MODAL ANALYSIS OF A STRAIGHT PIPE WITH AND WITHOUT FLUID

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CERTIFICATE OF APPROVAL

MODAL ANALYSIS OF A STRAIGHT PIPE WITH AND WITHOUT FLUID

by

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A project dissertation submitted to the Mechanical Engineering Department Universiti Teknologi PETRONAS in partial fulfilment of the requirements for the Bachelor of Mechanical Engineering with Honours

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CERTIFICATE OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the reference and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

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ABSTRACT

This study outlined a simple approach for studying a free vibrational analysis of a straight pipe with different types of support conditions and materials. The normal practices by refineries is that the vibration assessments are only done on the vibrating rotating equipment outlined by standard guidelines. This measurement conducted only gives an equivocal indication of vibratory states of the mechanical rotating equipment ignoring the vibrating pipes near them. The premise of the project is simple which is to investigate the vibrational behaviour that are natural frequencies and mode shapes of the pipes with respect to the pipe length towards the pipe containment, pipe support condition and the materials of the pipe. In the given conditions, the natural frequency and mode shape of the pipe are being discussed thoroughly. This study uses modal analysis by means of FEA method through ANSYS Workbench and validated with analytical approach. A simple regression analysis is performed to deduce the relationship between the length of the pipe and the conditions manipulated accordingly throughout the research. This modal analysis gives a platform for future study in response spectrum analysis, harmonic analysis or structural analysis in order to study deep further on vibrational behaviour and structural deflection.

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CHAPTER 1 INTRODUCTION

1.1 Background Study

Piping vibration is a common issue when it comes to refining industry. Vibration itself is a complex issue to discuss and a small number of researches had been done especially for vibrating pipes. The lack of interest and awareness on how big the vibration fatality is still low among industries personnel. On the fundamental understanding, vibration is a repeating motion or as known as oscillatory motion at an equilibrium position. According to Becht and Engineers [1], the process piping are built according to static analysis with little to no regard to vibration behaviour. A vibratory system in general view, includes a means for storing potential, a means for storing kinetic energy and a means for energy loss.

With a constant dynamic loading, the vibration force with respect to a certain period, it will introduce an ad hoc failure which is vibration induced fatigue failure. This failure could cause a huge impact towards people, environment, asset and reputation of the industry. Fatigue failure is a niche of concern for oil and gas industries as it gives negative impact towards process safety, production downtime, maintenance and operational costs, environmental impact and company's assets [2]. It also associated with Loss of Primary Containment (LOPC) for some giant companies. The LOPC is one of the most distress "mantra" for refining industry as it creates a huge loss of profit and maintenance cost [3].

Analysis used in this study is Modal Analysis. This analysis is chosen because it can evaluate and superimpose the vibration mode shape and visualise displacement deformation. The approach can be used with subsequent dynamic and vibration analyses for further studies such as response spectrum analysis, harmonic analysis or even structural analysis. Modal analysis gives a platform and simplified the methods instead of solving a large and a long transient together in one model simulation for response spectrum analysis, it can be desirably to approximate the maximum response spectrum by using the results from modal analysis before and a known input of spectral load to calculate the desired vibration behaviour of the thence model.

Thus, this study will emphasise the effect of the vibration behaviours which are the natural frequency and the pipe vibrating mode shape with respect to its length of pipe span, pipe support conditions, and the materials of the pipe.

1.2 Problem Statement

The current practices in refinery plants for vibration assessments are only done on the rotating equipment by operators and rotating engineers based on standard guidelines for vibration on rotating equipment. The vibration measurements conducted only just give a rough indication of vibratory states of the mechanical rotating equipment, but not the vibration energy that transmitted to the process piping. Furthermore, currently there are lack of analysis towards vibration behaviour which are the mode shape and natural frequency with respect to different support conditions for piping system and its length. Plus, vagueness of guidelines for industries in selecting materials that take into consideration for different vibration conditions to avoid ad hoc failure – fatigue crack. These portrayed that the lack of interest in knowing the vibration behaviour which are natural frequency and mode shape in real life situation as it could give a great insight on how severe the vibration of the pipe will be, and the deformation pattern will be formed at specific frequency.

1.3 Objectives

The supreme objective for this project is to model a vibrating straight pipe with and without fluid using modal analysis. Deliberately the objectives of this study are:

- To analyse the effect of vibrating behaviour which are the mode shape of the pipes and natural frequency towards the length of span and acquire governing equations for each of the conditions.
- To study the effect of the support condition towards its mode shape and natural frequency.
- To investigate the vibration behaviour for different types of process piping materials.

1.4 Scope of Study

This project is aiming to study the vibration behaviour on vibrating straight pipes with and without fluid by modal analysis approach. Thus, the scope of study for this research are as stated:

- The study is analysing free vibration motion of 0 m to 10 m length of straight pipe (i.e. 1m, 2m, 4m, 6m, 8m, and 10m).
- The pipe used for this research is 160 mm for outside diameter with 10 mm thickness.
- Only first natural frequency factor and its mode shape are analysed.
- As of pipe containment, there are Empty Pipe (Without Fluid) and Filled Pipe (With Fluid). For Filled Pipe, the fluid filled in the pipe is assumed full.
- As of support condition, this study uses Fixed-Fixed and Fixed-Supported.
- Materials used for this study are Steel, ASTM A106 Gr B and SS 304L

CHAPTER 2 LITERATURE REVIEW

2.1 Concept of Vibration

As aforementioned vibration case is a complex subject to discuss. The source of study is limited, and small number of researches had been done for the past years especially vibration related to process piping. A simple and ideal vibration concept for fundamental understanding is a spring-mass system which exhibit a harmonic motion and frequency.



Figure 1: Spring-Mass System [4]

Vibration is known as a movement in rotational, swinging or linear motion [5]. One complete vibration cycle is called period. *T*, and maximum amplitude is X_0 . Inverse of the period, $\frac{1}{T}$ is called frequency, *f* in which usually express in cycles per second (cps) or Hertz (Hz). A harmonic function is the simplest type of periodic motion as shown in Equation 1.

$$X = X_0 \sin(\omega t) \tag{1}$$

where

X = Vibration displacement, $X_0 =$ Maximum displacement, $\omega =$ angular frequency (in rad/sec) t = Time (in sec)

The corresponding oscillation frequency is identified as the natural system frequency that is controlled by system's mass and stiffness (see: Equation 2). A very little amount of energy is required to excite the natural frequency of a system. Naturally the system will respond at a specific frequency or as known as threshold frequency. This causes resonance where the exciting frequency coincide the natural frequency of the system. If damping system exists, then the dynamic energy is then will dissipates and reducing vibrational response.

Natural frequency:
$$f_n = \frac{1}{2\pi} \sqrt{\frac{\text{spring stiffness}}{\text{mass}}}$$
 (2)

The thence vibration can be well-defined in terms of displacement, velocity, and acceleration. The amplitude depends on the frequency domain for certain parameters. Displacement is a frequency dependent, resulting in large displacements at low frequencies and small displacements at high frequencies for the same energy intensity. Contrarywise, acceleration is weighed such that the highest amplitude occurs at the highest frequency. Velocity gives a more uniform weighting over the required range and it is the most directly related to the resulting dynamic stress and therefore it is most commonly used as the measurement of vibration. Hence, the reason why visual inspection on assessing piping vibration is not a dependable technique in evaluating the severity of the vibration.



Figure 2: Comparison of The Relative Amplitude of Displacement, Velocity, and Acceleration in Frequency Domain [4]

2.2 Causes of Vibration

Prior studies have identified that vibration could lead to fatigue failure – an ad hoc failure that requires numerous of dynamic stresses and welded joints and pipe junctions are the most vulnerable area. Vibration is a phenomenon that need a cause or an actuator for it to

happen. As mentioned by Mobley [6], "Vibration does not just happen". There must be some sources or common causes of piping to vibrate. The forms of excitation mechanisms common to pipework are as follows: (i) Mechanical Excitation, (ii) Flow Induced Turbulence; and (iii) Pulsation from Reciprocating, Rotary Equipment and Periodic Flow Induced Excitation [4, 7]. The details of these causes are discussed in the next subsections.

2.2.1 Mechanical Induced Vibration

Mechanical excitation is always encountered with pumps and compressors. This cause of vibration occurs when the process piping is located near to a running pump or compressor. Naturally, the rotating machineries itself has their own frequency of vibration usually on mega-Hertz, MHz but when it passes the threshold of piping natural frequency, it causes resonance and vibration will occurs and may cause the piping support to vibrate as well [4]. To demonstrate that there can be no coupling, the threshold frequency including the frequency of harmonics should not be within $\pm 20\%$ of the normal structural frequency. Problems arises where piping shares their support system with either the machinery system or the associated affected pipework.

2.2.2 Flow Induced Vibration

Turbulence itself exists in most of pipework system. In straight pipes, turbulence is generated by the boundary layer of the walls. The severity of the turbulence is depending the flow regime as defined by Reynolds number [8] (see: Equation 3). Turbulence also occurs because of flow cut-offs such as resistances in process equipment, valves, bends, tees, and reducers.

Reynolds Number,
$$Re = \frac{\rho v l}{\mu} = \frac{v l}{v}$$
 (3)

where

 ρ = Density of fluid v = Velocity of the fluid

- l = The characteristic length (or diameter)
- μ = The dynamic viscosity of the fluid
- v = The kinematic viscosity of the fluid

The cuts-off however will potentially generates a high level of broadband kinetic energy local to the turbulent source. This will lead to excitation of a low frequency vibration modes in pipework or in many cases it can be visually seen the motion of vibration or vibrating pipe supports.



Figure 3: An Example Of Kinetic Energy Distribution Due To Turbulence Caused By Tee [4]

2.2.3 Pulsation

Pulsation is the same way as structures exhibit natural frequencies and the fluid within the piping system also exhibits acoustic natural frequency [4]. Acoustic frequencies can amplify low levels of pressure pulsation which cause high pressure pulsation with high amplitudes. This cause to extreme vibrating forces. Pressure pulsation is a tonal form of excitation where dynamic variations are caused in process fluids are different frequencies. The pulsation happens usually at flow discontinuities like bends, tee, reducers etc. For more serious vibration problems, the frequency of excitation, acoustic natural frequency and structural frequency must associate with one another. Yet, if large threshold rates are present, high levels of non-resonant vibration can be experienced.

For reciprocating or rotating machinery, disparities in the process fluid are typically caused by oscillating pressure based on the way they behave. This can result in a high level of dynamic pressure generating shaking and vibrating forces which causes a problem of forced vibration. Nevertheless, extreme vibration rates can be generated when fluke occurs with the piping system's normal structural frequency.

According to Equation 4, flow over a body allows the vortices to shed at different frequencies.

$$f = \frac{Sv}{d} \tag{4}$$

Where,

f = Frequency

S =Strouhal number¹

v = Fluid velocity

d = dimension of the component

2.3 Modal Analysis

Modal analysis is an analysis that evaluates and superimpose a free vibration mode shapes and natural frequencies to characterise the displacement deformation patterns. All structural body or even piping have their own natural frequency depending on its mass distribution and stiffness of the support throughout the system [10]. Finite Element Analysis (FEA) is used in preparation of the model in ANSYS Workbench. The prime goal of this Modal Analysis is to acquire natural frequency and the mode shapes of the system. The natural frequency of the pipe span must not coincide with the external

¹ Strouhal Number is a dimensionless value, Strouhal Number, $St = \frac{\omega l}{v}$, where ω is oscillation frequency, l is characteristic length, and v is fluid flow velocity. Strouhal number usually uses in analysing and understanding oscillating unsteady fluid flow dynamics. Strouhal number depends on system structure and flow regime [9] E. Toolbox. "Strouhal Number." https://www.engineeringtoolbox.com/strouhal-number-d_582.html (accessed 18 April 2019, 2019).

excitation frequencies spectrum such as earthquake, rotating equipment, and flow induced turbulence. If the natural frequency of the system overlaps with the excitation frequency, resonance occurs thus, directing making the pipe to vibrate according to the severity outlined by Energy [4]. For instance, a motor with 1485 rpm that is approximately 24.75 Hz in normal operating condition will affect the neighbouring pipe to vibrate if the thence pipe natural frequency coincide the frequency of the operating motor. This is caused by resonance and vibration could occur. Modal analysis solved the undamped eigenvalue problem which identifies frequencies and mode shapes in help for a better understanding on the motion of the system [11]. Correspondingly, the analysis uses eigenvalues to uncouple and diagonalize the original set of coupled equations. The mathematical modelling are as follows [10, 12, 13]:

Consider the matrix form of the equation of vibration where it is the basis of vibration oscillation equation,

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = 0$$
(5)

Where,

m = mass of pipe (plus with fluid);

c = damping coefficient of the structure

k = stiffness matrix of the structure

By neglecting the damping coefficient of the piping model, the equation obtained is in the form of Equation 6

$$m\ddot{x}(t) + kx(t) = 0 \tag{6}$$

2.3.1 Finite Element Analysis

Finite Element Analysis (FEA) in general is a study using simulation tools for any physical phenomenon and given components using numerical approach. It is widely used in giant industries to reduce the number of expensive, tedious of prototypes and experiments in order to feign a real-life event. In this study the researcher had used FEA method using

ANSYS Workbench to obtain the natural frequencies and mode shapes for pipe span with different conditions that are the pipe span length, support conditions, pipe containment, and the materials of the pipe (steel, ASTM A106 Gr B, and SS 304L). As for pipe containment, the Empty Pipe is a pipe without fluid while Filled Pipe is a pipe with fluid in its boundaries. Mode shape is acquired in the FEA for modal analysis. This is to observe and study the deformation pattern of the pipe when the natural frequency is coincided with excitation external frequency. The deformation calculation is defined in colour coded from modal analysis – red coloured shows the localisation of high stresses areas while blue is low stress region. This gives enough insight on how the structure or pipe is deflected in the specific natural frequency [14].

Figure 4 shows an example of 2m steel filled pipe with Fixed-Supported support conditions with total deformation portrayed by ANSYS software. This shows the maximum stress located colour in red with the highest stress of 0.14879 Pa and the lowest is 0 Pa. This will be a great platform for further structural and stress analysis for deflection of the thence structure or pipe cause by vibration at the specific natural frequency.



Figure 4: Example Of 2m Steel Filled Pipe with Fixed-Supported Conditions with Total Deformation Portrayed by ANSYS Software

Several studies have used ANSYS Workbench for Modal Analysis in order to acquire the natural frequency and mode shape. Ashrafizadeh, et al. [15] had investigated a case study on a failure of a 36" high pressure gas pipe due to 1m crack. The stress and modal analysis using FEA was done after visual inspection and metallurgical approach of the cut samples of the crack pipe. The prime goal for this is to find the crack initiation and to evaluate the stress analysis and distribution of the caused by pressure inside the pipe due to vibration of valve. Sollund and Vedeld [16] also uses analytical method using ABAQUS to compare the natural frequencies obtained by FEA. A tailor-made DEA tool is developed from modal analysis of a multi-span of subsea pipelines to observe the responses of the pipes due to different type of beam and configurations.

A study on the effects of natural frequencies towards empty pipe, static fluid, and fluid flow conditions with different type of pipe support materials was done by Sutar, et al. [17]. A mathematical model was introduced using ABAQUS to study the dynamics and stability of filled pipe with fluid with guided support. The results are then compared with FEA modelled in I-DEAS. The comparison between mathematical model and FEA was a close agreement concluding that both mathematical and FEA can be use as reference. The pipe is following Euler-Bernoulli beam and the equation is derived by using Hamilton's variation approach. The same type Euler-Bernoulli beam was used by Madhurya [18] for different materials to obtain natural frequencies and mode shape. The modal analysis is using FEA method by ANSYS Workbench software to acquire natural frequency for different of crack depth initiated.

Other than that, Kudus, et al. [19] had done their research on a case study of a thinning wall pipes due to corrosion with vibration spectra due to natural phenomenon i.e. earthquake. FEA was performed for modal analysis to study the effect of the wall thickness damage of the pipe towards vibration characteristic other than to acquire natural frequencies and mode shapes of the pipe. The localisation of the mode shape is then found near to the damaged pipe structure. The localisation of the mode shape prompted by FEA had given an adequate evidence concerning the location and the size of damage at the pipe structure. This shows FEA could help decisive the location of the crack or thinning at the

pipe structure with respect to the vibration parameters. Vibration and modal analysis of a filter system had done by Liu, et al. [20]. The objective of the study is to demonstrate the effect of vibration on pipe structure integrity. Same research had been done by Jiang and Zhu [21] for a filled U-shaped pipeline to obtain the natural frequency and mode shape of the vibrating U-shaped pipe. The FEA is done by ANSYS Workbench and validated by experimental setup. The results from both experiment and FEA modal analysis were close agreement.

Sekacheva, et al. [10] using modal analysis for FEA by ANSYS Workbench software to obtain natural frequencies towards the effect of diameter, length and wall thickness of the straight pipeline. A method was anticipated to study the probability of occurrence on increasing vibration in pipelines. Regression analysis was done to represent the relationship between the natural frequency and diameter, length and wall thickness.

2.3.2 Analytical Modal Analysis

An analytical approach is outlined, and the approach can be approximated using uniform distributed beam vibration theory. The natural frequency acquired is then validated and compare with analytical method that deliberately explained later in this subchapter. There are studies that uses analytical approach to validate the natural frequency obtained FEA method. Sollund and Vedeld [16] had used ABAQUS to validate the FEA using Specific Purpose Finite Element Analysis (SPFEA) after performing modal analyses towards the multi-span pipelines. Sutar, et al. [17] had performed FEA using FORTRUN and validated the natural frequencies for modal analysis using ABAQUS software. Reason being the validation process is to ensure that the results compared in FEA and analytical or numerical approach must be in close agreement with low percentage of difference. Liu, et al. [22] had used ABAQUS to verify the natural frequency from FORTRUN on multi-span pipe length with different type of pipe support.

For this study, Equation 7 is used to calculate the natural frequency of a straight and uniform pipe [7, 23, 24].

$$\omega_n = f_o = \frac{\lambda}{2\pi} \sqrt{\frac{gEI}{\mu l^4}}$$
(7)

Where:

 $\omega_n = f_0$ = Natural frequency, Hz

g = Gravitational constant, 386 in/sec^2

E = Modulus of elasticity, psi

I = Moment of inertia, in^4

l =length of pipe, in

 λ = Frequency factor, dimensionless

 μ = Weight per unit length, lbs/in. if weight of fluid and insulation is negligible, then it is equal to ρA

 ρ = Density, lbs/in³

A = Pipe cross sectional area, in^2

The equation does not consider fluid that filled in the pipe either or both of insulation that covers to the pipe of interest. The natural frequency of filled pipe is a multiplication of ration empty pipe weight per length and pipe with liquid weight per length with natural frequency of empty pipe with respect to both density and liquid of each condition [7, 23]. As of frequency factor, the value is according to the types of support condition of the interest pipe. Usually the first frequency factor is considered in structural analysis to obtain pipe natural frequency and its mode shape. This can portray the vibrational behaviour of the pipe to prevent any failure or crack occurrence. According to Amaechi [14], the first natural frequency factor is the most important mode and mostly it is considered in design stage to study the deformation of the structure when vibrations is applied. The frequency factor and pipe configuration are as Table 1:

No.	Support Condition	Pipe Configuration	1 st Natural Frequency Factor (Hz)
1.	Fixed – Fixed		22.4
2.	Fixed – Supported		15.4

Table 1: The First Two Frequency Factor according to the Support Condition [23]

2.3.3 Regression Analysis

Regression Analysis is another numerical approach to deduce a relationship between the length of the pipe and pipe containment, pipe support conditions and the materials of the pipe. The significance of applying the regression analysis is to determine the analytical form relationship between the pipe conditions and the natural frequency obtained [10].

The equation used will be used to calculate the natural frequency for different type of length of the vibrating pipes if FEA is not desirably used in future. The Regression Analysis is compiled to obtain R^2 value. The acceptance of R^2 value is $R^2 \ge 0.80$ $(0.80 \le R^2 \le 0.99)$ where when the R^2 is near to 1, it is an ideal model where the coupling and the conditions is agreed accordingly – the model explains 100% dependence [10]. By acquiring both analytical and FEA natural frequency, a graph is plotted to portray the trend of natural frequency with respect to its length, support conditions and the conditions of the pipe. Equation of the graph is generated by using MS Excel. This equation can be used to calculate the natural frequency for different type of length of the vibrating pipes without using the equation nor simulation. The equation portrays the dependency the natural frequency of the pipe towards the length of the pipe span with respect to the conditions which are pipe containment, support conditions, and materials of the pipes. These results carry a significant meaning for vibration analysis and modal analysis is a first platform before any subsequent dynamic analysis is performed.

2.4 Case Study from Literature Review

Liu, et al. [22] had modelled a multi span pipe with different support condition by using FEA method via ABAQUS and validated by analytical approach using FORTRUN software for validation purpose. The aim of the paper was to provide an efficient method of vibration characteristic analysis in order to design a multi-span pipeline with different span length and type of supports. The author had conducted the analysis by using elastic support at each of the span with spring coefficient, k = 43 kN/m with rigid bearing support at both end of the pipe. The significant of this paper to this project is to validate and verification of method of Modal Analysis used afterwards. The pipe has its own parameter and it is B21 element. The parameters of the pipe used are as Table 2.

Table 2: Physical Parameters and Geometry of the Pipe

Elastic Modulus	Pipeline Density	Oil Density	Pipe Outside Diameter	Pipeline Wall Thickness
(GPa)	(Kg/m^3)	(Kg/m^3)	(m)	(m)
200	7850	900	0.16	0.01

Below is the drawing of the pipeline with unequal span length and pipe span.



Figure 5: Multi-span Pipe with Pipe Supports [22]

CHAPTER 3 METHODOLOGY

3.1 Project Methodology

The nature of piping vibration is usually a complex topic to be debated. Based on the literature review, the researcher had outlined methods for this project in order to achieve all objectives aforementioned. The premise of this project is simple – to investigate the mode shape and natural frequency of the pipe towards length of the pipe span, the pipe support conditions and the materials of the pipe. This project flowchart is as displayed as Figure 6.



Figure 6: Project Flowchart

3.2 Method Verification

First and foremost, data collection from the literature review was done. This is to ensure that the method used for Modal Analysis is verified following the specifications and standards and the literature review journal is act as a control reference. Thus, errors can be avoided or as minimal it can be besides producing reliable results. Literature from Liu, et al. [22] is chosen for Modal Analysis method verification. Reason being is that the Liu, et al. [22] had used FORTRUN as their analytical approach to validate the natural frequency obtained by FEA method using ABAQUS. For this research, the same pipe configuration and properties are modelled and using the exact same support condition which is Fixed-Fixed with elastic support at each of the pipe-span with spring coefficient, k = 43 kN/m. The analysis was performed using ANSYS Workbench and the data obtained were validated.

The pipe configuration and material properties of the pipe and fluid contain are as Figure 7 and Table 3. According to Liu, et al. [22], the goals is to achieve natural frequency of the pipe where the support conditions are Fixed-Fixed condition at both end of the pipe, and for every span length an elastic support with stiffness coefficient, k = 43kN/m supports the pipe. Then, the natural frequency obtained from the Finite Element Analysis is compared with the journal paper.



Figure 7: Multi-Span Pipe Used In This Study [22]

Table 3: Physical Parameters and Geometry of the Pipe

Elastic modulus (GPa)	Pipeline density (kg/m ³)	Oil Density (kg/m ³)	Pipe outside diameter (m)	Pipeline wall thickness (m)	Stiffness Coefficient, k (kN/m)
200	7850	900	0.16	0.01	43

3.3 Modal Analysis

The primary objective of this Modal Analysis is to obtain natural frequency and system mode shapes. All structural or even piping systems have their own natural frequency, depending on their mass distribution and the rigidity of the support in the network [10]. As aforementioned in Literature Review, the normal frequency of the pipe span shall not correspond with the spectrum of external excitation frequencies such as earthquake, rotating machinery, and turbulence caused by flow. When the normal frequency of the system overlaps with the frequency of excitation, the vibration occurs, and the pipe vibrates according to the intensity. In this study, ANSYS Workbench is used for FEA method and formula mentioned in Equation 7 for analytical approach.

3.3.1 Finite Element Analysis (FEA)

A model pipe is developed using CATIA V5 with different length of pipe span i.e. 1m, 2m, 4m, 8m, and 10m. The model will be evaluated under these two types of conditions which are: (i) Empty Pipe; and (ii) Filled Pipe. Empty Pipe is a pipe without fluid while Filled Pipe is a pipe with fluid in its containment. The fluid is assumed to be oil and filled full throughout the pipe length. The properties of fluid inside the pipe are as of Table 4.

Table 4:	Fluid	Properties
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Oil Density, ρ (kg/m ³)	Poisson Ratio	Modulus of Elasticity, E (Pa)	Bulk Modulus (Pa)	Shear Modulus (Pa)
900	0.3	1.3E+09	1.0833E+09	5.00E+08

Modal Analysis is performed with temperature of environment constant at 27°C for both empty and filled modelled pipe by using ANSYS Workbench for 1m, 2m, 4m, 8m, and 10m Steel pipe with Fixed-Fixed support condition. As mentioned in scope of this study, only the first natural frequency factor and the mode shape are attained. The results from FEA are then compared with analytical approach for validation and Regression Analysis is done to acquire the trend and equation of the goodness of fit according to the conditions with respect to the pipe length.

The equation used will be used to calculate the natural frequency for different type of length of the vibrating pipes if FEA is not desirably used in future. The significance of Regression Analysis is to obtain R^2 value done by MS Excel. The acceptance plotting is when R^2 value is $R^2 \ge 0.80$ ($0.80 \le R^2 \le 0.99$) where when the R^2 is near to 1. This shows the dependency from the length of the pipe towards pipe conditions.

Then, the FEA steps are repeated for Fixed-Supported support conditions. In ANSYS Workbench, Fixed-Supported for one end and at the other end Displacement Support with x-direction = 0, y-direction = 0, and z-direction = free, (x, y, z = 0, 0, free) is used. The steps are also repeated for two different materials that are widely used in the oil and gas industry which are ASTM A106 Gr B (or in ANSYS known as Low Carbon Steel, Annealed), and Stainless Steel, SS 304L (or as known as Stainless Steel, Austenitic in ANSYS). The properties of the materials are as Table 5. These materials were chosen because it is the most common pipe in refinery or plant industry. Carbon steel are known for high temperature resistant while stainless steel and steel pipes are known as general corrosive resistant.

Table 5: Material	Properties	of the Pi	pe
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No.	Material Properties	Steel	ASTM A106 Gr B	SS 304L
1.	Density (kg/m ³)	7850	7850	7850
2.	Elastic Modulus (GPa)	200	210	215
3.	Poisson Ratio	0.3	0.3	0.3

4.	Thermal Expansion (10 ⁻⁵ /K)	1.2	1.6	1.6
5.	Specific Heat (J/Kg-K)	465	450	490
6.	Tensile Strength (MPa)	420	413	510
7.	Yield Strength (MPa)	350	242	190

3.3.2 Analytical Modal Analysis

Validation purposes must be considered by calculating analytical natural frequency [7, 23, 24]. The equation calculated are as Equation 7.

$$\omega_n = f_o = \frac{\lambda}{2\pi} \sqrt{\frac{gEI}{\mu l^4}}$$
(7)

Where:

 $\omega_n = f_o$ = Natural frequency, Hz

g = Gravitational constant, 386 in/sec^2

E = Modulus of elasticity, psi

 $I = Moment of inertia, in^4$

l =length of pipe, in

- λ = Frequency factor, dimensionless
- = Weight per unit length, lbs/in. if weight of fluid and
- μ insulation is negligible, then it is equal to ρA

 ρ = Density, lbs/in³

A = Pipe cross sectional area, in^2

Note that the equation does not ponder any fluid and insulation of the pipe. To calculate the filled pipe, the natural frequency calculated for empty pipe is multiply with the ratio of empty pipe weight (kg/m) and the pipe with liquid weight (kg/m) with respect to the both density of pipe and the liquid [23]. As for frequency factor, the value is different according to the support condition of the pipes. The first two frequencies (or as known as the first two mode shape) are interested in calculating the pipe natural

frequency. The two different types of support conditions that considered namely Fixed-Fixed, and Fixed-Supported. These type of support conditions are mainly used in actual plant where a pipe is constructed to or from a pump to another column or vessel (Fixed-Fixed) or subjugated with a pipe support at the end (Fixed-Supported). The frequency factor and pipe configuration are as Table 6:

No.	Support Condition	Pipe Configuration	1 st Natural Frequency Factor (Hz)
1.	Fixed – Fixed		22.4
2.	Fixed – Supported		15.4

Table 6: The First Two Frequency Factor according to the Support Condition [23]

In terms of materials used in the study, three types of material were chosen that are steel, ASTM A106 Gr B (or generically known as Low Carbon Steel) and SS 304L (or generically known as Austenitic Stainless Steel). The material properties of the pipes for each of the materials are tabulated as Table 5. For this study, different materials are chosen because to examine the effect of natural frequency and the deformation on mode shape towards vibration. It should give a great platform for future study on stress structural analysis.

CHAPTER 4 RESULTS AND DISCUSSIONS

4.1 Method Verification

As mentioned in Methodology, the method verification is done to verify and check that the tool use is reliable. Liu, et al. [22] uses ABAQUS to obtain natural frequency by FEA method and validated using computer programme FORTRUN for analytical approach. The tool uses for this research are ANSYS Workbench and MS Excel. Figure 8 shows the pipe geometry for FEA with fixed support at both ends and elastic support with spring coefficient, k = 43kN/m.



Figure 8: Multi-Pipe Span with Elastic Support



Table 7: Mode Shape and Natural Frequency for Finite Element Analysis of the Multi Span Pipe



The comparison for ANSYS Workbench FEA and the literature review are compared in Table 8:

Mode	Natural Frequency (Hz) (FEA Simulated: ANSYS)	Natural Frequency (Hz) (Literature Review: ABAQUS)	Percentage of Error (%)
1	14.980	15.289	2.08
2	40.711	40.764	0.13
3	79.027	78.866	0.23
4	129.190	128.700	0.38
5	190.510	189.610	0.47
6	262.140	260.780	0.52

Table 8: Comparison between FEA and Journal Paper

Based on the percentage differences portrayed in Table 8, it is clearly indicates that the tool uses for simulation in ANSYS is acceptable as there are closed with the journal paper and the error is less than 5%. It appears that the Modal Analysis presented in this research are profoundly identical to those from the ABAQUS, in the literature review. Thus, the model and the step have been validated and further steps can be continued.

4.2 Vibration Behaviour Towards Pipe Length

In this subchapter, a filled steel pipe with Fixed-Supported support condition is used to obtain the vibration behaviour of the pipe with difference of pipe length. An example of pipe geometry for filled steel pipe with Fixed-Support support conditions are as Figure 9.



Figure 9: 2m Filled Steel Pipe with Fixed-Support Support Condition

4.2.1 Natural Frequency Towards Pipe Length

The natural frequency as aforementioned can be obtain by calculation as per Equation 7 where frequency factor and density of the pipe is calculated and specified beforehand. As of simulated natural frequency can be calculated using Finite Element Analysis by ANSYS Workbench.

To portray the effect of natural frequency with respect to the pipe length, a filled steel pipe with Fixed-Supported support condition. The hypothesis for this case is the longer the pipe, the lower the natural frequency produces. This is because the natural frequency is inversely proportional to the length of the pipe. Thus, reducing the natural frequency causes the pipe to vibrate even more susceptible and lead to failure due to vibration such as vibration fatigue failure.

An example of analytical calculation for 1m Steel Pipe is demonstrated as Equation 7:

$$\omega_n = f_o = \frac{\lambda}{2\pi} \sqrt{\frac{gEI}{\mu l^4}} \tag{7}$$

Where:

$$\begin{split} \omega_n &= f_o = \text{Natural frequency, Hz} \\ g &= 386 \text{ in/sec}^2 \\ E &= 2.90\text{E}+07 \text{ psi} \\ I &= 31.9834 \text{ in}^4 \\ l &= 1 \text{ m} = 39.37 \text{ in; } 2 \text{ m} = 78.7402 \text{ in; } 4 \text{ m} = 157.4803 \text{ in;} \\ 6 \text{ m} &= 236.2205 \text{ in; } 8 \text{ m} = 314.9606 \text{ in;} \\ \text{and } 10 \text{ m} &= 393.7008 \text{ in} \\ \lambda &= 1 \text{ st Frequency Factor (for Fixed- Supported)} = 15.4 \\ \mu &= 2.07 \text{ lbs/in} \\ \rho &= 0.2835992 \text{ lbs/in}^3 \\ A &= 7.3042 \text{ in}^2 \end{split}$$

Substitute with respect to length and frequency factor,

$$\omega_n = f_o = \frac{15.4}{2\pi} \sqrt{\frac{386(2.9E + 07)(31.9834)}{(2.07)(39.37)}}$$

$$f_o = 657.505$$
 Hz for Empty Pipe

To calculate Filled Pipe, the obtained natural frequency, f_o is multiplied by the ratio of empty pipe weight per length and filled pipe weight per length.

Empty pipe weight per =
$$1 \text{ m} = 37 \text{ kg/m}$$
; $2 \text{ m} = 74 \text{ kg/m}$;
length, W_{empty} (kg/m) $4 \text{ m} = 148 \text{ kg/m}$; $6 \text{ m} = 222 \text{ kg/m}$;
 $8 \text{ m} = 296 \text{ kg/m}$; and $10 \text{ m} = 508 \text{ kg/m}$
Filled pipe weight per = $1 \text{ m} = 50.8 \text{ kg/m}$; $2 \text{ m} = 101.6 \text{ kg/m}$;
length, W_{filled} (kg/m) $4 \text{ m} = 203.2 \text{ kg/m}$; $6 \text{ m} = 304.8 \text{ kg/m}$;
 $8 \text{ m} = 406.4 \text{ kg/m}$; and $10 \text{ m} = 508 \text{ kg/m}$

Thus for 1st Frequency Factor of 1m Steel Pipe,

$$f_o = 657.505 \ (\frac{W_{empty}}{W_{filled}})$$

 $f_o = 657.505 \ (\frac{37}{50.8})$
 $f_o = 561.136 \text{ Hz for Filled Pipe}$

∴ Empty Pipe Natural Frequency, $f_o = 657.505$ Hz Filled Pipe Natural Frequency, $f_o = 561.136$ Hz

The trend of the natural frequency of filled steel pipe with Fixed-Supported support condition are as Table 9. There is a good agreement for analytical approach and FEA as it has maximum percentage difference 5% for 2m and above. This shows that the pipe for 2m and above is a good model for portraying vibration behaviour with respect to the length of the pipe. It is clearly depicted the natural frequency of the pipe decreases with increasing pipe length. The length of the pipe is inversely proportional to the natural frequency of the pipe as per analytical natural frequency definition (see: Equation 7).

Length of pipe (m)	Analytical Natural Frequency (Hz)	FEA Natural Frequency (Hz)	Percentage Difference (%)
1	561.136	480.430	14.38
2	140.284	134.945	3.81
4	35.071	34.879	0.55
6	15.587	15.602	0.09
8	8.768	8.796	0.32
10	5.611	5.636	0.44

Table 9: Effect of Natural Frequency Towards Pipe Length for A Filled Steel Pipe With Fixed-Supported Support Condition

4.2.2 Regression Analysis Towards Pipe Length

A simple regression analysis was performed in order to deduce a relationship of the natural frequency and the length of the pipe for a filled steel pipe with Fixes-Supported support condition. The graph for a filled steel pipe with Fixed-Supported support condition are as follows producing equation and regression analysis using MS Excel. Figure 10 portrays the pattern of the natural frequency and pipe length. This regression used power function in order to obtain $R^2 \ge 0.8$. The R^2 for FEA method filled steel pipe with Fixed-Supported support condition is $R^2 = 0.9997$, while analytical method with the same pipe condition. The analytical method shows an accurate model with 100% dependence and ideal. The equation for filled steel pipe with Fixed-Supported for analytical and FEA approach are as Equation 8 and Equation 9 respectively.



Figure 10: Effect of Natural Frequency Towards Pipe Length for a Filled Steel Pipe With Fixed-Supported Support Condition

Equation for Analytical Method for Filled Steel Pipe with Fixed-Supported Support Condition:

$$f_o = 561.14l^{-2}, R^2 = 1 \tag{8}$$

Equation for FEA Method Filled Steel Pipe with Fixed-Supported Support Condition:

$$f_o = 498.09l^{-1.936}, R^2 = 0.9997 \tag{9}$$

Where: f_o = Natural Frequency, Hz l = Length of the Pipe

4.2.3 Mode Shape of the Pipe Towards Pipe Length

Mode shape describes the deformation pattern of the pipe span at specific natural frequency. The mode shape shows the localisation of high stress and deflection area at the pipe for future and further structural analysis. Table 10 shows the mode shape of the filled steel pipe length with Fixed-Supported support condition with respect to pipe length.

Length of Pipe (m)	FEA Natural Frequency (Hz)	Mode Shape
1	480.430	ANSYS The Tail Demonstrating Tail Tail Demonstrating Tail Tail Demonstrating Tail Tail Demonstrating Tail Tail Tail Tail Tail Tail Tail Tail
2	134.945	C. Mod 2n = fiél Trai Elefension Trai Series Units Trai Series Units Trai Series Trai Se
4	34.879	Ansys Ansys Answer

Table 10: Effect of Mode Shape Towards Pipe Length for a Filled Steel Pipe with Fixed-Supported support condition



4.3 Vibration Behaviour Towards Pipe Containment

In this subchapter, steel pipe with Fixed-Supported support condition act as control variable in order to obtain the vibration behaviour of empty and filled pipe with respect to the various pipe span length which are 1m, 2m, 4m, 8m, and 10m. The fluid is assumed to be fully occupied. Examples of pipe geometry for 2m empty and filled steel pipe with Fixed-Supported support conditions are as Figure 11 and Figure 12 respectively.



Figure 11: 2m Empty Steel Pipe with Fixed-Supported Support Condition

Figure 12: 2m Filled Steel Pipe with Fixed-Supported Support Condition

4.3.1 Natural Frequency Towards Pipe Containment

From the analysis done for steel Fixed-Supported support conditions, the trend of the natural frequency of the pipe containment are as Table 11. It is found that the analytical and FEA natural frequencies results are having a good agreement with maximum percentage difference is less than 5% except for both empty and filled 1m pipe. This shows that the pipe for 2m and above is a good model for portraying vibration behaviour in terms of pipe containment.

As to compare the pipe containment, the empty pipe natural frequency is higher than the filled pipe. This is because the fluid exists in the pipe act as damping and effecting the pipe mass that added to the total mass and acting as dampener. The weight of the pipe is inversely proportional to the natural frequency as per Equation 7. Thus, adding more mass into the pipe making the pipe has lower natural frequency. Natural frequency for Fixed-Fixed support condition with different type of materials are as per displayed at Appendix A.

Landh	Empty Steel P Sup	ipe with Fixed	-Supported n	Filled Steel Pipe with Fixed-Supported Support Condition		
of pipe (m)	Analytical Natural Frequency (Hz)	FEA Natural Frequency (Hz)	Percentage Difference (%)	Analytical Natural Frequency (Hz)	FEA Natural Frequency (Hz)	Percentage Difference (%)
1	657.505	557.680	15.18	561.136	480.430	14.38
2	164.376	157.180	4.38	140.284	134.945	3.81
4	41.094	40.674	1.02	35.071	34.879	0.55
6	18.264	18.197	0.37	15.587	15.602	0.09
8	10.274	10.259	0.15	8.768	8.796	0.32
10	6.575	6.572	0.05	5.611	5.636	0.44

Table 11: Natural Frequency for Empty and Filled Steel Pipe with Fixed-Supported Support Condition Against Length of The Pipe

4.3.2 Regression Analysis Towards Pipe Containment

As mentioned in Literature Review, Regression Analysis is performed to deduce a relationship between the length of the pipe and the pipe containment for this subchapter. A simple regression analysis had done for obtaining equations of natural frequency towards pipe containment with respect to the pipe span length. The regression for the current control condition which is steel pipe with Fixed-Support support condition are as Figure 13. Equation 10 depicted empty steel pipe with Fixed-Supported support conditions while Equation 11 is for filled steel pipe. The regression is using power function to obtain $R^2 \ge 0.8$ and for both cases, the R^2 complies the condition which are $R^2 = 0.9997$ for both empty and filled steel pipe.



Figure 13: Natural Frequency for Empty and Filled Steel Pipe with Fixed-Supported Support Condition Against Length of The Pipe

Equation for Empty Steel Pipe with Fixed-Supported Support Condition:

$$f_o = 578.83l^{-1935}, R^2 = 0.9997 \tag{10}$$

Equation for Filled Steel Pipe with Fixed-Supported Support Condition:

$$f_o = 498.09l^{-1.936}, R^2 = 0.9997 \tag{11}$$

Where: $f_o =$ Natural Frequency, Hz

l = Length of the Pipe

4.3.3 Mode Shape of the Pipe Towards Pipe Containment

As mentioned in Chapter 2.3.1: Finite Element Analysis, the mode shape describes the deformational pattern of the pipe span at specific natural frequency. From Table 12, the mode shape of empty and filled steel pipe with Fixed-Supported support condition is listed according to the length of pipe. Visually there is no significance difference for empty and filled pipe.

Table 12: Mode Shape of Natural Frequency for Empty and Filled Steel Pipe with Fixed-Supported Support Condition Against Length of The Pipe

	Empty Steel Pi	pe with Fixed-Supported	Filled Steel Pipe with Fixed-Supported Support Condition			
Length of	Supp	oort Condition				
Pipe (m)	FEA Natural	Mada Shana	FEA Natural	Mada Shana		
	Frequency (Hz) Mode Shape Frequency (Hz) Frequency (Hz)		Mode Shape			
1	557.680	Mit of the second secon	480.430			
		· · ·		• <u>•</u> ••		
2	157.180	Handler Handle	134.945			
		120		€ ¹⁹²⁰		
4	40 674	And a second sec	34 879			
-	-0.07-	i i i i i i i i i i i i i i i i i i i	51.075	E.		

6	18.197	15.602	
8	10.259	8.796	
10	6.572	5.636	

4.4 Vibration Behaviour Towards Pipe Support Condition

To study the vibration behaviour towards type of pipe support condition with respect to the length of pipe span, the control variable is filled steel pipe. Examples of pipe geometry for 2m Filled Steel Pipe with Fixed-Fixed and Fixed-Supported support conditions are as Figure 14 and Figure 15 respectively.



Figure 14: 2m Filled Steel Pipe with Fixed-Fixed Support Condition



Figure 15: 2m Filled Steel Pipe with Fixed-Supported Support Condition

4.4.1 Natural Frequency Towards Pipe Support Condition

The trend of the natural frequency of filled steel pipe for Fixed-Fixed and Fixed-Supported support conditions are as Table 13. It is found that the agreement for analytical and FEA is good for Fixed-Supported support conditions as it has a percentage difference below 5% for pipe span 2m and above. As for Fixed-Fixed Support conditions, the agreement comes after 4m pipe span length. The accuracy of the FEA might be highlighted for Fixed-Fixed support conditions.

The natural frequency for Fixed-Fixed support condition is higher compare to Fixed-Supported support condition. Reason being is the stiffness of the pipe for Fixed-Fixed support condition is higher than Fixed-Supported. This makes Fixed-Fixed support condition more rigid as it does not allow the pipe to move or vibrate. Thus, reducing the flexibility of the pipe. Rigid pipes are usually prone to fatigue cracks due to vibration. For Fixed-Supported, the pipes are freely to move in *z*-direction reducing the stiffness of the pipe. Natural frequency for different type of materials and pipe containment conditions are as per displayed at Appendix A.

Table 13: Natural Frequency for Filled Steel Pipe with Fixed-Supported and Fixed-Fixed Support Condition Against Length of The Pipe

Longth	Fixed – Fixe Fi	d Support Cor lled Steel Pipe	ndition for	Fixed – Supported Support Condition for Filled Steel Pipe			
of pipe (m)	Analytical Natural Frequency (Hz)	FEA Natural Frequency (Hz)	Percentage Difference (%)	Analytical Natural Frequency (Hz)	FEA Natural Frequency (Hz)	Percentage Difference (%)	
1	816.197	631.935	22.58	561.136	480.430	14.38	
2	204.049	189.660	7.05	140.284	134.956	3.81	
4	51.012	50.205	1.58	35.071	34.879	0.55	
6	22.672	22.570	0.45	15.587	15.602	0.09	
8	12.753	12.756	0.02	8.768	8.796	0.32	
10	8.162	8.176	0.17	5.611	5.636	0.44	

4.4.2 Regression Analysis Towards Pipe Support Condition

Regression analysis was performed to study the relationship between the natural frequency and pipe span length for different types of support condition which are Fixed-Fixed and Fixed-Supported. Figure 16 portrays the pattern of the natural frequency with respect to pipe span length. The regression used is power function in order to obtain $R^2 \ge 0.8$. The R^2 for Fixed-Fixed support condition is $R^2 = 0.9993$ meanwhile for Fixed-Supported support conditions is $R^2 = 0.9997$. The R^2 value for both support

condition is accepted as they are obeying the regression condition of $R^2 \ge 0.8$. The equation for both Fixed-Fixed and Fixed-Supported support condition with filled steel pipe are as Equation 12 and Equation 13 respectively.



Figure 16: Natural Frequency for Filled Steel Pipe with Fixed-Supported and Fixed-Fixed support Condition Against Length of The Pipe

Equation for Filled Steel Pipe with Fixed-Fixed Support Condition:

$$f_o = 668.2l^{-1.897}, R^2 = 0.9993 \tag{12}$$

Equation for Filled Steel Pipe with Fixed-Supported Support Condition:

$$f_o = 498.09l^{-1.936}, R^2 = 0.9997 \tag{13}$$

Where: f_o = Natural Frequency, Hz

l = Length of the Pipe

4.4.3 Mode Shape of the Pipe Towards Pipe Support Condition

The deflection pattern of Fixed-Fixed and Fixed-Supported support condition with respect to the pipe span length are shown as Table 14. When comparing the pipe mode shape between Fixed-Fixed and Fixed-Supported support conditions, the pattern of deflections is similar, and it show no connotation. However, the natural frequency differences give huge merit in understanding vibrational behaviour and have mentioned in previous chapter.

Table 14: Mode Shape for Filled Steel Pipe with Fixed-Supported and Fixed-Fixed Support Condition Against Length of The Pipe

	Fixed – Fixed	Support Condition for	Fixed – Support	ed Support Condition for		
Length of Filled Steel Pipe			Filled Steel Pipe			
Pipe (m)	FEA Natural	Mode Shape	FEA Natural	Mode Shape		
	Frequency (Hz)	Woue Shape	Frequency (Hz)	moue shape		
1	631.935		480.430			
2	189.660		134.945			
4	50.205		34.879			
6	22.570		15.602			
8	12.756		8.796			
10	8.176		5.636			

4.6 Vibration Behaviour Towards Pipe Materials

To study the vibrational behaviour for different pipe materials, filled steel pipe with Fixed-Supported support condition is chosen as control variable. The material chosen for this study are Steel pipes, ASTM A106 Gr B (Low Carbon Steel) pipes, and SS 304L (Austenitic Stainless Steel) pipes. These pipes are widely used in oil and gas field and its application had been explained in Chapter 3.3.2: Finite Element Analysis. Examples of 2m Fixed-Supported Support Conditions pipe geometry for Filled Steel pipes, ASTM A106 Gr B pipes and SS 304L pipes with respect to pipe length are as Figure 17, Figure 18, and Figure 19 respectively.





Figure 17: 2m Fixed-Supported Support Conditions for Filled Steel Pipe

Figure 18: 2m Fixed-Supported Support Conditions for Filled ASTM A106 Gr B (Carbon Steel) Pipe



Figure 19: 2m Fixed-Supported Support Conditions for Filled SS 304L (Austenitic Stainless Steel) Pipe

4.6.1 Natural Frequency Towards Pipe Material

From FEA and analytical approach, the natural frequency for filled steel pipe, ASTM A106 Gr B pipe and SS 304L pipe with fixed-supported support condition are as Table 15. It is found that the analytical approach and FEA method comes with a good agreement with maximum percentage difference is below 5% for 2m pipe and above. This shows that the modelling in FEA method is good and acceptable in portraying vibrational behaviour.

In order to compare the natural frequencies between the materials, ASTM A106 Gr B has the highest natural frequency followed by Steel and SS 304L. Reason being is that ASTM A106 Gr B has the highest stiffness as compare to Steel and SS 304L. Herewith, the natural frequency of the pipe is depending on the material stiffness. Natural frequency for Fixed-Fixed support condition with different type of pipe containment are as per displayed at Appendix A.

Len	Filled Steel Pipe with Fixed-Supported Support Condition		Filled ASTM A106 Gr B Pipe with Fixed-Supported Support Condition			Filled SS 304L Pipe with Fixed-Supported Support Condition			
gth of pipe (m)	Analytical Natural Frequency (Hz)	FEA Natural Frequency (Hz)	Percentage Difference (%)	Analytical Natural Frequency (Hz)	FEA Natural Frequency (Hz)	Percentage Difference (%)	Analytical Natural Frequency (Hz)	FEA Natural Frequency (Hz)	Percentage Difference (%)
1	561.136	480.430	14.38	574.969	492.310	14.38	561.112	478.790	14.67
2	140.284	134.945	3.81	143.742	138.245	3.82	140.279	134.300	4.26
4	35.071	34.879	0.55	35.936	34.231	4.74	35.070	34.704	1.04
6	15.587	15.602	0.09	15.971	15.983	0.07	15.586	15.523	0.41
8	8.768	8.796	0.32	8.984	9.011	0.30	8.767	8.752	0.18
10	5.611	5.636	0.44	5.748	5.774	0.42	5.611	5.608	0.06

Table 15: Natural Frequency for Filled Steel Pipe, ASTM A106 Gr B Pipe, and SS 304L Pipe with Fixed-Supported Support Condition Against Length of The Pipe

4.6.2 Regression Analysis Towards Pipe Containment

For this case, the regression analysis was performed due to study the relationship between natural frequency and the length of the pipe for three different materials with conditions of filled pipe with Fixed-Supported support conditions. Figure 20 shows the pattern of the natural frequency with respect to its pipe length. This regression analysis used power function in order to obtain as mentioned in methodology the acceptance value of R^2 is $R^2 \ge 0.8$. the R^2 value for Steel pipe is $R^2 = 0.9997$ while for both ASTM A106 Gr B and SS 304L are $R^2 = 0.9998$ each. The equation for filled steel pipe, ASTM A106 Gr B pipe and SS 304L pipe with Fixed-Supported support condition are as Equation 14 Equation 15 and Equation 16 respectively.



Figure 20: Natural Frequency for Filled Steel Pipe, ASTM A106 Gr B Pipe, and SS 304L Pipe with Fixed-Supported Support Condition Against Length of The Pipe

Equation for Filled Steel Pipe with Fixed-Supported Support Condition:

$$f_o = 498.09l^{-1.936}, R^2 = 0.9997 \tag{14}$$

Equation for Filled ASTM A106 Gr B with Fixed-Supported Support Condition:

$$f_o = 506.81l^{-1.937}, R^2 = 0.9998 \tag{15}$$

Equation for Filled SS 304L Pipe with Fixed-Supported Support Condition:

$$f_o = 496.16l^{-1.937}, R^2 = 0.9998$$
(16)

Where: f_o = Natural Frequency, Hz l = Length of the Pipe

4.6.3 Mode Shape of the Pipe Towards Pipe Containment

Mode shape displays the deformation pattern of the pipe, in this case it shows the deformation pattern of three different materials which are Steel, ASTM A106 Gr B and SS 304L for filled pipe with Fixed-Supported support condition as per Table 16, Table 17 and Table 18 respectively. As mentioned in previous conditions and cases, the mode shape of these different materials show no connotation when it compares between materials as the deformation patterns are similar. The main reason is the location of the high deformation point for future structural analysis.

Length of Pipe (m)	FEA Natural Frequency (Hz)	Mode Shape
1	480.430	C Mode In w Hall The The Editorianses The The Editorianses The The Editorianses The The Editorianses The The The The The The The The The The

Table 16: Mode Shape for Filled Steel Pipe with Fixed-Supported Support Condition Against Length of The Pipe



Table 17: Mode Shape for Filled ASTM A106 Gr B (Low Carbon Steel) Pipe with Fixed-Supported Support Condition Against Length of The Pipe

Length of Pipe (m)	FEA Natural Frequency (Hz)	Mode Shape
1	492.310	Name ANSYS
2	138.245	And the work of the formation of the for
4	34.231	Ansys The first start of the first start Start start start start Start start start start Start start start start Start start star
6	15.983	Image: Mill demonstration ANSYS Type: Mill demonstration Support Type: Mill demonstration Support Type: Mill demonstration Support Type: Mill demonstration Support Constration Support Constration Support Constration Support Constration Support Constration Support Constration Support Support Support
8	9.011	Windd Tyre field defauntation Tyre fi

10	5.774	X Model New Wold X Model New Wold X Model New Wold X Model X	NSYS 2019 R3
		0 3 Min 8 000 <u>1.500</u> <u>1.000 (m)</u> 0.510 <u>2.500</u>	

Table 18: Mode Shape for Filled SS 304L (Austenitic Stainless Steel) Pipe with Fixed-Supported Support Condition Against Length of The Pipe

Length of Pipe (m)	FEA Natural Frequency (Hz)	Mode Shape
1	478.790	At Acad I an RMA Tad Germanian throadyn (4797tr throadyn (4797tr) throadyn (4797
2	134.300	ANSYS Anticide devices and Answer A
4	34.704	ANSYS 2019 PB 20200 2134 AM 20200 PB 20200 PB 202000 PB 20200 PB 20200 PB 20200 PB 20200 PB 20200 PB 2020



CHAPTER 5 CONCLUSION AND RECOMMENDATION

5.1 Conclusion

In conclusion, this research had given a huge prospect as a simple approach is presented for studying a free vibrational analysis of a straight pipe with different type of support conditions and pipe properties for both with and without fluid pipes. The premise of this project is simple – to investigate the effect of the vibrational behaviour which are natural frequency of the pipes and its mode shapes towards the pipe span length, support conditions and the materials of the pipes. From the undergone Regression Analysis, it is clearly portrayed that the FEA approach using ANSYS Workbench is having a good agreement with the numerical and analytical approach for all of the stated conditions for 2m pipe and above. The natural frequency of a pipe will decrease when the length of the pipe is increasing.

With regards to the pipe containment conditions, the filled pipe has lower natural frequency than empty pipe. Reason being is because the fluid occupied in the pipe adding the pipe weight and act as damper. To associate between Fixed-Fixed and Fixed-Supported support condition, the Fixed-Supported support condition have lower natural frequency compare to Fixed-Fixed support condition due to less rigidity, and high flexibility for Fixed-Supported support condition. Lastly, as for materials comparison, the

ASTM A106 Gr B has the highest natural frequency followed by Steel and SS 304L due to difference in material stiffness

To ebb and to flow, this research had given a great insight as the simulation would help to visualise the behaviour of the vibration in depth. Also, from the simulation the researcher obtained the required and desired results for vibration characteristic according to the objectives outlined. The results attained from the simulation would help to understand further the vibration behaviour of the pipe after manipulating with the conditions specifically the length of pipe span, pipe containment, the support conditions and the materials of the pipe. As pointed out before, Modal Analysis is the first step for complex dynamic either or both of vibrational analyses such as Harmonic Response or Response Spectrum Analysis or even Structural Analysis where it introduces excitation force or mechanical induced vibration that gives a frequency domain with respect to spectral acceleration or displacement (for Response Spectrum Analysis). By this, it will give a great prospect for further rectification work in order to reduce the vibration of the pipe to prevent fatigue failure.

5.2 Recommendation

Future study shall be continued in order to obtain results for materials that widely used in refinery other than mentioned in this research such as aluminium, Incoloy, and Hastelloy. Other than, different type of pipe configurations such as bends, tee, and elbow can be done in modal analysis to obtain more varieties of pipe used in plant. A more detailed research can be pursued for several cases height of the liquid in pipe can be done to obtain its vibrational behaviour. Plus, different types of pipe support condition can be used for obtaining the natural frequency of the pipe and its mode shape.

Regression analysis and graph plotting for future understanding the nature behaviour of the natural frequency with respect to the environment for further deducing relationship. Structural analysis and response spectrum analysis can be done in future as modal analysis act as a platform further vibrational study.

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APPENDIX

NATURAL FREQUENCY DATA

The following table is natural frequency obtained by FEA and Anaytical Approach for all of the conditions which are pipe containment, pipe support condition, and pipe materials with respect to the pipe length.

Table 19: Natural frequency for FEA and Analytical Approach of Steel Pipe for Different Pipe Support condition and Pipe Containment with Respect to the Length of Pipe

	STEEL											
		Fixed-	Fixed Su	pport Cond	ition	Fixed- Supported Support Condition						
	E	mpty Pipe		Filled Pipe			Empty Pipe			Filled Pipe		
Length of pipe (m)	Analytical Natural Frequency (Hz)	FEA Natural Frequency (Hz)	Percentage Difference (%)	Analytical Natural Frequency (Hz)	FEA Natural Frequency (Hz)	Percentage Difference (%)	Analytical Natural Frequency (Hz)	FEA Natural Frequency (Hz)	Percentage Difference (%)	Analytical Natural Frequency (Hz)	FEA Natural Frequency (Hz)	Percentage Difference (%)
1	956.370	732.430	23.42	816.197	631.935	22.58	657.505	557.680	15.18	561.136	480.430	14.38
2	239.093	220.705	7.69	204.049	189.660	7.05	164.376	157.180	4.38	140.284	134.945	3.81
4	59.773	58.338	2.40	51.012	50.205	1.58	41.094	40.674	1.02	35.071	34.879	0.55
6	26.566	26.311	0.96	22.672	22.570	0.45	18.264	18.197	0.37	15.587	15.602	0.09

8	14.943	14.859	0.56	12.753	12.756	0.02	10.274	10.259	0.15	8.768	8.796	0.32
10	9.564	9.527	0.39	8.162	8.176	0.17	6.575	6.572	0.05	5.611	5.636	0.44

Table 20: Natural frequency for FEA and Analytical Approach of ASTM A106 Gr B Pipe for Different Pipe Support condition and Pipe Containment with Respect to the Length of Pipe

	ASTM A106 Gr B												
		Fixed	- Fixed Su	pport Con	dition	Fixed- Supported Support Condition							
	I	Empty Pip	e		Filled Pipe	e	I	Empty Pip	e	-	Filled Pipe	<u>,</u>	
Length of pipe (m)	Analytical Natural Frequency (Hz)	FEA Natural Frequency (Hz)	Percentage Difference (%)	Analytical Natural Frequency (Hz)	FEA Natural Frequency (Hz)	Percentage Difference (%)	Analytical Natural Frequency (Hz)	FEA Natural Frequency (Hz)	Percentage Difference (%)	Analytical Natural Frequency (Hz)	FEA Natural Frequency (Hz)	Percentage Difference (%)	
1	979.947	751.210	23.34	836.318	647.685	22.56	673.713	571.780	15.13	574.969	492.310	14.38	
2	244.987	226.195	7.67	209.079	194.305	7.07	168.428	161.080	4.36	143.742	138.245	3.82	
4	61.247	59.964	2.09	52.270	51.430	1.61	42.107	41.679	1.02	35.936	34.231	4.74	
6	27.221	26.960	0.96	23.231	23.120	0.48	18.714	18.646	0.37	15.971	15.983	0.07	
8	15.312	15.225	0.57	13.067	13.059	0.07	10.527	10.512	0.15	8.984	9.011	0.30	
10	9.799	9.761	0.39	8.363	8.375	0.15	6.737	6.734	0.05	5.750	5.774	0.42	

	SS 304L											
		Fixed-	Fixed Su	pport Cond	lition		Fixed- Su	pported S	Support Co	ndition		
	E	mpty Pipe		F	illed Pipe		Ε	mpty Pipe		F	illed Pipe	
Length of pipe (m)	Analytical Natural Frequency (Hz)	FEA Natural Frequency (Hz)	Percentage Difference (%)	Analytical Natural Frequency (Hz)	FEA Natural Frequency (Hz)	Percentage Difference (%)	Analytical Natural Frequency (Hz)	FEA Natural Frequency (Hz)	Percentage Difference (%)	Analytical Natural Frequency (Hz)	FEA Natural Frequency (Hz)	Percentage Difference (%)
1	956.330	730.830	23.58	816.163	625.420	23.37	657.477	555.860	15.46	561.112	478.790	14.67
2	239.082	219.730	8.09	204.041	188.810	7.46	164.369	156.445	4.82	140.278	134.300	4.26
4	59.771	58.224	2.59	51.010	49.951	2.08	41.092	40.470	1.52	35.069	34.704	1.04
6	26.565	26.176	1.47	22.671	22.453	0.96	18.263	18.104	0.87	15.586	15.523	0.41
8	14.943	14.782	1.08	12.753	12.683	0.55	10.273	10.206	0.65	8.767	8.752	0.18
10	9.563	9.478	0.90	8.162	8.134	0.34	6.575	6.538	0.55	5.611	5.608	0.06

Table 21: Natural frequency for FEA and Analytical Approach of SS 304L Pipe for Different Pipe Support condition and Pipe Containment with Respect to the Length of Pipe