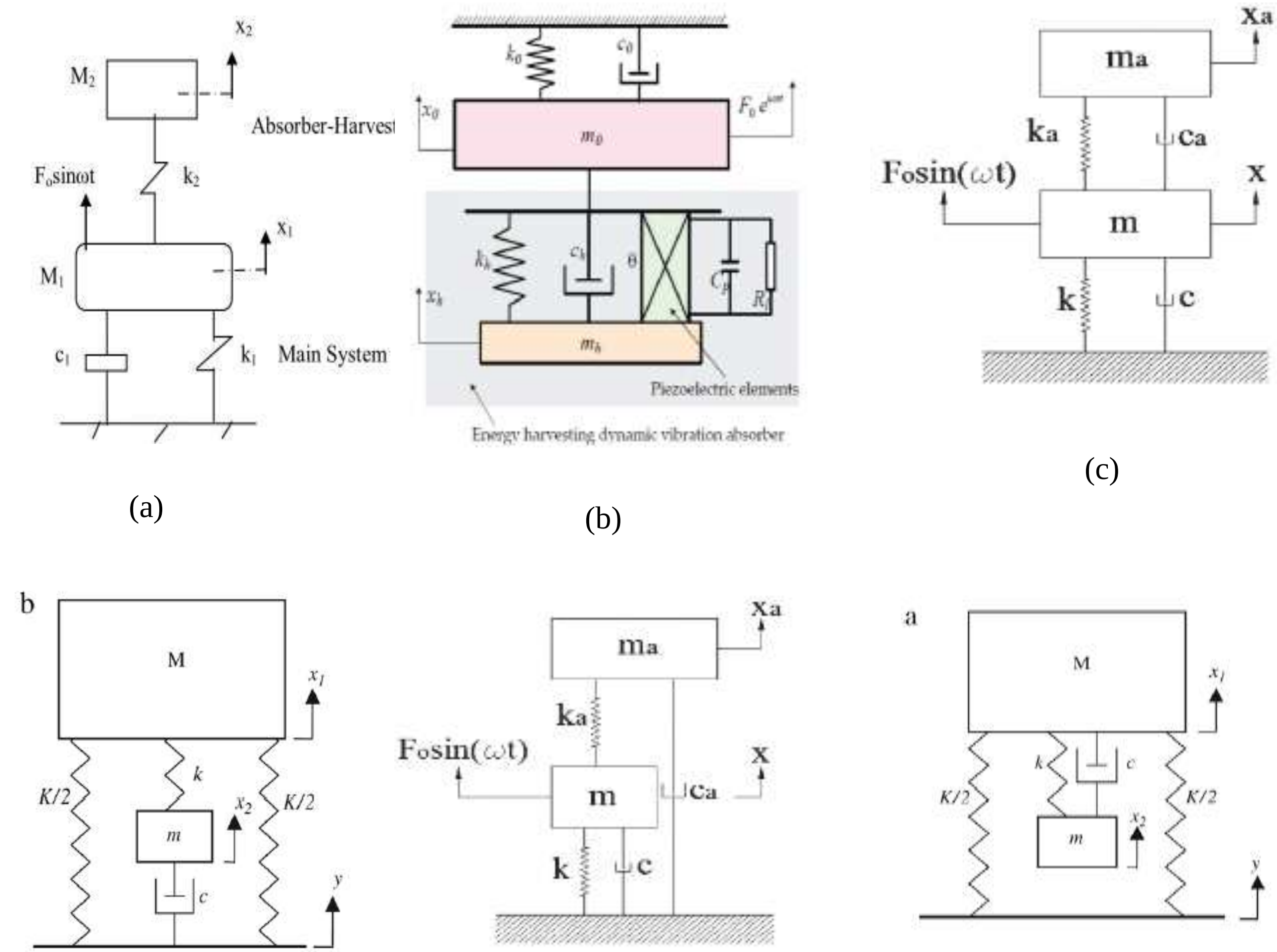

M₂ should not be used; see below

Table 2.1: Summary of the literatures on absorber system design

Author/Year	**Objective of the paper**	**Parameters studied**	**Remarks**
Hassaan (2014) (a)	- The aim is to tune the absorber to cancel the machine vibration and produce maximum vibration of the absorber mass to act as an energy harvester.	- Optimal operation of the proposed system is investigated for damping ratio between 0.1 and 0.4 and mass ratio between 0.05 and 4.5. - Natural frequency ratio, β	- The absorber succeeds to eliminate completely the main system vibrations at a tuning condition and provides high vibration amplitude at the absorber mass at the new defined harvesting frequency.
Ali and Adhikari (2013) (b)	- To control the vibration of the host structure and the secondary goal is to harvest energy out of the dynamic vibration absorber. - Approximate fixed-point theory is used to find a closed form expression for optimal frequency ratio of	- Frequency ratio - Mass ratio - Optimal damping factor	- Good choice of harvester parameter a broadband energy harvesting can be obtained with vibration reduction in the primary structure.

	the vibration absorber.		
Zuo et. al. (2011) (c)	- To study a novel piezoelectric energy harvester with a multi-mode dynamic magnifier which capable increasing the bandwidth and the energy harvested from ambient vibration. - The design comprises a multi-mode intermediate beam with a tip mass, called a 'dynamic magnifier', and an 'energy harvesting beam' with a tip mass.	- Six mode of frequency - Beam dimension - Weight of the tip masses	- The experiment demonstrates 25.5 times more energy harvesting capacity than the conventional cantilever type harvester in the frequency range 3–300 Hz, and 100–1000 times more energy around all the first three resonances of the harvesting beam.
Liu and Coppola (2010) (d) and (e)	- Compared two different design of a dynamic vibration absorber: (i) conventional design, and (ii) absorber damper directly connected to the main supporting structure.	- Damping ratio - Mass ratio - Optimum tuning parameter - Optimum damping ratio	- Model in figure D shows a good numerical result. - Model in figure E result differ from the analytical ones when mass ratio increases. Overcome by modify Chebysev's equioscillation theorem
Cheung and Wong	- Proposed new design for dynamic vibration absorber.	- Frequency ratio	- Analytically prove proposed the absorber provides a larger

(2007) (f) and (g)	- Compare the reduction of transmission of motion from support to the mass of the structure with traditional absorber design.	- Mass ratio	suppression of resonant vibration amplitude of the primary system excited by ground motion than the traditional absorber. - The comparison revealed that though model B requires a larger amount of damping than model A, the resonant vibration amplitude under the optimized condition of model B is always less than that of model A.

2.1.2 Absorber Modeling and Optimization.

This section shows the optimization that have been done to the absorber mass system. Brown et al. (2015) in his research doing optimization for a range of frequencies. It stated that when normalized frequency range below 0.405 the nominal frequency, this will minimized the maximum main mass displacement magnitude to a lower value. Hassaan (2014) optimize the mass ratio and frequency of his system. By doing that, the vibration amplitude of the absorber harvester mass increase as the damping ratio of the main system and main ratio decreases. Liu (2010) and Coppola stated by increasing the damping ratio m, the optimum tuning parameter decreases and the optimum damping ratio increases. Asami (2002) presents the analytical solutions for the $H_{(infinity)}$ and H_2 optimization of the DVA attached to the damped primary systems. $H_{(infinity)}$ optimization the DVA is designed such that the maximum amplitude magnification factor of the primary system is minimized, whereas in the H_2 optimization the DVA is designed such that the squared area under the response curve of the primary system is minimized. Summary of the optimization are included in Table 2.

Table 2.2: Summary of the literatures on absorber modeling and optimization

Author/Year	**Objective of the paper**	**Parameters studied**	**Remarks**
Brown et. al(2015)	- Optimization for a range of frequencies	- Vibration absorber mass ratio - Normalized frequency range	- The method shows improvement over results by utilizing knowledge about the expected forcing frequency range for a damped main mass system. - For the primary system damping z_1=0.1, when the normalized frequency range about the nominal frequency remained below 0.405, this method minimized the maximum main mass displacement magnitude to a lower value.
Hassaan (2014)	- To optimize Mass ratio and frequency ratio	- Mass ratio - Frequency ratio	- It is possible to reduce the vibration amplitude of the main vibrating to zero (eliminating completely its vibrations) by tuning the exciting frequency to the absorber- harvester

			natural frequency. - At the harvesting frequency, the vibration amplitude of the absorber-harvester mass is maximum depending on the damping ratio of the main system and the absorber mass ratio. - The vibration amplitude of the absorber-harvester mass increased as the damping ratio of the main system and mass ratio of the absorber decrease.
Liu and Coppola (2010)	- Optimal design of the dynamic vibration absorber for a damped system - Results from two different optimization methods are compared against analytical solution.	- Two tuning parameters (beta and zeta) were considered for optimization - Optimum design, beta	- Increase of the damping ratio, or the mass ratio m, the optimum tuning parameter decreases and the optimum damping ratio increases.

Cheung and Wong (2008)	- Optimum tuning condition including frequency and damping ratio	- Frequency ratio - Mass ratio	- Optimum tuning condition including the frequency and damping ratios of the proposed absorber has been derived based on the fixed-points theory.
Asami et al. (2002)	- Optimization absorber parameters for a damped system - - Presents the analytical solutions for the H (infinity) and H_2 optimization of the DVA attached to the damped primary systems.	- Optimized damping constant of the absorber	- In the H (infinity) optimization the DVA is designed such that the maximum amplitude magnification factor of the primary system is minimized; whereas in the H_2 optimization the DVA is designed such that the squared area under the response curve of the primary system is minimized. - The solution provided which is second order approximation solution has virtually no error up to zeta = 0.15 which sufficient to use in most DVA system.

Table 2.3: Commonly used optimization criteria in absorber system design [Asami et al. 2002]

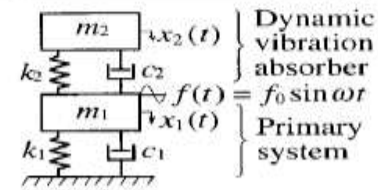
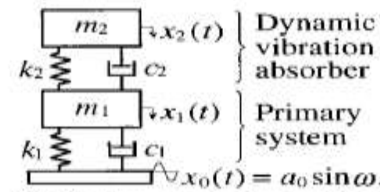
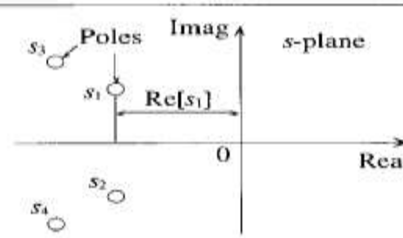
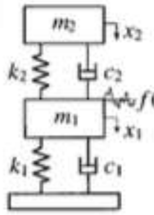
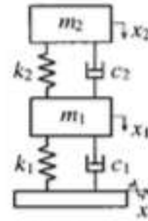
No.	Optimization criterion	Performance index	Objective / Who/When proposed?	Analytical model	Definition of the symbols
1	H_∞ Optimization	$A_{1\max} = \left\| \frac{x_1}{x_{st}} \right\|_{\max}$ or $T_{1\max} = \left\| \frac{x_1}{x_0} \right\|_{\max}$	to minimize the maximum amplitude response of the system Ormondroyd, J & Den Hartog, J. P., 1928	 (a) Force excitation system	$\omega_1 = \sqrt{\frac{k_1}{m_1}}$ $\omega_2 = \sqrt{\frac{k_2}{m_2}}$ $\mu = \frac{m_2}{m_1}$
2	H_2 Optimization	$I_1 = \frac{E[x_1^2]}{2\pi S_f \omega_1 / k_1^2}$ or $I_t = \frac{E[x_1^2]}{2\pi S_0 \omega_1}$	to minimize the total vibration energy of the system over all frequencies Crandall, S. H. & Mark, W. D., 1963	 (b) Motion excitation system	$\nu = \frac{\omega_2}{\omega_1}$ $\zeta_1 = \frac{c_1}{2m_1\omega_1}$ $\zeta_2 = \frac{c_2}{2m_2\omega_2}$
3.	Stability Maximization	$\Lambda = -\max_i(\mathrm{Re}[s_i])$	to attenuate the transient vibration of the system as soon as possible Yamaguchi, H., 1988 Nishihara,O. & Matsuhisa, H., 1997		$x_{st} = f_0 / k_1$ $E[\ ]$: ensemble mean S_f, S_0: uniform power spectrum density of the excitation

Table 2.4: (a) A series solution for H2 optimization of the dynamic vibration absorber for the force excitation system [Asami et al. 2002] (b) A series solution for H2 optimization of the dynamic vibration absorber for the

(a)

Analytical model and Performance index I_1	 $f(t)$ = white noise $I_1 = \dfrac{E[x_1^2]}{2\pi S_f\,\omega_1/k_1^2} = \dfrac{<x_1^2>}{2\pi S_f\,\omega_1/k_1^2} = \dfrac{1}{2\pi}\int_{-\infty}^{\infty}\left\|\dfrac{x_1}{f_0/k_1}\right\|^2 d\lambda$ $E[\ ]$: Ensemble avarage $<\ >$: Temporal avarage S_f: Uniform power spectrum density of the excitation, N^2s/rad
Definition of the symbols	$\omega_1=\sqrt{k_1/m_1}$, $\omega_2=\sqrt{k_2/m_2}$, $\mu=m_2/m_1$, $\nu=\omega_2/\omega_1$ $\zeta_1=c_1/(2m_1\omega_1)$, $\zeta_2=c_2/(2m_2\omega_2)$, $\lambda=\omega/\omega_1$
Optimum tuning ν_{opt}	$\dfrac{1}{1+\mu}\sqrt{1+\dfrac{\mu}{2}}-\zeta_1(4+\mu)\sqrt{\dfrac{\mu}{8(1+\mu)^3(2+\mu)(4+3\mu)}}$ $+\zeta_1^2\dfrac{\mu(192+304\mu+132\mu^2+13\mu^3)}{8(1+\mu)^2(4+3\mu)^2\sqrt{2(2+\mu)^3}}$ $-\zeta_1^3\dfrac{b_1}{16}\sqrt{\dfrac{\mu^3}{2(1+\mu)^5(2+\mu)^5(4+3\mu)^7}}$ where $b_1=4096+13056\mu+15360\mu^2+8080\mu^3+1780\mu^4+101\mu^5$
Optimum damping ζ_{2opt}	$\sqrt{\dfrac{\mu(4+3\mu)}{8(1+\mu)(2+\mu)}}-\zeta_1\dfrac{\mu^3}{4(1+\mu)(4+3\mu)\sqrt{2(2+\mu)^3}}$ $+\zeta_1^2\dfrac{-64-80\mu+15\mu^3}{32}\sqrt{\dfrac{2\mu^5}{(1+\mu)^3(2+\mu)^5(4+3\mu)^5}}$ $+\zeta_1^3\dfrac{\mu^3 b_2}{32(1+\mu)^2(4+3\mu)^4\sqrt{2(2+\mu)^7}}$ where $b_2=2048+6912\mu+8064\mu^2+3616\mu^3+288\mu^4-125\mu^5$

(b)

Analytical model and Performance index I_t	 $x_0(t)$ = white noise $I_t = \dfrac{E[x_1^2]}{2\pi S_0\omega_1} = \dfrac{<x_1^2>}{2\pi S_0\omega_1} = \dfrac{1}{2\pi}\int_{-\infty}^{\infty}\left\|\dfrac{x_1}{x_0}\right\|^2 d\lambda$ $E[\ ]$: Ensemble avarage $<\ >$: Temporal avarage S_0: Uniform power spectrum density of the excitation, m^2s/rad
Definition of the symbols	$\omega_1=\sqrt{k_1/m_1}$, $\omega_2=\sqrt{k_2/m_2}$, $\mu=m_2/m_1$, $\nu=\omega_2/\omega_1$ $\zeta_1=c_1/(2m_1\omega_1)$, $\zeta_2=c_2/(2m_2\omega_2)$, $\lambda=\omega/\omega_1$
Optimum tuning ν_{opt}	$\dfrac{1}{1+\mu}\sqrt{1+\dfrac{\mu}{2}}-\zeta_1(4+\mu)\sqrt{\dfrac{\mu}{8(1+\mu)^3(2+\mu)(4+3\mu)}}$ $+\zeta_1^2\dfrac{\mu(704+1328\mu+804\mu^2+157\mu^3)}{8(1+\mu)^2(4+3\mu)^2\sqrt{2(2+\mu)^3}}$ $+\zeta_1^3\dfrac{b_1}{16}\sqrt{\dfrac{\mu}{2(1+\mu)^5(2+\mu)^5(4+3\mu)^7}}$ where $b_1=65536+241664\mu+369920\mu^2+305664\mu^3$ $+148720\mu^4+43500\mu^5+7339\mu^6+576\mu^7$
Optimum damping ζ_{2opt}	$\sqrt{\dfrac{\mu(4+3\mu)}{8(1+\mu)(2+\mu)}}-\zeta_1\dfrac{\mu^3}{4(1+\mu)(4+3\mu)\sqrt{2(2+\mu)^3}}$ $+\zeta_1^2\dfrac{4096+13760\mu+18608\mu^2+12640\mu^3+4287\mu^4+576\mu^5}{32}$ $\times\sqrt{\dfrac{2\mu^3}{(1+\mu)^3(2+\mu)^5(4+3\mu)^5}}$ $+\zeta_1^3\dfrac{\mu b_2}{32(1+\mu)^2(4+3\mu)^4\sqrt{2(2+\mu)^7}}$

where $b_2=524288+2818048\mu+6621184\mu^2+8864512\mu^3$
$+7377280\mu^4+3896224\mu^5+1271168\mu^6+233491\mu^7+18432\mu^8$

2.2 Vibration to Electrical Conversion

2.2.1 Transducer - Piezoelectric

A transducer is a device that converts a signal in one form of energy to another form of energy [6]. For this project, the function of the transducers is to convert the mechanical energy into electrical energy from the vibration source. According to Zuo & Tang various transducer have been tested by a researcher to harvest vibration energy, including piezoelectric materials, linear and rotational electromagnetic motors, electrostatic generators and dielectric generators. Among these type transducer, piezoelectric materials have more potential for large scale vibration energy harvesting compared to the other [2].

Piezoelectric material is a force or stress induced transducer. This will enable them to convert mechanical energy into electrical energy when a pressure or force are exerted on it. Piezoelectric materials also have large energy density and it is suitable for the applications where weight or space is concern. Piezoelectric materials normally generate very high voltage and more suitable for vibration with large force and small deformation [2].

2.2.2 AC to DC Conversion

The electricity generated by the vibration energy harvesting system is usually in AC. In order to convert it to DC, traditional rectifier (Figure 3) consist of four diodes can be used. It is because AC cannot power the electronic devices directly as it need to be converted.

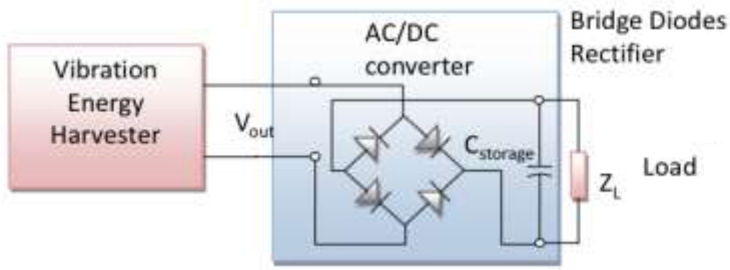


Figure 2.2: Basic circuit AC to DC

2.2.3 Energy Harvesting Concepts

This section shows the energy harvesting concept use by the researcher in harvesting energy from the absorber system and how they maximize the output of the harvested energy. Basically piezoelectric is used in order to harvest the energy. Summary of the information given in Table-2.5. Design energy harvesting concept can be seen from Figure-2.3.

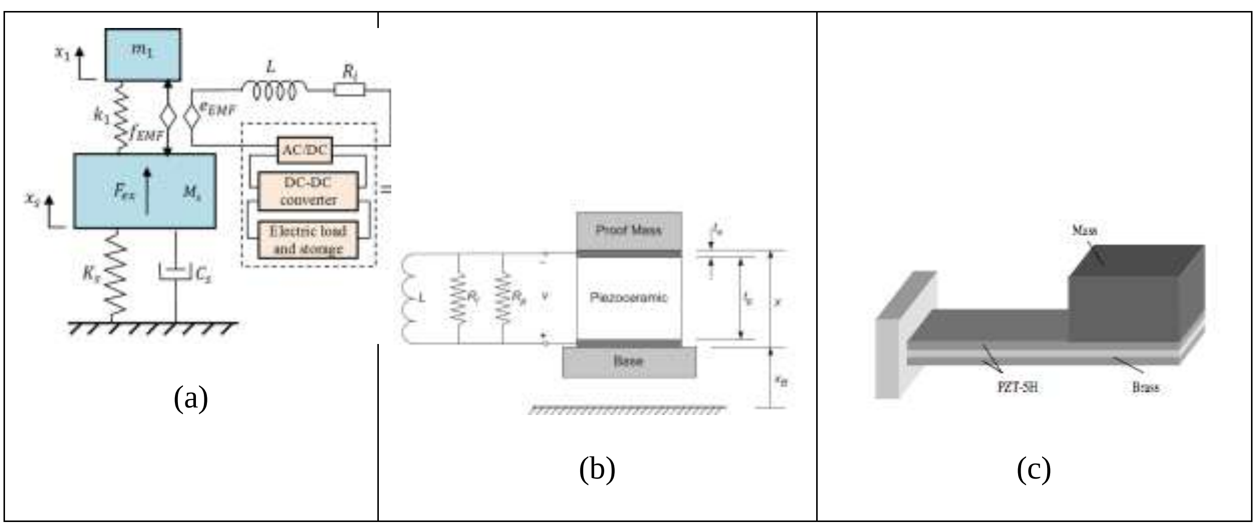


Figure 2.3 Energy harvesting concepts: (a) Zuo and Cui design (b) Renno design (c) Beeby et al.

Table 2.5 Summary of the literatures on harvested energy optimization

Author/Year	Objective of the paper	Parameters studied	Remarks
Zuo and Cui (2013) (a)	- Approach for dual-functional energy-harvesting and robust vibration control by integrating the tuned mass damper (TMD) and electro- magnetic shunted resonant damping.	- Decentralized H_2 control and gradient based methods are used for the optimization of individual parameters - Frequency response - Stroke - Mass - Stiffness - Damping	- Tuning the TMD resonance and circuit resonance close to the primary structure, the electromagnetic resonant-shunt TMD achieves the enhanced effectiveness and robustness of double-mass series TMDs, without suffering from the significantly amplified motion stroke. - The damping is implemented with an electromagnetic transducer shunted with a resistive circuit - The study indicate electromagnetic shunt series TMD provides better vibration control and energy harvesting.

Renno et al. (2009) (b)	- To optimize the power harvested by the absorber system - Utilizes a harvesting circuit employing an inductor and a resistive load - Explore the impact of damping on power optimality - Examined the effect of adding an inductor to the circuit. - In this paper the effect of electromechanical coupling demonstrating that the harvested power decreases beyond an optimal coupling coefficient. So that this result challenges previous literature suggesting that higher coupling coefficients always culminate in more efficient energy harvesters.	- Bifurcation damping ratio - Bifurcation coupling coefficient - Optimal power - Optimal voltage and current - Optimal resistance and inductance	- The addition of the inductor provides substantial improvement to the performance of the energy harvesting device. - It is shown damping ratios that are below a bifurcation damping ratio, the power has two maxima (at the resonance and anti-resonance frequencies) and one minimum. - Beyond the bifurcation damping ratio, the power exhibits only one maximum. - It is found that materials with higher electromechanical coupling coefficients do not necessarily yield higher output power. - Employing an optimal inductor in the circuit, can substantially enhance the harvested power

			- On the other hand, when the damping ratio is higher than the bifurcation damping ratio, the harvested power using an inductor can be much higher than that obtained via a purely resistive circuit.
Beeby et. al (2006) (c)	- Show the method in harvesting energy from vibration.	- Cantilever-based piezoelectric generators	- A cantilever structure with piezoelectric attached to it surface. - The structure is designed to operate in a bending mode thereby straining the piezoelectric films and generating a charge. - When operated in its fundamental bending mode at a frequency of 80.1 Hz, it produced up to 3 μW of power into an optimum resistive load of 333 kOhm.

CHAPTER 3 : METHODOLOGY

3.1 Introduction

This chapter illustrates the overall process of the project until its completion. The research methodologies applied for the study are elaborated, followed by the time allocation for each steps as shown in the Gantt chart. Furthermore, the calculation required in the project are also mentioned in this chapter.

3.1.1 Literature Review

The literature review present basic theory and information regarding the project based on the articles, books and previous research papers. The information gathered will be used as a supporting element in this project.

3.1.2 Modeling and Simulation

Modeling the mechanical absorber is needed before any work can be done. The absorber will be model base on equation of motion of undamped, damped and forced vibration.

3.1.3 Benchmark Data

Benchmark data are taken from previous research paper as a reference and the result will be compared with the output result of work done on this project.

3.1.4 Parametric Study

Investigate the effects of bandwidth, mass, damping, and stiffness towards the absorber system. These parameter will be change accordingly and the result will be recorded and analyze.

3.1.5 Flow Chart and Gantt Chart

A proposed project workflow is as illustrated in Figure below:

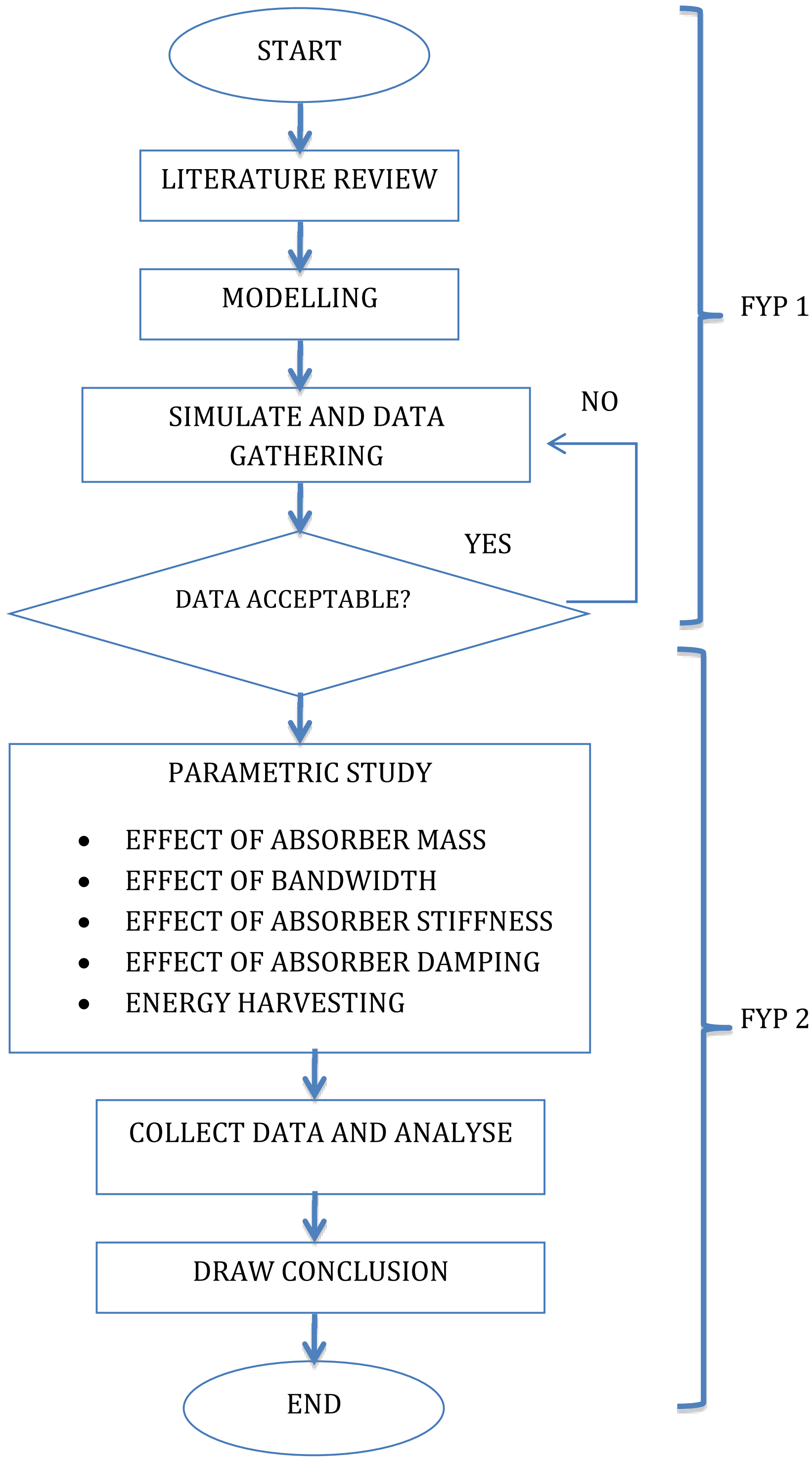


Figure 3.1: Project flow chart

Table 3.1: Gant Chart for FYP 1

No	Week / Activities	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	**Topic selection**														
2	**Literature review study**														
3	**Extended proposal submission**														
4	**Modeling absorber system**														
5	**Proposal defense**														
6	**Testing simulation and analysis**														
7	**Submission interim report**														
8	**Weekly meeting with supervisor**														

Milestone FYP 1	Completion Date
Absorber system modeling	16/4/2015
Simulation and Analysis	20/4/2015

Table 3.2: Gant Chart FYP 2

No	**Week / Activities**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
1	**Testing simulation and analysis**														
2	**Parametric study**														
	- Effect of mass, stiffness, bandwidth and damping														
3	**Energy Harvesting**														
4	**Analysis Data**														
5	**Submission project dissertation**														

Milestone FYP 2	**Completion Date**
Construction of a two degree of freedom absorber system	3/6/2015
Setting up MATLAB to gather experimental data	11/6/2015
Data analysis	24/10/2015

3.2 Modeling of Undamped Vibration Absorber

The theoretical result is calculated based on the mathematical formulation of second degree of freedom system. The calculation are divided into two category which first category the mass of the beam is neglected while mass of the beam is taken into calculation for second category.

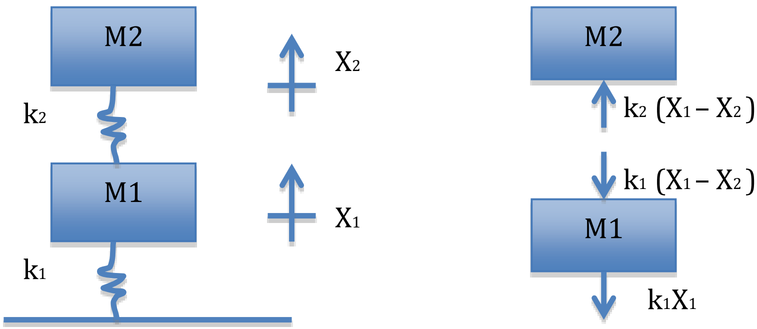


Figure 3.2: Free body diagram

$$X_1 > X_2$$

(1): Assume Vertically Upward Positive

$$\sum Fy = M_1\ddot{X}_1$$

$$M_1\ddot{X}_1 = -M_1X_1 - k_2(X_1 - X_2)$$

(2): Assume Vertically Upward Positive

$$\sum Fy = M_2\ddot{X}_2$$

$$M_2\ddot{X}_2 = k_2(X_1 - X_2)$$

$$0 = M_2\ddot{X}_2 - k_2X_1 + k_2X_2$$

From 1:

$$M_1\ddot{X}_1 + k_1X_1 + k_2X_1 - k_2X_2 = 0$$

$$\begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix}\begin{Bmatrix} \ddot{X}_1 \\ \ddot{X}_2 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = 0$$

Let:

$$X_1 = X_1 \sin(\omega t + \theta)$$

$$X_2 = X_2 \sin(\omega t + \theta)$$

$$\begin{bmatrix} k_1 + k_2 - \omega^2 M_1 & -k_2 \\ -k_2 & k_2 - \omega^2 M_2 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = 0$$

Characteristic Equation

$$\begin{bmatrix} k_1 + k_2 - \omega^2 M_1 & -k_2 \\ -k_2 & k_2 - \omega^2 M_2 \end{bmatrix} = 0$$

$$M_1 M_2 \omega^4 - \omega^2\big(M_1 k_2 + M_2(k_1 + k_2)\big) + k_1 k_2 = 0$$

3.3 Modeling of Damped Vibration Absorber.

$$\begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} \begin{Bmatrix} \ddot{X}_1 \\ \ddot{X}_2 \end{Bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} \begin{Bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = 0$$

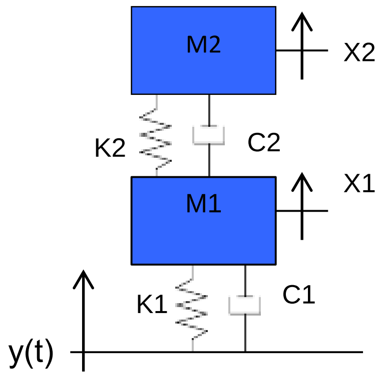
3.4 Solutions of the Equations of Motion

3.4.1 Free Vibration

$$\begin{bmatrix} k_1 + k_2 - \omega^2 M_1 & -k_2 \\ -k_2 & k_2 - \omega^2 M_2 \end{bmatrix} = 0$$

3.4.2 Forced Vibration

Below shows the calculation of the forced vibration. Using base Excitation models the behavior of a vibration isolation system. The base of the spring is given a prescribed motion, causing the mass to vibrate.



$$y(t) = A\sin(\omega\theta)$$

Assumption:

$$y(t) > X_1(t) > X_2(t)$$

(1): Mass 1 assume Vertically Upward Positive

$$\sum Fext = M_1\ddot{X}_1$$

$$M_1\ddot{X}_1 = -k_1X_1 - c_1X_1 - c_1 + k_1y + c_1\dot{y} - k_2(x_1 - x_2) - c_2(\dot{x}_1 - \dot{x}_2)F(t)$$

(2): Mass 2 assume Vertically Upward Positive

$$\sum Fext = M_2\ddot{X}_2$$

$$M_2\ddot{X_2} = k_2(X_1 - X_2) + c_2(\dot{x_1} - \dot{x_2})$$

Matrix form:

$$M_1\ddot{X_1} + k_1X_1 + c_1\dot{x_1} + k_2(X_1 - X_2) + c_2(\dot{x_1} - \dot{x_2}) = F(t)$$
$$M_2\ddot{X_2} - k_2(X_1 - X_2) + c_2(\dot{x_1} - \dot{x_2}) = 0$$

$$\begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix}\begin{Bmatrix} \ddot{X_1} \\ \ddot{X_2} \end{Bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}\begin{Bmatrix} \dot{x_1} \\ \dot{x_2} \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} F(t) \\ 0 \end{Bmatrix}$$

Where: $F(t) = k_2 y(t) + c_1\dot{y}(t)$

Forcing function details:

$$F(t) = R\sin(\propto + \omega t)$$

$$R = \hat{y}_b\sqrt{k_1^2 + (\omega C_1)^2}$$

$$\propto = tan^{-}(\frac{\omega c_1}{k_1})$$

Equation of motion:

$$\begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix}\begin{Bmatrix} \ddot{X_1} \\ \ddot{X_2} \end{Bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}\begin{Bmatrix} \dot{x_1} \\ \dot{x_2} \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} F(t) \\ 0 \end{Bmatrix}$$

$$F(t) = R\sin(\propto + \omega t)$$

$$X_1 = X_1\sin(\omega t + \theta)$$
$$X_2 = X_2\sin(\omega t + \theta)$$

Bypass $F(t) = R\sin(\propto + \omega t)$using assumption of

$$F(t) = \hat{R}e^{i(\alpha + \omega t)}$$

Where Euler's Formula state that:

$$e^{i(\theta)} = cos\theta + sin\theta$$

Solving

$$F(t) = \hat{R}e^{i(\alpha + \omega t)}$$

$$x(t) = \hat{X}e^{i(\alpha + \omega t)}$$

$$\dot{x}(t) = i\omega\hat{X}e^{i(\alpha + \omega t)}$$

$$\ddot{x}(t) = i\omega\hat{X} * i\omega e^{i(\alpha+\omega t)} - \omega^2 i\omega e^{i(\alpha+\omega t)}$$

$$-\omega^2 \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} \hat{X} e^{i(\alpha+\omega t)} + i\omega \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} \hat{X} e^{i(\alpha+\omega t)} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \hat{X} e^{i(\alpha+\omega t)} = \begin{Bmatrix} Re^{i(\alpha+\omega t)} \\ 0 \end{Bmatrix}$$

$$-\omega^2 \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} \hat{X} + i\omega \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} \hat{X} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \hat{X} = R$$

$$\{-\omega^2 \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} + i\omega \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\} \hat{X} = R$$

$$\begin{bmatrix} -\omega^2 m_1 + i\omega(c_1 + c_2) + k_1 + k_2 & -i\omega c_2 - k_2 \\ -i\omega c_2 - k_2 & -\omega^2 m_2 + i\omega c_2 + k_2 \end{bmatrix} \hat{X} = R$$

$$A * \hat{X} = R$$
$$\hat{X} = A^{-1} R$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

To find determinant:

$$A^{-1} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} 1/\det(A)$$

Determinant (A) = ad-bc

$$= [(k_1 + k_2) - \omega^2 m_1 + i\omega(c_1 + c_2)] * [k_2 - \omega^2 m_2 + i\omega c_2] - [-i\omega c_2 - k_2] * [-i\omega c_2 - k_2] = 0$$

$$= m_1 m_2 \omega^4 - [k_2 m_1 + (k_1 + k_2) m_2]\omega^2 + (k_1 + k_2) - i[(-m_1 c_2 - m_2 c_1 + m_2 c_2)\omega^3 + (k_1 c_2 + k_2 c_1)\omega]$$

Determinant (A) = x + iy

Where:

$$x = m_1 m_2 \omega^4 - [k_2 m_1 + (k_1 + k_2) m_2]\omega^2 + (k_1 + k_2)$$
$$y = -[(m_1 c_2 + m_2 c_1 + m_2 c_2)\omega^3 + (k_1 c_2 + k_2 c_1)\omega]$$

Characteristic Equation:

$$\begin{bmatrix} k_1 + k_2 - \omega^2 M_1 & -k_2 \\ -k_2 & k_2 - \omega^2 M_2 \end{bmatrix} = 0$$

$$M_1 M_2 \omega^4 - \omega^2\big(M_1 k_2 + M_2(k_1 + k_2)\big) + k_1 k_2 = 0$$

$$A^{-1} = 1/det \begin{bmatrix} k_2 + \omega m_2 - i\omega c_2 & k_2 + i\omega * c_2 \\ k_2 + i\omega * c_2 & (k_1 + k_2 - \omega^2 M_1 + i\omega(c_1 + c_2) \end{bmatrix} = 0$$

Where F = $\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = [\begin{smallmatrix} R \\ 0 \end{smallmatrix}]$

$$\hat{X} = A^{-1} R$$

$$X_1 = \frac{[k_2 - \omega^2 m_2 + i\omega c_2]R}{x + iy}$$

$$X_2/R = \frac{[k_2 - \omega^2 m_2 + i\omega c_2]}{x + iy}$$

Magnitude and Phase calculation:

$$\hat{X}_1 = \frac{[k_2 - \omega^2 m_2 + i\omega c_2]R}{x + iy} * \frac{x - iy}{x - iy}$$

Where:

$$R = \hat{y}_b \sqrt{k_1^2 + (\omega c_1)^2}$$

$$\hat{X}_1 = \frac{[k_2 - \omega^2 m_2 + i\omega c_2]\hat{y}_b \sqrt{k_1^2 + (\omega c_1)^2}}{x + iy} * \frac{x - iy}{x - iy}$$

$$\hat{X}_1/\hat{y} = \frac{[k_2 - \omega^2 m_2 + i\omega c_2]\sqrt{k_1^2 + (\omega c_1)^2} * (x - iy)}{x^2 - y^2}$$

$$\theta = \sqrt{k_1^2 + (\omega c_1)^2}$$

$$\hat{X}_1/\hat{y} = \frac{(k_2 - \omega^2 m_2)x + i\omega c_2 y + i(\omega c_2 - (k_2 - \omega^2 m_2)y)}{x^2 - y^2}$$

Magnitude Mass 1:$e = \frac{(k_2 - \omega^2 m_2)x + i\omega c_2 y}{x^2 - y^2} * R$

$Phase\ angle\ Mass\ 1: f = \frac{\omega c_2 - (k_2 - \omega^2 m_2)y}{x^2 - y^2} * R$

$$\hat{X}_2 = \frac{[k_2 + i\omega c_2]}{x + iy}$$

$$\hat{X}_2/R = \frac{[k_2 + i\omega c_2]}{x + iy} * \frac{x - iy}{x - iy}$$

$$\hat{X}_2/R = \frac{[k_2 + i\omega c_2] * x + iy}{x^2 - y^2}$$

$$\hat{X}_2/R = \frac{[k_2 x + \omega c_2 y] * i(\omega c_2 x - k_2 y)}{x^2 - y^2}$$

Magnitude Mass 2: $w = \frac{[k_2 x + \omega c_2 y]}{x^2 - y^2} * R$

Phase angle Mass 2: $u = \frac{[\omega c_2 x - k_2 y]}{x^2 - y^2} * R$

Where R = $\sqrt{k_1^2 + (\omega c_1)^2}$

3.5 Design of a Damped Dual Mass Vibration Absorber Using Bandwidth

Under this section, shows the design calculation of the absorber based on bandwidth and desired frequency. The system is assumed damped.

$$\begin{bmatrix} k_1 + k_2 - \omega^2 M_1 & -k_2 \\ -k_2 & k_2 - \omega^2 M_2 \end{bmatrix} = \begin{bmatrix} f_0 sin\,(\omega t) \\ 0 \end{bmatrix}$$

The first process in designing the absorber based on bandwidth is to identify the location where the absorber will be installed. By knowing the natural frequency of the system where the absorber will be installed, the main mass parameter can be calculated. For example cases which study carried on the frequency of the human motion. The human motion frequency is from 1 to 10Hz [17]. By tuning the absorber frequency equal to the frequency of the human motion, certain amount of energy can be harvested from the absorber. By calculating the amplitude of X1 and X2 at the particular operating frequency, the output power can be calculated.

$$X_1 = \frac{(k_2 - m_2\omega^2)f_0}{(k_1 + k_2 - m_1\omega^2)(k_2 - m_2\omega^2) - k_2^2}$$

$$X_2 = \frac{k_2 f_0}{(k_1 + k_2 - m_1\omega^2)(k_2 - m_2\omega^2) - k_2^2}$$

The output power (P) is depends on the difference between the amplitudes X1 and X2 of the two masses and the damping value(C_2) by using following formula:

$$P = c_2\omega^2(X_2 - X_1)^2$$

The parameter value of mass 1, mass 2 , stiffness 1 and stiffness 2 can be calculated from the following equation as the value of the two resonance frequency are already known :

$$\omega_1^2 \times \omega_2^2 = \frac{k_1}{m_1} \times \frac{k_2}{m_2}$$

$$\omega_1^2 + \omega_2^2 = \frac{k_1}{m_1} + \frac{k_2}{m_2} + \frac{k_2}{m_1}$$

CHAPTER 4 : RESULT AND DISCUSSION

4.1 Introduction

This section will present the result collected from the simulation and also the sample calculation work.

4.2 System Characteristics

Table 4.1 shows the material and dimension used in the simulation.

Table 4.1: Material dimension

Type	Weight (Kg)	Length (m)	Width (m)	Thickness (m)	Area (m^2)
Beam Primary	0.454	0.5	0.126	0.003	63
Mass 1 x 4 pieces	0.042	0.060	0.126	0.002	7.56
Mass 2 x 4 Pieces	0.062	0.060	0.126	0.003	7.56
Washer	0.002	/	/	/	/
Screw 1 x 2	0.007	/	/	/	/
Screw 2 x 2	0.010	/	/	/	/
Nut x 4	0.004	/	/	/	/
Beam Secondary	0.044	0.25	0.126	0.001	31.5
Whole structure (Mass 1& 2 +Screw 4 + Beam secondary +Beam 3+ Washer 5)	1.074	/	/	/	/

Material Properties:

Material: Aluminum

Aluminum density: 2712 Kg/m^3

Young modulus: E = 69 x 10^9 Gpa / N/m^2 = 6900 N/mm^2

Stiffness for beam: k = $3EI/L^3$

Moment of inertia of beam: I = $bh^3/12$

Dimension of full setup

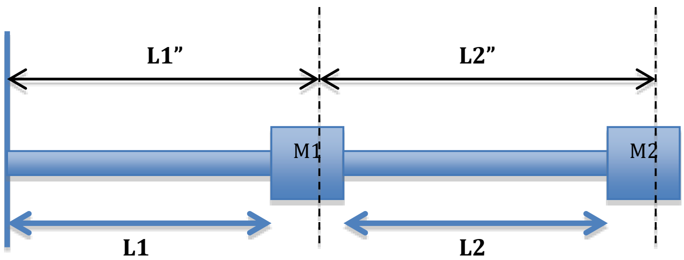


Figure 4.1 : Free body diagram of full setup

Mass at Tip 1: 0.368 Kg

Mass at Tip 2: 0.212 Kg

Length 1: 0.357 m

Length 2: 0.130 m

Length 1”: 0.387 m

Length 2”: 0.190 m

4.3 Free Vibration Analysis for Undamped System sample calculation

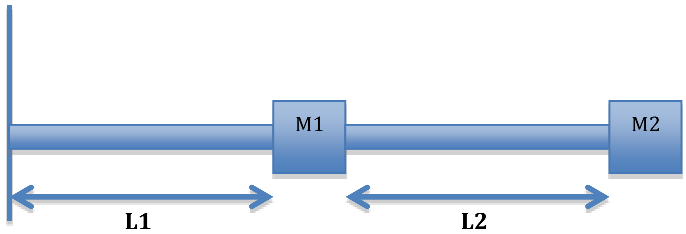


Figure 4.2: Free body diagram 1

***neglect mass of the beam.**

Mass 1 = 0.368 Kg ; Length 1: 0.357 m ; E: 69 x 10^9 N/m^2

Mass 2 = 0.212 Kg ; Length 2: 0.130 m ;

Moment of inertia;

I = BH^3/12

I 1 = (0.126) (0.003) ^3/12 ; I2 = (0.126) (0.001) ^3/12

= 2.835 x 10^-10 m^4 ; = 1.5 x 10^-11 m^4

Stiffness;

k = 3EI/L^3

k1 = 3(69 x 10^9) (2.835 x 10^-10)/ (0.357) ^3

= 1289.8 N/m

k2 = 3(69 x 10^9) (1.5 x 10^-11)/ (0.130) ^3

= 989 N/m.

Characteristic equation

$$\begin{bmatrix} k_1 + k_2 - \omega^2 M_1 & -k_2 \\ -k_2 & k_2 - \omega^2 M_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1289.8 + 989 - \omega^2(0.368) & -989 \\ -989 & 989 - \omega^2(0.212) \end{bmatrix} = 0$$

$$\begin{bmatrix} 2278.8 - \omega^2(0.368) & -989 \\ -989 & 989 - \omega^2(0.212) \end{bmatrix} = 0$$

$$(2278.8 - 0.368\omega^2)(989 - 0.212\omega^2) - (-989)^2 = 0$$

$$2.25 \times 10^6 - 483.1\omega^2 - 363.9\omega^2 + 0.07802\omega^4 - 978121 = 0$$

$$0.07802\omega^4 - 847\omega^2 + 1.27 \times 10^6 = 0$$

Let a = ω^2

$$a_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a_{1,2} = \frac{-(-847) \pm \sqrt{(-847)^2 - 4(0.07802)(1.27 \times 10^6)}}{2(0.07802)}$$

$$a_{1,2} = \frac{847 \pm 567}{0.15604}$$

$$a_1 = \omega_1^2 = 9062 \quad ; \quad a_2 = \omega_2^2 = 1794$$

$$\omega_{1max} = 95.2 \ rad/sec \quad ; \quad \omega_{2max} = 42.36 \, rad/sec$$

2)

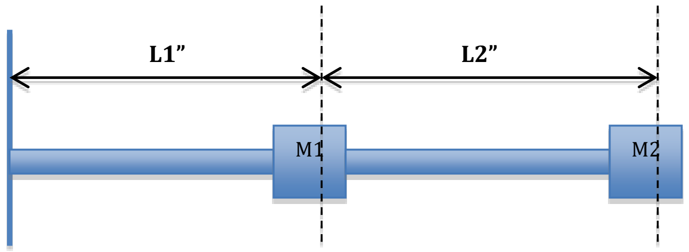


Figure 4.3: Free body diagram 2

***neglect mass of the beam.**

Mass 1 = 0.368 Kg ; Length 1": 0.387 m ; E: 69 x 10^9 N/m^2

Mass 2 = 0.212 Kg ; Length 2": 0.190 m ;

I 1 = 2.835 x 10^-10 m^4

I 2 = 1.5 x 10^-11 m^4

Stiffness;

k = 3EI/L^3

k1 = 3(69 x 10^9) (2.835 x 10^-10)/ (0.387) ^3

= 1012.5 N/m

k2 = 3(69 x 10^9) (1.5 x 10^-11)/ (0.190) ^3

= 316 N/m.

Characteristic equation

$$\begin{bmatrix} k_1 + k_2 - \omega^2 M_1 & -k_2 \\ -k_2 & k_2 - \omega^2 M_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1012.5 + 316 - \omega^2(0.368) & -316 \\ -316 & 316 - \omega^2(0.212) \end{bmatrix} = 0$$

$$\begin{bmatrix} 1328.5 - \omega^2(0.368) & -316 \\ -316 & 316 - \omega^2(0.212) \end{bmatrix} = 0$$

$$(1328.5 - 0.368\omega^2)(316 - 0.212\omega^2) - (-316)^2 = 0$$

$$419.8 \times 10^3 - 281.6\omega^2 - 116.3\omega^2 + 0.07802\omega^4 - 99856 = 0$$

$$0.07802\omega^4 - 398\omega^2 + 320 \times 10^3 = 0$$

Let a = ω^2

$$a_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a_{1,2} = \frac{-(-398) \pm \sqrt{(-398)^2 - 4(0.07802)(320 \times 10^3)}}{2(0.07802)}$$

$$a_{1,2} = \frac{398 \pm 242}{0.15604}$$

$$a_1 = \omega_1^2 = 4102 \quad ; \quad a_2 = \omega_2^2 = 1000$$

$$\omega_{1max} = 64 \ rad/sec \quad ; \quad \omega_{2max} = 31.6 \, rad/sec$$

3)

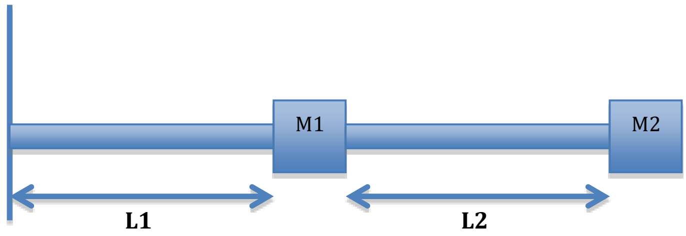


Figure 4.4: Free body diagram 3

***mass of the beam is taken into the calculation**

$$M_1 = M_1 + \frac{M_{b1}}{3}$$

$$M_1 = 0.368 + \frac{0.454}{3}$$

$$M_1 = 0.519\ Kg$$

$$M_2 = M_2 + \frac{M_{b2}}{3}$$

$$M_2 = 0.212 + \frac{0.044}{3}$$

$$M_2 = 0.227\ Kg$$

Length 1 = 0.357 m	;	I 1: 2.835x10^-10 m^4	;	k1 = 1289.9 N/m
Length 2 = 0.130 m	;	I 2: 1.5 x 10^-11 m^4	;	k2 = 989 N/m.

Characteristic equation

$$\begin{bmatrix} k_1 + k_2 - \omega^2 M_1 & -k_2 \\ -k_2 & k_2 - \omega^2 M_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1289.8 + 989 - \omega^2(0.519) & -989 \\ -989 & 989 - \omega^2(0.227) \end{bmatrix} = 0$$

$$\begin{bmatrix} 2278.8 - \omega^2(0.519) & -989 \\ -989 & 989 - \omega^2(0.227) \end{bmatrix} = 0$$

$$(2278.8 - 0.519\omega^2)(989 - 0.227\omega^2) - (-989)^2 = 0$$

$$2.25 \times 10^6 - 517.3\omega^2 - 513.3\omega^2 + 0.118\omega^4 - 978121 = 0$$

$$0.118\omega^4 - 1030.6\omega^2 + 1.27 \times 10^6 = 0$$

Let a = ω^2

$$a_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a_{1,2} = \frac{-(-1030.6) \pm \sqrt{(-1030.6)^2 - 4(0.118)(1.27 \times 10^6)}}{2(0.118)}$$

$$a_{1,2} = \frac{1030.6 \pm 680}{0.2356}$$

$$a_1 = \omega_1^2 = 7261 \quad ; \quad a_2 = \omega_2^2 = 1488$$

$$\omega_{1max} = 85.2 \ \frac{rad}{sec} \quad ; \quad \omega_{2max} = 38.6 \ \frac{rad}{sec}$$

4)

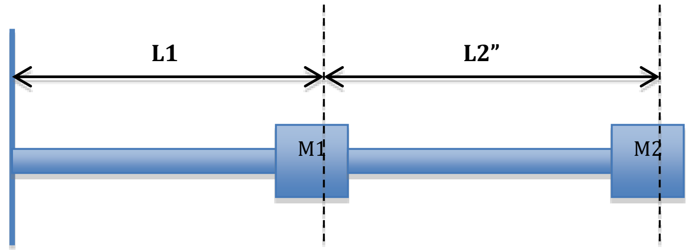


Figure 4.5: Free body diagram 4

***mass of the beam is taken into the calculation**

$M_1 = 0.519\ Kg$; $M_2 = 0.227\ Kg$

Length 1 = 0.387 m ; I 1: 2.835x10^-10 m^4 ; k1 = 1012.5 N/m

Length 2 = 0.190 m ; I 2: 1.5x10^-11 m^4 ; k2 = 316 N/m.

Characteristic equation

$$\begin{bmatrix} k_1 + k_2 - \omega^2 M_1 & -k_2 \\ -k_2 & k_2 - \omega^2 M_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1328.5 - \omega^2(0.519) & -316 \\ -316 & 316 - \omega^2(0.227) \end{bmatrix} = 0$$

$$(1328.5 - 0.519\omega^2)(316 - 0.227\omega^2) - (-316)^2 = 0$$

$$419806 - 310.6\omega^2 - 164\omega^2 + 0.1178\omega^4 - 9796 = 0$$

.

$$0.1178\omega^4 - 447.6\omega^2 + 410010 = 0$$

Let a = ω^2

$$a_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a_{1,2} = \frac{-(-447.6) \pm \sqrt{(-447.6)^2 - 4(0.1178)(410010)}}{2(0.1178)}$$

$$a_{1,2} = \frac{447.6 \pm 84.55}{0.2356}$$

$$a_1 = \omega_1^2 = 2259 \quad ; \quad a_2 = \omega_2^2 = 1540$$

$$\omega_{1max} = 47.5\ \frac{rad}{sec} \quad ; \quad \omega_{2max} = 39\,\frac{rad}{sec}$$

4.4 Simulation Result

4.4.1 Case 1 : Effect of damping (Benchmark Data Based on Zuo (2011)

The graph in figure 4.6 shows the effect of damping for the system with absorber and without absorber attached to it. There are three difference damping used in order to observed how its affect the system performance. The damping used is 0.1, 0.01 and 0.001. At the operating frequency which is 8 Hz, it can be seen that high vibration amplitude occur at the main system when no absorber attached to it. When the absorber are attached to the system, the vibration amplitude of the main system is eliminated. In order to successfully eliminate the vibration of the main system, the natural frequency of the absorber need to be tuned equal to the natural frequency of the main system. The larger difference between the amplitude at particular operating frequency will result in higher energy harvested from the system. As for the damping effect, it can be seen that at damping 0.001 the amplitude is higher and narrower operating region. While higher damping which is 0.1 will lead to lower peak and wider operating region.

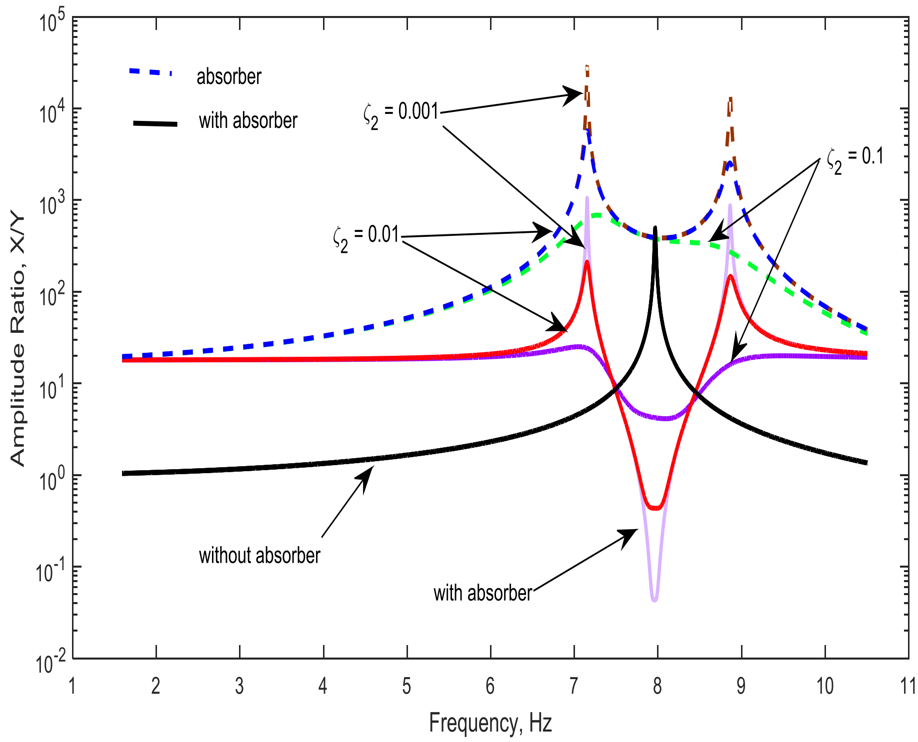


Figure 4.6: Effect of damping 0.1, 0.01 and 0.001

4.4.2 Case 2 : Effect of absorber mass ratio (Benchmark Data Based on Zuo (2011))

The figure in 4.7 shows the effect of difference absorber mass ratio towards the system. Higher absorber mass ratio will eliminate the main system vibration at different frequency as it shifted to the lower frequency region and only allow small portion of energy to be harvested from the desired operating frequency. At lower mass ratio which is 0.5, lesser energy can be harvested compare to higher absorber mass ratio. Even at certain frequency region the vibration of the main mass is amplify so this is not good for the system.

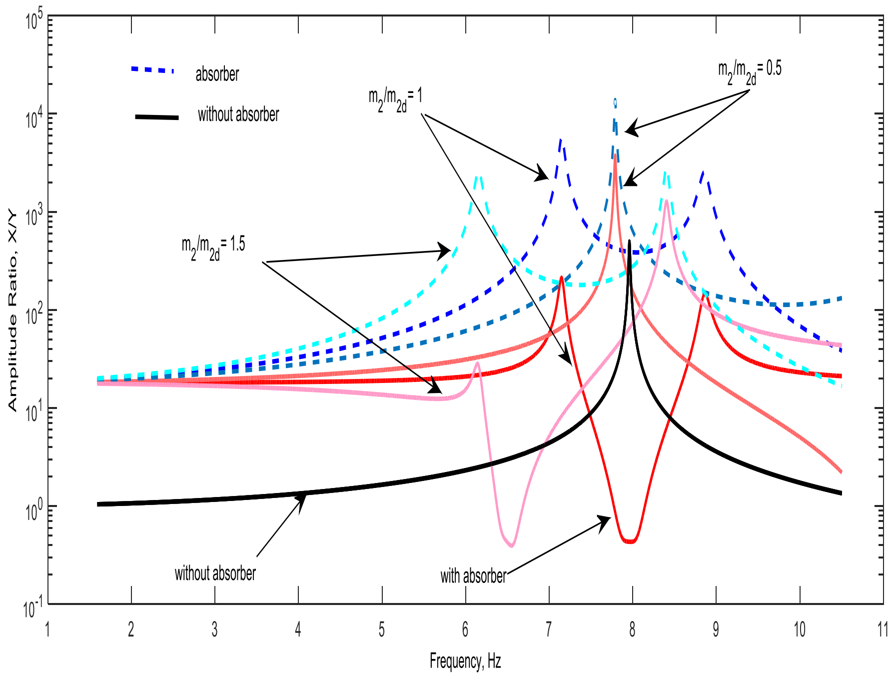


Figure 4.7: Effect of absorber mass ratio 1, 0.5 and 1.5

4.4.3 Case 3 : Effect of mass ratio (M2/M1) (Benchmark Data Based on Zuo (2011) and Hartog (1985))

Figure 4.8 shows the effect of 5% and 10% mass ratio (M2/M1). The brown and purple line show the value of 10% mass ratio while blue and red is using 5% mass ratio. Hartog stated that the 10% is the optimum mass ratio in order to eliminate the main system vibration. Zuo in his article also try the same method but with lower mass ratio also able to eliminate the main system vibration. Higher mass ratio will lead to amplifying the vibration on the main system or the vibration is eliminated at different frequency.

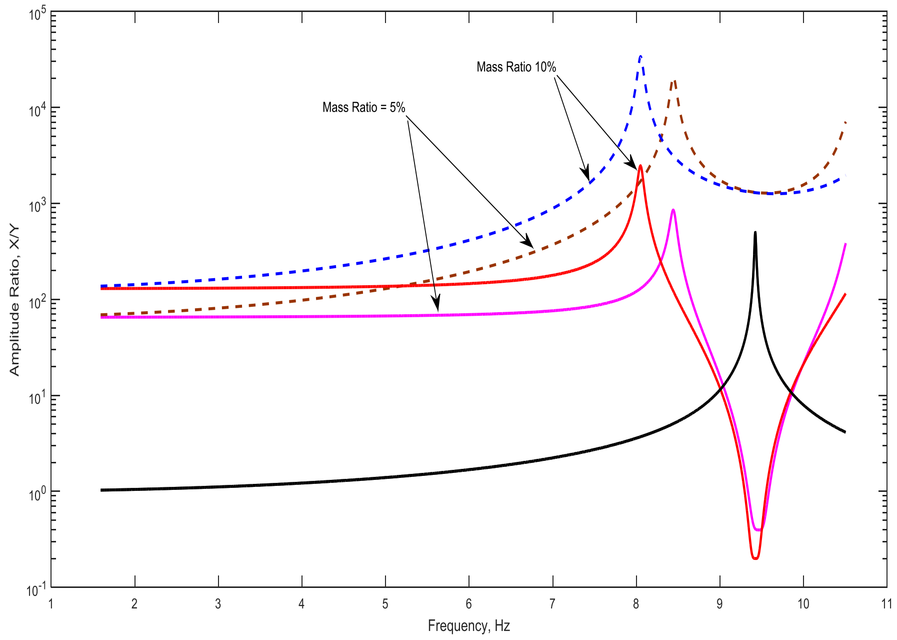


Figure 4.8: Mass Ratio 5% and 10%

4.4.4 Case 4 : Effect of different bandwidth ratio

Below shows the behavior of the system when different percentage of the bandwidth is used. As stated earlier, the higher difference between two amplitudes (X2-X1) the higher the generated power. In figure 4.9 5% of bandwidth is used. It can be seen that the bandwidth is narrower which produced very large difference between two amplitude is. This mean more energy can be harvested from the system, but the drawbacks is the energy only can be harvested at limited frequency region. In figure 4.10 10% of bandwidth is used. The bandwidth gap become slightly wider which is energy can be harvested at wider operating frequency but the energy can be harvested slightly lower. In figure 4.11 20% bandwidth is used. The bandwidth become wider as the percentage increases. The higher percentage of the bandwidth, the lower the difference between the two amplitudes (X2-X1) thus lower power will be generated. In other words, there is tradeoff between the generated power and the bandwidth regarding the amplitude difference.

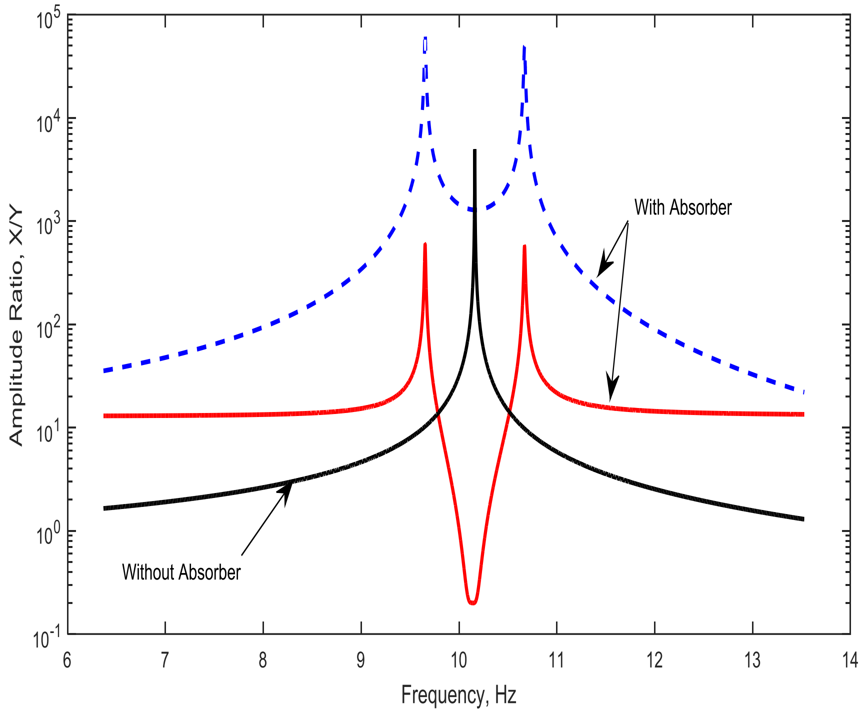


Figure 4.9: 5% Bandwidth

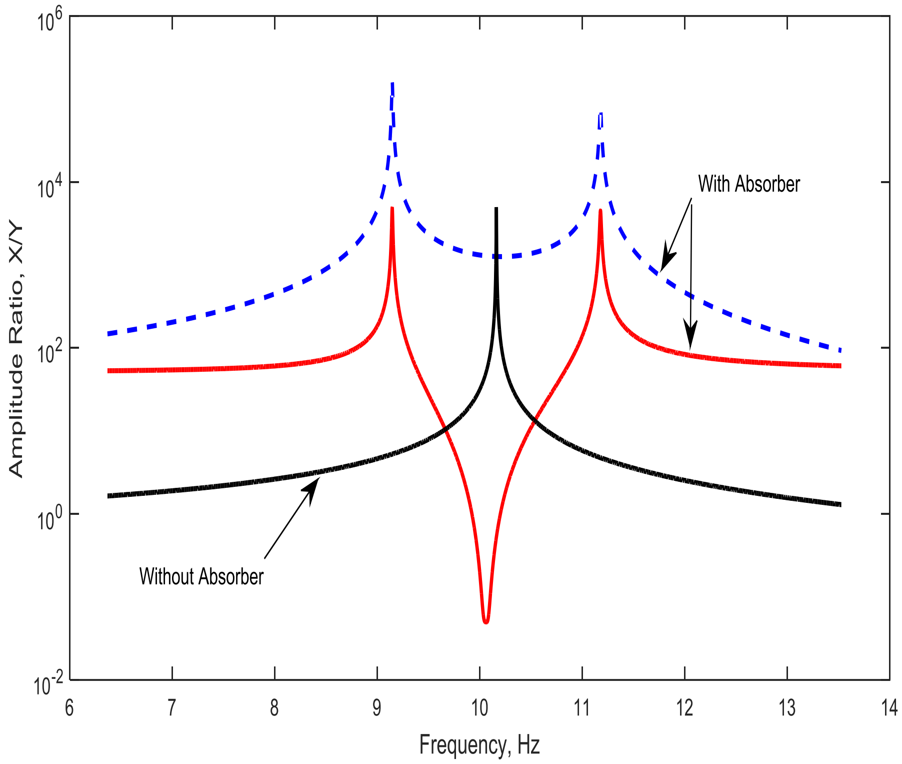


Figure 4.10: 10% Bandwidth

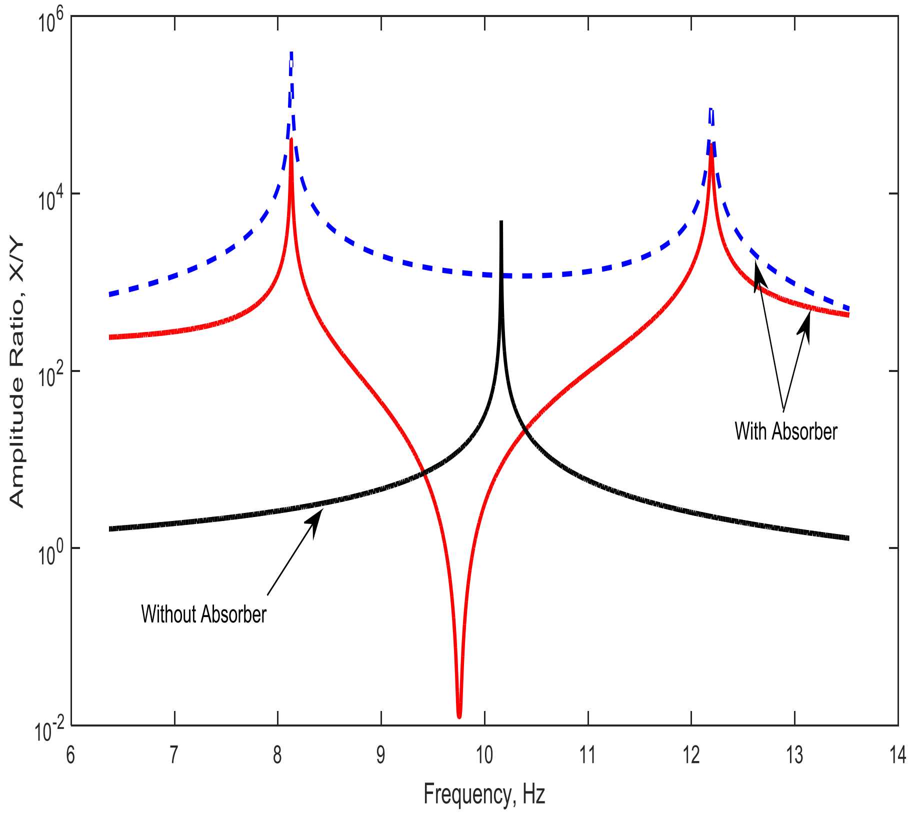


Figure 4.11: 20% Bandwidth

4.4.5 Power Output.

Figure 4.12 shows the power output generated from the previously chosen bandwidth. It is showing the comparison of power output from 5% to 50% bandwidth. At 5% bandwidth, energy can be harvested at limited operating frequency region which approximately at 10 Hz. As the bandwidth is increased to 10% and 20%. The power output slightly started to decreases but it gives wider range to harvest energy at different frequency region. When the bandwidth is increases to 40% and 50% value of power output can be harvested become lesser.

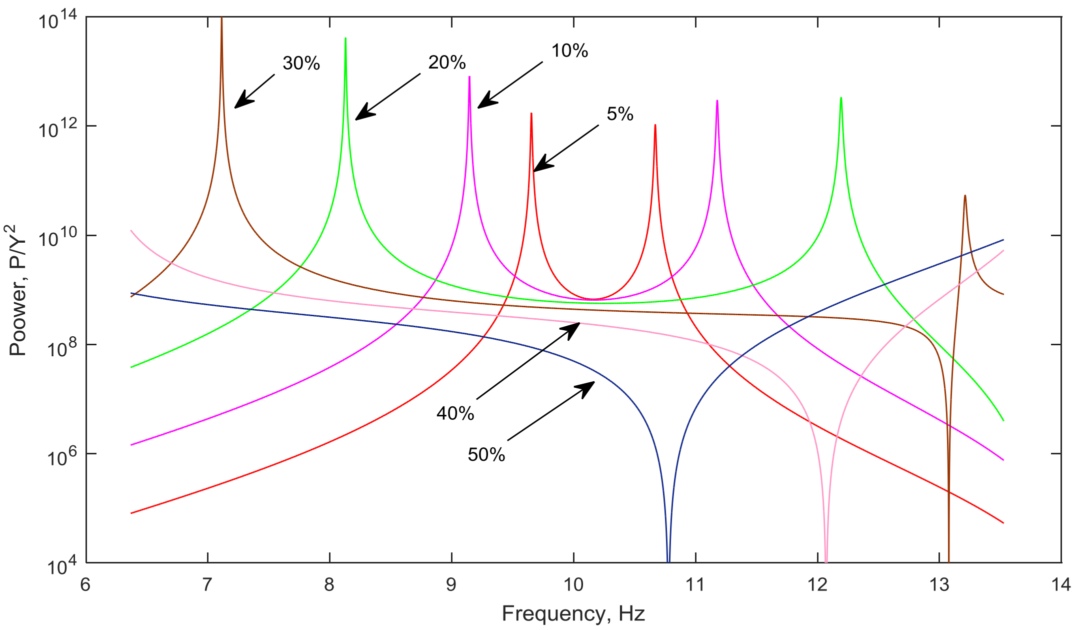


Figure 4.12: Power Output

CHAPTER 5 : CONCLUSION AND RECOMMENDATION

5.1 Conclusion

The thesis proposed the idea of dynamic vibration absorber as an energy harvesting system. The purpose of the absorber is to reduce the vibration of the primary structure while harvest energy from the vibration of an attached absorber. The study on effect of damping, mass, stiffness and bandwidth are carried out by using the numerical method. The numerical method is done by deriving the equation of motion of damped, undamped and forced vibration base on second degree of freedom vibration system. As this completed the first objective of the project which develop the mathematical model for absorber and main mass system. For the design recommendation of the system, it can be concluded that:

- Larger difference between two amplitude (X2 – X1), will give higher power output.
- Higher percentage of bandwidth, the difference between the two amplitude (X2 – X1) will be lower, so less power output will be harvested from the system.
- 5% to 30% ratio is acceptable range to increase the bandwidth, as higher than 30% will lead to decreasing the power output from the system.
- In addition, higher damping causing the amplitude of the system decrease. While lower damping will increase the amplitude.
- For the optimum mass ratio M2/M1 the best range should be between 5% - 10% of mass ratio.

In the future work, experimental work can be done to compare the result and validate the analysis.

5.2 Recommendation

In order to validate the numerical analysis, it is better to compare the data gathered with the experimental work. In this section, how the experimental work can be done will be explain. The setup consists of a cantilever beam with a tip mass attached at the end of the beam. The cantilever beam replicate the concept of conventional energy harvester which consist of mass and spring attach to the source of vibration base. The beam will be mounted on the vibration excitation device which is vibration shaker. Accelerometer attached along the beam are used to capture data with the help of the graphical programming software Lab view. The data from the experimental work will be collect and compare with the theoretical data.

5.2.1 Design Single Degree of Freedom Structure

The metal plate mounted on the vibration shaker. Single beam represent the single degree of freedom. Figure shows the experiment design using 3D modeling software.

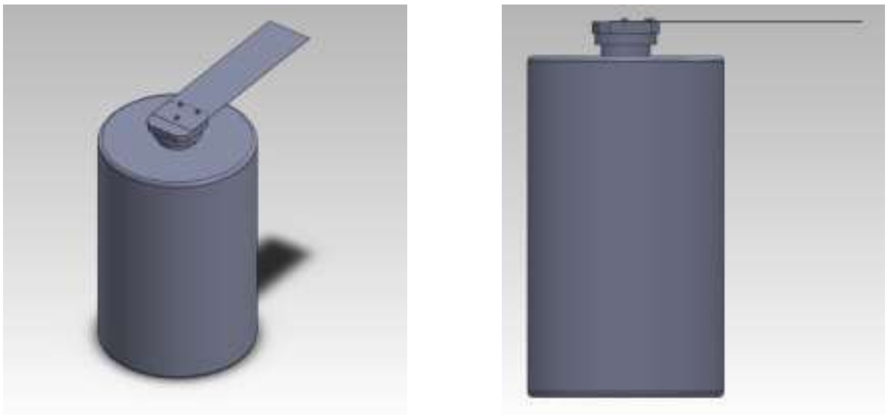


Figure 5.1: Single degree of freedom view

5.2.2 Two Degree of Freedom Structure

Figure shows another metal plate are mounted on the tip of the first plate. Two beam stacked together represent second degree of freedom vibration. At the end of the second plate another small plate are attached act as a mass to the system.

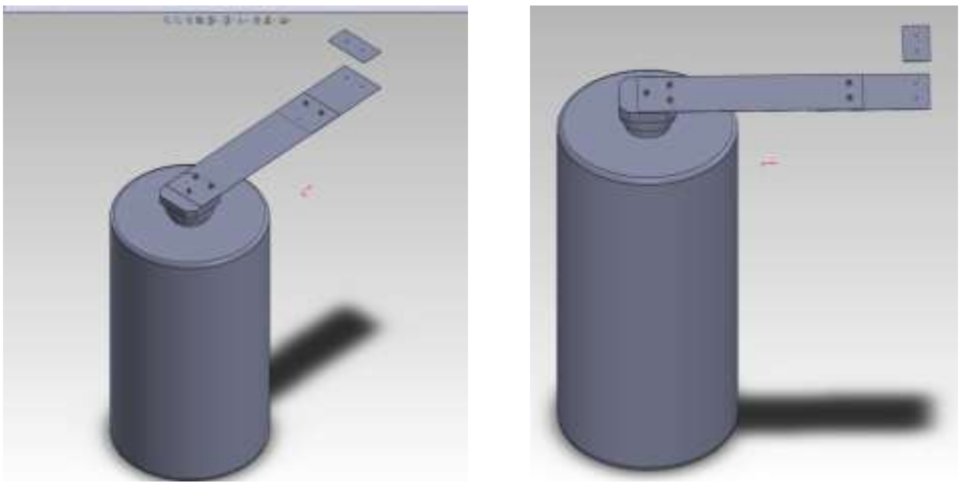


Figure 5.2: Second degree of freedom structure view

5.2.3 Actual Setup

Figure below shows the actual setup of the absorber system when mounted on the shaker.

(a) **(b)**

Figure 5.3: Single Degree of freedom actual setup

5.2.4 Experimental procedure

1 – The plate is mounted on the vibration shaker.

2 – Accelerometer are attached to the vibration shaker, tip of the primary beam and tip of the secondary beam to measure the input acceleration, amplitude and time.

3 – The vibration shaker excitation are set in sweep sine mode in the frequency range 3 – 25 Hz.

4 – The frequency response from the base acceleration to the tip acceleration of the primary beam will be gather and compare with the benchmark data.

5.2.5 Tools and Equipment

Below shows the equipment used in conducting the experiment.

1. IMV Shaker

Provide source of vibration to the mounted plate.

Figure 5.4: Vibration shaker

2. Accelerometer

An instrument for measuring motion and vibration that involved in the machine, building, or structure. This device will be attached to the constructed plate in order to measure its acceleration.

Figure 5.5: Accelerometer (a device to measure motion and vibration)

3. Lab view software

Lab view software is used to extract data from the accelerometer and write it in the digital format such as excel or word to make it easy for analysis purpose. From figure 5.6, DAQ Assistant function to set the sampling frequency and sample taken from the accelerometer that is connected to the computer. Data icon show the acceleration graph gathered from the DAQ Assistant. Write to measurement Box function to write the data in digital format as set by the user. Spectral measurement box is to convert the data from the DAQ Assistant to spectral measurement data or fast Fourier transform and the graph will be display in waveform graph box.

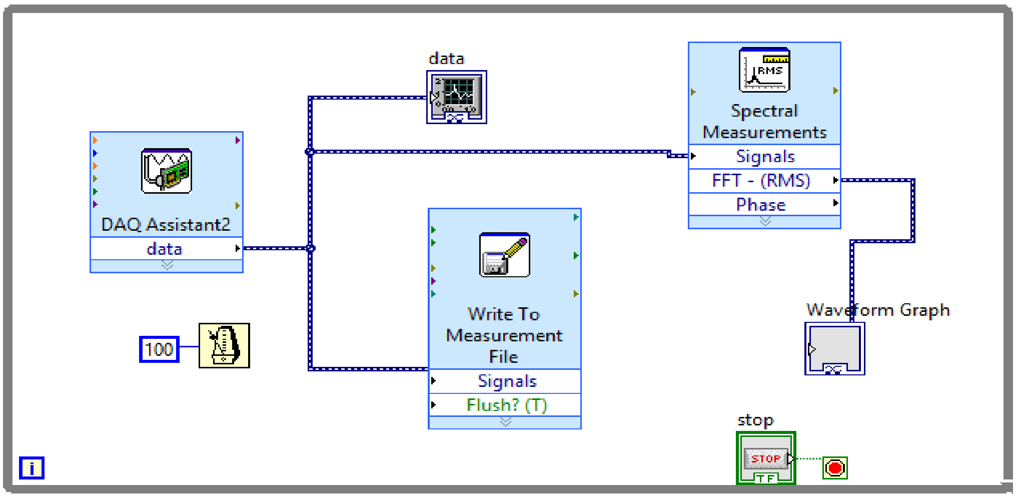


Figure 5.1: Lab view graphical programming

REFERENCES

[1] Rao, S.S, (2004). *Mechanical Vibrations*. 5th ed. : Pearson Education.

[2] Zuo, L & Tang, X, (2013). Large-scale vibration energy harvesting. *Journal of Intelligent Material Systems and Structures*, pp.1

[3] Adhikari,S & Ali, S.F, (2013). Energy Harvesting Dynamic Vibration Absorbers. *Journal of Applied Mechanics*. 80 (1), pp.1

[4] Hassaan, G.A, (2014). Optimal Design Of a Vibration Absorber-Harvester Dynamic System. *International Journal of Research in Engineering and Technology*. 3 (6), pp.325

[5] Hartog, J.P.D, (1985). *Mechanical Vibrations*. New York: Dover.

[6] Agarwal, Anant. Foundations of Analog and Digital Electronic Circuits.Department of Electrical Engineering and Computer Science, Massachusetts Institute of Technology, 2005, p. 43

[7] Triplett, A.L, (2011). Vibration-Based Energy Harvestung With Essential Non-Linearities. 1st ed. e.g. England: ProQuest.

[8] Beeby, S.P et. al, (2006). Energy harvesting vibration sources for microsystem applications. 17 (1), pp1

[9] Zhou, W, Reddy, G & Zuo, L. (2011). An efficient vibration energy harvester with a multi-mode dynamic magnifier. *Smart Materials and Structures*, 21(10).

[10] Asami, T, Nishihara, O & M baz, A. (2002). Analytical Solutions to H and H2 Optimization of Dynamic Vibration Absorbers Attached to Damped Linear Systems. *Journal of Vibration and Acoustic*, 124.

[11] Beeby, S.P et. al, (2007). A micro electromagnetic generator for vibration energy harvesting. *Journal Of Micromechanics and Microengineering*, 17.

[12] B. Brown, & T. Singh, Minimax design of vibration absorbers for linear damped systems, Journal of Sound and Vibration (2010), doi:10.1016/j.jsv.2010.12.002.

[13] Liu, K & Coppola, G. (2010). Optimal Design of Damped Dynamic Vibration Absorber for Damped Primary system. *Journal of sound and vibration*, 34(1).

[14] Renno, J, Daqaq, M & Inman, D. (2008). On the optimal energy harvesting from a vibration source. *Journal of sound and vibration*, 320.

[15] Wong, W & Cheung, Y. (2007). Optimal design of a damped dynamic vibration absorber for vibration control of structure excited by ground motion. *Engineering structure*, 30.

[16] Zuo, L & Xiudong, T. (2013). Large-scale vibration energy harvesting. *Journal*

***of Intelligent Material Systems and Structures*, 24(11).**

[17] Magdy, M.M., et al. (2014) Design Methodology of a Micro-Scale 2-DPF Energy Harvesting Device for Low Frequency and Wide Bandwidth. Journal of Sensor Technology,4 37-47.

APPENDICES

A. Benchmark Data

The data shown in this section is the benchmark data. The experiment will be conducted and the data gather will be compare with this data.

Dimension of the double beam structure for the finite element analysis.

Table 1A : Dimension used for double structure . Zhou et. al (2011)

Beam	Width b (mm)	Thickness t (mm)	Length L (mm)	Tip mass M1 and M2 (g)
Dynamic magnifier	45.47	3.18	401.3	165
Energy harvester	25.40	0.64	185.4	11.15

B. MATLAB Program

Bandwidth Program

```
clear all; clc
format short
%% Design of the absorber for a given frequency
% This can be done assuming undamped system.
% Example cases:
%  (1) Human motion: 1 to 10 Hz
%  (2) Human body movements: 0.5 to 4 Hz (source: Yun et al.(2014))
%  (3) Approximate Electrical Motor Speed (RPM)
%      ------------------------------------------------------------
-------
%      No. Poles    Speed with Rated Load   Synchronous Speed (no
Load)
%      ------------------------------------------------------------
-------
%                   60 hz       50 hz              60 hz      50 hz
%                   ------------------------------------------------
-------
%       2 Pole      3450        2850               3600       3000
%       4 Pole      1725        1425               1800       1500
%       6 Pole      1140        950                1200       1000
%       8 Pole      850         700                900        750
%      ------------------------------------------------------------
-------
%  (4) Arbitrary resonance frequency

%% Calculation for case(1)
% Ns = 5; %
% Omega = 2*pi*Ns; % in rad/s
```

```
% wn = Omega;
% wn1 = 0.8*Omega;
% wn2 = 1.2*Omega;

%% Calculation for case(2)
% Ns = 2; %
% Omega = 2*pi*Ns; % in rad/s
% wn = Omega;
% wn1 = 0.8*Omega;
% wn2 = 1.2*Omega;

%% Calculation for case (3)
% Ns = 1200; % 6 pole & 60 Hz
% Omega = 60/(2*pi)*Ns; % in rad/s
% wn = Omega;
% wn1 = 0.8*Omega;
% wn2 = 1.2*Omega;

%% Calculation for case(4)
% Main mass properties
rho_Al = 2402.1; % density of Aluminium in kg/ m^3
E_Al = 69e+9; % Young's modulus for Aluminium
L1 = 0.357; % in m
b1 = 0.126; % in m
h1 = 0.003; % in m
mL1 = 2402.1*0.5*0.126*0.003; % in kg
m1tip = 0.212; % in kg
m1 = m1tip + 0.23*mL1; % in rad/s (source: S.Rao Vibration)
I1 = b1*h1^3/12; % based on 3 mm thick plate
k1 = 3*E_Al*I1/L1.^3;
wn_sm = sqrt(k1/m1); % natural frequency of single mass
wn_Hz = wn_sm/(2*pi);

bw = 0.50; % desired band width
wn1 = (1-bw)*wn_sm;
wn2 = (1+bw)*wn_sm;

% Absorber mass properties
k2 = m1*(wn1^2+wn2^2 - k1/m1-(m1/k1)*wn1^2*wn2^2);
m2 = k2/(m1/k1*wn1^2*wn2^2);
b2 = 0.126; % in m
h2 = 0.001; % in m
I2 = b2*h2^3/12; % based on 1 mm thick plate
L2 = power(3*E_Al*I2/k2,1/3); % in m
mL2 = 2673.4*L2*0.126*0.001; % in kg
m1tip = m2 - 0.23*mL2; % in kg


%% Calculation of damping
zeta1 = 0.0001; zeta2 = 0.001;
c1 = 2*zeta1*sqrt(m1*k1);
c2 = 2*zeta2*sqrt(m2*k2);
mu = m2/m1 % mass ratio
beta = wn2/wn1;

%% Frequency vector
w = linspace(40,85,20000); % frequency

%% Response of the main mass alone
```

```
Xhat_Yhat = power(k1^2+(c1.*w).^2,1/2)./power((k1-
m1.*w.^2).^2+(c1.*w).^2,1/2);
alpha = 180/pi.*atan(c1/k1.*w);
phi = 180/pi.*atan(-(c1.*w)./(k1-m1.*w.^2));
Xhat_phase = alpha + phi;

% figure (1)
% semilogy(w,Xhat_Yhat,'r','LineWidth',2); grid on

% figure (2)
% plot(w,Xhat_phase,'r','LineWidth',2); grid on; hold on
% plot(w,alpha,'r','LineWidth',2);
% plot(w,phi,'b','LineWidth',2);

%% Response to Main mass & Absorber (Damped System)
Ys = sqrt(k1^2 + (c1.*w).^2); % frquency dependent variation of Y(w)
A1 = power((k2 - m2.*w.^2),2);
B1 = c2.*w;
A2 = (k1 - m1.*w.^2).*(k2 - m2.*w.^2)-(m2*k2).*w.^2 - (c2*c1).*w.^2;
B2 = (c2.*(k1-(m1+m2).*w.^2)+c1.*(k2 - m2.*w.^2)).*w;
C1 = k2^2;
D1 = c2.*w;

X1_Y = sqrt((A1.^2+B1.^2)./(A2.^2+B2.^2)).*Ys;
X2_Y = sqrt((C1.^2+D1.^2)./(A2.^2+B2.^2)).*Ys;

Phi_X1 = 180/pi.*atan((-A1.*B2+A2.*B1)./(A1.*A2+B1.*B2));
Phi_X2 = 180/pi.*atan((-C1.*B2+A2.*D1)./(C1.*A2+D1.*B2));

figure (3)
semilogy(w./(2*pi),X1_Y,'r','LineWidth',2);  hold on
semilogy(w./(2*pi),X2_Y,'--b','LineWidth',2);  hold on
semilogy(w./(2*pi),Xhat_Yhat,'k','LineWidth',2);
xlabel('Frequency, Hz')
ylabel('Amplitude Ratio, X/Y')

% figure (4)
% plot(w./(2*pi),Phi_X1+alpha,'r','LineWidth',2);  hold on
% plot(w./(2*pi),Phi_X2+alpha,'--b','LineWidth',2);  hold on
% plot(w./(2*pi),alpha + phi,'k','LineWidth',2);  hold on
% xlabel('Frequency, Hz')
% ylabel('Phase,degrees')

% figure (5)
% semilogy(w./(2*pi),X2_Y - X1_Y,'r','LineWidth',2);  hold on
%%  plotting the power that can be harvested.
con_c2 = 0.1;
Power = (con_c2.*w.^2).*(X2_Y - X1_Y).^2;
figure (6)
semilogy(w./(2*pi),Power,'k')
xlabel('Frequency, Hz')
ylabel('Poower, P/Y')

%% 3D Plots
zz = linspace(0.0001,0.75,200);
ww = linspace(40,85,200);
[X,Y] = meshgrid(zz,ww);
for i=1:200
    for j=1:200
        c2 = 2*X(i,j)*sqrt(m2*k2);
```

```
        w = Y(i,j);
        Ys = sqrt(k1^2 + (c1.*w).^2); % frquency dependent variation
of Y(w)
        A1 = power((k2 - m2.*w.^2),2);
        B1 = c2.*w;
        A2 = (k1 - m1.*w.^2).*(k2 - m2.*w.^2)-(m2*k2).*w.^2 -
(c2*c1).*w.^2;
        B2 = (c2.*(k1-(m1+m2).*w.^2)+c1.*(k2 - m2.*w.^2)).*w;
        C1 = k2^2;
        D1 = c2.*w;

        X1_Yn(i,j) = sqrt((A1.^2+B1.^2)./(A2.^2+B2.^2)).*Ys;
        X2_Yn(i,j) = sqrt((C1.^2+D1.^2)./(A2.^2+B2.^2)).*Ys;

        Phi_X1n(i,j) = 180/pi.*atan((-
A1.*B2+A2.*B1)./(A1.*A2+B1.*B2));
        Phi_X2n(i,j) = 180/pi.*atan((-
C1.*B2+A2.*D1)./(C1.*A2+D1.*B2));
        Power_3d(i,j) = (con_c2.*w.^2).*(X2_Yn(i,j) -
X1_Yn(i,j)).^2;
    end
end

%%
figure (300)
contour(Y./(2*pi),X,log10(Power_3d),30,'LineWidth',2);  hold on
colorbar;
xlabel('Frequency, Hz')
ylabel('Absorber damping ratio')

figure(301)
SObject = surface(Y./(2*pi),X,log10(Power_3d)); hold on; grid on
shading('flat'); colormap(jet);view(30,45);
xlabel('Frequency, Hz')
ylabel('Absorber damping ratio')
zlabel('log10(Power)')
```

Damping and Mass Ratio

```
%% Input parameter
zeta1 = 0.001; zeta2 = 0.0001;
m1 = 0.156; k1 = 390.8; c1 = 2*zeta1*sqrt(m1*k1);
wn1 = sqrt(k1/m1);
m2 = 0.0072*1.5; k2 = 18; c2 = 2*zeta2*sqrt(m2*k2);
wn2 = sqrt(k2/m2);

mu = m2/m1; % mass ratio
beta = wn2/wn1

%% Frequency vector
w = linspace(10,66,20000); % frequency

%% Response of the main mass alone
Xhat_Yhat = power(k1^2+(c1.*w).^2,1/2)./power((k1-
m1.*w.^2).^2+(c1.*w).^2,1/2);
alpha = 180/pi.*atan(c1/k1.*w);
phi = 180/pi.*atan(-(c1.*w)./(k1-m1.*w.^2));
Xhat_phase = alpha + phi;
```

```
% figure (1)
% semilogy(w,Xhat_Yhat,'r','LineWidth',2); grid on

% figure (2)
% plot(w,Xhat_phase,'r','LineWidth',2); grid on; hold on
% plot(w,alpha,'r','LineWidth',2);
% plot(w,phi,'b','LineWidth',2);

%% Response to Main mass & Absorber (Damped System)
Ys = sqrt(k1^2 + (c1.*w).^2); % frquency dependent variation of Y(w)
A1 = power((k2 - m2.*w.^2),2);
B1 = c2.*w;
A2 = (k1 - m1.*w.^2).*(k2 - m2.*w.^2)-(m2*k2).*w.^2 - (c2*c1).*w.^2;
B2 = (c2.*(k1-(m1+m2).*w.^2)+c1.*(k2 - m2.*w.^2)).*w;
C1 = k2^2;
D1 = c2.*w;

X1_Y = sqrt((A1.^2+B1.^2)./(A2.^2+B2.^2)).*Ys;
X2_Y = sqrt((C1.^2+D1.^2)./(A2.^2+B2.^2)).*Ys;

Phi_X1 = 180/pi.*atan((-A1.*B2+A2.*B1)./(A1.*A2+B1.*B2));
Phi_X2 = 180/pi.*atan((-C1.*B2+A2.*D1)./(C1.*A2+D1.*B2));

figure (3)
semilogy(w./(2*pi),X1_Y,'r','LineWidth',2);  hold on
semilogy(w./(2*pi),X2_Y,'--b','LineWidth',2);  hold on
semilogy(w./(2*pi),Xhat_Yhat,'k','LineWidth',2);
xlabel('Frequency, Hz')
ylabel('Amplitude Ratio, X/Y')

figure (4)
plot(w./(2*pi),Phi_X1+alpha,'r','LineWidth',2);  hold on
plot(w./(2*pi),Phi_X2+alpha,'--b','LineWidth',2);  hold on
plot(w./(2*pi),alpha + phi,'k','LineWidth',2);  hold on
xlabel('Frequency, Hz')
ylabel('Phase,degrees')

%% Plotting power that can be harvested.

con_c2 = 0.1;
Power = (con_c2.*w.^2).*(X2_Y - X1_Y).^2;
figure (6)
semilogy(w,Power,'k')

figure (7)
semilogy(w,X2_Y - X1_Y,'k')
```