Reserves Estimation and Economic Modelling Of Tight Gas/ Unconventional Reservoir

By

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14991

Dissertation submitted in partial fulfilment of

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CERTIFICATION OF APPROVAL

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A project dissertation submitted to the

Petroleum Enginerring Programme

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Approved by,

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(A.P. Dr. Syed Mohammad Mahmood)

UNIVERSITI TEKNOLOGI PETRONAS

TRONOH, PERAK

JANUARY 2015

CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

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(LEE SHAU MENG)

ABSTRACT

Reserves estimation for oil and gas reservoir has always been an interesting topic in the effort of estimating the volume of hydrocarbon in the world, which can be recoverable and economically produced. Recently, unconventional gas reservoirs have becoming hot topic of interest and playing an ever increasing role towards satisfying current and future energy demands, due to their high possibility for gas production. Therefore, this paper will discuss about the reserves estimation for tight gas/ unconventional reservoirs alongside with the economic modelling. Numerous methods of reserves estimation has been introduced decades ago to accurately estimate the hydrocarbon volumes. However, not all methods of reserves estimation is suitable for tight gas/ unconventional reservoirs. With the advances of technology, sophisticated methods or techniques are being applied to explore the hydrocarbon world in the extent that we never encounter before, especially in the context of unconventional resources. Therefore, comprehensives information regarding tight gas/ unconventional reservoirs will be discussed in this project. Ultimately, our goal in this project is to identify different methods of reserves estimation for tight gas/ unconventional reservoirs and study the best available method as well as introduce any other approach or modification made for the method of reserves estimation.

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ABBREVIATIONS & NOMENCLATURES

- DCA = Decline Curve Analysis
- EUR = Estimated Ultimate Recovery
- A = decline factor
- ao =initial instantaneous decline factor
- e = exponent
- q = flow rate, Mcf/D
- qo = initial flow rate
- t = time, hours or days

CHAPTER 1

1. INTRODUCTION

1.1 Background Study

The abundant energy demand as well as the higher depletion rate of the existing hydrocarbon reserves in the worldwide scale have caused a disparity between required energy and supply (Zahid et al., 2007). Approximately trillions cubic feet (TCF) of gas are exist in the world that could be produced to fulfill the world energy demand. Unconventional reservoirs which contain tight gas, gas hydrates, shale gas, coal bed methane and others will be an important part of the global energy mix for decades to come. Mainly because of the characteristic of low permeability, these unconventional reservoir systems are identified as unique and tough challenge as they are difficult for characterize and production. To overcome the energy demand issue, while at the same time maintaining a continuous supply of energy, the global oil and gas industry is now focusing and keep their eyes on the Exploration & Production (E&P) segment of the unconventional reservoir resources. Vast amount of reserves, very high potential across the time, economics such as gas price and cost are all the influencing factors that plays vital role on these unconventional resources in the upcoming future time.

According to the data, the total unconventional gas covers a large amount across the world of about thirty-two thousand trillion cubic feet (TCF) of gas-in-place (GIP) while seven thousand and four hundred TCF in tight gas reservoir (Dr. Zillur Rahim et al., n.d.). Tight gas is the normally known to people as the low permeability reservoirs which yield dry natural gas. Despite that to produce tight gas is not an easy task, however by looking at long term perspective, the large quantity of these unconventional gas is considered as the most important energy source for the future of mankind. It is believed that producing tight gas can help us to mitigate and balancing the supply and demand of energy in the next five to twenty years.

Unconventional gas resources can produce significant potential growth of gas production in the future, which currently accounting for 43% of the United States (U.S.) gas production (Abdelaziz et al., 2010). Therefore, it is very vital to estimate the gas reserves and to predict future performance of these tight gas or unconventional reservoir for optimize production in order to meet with the world energy demand. Depends on the availability of data, the most commonly used methods to estimate reserves are shown as below:

- Volumetric
- Material balance
- Decline Curves
- Reservoir Models

All of these methods will be discussed in details, as of their advantages and limitation especially when used for unconventional or tight gas reservoirs. Besides that, any other approach or modification made to the methods mentioned above will also be attempted and carried out in the case study throughout this project. In addition, economic aspects or economical modelling should also studied and investigated at the end of this project.

1.2 Problem Statement

This research is expected to have further understanding and perception on the following perspectives:

1. What are the methods for reserves estimation for tight gas/ unconventional reservoir and their respective advantages and limitation of each of the methods?
2. Is there any other approach or modification made to obtain more accurate results for reserves estimation of tight gas/ unconventional reservoir?
3. What are the economic aspects or economic modelling of tight gas/ unconventional reservoir?

1.3 Objectives

Basically, the aim and objectives of this Final Year Project (FYP) includes:

- To identify different methods of reserves estimation for tight gas/ unconventional reservoir & their respective advantages and limitation of each methods
- To develop case study of other approach or modification made to estimate reserves for tight gas/ unconventional reservoir
- To investigate economic aspects and economic modelling of tight gas/ unconventional reservoir

1.4 Scope of Study

Unlike conventional reservoirs, the so called tight gas or unconventional reservoir normally does not follow the existing models, methods and physical properties that applied in conventional exploration and production engineering. What makes the differences is the characterization of the reservoirs and production physics. While conventional reservoirs show a certain porosity and permeability in millidarcy, whereas in unconventional these parameters can be much smaller such as microdarcy or nanodarcy.

Therefore, the scope of study of this project is to investigate these reservoir systems and describe methods or technique to estimate the reserves of these tight gas reservoir systems. Some economic aspects will also be introduced and discussed in this project. Last but not least, effort on identifying other approach or modification for reserves estimation will be discussed by the end of this project.

CHAPTER 2

2. LITERATURE REVIEW

2.1 Conventional Vs. Unconventional Reservoir

First of all, conventional reservoir is considered simple and easy in terms of geological formations because they do not require sophisticated technology for development. Due to the facts that it is easier and economically to produce them, conventional reservoir has been the focus of oil and gas industry since long time ago. Conventional gas normally accumulate in reservoir that have relatively high porosity. The reservoir may exist in different rock formations such as sandstones, carbonates and siltstones which occur naturally to enable the gas permeable enough to flow to the wellbore on itself. On the other hands, unconventional reservoir has very low permeability, which by other words, the oil or natural gas is unable to flow through the rock and into the producing well naturally. The solution for unconventional reservoir is that oil and gas industry will use some well stimulation technique such as hydraulic fracturing, the purpose of fracking is to fracture the rock formation so that the oil or natural gas can flow through. The drawback is that more time and cost is required to carry out the stimulation for unconventional reservoir as compared to conventional reservoir which do not require that.

However, most of the natural gas resources nowadays are found from the unconventional reservoir. Unconventional gas reservoirs are those natural gas that deposited in impermeable rock formation such as shale, coal bed and tight sand. To gain access to these resources, the method that can be utilised is horizontal drilling and hydraulic fracturing. Therefore, it is essential for us to understand the geology of unconventional reservoir before the drilling and production. FIGURE 1 below shows the difference between conventional and unconventional reservoir.

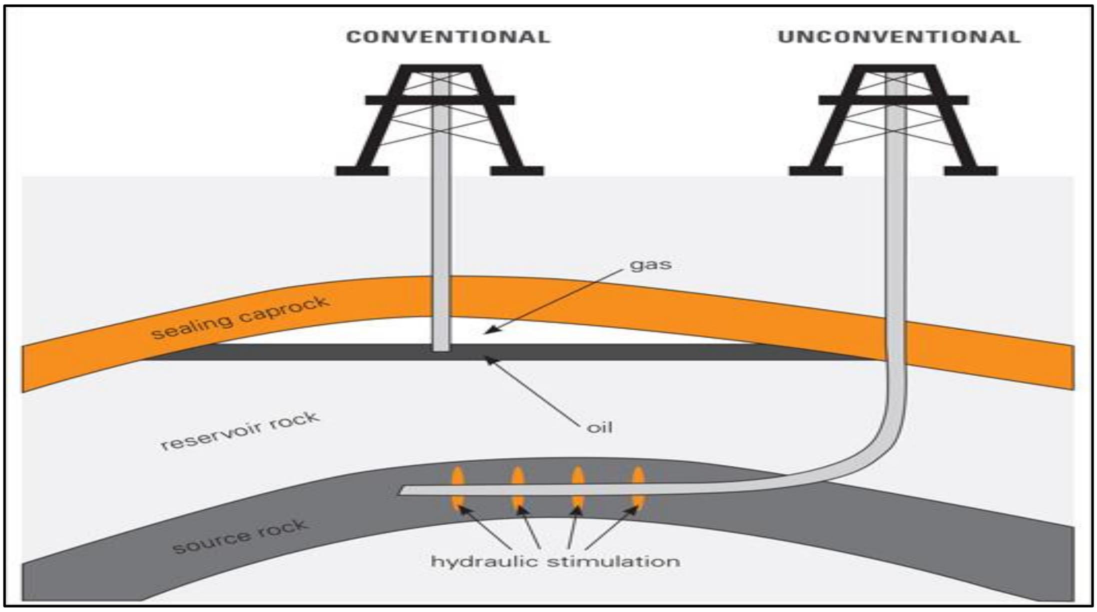


FIGURE 1. Conventional Vs Unconventional Reservoir (SGS, 2014)

Besides, some might define unconventional resources using the parameter of viscosity and permeability (Cander, 2012). As shown in the FIGURE 2 below, unconventional resources can be portrayed by using viscosity vs. permeability graph, which clearly shows the differences between unconventional and conventional reservoirs. In order to produce economically at certain flow rate, advanced technology is needed to increase the permeability or decrease the viscosity.

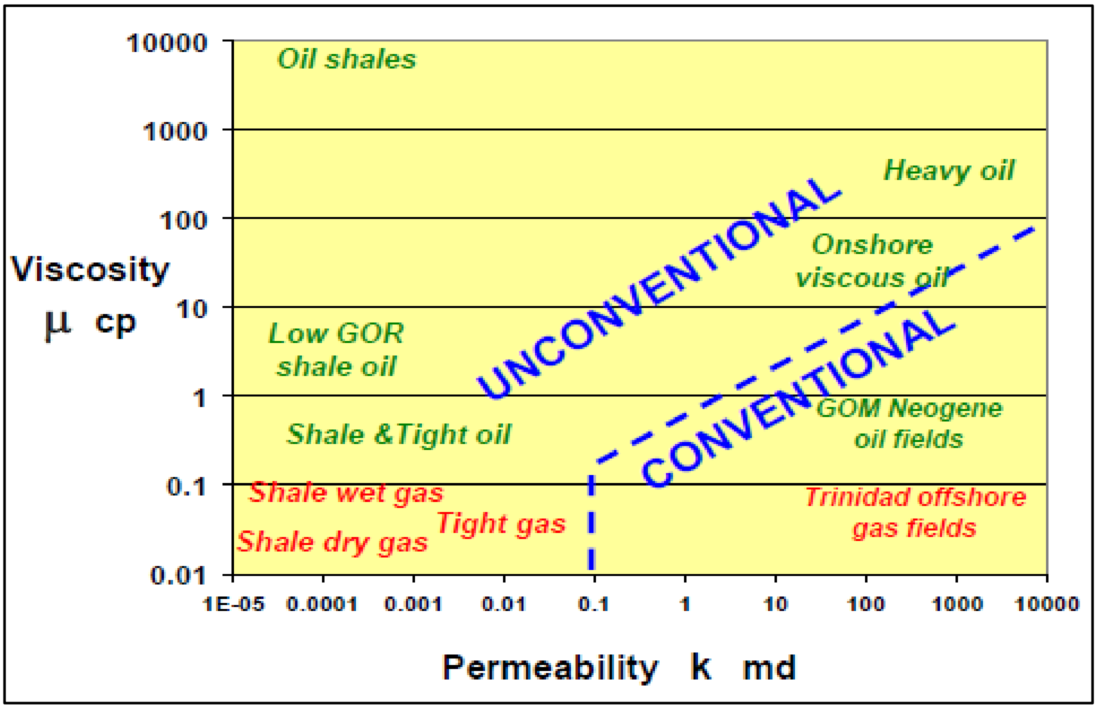


FIGURE 2. Viscosity Vs Permeability (Cander, 2012)

2.2 Tight Gas/ Unconventional Reservoir

Out of the total unconventional resources and large amount of reserves that have not been developed, tight gas makes up about three quarters out of it (Abdelaziz et al., 2010). Nevertheless, economical production of tight gas is considered difficult because of their low permeability and porosity characteristic although it is high potential for future development and demand. Therefore, lower production rates is comes from tight gas reservoirs due to its lower permeability. Majority of the tight gas reservoirs are recognised by high value height (100 to 1000 feet range) and multi-layered where we can use hydraulic fracturing to improve the production rates. Although with the advanced and state-of-the-art technology such as drilling, completion and stimulation technique, unconventional gas reservoir is considered as sophisticated and complicated in which the results are always unpredictable.

Nevertheless, the first important thing is to understand about the tight gas. So, what is tight gas? Normally, the definition of tight gas is low permeability reservoirs and most of its production is dry natural gas. Basically, optimum gas production can be achieved by some major hydraulic fracture treatment. Besides, horizontal wells can also be drilled and stimulated especially in some tight gas reservoirs that are already naturally fractured. Tight gas reservoir can be best defined as "the reservoirs that require massive hydraulic fracture treatment by horizontal or multilateral wellbores in order to produce natural gas economically" (Holditch, 2006). Tight gas reservoir could be a single or multiple layers, regardless of its depth, pressure or temperature, lenticular or blanket, homogeneous or naturally fractured. (Holditch, 2006).

2.3 Resource Triangle

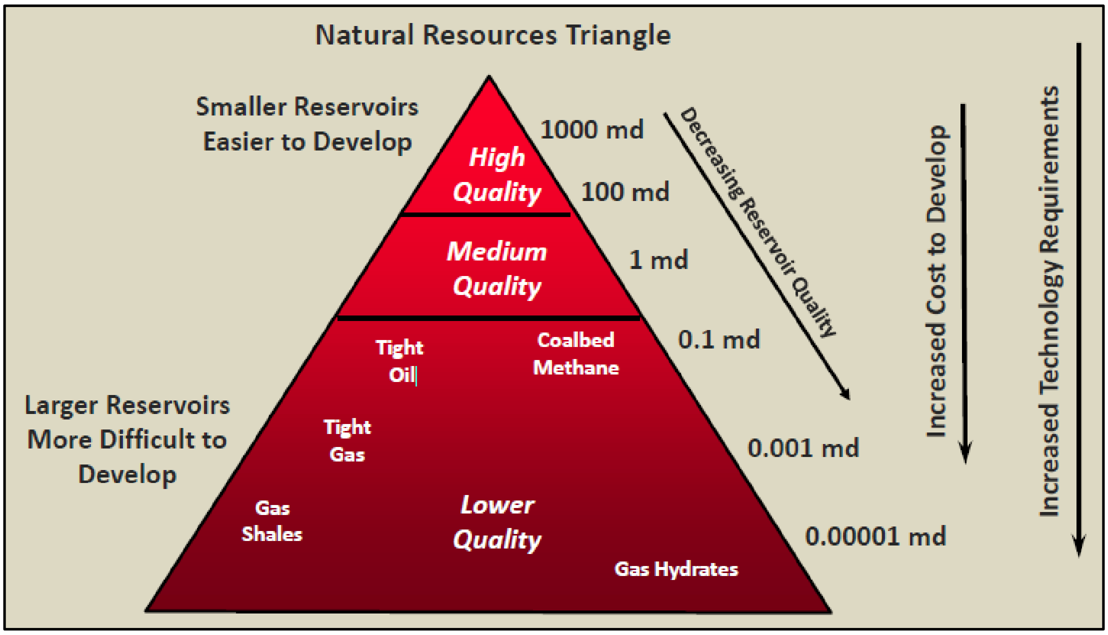


FIGURE 3. Resource triangle for natural gas (Schmezl, 2009)

In 1970s, Masters (1979) has introduced a concept called resource triangle to allocate the big gas field, which they believed that all natural resources are spread out naturally in log. FIGURE 3 clearly shows the resource triangle. According to the triangle, from top to bottom, the reservoirs are decreasing in permeability and grade. Higher quality reservoirs or in another words higher permeability reservoirs are positioned at the top of the triangle, but smaller in size. On the other hand, the low permeability reservoirs at the bottom of the triangle, however, are much larger in size. Different values of formation permeability for natural gas are also shown in the scale at the right side of the figure, which range from 100md to 0.0001md. Normally, unconventional resources are those less than 0.1md. Example of unconventional resources including tight gas, shale gas, coal gas, gas hydrates and others. Hence, it can be concluded that, from the top to the bottom of triangle, it is increasing in size or volume, technical challenge or technology and higher costs while at the same time decreasing in terms of grade or quality and also permeability. In the world, every hydrocarbon-producing basin can using the concept of resource triangle. Moreover, it is also possible to know the amount of hydrocarbon trapped in the low quality reservoirs, given condition that volumes of oil and gas inside the high permeability and quality reservoirs in a certain basin is known as well.

CHAPTER 3

3. METHODOLOGY AND PROJECT WORK

3.1 Research Methodology

Some methodologies are identified and investigated during the progression of this project. For instances:

3.1.1 Analysis

Collect and analyse the information regarding the tight gas or unconventional reservoirs to fully understand its characteristic and properties. Comprehensive literature study on tight gas, unconventional reservoirs and resources triangle will be discussed in this research.

3.1.2 Case Study

Conduct a few case studies on the methods of reserves estimation. Investigate and compare between all available options to select the best method to estimate reserves more effectively. Decline Curve Analysis (DCA) will be the main focus in this research due to the limited data available to carry out reservoir simulation of tight gas/ unconventional reservoir. Detailed study on DCA will be carried out and differences between conventional & unconventional reservoirs in terms of DCA will be compared and contrasted in this research.

3.1.3 Evaluation

Evaluate the best method for reserves estimation and economic modelling for tight gas/ unconventional reservoir. For example, DCA is currently the most suitable and available method for this research while more sufficient data is required for reservoir simulation method. Some other approaches or modifications made for the methods of reserve estimation also will be introduced in this report.

3.2 Project Activities

Based on the methodologies stated above, firstly it is essential to understand the concept of tight gas/ unconventional reservoir. Therefore, critical analysis on tight gas/ unconventional reservoir is being studied in literature review chapter as well as the concept of resource triangle is being introduced. Besides, the major reserves estimation methods such as Volumetric, Material Balance, Decline Curve and Reservoir Simulation Models are being discussed in the literature and the main focus in this research will be Decline Curve Analysis (DCA). In addition, studies on economic aspects and modelling also being carried out. In the results & discussion chapter, evaluation on Decline Curve Analysis (DCA) has been implemented and a simple case study has also been conducted, mainly on the differences of decline curve between conventional and unconventional reservoirs. More complicated and sophisticated case studies will be carried out in the upcoming research work. Furthermore, some discussion on the economical aspects and modelling also will be discussed in the result and discussion chapter. Lastly, any other approach or modification made for the method of reserves estimation will be introduced throughout this research project.

3.3 Key Project Milestones

FIGURE 4 below shows the key milestone for overall progress of the methodology in order to conduct Final Year Project (FYP) 1 smoothly and efficiently.

Week 1 - 2

- Project topic selection: " Reserves Estimation & Economic Modelling of Tight Gas/ Unconventional Reservoir".

Week 3 - 5

- Preliminary research work.
- Search and analyze all the articles/journals/ research paper.

Week 6 - 7

- Literature Review and Research Work.
- Propose suitable proposal for our topic and completed the Extended Proposal report.

Week 8 - 11

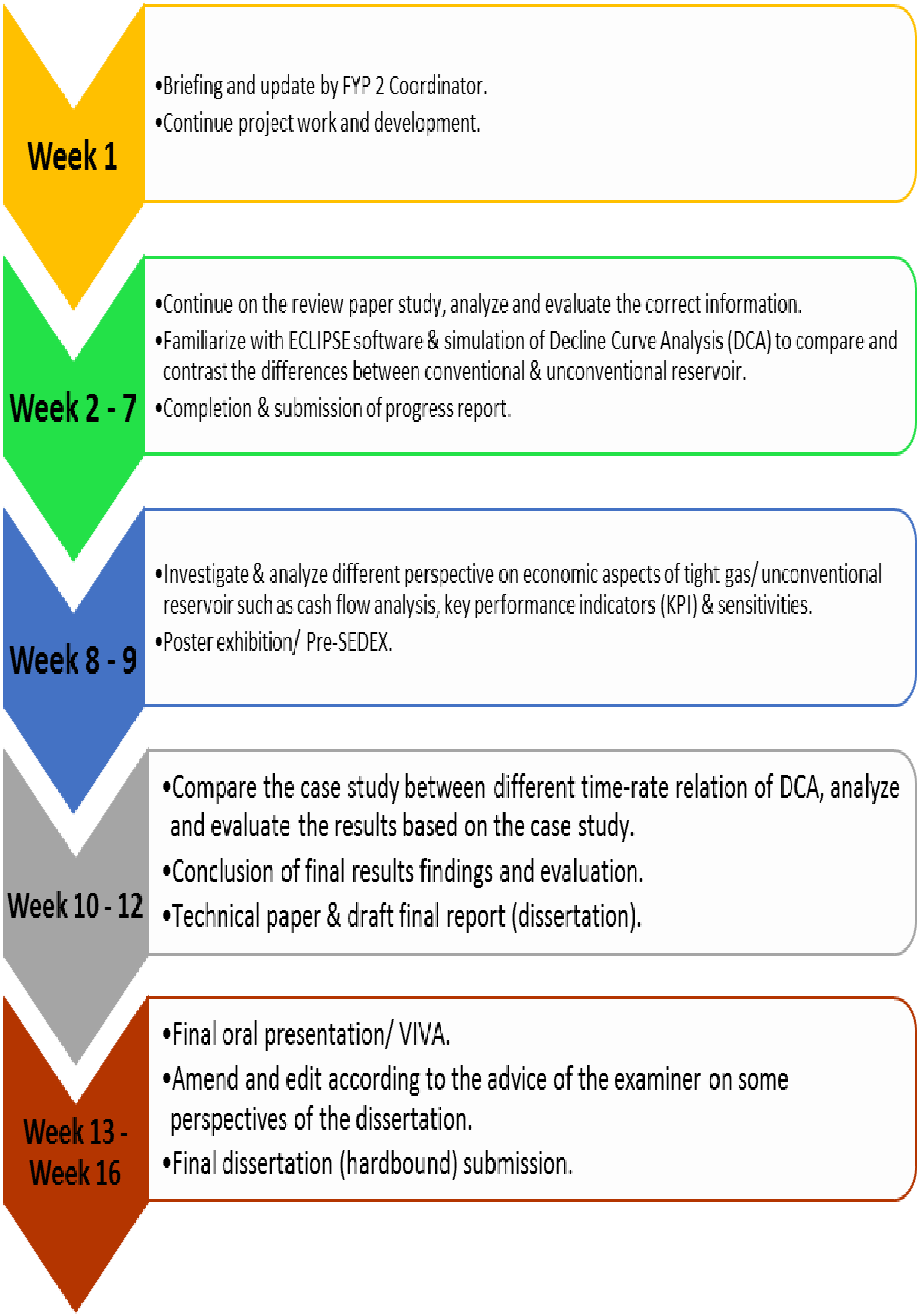
- Proposal Defence and progress evaluation.
- Literature Review on tight gas/ unconventional reservoir, resources triangle & methods of reserve estimation.
- Analyze on data and case of economic modelling and economic aspects.

Week 12 - 14

- Analyze the Decline Curve Analysis (DCA) method as the most suitable method reserves estimation for tight gas/ unconventional reservoir.
- Develop & evaluate a case study of DCA and provide relevant example.
- Completed the interim report and submission.

FIGURE 4. Key Milestone for FYP 1

FIGURE 5 below shows the key milestone for overall progress of the methodology in order to conduct Final Year Project (FYP) 2 smoothly and efficiently.



Week 1

- Briefing and update by FYP 2 Coordinator.
- Continue project work and development.

Week 2 - 7

- Continue on the review paper study, analyze and evaluate the correct information.
- Familiarize with ECLIPSE software & simulation of Decline Curve Analysis (DCA) to compare and contrast the differences between conventional & unconventional reservoir.
- Completion & submission of progress report.

Week 8 - 9

- Investigate & analyze different perspective on economic aspects of tight gas/ unconventional reservoir such as cash flow analysis, key performance indicators (KPI) & sensitivities.
- Poster exhibition/ Pre-SEDEX.

Week 10 - 12

- Compare the case study between different time-rate relation of DCA, analyze and evaluate the results based on the case study.
- Conclusion of final results findings and evaluation.
- Technical paper & draft final report (dissertation).

Week 13 - Week 16

- Final oral presentation/ VIVA.
- Amend and edit according to the advice of the examiner on some perspectives of the dissertation.
- Final dissertation (hardbound) submission.

FIGURE 5. Key Milestone for FYP 2

3.4 Project Timeline (Gantt Chart)

TABLE 1. Final Year Project (FYP) 1 Gantt Chart

Details	**Week**													
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
Project Title Selection and Confirmation	■	■												
Literature Review and Research		■	■	■	■	■	■							
Extended Proposal Submission						■								
Preparation for Proposal Defence							■							
Proposal Defence and Progress Evaluation								■	■					
Literature Review & Project Development								■	■	■	■	■		
Data Analyse & Evaluation										■	■	■		
Development & Evaluate the Case Study											■	■	■	
Interim Report Submission														■

TABLE 2. Final Year Project (FYP) 2 Gantt Chart

Details	**Week**															
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
Briefing & update by FYP 2 Coordinator	🟥															
Review Paper Study, Analyse & Evaluation	🟥	🟥	🟥	🟥	🟥	🟥	🟥									
Familiarize & Simulation of ECLIPSE		🟥	🟥	🟥	🟥	🟥	🟥									
Submission of Progress Report							🟥									
Investigate & Analyse on Economic Aspects								🟥	🟥	🟥	🟥	🟥				
Poster Exhibition/ Pre-SEDEX									🟥							
Compare Case Study & Evaluate Results										🟥	🟥	🟥				
Submission of Dissertation & Technical Paper												🟥				
Preparation for Final Oral Presentation/ Viva													🟥	🟥		
Amend and Final Edit of Dissertation & Submission of Hardbound Copy														🟥	🟥	🟥

CHAPTER 4

4. RESULTS AND DISCUSSION

4.1 Reserves Estimation Methods

Estimating reserves for unconventional reservoirs often encounter with challenges due to the limitation of conventional methods. This issue is obvious because hydrocarbon reserves are our main concern in reservoir planning and development. (Currie et al., 2010). TABLE 3 below shows the different conventional methods for reserves estimation and how these methods differentiate between conventional and unconventional reservoir.

TABLE 3. Reserves Estimation Methods for Tight Gas/ Unconventional Reservoir (Holditch, 2006)

METHOD	CONVENTIONAL GAS RESERVOIR	TIGHT GAS/ UNCONVENTIONAL RESERVOIR
Volumetric	Accurate in blanket reservoirs	Used only when no wells have been drilled
Material Balance	Accurate in depletion-drive reservoirs	Should never be used
Decline Curves	Exponential decline usually accurate	Hyperbolic decline must be used
Reservoir Models	Simulation of the field	Used to simulate individual wells

For conventional reservoirs with characteristic such as high permeability reservoir or blanket reservoir, volumetric method can best be used because the drainage area and the efficiency of recovery gas are known and hence volumetric method can provide precise estimation of reserves. However, volumetric method basically is not as reliable for tight gas or unconventional reservoirs due to the facts that drainage area and gas recovery efficiency of these reservoirs are not easy to be estimated and hence this method should only be used before any well is drilled and only act as last approach. After drilling and we can get production data, the analyst should use the production data available for reserves estimation.

Besides that, same theory applied to material balance method just like volumetric method, which is applicable only for high permeability gas reservoirs. Material balance method require precise gas production data and also reservoir pressure, in which accurate estimation of reservoir pressure is viable using Horner graphs as long as the well can be shut in for few hours or few days. On the other hands, material balance method should not be used in unconventional reservoir. It is because the current average reservoir pressure hardly can determined by shutting in the well for short period. In unconventional reservoir, it is essential to shut-in well or reservoir for long enough time as to collect the pressure data for estimation of the average reservoir pressure. Therefore, the outcome will results in underestimate of average reservoir pressure and thus affecting the ultimate gas recovery factor.

Hence, theoretically, the current best available method for reserves estimation in unconventional reservoirs is by using Decline Curves Analysis (DCA). DCA works well in both conventional and unconventional reservoir. For most of the gas reservoir, ultimate gas recovery can be found using exponential decline curve, in which the straight line should be extrapolated to the extent of economic limit or a fixed well life. In addition, it is common that large hydraulic fracture must be used to stimulate layered tight gas reservoirs before DCA can be applied. In this case, hyperbolic equation in DCA must be used to match fit the data and its economic limit is reached by extrapolation.

Last but not least, reservoir modelling method is known as the most precise and suitable method of reserves estimation especially for tight gas reservoirs. For instances, numerical-reservoir model or semi analytical model is used to match the production data. However, due to limitation of the data in this research study, only Decline Curve Analysis (DCA) will be given more focus at this stage. More details about how DCA works for tight gas or unconventional reservoirs will be discussed in this project, while reservoir modelling method or other alternatives for reserve estimation might be studied further in the future time.

4.2 Decline Curve Analysis

One of the most traditional method of reserves estimation is known as Arps decline curves. Engineers always modify the Arps decline curve to correlate with the production history (Arps, 1945). It is often carried out on the unconventional reservoir in order to estimate the reserves. Further study on Decline Curve Analysis (DCA) is carried out in this research. Evolution of DCA for the past few centuries was reviewed by J.J. Arps (1945) and then he proposed the mathematical relationships for exponential, hyperbolic and harmonic declines with the decline constant b ranging between 0 and 1 (Melvyn et al., 2012). In terms of tight gas reservoirs, it is essential to use hyperbolic equation. It is because the data is needed to be curve match as well as the economic limit being reached by data extrapolation. The hyperbolic decline equation is shown below:

$$q = \frac{q_o}{\left(1 + \frac{a_o}{2}t\right)^2},$$

where

a_o = initial instantaneous decline factor. The decline factor, a, is decreasing with time, where

$$a = \frac{a_o}{1 + \frac{a_o}{2}t}.$$

When near the end life of the well, the decline curve becomes exponential again. Normally, the decline rate is to be kept constant by the user for the remaining well life if the decline rate decreases below 6% to 8%. Example of a tight gas well that exhibit exponential decline is shown in the FIGURE 6 below.

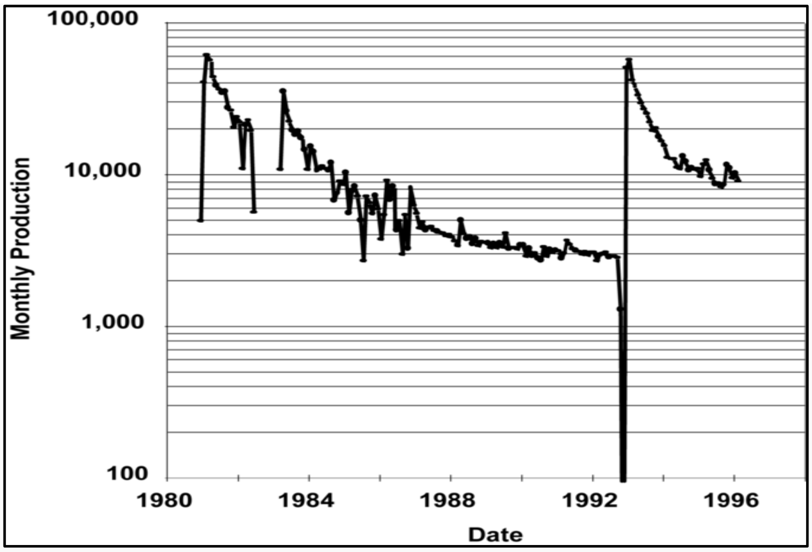


FIGURE 6. Decline curve for a tight gas well (Petrowiki, 2012)

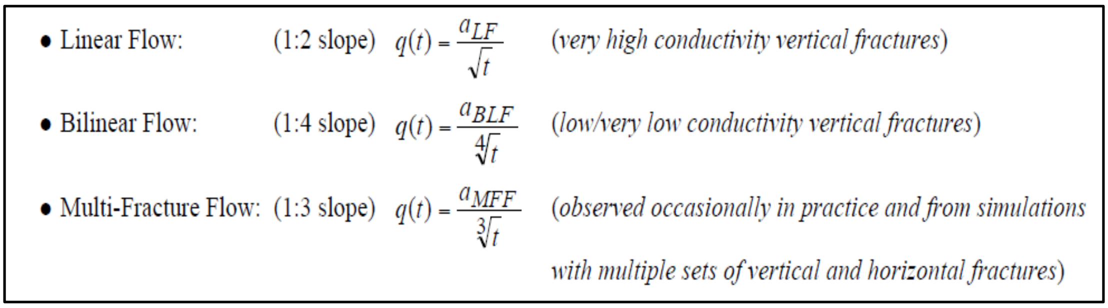
However, it is needed to analyse all the production data with caution even when using the hyperbolic equation on tight gas reservoirs. Sometimes, overestimate of reserves can occurred due to the unconstrained hyperbolic relation in Arps decline curve, which is only can use for boundary dominated flow regime (Rushing et al., 2007). For instances, it is common that most of the wells will have high gas flow rates and also high flowing tubing pressure at the beginning of production. Next, both the flow rate and the pressure will decline during the first few weeks or months. The extrapolation of the data for the future will be optimistic if only the gas flow-rate is being analysed. The flowing tubing pressure will stop declining when it has reach the pipeline pressure and at this time, decline rate of the gas flow-rate will be increases. Therefore, the most ideal method is calculate the values of flow rate divided by pressure drop during the period in which both the gas flow rate and the flowing tubing pressure are declining. In order to match both the decline in gas flow rate and flowing tubing pressure, the decline curve model is used.

4.3 Other Approach or Modification - Time-Rate Relations

Both hyperbolic & exponential Arps relations are the most traditional relations to evaluate the Estimated Ultimate Recovery (EUR) in oil and gas industry since decade ago. However, when these relations are used for unconventional resources which is extremely low permeability, the produced results might not be so accurate because of some incorrect assumptions such as:

- Assumption of constant bottom-hole pressure
- Assumption of boundary-dominated flow regime

According to the works by Rushing et al. (2007) and Lee and Sidle (2010), they showed that the inappropriate use of Arps' relations generally will produce overestimates of reserves. This normally happened when Arps b-value is larger than value 1 and the extrapolation of hyperbolic relation is unconstrained. The b-value greater than value 1, which is mainly due to the early time flow regime for horizontal well coupled with certain amount of hydraulic fracture stages, as shown below:



- Linear Flow: (1:2 slope) $q(t) = \frac{a_{LF}}{\sqrt{t}}$ (*very high conductivity vertical fractures*)
- Bilinear Flow: (1:4 slope) $q(t) = \frac{a_{BLF}}{\sqrt[4]{t}}$ (*low/very low conductivity vertical fractures*)
- Multi-Fracture Flow: (1:3 slope) $q(t) = \frac{a_{MFF}}{\sqrt[3]{t}}$ (*observed occasionally in practice and from simulations with multiple sets of vertical and horizontal fractures*)

All of the above are power law flow regimes, the rate is related to time raised to an exponent. Under some certain circumstances, the main idea here is we can reduce the Arps hyperbolic time-rate relation to power law form. The main error will normally occurred here when we use Arps hyperbolic time-rate relation for analysis and extrapolation of early-time production data in terms of power-law flow regime, which will then results in overestimation of EUR. According to Arps (1945) &

Johnson and Bollens (1927), the Arps hyperbolic time-rate relation is:

$$q(t) = \frac{q_i}{(1 + bD_i t)^{1/b}}$$

Empirically, substitute:

- b=2 into Eq. 1 (and assume that bDit >> 1); then we obtain the square-root time relation (linear flow)
- b=4 into Eq. 1 (and assume that bDit >> 1); then we obtain the fourth-root time relation (bilinear flow)
- b=3 into Eq. 1 (and assume that bDit >> 1); then we obtain the third-root time relation (multi-fracture flow)

The inappropriate use of Arps' relations generally will produce overestimates of reserves. This normally happened when Arps b-value is larger than value 1 and the extrapolation of hyperbolic relation is unconstrained. "Modified hyperbolic" relation has been introduced. During early times, it starts with initially unconstrained hyperbolic trend. Next, it is continued with an exponential decline trend using a standard terminal decline. Nevertheless, we have to clarify that this is a practice-based approach and research work is carried out to investigate the effectiveness of the modified hyperbolic time-rate relation for reserves estimation.

It is believed that the modified hyperbolic relation can be used for production extrapolation and prediction of the EUR. Nevertheless, diagnostic interpretations of the data is compulsory and needed to define the analyses. Some authors such as Ilk et al. (Power Law Exponential, 2008), Valkó (Stretched Exponential, 2009), Clark et al. (Logistic Growth Model, 2011), and Duong (2011) have propose some different rate decline relations due to the issues of Arps' rate decline relations. They are trying to modelling the early transient and transitional flow behavior.

Nevertheless, no one of these equations fulfil the criteria of forecasting the production for all the unconventional plays. It is because different play have its own characteristics, condition of operation and especially different time-rate equation behavior. In a simple term, maybe one of the equation can works well for one play but not all the other plays. Therefore, it is vital for us to comprehend the different equation behaviour and how to utilise it correctly for reserves estimation.

The new approach or modification made for DCA has been introduced by various authors. These recently developed time-rate relations including:

- Power-Law Exponential Model by Ilk et al. (2008, 2009), which is similar to the relations by Jones (1942)
- Stretched Exponential Model by Valkó (2009) & Kisslinger (1993) and Kohlrausch (1854)]
- Logistic Growth Model by Clark et al. (2011)
- Duong Model by Duong (2011)

The main assumption here is that every relation has its own justification and all the wok here is by empirically, which means that except analogy, these relations have not, by this time, relates to any reservoir engineering theory. For example, the infinite sum of exponential, the Stretched Exponential model can be considered to be “defined” by adding it with the absolute exponential decline as an analog.

Application of the Time-rate Models to Long Term Production Data

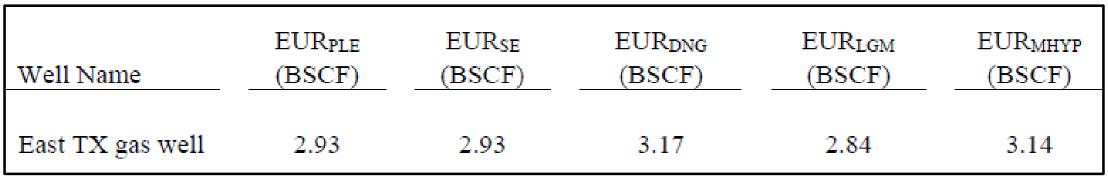
The methodology flow in this works are:

- To apply the "*Db*" "*β*-derivative," and "*q/Gp*" diagnostic plots to each data.
- To apply each model to a given data set and provide:
  - EUR predictions
  - Production projections
- To investigating model behavior and compare the EUR predictions obtained from each model

In this work, the case study is based on a tight-gas well in East Texas. The permeability of this tight gas well is approximately 7.0 μD and the provided production data is about 7 years. Long term production data is being used in this work in order to study the different rate decline equations and their respective behavior. For this case, diagnostic interpretation are implemented by matching data and estimate EUR. A complete summary of the time-rate analysis relations is provided in **Appendix B**.

The outcome of this case study is shown in the FIGURE 15 – 18 (**Appendix A**). All the time-rate relation models are matched with the production data provided for the tight gas well in East Texas. By using the diagnostic plots as guide, we make sure that all matches are carried out at the same time by calibrating the model parameters. We computed the case study for 30 years range and the EUR values obtain are shown in Table 4 below.

TABLE 4. Time-rate analyses results for the East TX gas well (long term production data for all models) (Okouma, 2012)



Well Name	EUR_{PLE} (BSCF)	EUR_{SE} (BSCF)	EUR_{DNG} (BSCF)	EUR_{LGM} (BSCF)	EUR_{MHYP} (BSCF)
East TX gas well	2.93	2.93	3.17	2.84	3.14

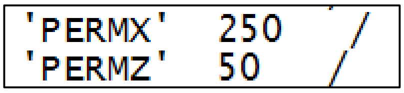
As seen from the results, highest EUR value (3.17) is produced by Duong's model while logistic growth model produced the lowest EUR value (2.84). Five percent (5%) of terminal decline value is applied for the modified hyperbolic equation. This results in decreasing of the predicted EUR while we increasing the terminal decline value. Therefore, based on the case study developed, we can conclude that:

- The primary diagnostic plot used to establish the well-reservoir character is the "*Db*" diagnostic plot
- The diagnostic analyses should be coupled with "*q/Gp*" diagnostic plot because it is good in data check but the expectation of a completely linear trend [Duong (2011)] is optimistic.
- The "β-derivative" diagnostic plot is suitable for establishing the existence of "power-law" flow regimes.

4.4 Compare & Contrast Conventional & Unconventional Reservoir Using Simulated Decline Curve

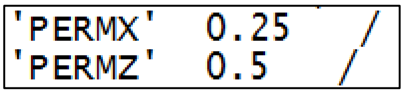
To have a better understanding about the usage of decline curve analysis (DCA) on tight gas/ unconventional reservoirs, it is essential to compare and contrast the DCA for both conventional and unconventional reservoirs. Therefore, Schlumberger Reservoir Simulator, ECLIPSE is used to simulate the production data curve according to DCA for both conventional and unconventional reservoirs. Due to limited data for tight gas or unconventional reservoirs in Malaysia, it is difficult to obtain production history data of unconventional reservoirs. Hence, the case model to be run in ECLIPSE is a simple and random model obtained in the simulator.

To distinguish the feature between conventional and unconventional reservoirs, the most important parameters that needed to be changed is the permeability (M. Rafiqul Islam, 2014). As an example, the first case model being run is a very simple model which contain only one single permeability value for both X & Z-direction. As shown below, the permeability in X-direction is 250 millidarcy (mD) and permeability in Z-direction is 50 millidarcy (mD).



```
'PERMX'   250    /
'PERMZ'   50    /
```

The value shown above will be assumed as the case model for conventional reservoirs and its decline curve graph will be shown later. On the other hands, for unconventional reservoirs, the permeability will be changed to a much lower value as shown below.



```
'PERMX'   0.25    /
'PERMZ'   0.5    /
```

As shown above, the permeability for unconventional reservoirs has been modified to a very low permeability value. In this case, the permeability in X-direction is 0.25 millidarcy (mD) and permeability in Z-direction is 0.5 millidarcy (mD). Therefore, by running the both case model using the reservoir simulator, ECLIPSE, decline curve graph for both cases can be obtained.

Both graphs are shown below. The graphs shown are both Oil Production Rate (STB/day) vs Time (days). Based on the graphs below, both graphs conform to the decline curve as the oil production rate is declining as the time getting longer. FIGURE 7 shows the decline curve for conventional reservoir while FIGURE 8 is for the unconventional reservoir. By comparing both graphs, it can be seen that the oil production rate for conventional reservoir is higher than the unconventional reservoir, providing that the only difference between them is the changes in permeability only.

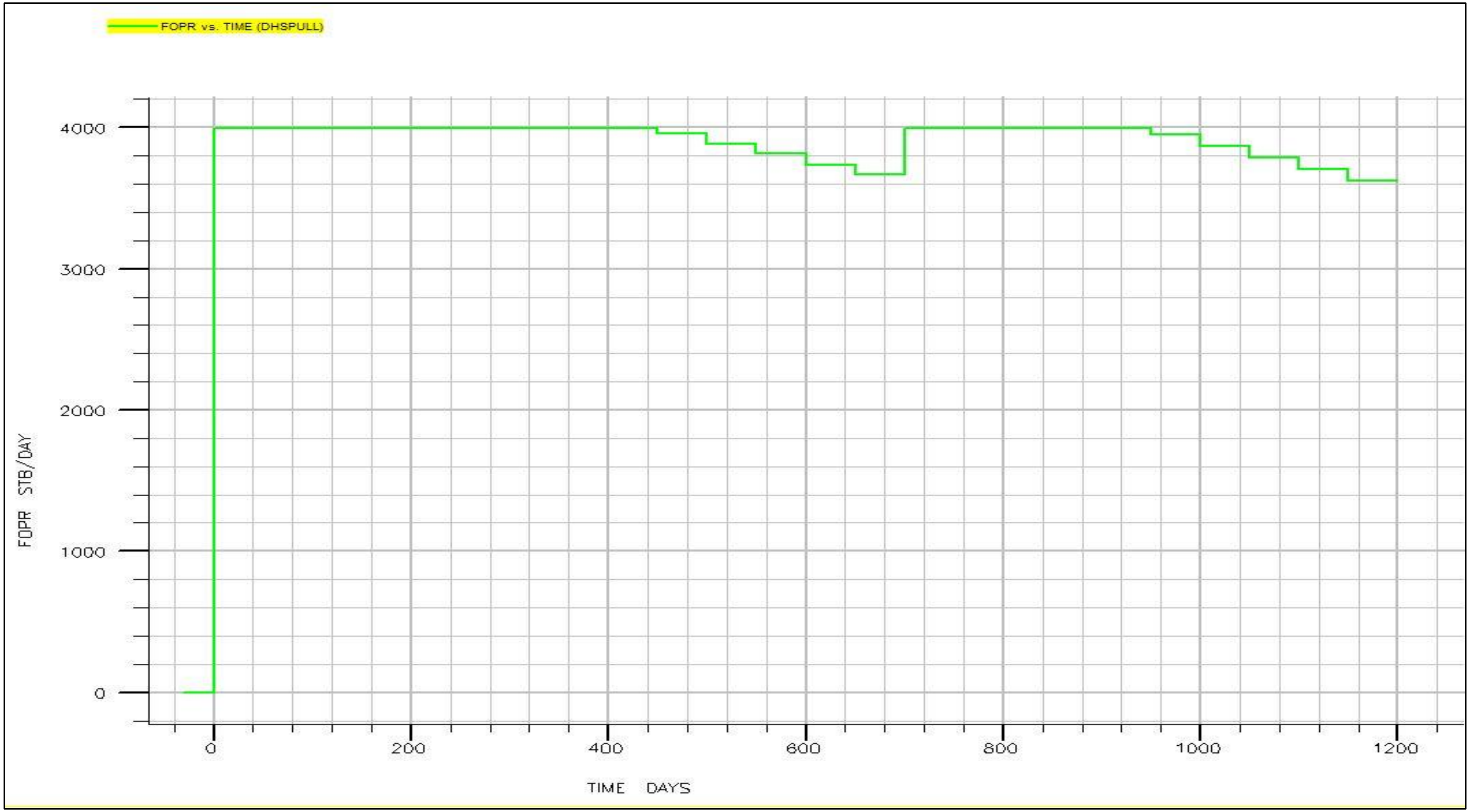


FIGURE 7. Oil Production Rate (STB/day) vs Time (days) [Conventional Reservoir First Case]

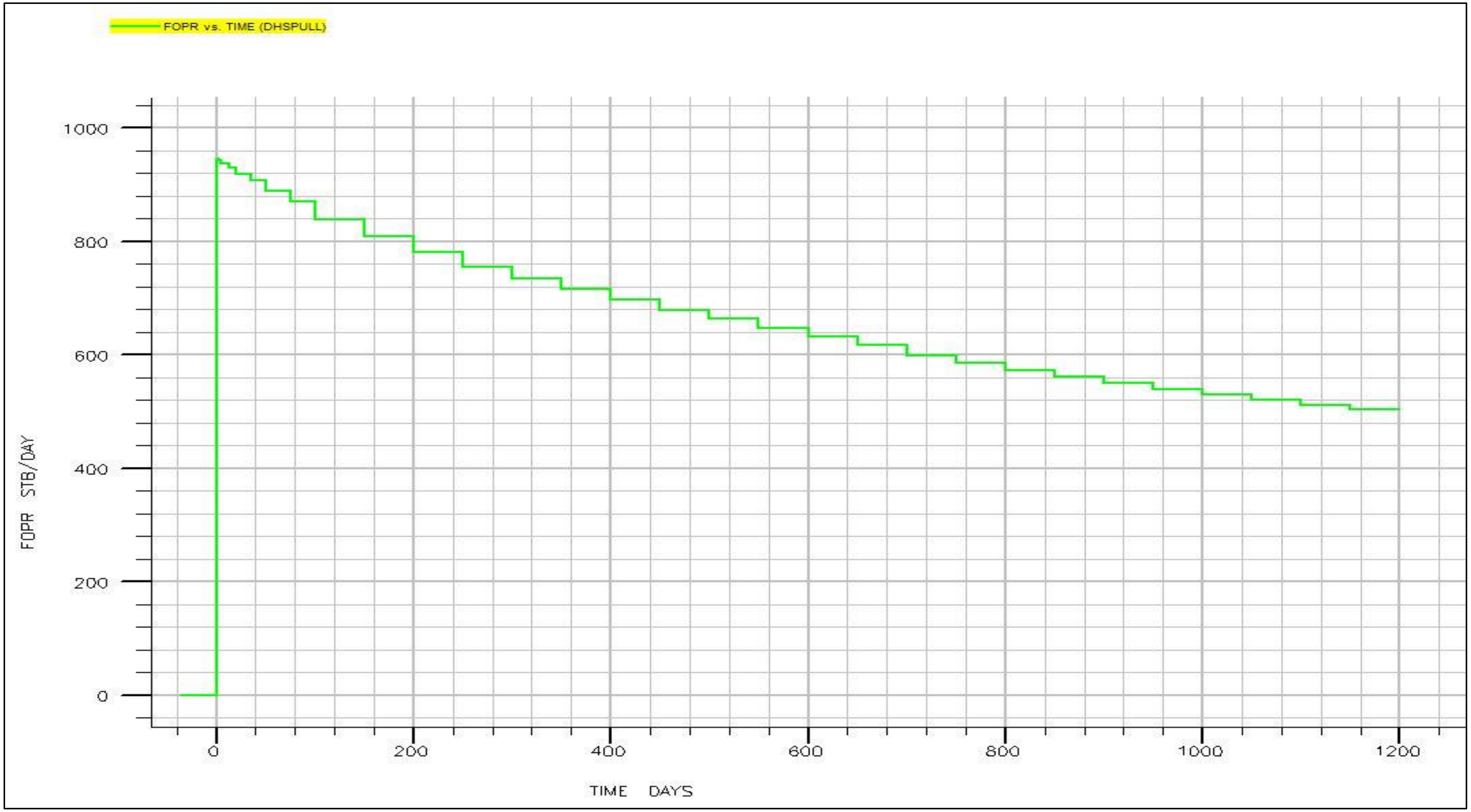
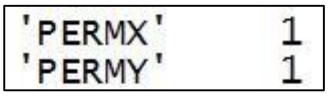


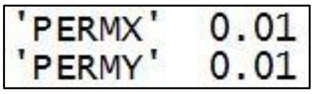
FIGURE 8. Oil Production Rate (STB/day) vs Time (days) [Unconventional Reservoir First Case]

The second case model also contain only one single permeability value for both X & Y-direction. As shown below, the permeability in X-direction is 1 millidarcy (mD) and permeability in Y-direction is 1 millidarcy (mD).



```
'PERMX'     1
'PERMY'     1
```

The value shown above will be assumed as the case model for conventional reservoirs and its decline curve graph will be shown later. On the other hands, for unconventional reservoirs, the permeability will be changed to a much lower value as shown below.



```
'PERMX'  0.01
'PERMY'  0.01
```

As shown above, the permeability for unconventional reservoirs has been modified to a very low permeability value. In this case, the permeability in X-direction is 0.01 millidarcy (mD) and permeability in Y-direction is 0.01 millidarcy (mD). Therefore, by running the both case model using the reservoir simulator, ECLIPSE, decline curve graph for both cases can be obtained.

Both graphs are shown below. The graphs shown are both Oil Production Rate (STB/day) vs Time (days). Based on the graphs below, both graphs conform to the decline curve as the oil production rate is declining as the time getting longer. FIGURE 9 shows the decline curve for conventional reservoir while FIGURE 10 is for the unconventional reservoir. By comparing both graphs, it can be seen that the oil production rate for conventional reservoir is higher than the unconventional reservoir, providing that the only difference between them is the changes in permeability only.

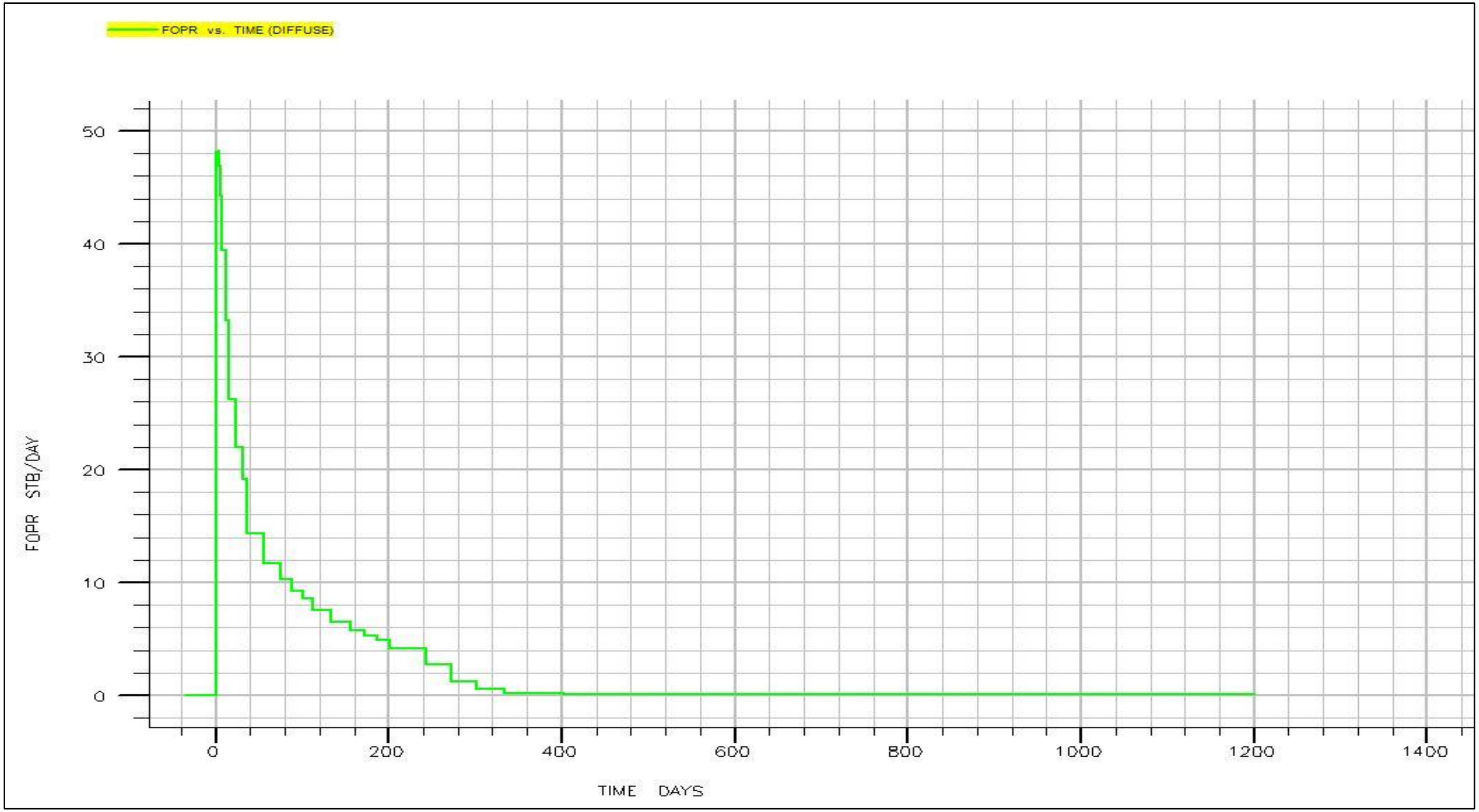


FIGURE 9. Oil Production Rate (STB/day) vs Time (days) [Conventional Reservoir Second Case]

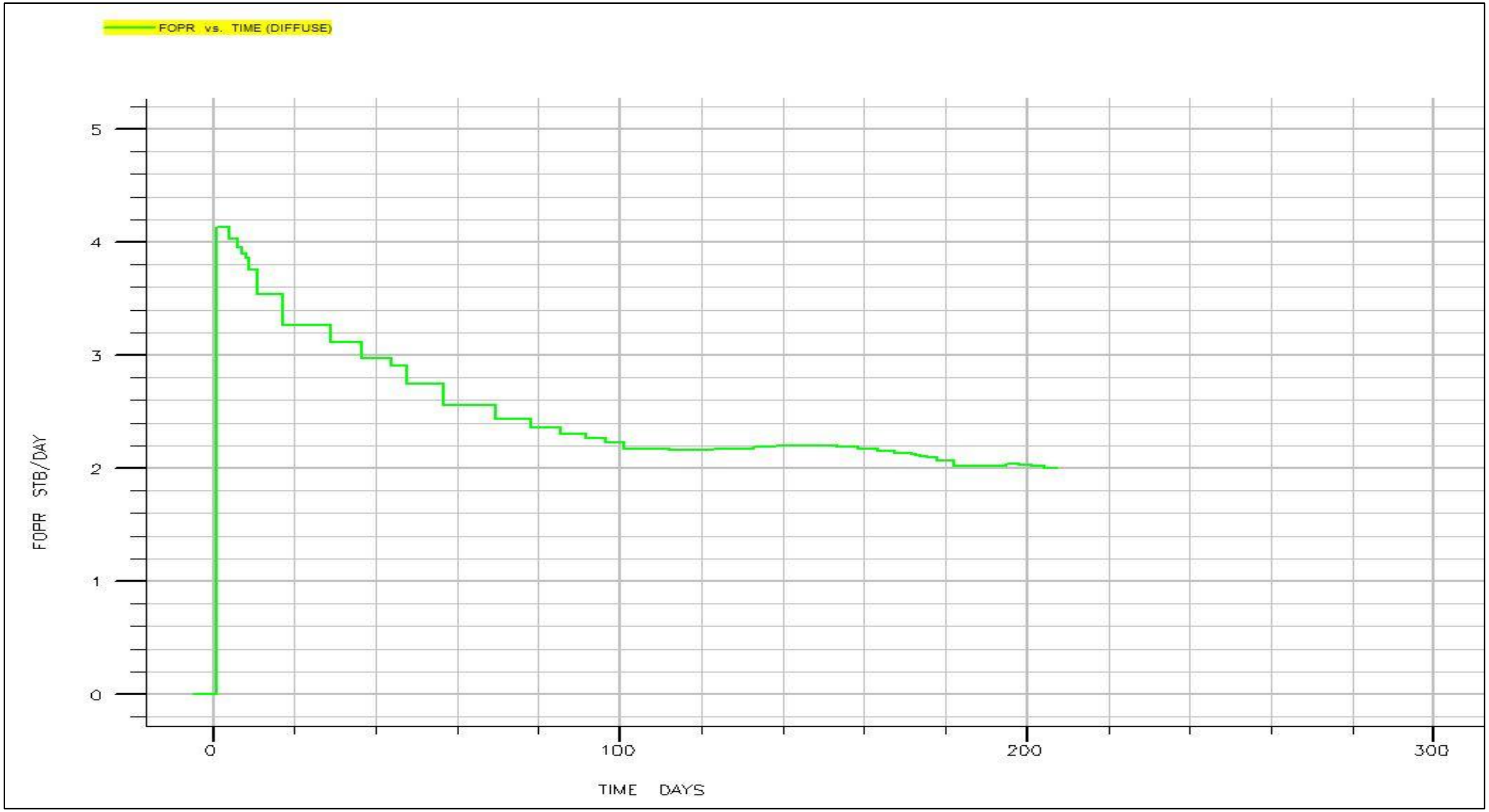
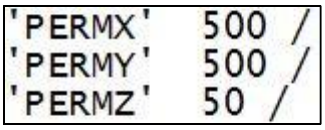


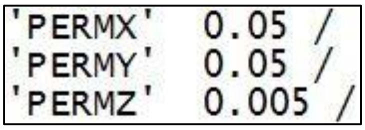
FIGURE 10. Oil Production Rate (STB/day) vs Time (days) [Unconventional Reservoir Second Case]

The third case model also contain only one single permeability value for all the X, Y & Z-direction. As shown below, the permeability in X-direction is 500 millidarcy (mD), permeability in Y-direction is 500 millidarcy (mD) and permeability in Z-direction is 50 millidarcy (mD).



```
'PERMX'  500 /
'PERMY'  500 /
'PERMZ'  50 /
```

The value shown above will be assumed as the case model for conventional reservoirs and its decline curve graph will be shown later. On the other hands, for unconventional reservoirs, the permeability will be changed to a much lower value as shown below.



```
'PERMX'  0.05 /
'PERMY'  0.05 /
'PERMZ'  0.005 /
```

As shown above, the permeability for unconventional reservoirs has been modified to a very low permeability value. In this case, the permeability in X-direction is 0.05 millidarcy (mD), permeability in Y-direction is 0.05 millidarcy (mD) and permeability in Z-direction is 0.005 millidarcy (mD). Therefore, by running the both case model using the reservoir simulator, ECLIPSE, decline curve graph for both cases can be obtained.

Both graphs are shown below. The graphs shown are both Oil Production Rate (STB/day) vs Time (days). Based on the graphs below, both graphs conform to the decline curve as the oil production rate is declining as the time getting longer. FIGURE 11 shows the decline curve for conventional reservoir while FIGURE 12 is for the unconventional reservoir. By comparing both graphs, it can be seen that the oil production rate for conventional reservoir is higher than the unconventional reservoir, providing that the only difference between them is the changes in permeability only. Therefore, based on three case model being simulated and all graphs obtained, it can be concluded that low permeability reservoir such as tight gas/ unconventional reservoir has lower production rate as compared to conventional reservoir.

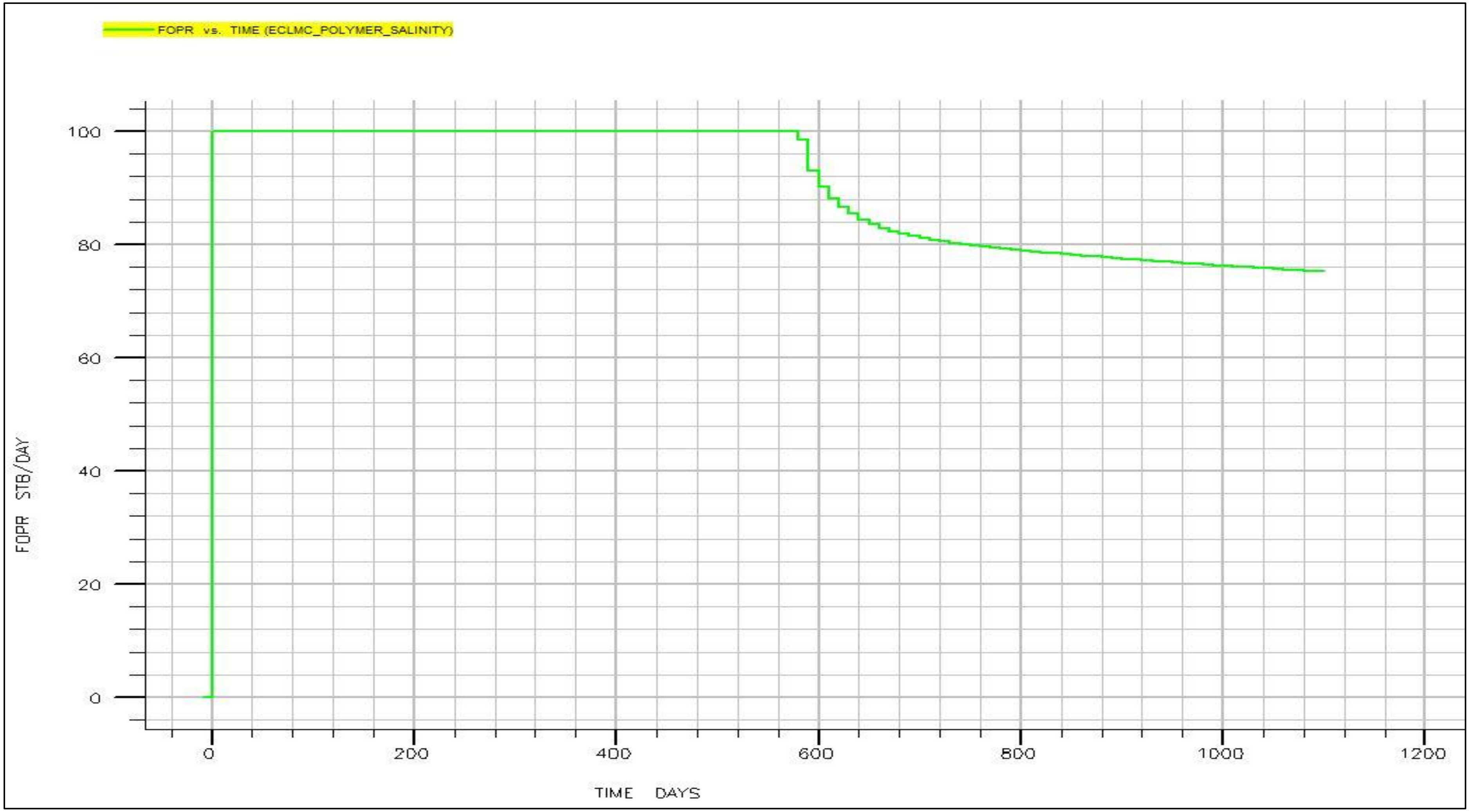


FIGURE 11. Oil Production Rate (STB/day) vs Time (days) [Conventional Reservoir Third Case]

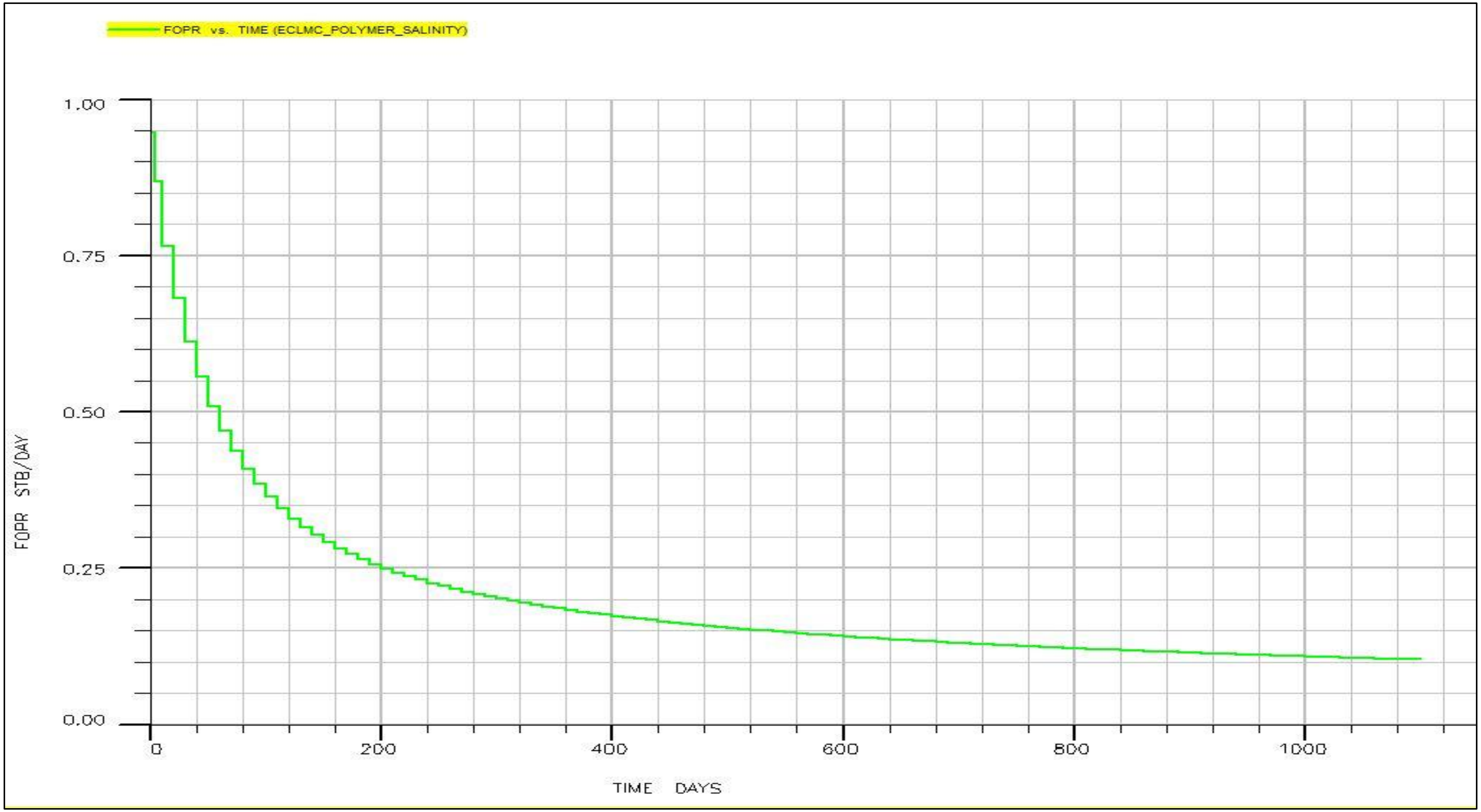


FIGURE 12. Oil Production Rate (STB/day) vs Time (days) [Unconventional Reservoir Third Case]

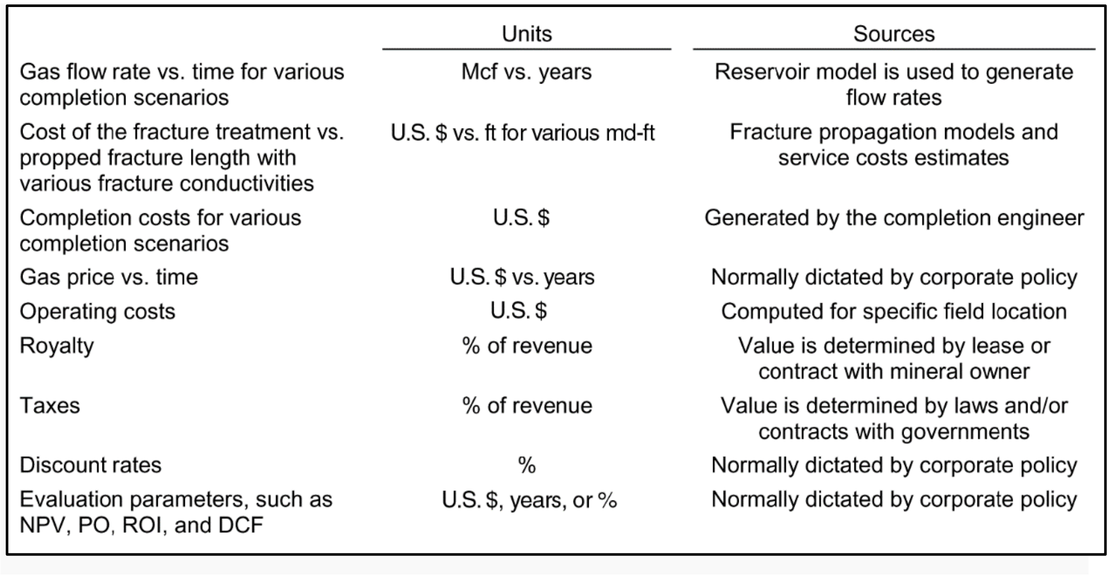
4.5 Economical Aspects & Modelling

It is difficult to estimate technically recoverable and economical reserves. There are specific data needed for an economics model in each and every conditions which depends on many factors or other variables. Basically, the factors that affecting the economics of tight gas or unconventional reservoirs are:

- Average long term gas prices
- Amount of recoverable gas for each reservoir or well
- Operating costs
- Infrastructure costs (includes well & completion cost)
- Royalty payments & Taxes
- Terms & conditions

As shown in the TABLE 5 below is the example of data required to run an economic model:

TABLE 5. Sources of Data for an Economic Model (Petrowiki, 2012)



	Units	Sources
Gas flow rate vs. time for various completion scenarios	Mcf vs. years	Reservoir model is used to generate flow rates
Cost of the fracture treatment vs. propped fracture length with various fracture conductivities	U.S. $ vs. ft for various md-ft	Fracture propagation models and service costs estimates
Completion costs for various completion scenarios	U.S. $	Generated by the completion engineer
Gas price vs. time	U.S. $ vs. years	Normally dictated by corporate policy
Operating costs	U.S. $	Computed for specific field location
Royalty	% of revenue	Value is determined by lease or contract with mineral owner
Taxes	% of revenue	Value is determined by laws and/or contracts with governments
Discount rates	%	Normally dictated by corporate policy
Evaluation parameters, such as NPV, PO, ROI, and DCF	U.S. $, years, or %	Normally dictated by corporate policy

On the other hands, another important factor that affects the economics of the unconventional gas reservoir is the fiscal regimes for gas reservoir development, this will then affect and decide whether the certain unconventional reservoir will be developed (Melvyn et al., 2012). A simple example can be portrayed with some calculations by input some typical well costs and then evaluate the gas price needed to breakeven, while the possible ultimate recovery is varies for every well for different fiscal regimes. It can be shown in FIGURE 13 below. Generally, with the current technology, majority of the unconventional gas reservoir only manage to produce around 2 to 4 BCF of gas per well, and hence it will not be economically viable except for very high gas prices. In some cases, to enable the project to be viable, extremely high ultimate recoveries are required at reasonable gas prices. Therefore, the volumes of unconventional or tight gas need to be considered in terms of risked economically accessible volumes. However, it is not easy to quote the numbers because it is common that gas price varies with time.

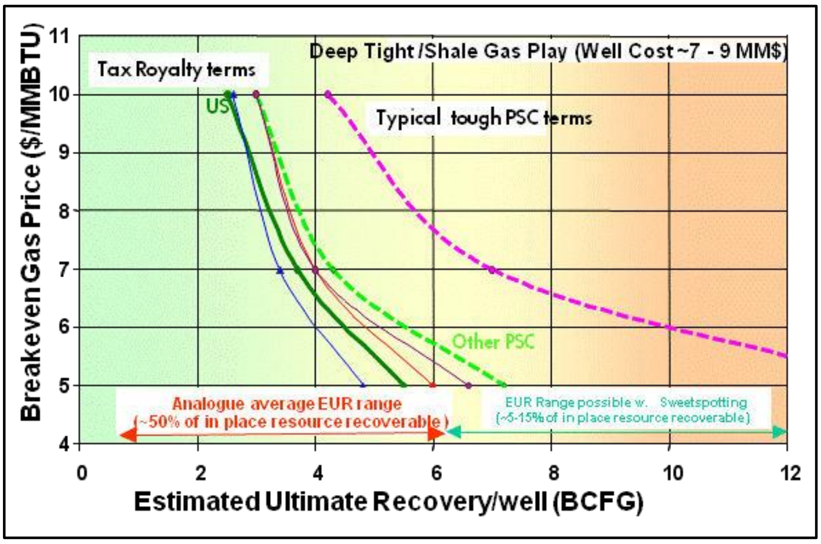


FIGURE 13. Breakeven Gas Price vs. Ultimate Recovery/Well for Various Global Fiscal Regimes (Melvyn et al., 2012)

Last but not least, FIGURE 14 shows an inverted pyramid in contrast to the resource pyramid mentioned beforehand. This inverted pyramid is used to describe the economically recoverable volumes. Nowadays, as the technology is improving, it is easy to increase the economically recoverable volumes of gas either by enhancing recovery factor or minimize the other costs.

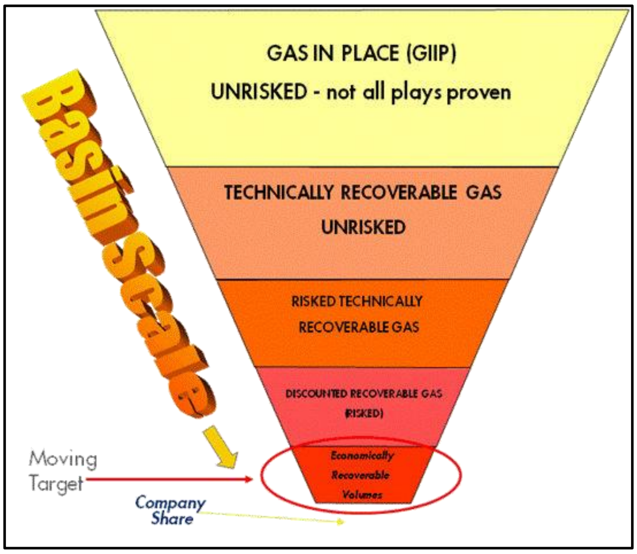


FIGURE 14. Relationship between gas in place and economically accessible volumes of unconventional gas (Melvyn et al., 2012)

CHAPTER 5

5. CONCLUSION AND RECOMMENDATION

In the nutshell, tight gas or unconventional reservoirs have been developed in our world since many years ago, and never was a new topic anymore. However, new and advanced technology is now being introduced every day to look for and develop all these unconventional reservoirs.

According to the resource triangle mentioned, it is known that all natural gas, are spread out naturally in log. Based on that statement, we can assure that if huge amount of natural gas production can get from conventional reservoirs, theoretically, more massive volumes of gas can get from the unconventional reservoirs, in the same basin. It is believed that in the next forty years, tight gas or unconventional reservoirs will definitely be more popular worldwide.

To fulfill the first objective, various method of reserves estimation had been studied, analyzed and evaluated. Based on the result findings, volumetric and material balance methods will not function properly due to their limitation. Hence, the focus will be on decline curves or reservoir simulation to analyse the production data. Reservoir simulation method will be carried out if sufficient data is available in the future time.

Decline Curve Analysis (DCA) and economic aspects of unconventional reservoirs have been discussed comprehensively throughout this research. In order to solve the issue by the unconstrained hyperbolic Arps decline curve which might sometimes led to overestimate of reserves, several approach or modification on various time-rate relation has been introduced by different author. To fulfill the second objective, a few case study has been carried out empirically to study and investigate the performance and behaviour of each of them. Throughout this work, all the other approaches or time-rate relation models are able to analyze and forecast the production data of well in very low permeability reservoirs, which represent the tight gas or unconventional reservoir. The main focus of this work is the application of the "*Db*", "*β*-derivative," and "*q/Gp*" diagnostic plots to be used as a guideline for the analysis in order to get the model parameters for all different time-rate relation. After that, we can extrapolate the production data and we can get the results of "estimated ultimate recovery" (EUR).

Throughout this case study work, we can come to conclusion that the diagnostic-based analysis can be driven from the production data. Therefore, it is believed that a reservoir engineer must be able to ensure the most accurate analyses of a sufficiently provided production data by making the full usage of the application for diagnostic workflow process.

Last but not least, it is not easy to construct a proper economic model especially when we do not have the relevant and sufficient data in order to complete the task. Nevertheless, some economic aspects has been investigated such as the factors that affecting the economics of tight gas or unconventional reservoirs, sources of data for an economic model, fiscal regimes for gas reservoir development and the economically recoverable volumes of tight gas or unconventional reservoirs.

The objectives of this research is recap again as below:

- *To identify different methods of reserves estimation for tight gas/ unconventional reservoir & their respective advantages and limitation of each methods*
- *To develop case study of other approach or modification made to estimate reserves for tight gas/ unconventional reservoir*
- *To investigate economic aspects and economic modelling of tight gas/ unconventional reservoir*

It is concluded that all the problem statement in this project have been solved and all the objectives have been achieved successfully.

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APPENDICES

Appendix A

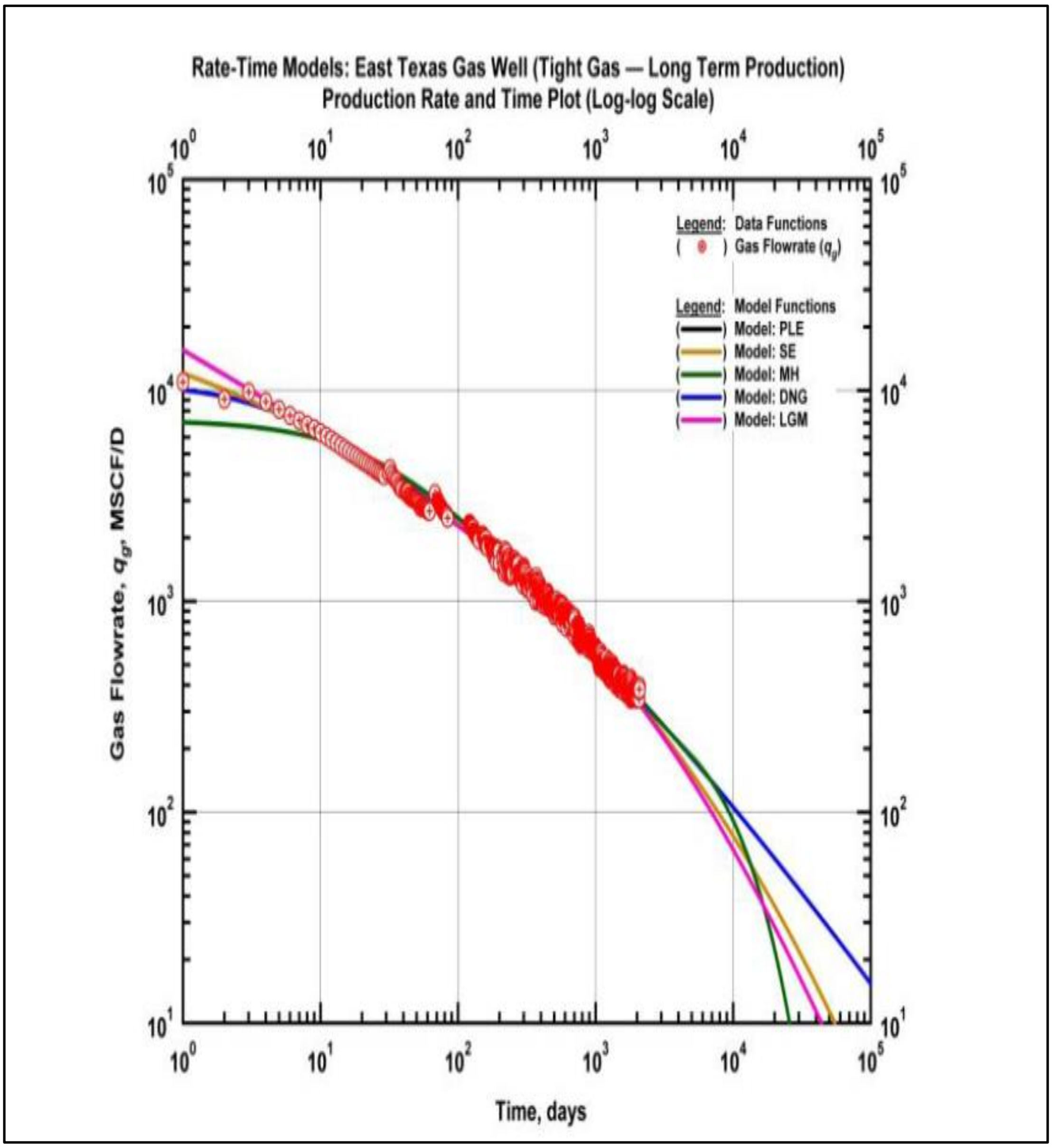

FIGURE 15. Time-rate analysis for East TX tight gas well - All models (rate and production time) (Okouma, 2012)

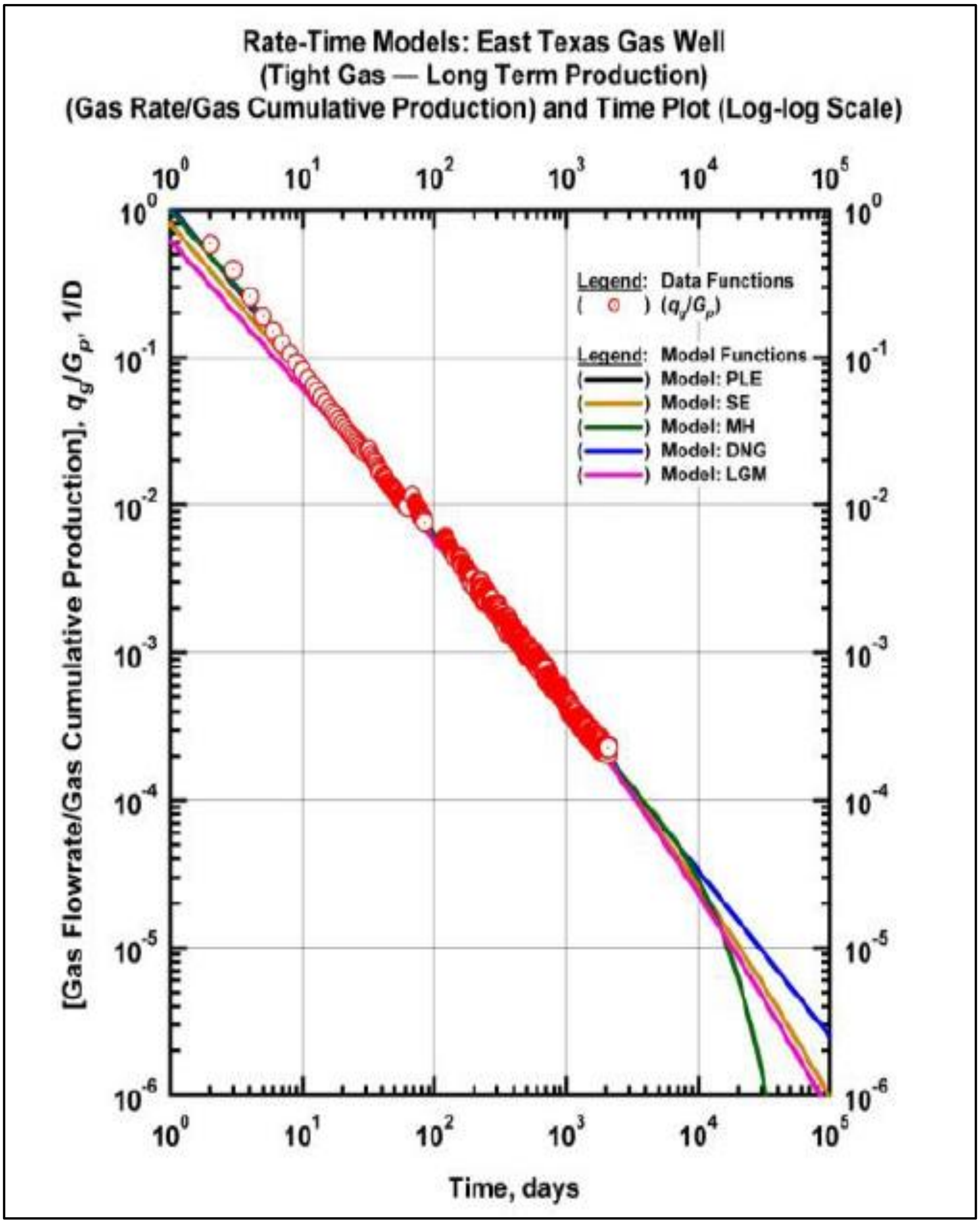

FIGURE 16. Time-rate analysis for East TX tight gas well - All models (gas rate/gas cumulative production and production time) (Okouma, 2012)

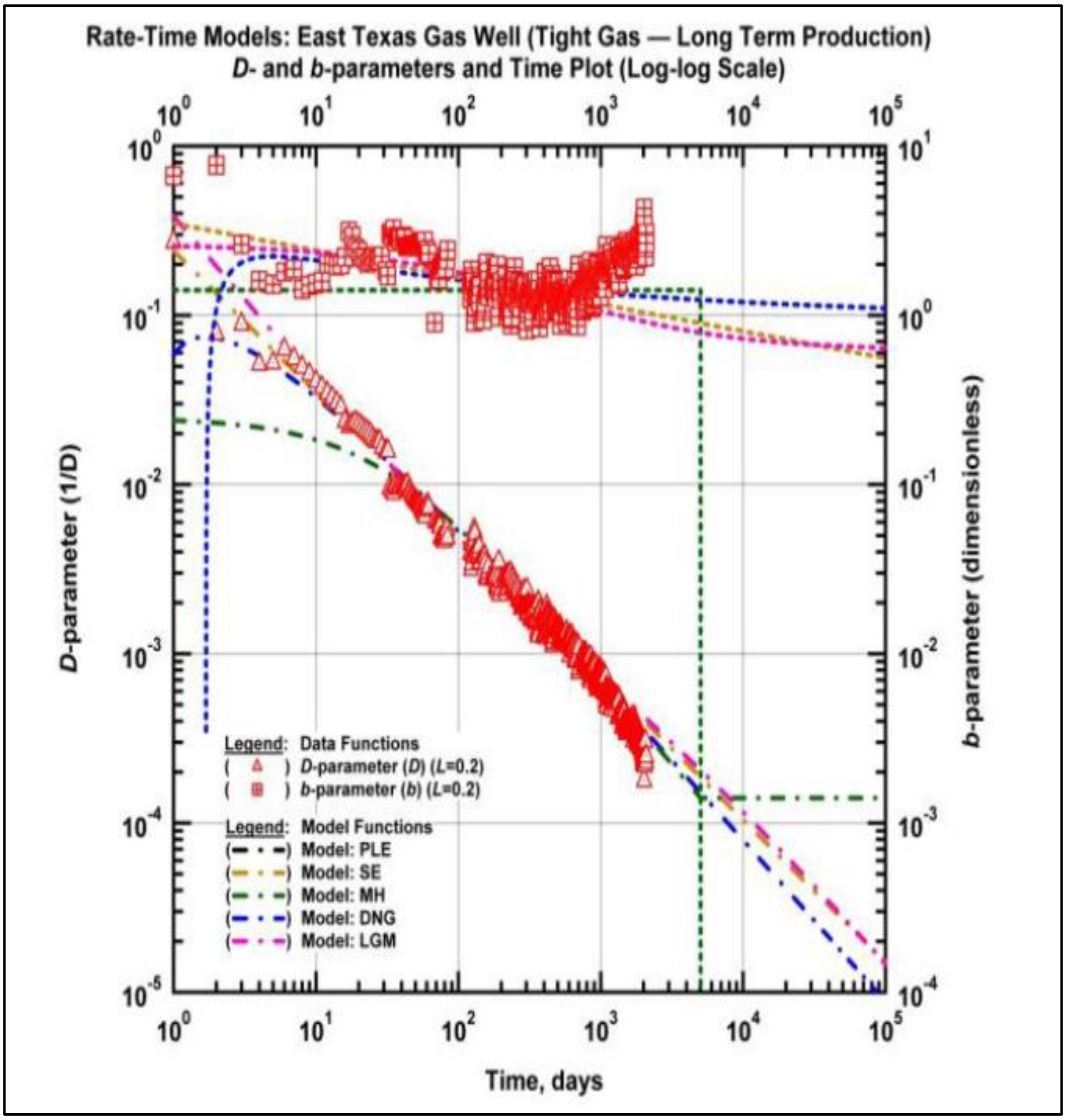

FIGURE 17. Time-rate analysis for East TX tight gas well - All models (computed D- and b-parameters and production time) (Okouma, 2012)

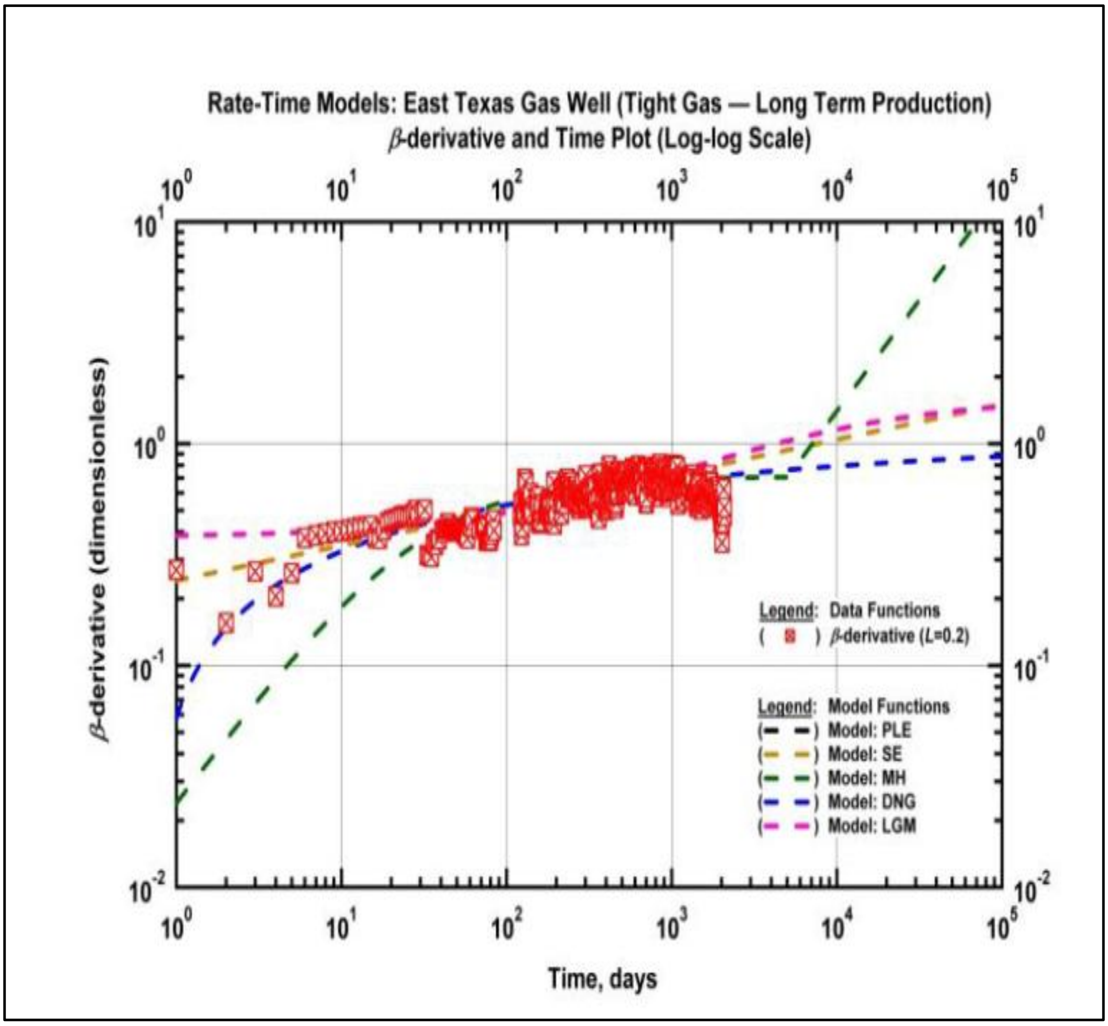

FIGURE 18. Time-rate analysis for East TX tight gas well - All models (β-derivative and production time) (Okouma, 2012)

Appendix B - Formulations for Diagnostic Functions

An inventory of the time-rate relations used in this work is provided, together with all the formulations for the various diagnostic functions (*i.e.*, $D(t)$, $b(t)$, $\beta(t)$ and $q/Gp(t)$) which are used in this paper:

Modified Hyperbolic Model:

This relation has two (2) trends — an "initial" trend that is hyperbolic (*i.e.*, for $t<t_{exp}$), and a "final" trend that is exponential (*i.e.*, for $t> t_{exp}$) — where "t_{exp}" is the time of change from hyperbolic to exponential.

For the hyperbolic function, we have:

$$q(t) = \frac{q_i}{(1+bD_i t)^{1/b}} \qquad (t<t_{exp}) \ldots\ldots \text{(A.1)}$$

$$D(t) = \frac{D_i}{(1+bD_i t)} \qquad (t<t_{exp}) \ldots\ldots \text{(A.2)}$$

$$b(t) = b \qquad (t<t_{exp}) \ldots\ldots \text{(A.3)}$$

$$\beta(t) = \frac{D_i t}{(1+bD_i t)} \qquad (t<t_{exp}) \ldots\ldots \text{(A.4)}$$

$$\frac{q(t)}{G_p(t)} = \frac{(1-b)D_i}{(1+bD_i t)^{(1/b)} - (1+bD_i t)} \qquad (t<t_{exp}) \ldots\ldots \text{(A.5)}$$

For the exponential function, we have:

$$q(t) = q_i \exp[-D_i t] \qquad (t> t_{exp};\ D_i = D_{lim}\ \text{[constant terminal decline]}) \ldots\ldots \text{(A.6)}$$

$$D(t) = D_i \qquad (t> t_{exp}) \ldots\ldots \text{(A.7)}$$

$$b(t) = 0 \qquad (t<t_{exp}) \ldots\ldots \text{(A.8)}$$

$$\beta(t) = D_i t \qquad (t> t_{exp}) \ldots\ldots \text{(A.9)}$$

$$\frac{q(t)}{G_p(t)} = D_i \frac{\exp[-D_i t]}{(1-\exp[-D_i t])} \qquad (t> t_{exp}) \ldots\ldots \text{(A.10)}$$

Power-law Exponential Model (PLE):

$$q(t) = \hat{q}_i \exp[-\hat{D}_i t^n - D_\infty t] \ldots\ldots \text{(A.11)}$$

$$D(t) = D_\infty + \hat{D}_i n t^{n-1} \ldots\ldots \text{(A.12)}$$

$$b(t) = \frac{\hat{D}_i (1-n) n t^n}{(D_\infty t + \hat{D}_i n t^n)^2} \ldots\ldots \text{(A.13)}$$

$$\beta(t) = D_\infty t + \hat{D}_i n t^n \ldots\ldots \text{(A.14)}$$

No $q/G_p(t)$ formulation is available for the "power-law exponential" relation as no closed form relation exists for the cumulative production function [$G_p(t)$]. We note that this is due to the complexity introduced by the D_∞ term.

Stretched Exponential Model (SE):

$$q(t) = \hat{q}_i \exp[-(t/\tau)^n] \tag{A.15}$$

$$D(t) = n\tau^{-n} t^{n-1} \tag{A.16}$$

$$b(t) = \frac{1-n}{n}\tau^{n} t^{-n} \tag{A.17}$$

$$\beta(t) = n\tau^{-n} t^{n} \tag{A.18}$$

$$\frac{q(t)}{G_p(t)} = \frac{n}{\tau}\,\frac{\exp\left[-\left[\frac{t}{\tau}\right]^n\right]}{\left[\Gamma\left[\frac{1}{n}\right] - \Gamma\left[\frac{1}{n}, \left[\frac{t}{\tau}\right]^n\right]\right]} \tag{A.19}$$

Duong Model:

$$q(t) = q_1 t^{-m_{Dng}} \exp\left[\frac{a_{Dng}}{(1-m_{Dng})}[t^{(1-m_{Dng})} - 1]\right] \quad \text{(time-rate model)} \tag{A.20}$$

$$D(t) = m_{Dng} t^{-1} - a_{Dng} t^{-m_{Dng}} \tag{A.21}$$

$$b(t) = \frac{m_{Dng} t^{m_{Dng}} [t^{m_{Dng}} - a_{Dng} t]}{[a_{Dng} t - m_{Dng} t^{m_{Dng}}]^2} \tag{A.22}$$

$$\beta(t) = m_{Dng} - a_{Dng} t^{(1-m_{Dng})} \tag{A.23}$$

$$\frac{q(t)}{G_p(t)} = a_{Dng} t^{-m_{Dng}} \quad \text{(base concept relation for Duong model)} \tag{A.24}$$

Logistic Growth Model (LGM):

$$G_p(t) = \frac{K\, t^{n_{LGM}}}{[a_{LGM} + t^{n_{LGM}}]}$$

(cumulative production formulation) .... (A.25)

$$q(t) = \frac{dG_p(t)}{dt} = \frac{K\, n_{LGM}\, a_{LGM}\, t^{(n_{LGM}-1)}}{[a_{LGM} + t^{n_{LGM}}]^2}$$

(rate formulation) .... (A.26)

$$D(t) = \frac{n_{LGM}(a_{LGM} - 1) + (n_{LGM} + 1)\, t^{n_{LGM}}}{t(a_{LGM} + t^{n_{LGM}})}$$

.... (A.27)

Logistic Growth Model (LGM): (continued)

$$b(t) = \frac{a_{LGM}^{\,2}(1 - n_{LGM}) - 2a_{LGM}(n_{LGM}^{\,2} - 1)t^{n_{LGM}} + (n_{LGM} + 1)t^{2n_{LGM}}}{[a_{LGM}(1 - n_{LGM}) + (n_{LGM} + 1)t^{n_{LGM}}]^2}$$

.... (A.28)

$$\beta(t) = \frac{n_{LGM}(a_{LGM} - 1) + (n_{LGM} + 1)\, t^{n_{LGM}}}{(a_{LGM} + t^{n_{LGM}})}$$

.... (A.29)

$$\frac{q(t)}{G_p(t)} = \frac{a_{LGM}\, n_{LGM}}{t(a_{LGM} + t^{n_{LGM}})}$$

.... (A.30)

Basic Definitions and Diagnostic Functions:

$$D(t) \equiv -\frac{1}{q(t)}\frac{dq(t)}{dt}$$

(Definition of the decline parameter) .... (A.31)

$$\frac{1}{D(t)} \equiv -\frac{q(t)}{dq(t)/dt}$$

(Definition of the loss-ratio) .... (A.32)

$$b(t) \equiv \frac{d}{dt}\left[\frac{1}{D(t)}\right] \equiv -\frac{d}{dt}\left[\frac{q(t)}{dq(t)/dt}\right]$$

(Derivative of the loss-ratio) .... (A.33)

$$\beta(t) \equiv \frac{1}{q(t)}\left| t\frac{dq(t)}{dt} \right| \equiv t\, D(t)$$

("Beta" function — relates rate and derivative function) .... (A.34)