ABSTRACT

Gantry crane system, a non-slewing-luffing crane system is most widely used in many work places. However, the heavier lifting capacities and the greater size of gantry crane, the vibrational motion become more significant during crane operations and it must be considered. The equations of motion of the system can be obtained by modeling the crane framework using finite element in conjuction with moving finite element method and gantry crane by using Lagrange's equations. The combinational direct integration technique, namely Newmark-β and fourth-order Runge-Kutta method is proposed to solve the coupled equations of motion.

Numerical simulation results show that the combination of flexibility of crane framework and hoist cable produces greater amplitudes and lower swing angles frequency compared to the gantry crane system with flexible hoist cable or crane framework only with respect to the rigid model. Furthermore, all the flexible models of gantry crane system have lower frequencies in the time histories of swing angles of payload with respect to the rigid model for all the parametric studies. The trends of maximum displacements of crane framework and hoist cable increase with the increase of payload mass and initial swing angle of payload. The increases are slightly linear for payload mass and nonlinear for initial swing angle of payload. Under the increase of structural damping, hoist cable stiffness, cross-sectional dimensions of crane framework and hoist cable length, the trends decrease for all the maximum displacements.

Control simulations clearly demonstrate that Zero-Vibration-Derivative-Derivative (ZVDD), Fuzzy Logic Controller (FLC) and Proportional-Integral-Derivative (PID) controllers have rough fluctuations in controlling flexible gantry crane with respect to their performances in controlling the rigid model of gantry crane. Compared to FLC and PID, ZVDD has larger steady state error.

ABSTRAK

Kren gantry, sistem kren yang *non-slewing-luffing* banyak digunakan dalam banyak tempat. Namun bagaimanapun, kapasiti yang semakin berat dan peningkatan ukuran kren *gantry* yang semakin besar menjadikan efek getaran semakin penting dan patut diambil kira semasa pengoperasian kren. Persamaan gerak dari pada sistem semacam itu diperolehi dengan menggunakan kaedah elemen hingga selaras dengan elemen hingga bergerak dan persamaan *Lagrage*. Persamaan gerak membentuk gandengan tidak linear diantara kren *framework* dan kren *gantry*. Teknik integrasi gabungan langsung, iaitu kaedah Newmark-β dan orde ke-empat Runge-Kutta diajukan untuk kemudian menyelesaikan persamaan gerak yang tergandeng.

Keputusan simulasi numerikal menunjukkan bahwasanya gabungan keanjalan kren *framework* dan tali *hoist* menghasilkan amplitud yang lebih besar dan frekuensi ayunan yang lebih rendah dibandingkan dengan model tegarnya. Tambahan lagi, keseluruhan sistem kren *gantry* yang anjal mempunyai frekuensi ayunan yang lebih rendah dibandingkan dengan model tegapnya untuk keseluruhan kajian parametrik. Kecenderungan perpindahan maksimum dari kren *framework* dan tali *hoist* meningkat selaras dengan kenaikan berat *payload* dan sudut awal ayunan *payload*. Kenaikannya adalah cukup *linear* untuk berat *payload* dan *nonlinear* untuk sudut awal ayunan *payload*. Selaras dengan kenaikan redaman struktur, kekakuan tali *hoist*, dimensi penampang kren *framework* dan panjang tali *hoist*, kecenderungannya menurun untuk semua perpindahan-perpindahan kren *framework*.

Hasil yang diperolehi daripada simulasi-simulasi kawalan dengan jelas menunjukkan bahwasanya teknik kawalan Zero-Vibration-Derivative-Derivative (ZVDD), Fuzzy Logic Controller (FLC) dan Proportional-Integral-Derivative (PID) memiliki fluktuasi yg kasar dalam mengawal kren gantry yang anjal. ZVDD mempunyai kesalahan keadaan stedi yang lebih besar dibandingkan dengan FLC dan PID.

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LIST OF SYMBOLS

$[M_{st}]$, $[C_{st}]$, $[K_{st}]$	Mass, damping and stiffness matrices of the crane framework
$\{q_{st}\},\{\dot{q}_{st}\}\{\ddot{q}_{st}\}$	Acceleration, velocity and displacement vectors for crane framework
$\{N_k\}$	Shape functions of space frame
x_s, ℓ_X	Local position of moving load, beam element length,
d_{si}	Displacements for the nodes of the space frame element
f_{0x}, f_{0y}, f_{0z}	Corresponding external force components in the x, y and z direction
μ	Membership function
$\otimes$, $\oplus$	T-norm, T-conorm operation
K_p, K_i, K_d	Gains of proportional, integral and derivative
$\theta,\dot{\theta},\ddot{\theta}$	Angle between the x_T-axis and $x_T y_T$-plane and its derivative
$\varphi,\dot{\varphi},\ddot{\varphi}$	Angle between the cable to $x_T y_T$-plane and its derivative: see Figure 3.1.
f_x, f_y, f_z	Applied input force for the *x*, *y* and *z* motions
r_T, r_p	The position vector of trolley and payload
ℓ, ℓ_p	Stretched length, unstretched length of hoist cable
k	Cable stiffness
$\delta,\dot{\delta},\dot{\delta}$	Hoist cable displacement and its derivative
K_T, K_P	Kinetics energy of the trolley and payload
P_T, P_P, P_K	Potential energy of the trolley, payload and hoist cable
m_T, m_P, g	Mass of the trolley, payload and the acceleration of gravity

u_T, v_T, w_T	Displacements of crane framework at position x_T and time t
x_T, $\dot{x}_T$, $\ddot{x}_T$	Position, velocity and acceleration of trolley
$[M]$, $[C]$, $[K]$	Total mass, damping and stiffness matrices of gantry crane system
ζ_m, ζ_n	Specific damping ratio
ω_m, ω_n	Specific natural frequency
β, γ	The two parameters of Newmark method
Δt	Time interval
a_i	Newmark's parameters
$\{\overline{K}\}$, $\{\overline{F}\}$	The effective stiffness matrix and the effective load vector
f_p, f_{MSD}	Natural frequency of the pendulum and MSD system
y_0	Initial displacement of MSD system
e_{x_T}, ∇e_{x_T}	The error of trolley position and its derivative
e_θ, ∇e_θ	The error of swing angle and its derivative
u_{x_T}, u_θ	Controller output for trolley and swing angle