EFFECT OF PDC BIT WEAR AND OPERATING PARAMETERS ON OIL-WELL DRILL-STRING DYNAMICS

by

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Dissertation submitted in partial fulfillment of
The requirements for the
Bachelor of Engineering (Hons)
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May 2015

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CERTIFICATION OF APPROVAL

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A project dissertation submitted to the
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In partial fulfillment of the requirement for the
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Approved by,

____

(Dr. Tamiru Alemu Lemma)

UNIVERSITI TEKNOLOGI PETRONAS

TRONOH, PERAK

May 2015

CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained here have not been undertaken or done by unspecified sources or persons.

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HAMZA TAYSEER YOUSEF HUSSEIN

ABSTRACT

Drill-string vibrations reduces the drilling performance, the bit wear negative effects on the drill-string dynamics is one of the elements which limit reaching the optimum drilling performance due to vibrations caused by the bit wear and other factors. To optimize the drilling parameters in order to reduce vibration related problems, real time analysis on the drill-string is needed. Due to extreme complexity of the vibration phenomenon it is difficult to construct a drill-string model which includes all types of vibrations thus this project presents a mathematical drill-string model which includes bit wear parameter or parameters in order to observe the impact that bit wear have on the drill-string dynamics. A parametric study is carried out to analyze the impact of operating parameter such as weight on bit (WOB) and rotary speed (RPM), other parameters which are related to bit wear such rock confined compressive strength and bit-specific coefficient of sliding were also studied, the parameters were tested using real field data. The study showed that increasing WOB increases stick/slip. However, increasing RPM seems to eliminate stick/slip. Relating CCS to bit wear allowed to observe the relationship between and stick/slip and was found that the severity of stick/slip increases as bit wear increases. The simulation results show similar trends as observed in the field. The simulation results show similar trends as observed in the field.

ACKNOWLEDGEMENT

In the Name of Allah the most Merciful, All perfect praise to Allah, the Lord of the worlds, and may the peace and blessings be upon the final Prophet Muhamad, upon his household, his companions, and following them all in goodness till the day of judgment. Praise and thanks due to Allah, for his blessings and guidance throughout my life and the success He granted me in completing this industrial internship program in ease and success.

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ABBREVIATIONS

PDA	Pile Driving Analysis
RPM	Rotation per Minute
TOB	Torque on Bit
WOB	Weight on Bit
BHA	Bottom-hole Assembly
NPT	Non-Productive Time
PDC	Polycrystalline Diamond Compact
HWDP	Heavy Weight Drill Pipe
ROP	Rate of Penetration
DOF	Degree of Freedom
BHP	Bottom hole pressure

NOMENCLATURE

$C_{b,d}$	Damping coefficient along the BHA and the drill-string, (Nms/rad)
C_{b1}	Damping coefficient at the bit (Nms/rad)
D_b	Bit diameter (m)
J_b	Moment of inertia of a bottom-hole assembly (kg .m^2)
J_d	Moment of inertia of a drill pipe (kg. m^2)
$k_{d,b}$	Torsional stiffness of drill-string and bottom-hole assembly (Nm/rad)
T_{fb}	Nonlinear friction torque at the bit (Nm)
T_b	Torque on bit (Nm)
W	Weight on bit (lb)
$\theta_{m, d, b}$	Angular displacement of motor, drill-string, and bit, respectively (rad)
μ	Friction coefficient (dimensionless)
λ	Decay factor
$\omega_{m ,d, b}$	Angular velocity of motor, drill-string, and bit, respectively (rad/s)
ρ	Density (kg/m3)
G	Shear Modulus (Gpa)
D_o	Outer Diameter (m)
D_i	Inner Diameter (m)
L	Length (m)
ε	Zita
CCS	Rock confined compressive strength (psi)
UCS	Rock unconfined compressive strength (psi)
DP	Differential pressure (psi)
FA	Rock internal angle of friction (rad)
W_f	Wear function (dimensionless)
K_{wf}	Wear function constant (dimensionless)
Nc	Number of cutters on bit face
A_w	Wear flat area underneath cutter (in^2)
ρ, τ	Wear function exponents (dimensionless)

CHAPTER 1

INTRODUCTION

1.1 Background

Drilling is defined as a part of the process of extracting oil & gas from the wells, the steps of the drilling process are as follows: budgeting > planning> designing> optimization> execution> PDA. Drilling is a twenty four hours process which can be done onshore or offshore. There are two types of wells in the drilling process, first is the exploratory well which is drilled to discover new reservoirs and the second is the development well which is drilled in order to exploit a known reservoir. Effective drilling occurs when using operational parameters at their optimum level such as: surface RPM, TOB, and WOB.

1.1.1 Drill-string

The drill-string which is a part of the drilling rig components consists of the drill pipes, the bottom-hole assembly (BHA) and the bit as seen in FIGURE 1.1.

FIGURE 1.1: Drill-string Components

Drill pipes: makes up most of the drill-string, each drill pipe contains a long tubular segment with a fixed outside diameter. The tool joints are a bigger diameter sections located at the end of the drill pipe. Each pipe has both pin and box in each; these allow drill pipe to be connected to each other.

BHA: made up of several components such drill collars which are used to apply weight on the bit, drill bit which is used to drill though formations and stabilizers which are used to maintain the drill-string in the center of the hole while drilling.

1.1.2 Vibration" Drill-sting vibration"

Drill-string vibration is one of the major problems that affect the drilling performance. The non-linear interactions between drill-string/borehole and bit/formation are the sources of vibration. Drill-sting vibrations can lead to down-hole tool failure, cause damage to the hole drilled and increase of NPT due to the frequent rig repair. Vibration can be classified into 3 types:

- Lateral (forward, backward, chaotic whirl) vibrations.
- Axial (bit bouncing).
- Torsional (stick-slip).

1.1.3 Bit wear

Wear is defined as the gradual failure of the cutting tool due the continuous operation of the tool. For the PDC bit wear it can be divided into two categories. The first category is abrasive wear which is a steady state wear that is usually related to the development of uniform wear-flats and the gradual degradation in ROP over the bit life. The second category is a result of the dynamic loading of the cutters, it is associated with broken, chipped and loss of cutters. It is caused by unexpected changes in the forces initiated by cutter/rock interaction or any sudden change in the surface drill-string control.

1.1.4 PDC bit

One of the most used drilling tools the used polycrystalline diamond compact cutter or PDC cutters. This type of cutters used a continuous shearing motion to drill through the rock. The cutters are in the form of a synthetic diamonds disks and the standard dimensions are ½ to 1 in. in diameter and 1/8 in. thick. The polycrystalline diamond compact bits are best used in drilling hard formations, most effectively when they're used along with oil-base muds.

1.2 Problem Statement

The PDC bit uses transverse shearing motion when used in a multi-formation, after a period of time bit wear occurs as the bit life comes to an end; due to this motion vibrations in the drill-string occur. However, vibration can be caused by many factors such as the rock/ bit interactions, or fault in arranging the drill pipes, etc.

It is both difficult and costly to identify the exact signal for bit wear when tested on a live site. By developing a model for drill-string dynamics which includes bit wear parameters and reversing the problem by creating bit wear and observing the effects on the drill-string dynamics, the vibration signal linked to bit wear can be observed and used for drilling optimization as it allows choosing optimum parameters in order to reduce vibrations.

1.3 Objectives

The main objectives of this project are:

- Investigate the effect of CCS on stick/slip
- To study the effect of operating parameters (WOB, pipe length and rotational speed) on drill-string dynamics and stick/slip.
- To investigate the effect of PDC bit wear on drill-string dynamics.
- To investigate the relationship between bit wear and stick-slip.

1.4 Scope of Study

This project is mainly about investigating the effects of PDC bit wear on drill-string dynamics, this is done by creating a drill-string model which includes a parameter or parameters related to bit wear and observing its effect by simulating the model. Due to the complexity of creating a drill-string model; a pre-existing model is chosen and validated for results. The simulation is done using ode15s solver in MATLAB R2013a.

CHAPTER 2

LITERATURE REVIEW

2.1 Drill-string Models

According to Dawson et al. [1] the interruption of downhole tool rotation causes the torque at the top drive during drilling to vary with time and is irregular due to the downhole friction factor. Dawson et al. [1] suggested that the stick-slip problem could be solved by decreasing the static friction. When the static friction factor is higher than the dynamic friction factor phenomenon of stick-slip arises. Such phenomenon can be observed when the drill-string rotates in continues motion as torsional energy is stored. Once the deposited energy surpasses the static friction, an acceleration of the bit movement starts, then the rotation reaches the maximum speed and relaxes the drill-string. TABLE 2.1 is a summary of some previous studies done related to drill-string and stick/slip.

TABLE 2.1: Studies on Modeling and Controlling of Torsional Drill-string Dynamics.

Researcher (year)	**Research**	**Approach**	**DOF**	**Remark**
Halsey et al. [2]	Modeling of stick–slip phenomena and torque feedback to cure stick–slip oscillations.	Theoretical	One	Model could not predict occurrences of stick– slip under given sets of condition.
Dykstra et al. [3]	Investigation of drilling performance by considering bit dynamics coupled with drill-string	Modeling	Six	
Rudat and Dashevskiy[4]	Developed model based stick–slip control system.	Theoretical and validation using field data	One	Model developed was adapted to actual drilling process using field data.

2.2 Bit Wear Models

Warren [5] developed a PDC bit model for both bit wear and performance. This model is checks cutter wear and proper cutter placement by assessing the mechanical designs of the bit which is based wear calculation and static cutter force. The paper also mentioned types of wear:

Abrasive wear: which is "a steady-state wear that is linked to the development of uniform wear-flats and gradual degradation in rate of penetration over the bit life" and it is a function of:

- Cutter properties
- Formation properties
- Cutter velocity
- Cutter temperature
- Force applied to the cutter

Dynamic loading: "triggered by rapid in the surface drill-string control or by forces induced by cutter/rock interactions". And it is characterized by:

- Chipped cutter
- Broken cutters
- Lost cutters

Deen, Wadel [6] mentioned that stick/slip can damage the PDC bits which shorten the bit's life. The continuous impact due the stick/slip causes breakage as well as accelerating the bit wear especially in hard formations. When damage is done to the PDC cutters the energy needed to maintain the rate of penetration becomes higher due to the lake of efficiency of the bit; that increases the severity of the stick/slip problem.

FIGURE 2.1 shows typical wear of PDC bit when drilling hard formations with no stick/slip. As in FIGURE 2.2 we see the impact damage done on the PDC bit when stick/slip is present.

FIGURE 2.1:Typical PDC wear due to drilling hard formations

FIGURE 2.2: Typical impact damage of stick/slip

Deen, Wadel [6] also mentioned that stick/slip can damage the PDC bits which shorten the bit’s life. The continuous impact due the stick/slip causes breakage as well as accelerating the bit wear especially in hard formations.

Caicwdo et al. [7] developed a method to calculate rock CCS of the rock on the bit; by using this method Caicwdo et al. [7] was able to develop a correlation to calculate the bit-specific coefficient of sliding friction μ for a PDC bit with more than 7 blades as a function of rock confined compressive strength.

Below is the equation for CCS:

- CCS = UCS + DP + 2DP * sin(FA)/(1-sin(FA)) (2.1)

Where:

CCS: rock confined compressive strength (psi)

UCS: rock unconfined compressive strength (psi)

DP: differential pressure (psi)

FA: rock internal angle of friction (rad)

The bit-specific coefficient of sliding friction μ for PDC bit was calculated using several tests which were conducted on PDC bits.

The tests used the following variables:

- Mud weight: 9.5 ppg
- BHP: 6,000 psi

The rock samples used along with their corresponding CCS were:

- Crab Orchard: 66,000 psi
- Catoosa shale: 18,500psi
- Carthage Marble: 36,226 psi

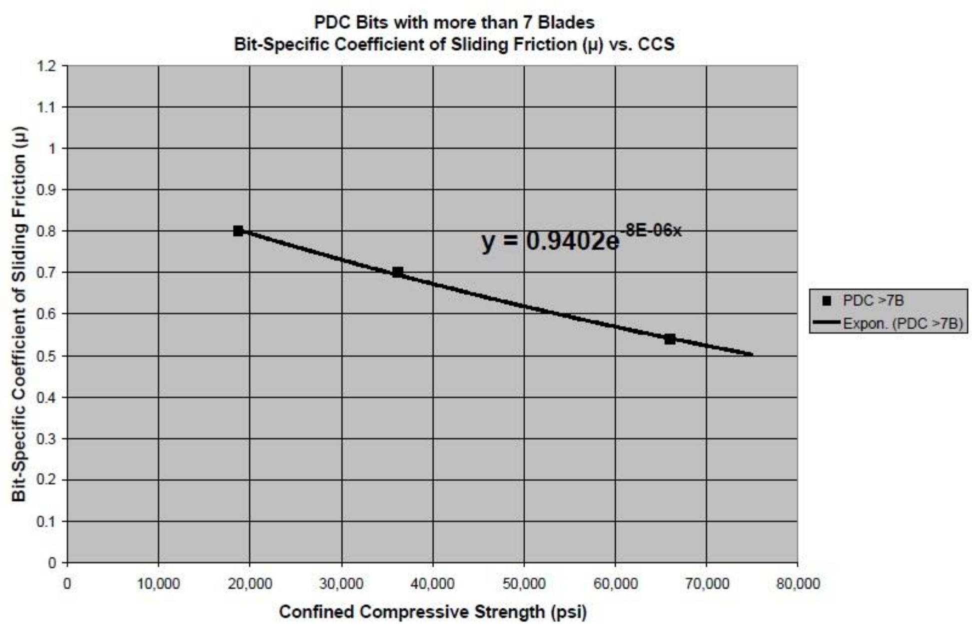


FIGURE 2.3: Rock CCS vs. μ

From FIGURE2.3 the correlation for μ is obtained:

$$\mu = 0.9402 \ * e^{(-8\times10^{-6}*CCS)} \quad (2.2)$$

Where:

- μ : Bit-specific coefficient of sliding friction
- CCS: rock confined compressive strength (psi)

Motahhari et al [8] developed an equation to calculate wear function for PDC bit:

$$W_f = k_{wf}(\frac{WOB}{Nc})^{\rho} \cdot \frac{1}{S^{\tau} \cdot Aw^{\rho+1}} \quad (2.3)$$

Where:

- W_f :wear function (dimensionless)
- K_{wf}: wear function constant (dimensionless)
- Nc: number of cutters on bit face
- A_w: wear flat area underneath cutter (in^2)
- ρ, τ: wear function exponents (dimensionless)
- WOB: Weight on Bit
- S: rock confined compressive strength (psi)

The development of the equation used a single PDC cutter experimental data from Glowka [9]. The equation relates the wear function to the rock confined comprehensive strength.

A model was developed by Cheknia et al [10] to study the tool shape variation that is caused by wear, it was based on rock elastic deformation and crushing, comprehensive consecrations of tool wear and rock brittle fracture. The model focused more on tool shape variation caused by wear but didn’t give much consideration to cutting performance variation of the PDC cutter under wear.

Moseley el al [11] reviewed wear and fracture mechanisms that uses PDC cutter for drilling. These include broken or chipped cutter, heat cracking, delamination, lost cutter and abrasive wear flatting. From these reviews it was found that there are four main mechanisms for PDC cutter wear, and they are:

1- Diamond cutter chipping which is caused by impact loading.
2- Abrasion which is caused by hard abrasives in the drilling rock.
3- Gross fracture induced by residual or thermos-elastic stresses
4- Thermal degradation when drilling temperature exceeds the point of degraded diamond properties.

The PDC cutter wear properties were studied on a microscopic level by Hibbs and Lee [12] by using PDC cutters a tubular core of sandstone on a lathe. They discovered that failure modes of the diamond crystals include a brittle fracture mode with big parts of the crystal breaking and a crushing mode with slight parts of the crystal being slowly chipped away.

Richard et al [13] presented a torsional model that accounts for stick/slip vibration for the PDC bits, stating that stick/slip vibrations increase bit wear and may lead to breakage of the bit or premature failure of the drill-string. Richard et al [13] suggested that decreasing the weight on bit or increasing the angular velocity eliminates stick/slip; Richard et al [13] supported this by field testing.

2.3 Simulation Models

The model displayed by Brett [14] presented that torsional vibration in the system is initiated by bit-rock interaction, and can be removed by controlling the gain in the rotary system at the surface. By using Runge-Kutta simulation approach for solving a simple model that uses 2 differential equations. The established model studies the behavior of the drill-string as a combined mass which is attached to a spring and the surface drive system.

2.4 Summary

This chapter mentions research papers related to the project, in which it includes papers about drill-string dynamics and bit wear models. The drill-string dynamics models analyze the stick/slip phenomena, recording the observation of its effect of the drill-sting and analyzing factors contributing to it. The bit wear models talk about types of bit wear along with its effect on the bit and the relationship between bit wear and stick/slip. Finally the simulation models talks about using the Runge-Kutta (appendix A.5) simulation approach to solve differential equations the represent a drill-sting model.

CHAPTER 3

METHODOLOGY

3.1 Literature Review

This is done by researching papers that are relevant to project to include in the literature review, such papers include drill-string models, and bit/rock interaction and PDC bit wear models. Certain parts in these papers that are seen to be remotely related were included in the literature review.

3.2 Lumped Parameter Modeling

The paper chosen as the benchmark for this project is the "Model Development of Torsional Drill-string and Investigating Parametrically the Stick-Slip Influencing Factors" by Patil et al [15].

The mathematical model used represented the drill-string as a simple torsional string with 2 DOF. Using MATLAB for parametric study of stick-slip influencing parameters; the model uses non-linear differential equations to represent the drill-string, BHA and non-linear bit/rock interaction.

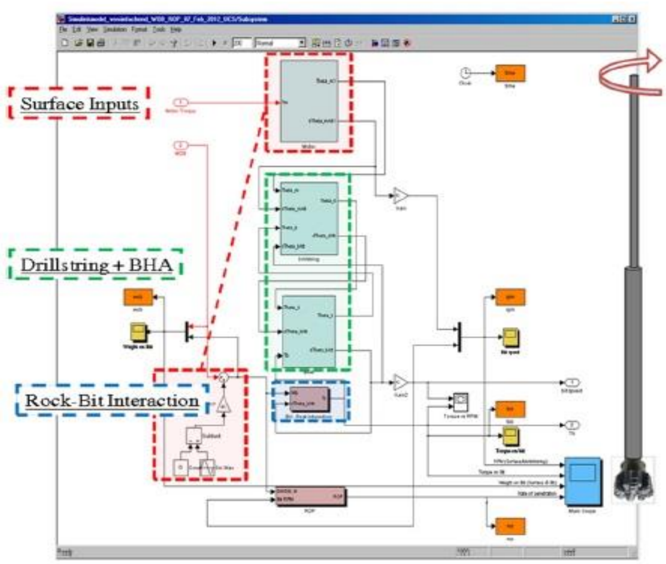


FIGURE 3.1: Torsional drill-string model using MATLAB/SIMULINK interface

FIGURE3.1 shows the Simulink blocks which are a representation of the coupled non-linear differential equations that connect the surface inputs with the drill-string and BHA along with the bit/rock interaction

The two main equations of motion used for simulation and modeling:

$$J_d\ddot{\theta}_d - C_d(\dot{\theta}_m - \dot{\theta}_d) - k_d(\theta_m - \theta_d) + c_b(\dot{\theta}_d - \dot{\theta}_b) - k_b(\theta_d - \theta_b) = 0 \quad (3.1)$$

$$J_b\ddot{\theta}_b - C_b(\dot{\theta}_d - \dot{\theta}_b) - k_b(\theta_d - \theta_b) = -T_b \quad (3.2)$$

Sub equations:

$$T_{fb} = \left[\mu_{cb}(\omega_b) + (\mu_{sb} - \mu_{cb}) * e^{-\lambda|\omega b|}\right] * D_b.W \quad (3.3)$$

$$T_b(w_b) = c_{b1} * w_b + T_{fb}(\omega_b) \quad (3.4)$$

Where:

- J_d: Inertia of Drill pipe
- C_d: Damping of Drill pipe
- J_b: Bit Inertia
- C_b: Bit Damping

3.3 Model the Benchmark and the Case Study

A case study will be given which for is project is K412 Drillship, the data from K412 Drillship will be used in the modeling and simulation to show that the model constructed from the benchmark is applicable.

3.4 Simulate the Benchmark Problem

A simulation of the benchmark will be done using MATLAB in order to compare the results from the benchmark to the results from the simulation done; this is done mainly to validate the benchmark. If the results from the model are similar then we proceed to the next step, if not then further modification is needed on the simulation in order to achieve validation of the benchmark.

In simulating the benchmark the results may not identical to the ones in the benchmark, the reason behind that may be due to MATLAB coding instead of MATLAB Simulink or may be due to using different solvers as the benchmark used ode45 and the solver used here is ode15s.

3.5 Simulate the Case Study

Upon the successful modeling and simulation of the benchmark problem, the model presented is verified and validated. Data from K421 Drilling will be used as inputs and will be simulated for results.

3.6 Parametric Study

Parametric study means changing one parameter while other parameters are constant, in this project the parametric study is done for both the benchmark and the case study and it is done in order to demonstrate occurs of the stick/slip phenomena over different weight on bit values for varies parameters, note that the parameters chosen are all related to the drill pipe while keep the BHA parameters constant.

3.7 Run the Simulation Analysis

The final step is to run the drill-string dynamics model using parameters related to bit wear, is order to observe the effect of bit wear on the drill-string dynamics and observe the relationship between bit wear and stick/slip. In case of an unsuccessful simulation the parameter chosen might need further modification.

3.8 Report Writing

The last remaining step is report writing, once all the modeling and simulation which are related to this project are complete, all the results obtained will be compiled and presented in the form of a written report.

3.9 Milestones

- Completion of the benchmark problem selection. Date: 07 - 11- 2014
- Completion of the benchmark problem simulation. Date: 28 - 11- 2014
- Completion of the parametric study. Date: 16 - 02 -2015
- Completion of report writing. Date: 13 - 04 – 2015

CHAPTER 4

RESULTS AND DISCUSSION

4.1 Data Gathering

The initial data needed in order to construct the drill-sting model are mostly available from the benchmark [15]; TABLEs 4.1, 4.2 and 4.3 list the data which will be used as inputs for the model. The data describe the torsional drill-string simulation parameters. Below are the data available from the benchmark:

TABLE 4.1: Drill-pipe Parameters Base Values

Drill-pipe parameters	**Symbol**	**Unit**	**Value**
Inertia	J_d	$kg.m^2$	600
Damping	C_d	Nms/rad	85
Torsional stiffness	K_d	Nm/rad	500
Diameter	D_p	in.	5
Length	L_p	m	5700
Nominal weight	Wt.	kg/m	26
Young modulus	E	N/m^2	200
Steel density	ρ	kg/m^3	7850

TABLE 4.2: BHA Parameters Base Values

BHA parameters	**Symbol**	**Unit**	**Values**
Inertia	J_b	$kg.m^2$	500
Damping	C_b	Nms/rad	50
Torsional stiffness	K_b	Nm/rad	900
Diameter	D_b	in.	6 ¾
Length	L_b	m	180
Nominal weight	Wt.	kg/m	105

TABLE 4.3: Bit-rock Parameters Base Values

Bit-rock	**Symbol**	**Unit**	**Value**
Static friction	μ_{sb}	NA	0.8
Coulomb friction	μ_{cb}	NA	0.5
Damping at bit	C_{b1}	Nms/rad	50
Decay factor	λ	NA	0.9
Bit diameter	D_b	in.	8 ½

4.2 Benchmark Problem

The simulation time used for the validation and further phases is 100 s; the dashed blue line represents the drill pipe and the red line represent BHA for all FIGUREs ahead. Only drill pipe parameters are changed while BHA parameters remain constant.

4.2.1 Effect of WOB on stick-slip

The simulation results from the benchmark problem showed that increasing WOB will result in increasing the severity of stick-slip. The below FIGUREs show the increasing severity of stick-slip when increasing WOB at constant RPM as in FIGURE 4.1 the angular velocity reaches around 230 RPM while in FIGURE 4.2 it reaches about 270 RPM and stick occurs for longer periods in FIGURE 4.2 as it clearly show.

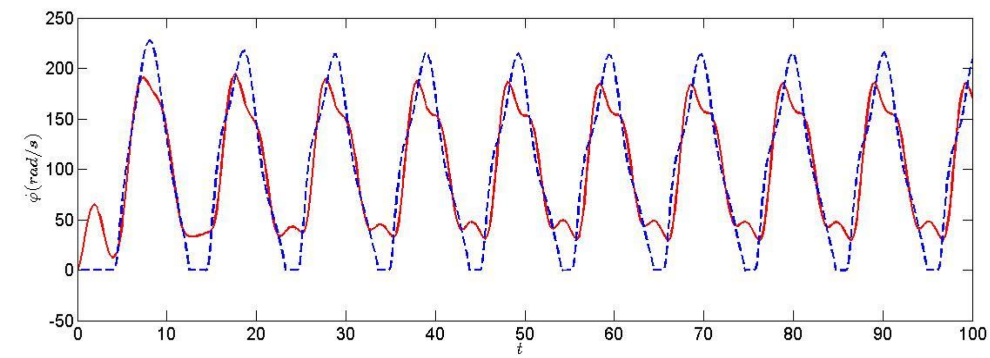


FIGURE 4.1: WOB 160 kN at 100 RPM

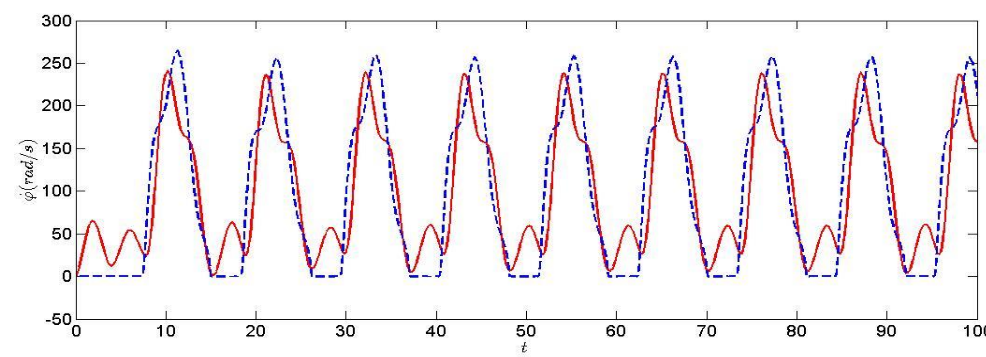


FIGURE 4.2: WOB 300 kN at 100 RPM

4.2.2 Effect of drill pipe stiffness on stick-slip

The benchmark simulation results showed that increasing drill-string inertia reduces stick-slip. The FIGUREs below show reduction of stick-slip as drill-string stiffness increases. As observed on FIGURE 4.4stick/slip occurs until about 65 s then the angular velocity starts to reach a steady form indicating no stick/slip but for FIGURE 4.3 stick/slip is seen along the entire simulation time which is 100 s.

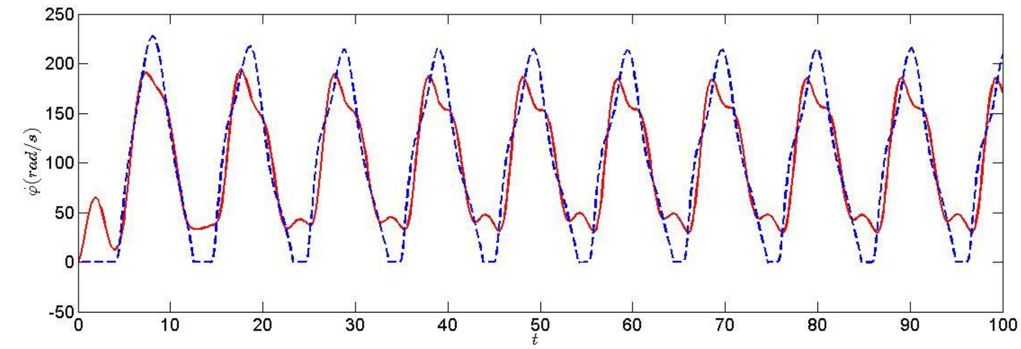


FIGURE 4.3: K_d 500 Nm/rad at WOB 160 kN

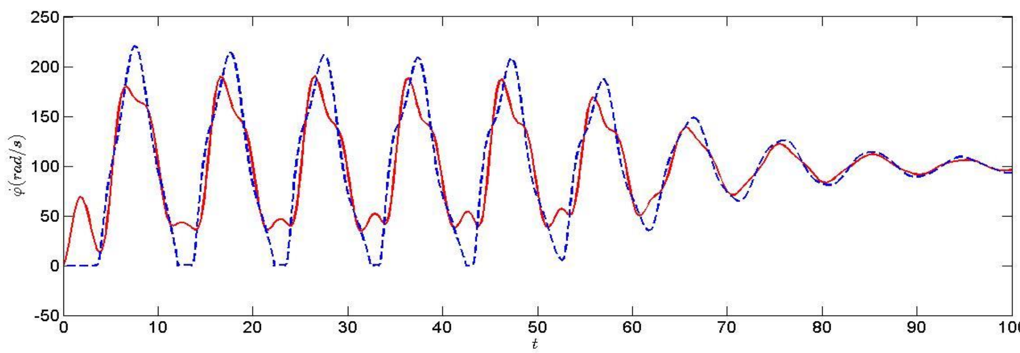


FIGURE 4.4: K_d 550 Nm/rad at WOB 160 kN

4.2.3Effect of drill pipe inertia on stick-slip

The benchmark simulation shows that increasing drill-sting inertia reduces stick-slip. The FIGUREs show that increasing drill-string inertia will result in reducing stick-slip. As observed on FIGURE 4.6stick/slip occurs until about 60 s then the angular velocity starts to reach a steady form indicating no stick/slip but for FIGURE 4.5stick/slip is seen along the entire simulation time which is 100 s.

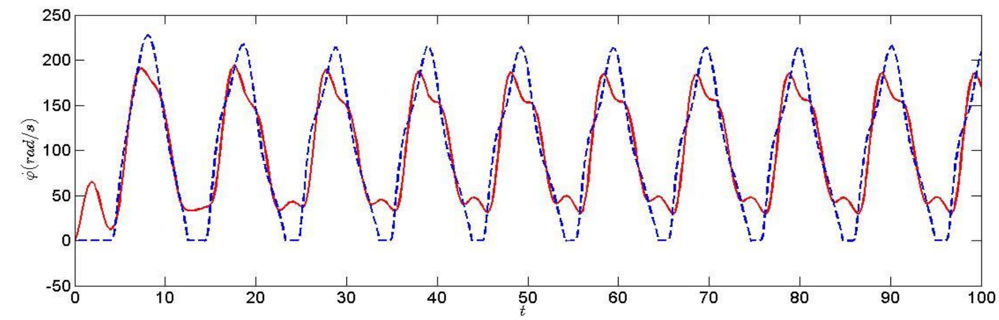


FIGURE 4.5: J_d 600 kg.m2 at WOB 160 kN

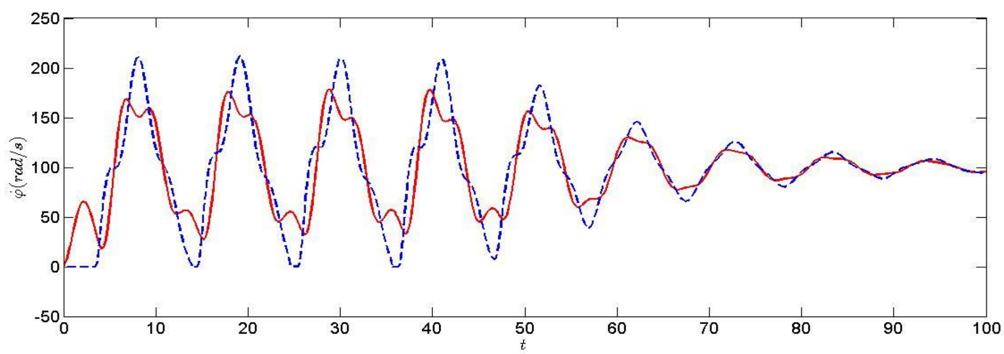


FIGURE 4.6: J_d 800 kg.m2 at WOB 160 kN

Observations:

- As weight on bit increase stick/slip increases
- Increasing the moment of inertia and stiffness reduces stick/slip
- Increasing rotary speed decreases stick/slip

These results are similar to the benchmark results, meaning that phase one which validation of the model is a success.

4.3 K421 Drillship

A case study was introduced with 3 sections which are:

- Sub-surface drilling: depth up to 1500 ft.
- Intermediate drilling: depth 1500 to 7500 ft.
- Deep drilling: depth 7500 to 11500 ft.
- The formation is assumed to be sandstone and the mud used is tap water.

From the case study data the moment of inertia, stiffness and damping of the pipe and BHA will be calculated manually then used as inputs in the MATLAB code to be simulated for results.

The first step is to list the data available and TABLEs 4.4 and 4.5 present the data available from the K421 Drillship.

TABLE 4.4: Material Propreties and Operating Parameters

Parameter	Symbol	Unit	Value
Young's Modulus	E	GPa	200
Shear Modulus	G	Gpa	77
Density	ρ	Kg/m^3	7850
Rotary speed	Ω	RPM	50
Weight on bit	WOB	kN	8.896 to 88.94
Rate of penetration	ROP	mm/s	4.23

TABLE 4.5: K421 Drillship Data

Parameter	Symbol	Unit	Subsurface Drilling	Intermediate Drilling	Deep Drilling
Pipe length	L_p	m	232.6	1940.2	3154.68
Outside diameter of the pipe	D_{po}	m	0.127	0.127	0.127
Inside diameter of the pipe	D_{pi}	m	0.109	0.1086104	0.1086104
Length of BHA	L_b	m	224.5	172.8	352.3
Outside diameter of BHA	D_{bo}	m	0.127	0.127	0.16764
Inside diameter of BHA	D_{bi}	m	0.073025	0.073025	0.0762
Bit diameter	D_b	m	0.4445	0.4445	0.2159

The second step is to calculate the moment of inertia, stiffness and damping of the pipe and BHA using the following equations:

- Moment of inertia $J = \frac{\rho G \pi}{32}(Do^4 - Di^4)$ (4.1)
- Stiffness $K = \frac{\pi G}{32\,L}(Do^4 - Di^4)$ (4.2)
- Damping $C = 2 * zeta * \sqrt{J * K}$ (4.3)

These equations are taken from the text book "Torsional Vibrations" and will be used for the drill pipe and BHA calculations which are considered hollow shafts.

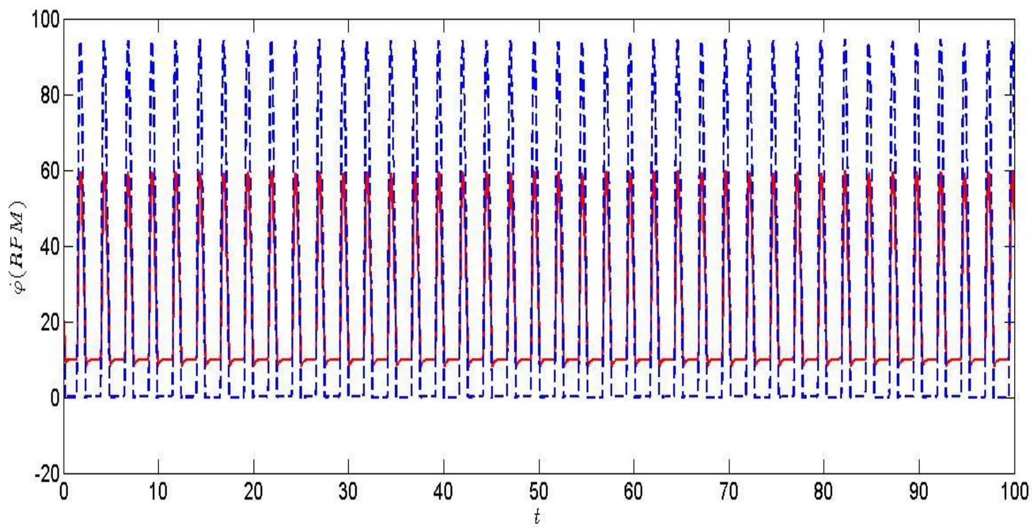
From Equations 4.1, 4.2 and 4.3 the values were determined as follows:

TABLE 4.6: K421 Drillship Computed Values

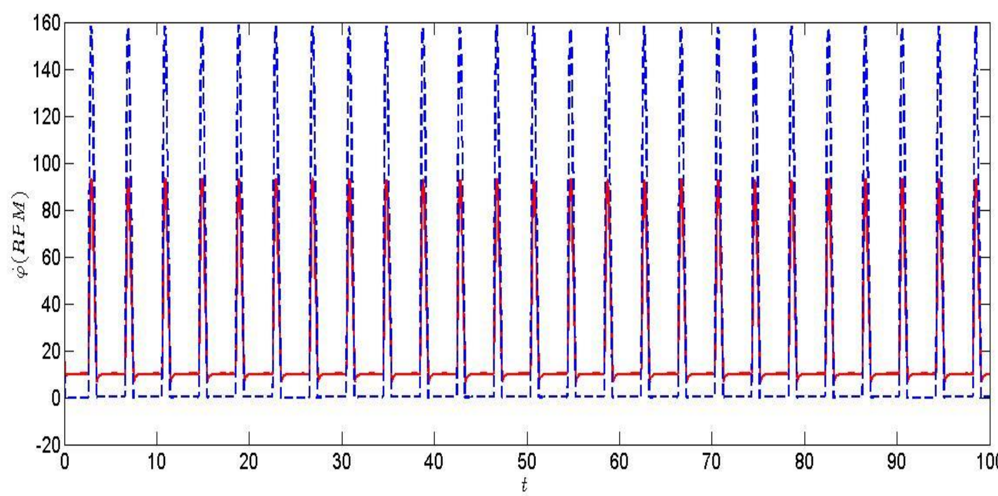
Parameter	**Symbol**	**Unit**	**Sub-surface Drilling**	**Intermediate Drilling**	**Deep Drilling**
Drill pipe inertia	J_d	$kg.m^2$	21.7	180.9	294.1
Drill pipe stiffness	K_d	Nm/rad	7862.7	942.8	579.8
Drill pipe damping	C_d	Nms/rad	0.0826	0.0826	0.0826
BHA inertia	J_b	$kg.m^2$	40.1	30.8	205.3
BHA stiffness	K_b	Nm/rad	7862.7	942.8	579.8
BHA damping	C_b	Nms/rad	0.0826	0.0826	0.0826

4.3.1 Effect of weight on bit on stick/slip

The FIGUREs shows severe stick/slip occurs over the weight on bit change. This level of stick/slip severity may be very destructive to the drill-sting. From FIGURE 4.7 a. rotary speed for slip seem to be around 90 RPM but for FIGURE 4.7 b. the rotary speed for slip exceeds 150 RPM indicating the negative effect of increasing WOB for a constant rotary speed.



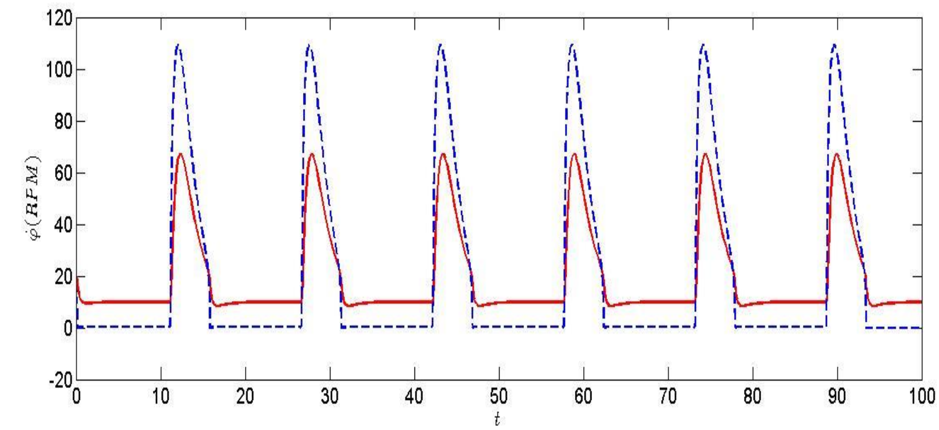
a. 20 RPM at WOB = 50 kN



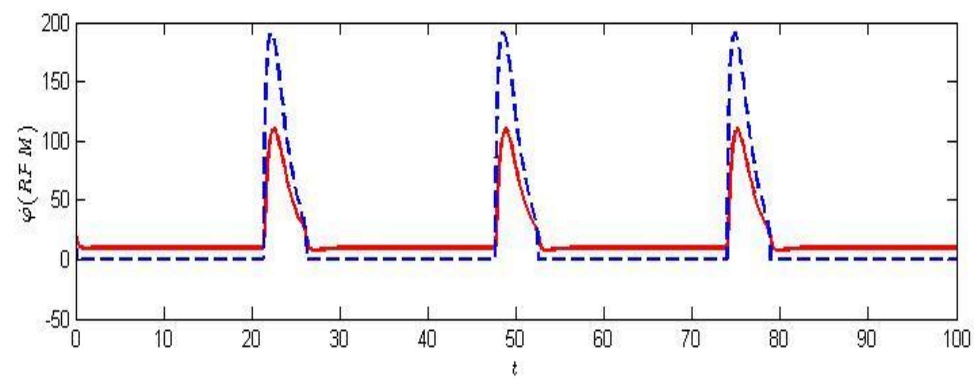
b. 20 RPM at WOB = 90 kN

FIGURE 4.7: Sub-surface Drilling

Stick/slip is observed for the intermediate drilling, though the level of severity observed here is less than the one in the sub-surface drilling this level of severity is also considered destructive to the drill-string, also stick/slip is in FIGUREs 4.8 a. and b. is not frequent as in sub-surface drilling, in FIGURE 4.8 a. the stick occurs for about 10 s followed by slip reaching over 100 RPM, this is due to the high amount of energy stored in the drill pipe.



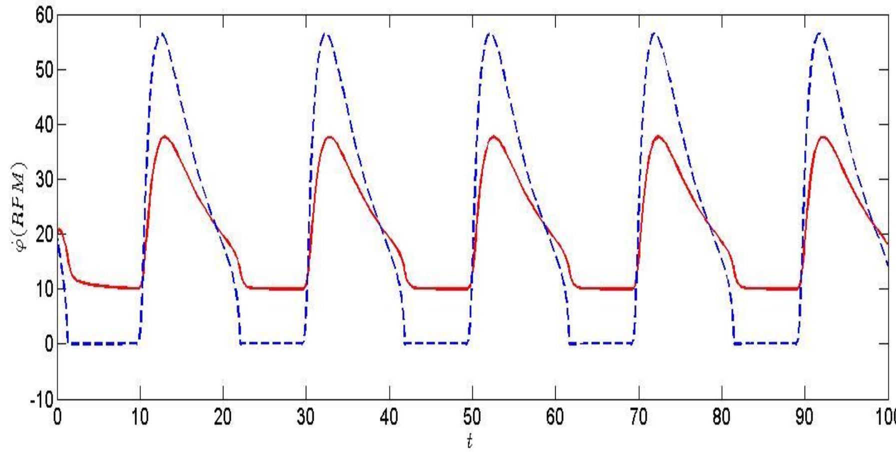
a. 20 RPM at WOB = 50 kN



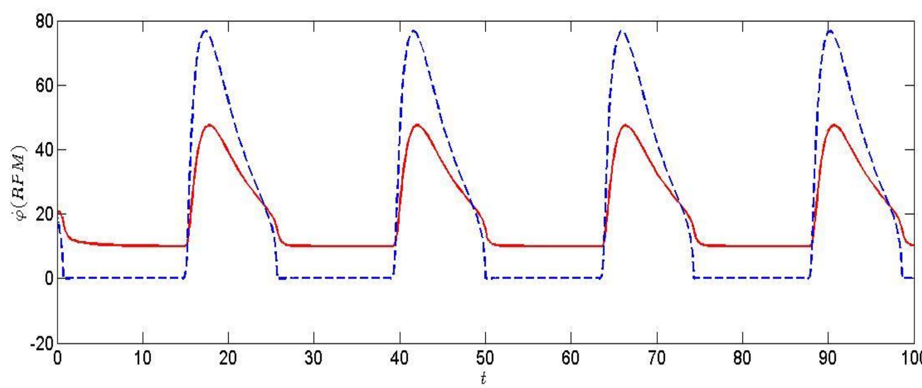
b. 50 RPM at WOB = 90 kN

FIGURE 4.8: Intermediate Drilling

Stick/slip is observed deep drilling as well. Though the severity is also less than the one for both sub-surface and intermediate drilling, elimination or reduction of the stick/slip is still needed in order to achieve drilling optimization. FIGUREs 4.9 a and b show stick/slip occurring at different WOB with a rotary speed of 20 RPM.



a. 20 RPM at WOB = 60 kN



b. 50 RPM at WOB = 90 kN

FIGURE 4.9: Deep Drilling

4.4 Further Simulation

The parametric study is done for both the benchmark and the case study; the reason for the parametric study to identify the optimum parameters for varies cases to try to avoid stick/slip situations. Several parameters such a drill pipe inertia stiffness and pipe length will be studied in the following section

4.4.1 Benchmark Problem

All cases were simulated over WOB = [100-200] kN with using base values from section 4.1, only drilling pipe parameters were used for the parametric study while BHA parameters remained constant. The graphs obtained were where from simulating each variable vs. WOB = [100-200] kN which is a combination of 90 simulation runs for each graph. The parameters that will be studied for the bench are weight on bit, rotary speed and drill pipe inertia.

Observations:

- High WOB values results in occurrence of stick/slip phenomena.
- At high RPM values (>150) no stick/slip were detected.
- An increase in RPM can reduce stick/slip for high WOB values.
- Increasing J_d values will reduce stick/slip.

1- Rotary speed change

TABLE 4.7 is transformed into FIGURE 4.10 to observe minimum rotary speed needed to avoid stick/slip for weight on bit values as stick/slip is the area is red it is shown. An example for that is taking 60kN and observing that stick occurs at 40 RPM and below thus in order to avoid stick/slip 60kN the rotary speed must exceed 40 RPM.

TABLE 4.7:WOB vs. Rotary Speed

RPM	**WOB**									
	100000	110000	120000	130000	140000	150000	160000	170000	180000	200000
30										
60										
80										
90										
100										
110										
120										
150										
200										

*The highlighted blocks represent stick/slip occurrence.

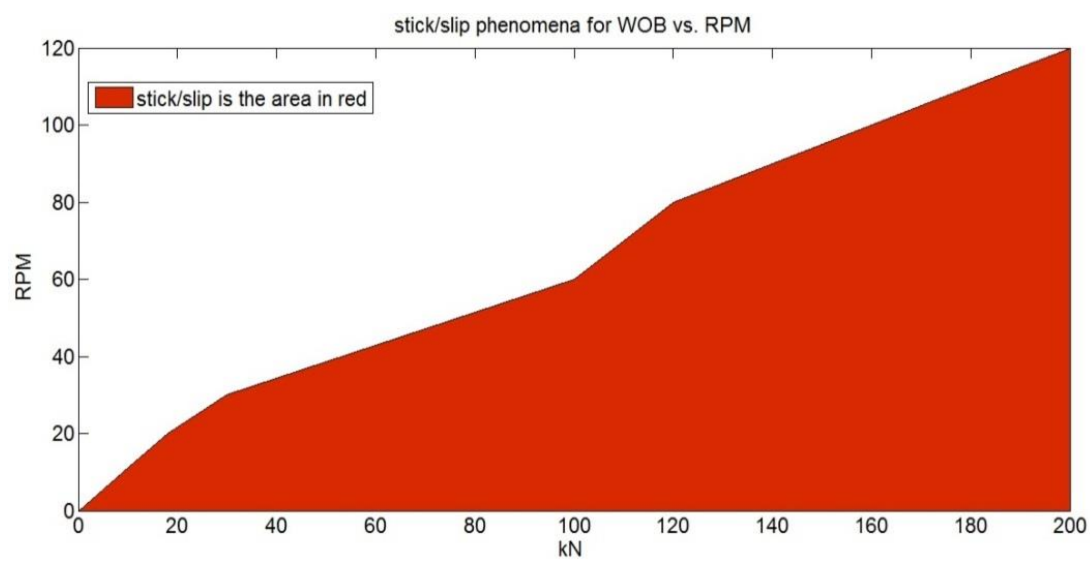


FIGURE 4.10:WOB vs. Rotary Speed

2- Drill pipe inertia change

TABLE 4.8 is transformed into FIGURE 4.11 to observe minimum drill pipe inertia needed avoid stick/slip for weight on bit values. For example, in order to avoid a stick/slip situation at WOB = 160kN the drill pipe inertia must exceed 700 $kg.m^{2.}$

TABLE 4.8:WOB vs. J_d

Jd	**WOB**									
	100000	110000	120000	130000	140000	150000	160000	170000	180000	200000
100										
200										
300										
400										
600										
700										
900										
1000										
1500										

*The highlighted blocks represent stick/slip occurrence.

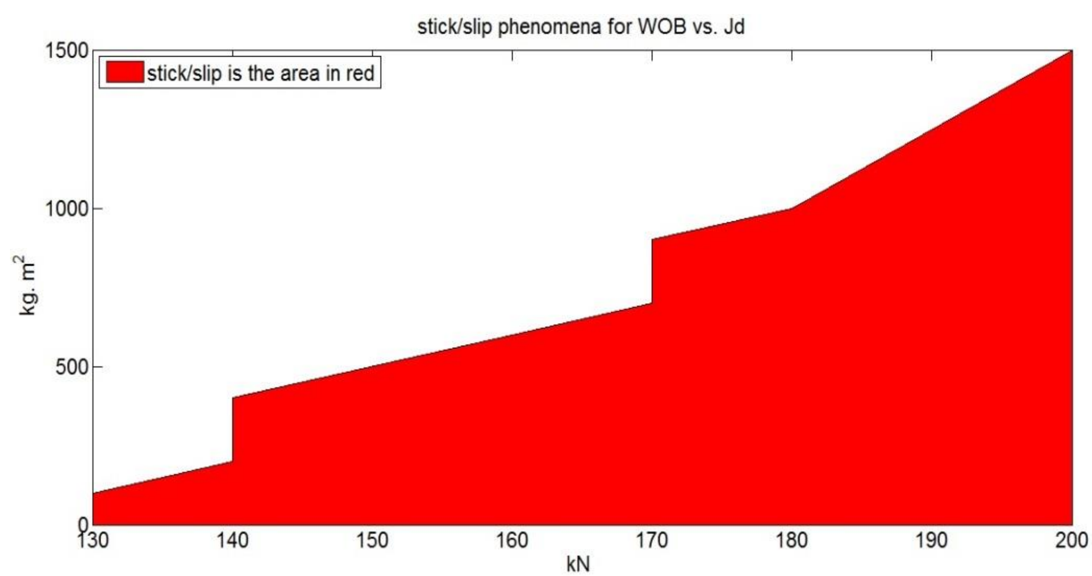


FIGURE 4.11: WOB vs. J_d

4.4.2 K421 Drillship

For the case study the only parameter that will be changed is the length of the drill pipe while keeping the BHA parameters constant. The parametric study was done on all three sections and reducing the length of the pipe showed reduction in the stick/slip phenomena.

TABLE 4.9 shows an inverse relationship between pipe length and WOB at which stick/slip start to occur. As seen when the length of the drill pipe is shortened it leads to elimination or reduction of stick/slip phenomena. From the observation the optimum length is 49.7 m. As long as the WOB does not exceed 90 kN, stick/slip phenomena won't occur.

TABLE 4.9 Sub-surface drilling pipe length vs WOB

No.	**Pipe length (m)**	**WOB which stick/slip starts to occur at kN**
1	49.7	90
2	80.2	70
3	110.7	70
4	141.2	40
5	171.6	40
6	202.1	30
7	232.6	30

The results for intermediate drilling are similar to sub-surface drilling, as shown in TABLE 4.10 shortening the pipe length leads to occurrence of stick/slip for higher WOB, at 940.2 no stick/slip was detected for the WOB used thus it is considered the optimum pipe length for the intermediate drilling section.

TABLE 4.10: Intermediate drilling pipe length vs WOB

No.	**Pipe length (m)**	**WOB which stick/slip starts to occur at kN**
1	940.2	NA
2	1140.2	90
3	1340.2	80
4	1540.2	70
5	1740.2	70
6	1940.2	70

The results for deep drilling as shown in TABLE 4.11 shortening the pipe length did not leads to occurrence of stick/slip for higher WOB but the opposite, meaning the optimum pipe length for the deep drilling case is 3154.68 as stick/slip starts to occur at 70kN while after shortening the length the stick/slip starts to occurs at lower WOB.

TABLE 4.11: Deep drilling pipe length vs WOB

No.	**Pipe length (m)**	**WOB which stick/slip starts to occur at kN**
1	554.6	60
2	1154.6	60
3	2154.6	60
4	3154.6	70

4.5 Effect of Bit Wear

The last aspect to be reviewed is bit wear in order to identify its effects on the drill-string dynamics, as the model introduced includes one parameter that relates to bit wear which is the bit-specific coefficient of sliding friction μ. The first step is to show how μ relates to bit wear and CCS:

From Chapter 2 the following formulas were introduced:

- $\mu = 0.9402 \ * e^{(-8\times10^{-6} * CCS)}$ (4.4)

The above equation shows μ as a function of CCS.

- $W_f = k_{wf}(\frac{WOB}{Nc})^{\rho} \cdot \frac{1}{S^{\tau} \cdot Aw^{\rho+1}}$ (4.5)

The above equation shows W_f as a function of CCS, assuming all the other parameters are constant. The relationship between W_f and CCS is an inverse relationship.
From TABLE 4.12 we observe that the relationship between rock CCS and μ is an inverse relationship. The μ values will be used for the parametric study to observe the effects that bit wear has on the model results.

TABLE 4.12: Rock Types with their corresponding CCS and μ Values

Rock Type	**CCS**	**μ**
Crab Orchard	66,000 psi	0.5545
Carthage Marble	36,226 psi	0.7037
Catoosa shale	18,500psi	0.8109

Since the model introduced doesn't include the cutter force, however considers one aspect of the bit/rock interaction which is μ and the equations found relates μ to CCS and CCS is to W_f, and finally μ to W_f.

From FIGURE 4.12 it is observed that low values of μ induce stick/slip at very high WOB, thus as μ values increase stick/slip occurs at lower WOB. As W_f has a direct relationship with μ, it shows a relationship between the wear function and stick/slip. FIGURE 4.13 indicates a direct relationship between μ and stick/slip for any WOB thus concluding that a direct relationship between bit wear and stick/slip exists Also as μ has an inverse relationship with the rock confined compressive strength, formations with high CCS values will have positive effect on drilling as stick/slip occurrence is expected to minimal.

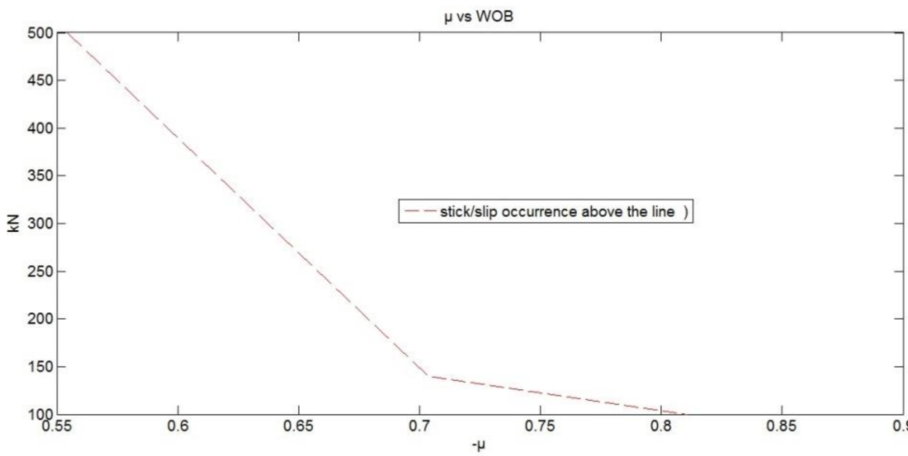


FIGURE 4.12:μ vs. WOB

FIGURE 4.13 supports the statement above as it shows an inverse relationship between CCS and stick/slip for any WOB, and as CCS has an inverse relationship with both W_f and μ. The statement above is supported.

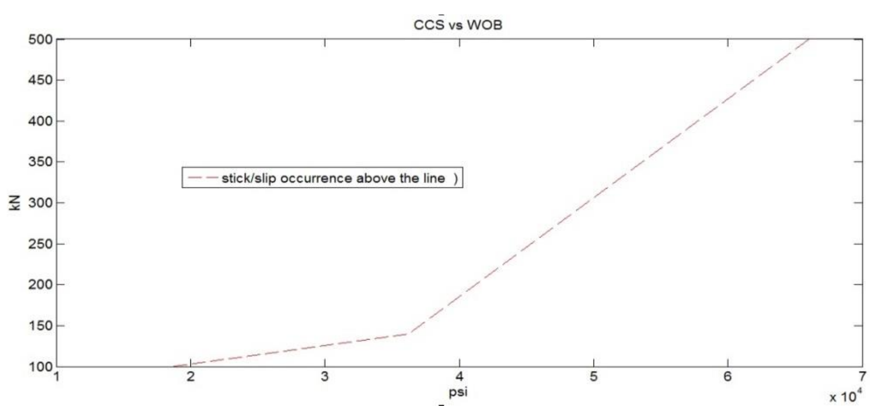


FIGURE 4.13: CCS vs. WOB

CHAPTER 5

CONCLUSION AND RECOMMENDATION

5.1 Conclusion

A drill-string model was developed using MATLAB R2013a, the model was validated and a parametric study was carried out to observe the effects that the operating parameters have on the stick/slip phenomena. The model used non-linear differential equations to represent the drill-string, BHA and the bit/rock interaction.

Parameters such as WOB, RPM and CCS were studied to observe the effect they have on the stick/clip phenomena and these parameters were tested using real field data. The study showed that increasing WOB increases stick/slip. However, increasing RPM seems to eliminate stick/slip. Relating CCS to bit wear allowed to observe the relationship between and stick/slip and was found that the severity of stick/slip increases as bit wear increases. The simulation results show similar trends as observed in the field.

5.2 Recommendation

As bit wear is one of many factor contributing to the vibrations of the drill-string, developing a model for the drill-string dynamics that identifies the effects of PDC bit wear will have a great impact on selecting optimum drilling parameters, thus further studies on bit wear parameters are recommended to provide a better understanding of the relationship between bit wear and the drill-string dynamics.

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APPENDICES

A1.Gantt chart

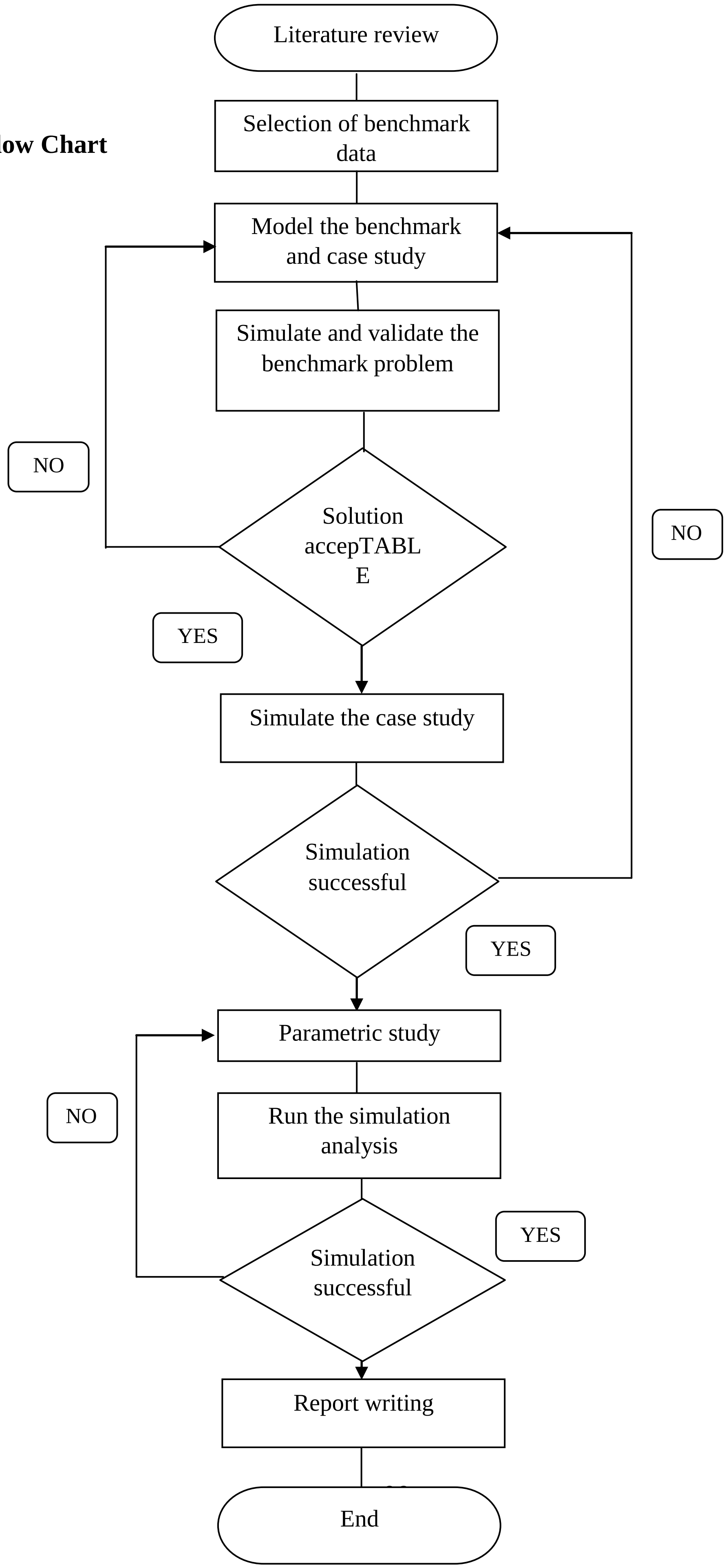
FYP 1

Description	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Literature review	■	■	■	■	■	■								
Selection of benchmark data						■	■	■						
Modeling the benchmark and case study problem								■	■	■	■			
Simulate the benchmark problem											■	■	■	■

FYP 2

Description	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Simulate the case study	■	■	■											
Parametric study				■	■	■								
Further simulation analysis							■	■	■	■	■			
Report writing											■	■	■	■

A2. Flow Chart



A3.MATLAB code for validation of benchmark

```
functionyout = fun_Patil2013()
% Model presented in Navarro-Lopez and Suarez (2004a)

%% Initialization
tspan = [0 100];
x0 = [0 0 0 0];
Omega = 100;
dthm = Omega*2*pi/60;
Wob = [ 10000   60000   80000   100000  120000  140000  160000  180000  200000
500000];
FIGURE (1)
fori=1:10
   [t,y] = ode15s(@SS_Patil,tspan,x0,[],Wob(i),dthm);
FIGURE (i)
subplot(1,2,1)
plot(t,60/(2*pi).*y(:,2),'r','LineWidth',1.5); hold on
plot(t,60/(2*pi).*y(:,4),'--b','LineWidth',2); hold on
xlabel('$t$','Interpreter','latex')
ylabel('$\dot{\varphi} (rad/s)$','Interpreter','LaTex')
subplot(1,2,2)
   plot3(y(:,1)-y(:,3),y(:,2),y(:,4),'r','LineWidth',1.5); hold on
%    plot3(y(:,2),y(:,4),y(:,6),'r','LineWidth',1.5); hold on
gridon
yout.y{i} = [t y];
end%% get info specific to the axes you plan to plot into
set(gcf,'Units','normalized')
set(gca,'Units','normalized')
ax = axis;
ap = get(gca,'Position');

%% annotation from 1,2 to 3,4
xo = t(25:26);
yo = y(25:26);
xp = (xo-ax(1))/(ax(2)-ax(1))*ap(3)+ap(1);
yp = (yo-ax(3))/(ax(4)-ax(3))*ap(4)+ap(2);
ah=annotation('arrow',xp,yp,'Color','k');

%% functions
function dx = SS_Patil(t,x,Wob,dthm)
thd = x(1); dthd = x(2);
thb = x(3); dthb = x(4);
```

```
Jd=600; %Kg.m^2
cd=85; %Nms/rad
kd=500;%Nm/rad

Jb= 500;%Kg.m^2
cb=50; %Nms/rad
kb=900; %Nm/rad

cb1 = 50;

mu_cb = 0.5;
mu_sb = 0.8;

Rb = 8.5*0.0254*0.5;

mu_cb = 0.5; mu_sb = 0.8;
Dv = 1e-6;
gammab = 0.9; vf = 1;

% calculation of driving torque, Tm
thm = dthm*t;
Tsb = mu_sb*Wob*Rb;
Tcb = mu_cb*Wob*Rb;
Tab = cb*dthb;
mu_b = mu_cb + (mu_sb - mu_cb)*exp(-gammab/vf*abs(dthb));
Teb = cb*(dthd - dthb) + kb*(thd - thb) - Tab;
% calculation of Tfb
if abs(dthb) <Dv
Tfb = Tsb;
Tfb = min(abs(Teb),Tsb).*sign(Teb);
elseif abs(dthb) >= Dv
Tfb = Wob*Rb*mu_b*sign(dthb);
end
Tb = Tab + Tfb;
% State matrix
dx(1,1) = dthd;
dx(2,1) = 1/Jd.*(cd*(dthm - dthd) + kd*(thm - thd) - cb*(dthd - dthb) - kb*(thd - thb));
dx(3,1) = dthb;
dx(4,1) = 1/Jb*(cb*(dthd - dthb) + kb*(thd - thb) - Tb);
% Last line
```

A4. MATLAB code used for the case study

- Calculation of Sub-surface drilling data

```
%% Drillstring parameters
G = 77e+9;
Rho = 7850;
mu_mud = 1;
% Pipe geometry
Lp = 763.3*12*0.0254;
Dp = 5*0.0254;
dp = 4.276*0.0254;

% BHA geometry
Lb = 736.7*12*0.0254;
Db = 5*0.0254;
db = 2.875*0.0254;

% Bit geometry
Dbit = 17.5*0.0254
```

- Calculation of intermediate drilling data

```
%% Drillstring parameters
G = 77e+9;
Rho = 7850;
mu_mud = 1;
% Pipe geometry
Lp = 6365.6*12*0.0254;
Dp = 5*0.0254;
dp = 4.276*0.0254;

% BHA geometry
Lb = 567.2*12*0.0254;
Db = 5*0.0254;
db = 2.875*0.0254;

% Bit geometry
Dbit = 17.5*0.0254
```

- Calculation of deep drilling data

```
%% Drillstring parameters
G = 77e+9;
Rho = 7850;
mu_mud = 1;
% Pipe geometry
Lp = 10350*12*0.0254;
Dp = 5*0.0254;
dp = 4.276*0.0254;

% BHA geometry
Lb = 1156*12*0.0254;
Db = 6.6*0.0254;
db = 3*0.0254;

% Bit geometry
Dbit = 8.5*0.0254;
```

- MATLAB code used for simulation

```
%% Calculation of modelK421 parameters
Jd = pi/32*Rho*Lp*(Dp^4 - dp^4);
kd = 2*pi/32*G/Lp*(Dp^4 - dp^4);
zeta = 0.0001;
cd = 2*zeta*sqrt(Jd*kd);
cd1 = 2*pi/(Dbit/2 - Dp/2)*mu_mud*(Dp/2)^3*Lp;

Jb = pi/32*Rho*Lb*(Db^4 - db^4);
kb = kd; % why the same ? diff di
cb = cd; % if diff di k value will be diff
cb1 = 2*pi/(Dbit/2 - Db/2)*mu_mud*(Db/2)^3*Lb + ...
pi*mu_mud*Db^2*(Db - 0.003)/(2*(Dbit - Db)) + pi*mu_mud/(32*0.003)*Db^2;

modelK421.Jd = Jd; %Kg.m^2
modelK421.cd = cd; %Nms/rad
modelK421.kd = kd;%Nm/rad
modelK421.cd1 = cd1;

modelK421.Jb = Jb;%Kg.m^2
modelK421.cb = cb;%Nms/rad
modelK421.kb = kb;%Nm/rad
modelK421.cb1 = cb1;
modelK421.Dbit = Dbit;
```

```
%% Calculation of initial conditions
Omega = 50;
thm = 0;
dthm = Omega*2*pi/60;
Wob = linspace(10000,90000,4);

%% Initialization
tspan = [0 100];
x0 = [0 0 0 0];
FIGURE (1)
fori=1:10
x0 = SS_K421_InitialCondition(Wob(i),dthm,thm,modelK421);
   [t,y] = ode15s(@SS_K421,tspan,x0,[],Wob(i),dthm,modelK421);
FIGURE (i)
subplot(1,2,1)
plot(t,60/(2*pi).*y(:,2),'r','LineWidth',1.5); hold on
plot(t,60/(2*pi).*y(:,4),'--b','LineWidth',2); hold on
xlabel('$t$','Interpreter','latex')
ylabel('$\dot{\varphi} (RPM)$','Interpreter','LaTex')
subplot(1,2,2)
   plot3(y(:,1)-y(:,3),y(:,2),y(:,4),'r','LineWidth',1.5); hold on
%     plot3(y(:,2),y(:,4),y(:,6),'r','LineWidth',1.5); hold on
gridon
yout.y{i} = [t y];
end%% get info specific to the axes you plan to plot into
% set(gcf,'Units','normalized')
% set(gca,'Units','normalized')
% ax = axis;
% ap = get(gca,'Position');

%% annotation from 1,2 to 3,4
% xo = t(25:26);
% yo = y(25:26);
% xp = (xo-ax(1))/(ax(2)-ax(1))*ap(3)+ap(1);
% yp = (yo-ax(3))/(ax(4)-ax(3))*ap(4)+ap(2);
% ah=annotation('arrow',xp,yp,'Color','k');

%% functions
function dx = SS_K421(t,u,W,dthm,modelK421)
thd = u(1); dthd = u(2);
thb = u(3); dthb = u(4);

Jd = modelK421.Jd; %Kg.m^2
cd = modelK421.cd; %Nms/rad
kd = modelK421.kd; %Nm/rad
cd1 = modelK421.cd1;
```

```
Jb = modelK421.Jb; %Kg.m^2
cb = modelK421.cb; %Nms/rad
kb = modelK421.kb; %Nm/rad
cb1 = modelK421.cb1;

mu_cb = 0.8; mu_sb = 0.5;
Rb = 0.5*modelK421.Dbit;
mu_cb = 0.5; mu_sb = 0.8;
Dv = 1e-6;
gammab = 0.9; vf = 1;

% calculation of driving torque, Tm
thm = dthm*t;
Tsb = mu_sb*W*Rb;
Tcb = mu_cb*W*Rb;
Tab = cb1*dthb;
mu_b = mu_cb + (mu_sb - mu_cb)*exp(-gammab/vf*abs(dthb));
Teb = cb*(dthd - dthb) + kb*(thd - thb) - Tab;
% calculation of Tfb
if abs(dthb) <Dv
Tfb = Tsb;
Tfb = min(abs(Teb),Tsb).*sign(Teb);
elseif abs(dthb) >= Dv
Tfb = W*Rb*mu_b*sign(dthb);
end
Tb = Tab + Tfb;
% State matrix
dx(1,1) = dthd;
dx(2,1) = 1/Jd.*(cd*(dthm - dthd) + kd*(thm - thd) - cb*(dthd - dthb) - kb*(thd - thb) -
cd1*dthd) ;
dx(3,1) = dthb;
dx(4,1) = 1/Jb.*(cb*(dthd - dthb) + kb*(thd - thb) - Tb);
% Last line
functionyout = SS_K421_InitialCondition(W,dthm,thm,modelK421)
Jd = modelK421.Jd; %Kg.m^2
cd = modelK421.cd; %Nms/rad
kd = modelK421.kd; %Nm/rad
cd1 = modelK421.cd1;

Jb = modelK421.Jb; %Kg.m^2
cb = modelK421.cb; %Nms/rad
kb = modelK421.kb; %Nm/rad
cb1 = modelK421.cb1;
Rb = 0.5*modelK421.Dbit;
```

```
mu_cb = 0.5; mu_sb = 0.6;
Dv = 1e-6;
gammab = 0.9; vf = 1;

% calculation of driving torque, Tm
Tsb = mu_sb*W*Rb;
Tcb = mu_cb*W*Rb;
Tab = cb1*dthm;
mu_b = mu_cb + (mu_sb - mu_cb)*exp(-gammab/vf*abs(dthm));
Tfb = W*Rb*mu_b;
Tb = Tab + Tfb;

% Initial state
thd = 1/kd*(-(cd1+cb1)*dthm - Tb);
dthd = dthm;
thb = -cd1/kd*dthm -(kb+kd)/(kb*kd)*Tb;
dthb = dthm;
yout = [thddthdthbdthb];

% Last line
```

A.5 Runge-Kutta Method:

Runge-Kutta method here after called as RK method is the generalization of the concept used in Modified Euler's method.

In Modified Eulers method the slope of the solution curve has been approximated with the slopes of the curve at the end points of the each sub interval in computing the solution. The natural generalization of this concept is computing the slope by taking a weighted average of the slopes taken at more number of points in each sub interval. However, the implementation of the scheme differs from Modified Eulers method so that the developed algorithm is explicit in nature. The final form of the scheme is of the form

$\mathbf{y_{i+1} = y_i +}$ **(**weighted average of the slopes**)** for $\mathbf{i = 0, 1, 2 \ldots}$

Where $\mathbf{h}$ is the step length and $\mathbf{y_i}$ and $\mathbf{y_{i+1}}$ are the values of $\mathbf{y}$ at $\mathbf{x_i}$ and $\mathbf{x_{i+1}}$ respectively.

In general, the slope is computed at various points $\mathbf{x_s}$ in each sub interval $\mathbf{[x_i, x_{i+1}]}$ and multiplied them with the step length $\mathbf{h}$ and then weighted average of it is then added to $\mathbf{y_i}$ to compute $\mathbf{y_{i+1}}$. Thus the RK method with $\mathbf{v}$ slopes called as v-stage RK method can be written as

$$\mathbf{K_1 = h\, f\, (x_i, y_i)}$$

$$\mathbf{K_2 = h\, f\, (x_i + c_2 h, y_i + a_{21} K_1)}$$

$$\mathbf{K_3 = h\, f\, (x_i + c_3 h, y_i + a_{31} K_1 + a_{32} K_2)}$$

. . .

. . .

. . .

$$\mathbf{K_v = h\, f\, (x_i + c_v h, y_i + a_{v1} K_1 + a_{v2} K_2 + \ldots + a_{vv-1} K_{v-1})}$$