APPENDIX

A. DERIVATION OF SIMPLIFIED GEOMETRIC OPTICS EQUATION

The rate of change of a vector field can be calculated as:

$$\frac{d\vec{A}}{ds} = \frac{dA_x}{ds}\vec{x} + \frac{dA_y}{ds}\vec{y} + \frac{dA_z}{ds}\vec{z}$$

$$= \vec{x}(\frac{\partial A_x}{\partial x}\frac{dx}{ds} + \frac{\partial A_x}{\partial y}\frac{dy}{ds} + \frac{\partial A_x}{\partial z}\frac{dz}{ds})$$

$$+ \vec{y}(\frac{\partial A_y}{\partial y}\frac{dx}{ds} + \frac{\partial A_y}{\partial y}\frac{dy}{ds} + \frac{\partial A_y}{\partial z}\frac{dz}{ds}) \qquad \text{(A.1)}$$

$$+ \vec{y}(\frac{\partial A_y}{\partial y}\frac{dx}{ds} + \frac{\partial A_y}{\partial y}\frac{dy}{ds} + \frac{\partial A_y}{\partial z}\frac{dz}{ds})$$

$$= \vec{x}(\frac{dr}{ds}.\nabla)A_x + \vec{y}(\frac{dr}{ds}.\nabla)A_y + \vec{z}(\frac{dr}{ds}.\nabla)A_z$$

$$= (\frac{dr}{ds}.\nabla)\vec{A}$$

Applying this property of $\frac{d\vec{A}}{ds}$ to calculate the $\frac{dr}{ds}$ derivative Equation 2.3 gives:

$$\frac{d}{ds}(\nabla L) = \frac{d}{ds}\left(n\frac{dr}{ds}\right)$$

$$\frac{d}{ds}\left(n\frac{dr}{ds}\right) = (\frac{dr}{ds}.\nabla)(\nabla L) \qquad \text{this is because } \frac{d\vec{A}}{ds} = (\frac{dr}{ds}.\nabla)\vec{A}$$

$$= \left(\frac{1}{n}(\nabla L).\nabla\right)(\nabla L) \qquad \text{this is because Equation 3.3 states } (\frac{dr}{ds} = \frac{\nabla L}{n})$$

$$= \frac{1}{2n}\nabla[\nabla L.\nabla L]$$

$$= \frac{1}{2n}\nabla[(\nabla L)^2]$$

$$= \frac{1}{2n}\nabla n^2 \qquad \text{based on Equation 3.2 } ((\nabla L)^2 = n^2) \qquad \text{(A.2)}$$

(A.2) gives

$$\frac{d}{ds}\left(n\frac{dr}{ds}\right) = \nabla n \qquad \text{(A.3)}$$

B. PSEUDO-CODE OF THE DEVELOPED DEFLECTION ESTIMATION VECTOR

Prompt the user to load set of calibration image (*Image__set 1)*
Prompt the user to load deformed background image *Image 2*
Compute the mean (*im1_mean)* of *Image1* and
im1_mean of every 10 successive images from *Image 2*
Load low pass filters (*Lo_D*)
Load high pass filters (*Lo_H*)
Load complex wavelet coefficients (*Lo_comp*)
Compute the complex wavelet x derivatives (*Lo_comp_x*)
Compute the complex wavelet y derivatives (*Lo_comp_y*)
Do until image decomposition level is 5
Load *im1_mean and im2_mean*
Compute the next image decomposition level (*im1_mean_L*)
& (*im2_mean_L*)
Median filter *im1_mean_L & im2_mean_L*

Assign Im_mean__l as mean of Im1_mean_L & Im2_mean_L

End do
Display the decomposed images
Calculate *I_psi* by convolving *Im_mean_l* with *Lo_comp*
Calculate *I_psix* by convolving *Im_mean_l* with *Lo_comp_x*
Calculate *I_psiy* by convolving Im*_mean_l* with *Lo_comp_y*
Compute the time derivative *It_psi* of each image level
Generate a diagonal unit matrix *Ident*
Initialize *known_ array* to zero
Input *I_psi, I_psix, I_psiy* and *Ident* into *known_array*
Compute *Real (known_array)*
Compute *Real (It_psi)*

Compute the SVD of *Real (known_array)*
Compute the inverse *(Real (known_array))*
Multiply inverse *(Real (known_array))* with *Real (It_psi)*
Display displacement vectors

C. GLADSTONE-DALE CONSTANT OF SPECIES VARIOUS GASES

Gas	**Gladstone –Dale constant (m3/kg)**
Ar	0.000158
H	0.00153
HO	0.000474
H_2	0.00147
H_2O	0.000316
He	0.000201
CO_2	0.000229
N	0.000253
N_2	0.000237
N_2O	0.000256
NO	0.000226
O	0.000153
O_2	0.000193
Ne	0.000075
Kr	0.000115
Xe	0.000119
CF_4	0.000122
CH_4	0.000617
SF_4	0.000113

Gladstone- Dale constant of air at different wavelengths

Gladstone-Dale Constant (m^3/kg)	**Wavelength (µm)**
0.0002239	0.9125
0.0002250	0.7034
0.0002259	0.6074
0.0002274	0.5097
0.0002304	0.4079

D. INDIVIDUAL GAS CONSTANTS AND SPECIES MOLECULAR WEIGHT

Specific gas Constant is given by the universal gas constant divided by the molar mass (M) of the gas or mixture.

Equation $R_{gas} = \vec{R}/M$ gives the specific gas constant for a particular gas is as follows:

where:

R_{gas} is the specific gas constant

$\vec{R}$ is the universal gas constant

M is the molar mass (or molecular weight) of the gas

The Individual Gas Constant for some common gases are listed below:

Gas	$\overleftrightarrow{R}$ (J/kgK)	**M(g/mole)**
Argon, Ar	208	39.94
Carbon Dioxide, CO_2	188.9	44.01
Carbon Monoxide, CO	297	28.01
Helium, He	2,077	4.003
Hydrogen, H_2	4,124	2.016
Methane - natural gas, CH_4	518.3	16.04
Nitrogen, N_2	296.8	28.02
Oxygen, O_2	259.8	32
Propane, C_3H_8	189	44.09
Sulfur dioxide, SO_2	130	64.07
Air	286.9	28.97
Water vapor	461.5	18.02

E. HOT WIRE ANEMOMETRY AND THERMOCOUPLE SPECIFICATIONS

Testo 425 Hotwire Anemometer Specifications	
Sensor	Thermal hot-wire air velocity, NTC temperature
Measuring range	0 to +4000 fpm, 0 to +20 m/s -4 to 160 °F, -20 to +70°C
Accuracy	±6 fpm +5% of reading ±0.03 m/s +5% of reading ±0.9 °F (32 to +140 °F) ±1.3 °F (remaining range)
Resolution	2 fpm, 0.01 m/s, 0.1 °F
Operating Temperature	-4 to 122 °F, -20 to +50°C
Weight	10 oz, 285 grams
Dimensions (LxWxH)	7 x 2.5 x 1.6 inches, 182 x 64 x 40 mm
Battery	9 volt
Battery Life	20 hours, typical

Thermocouple specifications		
Calibration type		K
conductors	Positive	Chromel (non magnetic)
	Negative	Alume l (magnetic)
Temperature range ^{o}C		$0^{o}C$ to$1250^{o}C$
Standard limits of error		$\pm 2.2^{o}C$ 0.75%
Response time		0.15 sec

F. CHEMICAL SPECIES CREATED DURING METHANE BURNING

Chemical species created
H_2 H O O_2 OH H_2O HO_2 H_2O_2 C CH CH_2 $CH_2(S)$ CH_3 CH_4
CO HCO CH_2O CH_2OH CH_3O CH_3OH C_2H C_2H_2 C_2H_3 C_2H_4
C_2H_5 C_2H_6 HCCO CH_2CO HCCOH N NH NH_2 NH_3 NNH NO
NO_2 N_2O HNO CN HCN H_2CN HCNN HCNO HOCN HNCO NCO
N_2 AR C_3H_7 C_3H_8 CH_2CHO CH_3 CHO C (graphite)