Unsteady Flow of Bingham Fluid Between Two Permeable Beds

By

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CERTIFICATION OF APPROVAL

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CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

MUHAMMAD HANIF HIDAYAT CHAI

ABSTRACT

As the application of chemical industry and oil and gas industry, fluid flow in the pipeline is governed by Navier-Stokes equations. Darcy's law is to express the fluid flow behaves in the porous medium. As the oil and gas industry interested on the heavy oil, the study focus on the heavy oil behaves in between permeable beds. The combination of Navier-Stokes and Darcy explain the behavior of heavy oil in between permeable beds. Pressure is assumed to vary exponentially with respect to time. Bingham fluid was deduced for velocity field between beds and between rigid walls, shear stress and mass flow rate for lower zones, upper zones and plug flow region. Findings have shown that permeable beds increases the velocity of the fluid flow compared to the rigid wall condition. It also found out low σ increases the fractional increase drastically after $\sigma<6$.

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Last but not least, the author wishes to show his appreciation to all parties involved in this project.

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NOMENCLATURE

x, y	:	Cartesian co-ordinates
t	:	Time
U	:	Velocity field in the plug flow region
u_1, u_2	:	Velocity components in x-direction in zones I and II respectively
k_1, k_2	:	Permeabilities of the lower and upper beds
σ_1, σ_2	:	Dimensionless parameters $\frac{h}{\sqrt{k_1}}, \frac{h}{\sqrt{k_2}}$
σ_{xy}	:	Shear stress
u_{B1}, u_{B2}	:	Slip velocities at the lower and upper beds
Q_1, Q_2	:	Darcy's velocities
p	:	Pressure
α	:	Slip parameter
σ_0	:	Yield stress
σ_1	:	Shear stress at lower bed
h	:	Width of the channel
c	:	Constant
Re	:	$\frac{\rho c h^2}{\mu}$, Reynold number
$(\ )^*$	:	Dimensionless quantity
μ	:	Viscosity coefficient
ρ	:	Density
ε	:	Porosity
α_0	:	Non-Newtonian parameter

CHAPTER 1

INTRODUCTION

1.1 Background

The analysis of Non-Newtonian fluids flow has been a popular area of research since several years ago. In order to understand the fluid flow in between permeable beds, several studies have been carried out. Bingham fluid flow is investigated bounded by permeable beds with different permeability under unsteady flow. Bingham fluid has often representing viscous fluid's behavior[2] and therefore these flows finds applications in chemical engineering and oil industry. As the nature of geological formed by layering, there is higher permeability in the x direction. For any exploration or production well nearby, fluid tend to flow in x direction due to the pressure difference. Hence, the high viscous fluid such as heavy oil will tend to move horizontally.

1.2 Problem Statement

The heavy oil in the reservoir is always be assumed as non-Newtonian fluid such as Bingham fluid. The other fluids such as drilling mud, cement, foam which are used in oil and gas industry are interrelated with the non-Newtonian fluid model. Therefore, the study focus more on Bingham fluid flow between permeable beds for better understanding on the heavy oil flow.

1.3 Objectives

The objectives of this study are defined as following:

- To formulate Bingham's unsteady flow in zone 1, zone 2 and plug flow region
- To formulate Bingham's fluid with equal permeability between two permeable beds, between two rigid walls, shear stress and mass flow rate
- To identify the velocity profiles under different σ

1.4 Scope of Study

This study investigates Bingham fluid in various velocity profile, shear stress and fractional mass flow rate under unsteady flow which bounded by permeable beds with different permeabilities. Under various assumptions, flow is assumed as incompressible, horizontal direction and driven by Pexp(ct) between homogeneous beds. Through Matlab programming, velocity profiles will show how the fluid flow and behaves. Upon validating the result, several studies will be compared for the result.

CHAPTER 2

LITERATURE REVIEW AND THEORY

Many scientists have been researching on the Newtonian fluid and non-Newtonian fluid flow in the porous media. It is necessary to understand the fluid flow in porous media, but it is more crucial to know that geology is usually in heterogeneous formation. Therefore, in this chapter, review and findings made by previous researchers, the different fluid flows between permeable beds and porous media.

2.1 Review of Previous Studies

It is crucial as a fundamental engineering application for understanding non-Newtonian fluid behaves in permeable beds[3]. Bingham fluid and power law fluid were studied widely by various literature until now. Wu [3] studied how Bingham fluid's displace and move in porous media. Slightly compressible Bingham fluid is discussed and new well-test-analysis method is developed.

Pascal [2] showed transient flow in porous medium by power law fluids. Poollen [1] mentioned that when injecting power-law type of fluid into a reservoir, the viscosity of the power-law fluid will decreases as rate of shear or flow rate increases. Poollen[1] formulate equations for stady-state linear, transient behavior results from a finite difference model of a radial system, and transient behavior results from a field test.

Vajravelu [4] investigated study of two immisicible conducting fluids between permeable beds with hydromagnetic unsteady flow. Results in the form of velocity distributions in the porous regions and mass flow rate are obtained.

Malathy [7] studied the pulsating flow of a hydromagnetic fluid between two permeable beds. Channel from the lower permeable bed is injected with fluids and sucked out at the upper permeable bed with the same velocity. Velocity field and volume flux are obtained as result.

Two immiscible conducting fluids under hydromagnetic unsteady flow between two permeable beds was studied with different permeabilities by Vajravelu[4]. Through a porous medium between permeable beds, hydromagnetic fluid flow is investigated by Prasad[8]. He exhibited different parameters and showed the velocity field and volume flux under graphical method.

2.2 Bingham fluid

Among the non-Newtonian fluid's model, Bingham fluid is one of them. Under an amount of force where beyond the yield stress, Bingham fluid's flow rate will increase proportional with shear stress. Mathematical expression of this model is $\sigma_{xy} = \sigma_0 + \mu \frac{\partial u_i}{\partial y}$. Having viscosity coefficient μ, and the yield shear stress σ_0 as parameters which characterize Bingham fluid. If the shear stress is lower than yield stress, these fluids act as rigid solids.

CHAPTER 3

METHODOLOGY

3.1 Formulation of the Problem

The flow region between two permeable beds is divided in three zones with unsteady flow of Bingham fluid. Zone I is bounded by $y = 0$ and $y = y_1$, plug flow region is divided by $y = y_1$ and $y = y_2$, and zone II is covered by $y = y_2$ and $y = h$. In zones I and II, $|\tau_{xy}| > \tau_0$. In plug flow region, $|\tau_{xy}| = \tau_0$. Zone I and II are ruled by Navier-Stokes equations. Darcy's law expresses the flow behavior in between permeable beds.[5]

The pressure is assumed to vary exponentially with respect to time.

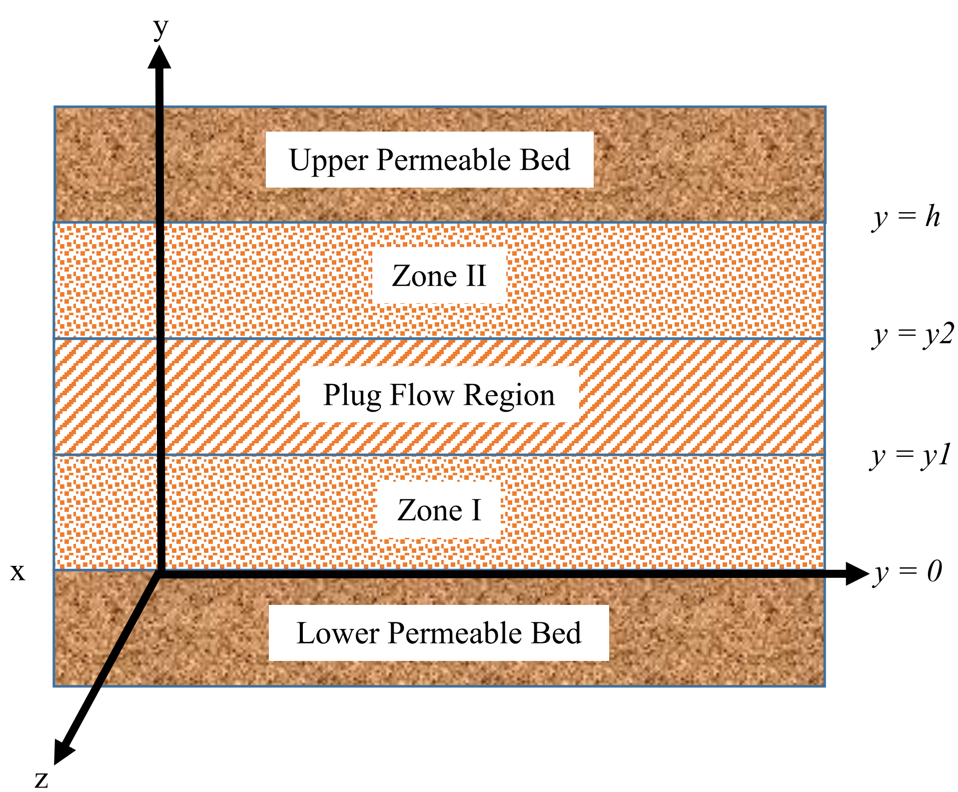

Figure 1 Physical Model

In order to derive the basic equations, some assumptions are made as follow :

1. The flow is unsteady and incompressible.
2. The flow is in x-direction.
3. All the physical quantities except the pressure are functions of y and t only. The velocity is given by $(u(y,t),0,0)$
4. The body forces are negligible.
5. Homogeneous lower and upper beds has constant permeabilities k_1 and k_2 respectively.
6. The flow is driven by Pexp(ct) which is a common time-dependent pressure gradient.

Flow Between Permeable Beds

Basic Equations :

$$\rho \frac{\partial u_i}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} \text{ where } i = 1,2 \tag{3.1}$$

where

$$\sigma_{xy} = \mu\left(\frac{\partial u_i}{\partial y}\right) \pm \sigma_0 \tag{3.2}$$

(+sign for zone-I and –sign for zone-II)

Boundary condition :

$$u_i = u_{Bi}, \frac{\partial u_i}{\partial y} = \pm \frac{\alpha}{\sqrt{k_i}}(u_{Bi} - Q_i) \tag{3.3}$$

at $y = 0$, where $i = 1,2$ (+ sign for $i = 1$, - sign for $i = 2$)

(Beavers and Joseph (1967) slip condition)

$$\sigma_{xy} = \sigma_1 \text{ at } y = 0$$

Flow in the Permeable Beds

Basic Equations :

$$\frac{\rho}{\varepsilon}\frac{\partial Q_i}{\partial t} = -\frac{\partial P}{\partial x} - \frac{\mu}{k_i} Q_i \quad (i = 1,2) \tag{3.4}$$

$i = 1$ corresponds to lower permeable bed

$i = 2$ corresponds to upper permeable bed

3.2 Non-Dimensionalization of the Flow Quantities

It is convenient to introduce the following non-dimensional quantities:

$$u_i^* = \frac{u_i}{u_{av}}; t^* = \frac{tu_{av}}{h};$$

$$x^* = \frac{x}{h}; y^* = \frac{y}{h};$$

$$P^* = \frac{P}{\rho u_{av}^2}; Q_i^* = \frac{Q_i}{u_{av}};$$

$$\sigma_{xy}^* = \frac{\sigma_{xy}}{\rho u_{av}^2}; \sigma_0^* = \frac{\sigma_0}{\rho u_{av}^2};$$

$$u_{Bi}^* = \frac{u_{Bi}}{u_{av}};$$

The asterisk (*) are neglected after dimensionless quantities are used in (3.1) – (3.4). The non dimensional form of (3.1) – (3.4) are

$$\frac{\partial u_i}{\partial t} = -\frac{\partial P}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} \tag{3.5}$$

$$\sigma_{xy} = \frac{1}{\mathrm{Re}}\frac{\partial u_i}{\partial y} \pm \sigma_0 \tag{3.6}$$

$$\frac{1}{\varepsilon_i}\frac{\partial Q_i}{\partial t} = -\frac{\partial P}{\partial x} - \frac{\sigma_i^2}{\mathrm{Re}} Q_i \tag{3.7}$$

$$u_i = u_{Bi}, \frac{\partial u_i}{\partial y} = \pm \alpha \sigma_i \left(u_{Bi} - Q_i\right) \text{ at y=0,1} \tag{3.8}$$

Based on assumption (6), we take

$$u_i(y,t) = v_i(y)e^{\lambda^2 t}$$

$$\frac{\partial p}{\partial x} = -Pe^{\lambda^2 t}$$

$$u_{Bi} = v_{Bi}e^{\lambda^2 t}$$

$$Q_i = v_{Qi}e^{\lambda^2 t}$$

$$\sigma_{xy} = \tau_{xy}e^{\lambda^2 t}$$

$$\sigma_0 = \tau_0 e^{\lambda^2 t}$$

The basic equations and boundary conditions above form as following:

Zone I

$$\frac{1}{\text{Re}}\frac{d^2 v_1}{dy^2} - \lambda^2 v_1 = -P \tag{4.1}$$

and
$$\tau_{xy} = \frac{1}{\text{Re}}\frac{dv_1}{dy} + \tau_0 \tag{4.2}$$

$$\tau_{xy} = \tau_1 \text{ at } y = 0 \tag{4.3}$$

and
$$v_1 = v_{B1}, \frac{dv_1}{dy} = \alpha\sigma_1\left(v_{B1} - v_{Q1}\right) \text{ at } y = 0 \tag{4.4}$$

Plug Flow Region

In this region, we defined velocity by $\left|\tau_{xy}\right| = \tau_0$ for $y_1 \le y \le y_2$

Zone II

$$\frac{1}{\mathrm{Re}}\frac{d^2v_2}{dy^2}-\lambda^2v_2=-P \quad (i=1,2) \tag{4.5}$$

$$\tau_{xy}=\frac{1}{\mathrm{Re}}\frac{dv_2}{dy}-\tau_0 \tag{4.6}$$

$$v_2=V \text{ at } y=y_2 \tag{4.7}$$

and

$$v_2=v_{B2},\frac{dv_2}{dy}=-\alpha\sigma_2\left(v_{B2}-v_{Q2}\right) \text{ at } y=1 \tag{4.8}$$

Flow in the Permeable Beds

$$v_{Qi}=\frac{P\varepsilon\,\mathrm{Re}}{\lambda^2\,\mathrm{Re}+\sigma_i{}^2\varepsilon} \quad (i=1,2) \tag{4.9}$$

3.3 Solution of the Problem

Zone I

From (4.1) with boundary condition of (4.3) and (4.4), we get

$$v_1=c_1e^{\lambda\sqrt{\mathrm{Re}}y}+c_2e^{-\lambda\sqrt{\mathrm{Re}}y}+\frac{P}{\lambda^2} \tag{5.1}$$

where

$$c_1=\frac{\mathrm{Re}\left(\tau_1-\tau_0\right)}{2}\left[\frac{\sqrt{\mathrm{Re}}}{\alpha\sigma_1}+\frac{1}{\lambda}\right]+\frac{1}{2}v_{Q1}-\frac{P}{2\lambda^2}$$

$$c_2=\frac{\mathrm{Re}\left(\tau_1-\tau_0\right)}{2}\left[\frac{\sqrt{\mathrm{Re}}}{\alpha\sigma_1}-\frac{1}{\lambda}\right]+\frac{1}{2}v_{Q1}-\frac{P}{2\lambda^2}$$

Use (5.1) in (4.2), shear stress is expressed as

$$\tau_{xy}=M\left[c_1e^{\lambda\sqrt{\mathrm{Re}}y}+c_2e^{-\lambda\sqrt{\mathrm{Re}}y}\right]+\tau_0 \tag{5.2}$$

Velocity in Zone I can be expressed as

$$v_1 = \left(v_{B1} - \frac{P}{\lambda^2} \right) \cosh \lambda \sqrt{\mathrm{Re}}\, y + \frac{\sqrt{\mathrm{Re}}\left(\tau_1 - \tau_0\right)}{\lambda} \sinh \lambda \sqrt{\mathrm{Re}}\, y + \frac{P}{\lambda^2} \tag{5.3}$$

where slip velocity v_{B1} is given by

$$v_{B1} = \frac{\mathrm{Re}\left(\tau_1 - \tau_0\right)}{\alpha \sigma_1} + v_{Q1} \tag{5.4}$$

Plug Flow Region

Use $v = V$ at $y = y_1$ in (5.3), velocity in plug flow region as

$$V = \left(v_{B1} - \frac{P}{\lambda^2} \right) \cosh \lambda \sqrt{\mathrm{Re}}\, y_1 + \frac{\sqrt{\mathrm{Re}}\left(\tau_1 - \tau_0\right)}{\lambda} \sinh \lambda \sqrt{\mathrm{Re}}\, y_1 + \frac{P}{\lambda^2} \tag{5.5}$$

Zone II

From (4.5), with boundary conditions (4.7) and (4.8), velocity in Zone II as

$$v_2 = \frac{\left\{ \left(\lambda^2 V - P \right) \sinh \lambda \sqrt{\mathrm{Re}} (1 - y) + P \sinh \lambda \sqrt{\mathrm{Re}} (1 - y_2) + (\lambda^2 v_{B2} - P) \sinh \lambda \sqrt{\mathrm{Re}} (y - y_2) \right\}}{\lambda^2 \sinh \lambda \sqrt{\mathrm{Re}} (1 - y_2)} \tag{5.6}$$

where slip velocity v_{B2} is given by

$$v_{B2} = \frac{\lambda^2 \sqrt{\mathrm{Re}} V + \alpha \sigma_2 v_{Q2} \lambda \sinh \lambda \sqrt{\mathrm{Re}} (1 - y_2) + \sqrt{\mathrm{Re}} P \cosh \lambda \sqrt{\mathrm{Re}} (1 - y_2) - \sqrt{\mathrm{Re}} P}{\alpha \sigma_2 \lambda \sinh \lambda \sqrt{\mathrm{Re}} (1 - y_2) + \lambda^2 \sqrt{\mathrm{Re}} \cosh \lambda \sqrt{\mathrm{Re}} (1 - y_2)} \tag{5.7}$$

3.4 Shear Stress

Plug flow region is not being affected by shear stress at the boundaries. At Zone I and Zone II, fluid is affected by shear stress as fluid constantly contact with solid which is the permeable beds. Therefore, determining y1 and y2 can find out the height of fluid affected by shear stress

Shear stress in Zone I is given by

$$\tau_{xy} = \frac{1}{\text{Re}} \frac{dv_1}{dy} + \tau_0 \tag{4.2}$$

then it is being substituted by the differential of (5.3)

$$\tau_{xy} = \left(v_{B1} - \frac{1}{M^2} \right) M \sinh My + (\tau_1 - \tau_0) \cosh My + \tau_0 \tag{6.1}$$

where the boundary of Zone I

$$\tau_{xy} = \tau_0 \text{ at } y = y_1 \tag{6.2}$$

For (6.2) in (6.1),

$$y_1 = \frac{1}{\lambda \sqrt{\text{Re}}} \tanh{-1} \left\{ \frac{\lambda \sqrt{\text{Re}} \alpha \sigma_1 (\tau_1 - \tau_0)}{P \alpha \sigma_1 - \lambda^2 v_{Q1} \alpha \sigma_1 - \lambda^2 \text{ Re}(\tau_1 - \tau_0)} \right\} \tag{6.3}$$

Shear stress in Zone II is given by

$$\tau_{xy} = \frac{1}{\text{Re}} \frac{dv_2}{dy} - \tau_0 \tag{4.6}$$

then it is being substituted by the differential of (5.6)

$$\tau_{xy} = \frac{1}{\text{Re}} \left[\frac{\left\{ \left(P - \lambda^2 V\right) \sqrt{\text{Re}} \cosh \lambda \sqrt{\text{Re}} (1 - y) + (\lambda^2 v_{B2} - P) \sqrt{\text{Re}} \cosh \lambda \sqrt{\text{Re}} (y - y_2) \right\}}{\lambda^2 \sinh \lambda \sqrt{\text{Re}} (1 - y_2)} \right] - \tau_0 \tag{6.4}$$

where the boundary of Zone II

$$\tau_{xy} = -\tau_0 \text{ at } y = y_2 \tag{6.5}$$

For (6.5) in (6.4)

$$y_2 = 1 - \frac{1}{\lambda \sqrt{\text{Re}}} \log \left\{ \frac{\alpha \sigma_2 (P - \lambda v_{Q2}) + \sqrt{\alpha^2 \sigma_2^2 (P - \lambda v_{Q2})^2 - \left(\alpha^2 \sigma_2^2 - \lambda^2 \text{ Re}\right)\left(P - \lambda^2 V\right)^2}}{\left(P - \lambda^2 V\right)\left(\alpha \sigma_2 - \lambda^2 \sqrt{\text{Re}}\right)} \right\} \tag{6.6}$$

As σ_1 and σ_2 tend to infinity, equation (6.3) and (6.6) becomes

$$y_1 = \frac{1}{\lambda \sqrt{\text{Re}}} \tanh{-1} \left\{ \frac{\lambda \sqrt{\text{Re}} (\tau_1 - \tau_0)}{P} \right\} \tag{6.7}$$

$$y_2 = 1 - \frac{1}{\lambda \sqrt{\text{Re}}} \log \left\{ \frac{P + \sqrt{P^2 - \left(P - \lambda^2 V\right)^2}}{\left(P - \lambda^2 V\right)} \right\} \tag{6.8}$$

3.5 Mass Flow Rate

The mass flow rate G of the Bingham fluid flow between permeable beds is given by

$$G = G_0 e^{\lambda^2 t} \tag{7.1}$$

$$G_0 = \int_0^{y_1} v_1 dy + \int_{y_1}^{y_2} V dy + \int_{y_2}^{1} v_2 dy \tag{7.2}$$

$$\begin{aligned} G_0 &= \left(v_{B1} - \frac{P}{\lambda^2}\right)\left[\frac{\sinh \lambda\sqrt{\mathrm{Re}}\, y_1}{\lambda\sqrt{\mathrm{Re}}} + (y_2 - y_1)\cosh \lambda\sqrt{\mathrm{Re}}\, y_1\right] \\ &+ \frac{\tau_1 - \tau_0}{\lambda^2}\left[\lambda\sqrt{\mathrm{Re}}(y_2 - y_1)\sinh \lambda\sqrt{\mathrm{Re}}\, y_1 + \cosh \lambda\sqrt{\mathrm{Re}}\, y_1 - 1\right] \\ &+ \frac{(\lambda^2 V + \lambda^2 v_{B2} - 2P)(\cosh \lambda\sqrt{\mathrm{Re}}(1 - y_2) - 1)}{\lambda^3\sqrt{\mathrm{Re}}\sinh \lambda\sqrt{\mathrm{Re}}(1 - y_2)} + \frac{P}{\lambda^2} \end{aligned} \tag{7.3}$$

Mass flow rate G_c of the Bingham fluid flow between rigid walls as equation (7.4) as σ_1 and σ_2 tend to infinity.

$$\begin{aligned} G_C &= \frac{P}{\lambda^2}\left\{1 - \frac{\sinh \lambda\sqrt{\mathrm{Re}}\, y_3}{\lambda\sqrt{\mathrm{Re}}} - (y_4 - y_3)\cosh \lambda\sqrt{\mathrm{Re}}\, y_3\right\} \\ &+ \frac{\tau_1 - \tau_0}{\lambda^2}\left\{\lambda\sqrt{\mathrm{Re}}(y_4 - y_3)\sinh \lambda\sqrt{\mathrm{Re}}\, y_3 + \cosh \lambda\sqrt{\mathrm{Re}}\, y_3 - 1\right\} \\ &+ \frac{\lambda\sqrt{\mathrm{Re}}(\tau_1 - \tau_0)\sinh \lambda\sqrt{\mathrm{Re}}\, y_3 - P\cosh \lambda\sqrt{\mathrm{Re}}\, y_3 - P)(\cosh \lambda\sqrt{\mathrm{Re}}(1 - y_4) - 1)}{\lambda^3\sqrt{\mathrm{Re}}\sinh \lambda\sqrt{\mathrm{Re}}(1 - y_4)} \end{aligned} \tag{7.4}$$

where

$$\lim_{\sigma\to\infty} y_1 = y_3 \quad \lim_{\sigma\to\infty} y_2 = y_4$$

The fractional increase in mass flow rate is given by

$$\phi = \frac{G_0 - G_C}{G_C} \tag{7.5}$$

3.6 Deductions of Two Different Situations

3.6.1 Bingham Fluid Between Two Permeable Beds of Equal Permeability

For $k_1 = k_2 = k$, (then $\sigma_1 = \sigma_2 = \sigma$), velocity is expressed as

Zone I

$$v_1 = \left(v_{B1} - \frac{P}{\lambda^2} \right) \cosh \lambda \sqrt{\mathrm{Re}}\, y + \frac{\sqrt{\mathrm{Re}}(\tau_1 - \tau_0)}{\lambda} \sinh \lambda \sqrt{\mathrm{Re}}\, y + \frac{P}{\lambda^2} \tag{8.1}$$

where slip condition of lower velocity

$$v_{B1} = \frac{\tau_1 - \tau_0}{\alpha\sigma} + v_{Q1} \tag{8.2}$$

Plug Flow Region

$$V = \left(v_{B1} - \frac{P}{\lambda^2} \right) \cosh \lambda \sqrt{\mathrm{Re}}\, y_1 + \frac{\sqrt{\mathrm{Re}}(\tau_1 - \tau_0)}{\lambda} \sinh \lambda \sqrt{\mathrm{Re}}\, y_1 + \frac{P}{\lambda^2} \tag{8.3}$$

Zone II

$$v_2 = \frac{\left\{ \left(\lambda^2 V - P\right) \sinh \lambda \sqrt{\mathrm{Re}}(1-y) + P \sinh \lambda \sqrt{\mathrm{Re}}(1-y_2) + (\lambda^2 v_{B2} - P) \sinh \lambda \sqrt{\mathrm{Re}}(y - y_2) \right\}}{\lambda^2 \sinh \lambda \sqrt{\mathrm{Re}}(1-y_2)} \tag{8.4}$$

where slip condition of upper velocity

$$v_{B2} = \frac{\lambda^2 \sqrt{\mathrm{Re}} V + \alpha\sigma v_{Q2} \lambda \sinh \lambda \sqrt{\mathrm{Re}}(1-y_2) + \sqrt{\mathrm{Re}} P \cosh \lambda \sqrt{\mathrm{Re}}(1-y_2) - \sqrt{\mathrm{Re}} P}{\alpha\sigma\lambda \sinh \lambda \sqrt{\mathrm{Re}}(1-y_2) + \lambda^2 \sqrt{\mathrm{Re}} \cosh \lambda \sqrt{\mathrm{Re}}(1-y_2)} \tag{8.5}$$

$$v_{Qi} = \frac{P \varepsilon_i \mathrm{Re}}{\lambda^2 \mathrm{Re} + \sigma^2 \varepsilon_i} = v_{Q1} = v_{Q2} \tag{8.6}$$

3.6.2 Bingham Fluid Flow Between Two Rigid Walls

For $k_1 \& k_2$ tend to zero, (then $\sigma_1 = \sigma_2 \to \infty$), velocity is expressed as

Zone I

$$v_1 = \frac{\sqrt{\mathrm{Re}}\left(\tau_1 - \tau_0\right)}{\lambda} \sinh \lambda \sqrt{\mathrm{Re}}\, y + \frac{P}{\lambda^2}\left(1 - \cosh \lambda \sqrt{\mathrm{Re}}\, y\right) \tag{6.7}$$

where

$$v_{Q1} = v_{Q2} = 0$$

Plug Flow Region

$$V = \frac{\sqrt{\mathrm{Re}}\left(\tau_1 - \tau_0\right)}{\lambda} \sinh \lambda \sqrt{\mathrm{Re}}\, y_1 + \frac{P}{\lambda^2}\left(1 - \cosh \lambda \sqrt{\mathrm{Re}}\, y_1\right) \tag{6.8}$$

Zone II

$$v_2 = \frac{\left\{\left(\lambda^2 V - P\right)\sinh \lambda \sqrt{\mathrm{Re}}(1-y) + P \sinh \lambda \sqrt{\mathrm{Re}}(1-y_2) + P \sinh \lambda \sqrt{\mathrm{Re}}(y - y_2)\right\}}{\lambda^2 \sinh \lambda \sqrt{\mathrm{Re}}(1-y_2)} \tag{6.9}$$

where

$$v_{B1} = 0$$

CHAPTER 4

RESULT AND DISCUSSION

Bingham model was chosen for derivation and understanding how unsteady fluid flow move between permeable beds over time. The derivation was derived from basic equations of Navier Stokes and Darcy law, velocity equations, shear stress, mass flow rate until different conditions applied. Matlab coding was created for the graphical of velocity profiles. It is attached as Appendix 4.

For the Bingham fluid flow between two permeable beds, velocity profiles are drawn in Figure (2-10). With the various value of τ_0, α and σ, different shapes of velocity graph can be seen in Figure (2-10). For a fixed σ, the velocity of the flow grows larger with the increment of y initially from lower permeable bed and take a constant value in the plug flow region. After the plug flow region, the velocity decreases with the continue of increment in y until the upper permeable bed. Therefore, the velocity is maximum in the plug flow region.

For a fixed y, the velocity of the curves decreases with the increasing of σ, and it reaches to a minimum when the σ becomes infinity. As the σ increases, the gap between velocity curves becomes smaller which indicates the effect of the σ toward velocity reduces.

For τ_0 increases from 0.1 to 0.3, for example, in Figure (2-4), the velocity reduces.

As the α increases, the width of plug flow region reduces. It indicates the α has a direct effect on the Zone I and Zone II which influenced by the shear stress.

Comparing the velocity curves between the two conditions of permeable beds and rigid walls, we found out that the effect of permeable beds is to increases the velocity in the channel.

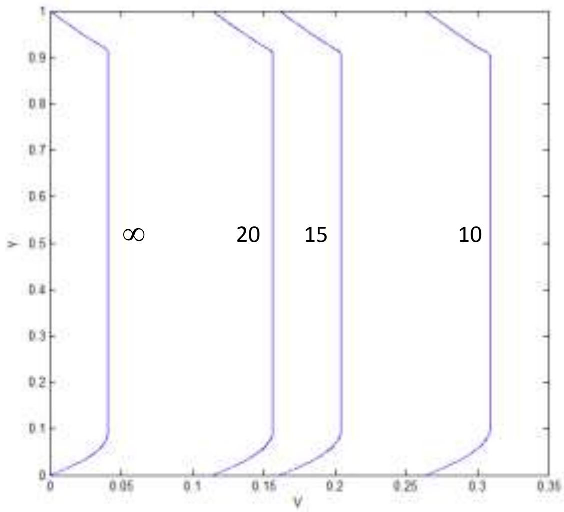

Figure 2 V against Y with τ_0=0.1,α=0.5 and different σ

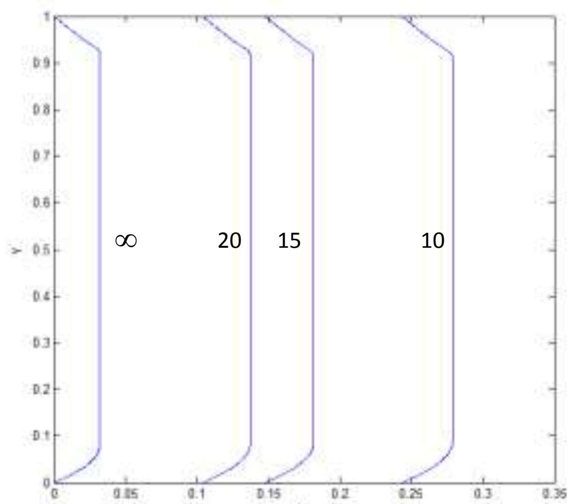

Figure 3 V against Y with τ_0=0.2,α=0.5 and different σ

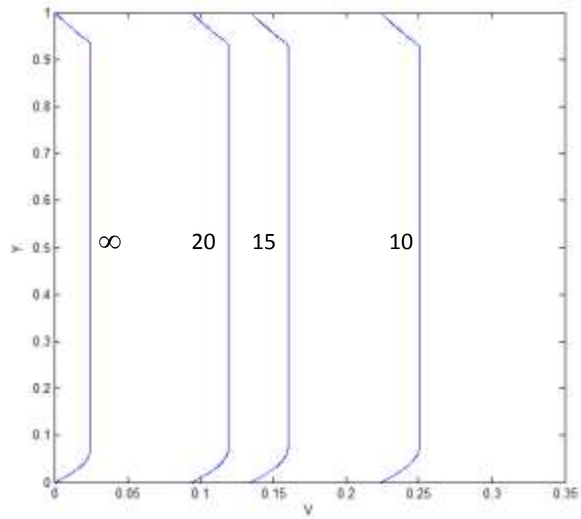

Figure 5 V against Y with τ_0=0.3,α=0.5 and different σ

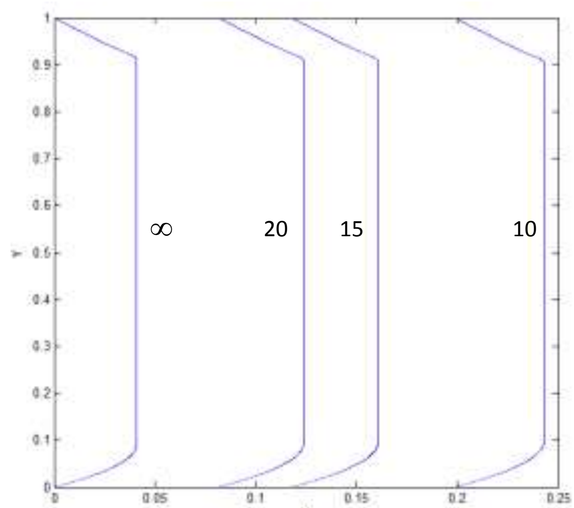

Figure 4 V against Y with τ_0=0.1,α=0.78 and different σ

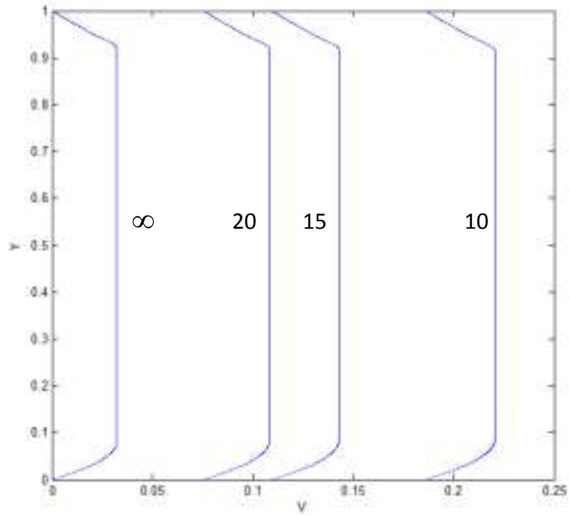

Figure 6 V against Y with τ_0=0.2,α=0.78 and different σ

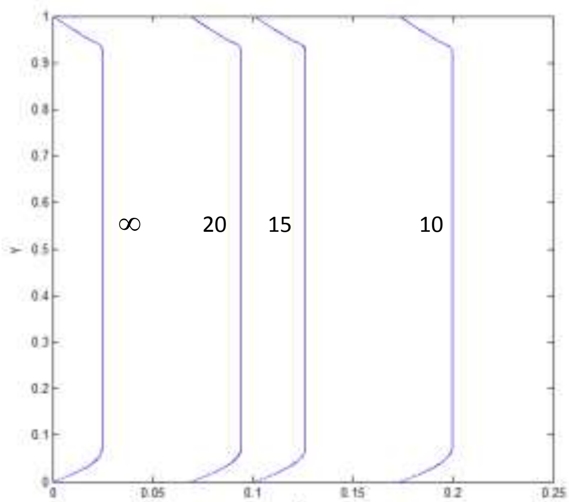

Figure 7 V against Y with τ_0=0.3,α=0.78 and different σ

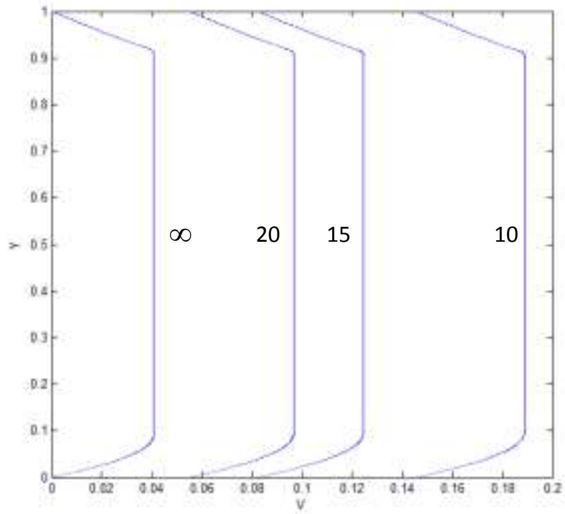

Figure 9 V against Y with τ_0=0.1,α=1.45 and different σ

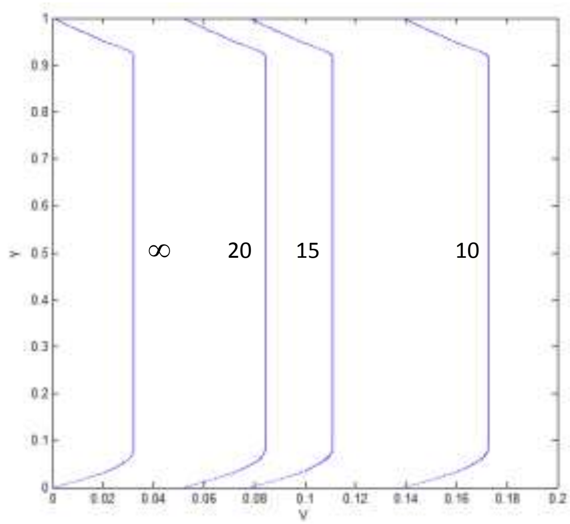

Figure 8 V against Y with τ_0=0.2,α=1.45 and different σ

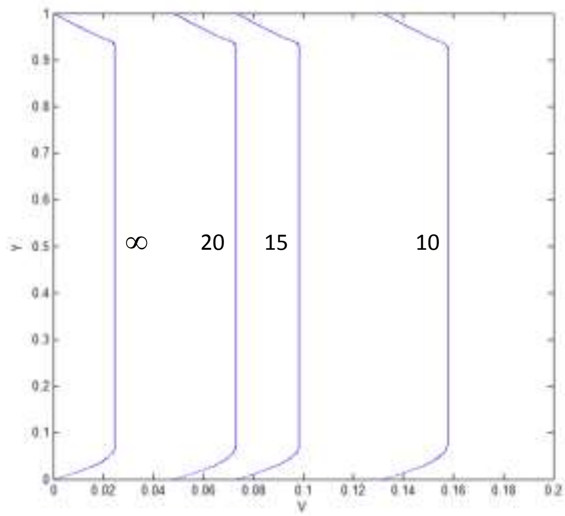

Figure 10 V against Y with τ_0=0.3,α=1.45 and different σ

In Figure (11),(12) and (13), those are graphs plotted for the variation of fractional increase in mass flow rate for different τ_0 and α. For the fixed α, fractional increases decreases with the increment of σ. For fixed τ_0, and σ, it decreases with the increment of α. For fixed α and σ, the fractional increases increases with the increment of τ_0.

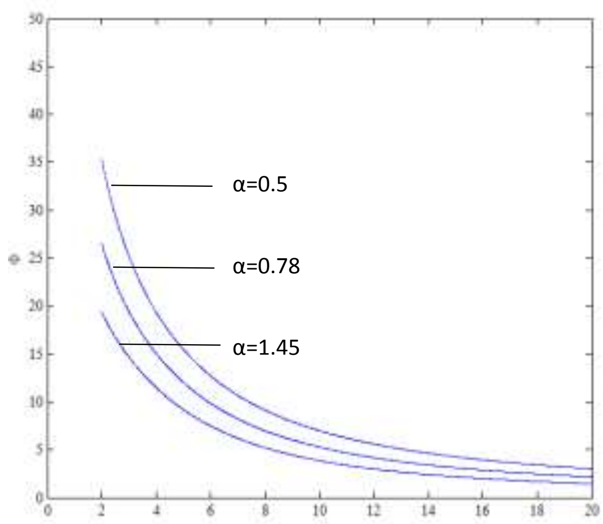

Figure 11 Fractional increase in mass flow rate with τ_0=0.1, and different α

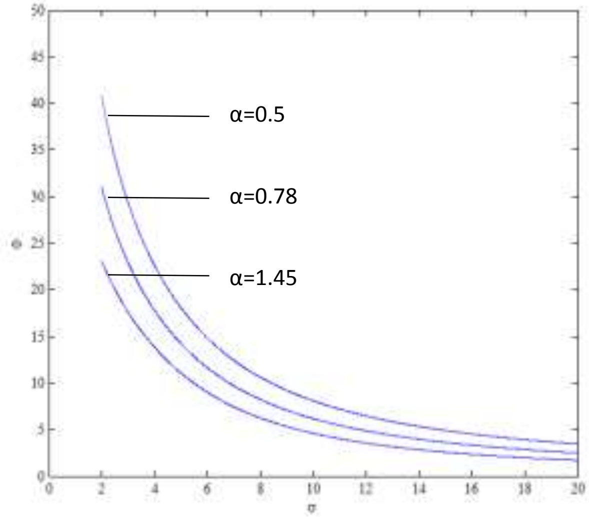

Figure 12 Fractional increase in mass flow rate with τ_0=0.2, and different α

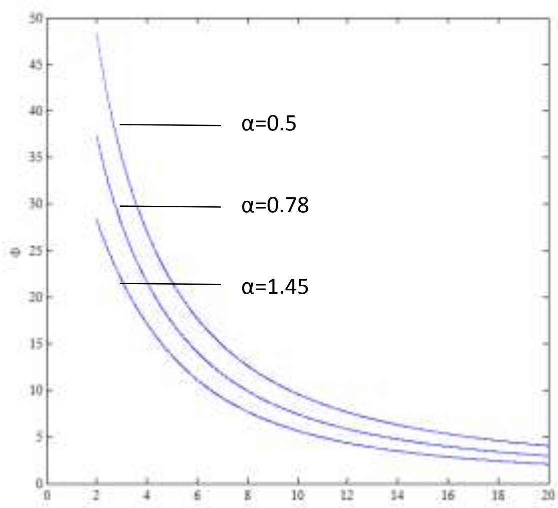

Figure 13 Fractional increase in mass flow rate with τ_0=0.3, and different α

CHAPTER 5

CONCLUSION & RECOMMENDATIONS

5.1 Conclusion

In the nutshell, this project studies hows the behavior of the velocity profile for the three regions, zone I, II and plug flow region and fractional increase in mass flow rate of with the variation of τ_0, α and σ parameters. The steps of derivation for the Bingham fluid between permeable beds is shown in the Chapter 3. Furthermore, Figure (2-10) have shown that permeable beds increases the velocity of the fluid flow compared to the rigid wall condition. Figure (11-13) have resulted low σ increases the fractional increase drastically after $\sigma<6$.

5.2 Recommendations

This study was entirely assumed the flow in x-direction, which limited the idea of natural behavior of the fluid flow in all direction. Futher study of 3D of the fluid flow should be carried out as fluid will flow upward as the pressure decreases. Other type of fluid flow between permeable beds such as power law and Herschel Bulkley should be studied for different situations. Different scenario of the fluid flow should be investigated for a better understanding of heavy oil behave in complex geology.

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APPENDICES

Appendix 1: Project Key Milestones

Project Key Milestones		Date
FYP 1	Project topic selection	16th Jan 2015
	Literature Review	23th Jan– 10th Feb 2015
	Derivation of Past Papers	25th Feb – 30th Mar 2015
	Matlab Simulation	1st Apr – 17th Apr 2015
FYP 2	Derivation of Bingham Fluid Model	18th Jul – 30th Jun 2015
	Matlab Simulation	1st Jul – 31st Jul 2015
	Simulation Result Collection and Analysis	20th Jul– 31st Jul 2015

Appendix 2: Project Timeline - Gantt Chart

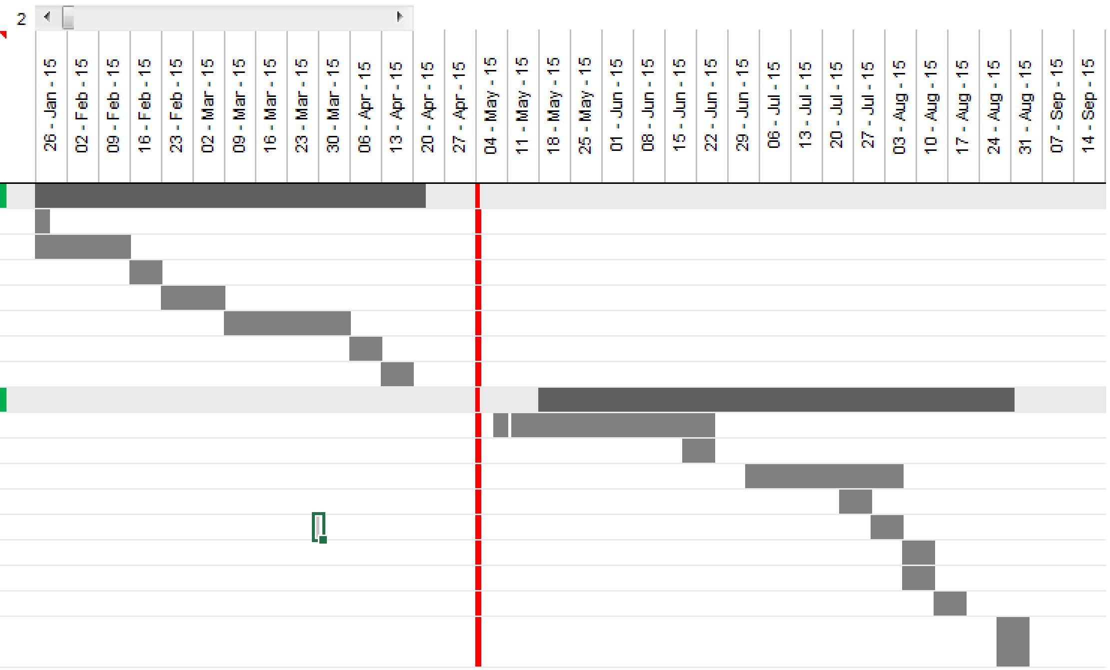
Final Year Project 1 (PCB4022): Unsteady Flow of Herschel-Bulkley Fluid Between Two Permeable Beds

Universiti Teknologi PETRONAS

Monday

Start Date: 26/1/2015 Monday

WBS	Tasks	Start	End	Duration (Days)
1	FYP 1	January 22, 2015	4/22/15	91
1.1	Selection of Project Topics	January 22, 2015	1/28/15	7
1.2	Preliminary Research Work	January 26, 2015	2/15/15	21
1.3	Submission of Extended Proposal	February 16, 2015	2/22/15	7
1.4	Proposal Defence	February 23, 2015	3/8/15	14
1.5	Project work continues	March 9, 2015	4/5/15	28
1.6	Submission of Interim Draft Report	April 6, 2015	4/12/15	7
1.7	Submission of Interim Report	April 13, 2015	4/19/15	7
2	FYP 2	May 18, 2015	8/31/15	106
2.1	Project work continues	May 8, 2015	6/25/15	49
2.2	Submission of Progress Report	June 19, 2015	6/25/15	7
2.3	Project work continues	July 3, 2015	8/6/15	35
2.4	Pre-SEDEX	July 24, 2015	7/30/15	7
2.5	Submission of Draft Report	July 31, 2015	8/6/15	7
2.6	Submission of Dissertation (soft bound)	August 7, 2015	8/13/15	7
2.7	Submission of Technical Paper	August 7, 2015	8/13/15	7
2.8	Oral Presentation	August 14, 2015	8/20/15	7
2.9	Submission of Project Dissertation (Hard Bound)	August 28, 2015	9/3/15	7



Appendix 3: Matlab Coding

1) Matlab coding: Velocity graph

```
clear all;
tao1=1;
A=1.45;          %alpha
S=10;            %sigma
tao0=0.3;        %change these value for another graph
L=2;             %lamda(constant)
P=10;            %pressure
E=0.2;           %porosity
Re=1;            %Reynold number

vq1=(P*E*Re)/(L^2*Re+S^2*E);
vq2=vq1;
x1=(L*sqrt(Re)*A*S*(tao1-tao0))/(P*A*S-L^2*vq1*A*S-L^2*Re*(tao1-tao0));
y1=(1/(L*sqrt(Re)))*atanh(x1);
vb1=(Re*(tao1-tao0)/(A*S))+vq1;

V=(vb1-(P/L^2))*(cosh(L*sqrt(Re)*y1))+((sqrt(Re)*(tao1-tao0)/L))*sinh(L*sqrt(Re)*y1)+(P/L^2);

x2=sqrt(A^2*S^2*(P-(L^2*vq2))^2-((A^2*S^2)-(L^2*Re))*(P-(L^2*V))^2);
y2=1-((1/(L*sqrt(Re)))*log((A*S*(P-L^2*vq2)+x2)/((P-L^2*V)*(A*S+L*sqrt(Re)))));
vb2=(L^2*sqrt(Re)*V+L*A*S*vq2*sinh(L*sqrt(Re)*(1-y2))+sqrt(Re)*P*cosh(L*sqrt(Re)*(1-y2))...
   -(sqrt(Re)*P))/(L*A*S*sinh(L*sqrt(Re)*(1-y2))+L^2*sqrt(Re)*cosh(L*sqrt(Re)*(1-y2)));

for y=0:0.001:1
if (0<=y)&&(y<=y1)
   V=(vb1-P/L^2)*cosh(L*sqrt(Re)*y)+(sqrt(Re)*(tao1-tao0)/L)*sinh(L*sqrt(Re)*y)+P/L^2;
elseif (y1<=y)&&(y<=y2)
```

```
   V=(vb1-P/L^2)*cosh(L*sqrt(Re)*y1)+(sqrt(Re)*(tao1-tao0)/L)*sinh(L*sqrt(Re)*y1)+P/L^2;
else
   V=(1/(L^2*sinh(L*sqrt(Re)*(1-y2))))*((L^2*V-P)*sinh(L*sqrt(Re)*(1-y))...
      +P*sinh(L*sqrt(Re)*(1-y2))+(L^2*vb2-P)*sinh(L*sqrt(Re)*(y-y2)));
end
plot(V,y,'b')
xlabel('V') % x-axis label
ylabel('Y') % y-axis label
hold on
end
```

2) Matlab coding for fractional increase in mass flow rate

```
clear all;
clc

tao1=1;
tao0=0.1;     %change these value for another graph
A=1.45;       %alpha
E=0.2;        %porosity
L=2;          %sigma
P=10;         %pressure
Re=1;         %Reynold number

for S=2:0.01:20
vq1=(P*E*Re)/(L^2*Re+S^2*E);
vq2=vq1;

x1=(L*sqrt(Re)*A*S*(tao1-tao0))/(P*A*S-L^2*vq1*A*S-L^2*Re*(tao1-tao0));
y1=(1/(L*sqrt(Re)))*atanh(x1);
vb1=(Re*(tao1-tao0)/(A*S))+vq1;

V=(vb1-(P/L^2))*(cosh(L*sqrt(Re)*y1))+((sqrt(Re)*(tao1-tao0)/L))*sinh(L*sqrt(Re)*y1)+(P/L^2);

x2=sqrt((A^2*S^2*(P-L^2*vq1)^2)-((A^2*S^2-L^2*Re)*(P-L^2*V)^2));
y2=1-((1/(L*sqrt(Re)))*log((A*S*(P-L^2*vq1)+x2)/((P-L^2*V)*(A*S+L*sqrt(Re)))));
vb2=(L^2*sqrt(Re)*V+L*A*S*vq1*sinh(L*sqrt(Re)*(1-y2))...
  +sqrt(Re)*P*cosh(L*sqrt(Re)*(1-y2))-(sqrt(Re)*P))/(L*A*S*sinh(L*sqrt(Re)*(1-y2))...
  +(L^2*sqrt(Re)*cosh(L*sqrt(Re)*(1-y2))));

y3=(1/(L*sqrt(Re)))*atanh(L*sqrt(Re)*(tao1-tao0)/P);
```

```
V1=(P/L^2)*(1-cosh(L*sqrt(Re)*y3))+(sqrt(Re)*(tao1-tao0)/L)*sinh(L^sqrt(Re)*y3);

y4=1-(1/(L*sqrt(Re)))*log((P+sqrt(P^2-(P-L^2*V1)^2))/(P-L^2*V1));


G=(vb1-P/L^2)*(sinh(L^sqrt(Re)*y1)/(L*sqrt(Re))+(y2-y1)*cosh(L*sqrt(Re)*y1))...
   +((tao1-tao0)/L^2)*(L*sqrt(Re)*(y2-y1)*sinh(L*sqrt(Re)*y1)+cosh(L*sqrt(Re)*y1)-1)...
   +((L^2*V+L^2*vb2-2*P)*(cosh(L*sqrt(Re)*(1-y2))-1))/(L^3*sqrt(Re)*sinh(L*sqrt(Re)*(1-
y2)))+P/L^2;

Gc=(P/L^2)*(1-(sinh(L*sqrt(Re)*y3)/(L*sqrt(Re)))-(y4-y3)*cosh(L*sqrt(Re)*y3))...
   +((tao1-tao0)/L^2)*((L*sqrt(Re)*(y4-y3)*sinh(L*sqrt(Re)*y3))+cosh(L*sqrt(Re)*y3)-1)...
   +((L*sqrt(Re)*(tao1-tao0)*sinh(L*sqrt(Re)*y3)...
   -P*cosh(L*sqrt(Re)*y3)-P)*(cosh(L*sqrt(Re)*(1-y4))-1))/(L^3*sqrt(Re)*sinh(L*sqrt(Re)*(1-
y4)));

phi=(G-Gc)/Gc;
plot(S,phi,'b')
set (gca,'FontName','Symbol')
xlabel('s') % x-axis label
ylabel('F') % y-axis label
hold on

end
```