Slot Flow Approximation for Herschel-Bulkley Drilling Fluid through Eccentric Annuli

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16832

Dissertation submitted in partial fulfillment of

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CERTIFICATION OF APPROVAL

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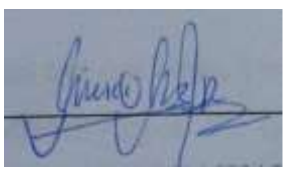
CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.



DWIN KURNIAWAN BIN JAFAR

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ABSTRACT

The studies of fluid flow through eccentric annulus has been started since early 1940's and numbers of solution in order to enhance the efficiency of the cutting transport to the surface have been developed. The solution developed so far by many investigators is either analytical or numerical and mostly involving parameters such as velocity distribution, shear stress, shear rates and pressure drop.

The aim of this study is to develop and analyze the numerical solution of fluid flow through eccentric annuli which is represented by slot of variable height with the use of Herschel-Bulkley drilling fluid. Developed numerical solution is then used to estimate the Velocity distribution in the eccentric annuli as well as to determine flow rate for a given drilling condition. In addition, the rheology of non-Newtonian fluid (Bingham, Power and Herschel-Bulkley) will be discussed.

By applying the correct assumption of eccentricity ratio (k=0.3) on the numerical solution developed earlier, velocity distribution were obtained. Apart from that, by representing the eccentric annuli as slot of variable height the accuracy of the solution obtained can be improved and no iterative computation needed.

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NOMENCLEATURE

r_o = hole radius (m)

r_i = pipe outer radius (m)

τ= Shear stress (Pa)

τ_y= Yield Stress (Pa)

n= Flow Behavior index (dimensionless)

K= Fluid Consistency index (Pa s^n)

c= radial clearance (m)

k=ratio of radius pipe to hole (dimensionless)

Δ= Pressure drop per unit length (Pa/m)

Q= fluid flow rate (m^3/s)

V= Fluid velocity (m/s)

ε= pipe to hole eccentricity (dimensionless)

h= slot height (m)

h_a= slot height from hole surface to h_a (m)

h_b= slot height from h_a to h_b

e=inner-pipe offset relative to hole center (m)

Θ= angular position (°)

L=length of annulus (m)

γ=Shear rate (1/s)

F=force acting on fluid

P= Pressure acting on fluid (Pa)

ξ=half of slot height (m)

$m = 1/n$ (dimensionless)

V_p=Plug velocity (m/s)

CHAPTER 1

INTRODUCTION

1.1 Background

In oil-well drilling operation, engineers often-encounter various problems related to fluid flow through the annular space. The major problem arises when the world start implementing the idea of directional drilling in order to obtain the hydrocarbon from the subsurface due to the restriction faced such as salt dome (geographical). In this case a great effort is required in order to achieve the target set earlier because of the condition of the annular space which is eccentric.

Apart from that, the selection of the drilling fluid that will be use during drilling also play an important role -i.e., the effectiveness for bottomhole cleaning- in the success of the operation. The non-Newtonian fluid that often used are the one with two parameter because of their simplicity such as Bingham plastic model (Bingham,1922) and Power law model (Govier and Aziz; Bourgoyne et al.,1991).

In this study, the difference in the rheology for non-Newtonian fluid (Bingham Plastic, Power and Herschel-Bulkley is presented as well as how the numerical solutions for Herschel-Bulkey drilling fluid flow in the eccentric annuli is obtained when it is treated as a slot.

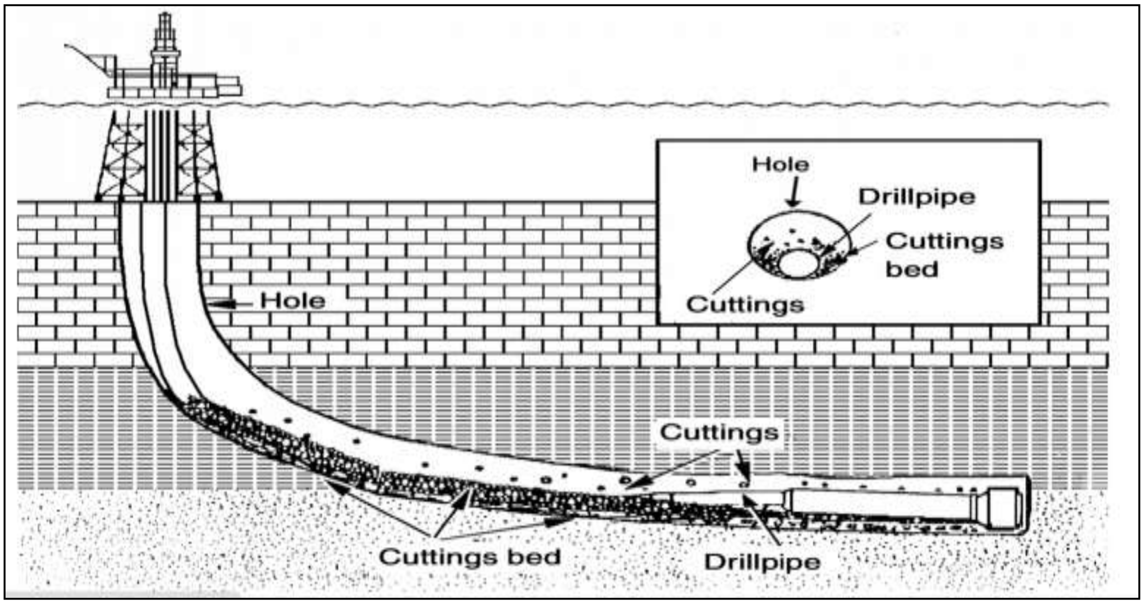


Figure1.0: Directional well and eccentric annulus

1.2 Problem Statement

In drilling operation, non-Newtonian fluid model such as Bingham plastic and Power law are the one that usually being used either for vertical wells or directional wells. However, the exploration of oil and gas nowadays become very difficult due to many factor such as uncertainty in geographical aspect and this matter urge investigators to step up and come with the best solution to solve this matter. Therefore, the study on more complex fluid model especially for case of directional drilling might provide a promising solution to this matter.

1.3 Objective & Scope of Study

The main objective of this project is to identify the numerical solution for Herschel-Bulkley drilling fluids through eccentric annuli by using slot flow solution in order to achieve better bottomhole cleaning during drilling operation.

This study will be focusing on:

- Discussion about the rheology of non-Newtonian fluid (Bingham Plastic, Power and Herschel-Bulkley).
- Obtaining the numerical solution (velocity and flow rate) using the slot flow slot flow solution through the eccentric annuli when Herschel-Bulkley drilling fluid is used in drilling operation.
- Selection of drilling fluid for eccentric case

CHAPTER 2

LITERATURE REVIEW

Numerous studies have been performed on fluid flow through annular space for many decades. Thus, it is necessary to understand the studies related to fluid flow in annular space especially the one with highly deviated well in directional drilling in order to get the overall idea before further studies can be start. In this chapter, review of the studies made by previous researcher, the rheology of non-Newtonian fluid and effect of pipe/hole eccentricity on cutting transportation will be discussed in details.

2.1 Review of Previous Studies

Tao and Donovan (1955) presented the narrow annuli as slot of variable height based on theoretical and experimental study on laminar and turbulent flow. From this study, solution for velocity profile is developed in analytical manner and showed that high flow velocity in the annulus can be achieved when the inner pipe is rotating.

Actually, in the early time the study about Newtonian fluid represented as a hydrodynamic is done by a group of mathematician. Heyda (1959) used Green's Function and bipolar coordinate system to carried out an investigation for the determination of velocity distribution through eccentric annulus.

Redberger and Charles (1962) analyze the velocity profile for non-Newtonian fluid by using Heyda's approach. The studies attempted to solved numerically the second order differential for point velocities.

On the other hand, Vaughn (1965) presented Power law fluid model studies by applying it to Tao and Donovan studies. The studied showed that concentric annulus can be represented by equivalent rectangular slot with a condition of k-ratio (inner-pipe/outer-pipe ratio) is at least 0.3. In the same year also, W.Snyder and G.A Goldstein investigated the flow of fully developed laminar for the eccentric annuli. At the end of the studied, these investigators able to presented the exact solution for velocity distribution.

The studies then continued by Mitsuishi and Aoyagi(1973) for the flow of non-Newtonian fluid through eccentric annuli which focused on velocity profile. The result of the studies shows that as the eccentricity increase, the pressure drop in the eccentric annuli will decrease.

Iyoho and Azar (1981) modified the works from the previous investigators to calculate the slot height in eccentric annuli. Based on the studies made by these investigators, the solution developed is free of iterative finite difference and the result shows that the velocity decreases drastically in the reduced region in the eccentric annuli.

2.2 Non-Newtonian Fluid Model

Non-Newtonian fluid is defined as any fluid that deviates from Newton's law of viscosity whereas rheological model describe the relationship between shear stress and shear rates. Adam T. Bourgoyne et al. (1986) believe that this fluid model is too complex to be describe as a single value of viscosity. Due to that reason, a few fluid model that describe the flow behavior of non-Newtonian fluid has been developed. In the following pages, the literature review related to two and three parameter non-Newtonian fluid (Bingham Plastic, Power law and Herschel-Bulkley) model is discussed.

2.2.1 Bingham Plastic Fluid Model

This model is categorized under two parameter fluid model. Plastic viscosity (PV) and Yield point (YP) are the two parameter used in this model. Since Bingham plastic fluid is non-Newtonian fluid, it will need some amount of force in order to initiate the flow which also means that the shear stress (τ_y) applied on this fluid must exceed the minimum amount of shear rate (γ) in order for the fluid to start flowing. The following is the general equation for Bingham Plastic Fluid Model:

$$\tau = \mu_p\gamma + \tau_y \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(1)}$$

where τ, τ_y are the shear stress and yield stress (yield point) respectively while μ_p, γ are plastic viscosity and shear rate. The shear stress ratio versus the shear rates ratio for Bingham Plastic fluid model is linear (Figure 2.0). The slope of the shear stress versus the shear rates is known as plastic viscosity (PV) whereas the Yield Point (YP) is the y-intercept. Yield point can be defined as the threshold stress which means the point at which the fluid will start to flow.

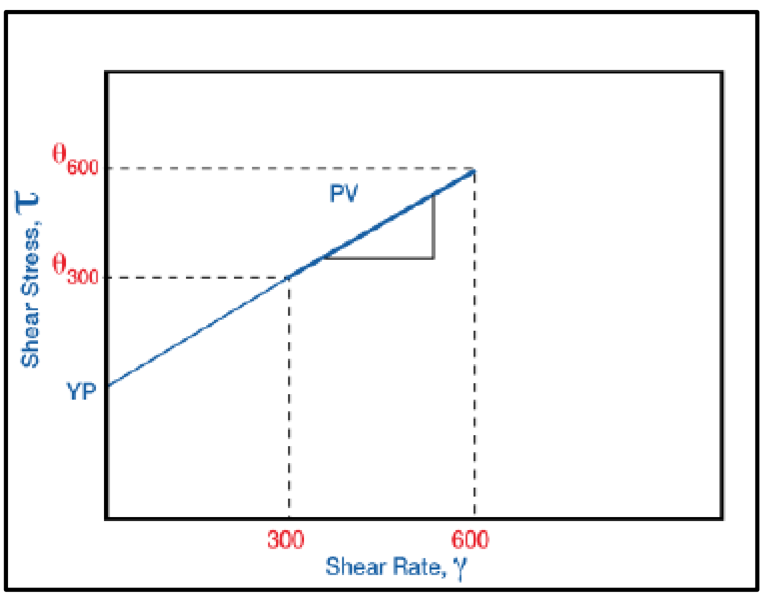


Figure 2.0: Graphical representation of Bingham plastic Fluid Model

2.2.2 Power Law Fluid Model

Like Bingham Plastic, Power Law fluid model also categorized under two parameter fluid model. The general equation for Power Law fluid model is as follows:

$$\tau = K\gamma^{n} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots .. (2)$$

where τ , γ , K , n are shear stress, shear rates, fluid consistency index and power law index. As shown in figure 2.1, this fluid model can be divided into three types and it's all depending on the Power-Law index (n) in which when n<1 it can represent Pseudo-plastic fluid (shear thinning), when n=1 it can represent Newtonian fluid and when n>1 it will represent dilatant fluid (shear Thickening). The value of Power law index will never be zero. Actually this model is developed to in order solved the problem of Bingham Plastic Fluid model at low value of shear rates.

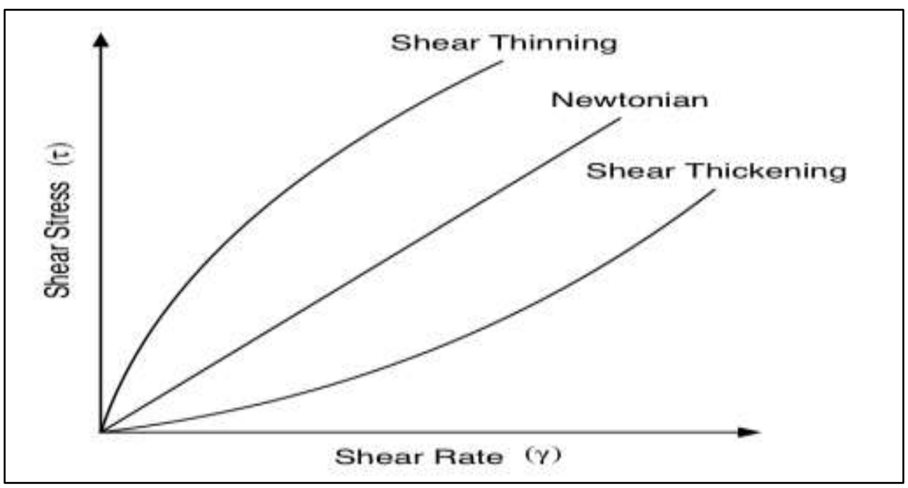


Figure 2.1: Graphical representation of Power Law Fluid Model (Adopted from https://neutrium.net/fluid_flow/viscosity/)

2.2.3 Herschel-Bulkley Fluid Model

This fluid model is the modification of Bingham Plastic and Power law fluid model and it also known as Yield-Power law fluid model. Unlike the first two fluid models, Herschel-Bulkley is categorized as three parameter fluid model. Kelessidis et al. (2006) believe that the more complex the fluid model become, the more accurate it able to predict the behavior of drilling fluids.

The Herschel-Bulkley model is represented by

$$\tau = \tau_y + K\gamma^n \quad ,\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

where τ, $\tau_{y,}$ K, γ and n are shear stress, yield stress, fluid consistency, shear rates and fluid behavior index. The condition for this equation to work well is that K>0, $\tau_y > 0$ and $0 < n < \infty$ must be imposed in order to avoid meaningless result. Nguyen and Boger (1987) agreed that the complexity of the derivation is the reason why three parameter fluid models rarely used.

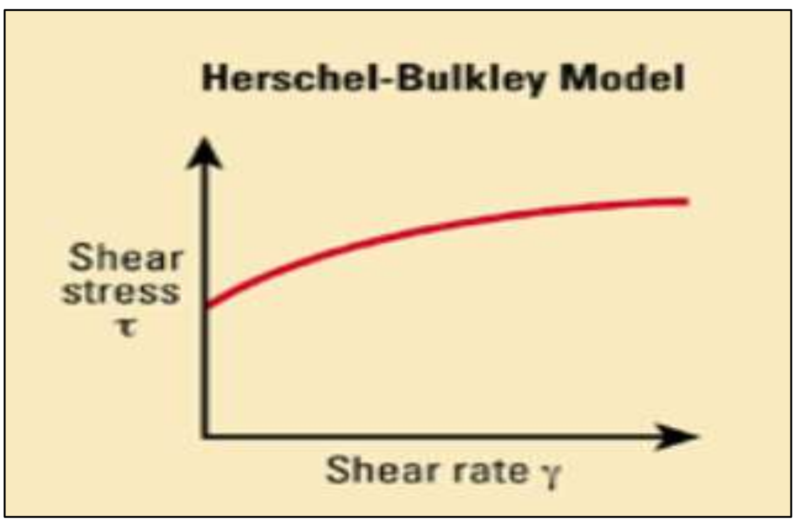


Figure 2.2: Graphical representation of Herschel-Bulkley Fluid Model (Modified from http://www.drillingformulas.com/)

2.3 Effect of Pipe/hole eccentricity on cutting transport

It is necessary for drill cuttings to be transported as fast as possible from the bottomhole to the surface in order to avoid any major problem during drilling operation. Especially in directional or deviated wells, cutting transportation is a major problem in oil and gas industry. There are many factor affecting cutting transport in directional well which are i) Effect of mud rheology ,ii) Effect of mud weight, iii) Effect of pipe/hole eccentricity, iv) Cutting size ,v) Rate of Penetration and vi) wellbore inclination angle.

Numerous studies has been made related to factor affecting cuttings transport which mentioned one of the factor is pipe/hole eccentricity. Eccentric is defined as out of center. This is a case when pipe is not precisely in the center of the wellbore. Iyoho and Azar (1981) mentioned that drillpipe tends to rest on the lower side of the hole in directional wells. This is due to gravitational effect.

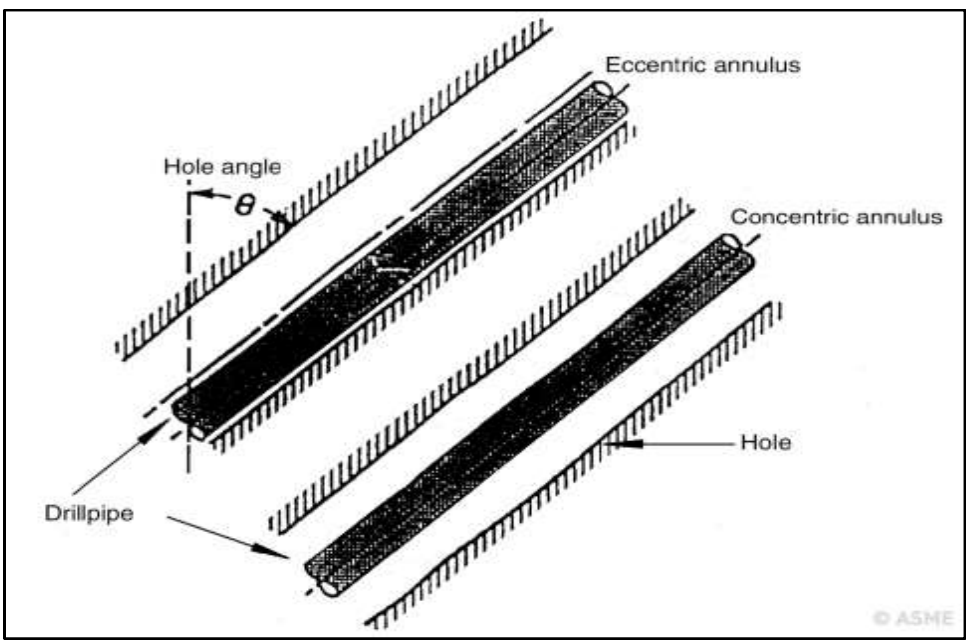


Figure 2.3: Eccentric and Concentric annulus for directional wells (Adopted from http://petrowiki.org/Hole_cleaning)

Requirement for hydraulic of hole cleaning decrease as the pipe moves towards the upper side of the wellbore (from positive eccentricity to negative eccentricity). Eccentricity will become more dominant with the using of viscous drilling fluids. The formulation of eccentricity shown below:

$$\varepsilon = 2e/h \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(4)}$$

where ε, e, h are the pipe/hole eccentricity, inner pipe offset relative to the hole center abd the slot height or local annular clearance. Unlike other parameters, ε is in fact dimensionless since it is the form ratio. For the case of concentric annulus, e=0 thus $\varepsilon=0$ while in fully concentric annulus $e=r_o-r_i$ and thus $\varepsilon=1$. Below is the figure showing concentric and eccentric annulus.

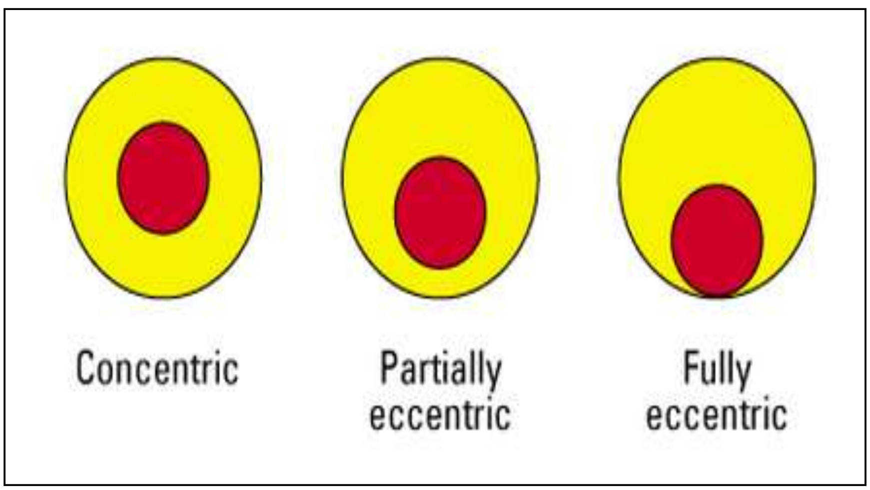


Figure 2.4: Concentric and eccentric pipe/hole orientation (Adopted from www.glossary.oilfield.slb.com

2.4 Theory

2.4.1 Representation of Annulus as a Slot

According to Iyoho and Azar (1981), eccentric annulus can be represented by a nonrectangular slot. The main reason behind this is that the equation of the slot flow is way more simpler and easier to be manipulated compared to the conventional annular-flow solution and the result generated also reasonably accurate as long as the ratio between the inner-pipe diameter to the outer-pipe diameter is greater than 0.3.

Below is the mathematical representation for the eccentricity of the annulus based on figure 7.0.

$$\varepsilon = 2e/h, \quad (4)$$

$$e = \varepsilon * c, \quad (5)$$

$$c = r_o - r_i, \quad (6)$$

where ε, e, h, c, r_o and r_i are pipe eccentricity, offset for inner pipe with respect to hole center, slot height, radial clearance , inner-pipe radius and outer pipe radius. To obtain a slot shape, it is assumed that the outer pipe or hole together with the inner pipe is cut symmetrically with respect to the origin. Then, the result is bend horizontally. For eccentric case, the slot will not have a constant h throughout the slot due to inner pipe position which is out of center.

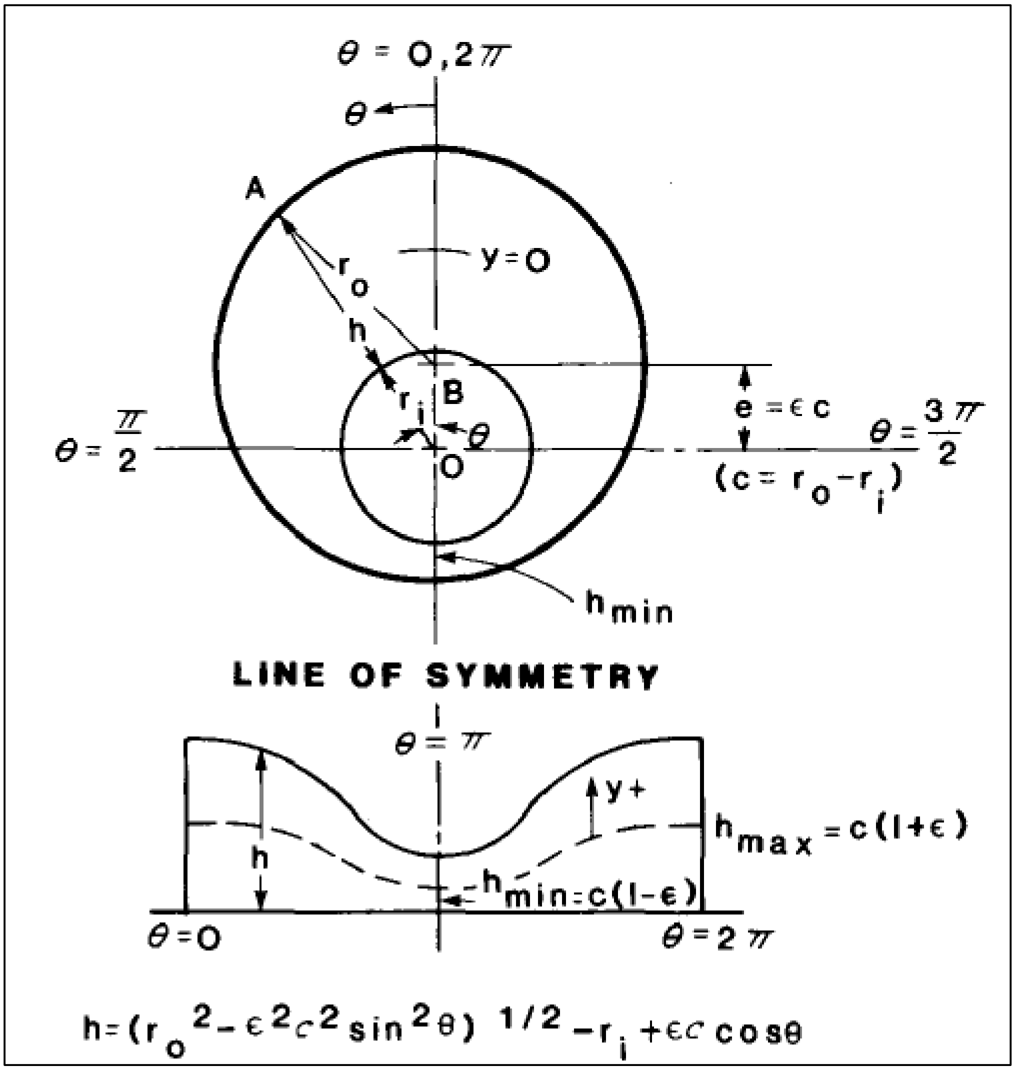


Figure 2.5: Eccentric Annulus and non-rectangular slot

For this study, the focus will be towards the slot flow approximation in the eccentric annuli with the use of Herschel-Bulkley drilling fluid in order to improve the cleaning process of the drill cutting from the bottomhole. The target parameters to be studied are velocity, flow rate and pressure drop.

CHAPTER 3

METHODOLOGY

To facilitate this project, a few methodologies strategies are adopted to ensure adequate information are obtained and thus help in achieving the project objective.

Literature Review

To understand the needs of the project, review on the literature related to problem under study from various sources is necessary. This part is crucial in order to ensure the objective which is set earlier can be achieved. The main reference to this literature study is Society of Petroleum Journal (SPEJ) and American Society of Mechanical Engineer Journal (ASME Journal).

Derivation of related equation

The work from the previous investigators being studied as a reference including the derivation of the equation that been done. Modified equations then produce to extend the study.

Create Mathematical Model (Matlab)

Based on the modified equation that being produce earlier, a mathematical program is created using MATLAB software to generate the analysis for parameter involved by using the field data from previous investigators.

Analysis

The result generated from Matlab is studied and comparison made with respect to previous study

3.1 Tools and Equipment Required

i. Software
- Microsoft Office
- MATLAB
- Endnote

ii. Hardware
- Personal Laptop

3.2 Gantt Chart

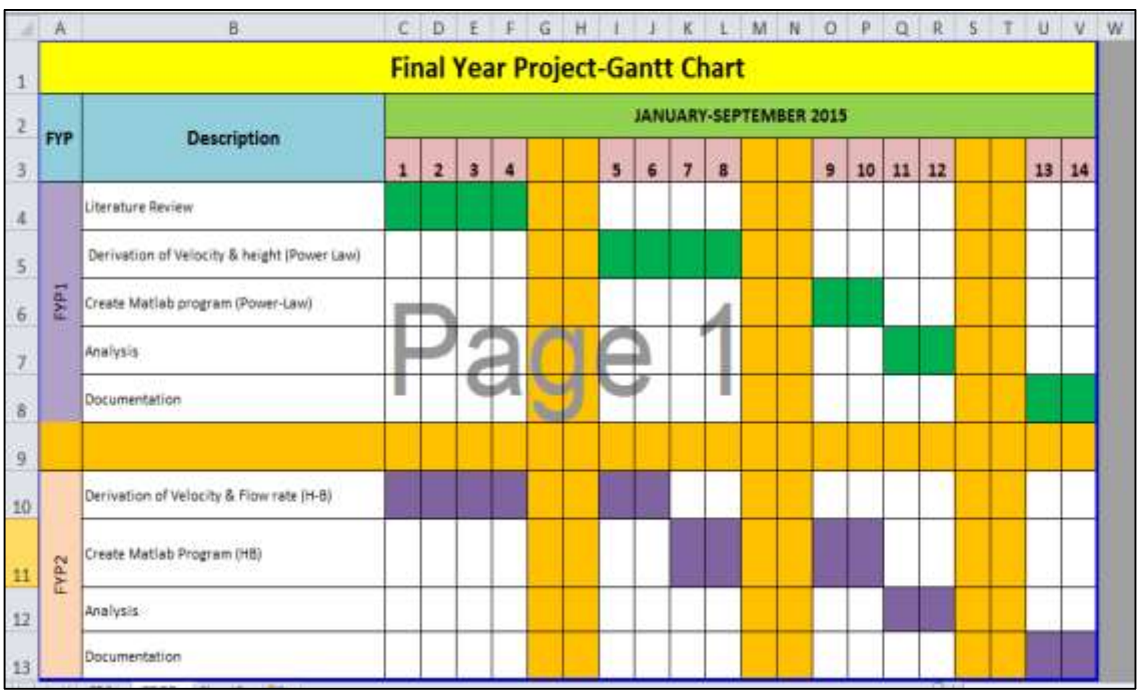


Table 3.0: Project Timeline

3.3 Key Project Milestone

Below are some of the key events that will take place during Final Year Project I and II.

JAN 2015	• Finalize project title • Research Literature • Identify Problem statement and Objective
FEB 2015	• Derivation of slot flow differential equation • Completion and submission of extended proposal report
MAC 2015	• Derivation of variable slot height equation • Proposal defense presentation • Start programming mathematical code using Matlab
APRIL 2015	• Validate result of Dimensionless Velocity • Result analysis • Completion and submission of Interim Report
MAY 2015	• Derive Bingham-plastic slot flow approximation for concentric annuli • Derive Power-law slot flow for concentric annuli
JUNE 2015	• Derive the Herschel-Bulkley slot flow approximation for concentric annuli
JULY 2015	• Kick-off mathematical programming using Matlab for Herschel-Bulkley slot-flow approximation through eccentric annuli • Completion of Progress report
AUG 2015	• Result analysis for point velocity and flow rate • Completion of Dissertation & Technical Paper

CHAPTER 4

DEVELOPMENT OF EQUATIONS

After finish with literature works related to topic of fluid flow through annular spaces, the idea on how to develop the slot flow equation for Herschel-Bulkley drilling fluid through eccentric annuli become clearer. There are two methods that can be used to developed the slot flow equation through eccentric annuli for Herschel-Bulkley drilling fluid which is by following the Bingham plastic method given by Bourgoyne et al.(1991) or following the Power-Law model given by Iyoho et al. (1981).In this study the development of equation using both method will be shown but only one equation which follows Bingham Plastic will be used to generate the result.

The assumption made in developing the equation is as follows:

- ✓ Single phase, incompressible fluid flow
- ✓ Constant Temperature
- ✓ No rotation for the inner pipe
- ✓ Slot solution is assumed
- ✓ No slip and gravitational effect

4.1 Development of equation for variable slot height (h) in eccentric annuli

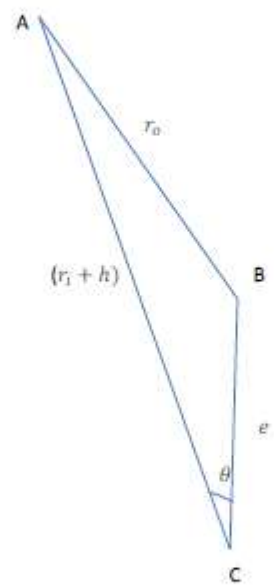
According to Vaughn (1965), the slot height (h) expression is represented by h= c + e cos θ where e = offset between the center of the inner and outer circle, c = concentric radial clearance and θ = eccentric angle. However this expression cannot be applied in real drilling operation due to the serious error that it can created to the velocity distribution value and thus a new expression need to be created to ease the work.

Based on figure 6:

$$C= r_o - r_i \quad (7)$$

$$\epsilon = \frac{e}{c} \quad (8)$$

$$e = \epsilon c \quad (9)$$



Apply cosine rule on triangle AOB above:

Thus:

$$ro = (h + ri)^2 + e^2 - 2(\mathrm{h} + \mathrm{ri})\mathrm{ecos}\,\theta \quad (10)$$

Expand and then re-arrange equation (4) into Quadratic form of equation:

$$h^2 + 2h(ri - e\cos\theta) + (e^2 - ro + ri^2 - 2er\cos\theta = 0$$

Use quadratic solution: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$,

Where a=1

b= 2(ri - ecosΘ)

c= e^2 - ro + ri^2 - 2ercosΘ

Solve for h and simplify:

$$h = -\left[\frac{2(ri - e\cos\theta) \pm \left[\left(4(ri - ecos\theta)^2 - 4(e^2 - ro^2 + ri^2 - 2ericos\theta)\right)\right]^{\frac{1}{2}}}{2}\right]$$

$$h = \sqrt{ro^2 - e^2sin^2\theta} - ri + ecos\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(11)}$$

Substitute equation (3) into (5) yield:

$$h = \sqrt{ro^2 - \epsilon^2c^2sin^2\theta} - ri + \epsilon ccos\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(12)}$$

The new slot height expression (12) can be use or substituted in slot flow equation of eccentric annuli for any type of drilling fluid.

4.2 Formulation of slot flow equation through eccentric annuli

The first formulation (4.2.1) will follow the Bingham Plastic method while the second formulation (4.2.2) will follow the Power Law method .The parameter investigated in this study is velocity and flow rate.

4.2.1 Herschel-Bulkley Drilling Fluid Model and Concentric Annuli

The Hershel-Bulkley fluid model is represented by Equation.13:

$$\tau = \tau_y + K\gamma^n \quad \text{...} \quad (13)$$

Where: $\gamma = \frac{\partial V}{\partial y}$

At the inner layer of the plug, h_a:

$$\tau_a = -\tau_y \quad \text{...} \quad (14)$$

The force balance performed on the Fluid element in this region gives;

$$-\tau_y = C_1 + h_a\left(\frac{\partial P_f}{\partial L}\right) \quad \text{..} \quad (15)$$

Where $\frac{\partial P_f}{\partial L} = \Delta \; and \; \tau_o = C_1 = integrating \; constant;$

$$-\tau_y = C_1 + h_a(\Delta) \quad \text{....................................} \quad (16)$$

Equate equation (15) and (16):

$$\tau_a = \tau_y = C_1 + h_a\Delta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(17)}$$

At the outer layer of the plug, h_b;

$$\tau_b = \tau_y \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(18)}$$

The force balance performed on fluid element in this region gives;

$$\tau_y = C_1 + h_b(\Delta) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(19)}$$

Equate equation (18) and (19) to yield:

$$\tau_b = \tau_y = C_1 + h_b\Delta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(20)}$$

Equate (13) and (17);

$$-\tau_y - K\left(\frac{\partial V}{\partial y}\right)^n = C_1 + h_a\Delta \ldots\ldots\ldots\ldots\ldots\ldots \text{(21)}$$

Separate and integrate (21) with respect to y will yield:

$$\int \partial V = \int \left[-\frac{\tau_y + C_1}{K}\right]^{\frac{1}{n}} \partial y + \int \left(-\frac{y\Delta}{K}\right)^{\frac{1}{n}} \partial y \ldots\ldots\ldots\ldots \text{(22)}$$

Let $\frac{1}{n} = m$;

Solve the integration for (22);

$$V = -\frac{K}{\Delta(m+1)}\left[\left(-\frac{y}{K}\Delta\right)^{m+1} + \left(-\frac{\tau_y + C_1}{K}\right)^{m+1}\right] + V_o \ldots\ldots\ldots \text{(23)}$$

Apply boundary condition of V=0 and y=0 on (23) will give:

$$V_o = \frac{K}{\Delta(m+1)}\left(-\frac{\tau_y - C_1}{K}\right)^{m+1}$$

Therefore velocity of the inner plug regions is:

$$V = -\frac{K}{\Delta(m+1)}\left[-\left(-\frac{\tau_y + C_1}{K}\right)^{m+1} + \left(-\frac{\tau_y + C_1}{K} - \frac{y\Delta}{K}\right)^{m+1}\right] \; ; \qquad 0 \le y \le h_a$$

Expressed the above equation in term of h_a,

$$V = \frac{1}{(m+1)} \times \left(\frac{\Delta}{k}\right)^{m} \times \lceil -(h_a)^{m+1} + (h_a - y)^{m+1} \rceil \quad ; \qquad 0 \le y \le h_a \ldots\ldots (24)$$

The Plug velocity, V_p occur at y= h_a, thus (24) become;

$$V_p = \frac{1}{(m+1)} \times \left(\frac{\Delta}{K}\right)^{m} \times (h_a)^{m+1} \; ; \qquad h_a \le y \le h_b \qquad \ldots\ldots\ldots\ldots\ldots\ldots (25)$$

The same procedure is done for the upper layer of the plug and yields:

$$V = \frac{1}{\Delta K^m (m+1)}\left[\left(\tau_o - \tau_y + h\Delta\right)^{m+1} - \left(C_1 - \tau_y + y\Delta\right)^{m+1}\right]; \quad h_b \le y \le h \ldots\ldots (26)$$

Expressing (26) in term of h_b :

$$V = \left(\frac{\Delta}{m}\right)^{m}\left(\frac{1}{m+1}\right)[(h - h_b)^{m+1} - (y - h_b)^{m+1}]; \quad h_b \le y \le h \ldots\ldots\ldots\ldots (27)$$

The plug velocity for will occur at y=h_b ;

$$V_p = \left(\frac{\Delta}{K}\right)^{m}\left(\frac{1}{m+1}\right)[(h - h_b)^{m+1}]; \quad h_a \le y \le h_b \ldots\ldots\ldots\ldots\ldots\ldots (28)$$

Equate (25) and (28) will yield;

$$(h_a)^{m+1} = (h - h_b)^{m+1}$$

By keeping only the positive value and not the power;

$$h_a = h - h_b$$

$$h = h_a - h_b \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(29)}$$

Substitute the equation of h_a and h_b in term of τ into (29) will give:

$$C_1 = -\frac{h}{2}\Delta$$

Therefore;

$$h_a = \frac{h}{2} - \frac{\tau_y}{\Delta} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(30)}$$

$$h_b = \frac{h}{2} + \frac{\tau_y}{\Delta} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(31)}$$

The flow rate is given by:

$$q = W\int_0^h Vdy = W\int_0^{h_a} Vdy + WV_p\int_{h_a}^{h_b} dy + W\int_{h_b}^{h} Vdy$$

Solve the integration for the left hand side first so it will ease the work:

$$q = W\int_0^h Vdy = -\int_0^{h_a} y\frac{dV}{dy}dy - \int_{h_b}^{h}\frac{dV}{dy}dy = -L_1 - L_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(32)}$$

Differentiate the Velocity equation derived previously based on the limit stated in equation above gives:

$$L_1 = \left(\frac{\Delta}{K}\right)^m \left[\frac{(h_a)^{m+2}}{(m+1)(m+2)}\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(33)}$$

$$L_2 = -\left(\frac{\Delta}{K}\right)^m \left[\frac{(h-h_b)^{m+2}}{(m+2)} + \frac{h_b(h-h_b)^{m+1}}{(m+1)}\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(34)}$$

Substitute (33) and (34) into (32) and expressed the resulting equation in term of τ_y and Δ:

$$q = \left(\frac{\Delta}{K}\right)^{m} \frac{W}{(m+1)(m+2)} [-(h_a)^{m+2} + (h-h_b)^{m+2}(m+1) + h_b(h-h_b)^{m+1}(m+2))]$$

$$q = \left(\frac{\Delta}{K}\right)^{m} \frac{W}{(m+1)(m+2)} \left[-\left(-\frac{\tau_y+\tau_o}{\Delta}\right)^{m+2} + \left(h - \frac{\tau_y-\tau_o}{\Delta}\right)^{m+2}(m+1) + \left(\frac{\tau_y-\tau_o}{\Delta}\right)(h - \frac{\tau_y-\tau_o}{\Delta})^{m+1}(m+2))\right] \ldots\ldots. (35)$$

Express q in term of annulus geometry:

$$q = \left(\frac{1}{(2)^m(2m+4)(m+1)}\right) \times \left(\pi(r_o^2 - r_i^2)(r_o - r_i)^{m+1}\left(\frac{\Delta}{K}\right)^{m}\right) \times \left(1 - \frac{\tau_y}{\left[(r_o-r_i)\times\frac{\Delta}{2}\right]}\right)^{m+1} \times \left[\left(\frac{\tau_y}{(r_o-r_i)\Delta} + (m+1)\right) \times \frac{1}{(2^m(2m+4)(m+1))}\right] \ldots\ldots\ldots.. (36)$$

Equation (24), (25), (27), (30), (31) and (36) will be used during mathematical programming in order to determine parameter V and q.

4.2.2 Development of Slot-Flow approximation for Herschel-Bulkley through eccentric annuli.

This derivation will follow the procedure of the Slot Flow differential equation for Power-Law drilling fluid through eccentric annuli given by Iyoho and Azar (1981).

The general equation for Herschel-Bulkley fluid model is as follows:

$$\tau = \tau_y + K\gamma^n \quad \ldots\ldots (37)$$

Where: $\gamma = \frac{dV}{dy}$

Thus, substitute the above equation into (37) to yield:

$$\tau = \tau_y + K\left(\frac{dV}{dy}\right)^n \quad \ldots\ldots (38)$$

At equilibrium, all forces acting on fluid element in the annulus are:

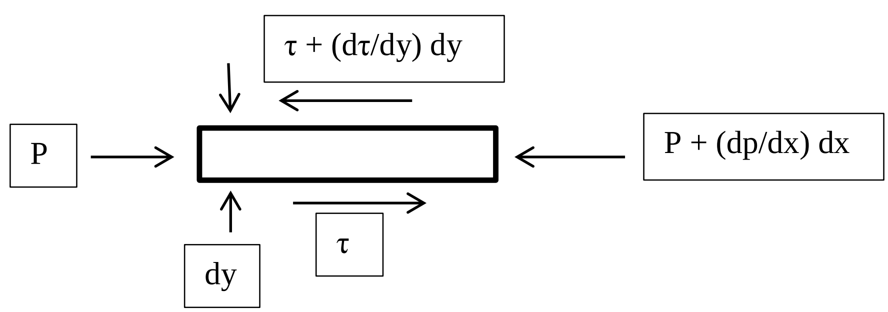


Figure 4.0: Representation of Fluid element and forces acting on it

$$F_{x1} + F_{x2} + F_{y1} + F_{y2} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(39)}$$

Substitute all forces involve into (39) and solved it:

$$\tau dx - \left(\tau + \frac{d\tau}{dy}dy\right)dx + Pdy - \left(P + \frac{dP}{dy}dx\right)dy = 0$$

$$-\frac{dP}{dx}dxdy = \frac{d\tau}{dy}dy\, dx \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(40)}$$

Divide both sides of (40) by dxdy:

$$-\frac{dP}{dx} = \frac{d\tau}{dy}$$

Let dp/dx = ΔP/L, therefore the above equation become:

$$-\frac{\Delta P}{L} = \frac{d\tau}{dy} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \text{(41)}$$

Integrate (41) with respect to dy:

$$\int \frac{d\tau}{dy} = -\int \frac{\Delta P}{L}dy$$

$$\tau = -\frac{\Delta P}{L}y + C' \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots... \text{(42)}$$

τ=0 at the middle of annulus which is when y = ε, thus (42) become:

$$C' = \frac{\Delta P}{L}\xi \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(43)}$$

Substitute (43) into (42) to yield:

$$\tau = \frac{\Delta P}{L}(\xi - y) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (44)$$

Equate (44) with (38) and re-arrange in term of dV/dy:

$$\frac{dV}{dy} = \left[\frac{\Delta P}{KL}(\xi - y) - \frac{\tau_y}{K}\right]^{\frac{1}{n}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (45)$$

For mathematical correctness,

Let V=V' and y=y':

$$\frac{dV'}{dy'} = \left[\frac{\Delta P}{KL}(\xi - y') - \frac{\tau_y}{K}\right]^{\frac{1}{n}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (46)$$

Integrate equation (46):

For y< ξ:

$$\int_0^V dV' = \left(\frac{\Delta P}{KL}\right)^{\frac{1}{n}} \int_0^y (\xi - y')^{\frac{1}{n}} dy' - \left(\frac{\tau_y}{K}\right)^{\frac{1}{n}} \int_0^y dy'$$

Let 1/n=m and ξ=h/2 .Substitute into above equation and solve it with respect to V:

$$V = -\left(\frac{1}{m+1}\right)\left(\frac{\Delta P}{KL}\right)^m \left(\frac{h}{2}\right)^{m+1}\left[\left(1 - \frac{2y}{h}\right)^{m+1} - 1\right] - y\left(\frac{\tau_y}{K}\right)^m \ldots\ldots\ldots\ldots\ldots\ldots (47)$$

For y> ξ;

Do the same procedure as for y< ξ and finally yield:

$$V = -\left(\frac{1}{m+1}\right)\left(\frac{\Delta P}{KL}\right)^{m}\left(\frac{h}{2}\right)^{m+1}\left[\left(\frac{2y}{h} - 1\right)^{m+1} - 1\right] - y\left(\frac{\tau_y}{K}\right)^{m} \ldots\ldots\ldots\ldots.. (48)$$

Translating the origin of the y-coordinates to the middle section of annulus, thus (47) and (48) becomes:

$$V = \left(\frac{1}{m+1}\right)\left(\frac{\Delta}{K}\right)^{m}\left[\left(\frac{h}{2}\right)^{m+1} - |y|^{m+1}\right] - y\left(\frac{\tau_y}{K}\right)^{m} \ldots\ldots\ldots\ldots\ldots\ldots\ldots. (49)$$

CHAPTER 5

RESULT AND DISCUSSION

In this chapter, the result from sensitivity analyses done with respect to base case are explained in detail. Apart from that, the results obtained in this study are compared with previous researcher result.

5.1 Base Case Model and Sensitivity Analysis

There are many factors that can affect the velocity and flow rate through the annulus. The effect of parameters such as Flow behavior index (n), Fluid Consistency index (K), Eccentricity ratio (ε), Yield Stress (τ_y), Pressure drop (Δ), angular position (ϴ) and radius (r) are presented here. The solution for velocity is assumed to have a plug flow pattern. Table 2.0 shows the data use for the Matlab simulation in this study compared to the experimental data from previous studies. Note that, some changes are made especially for the radius where the radius ratio (k) must be at least 0.3 (Iyoho and Azar, 1981; Bourgoyne et al., 1991).

Data	S13(NL)-Kelessidis et al. (2006)	Base Case
inner radius (m)	0.1270	0.1000
Outer radius (m)	0.4830	0.3000
n	0.4352	
ε	0 (concentric)	0.5

τ_y (Pa)	1.7020
K (Pa s^n)	1.2063
Δ (Pa)	44.000
Θ (degree)	0

Table 5.0: Comparison of Data between Kelessidis et al. (2006) and Base Case Model

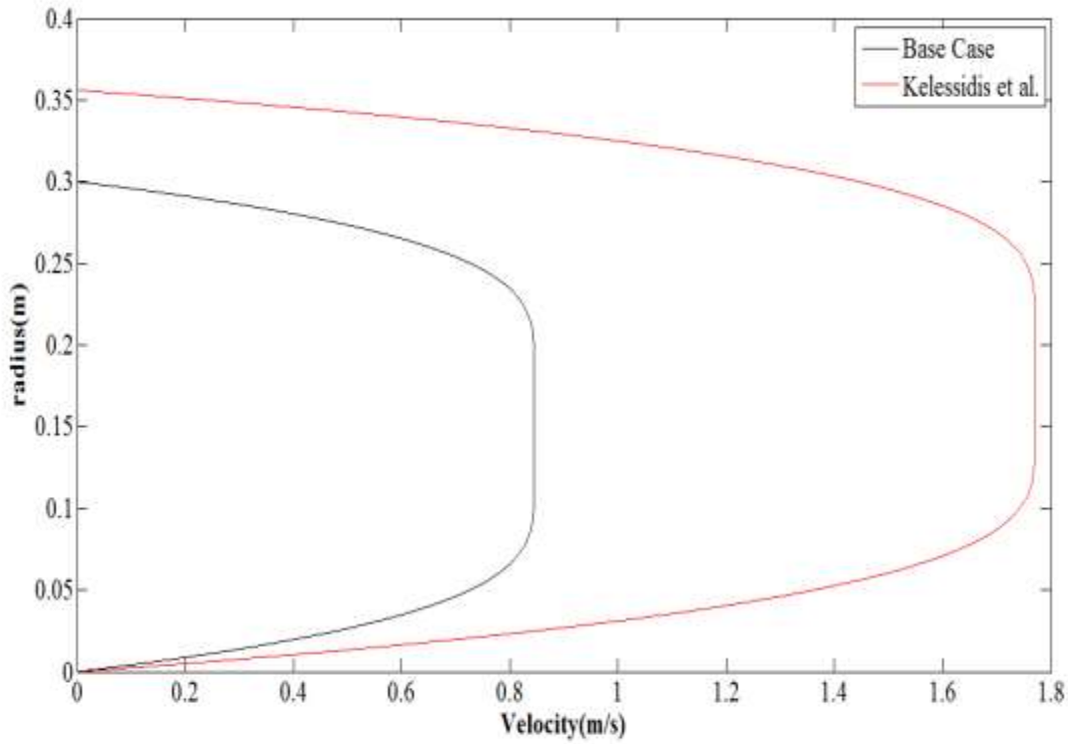


Figure 5.1: Velocity profile comparison between Kelessidis et al (2006) and Base case Model.

5.2 Special case comparison

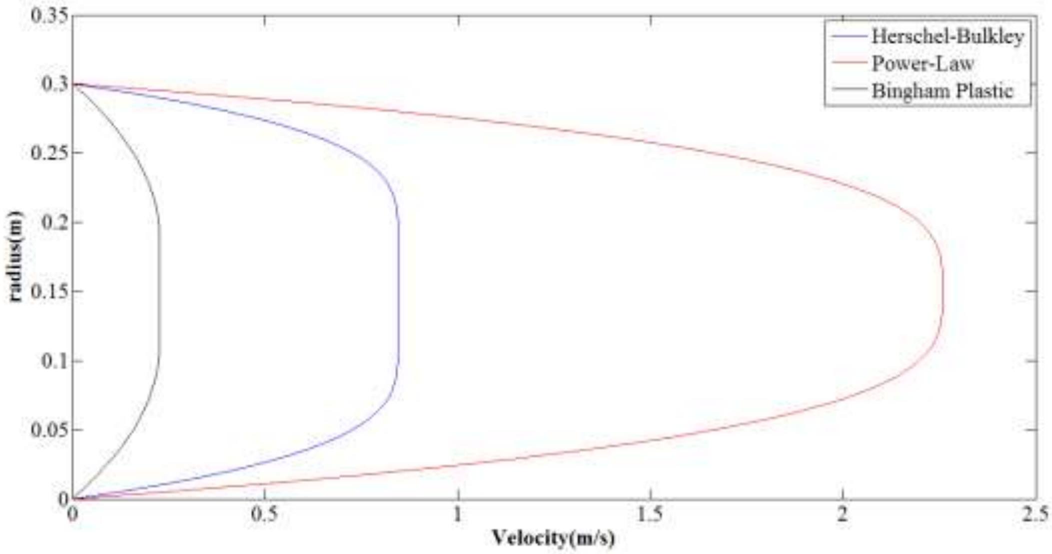


Figure 5.2: Comparison between fluid models.

Based on figure 5.2, the three fluid models were compared among one another. The basis equation used to generate this graph is the Herschel-Bulkley equation in eccentric annuli (refer 4.2.1). The condition to convert Herschel-Bulkley to Bingham plastic is to set the n value equal to one (n=1) while to obtained Power-Law is to set the Yield Stress equal to zero (τ_y= 0 Pa). The result shows that Power-Law fluid flows with greater velocity compare to the other two fluid models in eccentric annuli. However, in real drilling operation that involves eccentric case, it is better to use Herschel-Bulkley as the drilling fluid because of the greater plug profile which indicates the great ability to suspend more drill cuttings. In term of velocity, the requirement to ensure that it is large enough to transport the cutting is the reason to choose Herschel-Bulkley as drilling fluid compared to others even though it flows slower than Power-Law fluid.

5.3 Effect of changing flow behavior index (n) value

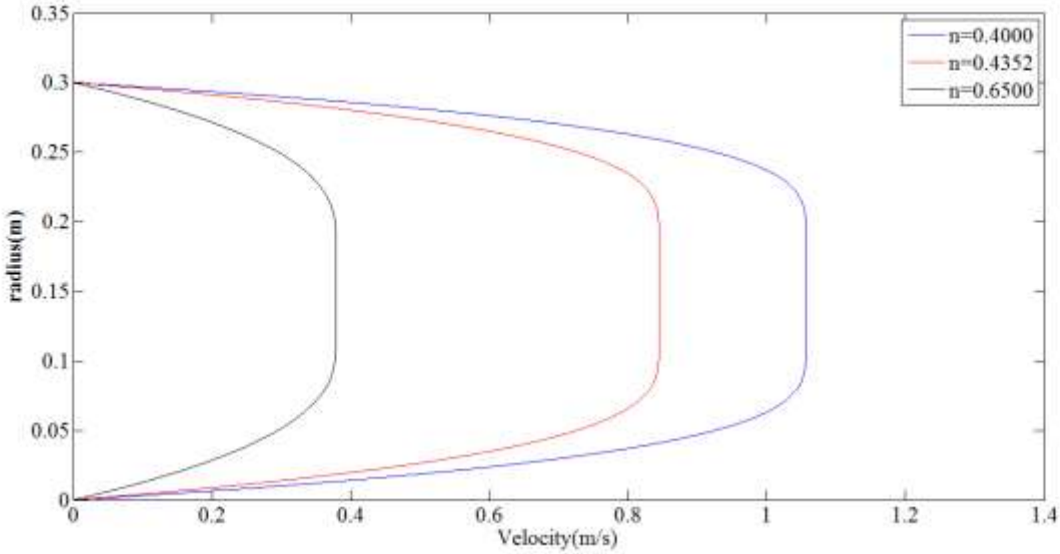


Figure 5.3: Effect of changing flow behavior index (n) on velocity profile of Base Case Model (wide region)

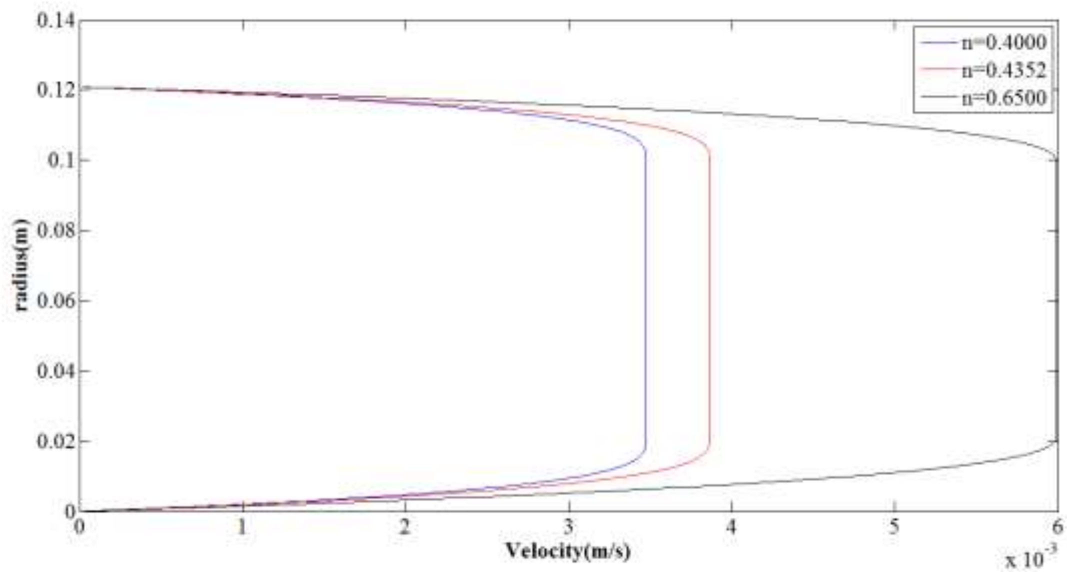


Figure 5.4: Effect of changing flow behavior index (n) on velocity profile of Base Case Model (Narrow region)

Figure 5.3 and 5.4 show the results of velocity profile computation when parameter of flow behavior index (n) is altered. With the use of Herschel-Bulkley drilling fluid as the medium, increasing 'n' value significantly decrease the velocity at the wider region whereas at the narrow region the result show the opposite. High restriction experienced in the narrowed region result in low velocity of the fluid. The existence of plug velocity profile for both positions is expected to increase the cuttings suspension to be transport to surface.

5.4 Effect of changing eccentricity

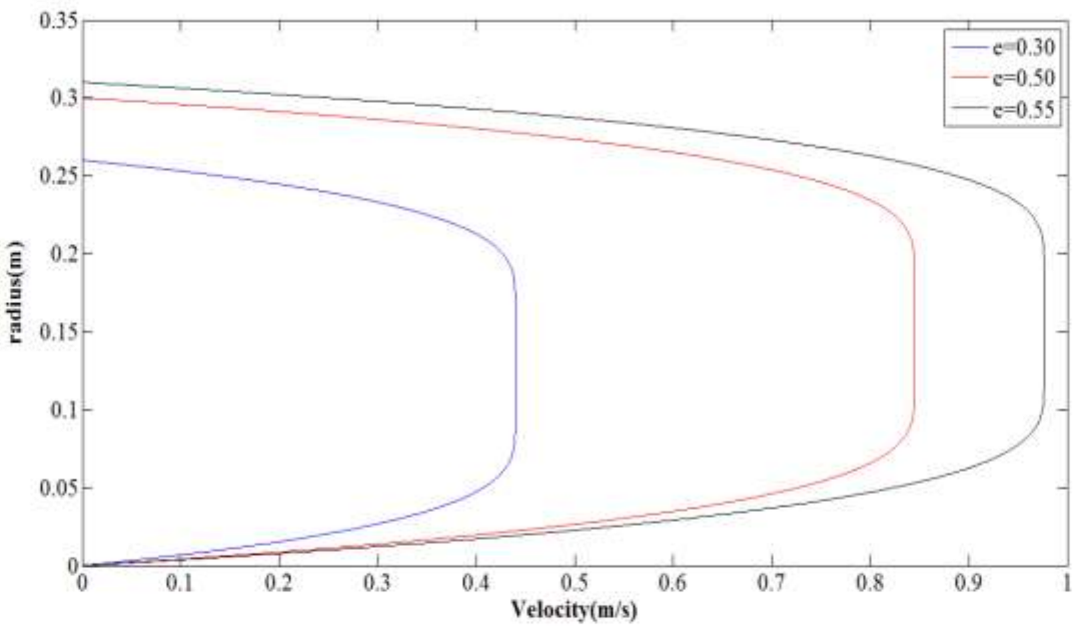


Figure 5.5: Effect of changing eccentricity (ε) on velocity profile of Base Case Model (Wide region)

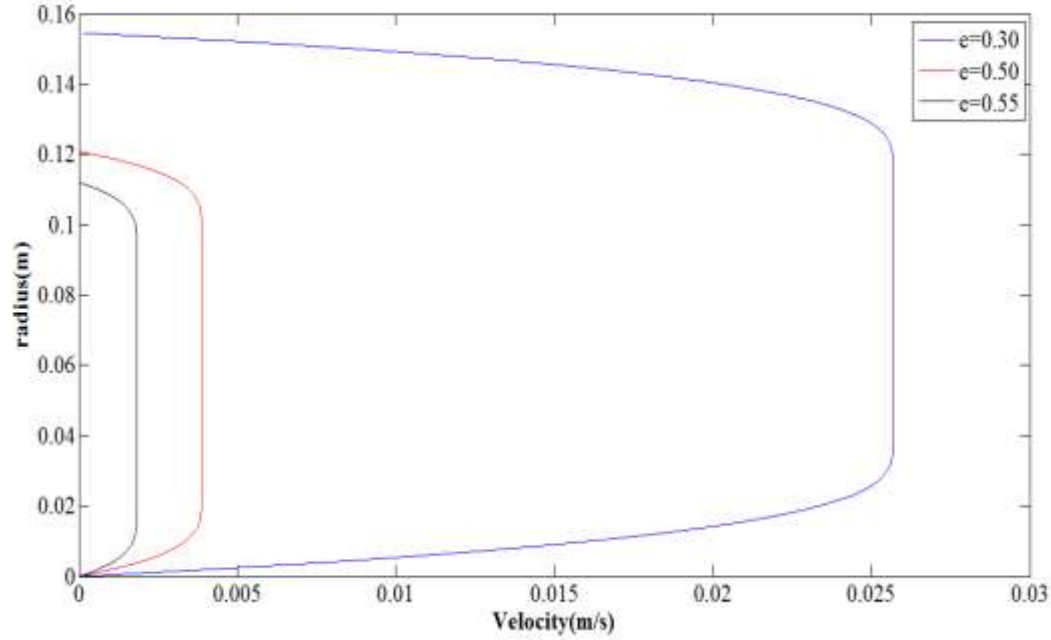


Figure 5.6: Effect of changing eccentricity (ε) on velocity profile of Base Case Model (Narrow region)

As the eccentricity of pipe to hole increases, the trend of the plug velocity profile also increase as shown in figure 5.5. This is true considering that the slot height is reduced (from wider to narrow region). Velocity will be higher at the wider region and the volume of cuttings transported through this region also will be greater than the narrowed. By comparing figure 5.4 and 5.6, it can be concluded that changing eccentricity will gives better velocity since it is the requirement to prevent the formation of cutting bed that can cause problem at the downhole.

5.5 Effect of changing Yield Stress

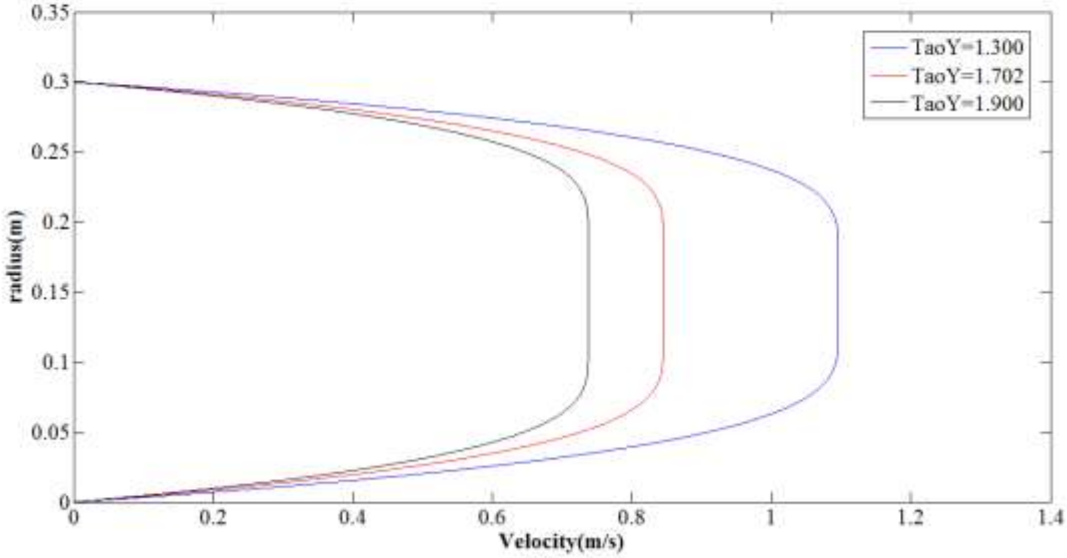


Figure 5.7: Effect of changing Yield Stress (τ_y) on velocity profile of Base Case Model (wide region)

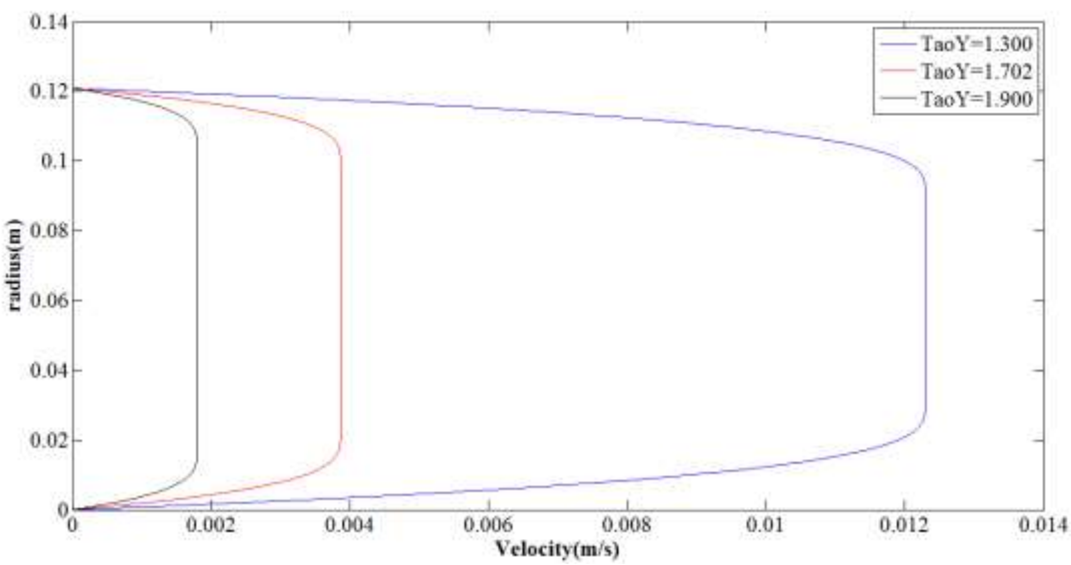


Figure 5.8: Effect of changing Yield Stress (τ_y) on velocity profile of Base Case Model (Narrow region)

The analyses that can be made from figure 5.7 is that as the value of yield stress increases, the velocity of the plug decreases from 1.1 m/s to 0.73 m/s. This is in agreement with the one in narrowed region (Figure 5.8) even though the magnitude of the velocity different. The effect of yield stress to velocity profile in these eccentric annuli can be said is a minor factor that contributes to change in velocity profile if compared to other parameters studied. For Herschel-Bulkley drilling fluid to start flowing it is convenient for the yield stress to have a force value greater than zero.

5.6 Effect of changing fluid consistency index

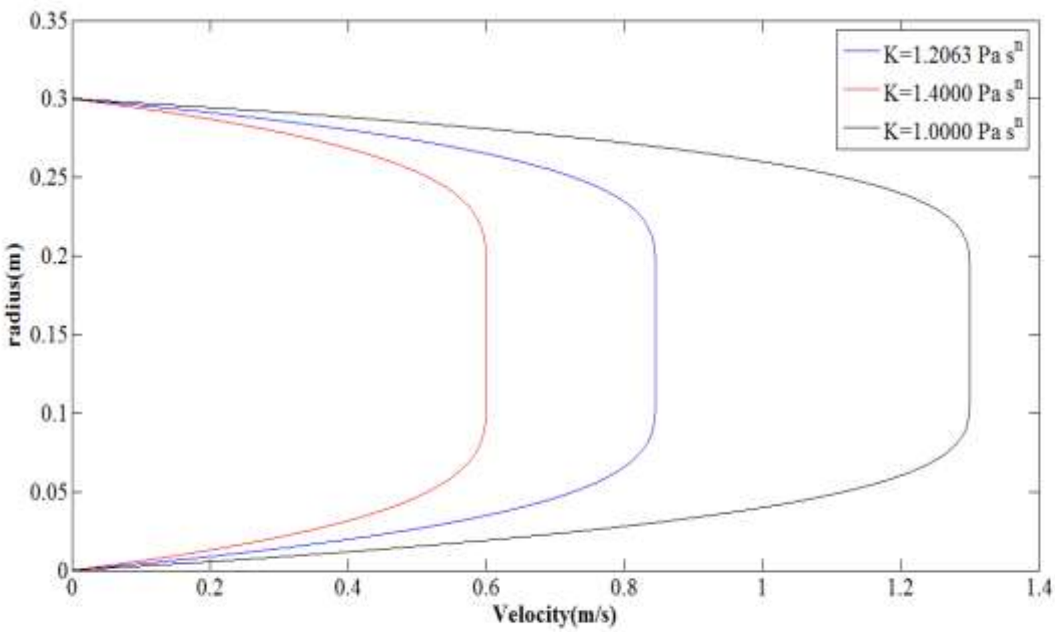


Figure 5.9: Effect of changing fluid consistency index (K) on velocity profile of Base Case Model (wide region)

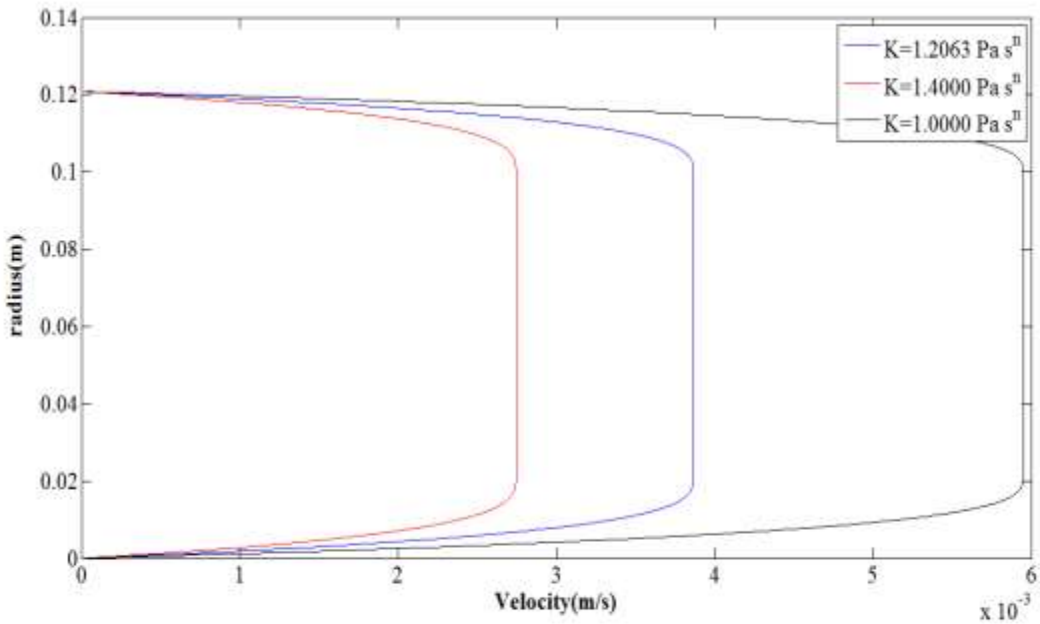


Figure 5.10: Effect of changing fluid consistency index (K) on velocity profile of Base Case Model (narrow region)

The variation of velocity profile due to the alteration made in the fluid consistency index at narrow region is much smaller compared to the one shown by the effect of yield stress. Since consistency index gives the idea how the fluid viscosity will be, it indirectly will affect the velocity at both narrowed and wider gap of the slot. As can be seen in Figure 5.9 and 5.10, the velocity profile of the plug is decreasing drastically when the consistency value increased from 1.0 Pa s^n to 1.2063 Pa s^n.

5.7 Effect of changing pressure drop

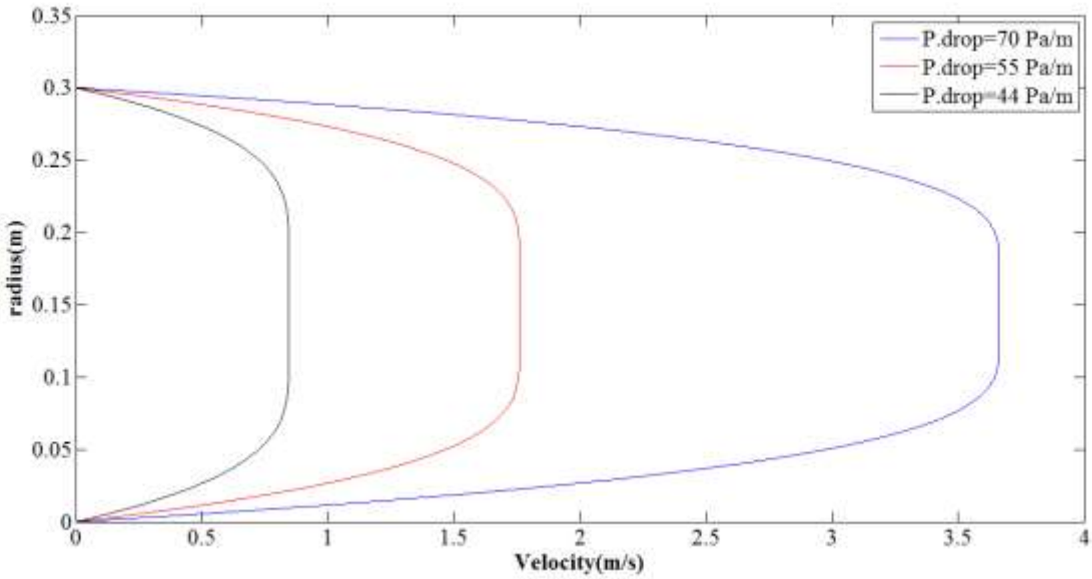


Figure 5.11: Effect of changing Pressure drop (Δ) on velocity profile of Base Case Model (Wide region)

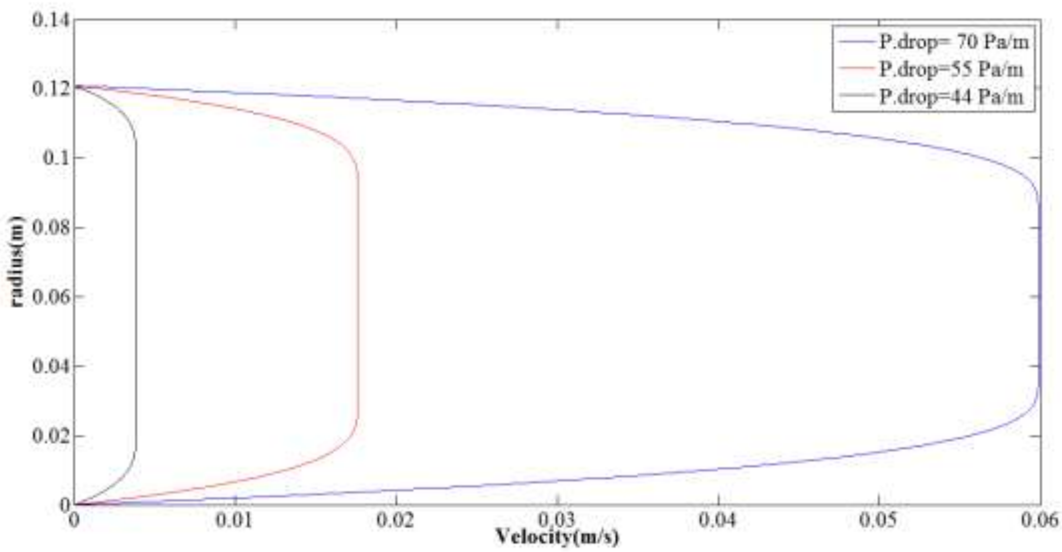


Figure 5.12: Effect of changing Pressure drop (Δ) on velocity profile of Base Case Model (Narrow region)

The observation from figure 5.11 and 5.12 indicate significant increase in velocity as the pressure drop change from 44 pa to 70 pa for about three to four times. This means that larger pressure drops are created by high velocity flow of the fluid in the annulus. The increase in velocity will eventually increase the flow rate too. In case of real drilling operation, most engineers want to minimize the pressure drop as much as they can starting from the surface equipment down to the bottomhole and back to the surface. Thus, it is necessary for every engineer that involved in this area to properly select the drilling fluid that will to be use.

From all the analyses made before, table 5.1 shows the summary of the effect parameter on velocity profile through the eccentric annuli based on the slot solution.

Parameters	Effects (major/ minor)
Flow behavior index, n	Major
Eccentricity, ε	Major
Yield Stress, τ_y	Minor
Fluid Consistency index,	Major
Pressure Drop, Δ	Major

Table 5.1: Summary of the effect of parameter on Velocity profile

5.8 Flow rate Estimation

There are two options in order to obtain the flow rate in this study.

- ✓ Manipulate the Velocity equation by multiplying by Area equation.
- ✓ Use the derived flow rate equation (refer Appendix A) and assume pressure drop to be the input data.

The second option is used in this study as the equation already derived previously. Based on the result obtained in Figure 5.13**,** it shows that the flow rate increases as the value of pressure drop become bigger. This is exactly true because the greater the flow rate applied from the surface, the more pressure drop there will be throughout the circulation system.

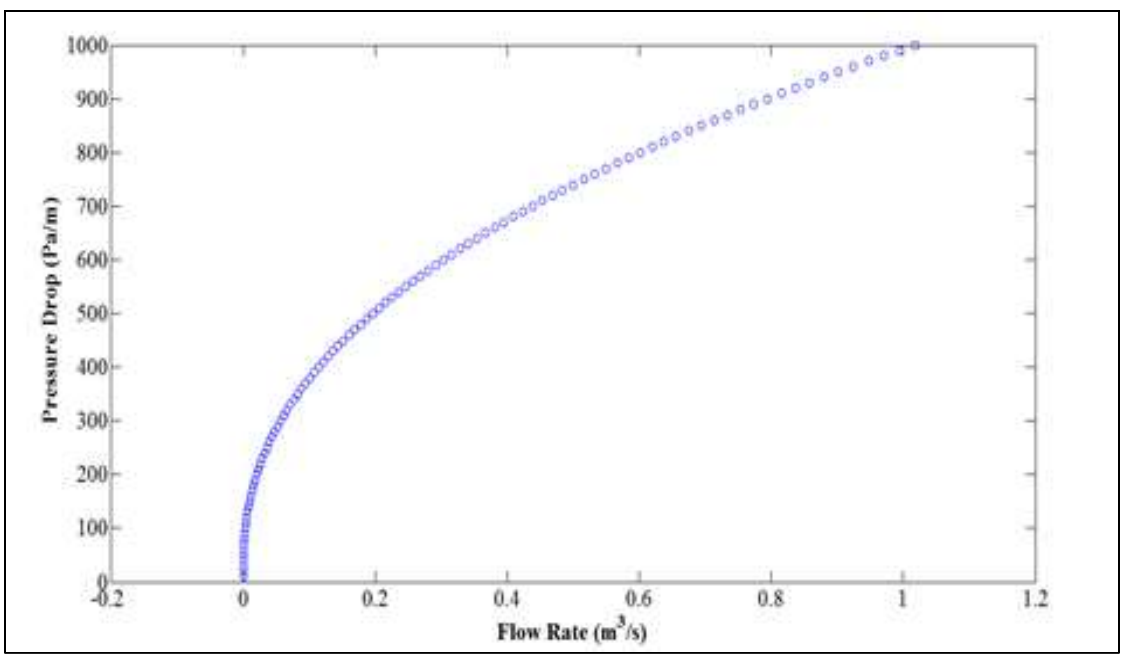


Figure 5.13: Pressure drop versus flow rate for the Base Case Model

CHAPTER 6

CONCLUSION AND RECOMMENDATION

In this study, a new fluid model (Herschel-Bulkley) is use for the analysis of fluid flow through eccentric annuli in which the eccentric annuli is represented as a slot. By using this model, the iterative finite difference method used by previous researchers can be avoided. The solution for hydraulic and drilling parameters such as velocity, pressure drop and flow rate can be obtained too by utilizing previous investigator data.

After obtaining all results, it can be conclude that Herschel-Bulkley drilling fluid is the suitable candidate to be used as drilling fluid especially in drilling operations that involves eccentric case such as in directional drilling as the plug flow can increase the volume of cuttings to be transported to surface and the velocity is not too small.

Hence, the engineers need pair more attention to the four main factors (flow behavior index, consistency index, eccentricity and pressure drop) which can influence the velocity distribution in the eccentric annuli.

As a recommendation, further studies should be done on a more complex fluid other than Bingham-Plastic, Power law and Herschel-Bulkley by considering inner pipe rotation in order to overcome the problem encountered in the future.

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APPENDICES

APPENDIX A: Development of Matlab code to Determine Dimensionless Velocity for Power Law fluid through eccentric annuli which treated as a slot of variable height.

To obtained solution of Dimensionless Velocity based on multiple radial position and angular position: (here ϵ is represented by e)

```
ri=3;

ro=10;

c=ro-ri;

n= input ('Key in n = ');

e= input ('Key in e = ');

y=[-1.0:0.2:1.0];

th=[0:22.5:180]';

P=(n+1)/n;

l1=length(y);

l2=length(th);

V=zeros(10,12);

V(1,2:12)=y(1,1:11);

V(2:10,1)=th(1:9,1);

for j=2:1:l1+1

for i=2:1:l2+1

h = (ro^2-e^2*c^2 * (sind(th(i-1)))^2)^0.5 - ri + e*c* cosd(th(i-1));

V(i,j)=(2*n+1)/(n+1) * ((h/2)^P-(abs(y(j-1)*h/2))^P)/(c/2)^P;

end
```

```
end

for i=1:1:l2+1

fprintf('%6.2f %6.2f %6.2f %6.2f %6.2f %6.2f %6.2f %6.2f %6.2f %6.2f %6.2f %6.2f
\r\n',V(i,:));

end
```

APPENDIX B: Development of Matlab code to Determine Velocity for Herschel-Bulkley fluid through eccentric annuli which treated as a slot of variable height.

```
Clear all

ri=input('key in ri= ');              % Outer radius of pipe
ro=input('Key in ro= ');              % Hole radius
c=ro-ri;                              % Radial clearence
n=input ('Key in n= ');               %Fluid behavior index
e=input ('Key in e= ');               %pipe/hole eccentricity in ratio
TaoY=input ('Key in TaoY= ');   %Yield point( should be greater than zero)
K=input('Key in K value= ');          %Fluid consistency index
delta=input ('Key in delta= ');       %Pressure drop
th=input ('key in th= ');             % theta or angular position

h =(ro.^2-e.^2*c.^2 * (sind(th)).^2).^0.5 - ri + e*c* cosd(th);
ya=((0.5*h)-(TaoY/delta));
yb=((0.5*h)+(TaoY/delta));
m=1/n;

y=0:0.00001:h;
V(0<=y & y<=ya)=(-(1/(m+1))*(delta/K).^m )*(-(ya.^(m+1))+((ya-y(0<=y &
y<=ya)).^(m+1)));
V(ya<=y & y<=yb)=((1/(m+1))*(delta/K).^m)*(ya.^(m+1));
V(yb<=y & y<=h)=((1/(m+1))*(delta/K).^m)*(((h-yb).^(m+1))-((y(yb<=y & y<=h )-
yb).^(m+1)));

plot(V,y,'r-')
xlabel('V(m/s)')    %x-axis label
ylabel('r(m)')      %y-axis label
hold on
```

APPENDIX C: Development of Matlab code to determine the Flow Rate for Herschel-Bulkley Drilling fluid in Eccentric Annuli

```
clear all

clc


ri=input('Key in ri=');

ro=input('Key in ro=');

K=input('Key in K = ');

n=input('Key in n=');

TaoY=input('Key in TaoY=');

m=1/n;


for d=0:10:1000

q=(((pi*(ro.^2-ri.^2)*(ro-ri).^(1+m)*(d/K).^m)/(2.^m*(m+1)*((2*m)+4)))*(1-
(TaoY/(((ro-ri)/2)*d))).^(1+m)*(((TaoY/((ro-ri)*d))+m+1)/(2.^m*(m+1)*((2*m)+4))));

plot(q,d,'bo')

xlabel ('Flow Rate (m^3/s)')

ylabel ('Pressure Drop (Pa/m)')

hold on

end
```