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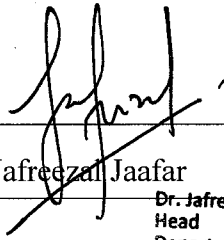
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USING TYPE-2 FUZZY

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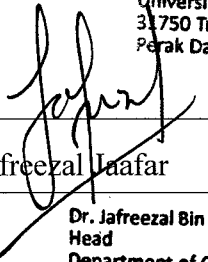


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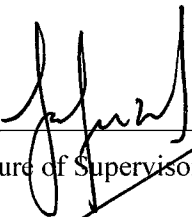
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DEDICATION

I dedicate this work to all my beloved big families.

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ABSTRACT

Significant wave height parameter plays an important role in ocean and coastal activities. However, the forecasting process of this parameter involved with uncertainty due to the nature of data. In past decades, fuzzy logic was introduced by Zadeh that enables a computer system to reason with uncertainty. This theory had widely applied in many applications, including wave parameter forecasting. Currently, wave height forecasting techniques were focusing on using Type-1 fuzzy method. This technique has a limitation in handling and minimizing uncertainties which affect the accuracy of the forecasting result. Therefore, this research is aimed to propose a method on forecasting significant wave height by using Type-2 fuzzy. The Type-2 fuzzy of has more degrees of membership function in fuzzy set which could model the uncertainties better than Type-1 fuzzy did. This method was done by assigning uncertainties to its own Type-1 counterpart. The performance of proposed method was scored with Root Mean Square Error (RMSE) and was compared with Type-1 fuzzy method. The proposed method results shown better performance compared with Type-1 fuzzy method with spread changes value of 0.15 and 0.2 from Type-1 counterpart for both locations. However, these spread changes values must have a cluster radius value of 0.35 in subtractive clustering phase. The proposed method had given an alternative tool for forecasting significant wave height with the improvement of 2% to 5% from Type-1 fuzzy method.

ABSTRAK

Tinggi ombak yang ketara memainkan peranan yang penting dalam bidang aktiviti laut dan pantai. Walau bagaimanapun, proses menjangka parameter ini melibatkan ketidakpastian yang disebabkan sifat data itu sendiri. Beberapa dekad yang lalu, kaedah kabur telah diperkenalkan oleh Zadeh yang membolehkan sebuah komputer mengendalikan ketidakpastian ini. Kaedah ini telah banyak diaplikasikan termasuk juga pengjangkaan parameter ombak. Kaedah yg digunakan pakai sekarang utk menjangka ombak hanya melibatkan kabur tahap pertama sahaja. Dimana, kaedah ini menghadapi kekurangan dalam mengendalikan ketidakpastian yang akan menjejaskan ketepatan keputusan jangkaan. Oleh yang demikian, kajian ini bertujuan untuk mencadangkan satu kaedah untuk menjangka tinggi ombak dengan menggunakan kaedah kabur tahap kedua. Kabur tahap dua ini memberikan lebih banyak kebebasan kepada fungsi keahlian dalam set kabur yang membolehkan ia mengendalikan ketidakpastian dengan lebih baik berbanding kabur tahap pertama. Kaedah ini dilaksanakan dengan menamba ketidakpastian kepada kabur tahap satu. Prestasi kaedah yang dicadangkan dinilai dengan menggunakan Ralat Punca Purata Kuasa Dua (RMSE) dan di bandingkan dengan kabur tahap pertama. Hasil keputusan kedah yang dicadangkan telah menunjukkan prestasi yang lebih baik berbanding prestasi kabur tahap satu dengan nilai perubahan penyebaran 0.15 dan 0.2 dari kabur tahap pertama di kedua-dua lokasi. Akan tetapi, nilai perubahan penyebaran ini mestilah mempunyai nilai radius kelompok 0.35 di fasa kelompok subtraktif. Kaedah yg dicadangkan boleh digunakan sebagai alat alternatif untuk menjangka ketinggian ombak dimasa akan datang dengan kadar peningkatan sebanyak 2% sehingga 5% daripada kabur tahap pertama.

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CHAPTER 1

INTRODUCTION

This chapter presents a general view of this research and problem statements that have been determined. The objectives of the research were identified based on the problem statements which are the important elements that guide along the research. Scope of study was specified and further thesis organization was briefly explained at the end of this chapter.

1.1 Introduction

Forecasting is important in many areas of sciences, industrial, commercial and economic activities. It helps the decision makers to deal with real problems by informing their expectations in the future [1]. Considering the imprecise and ambiguous nature of data and information, the ability to handle uncertainty is a crucial issue in forecasting. One of the techniques to solve the uncertainty problem is fuzzy method [2]. This method has successfully been applied in various fields such as pattern recognition [3], process control [4, 5], decision making [6] and forecasting [7]. All the studies are basically based on Type-1 fuzzy where a single degree of membership function has been used.

Recently, there has been a significant growth in Type-2 fuzzy where the fuzzy degree of membership function is introduced. Researchers such as [8-10] have started to use Type-2 fuzzy in modeling the uncertainties due to its ability to ensure the stability on the accuracy and robustness of forecasting algorithm [11]. Moreover, it also has a capability to handle linguistic uncertainties by unreliability of information [12]. Therefore, this research is proposing a method for forecasting significant wave

height parameter by using Type-2 fuzzy which has more capability in handling uncertainties in comparison with Type-1 fuzzy.

1.2 Background of Study

In the area of meteorological and oceanographic (Metocean), significant wave height parameter plays a vital role in ocean, coastal and marine activities [13]. A good forecasting model can ensure the operations performed safely, economically and efficiently. This makes the forecasting of the wave characteristics crucial. Various methods such as numerical [14], empirical [15], and soft computing [16] have been developed to produce the best wave parameter forecasting result in terms of forecasting accuracy with the smallest error.

Previously, numerical models [17] were used to forecast the wave parameter. However, these methods required high speed computers and variety of input data, which were not efficient in terms of economic point of view [18]. On the other hand, the empirical method such as Autoregressive Integrated Moving Average (ARIMA) method [15] is limited only to the requirement of stationary of time series, normality and independence of residuals [19]. Moreover, this method was unoccupied to solve forecasting problems in which the historical data were uncertainties such as linguistic value [20].

Due to the limitation in both methods, soft computing methods were introduced and practical in forecasting model. Ease of the application, good in handling uncertainty as well as less required computational time, has made these soft computing methods more suitable for wave modeling [21]. An example of soft computing method been used is Fuzzy Inference System (FIS). FIS is based on expertise expressed in terms of IF-THEN rules which can be used to predict uncertainty systems.

In 2005, the FIS was introduced to forecast the significant wave height in Lake Ontario [22] and also was used to investigate the relationship between wind and wave parameters in Pacific Ocean [23]. Then, subtractive clustering method was added in

FIS model to give initial and final values of antecedent part [24] in Northern Aegean Sea. However, most of FIS models used in wave domain were focused on Type-1 fuzzy model which might suffer in modeling the uncertainties. Therefore Type-2 fuzzy were applied, such as in the application of local meteorological forecasting [25].

1.3 Motivation of study

The motivation of study is due to the capabilities of fuzzy logic in modeling and handling uncertainties [8]. One of the advantages of FIS is it allows the users to include the unavoidable imprecision and uncertainty in the data through the degree of membership function in fuzzy set. As an analogy, man with 1.8 meter height might be considered tall in some countries and considered as short in other countries. In Type-1 fuzzy, the uncertainty is added in the fuzzy set with the present of degree of membership function. Belongingness of the value to the fuzzy set depends on the degree of membership function where if the value is near to 1, then the more belongingness of the value towards the fuzzy set. In Figure 1.1, with the tall of 1.8, it has a degree of 0.42 which means the value belongs to the tall fuzzy set with the degree of 0.42.

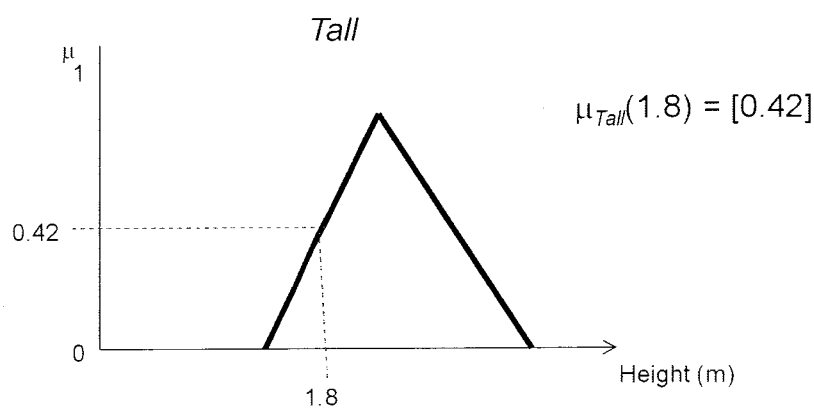


Figure 1.1: Type-1 fuzzy set

Currently, many fuzzy studies have put attention to Type- 2 fuzzy [26-29] which gives more degree of membership function to the fuzzy set. This extra degree of membership function is known as Footprint of Uncertainty (FOU) [2]. By using the

same example in Type-1 fuzzy, the tall fuzzy set has degrees of membership function more than one with Type-2 fuzzy. With 1.8 meter tall, the man has possibility to be categorized as tall or medium tall as the degrees of membership function is between 0.42 and 0.78 as shown in Figure 1.2.

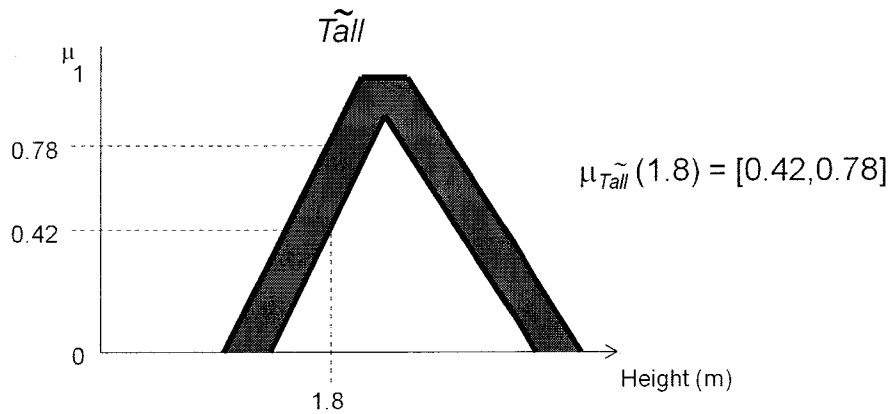


Figure 1.2: Type-2 fuzzy set

Therefore it is an interest to study Type- 2 fuzzy as they can model the uncertainties better than Type- 1 did by reducing the amount of uncertainties, which the Type- 2 fuzzy was characterized by fuzzy membership function gives more freedom in handling the uncertainties and reduced the error [12, 30]. Moreover, [12, 31] studies had shown that Type-2 fuzzy had outperformed the Type-1 fuzzy.

The proposed method is tested to forecast significant wave height because it is one of the essential parameter in Metocean environment for planning, operation and works related to Ocean Engineering especially in design, analysis and determination of the economical life of coastal and offshore structure [18]. Accurate value for significant wave height can affect the design of particular project structure [32]. Besides, significant wave height parameter was influenced by wind parameter [23] and these two parameters deal a lot with uncertainties [33]. Hence there is a need to find alternative ways to forecast which have capability in handling the uncertainties.

In addition, fuzzy model [23] has capability to directly forecast the wave height from wind parameter and past wave parameter. Based on literature review, the Type-2

fuzzy has a better capability in handling uncertainties compared to Type-1 fuzzy. Therefore, Type-2 fuzzy has a good potential for forecasting significant wave height.

1.4 Research Problem Statement

One of the aspects in forecasting the significant wave height is to determine a better and more accurate model with the smallest error generated. As the model is created by human, there are a lot of weaknesses that need to be identified on. According to [34], forecasting significant wave height is uncertain which the random process is not easy to be accomplished by using deterministic equation in empirical method. Meanwhile, fuzzy logic does not need the deterministic equations because it can forecast by using current measurement.

However, current wave parameter forecasting using fuzzy is just focusing on Type-1 fuzzy [22-24, 33, 35, 36]. This Type-1 fuzzy has limitation in handling and minimizing the uncertainties because Type-1 fuzzy set is certain as it has a crisp membership function for each fuzzy set [37]. When something is uncertain, like a measurement, it is difficult to determine its exact value. It is not reasonable to use an accurate membership function for something uncertain.

Measuring significant wave height is not an easy task. Therefore it can be achieved by using wind data as it is much easier to be obtained. However, the error may become large due to many uncertainties in the wind generation [38]. Basically, uncertainty can be a reflection of incompleteness, imprecision, missing information, or randomness in data and process [39]. These uncertainties can be reduced by modelling with fuzzy logic theory. In this case, Type-2 fuzzy sets is proposed. Therefore, the amount of uncertainty in a system can be reduced by using Type-2 fuzzy logic because it offers better capabilities to handle linguistic uncertainties by modeling vagueness and unreliability of information. This theory has been supported by [26] in which Type- 2 fuzzy set should offer a significant improvement on Type- 1 fuzzy set with more imprecise data.

1.5 Research Question

The main problem is the existing significant wave height forecasting model can only handle uncertainties with a crisp membership function. The research questions are:

1. How to extend Type-1 fuzzy to Type-2 fuzzy?
2. How Type-2 fuzzy can be used and applied for significant wave height forecasting?
3. How Type-2 fuzzy can improve significant wave height forecasting accuracy?

1.6 Research Objectives

The main objective of this research is to propose a method to forecast significant wave height by using Type-2 fuzzy. The formulation of Type-2 fuzzy methods is achieved by assigning uncertainties to cluster center of Type-1 fuzzy. Other related objectives of the research are listed below:

1. To extend Type-1 fuzzy to Type-2 fuzzy by assigning uncertainties to its own Type-1 counterpart.
2. To design and develop simulation of Type-2 fuzzy by using wave and wind data
3. To evaluate Type-2 fuzzy in term of accuracy of forecasting result by comparing Type-2 fuzzy with Type-1 fuzzy.

1.7 Scope of Study

The study scope must be identified in order to inform the limitation and the focusing area. The following scopes have been identified:

1. The study is focusing on the development of fuzzy logic itself where the Type-2 fuzzy was proposed and compared with Type-1 fuzzy.

2. The proposed method is developed and tested with the Metocean environment which focused on three variables; significant wave height, wind speed and wind direction.
3. The results shown in Chapter 4 are based on forecasting of significant wave height in 6 hours lead time.

1.8 Organization of Thesis

As mentioned in section 1.5, the main objective of this research is to forecast significant wave height parameter by using Type-2 fuzzy logic. Hence, the organization of this thesis is structured for the reader to have a better understanding about the topic.

In Chapter 2, a review of literature is provided on introduction about fuzzy logic theory which fuzzy set, membership function, fuzzy operator and also the differences between Type-1 fuzzy and Type-2 fuzzy are elaborated. This section is followed by an important topic that will be used in extending Type-1 fuzzy to Type-2 fuzzy where the previous methods are presented. Then, each phase in Fuzzy Inference System (FIS) and types of Inference System are generally discussed. The chapter concludes with a brief description of current wave height applications.

Chapter 3 describes the methodology of the proposed method. It consists of method used to forecast significant wave height based on phases available in FIS. Details explanations of each phases is elaborated with example of input from dataset.

In Chapter 4, the results obtained from the proposed method in chapter 3 are presented and analyzed. The performance of proposed method is compared with other fuzzy methods and the findings of the study are determined.

Chapter 5 presents conclusions based on results and analysis discussed in chapter 4, aligning with the problem statements and objectives stated in chapter 1. The chapter ends with the contributions of this study and recommendation for future works.

1.9 Summary

This chapter presents the summary of whole idea about the study of Type-2 fuzzy in forecasting significant wave height. The problems of wave height forecasting and current Type-1 fuzzy are identified. The objectives of study and research question are discussed based on the identified problem statement. These objectives are important as a guideline along the research. Ground knowledge of this study will be future elaborated in Literature Review chapter.

CHAPTER 2

LITERATURE REVIEW

This chapter elaborates the ground theory of fuzzy logic. It is important to have a good understanding in fuzzy environment before applying it in forecasting the significant wave height. This chapter starts with fundamental theory of fuzzy logic, where the fuzzy set, membership function and types of fuzzy are presented. Fuzzy Inference System (FIS) and current research applications that are involved with fuzzy and significant wave height are also discussed in this chapter.

2.1 Fuzzy Logic

Fuzzy logic was first developed by Lotfi A.Zadeh [40] in mid 1960s at Berkeley. He introduced fuzzy logic by mean to model the uncertainty of natural language [41]. It has a reasoning capability or inference system to work like human mind [42, 43]. The major advantages of fuzzy system models are their robustness and transparency. Fuzzy system modeling achieves robustness by using fuzzy sets which incorporates imprecision to the system models [44].

Fuzzy logic uses fuzzy set theory that generalizes from a crisp set. The fuzzy logic is made by replacing the bivalent membership functions of crisp logic with fuzzy membership function, where instead 0 or 1 value only, the value can be any in 0 to 1. Fuzzy logic provides a mean of calculating intermediate values between absolute true and false with resulting values ranging between 0.0 and 0.1. Furthermore, it calculates the shades of grey between black and white or true and false that is based on degrees of truth.

A fuzzy set is a set containing elements of universe of discourse with a specified range. Fuzzy set is mapped to real numbered value in the interval 0 to 1. Commonly

there are two ways to denote a fuzzy set. If X is the universe of discourse, and x is a particular element of X , then a fuzzy set A defined on X can be written as equation 1.

$$A = \{(x, \mu_A(x))\} \quad x \in X \quad (1)$$

$\mu_A(x)$ is the Membership Function (MF) of x in set A . It is also called as membership degree or membership grade. Fuzziness in fuzzy set is characterized based on the MF. In other words, the degree an object belongs to a fuzzy set is denoted by a membership value between 0 and 1. For example, the set A have a collection of the integers in equation 2.

$$A = \{(1, 1.0), (3, 0.7), (5, 0.3)\} \quad (2)$$

Thus, the second element of A expresses that 3 belongs to A to a degree of 0.7.

2.1.1 Membership Function

The idea of MF is to make a suitable decision when uncertainty occurs [45]. MF is a curve that defines how each point in the input space is mapped to a membership value or degree of membership between 0 and 1. The closer the membership degree is to 1, the more an element belongs to a given set. A membership degree of 0 means, that an element is clearly not a member of a particular set. Elements with a membership degree between 0 and 1 are more or less members of a particular set which represent fuzziness.

The MF definition is applicable to be expressed on mathematical formula. Mathematical formula is based on the shape of MF. There are several shapes that are usually used which are triangle, trapezoidal, Gaussian and others. The mathematical formula for Triangular MF, Trapezoidal MF and Gaussian MF are defined based on [46].

2.1.1.1 Triangular MF

This triangular MF is the most common MF used and applied in many areas of studies such as, a study of student results performance [47] for a creative work like calligraphy and art. The membership grade scales for triangular MF was adjusted and helped the teacher to establish the rule reasoning easily. The triangular MF is specified by three parameters (a, b, c) as follows:

$$\text{Triangle}(x; a, b, c) = \begin{cases} 0, & x \leq a. \\ \frac{x-a}{b-a}, & a \leq x \leq b. \\ \frac{c-x}{c-b}, & b \leq x \leq c. \\ 0, & c \leq x. \end{cases} \quad (3)$$

The parameters (a, b, c) with $a < b < c$ determine the x coordinates of the three corners of the underlying triangular MF. Examples of triangular MF application by using 'trimf' function.

```
x=0:0.1:10;  
y=trimf(x,[3 6 8]);  
plot(x,y)  
xlabel('trimf, P=[3 6 8]');
```

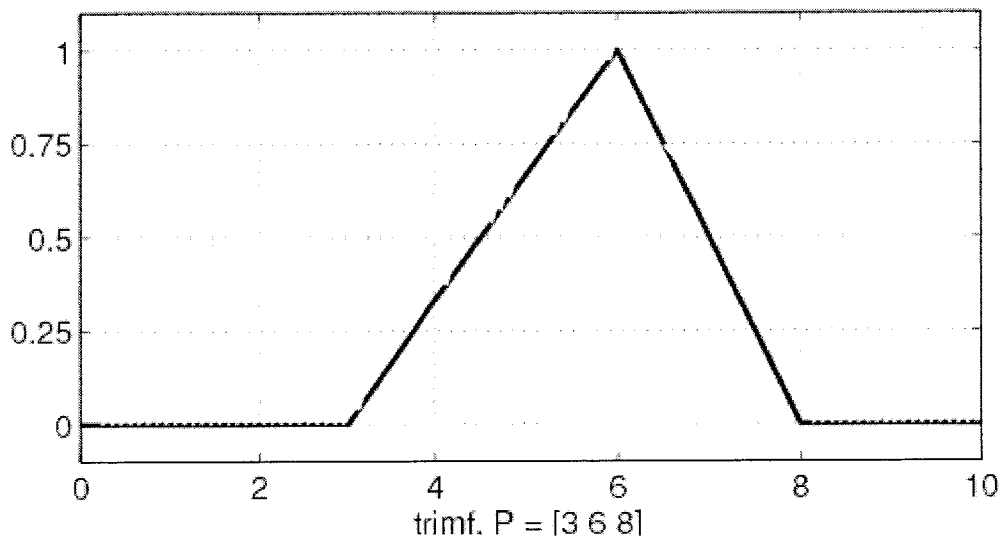


Figure 2.1: Triangular MF [46]

2.1.1.2 Trapezoidal MF

Studies [48-50] were applied trapezoidal MF. The trapezoidal MF is specified by four parameters (a, b, c, d) in equation 3.

$$\text{Trapezoid}(x; a, b, c, d) = \begin{cases} 0, & x \leq a. \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & b \leq x \leq c. \\ \frac{d-x}{d-c}, & c \leq x \leq d. \\ 0, & d \leq x. \end{cases} \quad (4)$$

Where parameter (a, b, c, d) with $a < b < c < d$ determine the x coordinate of the corners of underlying trapezoidal MF. Examples of trapezoidal MF application by using 'trapmf' function.

```
x=0:0.1:10;  
y=trapmf(x,[1 5 7 8]);  
plot(x,y)  
xlabel('trapmf, P=[1 5 7 8]');
```

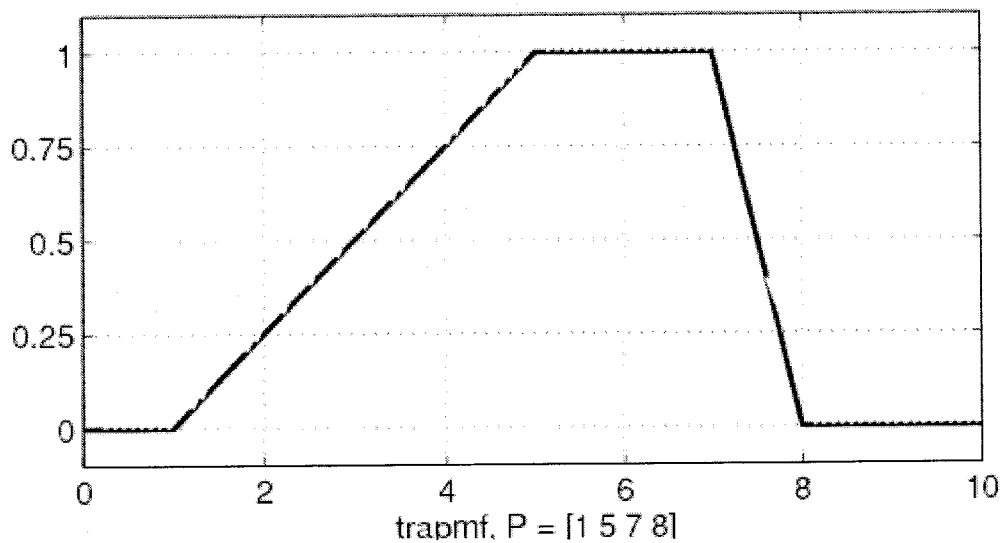


Figure 2.2: Trapezoidal MF [46]

2.1.1.3 Gaussian MF

Gaussian MF is suitable for problems which require continuously differentiable curves therefore smooth transitions [51]. The Gaussian MF is defined specified by two parameters (c , σ), which is one parameter less than the triangular MF. The formula is defined in equation 5 :

$$\text{gaussian}(x; c, \sigma) = e^{-\frac{1}{2\sigma^2}(x-c)^2} \quad (5)$$

Where c represents the MFs center and σ determines the MFs width or standard deviation. Here are the examples of Gaussian MF function by using ‘*gaussmf*’ function.

```
x=0:0.1:10;  
y=gaussmf(x,[2 5]);  
plot(x,y)  
xlabel('gaussmf, P=[2 5]');
```

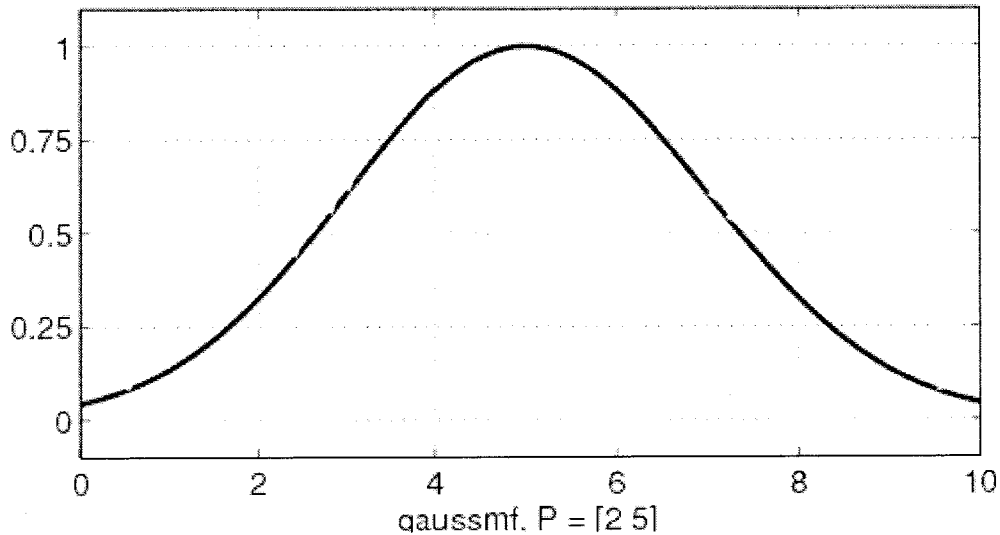


Figure 2.3: Gaussian MF [46]

2.1.2 Fuzzy Operation

Fuzzy operator works in an inference engine where it computes the fuzzy ‘IF-THEN’ rules. Fuzzy operator for intersection and union are defined in terms of their

membership function, which through min and max operators. These two operators; intersection and union are corresponding to ‘AND’ and ‘OR’ respectively in classical logic. The formula for these operators is defined in equation (6) and (7).

$$\text{Union: } \mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x)) \quad (6)$$

$$\text{Intersection: } \mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x)) \quad (7)$$

2.1.3 Uncertainties

In the real world, the uncertainty reflect incompleteness, imprecision, missing information, or randomness in data and a process [39]. Fuzzy logic was introduced to represent uncertainty and imprecise knowledge with fuzzy set. In case of numerical data, the uncertainties such as noise has been translated into uncertainty in MF and produced a degree of MF [46]. In other words, the lack of information, imprecise and vague data are the uncertainties that can be modeled by fuzzy set, which represents uncertainty by numbers in the range of 0 to 1. Figure 2.4 illustrates a simple structure on fuzzy logic system.

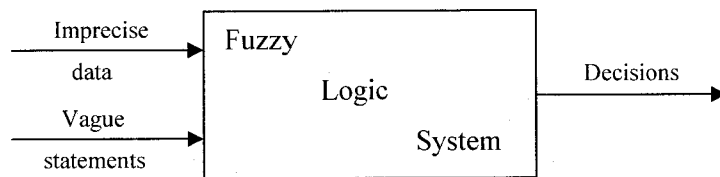


Figure 2.4: Fuzzy logic accepts imprecise data and vague statement and provides decision [45]

Uncertainty affects in decision making and can appear in a number of different forms. Uncertainties is an attribute of information [52] where the concept of information is fully connected with the concept of uncertainty. The most fundamental aspect of this connection is that the uncertainty involved in any problem solving situation which is a result of some information deficiency. This information can be incomplete, imprecise, fragmentary, not fully reliable, vague, contradictory, or deficient in some other way [12].

These uncertainties can also appear in Fuzzy Inference System (FIS) structure itself. [53] has defined the sources of uncertainty in FIS as follows:

1. Uncertainty about the consequent that is used in a rule
2. Uncertainty about the measurement that activates the Fuzzy Logic System (FLS)
3. Uncertainty about the data that are used to tune the parameters of the FLS
4. Uncertainty about the meanings of the words that are used in the rules

Then, [12] also has come out the sources of uncertainty in Type-2 FIS as below:

1. The meanings of the words that are used in the antecedents and consequents of rules can be uncertain (words mean different things to different people).
 2. Measurements that activate a Type-1 FLS and also the data used to tune the parameters Type-1 FLS may be noisy and therefore uncertain.
-

2.1.4 Type-1 Fuzzy Logic

Type-1 fuzzy logic is defined on a universe of discourse, which provides the set of allowable values for a variable and is characterized by a membership function (MF), $\mu_A(x)$ that takes only one degree of membership with value in the interval $[0, 1]$, which gives more values to the membership function rather than crisp set that only provides a value of 1 or 0. In other words, Type-1 fuzzy set maps an element of universe of discourse onto a precise number in the unit interval $[0, 1]$.

The main problem with Type-1 fuzzy logic is the fuzzy set has a crisp membership grade which certain in sense that for each input, therefore Type-1 fuzzy logic have difficulties in modeling and minimizing the effect of uncertainties [37]. It needs to continuously tune or handling by a group of Type-1 fuzzy because this single Type-1 fuzzy method cannot handle on its own uncertainties [43]. Hence, the Type-2 fuzzy was introduced to handle the uncertainties and has the potential to outperform its Type-1 counterparts.

2.1.5 Type-2 Fuzzy Logic

In 1975, Zadeh [46] had introduced Type-2 fuzzy set as extension of Type-1 fuzzy set which Type-2 fuzzy is not limited to handling uncertainties about linguistic variables but also present in the definition of the membership function. Handle here means can minimize the effect of uncertainties. The development of type-2 had successfully solved and overcome the limitation in type-1 fuzzy in many areas such as pattern recognition [54, 55], intelligent manufacturing [56], robotics [57] and automation [46].

The Type-2 fuzzy set is characterized by a fuzzy MF. It can have more than one degrees of membership function with the value between the interval [0, 1]. Definition of interval Type-2 characterized by \tilde{A} is [58]:

$$\tilde{A} = ((x, u), \mu_{\tilde{A}}(x, u)) \forall x \in X, \forall u \in J_x \subseteq [0, 1] \quad (8)$$

where the secondary membership function; $\mu_{\tilde{A}}(x, u) = 1$.

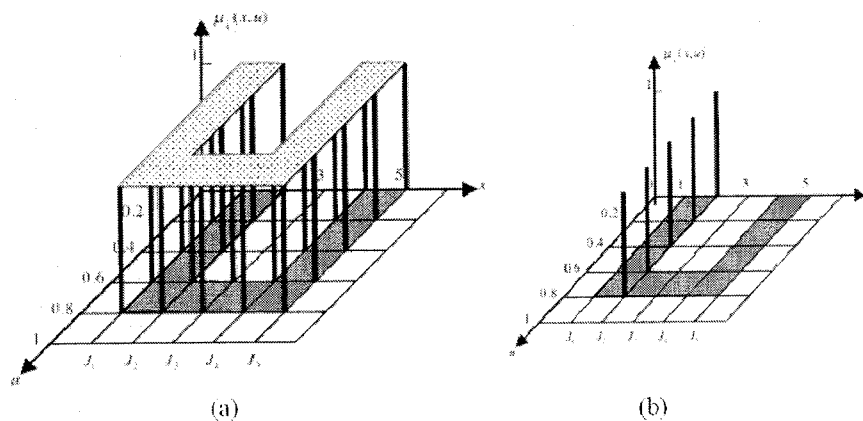


Figure 2.5: (a) Example of Type-2 fuzzy. The shaded area in the $u - x$ plane is Footprint of Uncertainty (FOU) (b) secondary MF has value of one [58]

Type-1 handles uncertainties about the meanings of words by using crisp or precise degree of MF. Once the Type-1 MF is chosen, all uncertainty about the words disappears, because Type-1 MF is totally precise. Meanwhile, Type-2 handles uncertainty about the meanings of the words that they modeling the uncertainties by using Type-2 membership function [39]. The transformation process of Type-2 fuzzy is illustrated in Figure 2.6.

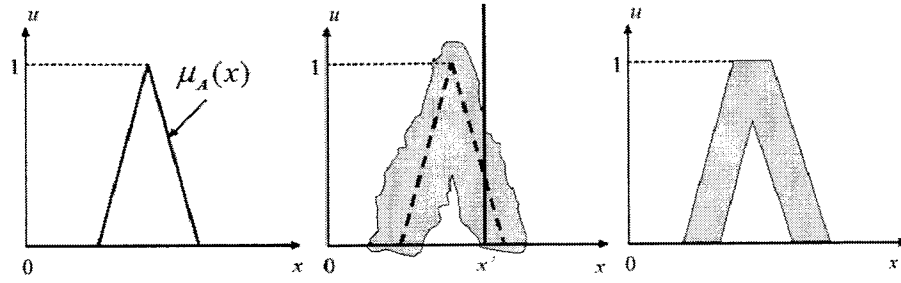


Figure 2.6: (a) Type-1 MF, (b) Blurred Type-1 MF, (c) FOU [39]

According to [58], the Type-1 MF in Figure 2.6 (a) is blurred by shifting the points on the triangle either to the left or to the right. At specific value of x , x' at 2.4 (b), the MF takes values wherever the vertical line intersect the blur. The FOU in Figure 2.6 (c) represents the set of possible primary MFs embedded in Type-2 fuzzy set that provides new degree of freedom that let uncertainties be handled by Type-2 fuzzy. Mathematically, embedded of Type-1 is a union of all primary MFs [46]. It can also be described through the concepts of lower and upper membership [25].

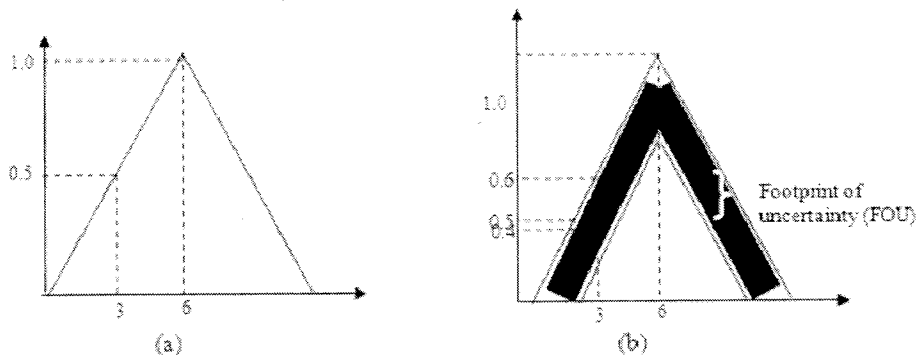


Figure 2.7: (a) Type- 1 fuzzy (b) Type- 2 fuzzy

A simple example in Figure 2.7 is showing the fuzzy MF in Type-2 fuzzy set. From Figure 2.7 (a), the $x = 3$ has a degree of memberships value of 0.5. With footprint of uncertainty, the $x = 3$ have more than one degree of membership values which are 0.4, 0.5 and 0.6 in Figure 2.7 (b). Fuzzy MF in Type-2 fuzzy set provides extra observations rather than Type- 1 fuzzy set with single observation [8].

In terms of membership function, Type-2 fuzzy logic also has several shapes. The main interest in this study is focusing on Gaussian MF, where the uncertainties in

Gaussian MF can be described at σ or c , or both. Definition for Gaussian MF with uncertainties in c shows in equation 9.

$$A(x) = e^{-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2} \quad (9)$$

2.2 Extension of Type 1-Fuzzy to Type-2 Fuzzy

Type-2 fuzzy logic method was used in many applications because it can handle uncertainty better than T1FL does [59]. However, the issue appears on how to extend Type-1 fuzzy to Type-2 fuzzy. In this section, four methods are presented. The first method generates Type-2 fuzzy by embedded all Type-1 fuzzy logic [43]. Second and third methods are using subtractive clustering, where in the second method extends Type-1 fuzzy by assigning uncertainties in cluster center, standard deviation and at consequent parameter [60]. Meanwhile, the third method extend the subtractive clustering algorithm by adding fuzziness parameter in the algorithm [55]. The fourth method combined Type-1 fuzzy with standard error method [61]. This standard error is used to create the uncertainty in Type-2 fuzzy set.

2.2.1 Type-2 Fuzzy Logic Controller through embedded Type-1 Fuzzy Logic

A study of Type-2 fuzzy generalized from embedded all Type-1 fuzzy set is presented [43]. The idea is that, a group of embedded Type-1 fuzzy set will create FOU in Type-2 fuzzy set that can capture and faced the uncertainties. From a single Type-1 fuzzy in Figure 2.8 produces Type-2 fuzzy sets in Figure 2.9 by embedding all Type-1 fuzzy set.

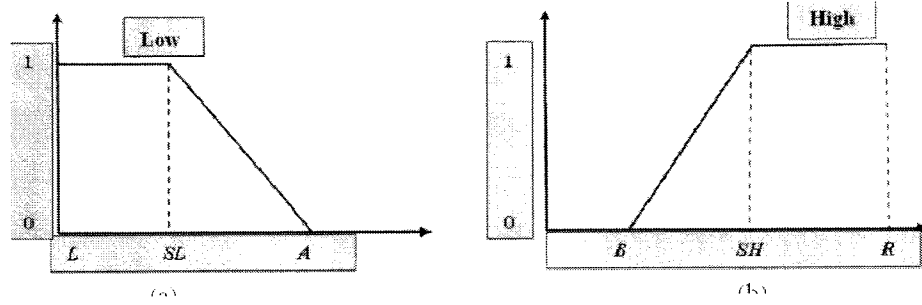


Figure 2.8: (a) Type-1 fuzzy set used to represent linguistic label Low (b) Type-1 fuzzy set used to represents linguistic label High [43]

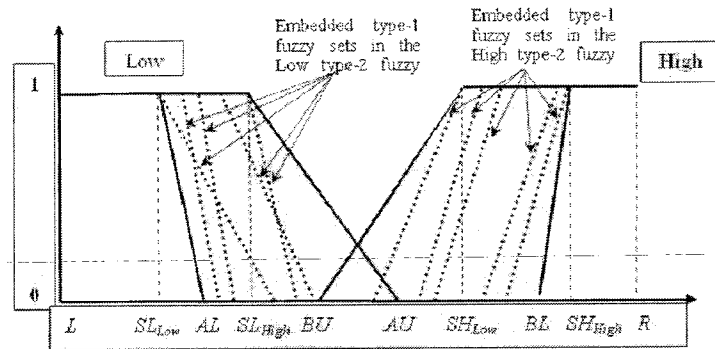


Figure 2.9: : The Type-2 MF function of each input in which each Type-2 fuzzy set embeds the used Type-1 fuzzy sets [43]

2.2.2 Type-1 Gaussian MF as Principal MF by using Subtractive Clustering

[60] has proposed to use Type-1 Gaussian MF as principal MF to extend Type-1 fuzzy to Type-2 fuzzy. Initially, Chiu's subtractive clustering [62] is used to recognize Type-1 fuzzy based on input and output data by listing cluster center and sigma for the antecedent part. Then, the consequent part is obtained through least squares estimation algorithm. The following steps of extending Type-1 fuzzy to Type-2 fuzzy are listed below [56]:

1. Spread cluster center to expand premise MFs from Type-1 fuzzy set to Type-2 fuzzy set as illustrated in Figure 2.10. The cluster center is fuzzy number as below:

$$\tilde{x}_v^{k*} = [x_v^{k*}(1 - a_v^k), x_v^{k*}(1 + a_v^k)] \quad (10)$$

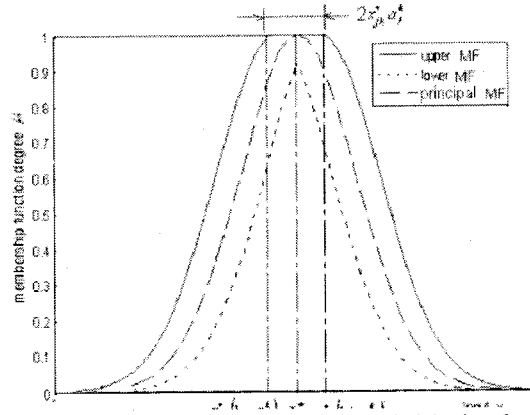


Figure 2.10: Spread of mean of the v th variable in the k th [53]

2. The deviation for each rule varies from each other in the fuzzy model to get the best model where constant σ^k is replaced by σ_j^k .

$$\tilde{Q}_v^k = \exp \left[-\frac{1}{2} \left(\frac{x_v - x_v^{k*} (1 \pm a_v^k)}{\sigma_j^k} \right)^2 \right] \quad (11)$$

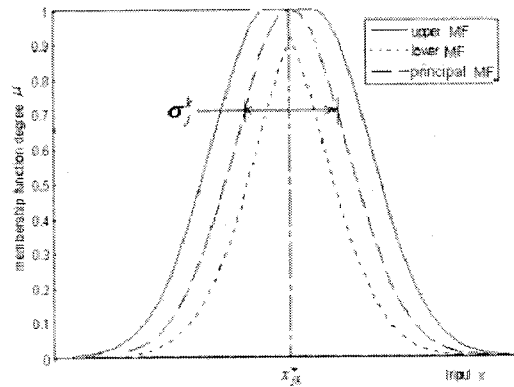


Figure 2.11: Standard deviation of Gaussian MF [53]

3. Spread the parameters of consequence to expand consequent parameters from constants to fuzzy numbers:

$$\tilde{p}_j^k = p_j^k (1 \pm b_j^k) \quad (12)$$

where b_j^k is the spread percentage of fuzzy number \tilde{p}_j^k .

2.2.3 Extension of Subtractive Clustering Algorithm with Fuzziness Parameter m

A study [55] is an extended subtractive clustering algorithm by using two fuzzifier m_1 and m_2 which create FOU. With the creation of FOU, it becomes Type-2 fuzzy and can manage to handle uncertainties better than Type-1 fuzzy. The idea of fuzziness parameters are from four pre-initialized parameters which are accept ratio, reject ratio, cluster radius and squash factor. They have been identified as uncertainties parameter in subtractive clustering algorithm. The steps of extending Type-1 fuzzy to Type-2 fuzzy by the helps of subtractive clustering are as follows:

1. Initialize four main parameter; accept ration, reject ratio, cluster radius, squash factor and add new fuzziness parameter m_1 and m_2 ($1 < m_1 < m_2$)
2. Calculate density for all points with two fuzzifiers m_1 and m_2 by using formula:

$$\bar{P}_1 = \sum_{j=1}^n e^{-\frac{4}{r_a^2} (x_j - x_i)^{\frac{2}{m_1 - 1}}} \quad (13)$$

$$\underline{P}_1 = \sum_{j=1}^n e^{-\frac{4}{r_a^2} (x_j - x_i)^{\frac{2}{m_2 - 1}}} \quad (14)$$

$$P_1 = \frac{\bar{P}_1 * m_1 + \underline{P}_1 * m_2}{m_1 + m_2} \quad (15)$$

Then, data point with the highest density is selected as the first cluster centroid.

3. Revise the density of all data points.
4. Identify next cluster centroids using similar subtractive clustering method.
5. The outputs of the clustering result are displayed.

2.2.4 Type-1 Fuzzy with Standard Error Method

This method was reported by [61] that combines Type-1 fuzzy with standard error method in order to generate interval Type-2 fuzzy. This standard error method is used to calculate and identify the uncertainties occurred. This uncertainty is treated as propagation error where it can be directly calculated using standard error method.

Interval Type-2 has a value of 1 at secondary membership function as shown in Figure 2.12 (b) and it can be expressed as equation 16.

$$\mu = \mu_{avg} \pm \Delta\mu \quad (16)$$

where the μ_{avg} is the mean value of μ and $\Delta\mu$ is represents the uncertainties in triangular MF. Therefore the μ_{avg} and $\Delta\mu$ in Figure 2.12 (a) can be calculated as

$$\mu_{avg} = \frac{\mu_H + \mu_L}{2} \quad (17)$$

$$\Delta\mu = \mu_H - \mu_{avg} = \mu_{avg} - \mu_L \quad (18)$$

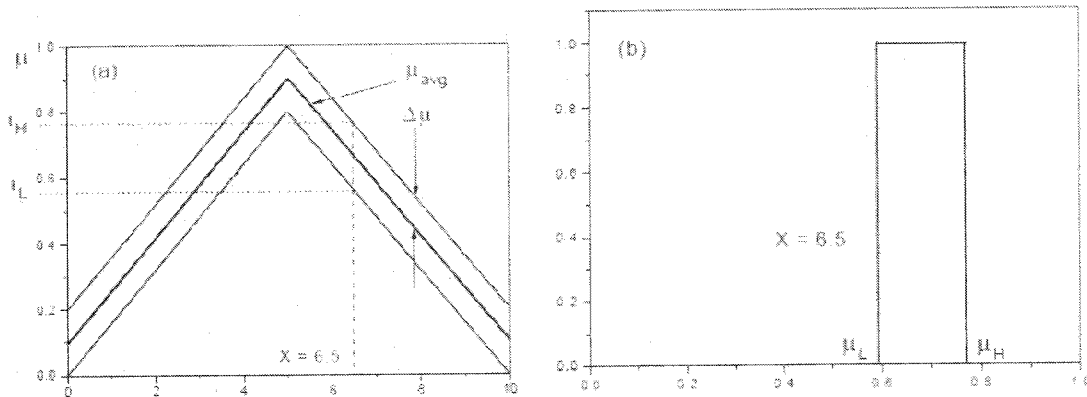


Figure 2.12: (a) primary and (b) secondary membership function for interval Type-2 fuzzy [61]

2.3 Fuzzy Inference System

Fuzzy Inference System (FIS) is a system that can perform approximate reasoning more like a human brain. Fuzzy inference maps a given set of input variable to an output based on a set of fuzzy rules. FIS is based on expertise expressed in terms of ‘IF–THEN’ rules, which can be used to forecast uncertain system [33]. Generally, Type-1 and Type-2 FIS involve with three processes that consist of fuzzification, inference engine and defuzzification.

Type-2 FIS has additional stages named as Type Reducer where it is not applicable for Sugeno inference engine [46]. The type reducer is basically transformed Type-2 fuzzy set to Type-1 fuzzy set. Centre-of-sets is a common method used in type reducer which combines all the Type-2 output sets and then performs a

center-of-sets calculation to produce a Type-1 set [15]. The type reducer stage and sequence in FIS are shown in Figure 2.13.

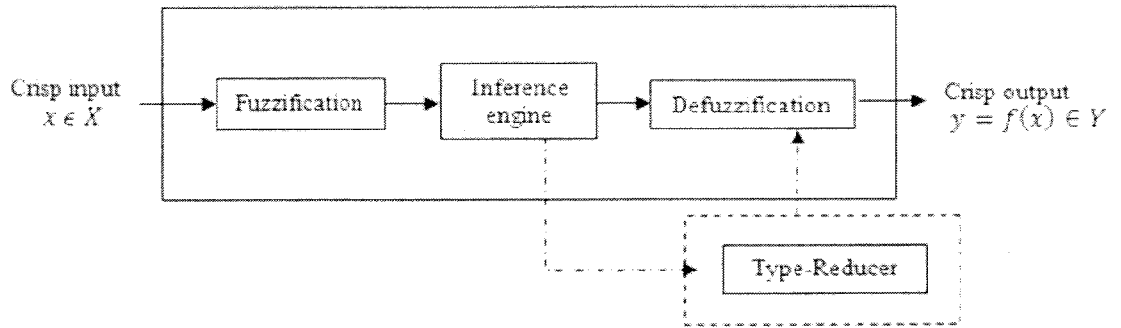


Figure 2.13: Type-1 and Type-2 FIS

2.3.1 Fuzzification

The FIS starts with fuzzification stages. The fuzzification is a process of converting crisp value into membership values in the fuzzy sets [63]. The conversion is performed using membership functions that provide fuzzy terms with specific meaning. Due to complexity to partition the crisp values into groups of fuzzy set, several methods are developed such as neural network-based [64], genetic algorithm [37] and fuzzy clustering [23].

Fuzzy clustering involves the creation of clusters in data space and transformation of these clusters into fuzzy model rules. This method will cluster data into groups, which has a related behavior or similarities. There are varieties of cluster algorithm methods that have been proposed, including k-mean, fuzzy c-mean, mountain and subtractive clustering [65].

2.3.1.1 Subtractive Clustering

Subtractive clustering is introduced by Chui [62] to compute the optimal data point that defines a cluster center based on the density of surrounding data points or its distance to other data points [66, 67]. The advantages of using subtractive clustering is to reduce the computational complexities, the initialization is simple and

gives better distribution of cluster centers compared to other clustering algorithms by considering the data points, rather than a grid point [60, 67]. Basically, the subtractive clustering algorithm is as follows:

1. Select the data point with the highest potential to be the first cluster center.
2. Remove all data points in the vicinity of the first cluster center (as determined by cluster radius), in order to determine the next data cluster and its center location.
3. Continue this process until all of the data is the within area of a cluster center.

Furthermore, subtractive clustering is suitable to find the number of clusters and extract the fuzzy rules from input or output data. The number of clusters generated from subtractive clustering determines the number of rules generated in inference engine. Deep explanation of subtractive clustering is presented in Methodology Chapter.

2.3.2 Inference Engine

A fuzzy rule has two parts the *antecedent* and the *consequence*. The rules provide a transition between input and output fuzzy set. Inference engine computes output set corresponding to each rule. The fuzzy "and" is used to combine the membership functions to compute the rule strength. The knowledge has been represented by propositions as *IF premise(antecedent), THEN conclusion(consequent)* that commonly referred to as an IF-THEN rule [68].

Generally, there are two types of commonly used FIS in Fuzzy Logic toolbox which are Mamdani and Sugeno method. They differ in the consequent part of their fuzzy rules, aggregations and defuzzification process. Mamdani method is proposed by Ebrahim Mamdani attempt to control a steam engine and boiler in 1975 [69]. Both antecedents and consequent in Mamdani inference system are in a form of fuzzy set [59]. Mamdani fuzzy rule-base in IF-THEN forms is shown below:

$$\text{If } X_1 \text{ is } A_1 \text{ and } \dots X_n \text{ is } A_n \text{ then } Y \text{ is } B \quad (19)$$

Where X_l , X_n and Y_l are the input and output variables, respectively and A_l , A_n and B are the fuzzy set. Figure 2.14 illustrates the whole idea of Mamdani inference engine process.

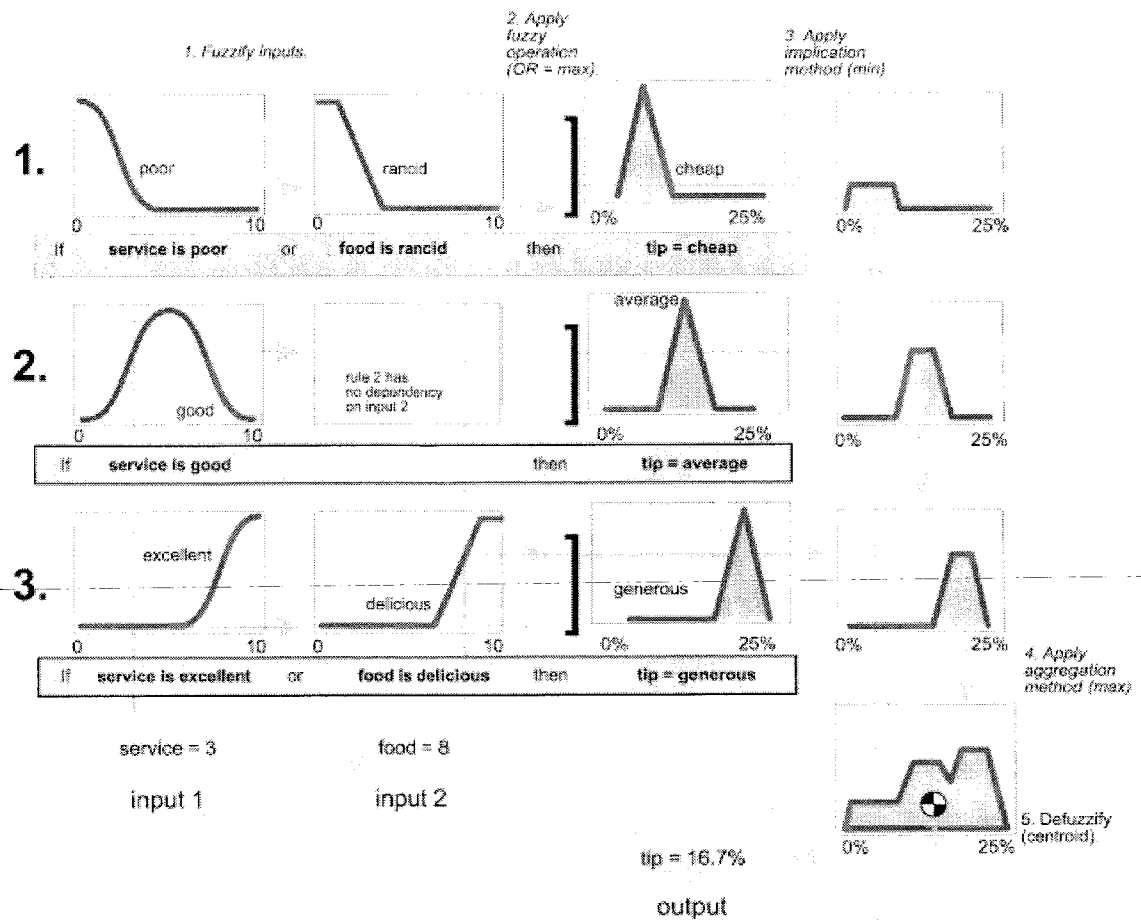


Figure 2.14: Mamdani FIS [70]

Mamdani is known widely as having a capability to capture the expert knowledge that allows describing the expertise in human-like manner. However, Mamdani is having a substantial computational burden [71]. Meanwhile, Sugeno method is computationally efficient and it can produce near to accurate solution of wave characteristic [24]. Sugeno FIS was proposed to develop a systematic approach to generate fuzzy rules from a given input-output data [69]. Sugeno method is proposed by Takagi and Sugeno in 1985 [68]. Sugeno FIS has fuzzy input in antecedent and a crisp output in consequent part. The output function can be either constant or linear. Sugeno fuzzy rule-base in IF THEN forms is shown below:

$$\text{If } X_l \text{ is } A_l \text{ and } \dots X_n \text{ is } A_n \text{ then } Y = c_0^l + c_1^l x_1 + \dots + c_m^l x_m \quad (20)$$

Where X_l, X_n and Y_l are the input and output variables, respectively. A_l and A_n are the fuzzy set and $c_0^l + c_1^l x_1 + \dots + c_m^l x_m$ is the output in function form. Figure 2.15 illustrates the whole idea of Sugeno inference engine process.

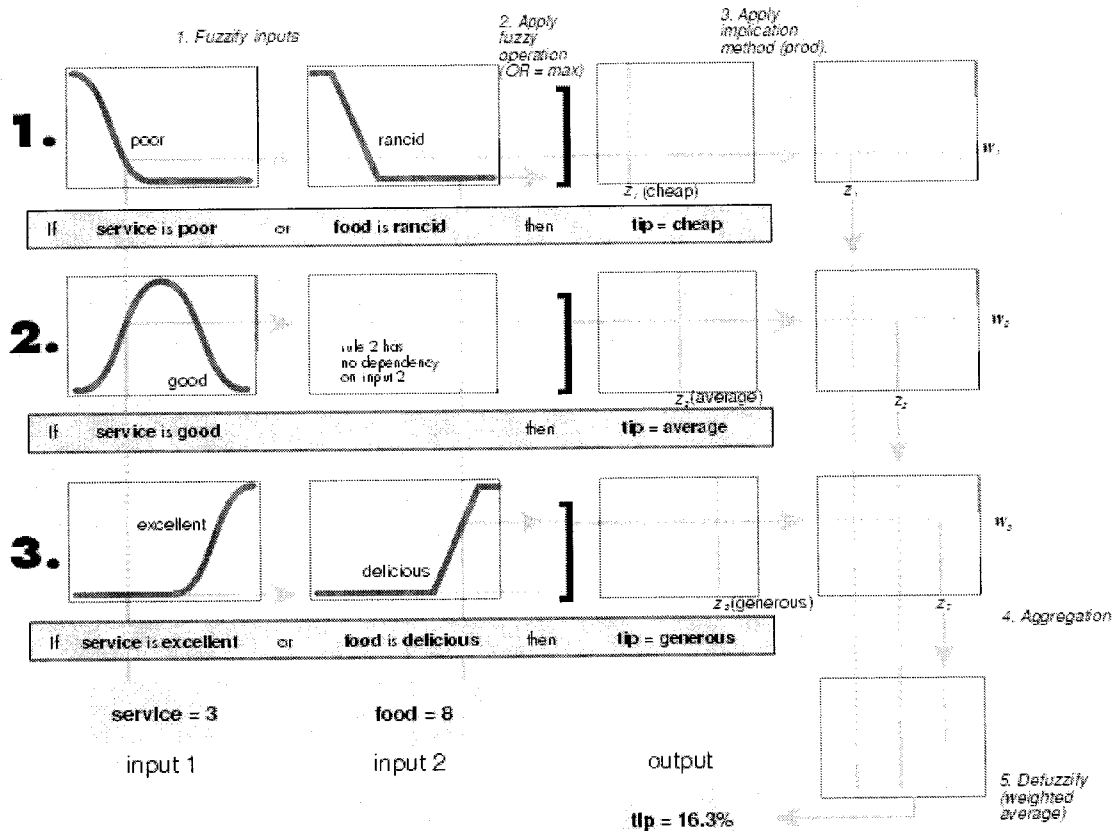


Figure 2.15: Sugeno FIS [72]

2.3.3 Defuzzification

Then, the defuzzifier computes a crisp output from these rule output set. By mean of a numeric value is extracted from a fuzzy set as a representative value. The common defuzzification technique uses the centroid method, which defines the crisp output as the center of gravity or center of area of the distribution of possible actions. Other techniques are max-membership principle, where the defuzzified value, Z , equals the x -value with the highest membership degree. Weighted average method and mean-max membership are among the defuzzifier techniques.

2.4 Wave Height Forecasting

Generally, forecasting is a technique to forecast or to estimate the future by using historical data and knowledge of any future events that might impact the forecasts as accurate as possible [73]. In focusing on wave height forecasting, this parameter plays an important role in activities that involved ocean engineering applications. Basically, significant wave height helps in designing, analyzing and determination of the economical life of coastal and offshore structure [18].

There are many forecasting techniques depend on the forecast subject. These techniques can be defined as numerical method [14], empirical method [7], soft computing [24] and others. Through the listed forecasting techniques, various forecasting significant wave height methods were rapidly grown in order to achieve the best forecasted value that can improve the Metocean environment activities.

Previously, well-known numerical models Wave Model (WAM) [74] and Third Generation Wave Model (WAVEWATCH) [17] were used to forecast the wave parameter. However, these methods required high speed computers and variety of input data, which were not efficient in terms of economic point of view [18]. The empirical methods such as Autoregressive Integrated Moving Average (ARIMA) model, Autoregressive Moving Average (ARMA) model approaches which are highly famous in conventional forecasting [75].

However, these methods were limited only to the requirement of stationary of time series, normality and independence of residuals [19]. This occurs due to the implementation of statistical or mathematical concepts in the model in which it usually assumes that time series are stationary. In other words, it fluctuates more or less uniformly around a time-invariant mean where the relation exists between time t and its prior time $t-1$ is all the same. Most importantly is that the empirical methods have a difficulty in forecasting when the historical data involved with uncertainties [20].

Other than that, [76] has introduced genetic algorithms (GA) methods at Bay of Bengal Ocean which can forecast up to 48 hours by using wind speed and wind

direction. Then, [18] has combined GA and Kalman Filtering (KF) method at swallow lake with wind speed as input. Wave parameter also was forecasted with classification and regression method [34]. Classification and regression method were compared with ANNs model. Its result has shown that ANNs model was marginally more accurate than classification method and the error statistics of ANNs and regression tree were similar.

Research on finding more accurate significant wave height forecasting has lead to introduced Multiple Linear Regression (MLR) [13] with the main concern of input parameters of wave forecasting and output error. Here, the MLR has advantages by less numbers of parameter when compared to ANN. The ANN method has advantage of small error rate. This method succesfully showed a good result when forecasting with wind speed input only or less input parameters.

Eventhough ANNs seems to perform better than [34], [13], [77], ANNs method is time consuming to perform trial and error in order to find the best topology, both number of the hidden layers and number of neurons in each hidden layer. Therefore, both architecture to be assumed as fixed [76]. The neural networks works evidently more like a black box. Due to limitation in methods, fuzzy methods were introduced and practical in forecasting models with the ease of the application, as well as less required computational time, has made this method more suitable for wave modeling [21].

2.4.1 Wave Height Forecasting using Fuzzy Theory

In 2005 [22], the wave parameter was determined from wind speed and fetch length by using fuzzy logic method. It had used Adaptive Network based on Fuzzy Inference System (ANFIS). ANFIS was used to cover the limitation of Takagi Sugeno model which is one of the Fuzzy models in identified parameters. The ANN has a capability to learn relationship between input and output variables. Meanwhile, the precondition of FIS model did not require knowledge of the core physical process and could be used to forecast uncertain systems.

The involvement of fuzzy logic in wave parameter starts to grow parallel with the development of fuzzy logic itself. Based on [23], the paper proposed fuzzy approach in 2007 to determine the relationship between wave and wind speed parameter in wave generation system. Fuzzy method was proposed because from the rule available at inference engine, people can have a better interpretation. The input and output of fuzzy system for this method can be clearly interpreted in figure below.

Rules	Description
1	IF $wsp(t)$ is Low and $H_s(t)$ is Low THEN $H_s(t+1) = 0.1144 \times wsp(t) + 0.879 \times H_s(t) + 0.2757$
2	IF $wsp(t)$ is Medium and $H_s(t)$ is Medium THEN $H_s(t+1) = -0.02215 \times wsp(t) + 0.9969 \times H_s(t) + 0.2757$
3	IF $wsp(t)$ is High and $H_s(t)$ is High THEN $H_s(t+1) = 0.08978 \times wsp(t) + 0.8118 \times H_s(t) + -0.1511$

Figure 2.16: Fuzzy rules with one step ahead for significant wave height forecasting

From the Figure 2.16, it has shown the forecasting of significant wave height by using wind speed and past significant wave height. Three years data were used from January 1, 2001 to December 31, 2004 as this inference system required a long-term data. The last 6 months data were used to test the validation of model. The fuzzy approach result had outperformed Auto Regressive Moving Average with exogenous input (ARIMAX) model.

The wind-wave modelling with fuzzy inference system method was continued by [24] to forecast the impact of wind speed on deep sea wave parameter. It has been stated that proposed model can be a valuable tool for operational forecasting of wave characteristics in Northern Aegean Sea. This model also was operated by using Takagi Sugeno model but expended to determine a suitable input variable, where the wind direction was added as an input. In training part, May 25,2000 to December 31, 2005 data were considered, while January 1, 2006 to June 15, 2006 was left for validation. This paper further describes the ANFIS method with subtractive clustering where the pre-initialized parameters were set in Table 2.1.

Table 2.1: Pre-initialized parameter

Parameter	Value
Accept ratio	0.1
Reject ratio	1.0
Squash factor	0.5
Cluster radius	2.0

From the description of all fuzzy model, it can be said that fuzzy inference system can be a good tool to forecast the wave characteristics. Furthermore, in terms of the influence of input parameters towards the wave forecasting, most of the papers had highlight wind speed parameters. Indeed, it is unavoidable to consider wind speed for the forecast of significant wave height. This is due to the wave has a strong relation with wind as wind influence the creation of wave.

However, it is not enough to consider the wind speed as the only predictor for the wave height. Complex combinations of meteorological factors such as wind speed, wind direction, air pressure, air temperature and sea temperature are effective on wave generation in the concerned region [13]. Therefore, this study is focusing on two input parameters which are wind speed and wind direction with the supported from [78] which mention that wind speed and wind direction were the most important parameters in wave characteristic. These two parameters deals a lot with uncertainties [33]. Hence there is a need to find alternative ways to forecast which have capability in handling the uncertainties.

2.5 Summary

The explanation of this chapter is crucial to help in development of proposed method in chapter 3. This literature review chapter starts with the basic theory of fuzzy logic and Fuzzy Inference System (FIS). The existing methods on wave height forecasting are studied based on the advantages and disadvantages of the methods. Fuzzy logic were used as it has advantage of a better modelling of uncertainties, ease to use and good interpretation with rule inference compared to other methods. It has also been highlighted that wind speed and wind direction plays an important role in forecasting significant wave height. In order to use Type-2 fuzzy, the second method was adapted in proposed method which is Type-1 Gaussian MF as principal MF by using subtractive clustering. The reason is because this method is using subtractive clustering where the best method for fuzzy clustering in fuzzification stage.

CHAPTER 3

METHODOLOGY

This chapter presents the proposed method for forecasting the significant wave height parameter by using Type-2 fuzzy. The propose method starts with fuzzification phase, inference engine phase, output phase and concludes with the evaluation method for proposed method.

3.1 Introduction

The proposed method is focused at antecedent part in inference engine phase where the Type-2 fuzzy set was used. The Type-2 fuzzy was designed by assigning uncertainties to its own Type-1 cluster center [60]. This method has been applied in [79]. A complete program using Fuzzy Logic Toolbox and *subclust* function in MATLAB programming was utilized to achieve the objectives of the presented work.

3.2 Proposed Method

Proposed method starts with fuzzification phase by performing subtractive clustering for input-output data. The values of cluster center and sigma were produced to create the membership function in antecedents later on. The subtractive clustering method was combined with least square estimation to estimate parameters to build the Fuzzy Inference System where these methods were also used by [4, 55, 79]. Then, the antecedent membership function was extended to Type-2 fuzzy by assigning uncertainties to cluster center of its own Type-1 membership function.

The new antecedent membership functions were used at inference engine phase and produce a set of rules. After inference engine phases, the data when through the

output phase and the forecasted significant wave height values were evaluated to see the accuracy of the proposed method. Overall steps of proposed method are shown in Figure 3.1.

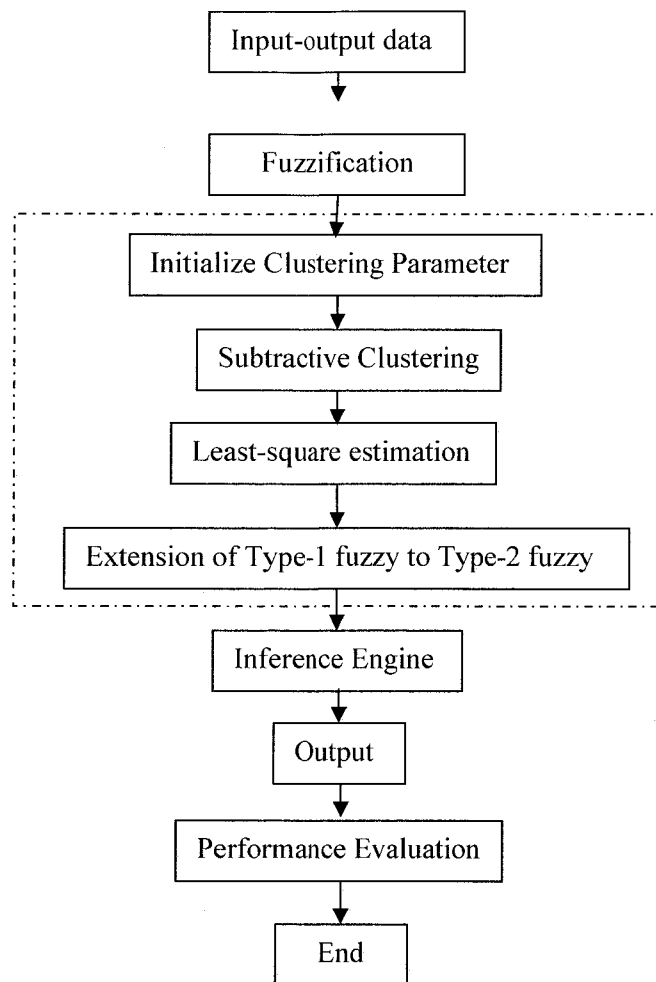


Figure 3.1: Proposed methodology

3.2.1 Fuzzification

Fuzzification phases transformed the crisp of inputs values into membership values in the form of fuzzy sets. The belongingness of the crisp value to the fuzzy set depends on its grade of membership function, where the value of degree is between 0 and 1. Subtractive clustering was proposed in order to do the fuzzification task [57]. Along the process, the subtractive clustering was combined with least square estimation method to produced fuzzy model by providing cluster center, sigma and coefficient values.

3.2.1.1 Subtractive Clustering

Subtractive clustering had given the potential of a data point as a cluster center based on the distance between the data point with the remaining data points [79]. This process was performed by using *subclust* function which is available in MATLAB software. The *subclust* function determined the number of rules and the value of cluster center and sigma to define the antecedent membership function.

According to [56, 60], four parameter had been initialized in the beginning of subtractive clustering phase. Each parameter definition and its values used in the proposed method were shown in Table 3.1. The squash factor, reject ratio and accept ratio were set as default meanwhile cluster radius was set into three different value. A study of the effects of parameters of subtractive clustering was done and the cluster radius had given a big influence towards the performance of model [66]. [62] had suggested the suitable cluster radius value was between 0.2 and 0.5.

Table 3.1: Pre-initialization parameter

Parameter	Definition	Value
Cluster radius r_a	A vector that specifies a cluster center's range of influence in each of the data dimensions	0.2, 0.3 and 0.5
Squash factor $\eta = \frac{r_b}{r_a}$	r_b denotes the neighbourhood which will have the measurable reductions in potential	1.25
Reject ratio $\underline{\varepsilon}$	Specifies a threshold for the potential above which the data point definitely accepted as a cluster center	0.15
Accept ratio $\bar{\varepsilon}$	Specifies a threshold for the potential below which the data point is definitely rejected	0.5

Each data point is considered as a potential cluster center. A measure of the potential is associated to each point according to its neighborhood where each had been defined by a cluster radius value, r_q . For the point i , it is written in equation (21).

$$P_i = \sum_{j=1}^n e^{-\alpha \|x_i - x_j\|^2}, \alpha = \frac{4}{r_a^2} \quad (21)$$

where $\| \cdot \|$ denotes the Euclidean distance.

P_i is the potential of i th data point, n is the total number of data points, x_i and x_j are data vectors in the data spaces including both input and output dimensions and r_a defining the cluster radius.

After the potential of every data point has been calculated, the data point with the highest potential was selected as the first cluster center. Assume x_1^* is the location of the first cluster center, and p_1^* is its potential value, then revise the potential of each data point x_j was revised by the formula:

$$p_i \leftarrow p_i - p_1^* e^{-\beta \|x_i - x_1^*\|^2} \quad (22)$$

where $\beta = 4/r_b^2$ and $r_b = n * r_a$.

Squash factor η is positive constant greater than 1. The positive constant r_b is greater than r_a and helps to avoid close spaced for cluster centers.

The process of acquiring new cluster center was based on the potential value; p_1^* in relation to an acceptance threshold; $\bar{\epsilon}$ and rejection threshold; $\underline{\epsilon}$. A data point with the potential greater than the acceptance threshold is directly accepted as a cluster center. The acceptance of a data point with a potential between the upper and the lower thresholds depends on the relative distance equation as

$$\frac{d_{min}}{r_a} + \frac{p_k^*}{p_1^*} \geq 1 \quad (23)$$

where d_{min} is the shortest distance between the candidate cluster center and all previously found cluster centers. The process of acquiring a new cluster center and revising the potentials were repeated until remaining of data points fall below some fraction of the potential of the first cluster center. The subtractive clustering was summarized based on [4, 56, 60, 80] as in Figure 3.2.

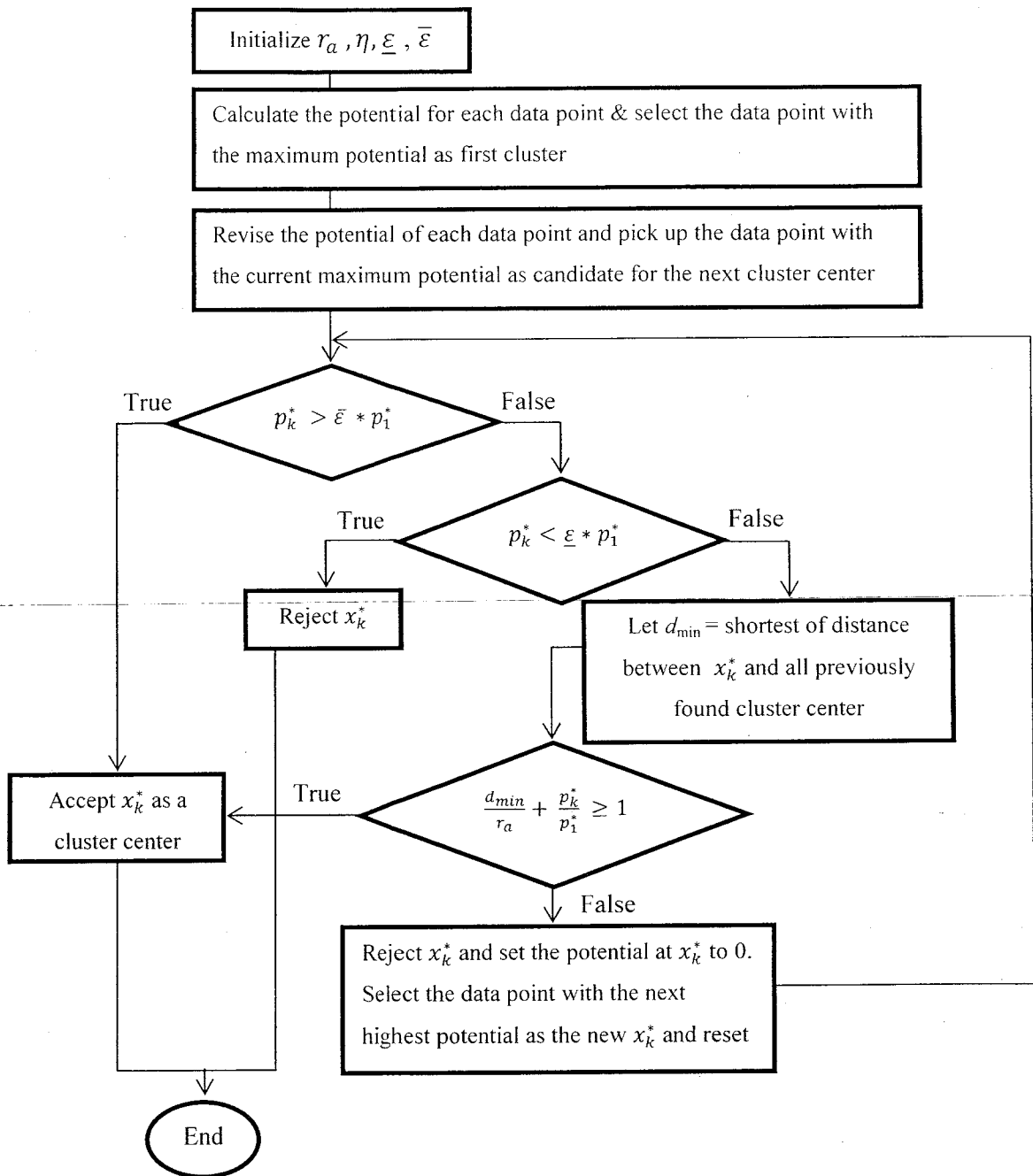


Figure 3.2: Subtractive clustering summary

The same steps were used to compute different values of cluster radius. From the three of selected cluster radius value, each of them had created different location and numbers of cluster center. There were 10 cluster center generated from the cluster radius of 0.2, 5 number of cluster center produced by cluster radius value 0.35 and lastly, only 2 number of cluster center generated by cluster radius of 0.5. The numbers

and values of cluster center depended on the cluster radius parameter and also the input and output dataset. Figure 3.3, Figure 3.4 and Figure 3.5 illustrated the location and the number of cluster center for both wind speed and wind direction.

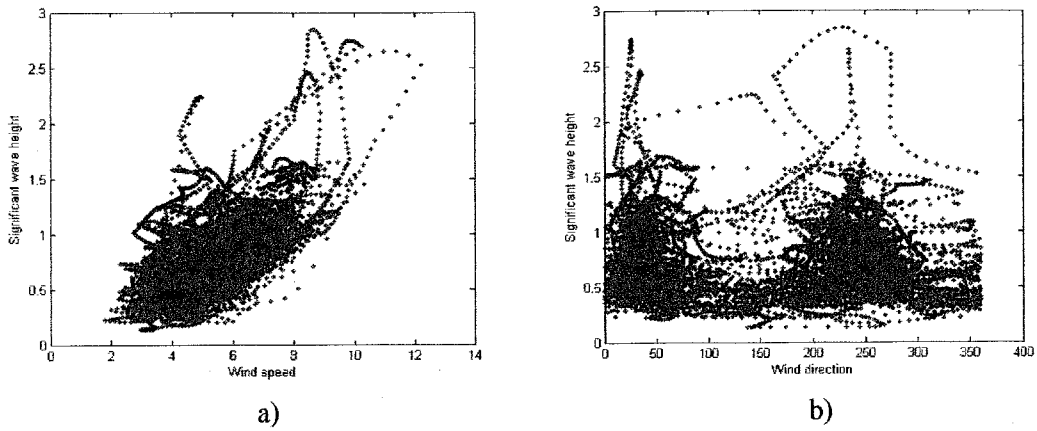


Figure 3.3 : Cluster centers with cluster radius of 0.2 a) wind speed b) wind direction

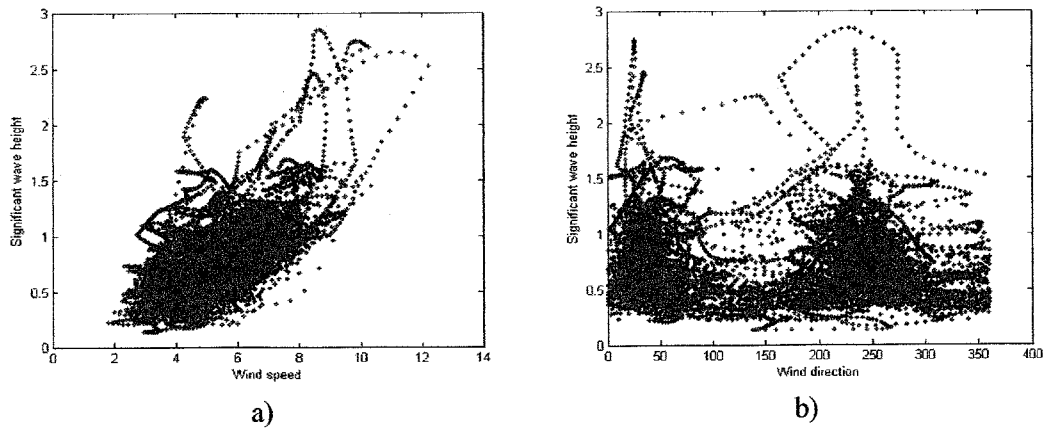


Figure 3.4 : Cluster centers with cluster radius of 0.35 a) wind speed b) wind direction

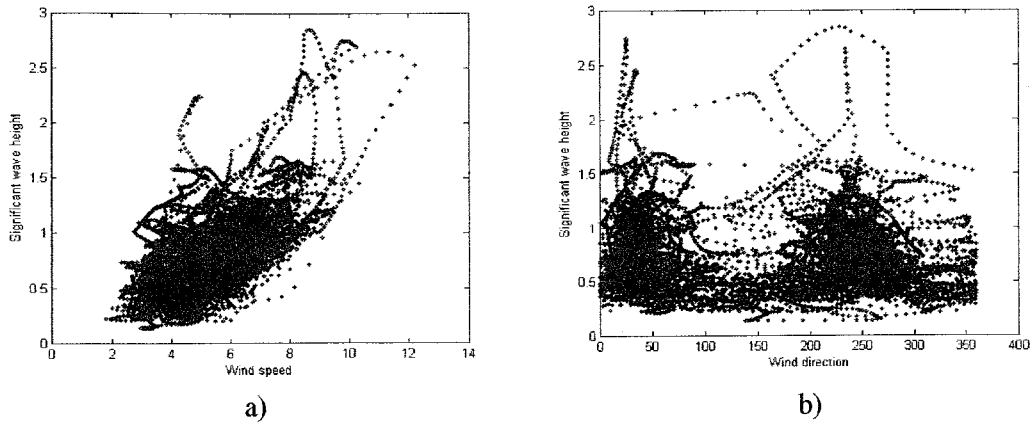


Figure 3.5 : Cluster centers with cluster radius of 0.5 a) wind speed b) wind direction

Table 3.2: Example of result from subtractive clustering phase

Variable	Wind direction	Wind speed
Cluster 1	249	4.83
Cluster 2	40.1	5.43
Cluster 3	233.4	7.07
Cluster 4	41.4	7.66
Cluster 5	114.7	3.42
Sigma	44.5	1.2894

Table 3.2 showed an example of cluster center and sigma values generated with the installation of cluster radius of 0.35 in the beginning of subtractive clustering phase. Five clustered were produced for inputs data; wind speed and wind direction and with one sigma value for each input.

The cluster center and sigma values were used to produce Gaussian MF where this Gaussian MF would fuzzify the crisp value to fuzzy set. Gaussian MF was selected because it works with subtractive clustering method and can produced good result as compared to other MF [24] . Gaussian MF formula is defined in equation 24.

$$\mu A(x) = e^{-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2} \quad (24)$$

where c represents the MF centre and σ determines the MF width or standard deviation. In order to adapt Gaussian MF formula, c is taken from cluster center and the sigma in Table 3.2 is used as σ in equation 24.

From the listed cluster center in Table 3.2, five fuzzy sets were identified for wind direction (A_{11} , A_{12} , A_{13} , A_{14} , A_{15}) and five wind speed (A_{21} , A_{22} , A_{23} , A_{24} , A_{25}) respectively. Fuzzy set A defines on U is $A = f_A(u_1)/u_1 + f_A(u_2)/u_2 + \dots + f_A(u_n)/u_n$ where $f_A(u_i)$ denotes the grade of membership of u_i fuzzy set A , $i = 1, 2, \dots, n$. Instead of labeling with A_{ij} , the fuzzy sets were named by linguistic value. Therefore, each fuzzy set had a meaningful name. Suggested linguistic name for wind speed and wind direction as follows:-

Wind Speed: (Very Low, Low, Medium, Fast, Very Fast)

Wind Direction: (West, Southwest, South, Southeast, East)

Table 3.3: Fuzzification of wind speed variable

Times	Wind speed	Fuzzy set	Linguistic value
10000	3.12	Cluster 5	Very low
10100	3.14	Cluster 5	Very low
11800	4.79	Cluster 1	Low
11900	5.03	Cluster 1	Low
12200	5.77	Cluster 2	Medium
12300	6.02	Cluster 2	Medium
20000	6.26	Cluster 2	Medium

Table 3.3 showed an example of fuzzifying the wind speed data based on degree of membership towards cluster in Table 3.2. For example the wind speed at time of 20000 was fuzzify to fuzzy set of cluster 2 with linguistic value of 'medium' and the membership degree was identify by using equation 24.

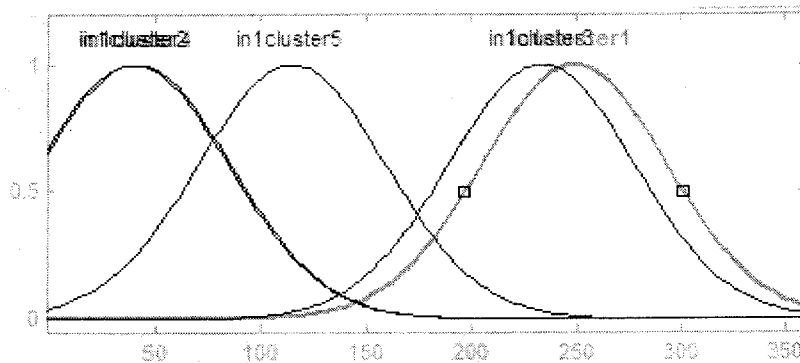


Figure 3.6: Type-1 Gaussian membership function for wind direction

For a better understanding, Figure 3.6 illustrates the wind direction membership function with cluster radius of 0.35. Each membership function curve defines each point in universe of discourse that mapped to membership value. Here, it has shown five groups of membership function based on its own cluster center. By using the same cluster center values, the Type-2 fuzzy set were created.

3.2.2 Extend Type-1 fuzzy set to Type-2 fuzzy set

After subtractive clustering stages, the Type-1 fuzzy sets were transformed to Type-2 fuzzy which can be obtained by assigning uncertainties to its own Type-1 membership function. In Type-2 fuzzy logic toolbox, there were many types of Gaussian membership function. *igaussmtype2* function was selected because it has uncertain value of mean as the proposed method was focused on the uncertain cluster center only. The formula for *igaussmtype2* function is shown in equation 25.

$$\mu_A(x) = e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad (25)$$

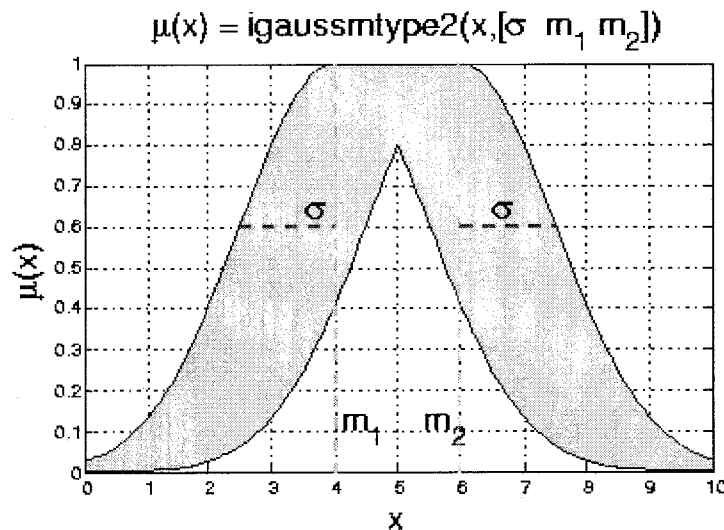


Figure 3.7: Gaussian membership function with uncertainty mean

Figure 3.7 gave a clear picture of *igaussmtype2* function. In equation 25, parameter c represents; cluster center for upper and lower membership function. These values can be found using following steps:

1. Calculate the mean for each variable

$$\mu = \frac{\sum x}{n} \quad (26)$$

where $\sum x$ is total submission of data and n is total of data

2. Calculate the spread changes

$$a_j^k = range * \mu \quad (27)$$

where *range* is the value between 0.05 and 0.2

3. Calculate the new cluster center

- Left cluster center : $x_{jk}^l = x_{jk}^* - a_j^k$, (28)

- Right cluster center : $x_{jk}^r = x_{jk}^* + a_j^k$, (29)

where x_{jk}^* is the Type-1 cluster center

Table 3.4: New Type-2 cluster center

Parameter	Type-1	Left: Type-2	Righ: Type 2
Cluster 1	249.0	233.2	264.8
Cluster 2	40.1	24.26	55.94
Cluster 3	233.4	217.6	249.2
Cluster 4	41.4	25.56	57.24
Cluster 5	114.7	98.86	130.5

Table 3.4 shows the list of new cluster center values for wind direction with the *range* value of 0.15. These values were applied in equation 25 to get Type-2 membership function for wind direction input. The process of transformation from Type-1 fuzzy to Type-2 fuzzy was illustrated in Figure 3.8, where with the wind direction value of 150 degree. This data point fall in Type-2 fuzzy set of ‘in1cluster1’

with membership function values between 0.2 and 1.00. The creation of Type-2 fuzzy set gives more membership function values rather than Type-1 fuzzy does. The whole Type-2 fuzzy set for wind direction input were shown in Figure 3.9, where there were five fuzzy sets.

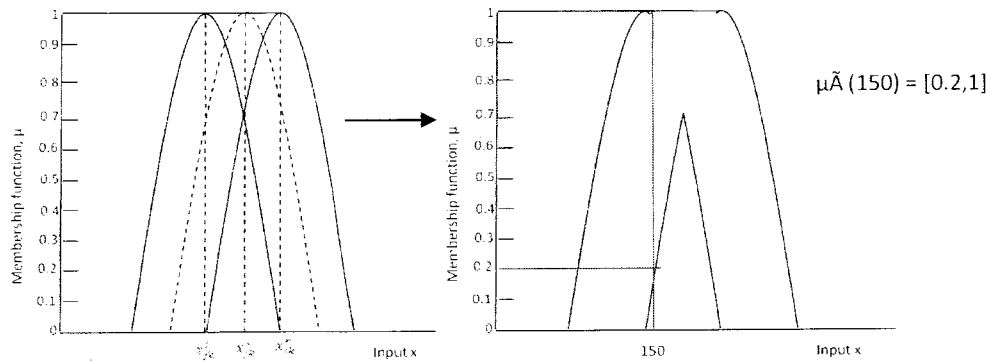


Figure 3.8: Example of Type-2 Gaussian MF for wind direction input

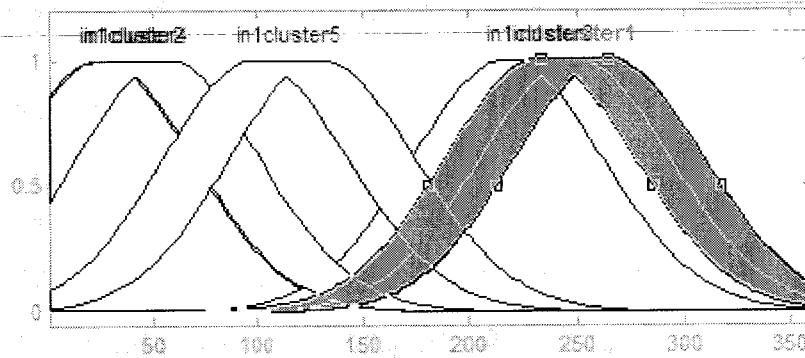


Figure 3.9: Type- 2 Gaussian membership function for wind direction

3.3 Fuzzy Inference System in Type-2 Fuzzy

Table 3.5: Type- 2 Fuzzy Inference System phase

Fuzzy Inference System phase	Methods
Meet: And method	Min
Join: Or method	Max
Reduction: Defuzzification	Avg

Initially, fuzzy inference system phases were determined as the system works according to methods selection. AND and OR methods are functions that worked in inference engine as operator. The AND method was selected to be used in IF-THEN

structure and it uses Min method calculation. Then, in defuzzification phase, weight average method was used and further elaboration is available at 3.4 Output.

The number of rules can be derived from the number of cluster center generated by the *subclust* function. As been stated by [65], the number of rules must equal to the number of cluster center with the aim of optimize the inference system of Sugeno model. Therefore, the number of rules has same number of cluster center.

For overall process in inference engine, if a data point is closer to the first cluster in which it has a strong membership degree to the first cluster. Then, rule one will fire with more firing strength than the other rules. Similarly, an input with strong membership to the second cluster will fire the second rule with more firing strength than the other rules and so on. Then, the output of rules (firing strength) was used to generate the output of the FIS through the output membership functions, where in Sugeno model, the output is in a function.

Basically, there are four components involved in inference engine which are antecedents, consequent, operator and rule. Antecedent has connection with consequence part that can be described by IF-THEN rules which represent input and output relations. Then, operator is used to combine all rules from input to output. Instead of union and intersection, Type-2 fuzzy operators involved are join (\sqcup) and meet (\sqcap). Its k th rule can be expressed as:

Rule: IF x_1 is A_{1k} and x_2 is A_{2k} and ... x_n is A_{nk} ,

THEN Z is $w^k = p_0^k + p_1^k x_1 + p_2^k x_2 + \dots + p_n^k x_n$

Where x_1, x_2, \dots, x_n and Z are linguistic variables, $A_{1k}, A_{2k}, \dots,$ and A_{nk} are the fuzzy sets and $p_0^k, p_1^k, \dots,$ and p_n^k are regression parameters.

Generally, IF-THEN rules for both Type- 1 and Type- 2 have a similar structure except for the fuzzy set element in antecedent and consequent. In this proposed method, it gave focus on changes in antecedent part only, where it used Type-2 fuzzy set. In forecasting the significant wave height, IF-THEN rules format as follows:-

IF wind direction is \tilde{A}_{11} and wind speed is \tilde{A}_{12} ,

THEN significant wave height = $p_0^k + p_1^k \text{wind direction} + p_2^k \text{wind speed}$.

The antecedent part has been defined in subtractive clustering process. Meanwhile the consequent part of Sugeno model had been calculated using function equation, where the parameter can be obtained by using least square estimation. Each rules have similar function but with a different regression parameters. Basically, this method will find the best-fitting curve to a given set of points by minimizing the sum of squares of the residuals of points from curve. Practically, it minimized the error of matching sum between expected results with the output from the fuzzy logic system [28]. Examples of rules generated from subtractive clustering and least square estimation process in fuzzy logic toolbox follows:

Rule 1:

IF wind direction is cluster₁₁ AND wind speed is cluster₁₂

THEN wave height = -0.0005302(wind direction) + 0.1131(wind speed) + 0.3221

Rule 2:

IF wind direction is cluster₂₁ AND wind speed is cluster₂₂

THEN wave height =0.004075(wind direction)+ 0.2155(wind speed) -0.4419

Rule 3:

IF wind direction is cluster₃₁ AND wind speed is cluster₃₂

THEN wave height = -0.0003328(wind direction)+ 0.2619(wind speed) -0.7086

Rule 4:

IF wind direction is cluster₄₁ AND wind speed is cluster₄₂

THEN wave height = -0.00633(wind direction) + 0.3803(wind speed) -1.599

Rule 5:

IF wind direction is cluster₅₁ AND wind speed is cluster₅₂

THEN wave height = -0.001169(wind direction) + 0.03704(wind speed) + 0.5101

3.4 Output

The final step is to do defuzzification with the appropriate defuzzification method in order to fired-rule output fuzzy to numeric value. For Sugeno FIS there are two methods to obtain the final output; weighted average or weighted sum. In this method, the weighted average was used to get a final output. The structure of final output was given as follows [81]:

$$Y = \int_{f^1 \in [\underline{f}^1, \bar{f}^1]} \dots \int_{f^M \in [\underline{f}^M, \bar{f}^M]} 1 / \frac{\sum_{j=1}^M f_j u_j}{\sum_{j=1}^M f_j} \quad (30)$$

Where \underline{f}^i and \bar{f}^i are given by:

$$\underline{f}_j(x) = \underline{\mu}_{Aj1}(x_1) * \dots * \underline{\mu}_{Ajn}(x_n) \quad (31)$$

$$\bar{f}_j(x) = \bar{\mu}_{Aj1}(x_1) * \dots * \bar{\mu}_{Ajn}(x_n) \quad (32)$$

the * represents the t-norm which is the *prod* operator.

The output of the fuzzy system is obtained by [81] :

$$Y = \frac{\left(\frac{\sum_{j=1}^M \underline{f}_j u_j}{\sum_{j=1}^M \underline{f}_j + \sum_{j=1}^M \bar{f}_j} + \frac{\sum_{j=1}^M \bar{f}_j u_j}{\sum_{j=1}^M \bar{f}_j + \sum_{j=1}^M \underline{f}_j} \right)}{2} \quad (33)$$

3.5 Performance Evaluation

The performance of proposed method was evaluated by using Root Mean Square Error (RMSE). RMSE is an error measurement of success for numeric forecasting as it computes the average of the squared differences between each forecasted value and its actual value. Before calculating the RMSE value, the generated proposed method need to be tested by using *evalifistype2* function that is available in MATLAB. This function was used to evaluate the FIS and gives the forecasted values for significant

wave height. From the result, the RMSE was used to compare the proposed method with the actual data.

In order to examine the improvements in forecasting accuracy after the implementation of proposed method, Type-1 Sugeno Fuzzy Inference System and Type-1 Mamdani Fuzzy Inference System were used as comparison methods as defined in the equation below, RMSE is used as an evaluation criterion to compare the forecasting performance of listing methods.

$$\sqrt{\frac{\sum_{t=1}^n (P(t) - \text{forecast}(t))^2}{n}} \quad (34)$$

where n is the time of forecasting, $P(t)$ is the actual significant wave height at time t , and $\text{forecast}(t)$ is the forecasting significant wave height at time t .

3.6 Summary

This chapter presents the methodology of proposed method where basically the Type-2 fuzzy inference system was used. The main step in methodology was to extend Type-1 fuzzy to Type-2 fuzzy. The Type-1 cluster center was shifted to left and right with certain amount of uncertainties defined by the spread changes. The output of proposed method performance is evaluated using Root Mean Square Error (RMSE). The results will be presented in the next chapter.

CHAPTER 4

RESULTS AND DISCUSSION

Chapter 4 presents the experiment setups, results and analysis that have been discussed in Chapter 3. The findings have been compared with Sugeno Type-1 Fuzzy Inference System (ST1FIS) and Mamdani Type-1 Fuzzy Inference System (MT1FIS) in order to validate the proposed method and to find the most accurate method. Each method used Root Mean Square Error (RMSE) indicator to demonstrate the method performance.

Then, detailed analyses of results obtained are discussed. There were three main aspects that have been considered in conducting the experiments which were the number of data during FIS development; cluster radius parameter; and also the change of spread percentage in cluster center in Type-1 fuzzy.

4.1 Data Description

Metoccean data covers meteorology and oceanography environment. This experiment specifically focuses on the forecasting of significant wave height parameter that was used to validate the proposed method. The data used in this study were obtained from SEAMOS-South Fine Grid Hindcast (SEAFINE) project, which was a joint industry project (JIP) administered by Oceanweather, Inc (OWI) [82]. Due to the present unavailability of measured data, hindcast results were employed in current research to develop forecast methodology. Besides, hindcast data is sufficient and acceptable when compared with measured data [83]. The data was collected from two locations in Malaysia with a different water depth. The details of both locations are shown in Table 4.1.

Table 4.1: Case study location

No	Latitude	Longitude	Water depth
A	5.35 ⁰ N	114.9 ⁰ E	35m
B	5.8 ⁰ N	104.15 ⁰ E	62.508m

Location A was located near to East Malaysia with water depth of 35m and location B near to Peninsular Malaysia with water depth of 62.508m. Peninsular Malaysia has a relatively harsh wave climate as compared to other coasts in Malaysia [32]. For both locations, the data covered the period of year 2000 to 2006. These data were divided into two groups; development data and testing data. The development data were grouped into three periods of time; 2005, from 2003 to 2005 and from 2000 to 2005. Then, the testing data that included 4338 records from January 2006 to June 2006 was used to verify the methods. The summary of the amount of data based on period of time is presented in Table 4.2.

Table 4.2: The number of development dataset for each variable

Period of time	Amount of data
2005	8760
2003 – 2005	26304
2000 – 2005	52608
Jan to June 2006	4338

There were three variables used to develop the proposed method where wind speed and wind direction have been chosen as inputs and significant wave height as an output. The input parameter selections were based on previous significant wave height [34] that justified the influences of wind speed and wind direction towards significant wave height forecasting. This is because the fetch length parameter depends on the wind direction. All of these three set of data were presented in time series format, in 1 hour interval of time. Preliminary statistical analysis was performed to obtain essential information for these three variables in the following section.

4.1.1 Significant Wave Height

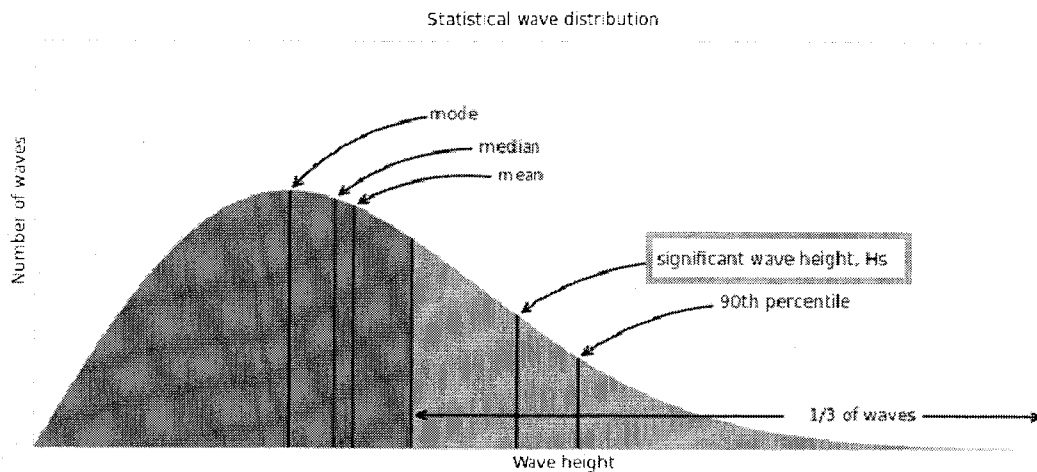


Figure 4.1: Statistical wave distribution

Significant wave height ($H_{1/3}$) is the average of the highest one-third of all waves occurring during a period as shown in Figure 4.1. $H_{1/3}$ is close to the ‘height’ of random waves that would be reported by an observer. It has also been stated by [84] as the most frequent wave statistic and oftenly appeared in ocean engineering environment. According to [82], this parameter is 4.000 times the square root of the total variance, in meter. Equation 35 is a common formula to forecast significant wave height, where U_{10} is the wind speed (in ms^{-1}) at a reference height of 10m. The formula involved as follows:

$$H_{1/3} = 0.0246 U_{10}^2 \quad (35)$$

Table 4.3: Minimum and maximum values for significant wave height in meter (m)

Year	Significant Wave Height (m)			
	Location A		Location B	
	Minimum	Maximum	Minimum	Maximum
2000 to 2005	0.133	3.134	0.208	3.578
2003 to 2005	0.133	2.849	0.208	3.578
2005	0.133	2.849	0.208	3.358
2006	0.152	2.576	0.199	2.84

Location A and location B in Table 4.3 shows range of 0.133m to 0.208m for minimum height of significant wave height and maximum range between 2.576m and

3.578m. Location B had the maximum of wave height with 3.578m but not much different with the maximum value in location A which is 3.134m.

4.1.2 Wind Speed

Second parameter is wind speed, where an hour average of the effective neutral wind at a height of 10 meters, units in meters/second. The wind speed used for this case study fall between 0.86m/s and 15m/s as shown in Table 4.4. In location A, the maximum value of wind speed is 15m/s with a minimum value of wind speed is 1.35m/s. Meanwhile, wind speed in location B has a maximum value of 13.8m/s and a minimum value of 0.86m/s. The wind speed in location A moves faster than in location B.

Table 4.4: Minimum and maximum values for wind speed

Year	Wind Speed (m/s)			
	Location A		Location B	
	Minimum	Maximum	Minimum	Maximum
2000 to 2005	1.79	15	0.86	13.8
2003 to 2005	1.79	12.24	1	13.8
2005	1.82	12.24	1	13.31
2006	1.35	11.32	1.22	11.96

4.1.3 Wind Direction

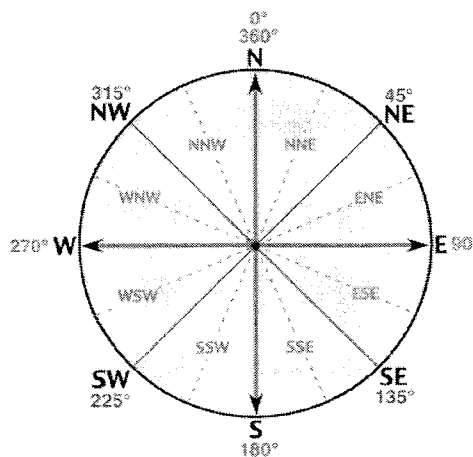


Figure 4.2: Wind direction compass [85]

Wind direction from which the wind is blowing and can be read as clockwise from true north in degrees as shown in Figure 4.2. The wind direction at location A and B had a similar pattern along the 5 years but differ pattern compare to one another. This is due to the location of place located. It can be clearly seen in Figure 4.3 and Figure 4.4.

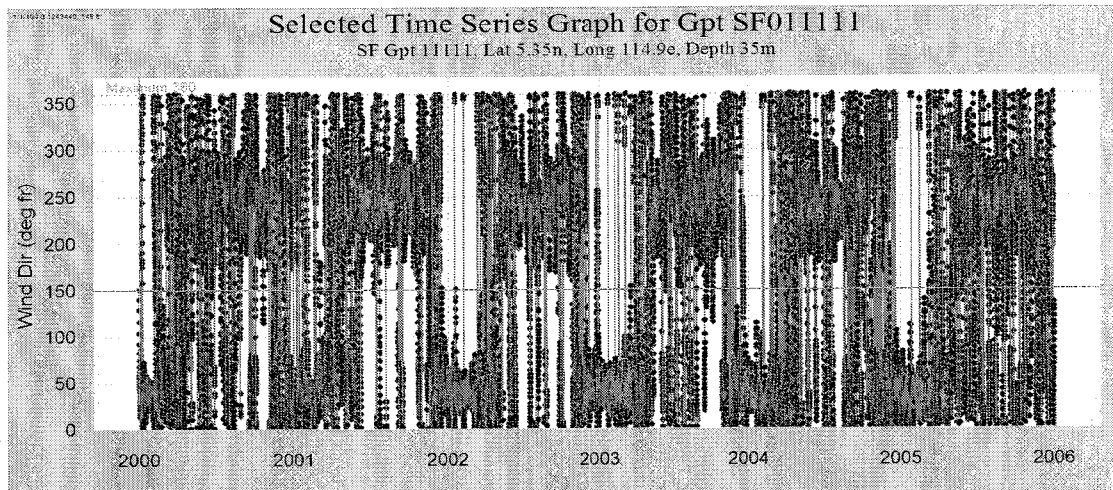


Figure 4.3: Wind direction for year 2000 to 2005 at location A

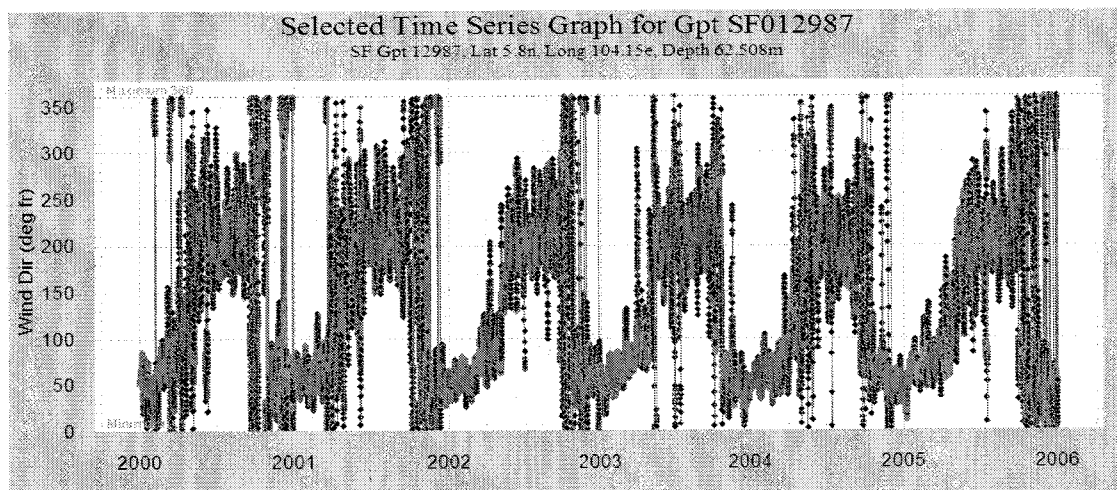


Figure 4.4: Wind direction for year 2000 to 2005 at location B

From these three parameters, each location had different values of maximum and minimum for wave height and wind speed parameters. It also has a different pattern in both locations. The differences can affect the results of FIS later on.

4.2 Parameter Setting

Parts from methodology chapter which are in the fuzzification phase, three main parameters were highlighted in order to see the impact towards Type- 2 fuzzy development. The parameters were dataset used to develop FIS, the value of cluster radius to be pre-initialized at subtractive clustering process and the optimal values of uncertainties in Type-2 fuzzy set. The list of parameters and its values were shown in Table 4.5.

Table 4.5: List of parameters

Parameter	Value
Development of FIS	2005, 2003-2005 and 2000-2005 data
Cluster Radius	0.2, 0.35 and 0.5
Spread percentage	0.05, 0.1, 0.15, 0.2

The first parameter value is the numbers of data in FIS development where the numbers of year selection are in the range of year 2000 to year 2005. The numbers of year will determine the amount of data input used during development. It was a concern to see the impact of amount of data used to develop the FIS towards the accuracy of the method. Details of this parameter had been described in section 4.1 Data Description.

The second parameter is cluster radius value. In the beginning, cluster radius has been selected to be installed at subtractive clustering phases. This cluster radius indicates the range of influence of a cluster when the data space is a unit hypercube. Cluster radius parameter gives an impact to the number of clusters, rules, and data in the cluster itself as well as the number of fuzzy set. In many cases, using small cluster radius will yield many small clusters in data, where many rules will be generated as results. Meanwhile, large cluster radius will usually yield a few large clusters in data. As a result, few rules will be generated [60].

Initially, location A has been tested based on radius parameter between 0.1 and 0.5. As been reported in Chapter 2, [62, 86] were introduced the suitable value for cluster radius is between 0.2 and 0.5. Therefore, in this experiment, the smallest and the largest reported cluster radius values were selected. Then, the last cluster radius

value is taken from the cluster radius values with the smallest RMSE result. In Figure 4.5, the smallest RMSE was resulted from cluster radius values of 0.3 and 0.35.

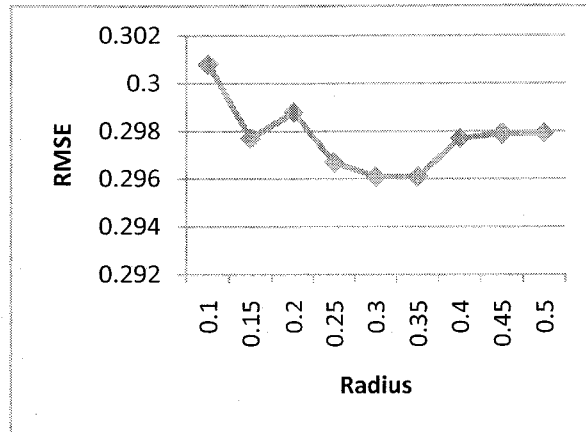


Figure 4.5: RMSE according to cluster radius values

After investigating the cluster radius values, the optimal values for uncertainties in Type- 2 fuzzy were discussed. This is one of the important parameter that shifted Type-1 fuzzy to Type-2 fuzzy. The uncertainties or called spread change were assigned to Type-2 fuzzy by assigning some values to Type- 1 fuzzy. The spread change values were dispersed to shift the Type-1 cluster center to both left and right side and produced FOU. This FOU gives extra degree of freedom of primary membership function to input parameters that lead to a better uncertainty modeling.

The more area involved in FOU, the more uncertainty occurred. Based on [87], the authors shifted the cluster center between 0.05 and 0.4 from the original cluster center. There is no guarantee that a Type-2 TSK FLS have the potential to outperform its Type-1 counterpart and it is difficult to determine the range for spreading changes of cluster centers [10]. These values were used to decide the size of bounded regions of the union of all antecedents in inference engine stages, one of fuzzy logic system components. All of these parameters were observed and analyzed to find the best amount of uncertainties used in both locations.

4.3 Forecasting results

This section presents the results from three different methods, which were T1SFIS, T1MFIS and Proposed method. Both Type-1 FIS methods were used Type-1 fuzzy set in both antecedent and consequent part. Meanwhile proposed method focused on Type-2 fuzzy set in antecedent part only. These three methods were applied to forecast the next 6 hours of significant wave height variable from January to June 2006. The equation of forecasting was as below:

$$Y(t) = X_1(t-6) + X_2(t-6) \quad (36)$$

where X_1 is wind direction and X_2 is wind speed.

In the beginning, the proposed method was applied to forecast significant wave height for 1 hour up to 24 hours of lead time. From the results, the lowest error was found when forecasting the following next 6 hours significant wave height values by using input of wind direction and wind speed. Therefore, RMSE for all methods in this section were resulted from forecasting the 6 hours lead time of significant wave height. Figure 4.6 and Figure 4.7 illustrate the lowest RMSE result at 6 hours lead time for Type-1 and proposed method.

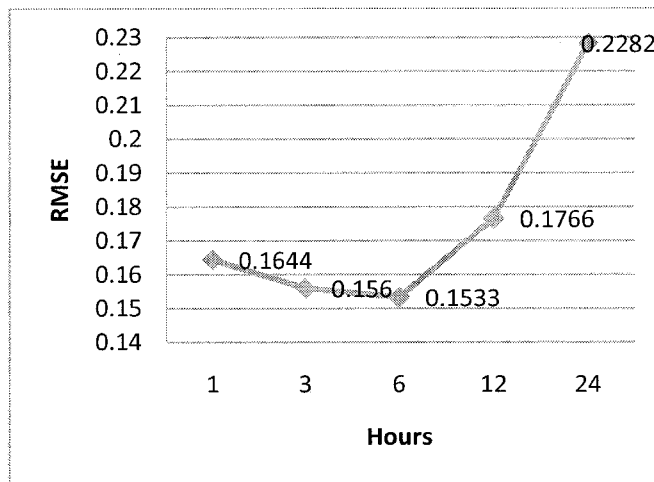


Figure 4.6: RMSE for 1 h, 3 h, 6 h, 12 h, 24 h lead time by using Type- 1 fuzzy; h for hour

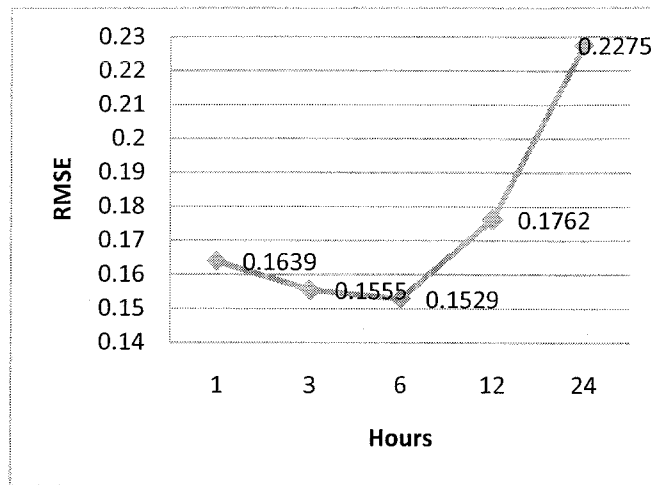


Figure 4.7: RMSE for 1 h, 3 h, 6 h, 12 h, 24 h lead time by using Type-2 fuzzy(proposed method);
h for hour

The accuracy of proposed methods towards T1SFIS and T1MFIS had been compared based on RMSE indicators with three characteristics presented in Parameter Setting section.

4.3.1 Sugeno Type-1 Fuzzy Inference System (ST1FIS)

T1SFIS was developed with fuzzy toolbox in MATLAB by choosing the ‘Sugeno’ inference system. *subclust* function was used to define the initial membership function for input and least square estimation method to generate consequent function. The fuzzy operator in antecedent part was set to Min method, where this operator combined the variables rules. The output consequent function was weighted by firing strength of the rule and defuzzify by weighted average method. FIS processes methods were defined in Table 4.6.

Table 4.6: Sugeno Type-1 FIS

Fuzzy Inference system phase	methods
Meet: and method	Min
Join: or method	Max
Reduction: Defuzzification	Weighted average

This setting was used in three experiments with different amount of data development. For each experiment, three dissimilar cluster radius values were used in *subclust* function. Data development and cluster radius values were defined in Parameter Settings section. After developing FIS models, testing data (4338, January to June, 2006) were used to calculate the RMSE. The results have been compared according to data development, cluster radius values and also both location A and B.

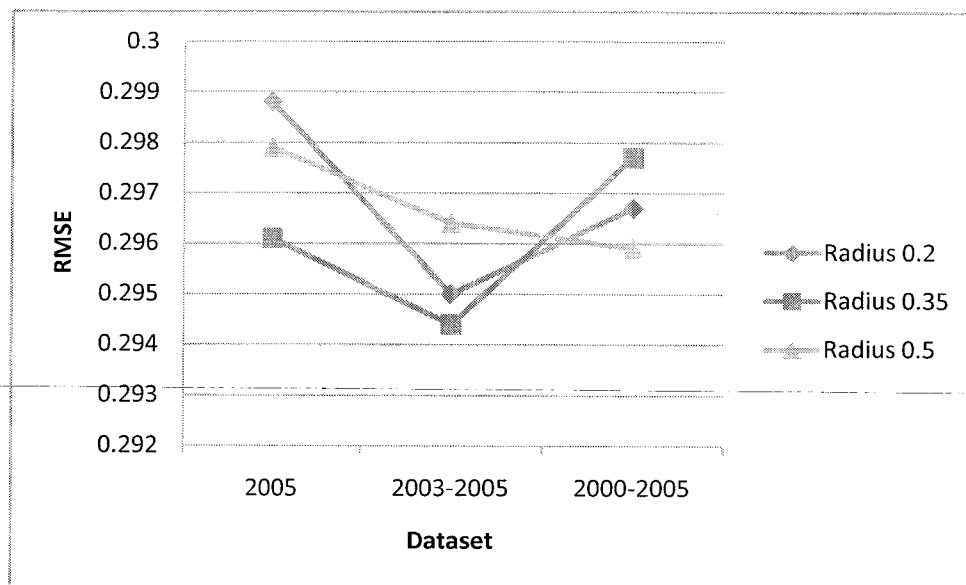


Figure 4.8: RMSE in location A based on number of dataset and cluster radius values

As presented above, results in Figure 4.8 were based on two characteristics, the dataset and cluster radius values which were 0.2, 0.35 and 0.5. In location A, the best result with RMSE 0.2944 was at year 2003 to 2005 dataset and cluster radius value of 0.35. Based on the dataset, three years dataset (2003 – 2005) had the smallest error for all cluster radius values except for radius 0.5, but with not much difference. In terms of cluster radius, value of 0.35 had a good performance at dataset year 2005 and year 2003 to 2005. However, 0.35 had the worst result when using with year 2000 to 2005 dataset.

Meanwhile, in location B, the RMSE had the best result at dataset of year 2003 to 2005 and cluster radius value of 0.2 which resulted in RMSE of 0.1384. Moreover, the cluster radius with the value of 0.2 has shown a good result in both dataset of year 2000 to 2005 and year 2003 to 2005, but not with year 2005 dataset. Location B had

quite similar result with location A where year 2003 to 2005 dataset had the lowest RMSE for all cluster radius value.

From both locations as shown in Figure 4.8 and Figure 4.9, 3 years of dataset (2003 to 2005) combined with 0.35 cluster radius for FIS development has produced better result which contrasts with 1 and 5 year of dataset. Cluster radius values of 0.35 also performed a stable result in both locations.

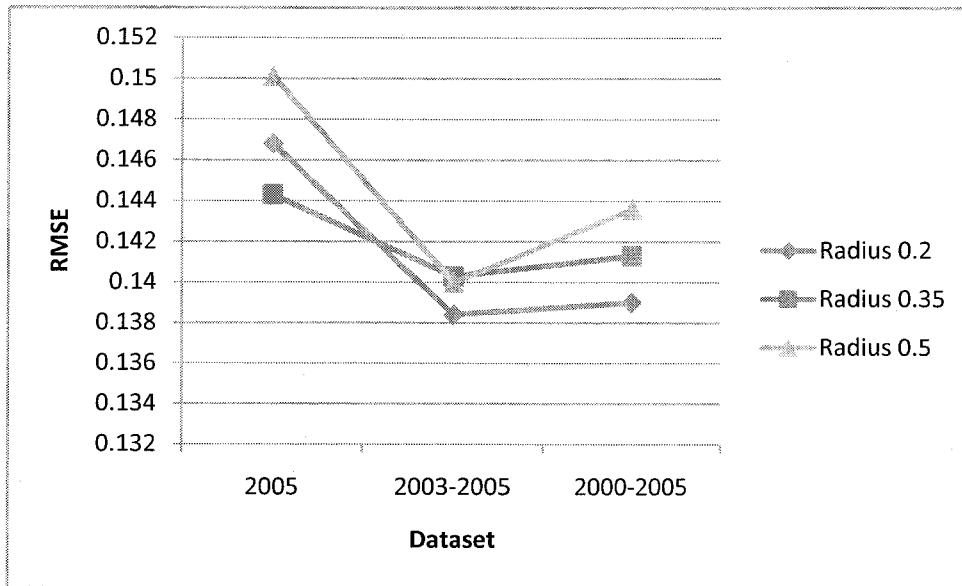


Figure 4.9: RMSE in location B based on number of dataset and cluster radius values

4.3.2 Mamdani Type-1 FIS (MT1FIS)

MT1FIS method was developed with Fuzzy Toolbox which is available in MATLAB. This method had a quite similar structure with Sugeno method but differs in terms of consequent part structure. It started by calling the fuzzy inference interface with 'fuzzy' word, choose the 'Mamdani' inference system and set the fuzzy inference system phase. The setting parameters were the same as Sugeno inference system as presented in Table 4.6 and with inclusion of Implication and Aggregation method as presented in Table 4.7.

Table 4.7: Mamdani Type-1 FIS

Fuzzy Inference system phase	methods
Meet: and method	Min
Join: or method	Max
Meet: Implication	Min
Join: Aggregation	Max
Reduction: Defuzzification	Centroid

The fuzzy operator in antecedent part was set to Min method, where this operator combined the variables rules. Then, Min method was applied to implicate from antecedent to consequent. The consequents were aggregated across the rules with Max method and defuzzify by Centroid method. The process of inference system was illustrated in Figure 4.10.

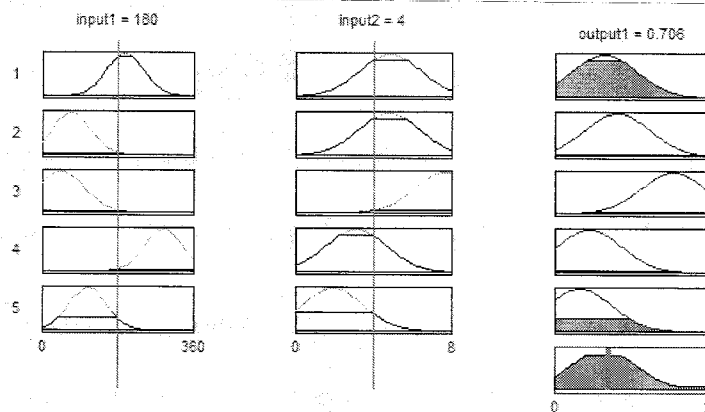


Figure 4.10: Type- 1 Mamdani Rule Reviewer

Then, two inputs and one output was selected. For each variable, five membership functions were created with Gaussian membership. Universe of discourse for each variable was identified. Only two dataset were used in FIS development; year 2005 and year 2003 to 2005. The number of rules were based on the number of fuzzy set generated. The rules were listed by matching the fuzzy set for each variable according to it fuzzy set order as below:

IF X_1 is A_1 AND X_2 is B_1 THEN Y_1 is C_1 ,

IF X_1 is A_2 AND X_2 is B_2 THEN Y_1 is C_2 ,

IF X_1 is A_3 AND X_2 is B_3 THEN Y_1 is C_3 .

IF X_1 is A_4 AND X_2 is B_4 THEN Y_1 is C_4 .

IF X_1 is A_5 AND X_2 is B_5 THEN Y_1 is C_5 .

Where X_1, X_2, Y_1 is wind speed, wind direction and significant wave height.

The accuracy of MT1FIS was tested with the same testing data from previous method (January to June, 2006). Based on Table 4.8, the results show poor performance with MT1FIS method when compared to ST1FIS method, which ST1FIS had RMSE below 0.3 for both locations and data development.

Table 4.8: Result for both location based on MT1FIS

Data development	Location	
	A	B
2005	0.3443	0.3161
2003-2005	0.3615	0.3030

4.3.3 Proposed method

The proposed method focused on Type-2 fuzzy method where the antecedent part was using Type-2 fuzzy set. Type-2 fuzzy set was caused by shifting the Type- 1 cluster center to the left and right. This method was using the same numbers of cluster and rules generated in ST1FIS method, but with extension of Type- 1 fuzzy set to Type- 2 fuzzy set. Details of development of proposed method development have been presented in Chapter 3 (Methodology).

The results of proposed method were analyzed based on three characteristics; dataset used to develop FIS, cluster radius value and the optimal values of uncertainties assigned in Type- 2 fuzzy. For each dataset, cluster radius values and uncertainty values were compared. The overall performance had been summarized at the end of section.

4.3.3.1 FIS data development: year 2005

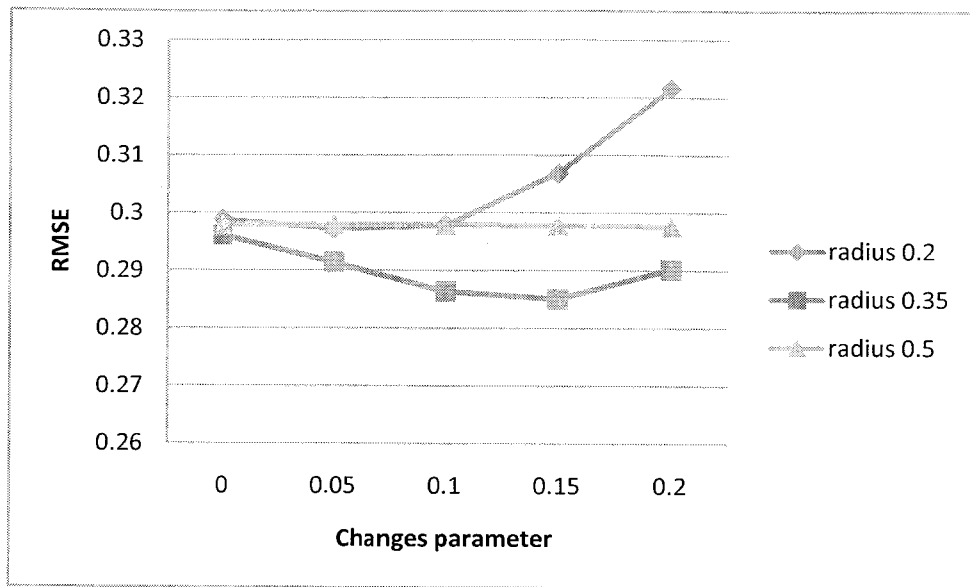


Figure 4.11: RMSE result based on cluster radius and spread changes in cluster center by using year 2005 data (location A)

Figure 4.11 portrays RMSE results for proposed method categorized by cluster radius values and spread changes. Three cluster radius values were 0.2, 0.35 and 0.5. The spread changes here refer to the uncertainties assign to Type-2 fuzzy where the values have been taken in interval of 0 to 0.2. The 0 value in graph represents Type-1 fuzzy. This spread percentage parameter has been used to extend Type-1 membership function to Type-2 membership function by assigning uncertainty in cluster center. For example, the value of 0.05 from the mean of the dataset has been shifted to produce the FOU areas in order to give more freedom to membership function of variables.

Obviously from Figure 4.11, cluster radius value of 0.35 with 0.15 spread changes generated the best RMSE for year 2005 dataset. By having the same characteristics of dataset and cluster radius value with T1SFIS, proposed method has improved the performance for 3.7% where the RMSE values decreased from 0.2961 to 0.2851. The cluster radius value with the smallest Type-1 RMSE result had given a better RMSE result for Type-2.

In terms of cluster radius value, 0.35 had maintained good results along with the amount of uncertainties assigned in fuzzy set. Different with cluster radius value of 0.2, the RMSE increased as the size of FOU getting bigger, especially after 0.1 spread changes. On the other side, cluster radius value of 0.5 had near to similar RMSE results for each spread changes.

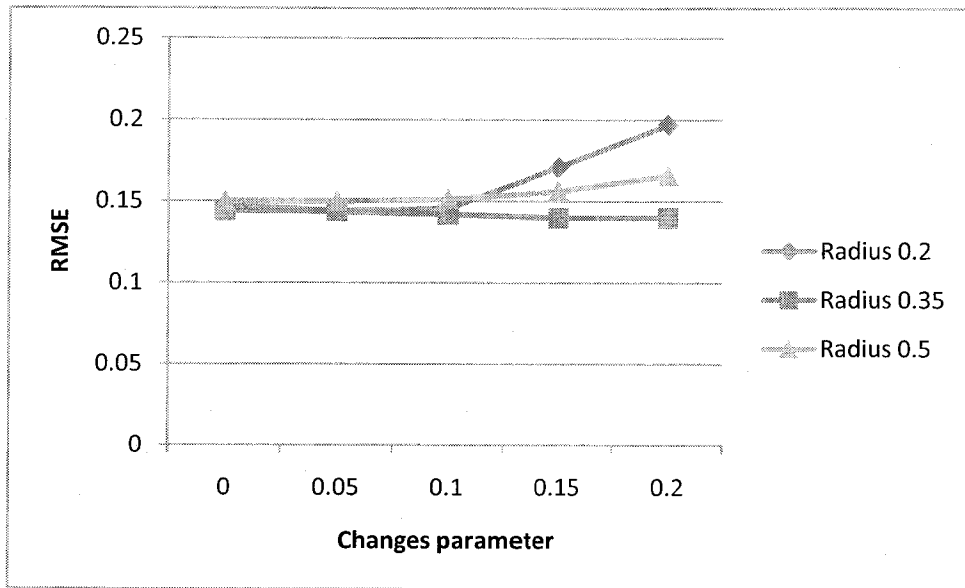


Figure 4.12: RMSE result based on cluster radius and spread changes in cluster center by using year 2005 data (location B)

Results in Figure 4.12 showed similar RMSE value for all cluster radius and start to distinguish after 0.1 spread changes. The proposed method produced the best RMSE result in location B at cluster radius 0.35 with 0.15 spread changes. The RMSE result caused an improvement of 3.19% from Type-1 fuzzy by 0.1443 decreased to 0.1397 in proposed method. The graph for cluster radius of 0.2 and 0.5 was increased as the size of spread changes enlarged but with a small enhancement in RMSE result as when compared to location A.

FIS dataset development by using year 2005 dataset had shown the lowest error with comparable result characteristics in both locations. The forecasting of significant wave height in both locations produced the best RMSE result at cluster radius of 0.35 with 0.15 of uncertainties assignment in its own Type- 1 fuzzy. Moreover, cluster radius of 0.35 had good results in all conditions for spread changes of 0.1 until 0.2 of. It has been reported that cluster radius of 0.2 given the biggest error in these two

locations. It can be seen that the amount of uncertainties assigned had a relation with the installed cluster radius value in subtractive clustering phase.

4.3.3.2 FIS data development: year 2003- 2005

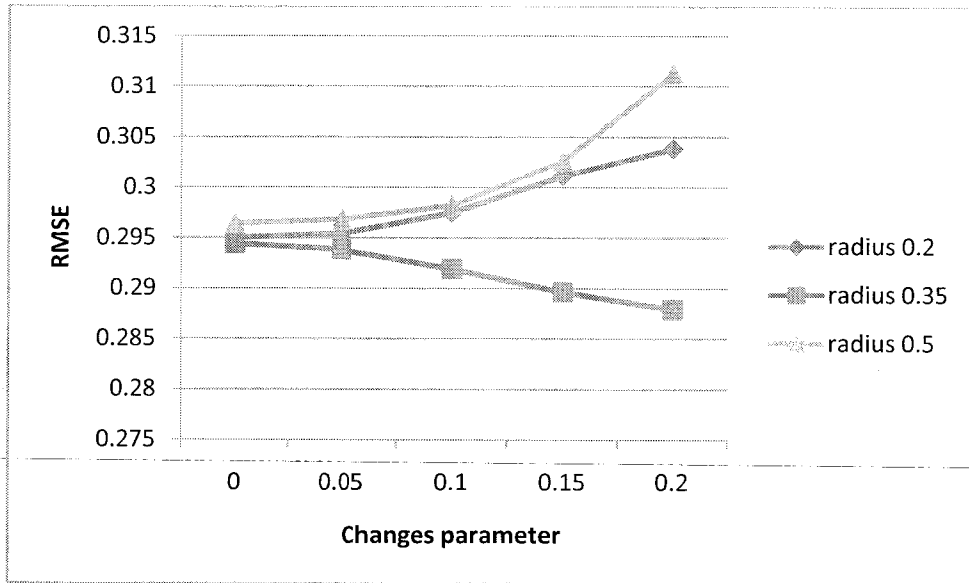


Figure 4.13: RMSE result based on cluster radius and spread changes in cluster center by using year 2003 to 2005 data (location A)

The graph in Figure 4.13 explained that the RMSE results for significant wave height forecasting by using year 2003 to 2005 dataset when considered the impact of cluster radius and spread changes in Type-2 FIS progress. The smallest RMSE value; 0.2879 in location A was at spread changes of 0.2 with cluster radius of 0.35. As compared to Type-1 RMSE result which was 0.2944, 2.21% improvement was achieved with proposed method. Distinct for cluster radius 0.2 and 0.5, both has shown poor result when the Type-2 fuzzy was implemented. The errors for both characteristics were kept on increasing with the added uncertainties to fuzzy set.

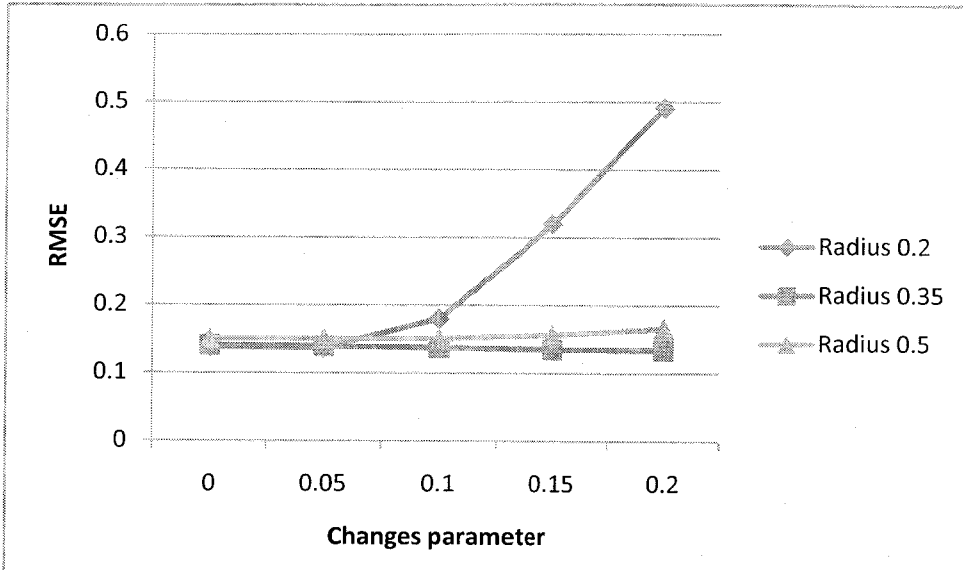


Figure 4.14: RMSE result based on cluster radius and spread changes in cluster center by using year 2003 to 2005 data (location B)

A big gap of RMSE result between cluster radius 0.2 and others cluster radius in location B can be observed in Figure 4.14. Initially, cluster radius value of 0.2 produced a closer result with cluster radius 0.35 and 0.5 when 0.05 of spread changes was implemented. However, the error keeps on getting bigger and the values were varied from the others. It happened because of the FOU generated from cluster radius 0.2 for each fuzzy set overlapped with one another in rule reviewer in Figure 4.15.

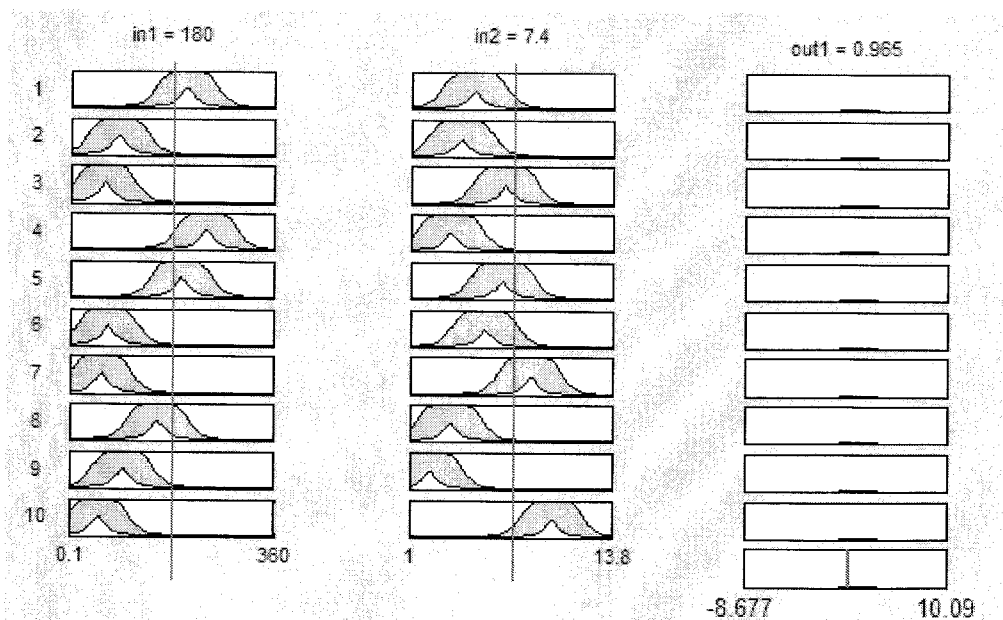


Figure 4.15: Type- 2 Rule Viewer with cluster radius value 0.2

Cluster radius of 0.35 with 0.2 spread changes generated the best RMSE result. From RMSE result of 0.1403 in Type-1 to 0.1339 in proposed method, Type-2 fuzzy had advancement of 4%. FIS development with dataset of year 2003 to 2005 has shown a different RMSE graph patterns in both locations. However, these two locations produced command characteristics for the lowest RMSE results in term of cluster radius and spread changes. Both were found at cluster radius 0.35 with 0.2 spread changes. Location A had the worst result at cluster radius 0.5 and location B at cluster radius 0.2 with a big different of RMSE value.

4.3.3.3 FIS data development year 2000 - 2005

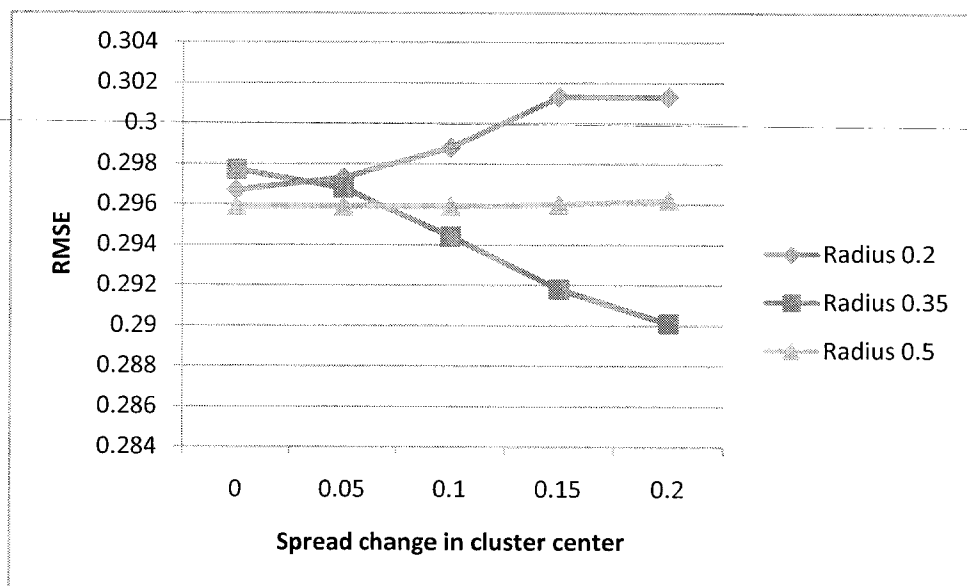


Figure 4.16: RMSE result based on cluster radius and spread changes in cluster center by using year 2000 to 2005 dataset (location A)

Figure 4.16 indicates that the RMSE result for FIS development with year 2000 to 2005 dataset. It has been compared between spread changes and cluster radius. Proposed method has recorded the best forecasting of significant wave height with the lowest RMSE of 0.2901. This record was obtained with cluster radius 0.35 at spread changes of 0.2. 2.6% improvement was achieved as compared RMSE result in Type-1 method; 0.2977.

In comparing results when using year 2005 dataset, the RMSE graph produced a similar trend with 2000 to 2005 dataset. The RMSE results maintain closer value along the spread changes. Implausible, cluster radius 0.2 had an increase of error as the uncertainties were assigned.

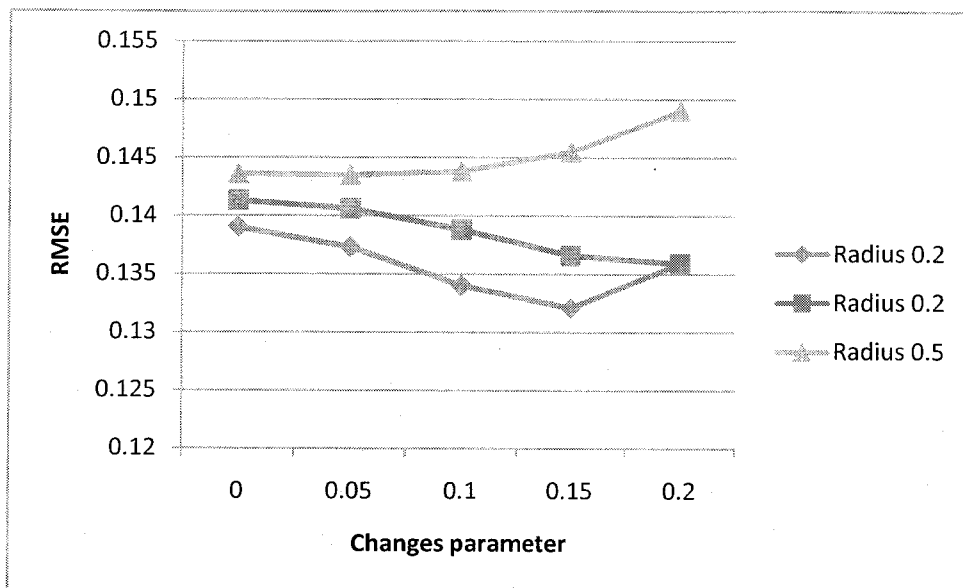


Figure 4.17: RMSE result based on cluster radius and spread change in cluster center by using 2000 to 2005 data (location B)

RMSE results in location B were slightly different with location A. The graph in Figure 4.17 demonstrates cluster radius 0.2 generated the smallest error with RMSE value of 0.1321 at 0.15 spread changes. As compared to Type-1 method where the RMSE was 0.1390, 4.96% of improvement was recorded in the proposed method. On the other hand, cluster radius value of 0.35 still maintains a good result by using year 2000 to 2005 for data development with 3.8% improvement. Cluster radius 0.5 was the only parameter with a poor RMSE result.

FIS development with dataset of year 2000 to 2005 in both locations were totally had a different characteristics for the smallest RMSE result. Location A was at cluster radius 0.35 with 0.2 spread changes whereas at location B was at cluster radius 0.2 with 0.15 spread changes where this cluster radius has shown the worst result at location A.

4.3.3.4 Overall performance for proposed method

Table 4.9: The best result for each location with its own characteristics

Location	Location A				Location B			
Year / parameter	Cluster radius	Changes parameter	%	RMSE value	Cluster radius	Changes parameter	%	RMSE value
2005	0.35	0.15	3.7	0.2851	0.35	0.15	3.19	0.1397
2003-2005	0.35	0.2	2.21	0.2879	0.35	0.2	4.0	0.1339
2000-2005	0.35	0.2	2.6	0.2901	0.2	0.15	4.96	0.1321

Table 4.9 summarizes the best results for both locations according to three characteristics; FIS dataset development, cluster radius and spread changes. Location A generated the best result when using year 2005 dataset for FIS development with cluster radius of 0.35. Meanwhile, location B produced the best result when using year 2000-2005 dataset with cluster radius of 0.2. Even though the best result in location B produced the best result with cluster radius of 0.2, the cluster radius 0.35 still obtained a good result with the same dataset by 3.8% of improvement. On the other side, cluster radius 0.5 was not showing a good result for all dataset. Most of RMSE performances were showing better result with cluster radius value of 0.35 rather than cluster radius of 0.2 and 0.5. [86] has supported the result of using cluster radius of 0.35 and also this value is within suggested value from [62].

In terms of spread changes at its own Type-1 counterpart, 0.15 to 0.2 of spread changes in Type-1 cluster center would produce good result with one condition; the subtractive clustering setting must have a cluster radius value of 0.35. Based on the results, it had been indicated that cluster radius value installed in subtractive clustering has relation with spread changes at Type-1 cluster center. The proposed method provided an improvement as compared with Type-1 fuzzy with the improvement between 2% and 5%.

4.3.4 Comparison between ST1FIS, MT1FIS and Proposed Method

This section presents the significant wave height forecasting values forecasted by proposed method, ST1FIS method, MT1FIS method and recorded data from database. These three methods were compared with recorded data from dataset. For illustration purposes, parts of results from January to June 2006 in both locations were displayed in Figure 4.18 and Figure 4.19 by using year 2003 to 2005 dataset.

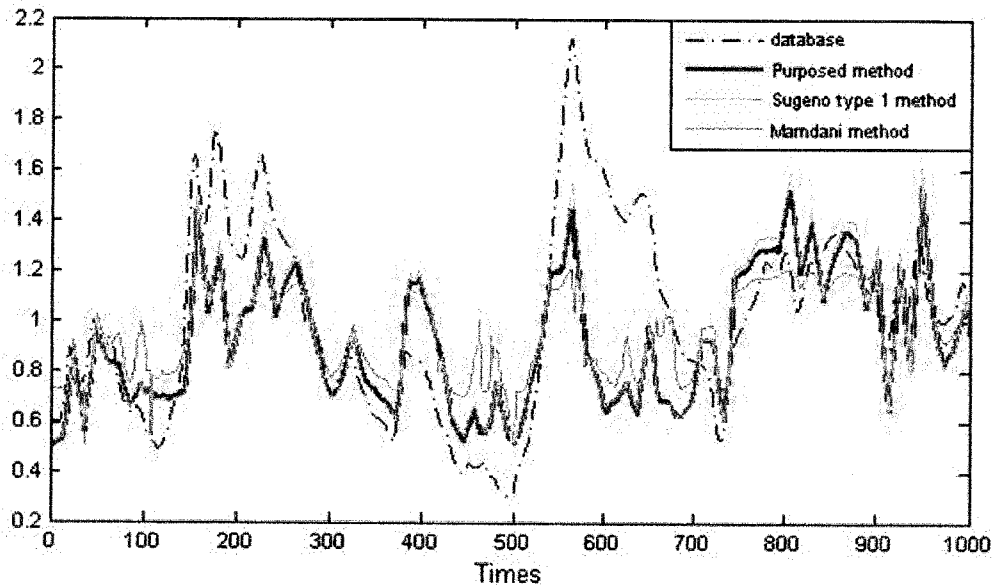


Figure 4.18: Comparison between methods for location A using year 2003-2005 dataset in FIS development

Figure 4.18 demonstrates recorded data from database as compared to three forecasting methods. In location A, 1000 times of data was selected to illustrate the results. All three methods were using year 2003 to 2005 to develop the FIS. The spread changes in cluster center for the proposed method were set to 0.15. Meanwhile, subtractive clustering procedure for ST1FIS method and the proposed method were initialized with cluster radius values of 0.35. The ST1FIS method follows the proposed method very closely and varies from recorded data at certain points especially at time of 500 to 700 in Figure 4.18.

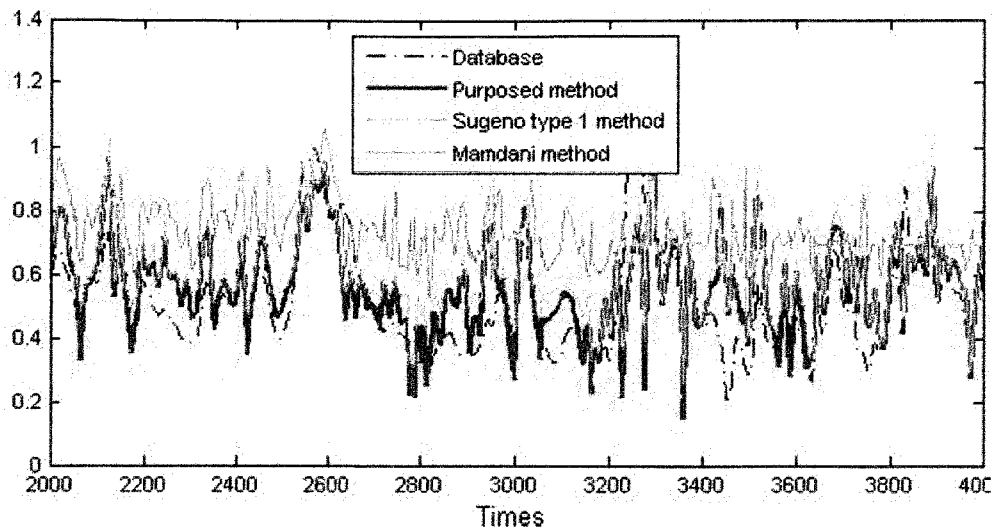


Figure 4.19: Comparison between methods for location B using year 2003-2005 data development

The settings for all methods in location B were the same as location A with 0.35 cluster radius, 0.15 spread changes and using year 2003 to 2005 dataset for FIS development. Graph in location B was more stable as compared to graph in location A. Location B graphs show better results compared to location A. As shown in Figure 4.19, ST1FIS method, proposed method and recorded data were closely moved along the graph and only the MT1FIS method varied from other methods.

Generally, even though significant wave height forecasting with the proposed method was not really closed to the recorded data in graphic view, however with root mean square error criteria, it can be clearly observed that the proposed method produced the forecasting with smaller errors as compared with ST1FIS method and MT1FIS method. It has also proven that the proposed method can provide improved forecasting than other two methods.

4.3.5 Summary

This chapter presents the details of data descriptions, parameter settings and performance of the proposed method in Chapter 3 as compared with Sugeno Type-1 FIS mainly and Mamdani Type-1 FIS. The performance of each method was evaluated by using RMSE indicator with three main aspects; number of dataset used to develop FIS, cluster radius value and spread changes of Type-1 cluster center.

The conclusion of the results can be summarized that Sugeno Type-1 FIS generated the smallest error in both location A and B by using year 2003 to 2005 dataset. However, location A used cluster radius value of 0.35 and location B used cluster radius of 0.2. For Mamdani Type-1 FIS the RMSE results had shown more error than Type-1 Sugeno FIS does with above 0.3 in both locations. Meanwhile, the proposed method produced the best results for both locations with the cluster radius value of 0.35 and spread changes values of 0.15 and 0.2.

CHAPTER 5

CONCLUSION AND FUTURE WORKS

This is the final chapter that summarizes the major finding of this research. The findings of the study match the objectives of study and research questions. The contributions of this study are emphasized and future works is proposed.

5.1 Summary

In this thesis, forecasting method for significant wave height parameter is proposed. Type-2 fuzzy set was introduced to solve the limitation of Type-1 fuzzy set in handling uncertainty. Currently, most of fuzzy forecasting technique for wave height forecasting technique is focusing on Type-1 fuzzy only. This proposed method provides forecasted values for significant wave height in 6 hours lead times with wind speed and wind direction as input parameter. All data were selected in two locations, which at latitude 5.35° N, longitude 114.9° E and latitude 5.8° N, longitude 104.15° E.

It is an interesting point of view to use Type-2 fuzzy in forecasting wave height as Type-1 fuzzy has a limitation. Therefore, the first objective had been determined where *to extend Type-1 fuzzy to Type-2 fuzzy by assigning uncertainties to its own Type-1 counterpart*. Four methods were written in Chapter 2 and one of the methods is selected to be used in this study. The spread of changes mention in Chapter 3 and Chapter 4 has created the amount of uncertainties that applied in Type-1 fuzzy. Related to Chapter 4, Results and Discussion, the spread changes with values of 0.15 and 0.2 had shown better results when comparing the proposed method with Type-1 fuzzy. This result is still in the range of suggested value by [53]. The objective is achieved by assigning uncertainties to its own Type-1 cluster center and had answered the research question of “*How to extend Type-1 fuzzy to Type-2 fuzzy?*”

The following objective is *to design and develop simulation of Type-2 fuzzy by using wave and wind data*. This objective is achieved in Chapter 3. The code of extending Type-1 fuzzy to Type-2 fuzzy is implemented with Interval Type-2 Fuzzy toolbox in MATLAB software. The proposed method is using subtractive clustering method to group the data and least square estimation method to optimize the output function parameter. Here, all the pre-initialized parameter in subtractive clustering is set as default except the radius parameter. The results have shown the best results with radius value of 0.35 with the supported by [86].

From the development of proposed method, Type-2 fuzzy was applied in the fuzzification phase where the input had more than one degree of membership function value. With an extra degree of membership function, the Type-2 fuzzy had provided better modeling of uncertainties rather than Type-1 fuzzy. Therefore, the wind parameter with a reliable value is used to forecast the significant wave height. This addresses the research question of *“How Type-2 fuzzy can be used and applied for significant wave height forecasting?”*

The Root Mean Square Error (RMSE) indicator is used to accomplish the final objective which is *to evaluate Type-2 fuzzy in term of accuracy of forecasting result by comparing Type-2 fuzzy with other methods*. The output of proposed method is compared with the original data from dataset. Small RMSE value indicates the method had less error and more accurate as compared to other methods. The performance comparison is applied between proposed method and other techniques. The research question of *“How Type-2 fuzzy can improve significant wave height forecasting accuracy?”* is answered with the significant improvement in the performance evaluation between 2% to 5%.

5.2 Contributions

This study is purposely designed to give some contribution in both of fuzzy and wave height environment itself. There are four contributions highlighted in this study.

1. To investigate the used of Type-2 fuzzy theory in forecasting significant wave height parameter which gives a better handling of uncertainties towards wave parameter compared with Type-1 fuzzy.
2. Give a better accuracy and improvement compared to Type-1 fuzzy method for wave height forecasting in short-term forecasting of 6 hours lead time. The improvement was about 2% to 5% from Type-1 fuzzy method.
3. Based on results, this study has highlighted the suitable uncertainties values to be assigned in Type-1 cluster center are 0.15 and 0.2. These values would produced good result with the condition of radius parameter value was set to 0.35 in the subtractive clustering.
4. It also gives an alternative tool to forecast the significant wave height which can help the civil engineers in analyzing future trend in building offshore infrastructure.

5.3 Future works

This research is to propose a method to forecast significant wave height by using type-2 fuzzy with the input of wind parameter. Even though overall performance of proposed method has shown a little improvement in term of accuracy, however, there is still a room of improvement. Some recommendations for future work in line with research are listed as below:

1. The relationship between wind and wave gives impact towards the accuracy of wave parameter forecasting has been highlighted in literature review in Chapter 2. This study used wind parameters to forecast wave parameter where wind speed and wind direction were involved. Other studies have used different kinds of

parameters as an input to forecast wave parameter such as in [13] that used wind gust, wind speed, air pressure, water temperature and air temperature. [21] study has used same input with proposed method but added up one parameter which is previous significant wave height. Other studies [24, 35] had used other related wind-wave parameter. These parameters can be applied in proposed method in future work.

2. Different methods will bring out different output result. This study has shown result of extension of Type-1 fuzzy to Type-2 fuzzy with the assignment of uncertainties to its own Type-1 counterpart. There are various methods of development in Type-2 fuzzy which were also had been mentioned in Chapter 2, Section 2.2. One of the methods is to use standard error method [61] to calculate and identify the uncertainties instead of try and error method in proposed method. Other than that, [55] was focusing on subtractive clustering development which concentrated on uncertainties of pre-initialized parameter of Type-2 fuzzy development. Applying those methods in future research might give different output towards wave height forecasting method.
3. The characterization of Type-2 fuzzy application is not limited to apply at both antecedent and consequent part of FIS. It can be either one as long as Type-2 fuzzy was applied, the application is considered used Type-2 fuzzy. This study has focused on applying Type-2 fuzzy at antecedent part only and the parameter for consequent part is taken from Type-1 fuzzy process. It would be an interesting method to apply Type-2 fuzzy in both parts in order to examine the accuracy of it.

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