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UNIVERSITI TEKNOLOGI PETRONAS  
RECURSIVE LEARNING ALGORITHM ON RBF NETWORKS FOR  
NONLINEAR SYSTEM IDENTIFICATION

by

NUR AFNY CATUR ANDRYANI

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RECURSIVE LEARNING ALGORITHMS ON RBF NETWORKS FOR  
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A Thesis

Submitted to the Postgraduate Studies Program as A requirement for The Degree of  
Master of Science in Electrical and Electronics Engineering

MASTER OF SCIENCE

ELECTRICAL AND ELECTRONICS ENGINEERING

UNIVERSITI TEKNOLOGI PETRONAS

BANDAR SERI ISKANDAR,

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April 2010

## DECLARATION OF THESIS

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**Recursive Learning Algorithms on RBF Network  
for Nonlinear System Identification**

I NUR AFNY CATUR ANDRYANI

hereby declare that the thesis is based on my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at UTP or other institutions.

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## ACKNOWLEDGMENTS

First and foremost, all praises and thanks are due to Allah, the almighty God, the source of life and hope for giving me the strength and wisdom to succeed in life.

My very special thanks go to my supervisor Dr. Vijanth S. Asirvadam, his support, encouragement, fully assistance, and unwavering patience rescued me from frustration and guided me throughout my research. Without his excellent guidance it would have been impossible to finish this work. His really nice personality makes me feel comfort to do the research work under his supervision. His very positive influence on my professional development will be carried forward into my future career.

I also would like to thank to Dr Nor Hisham Hamid for his co-supervision. His guidance as head department in Electrical and Electronics Engineering department is really worth for me throughout my master degree study.

A million thanks also for my husband and my big family for their endless support and for giving courage and strength to never give up even though we've been apart. They provided me sturdy motivation and strong emotional and moral support which I have needed throughout my life; they are always close to my heart.

Special thanks extended to all my best friends at Electrical and Electronic Engineering Department especially for 22-01-05 team and my entire Indonesian best friends. They are not only provided an excellent study environment but also a lot of great moments outside.

At the end, I would like to thank to all my Indonesian mates for their consistent support, help, and encouragement.

## ABSTRACT

Science and technology development has the tendency of learning from nature where human also try to develop artificial intelligent by imitating biological neuron network which is popularly termed Artificial Neural Network (ANN). It represents an interconnection among neuron which consists of several adjustable parameters which are tuned using a set of learning examples to obtain the desired function represent the actual system.

Radial Basis Function (RBF) networks, one of feed forward artificial neural network architecture, have recently been given much attention due to its good generalization ability. The RBF network is popular among scientist and engineer and used in a number of wide ranging signal and control applications which includes the area of system identification or estimation.

The learning approach, a process which updated the parameters of RBF networks, will be the most important issue in neural computing research communities. The learning method will determine the performance's capability of the networks for the system identification process which will be one of the key issues to be discussed in the thesis.

This thesis proposes derivative free learning, using finite difference, methods for fixed size RBF network in comparison to gradient based learning for the application of system identification. The thesis also try to investigate the influence of initialization of RBF weights parameters on the overall learning performance using random method and advanced unsupervised learning, such as clustering techniques, as a comparison. By taking advantage of localized Gaussian basis function of RBF network, a decomposed version of learning method using finite difference (or

derivative free) gradient estimate has been proposed in order to reduce memory requirement for the computation of the weight updates.

The proposed training algorithms discussed in this thesis are derived for fixed size RBF network and being compared with Extreme Learning Machine (ELM) as the ELM technique just randomly assigned centers and width of the hidden neurons and update the output connected weights. The proposed methods are tested using well known nonlinear benchmark problems and also evaluated for system with irregular sample time or known as *lost packets*. The finite difference based gradient estimate, proposed in this thesis, provides a viable solution only for identifying a system with irregular sample time.

## ABSTRAK

Perkembangan sains dan teknologi telah membawa kepada kecenderungan manusia untuk mempelajari daripada alam sejagat untuk mengembangkan kecerdikan buatan dengan meniru kepada rangkaian biologi neuro dimana lebih popular dikenali sebagai “rangkaian neural buatan”. Ianya mewakili rangkaian-rangkaian neuro yang mana terdiri daripada beberapa pembolehubah yang boleh diubah suai menggunakan set-set pembelajaran contoh untuk memperolehi fungsi tertentu yang mana ianya mewakili sistem yang sebenar.

Rangkaian Fungsi Asas Radial, adalah salah satu daripada senibina rangkaian saraf tiruan yang mana ianya mempunyai daya pengitlakan yang tinggi. Rangkaian Fungsi Asas Radial merupakan salah satu daripada senibina yang popular dikalangan para saintis dan jurutera dan digunakan secara meluas didalam bidang aplikasi isyarat dan kawalan.

Pendekatan pembelajaran ialah proses dimana pembolehubah dikemaskini didalam rangkaian Rangkaian Fungsi Asas Radial menjadi isu terpenting didalam komuniti kajian saraf perkomputeran. Pendekatan pembelajaran akan menentukan kebolehan prestasi sesuatu rangkaian didalam proses pengenalan sistem yang mana menjadi salah satu daripada aspek terpenting yang dibincangkan didalam tesis ini.

Tesis ini mencadangkan pembelajaran turunan bebas menggunakan kaedah perbezaan terhingga untuk Rangkaian Fungsi Asas Radial bersaiz tetap berbanding dengan menggunakan kaedah pembelajaran berdasarkan kecerunan bagi aplikasi pengenalan sistem. Tesis ini juga mencadangkan penyiasatan yang dipengaruhi oleh pemulaan pembolehubah Rangkaian Fungsi Asas Radial kepada keseluruhan prestasi pembelajaran menggunakan keadah rawak dan lanjutan pembelajaran tak yang diawasi seperti teknik kekelompokan. Dengan mengambil kelebihan daripada Rangkaian



Fungsi Asas Radial Gaussian, kaedah pembelajaran menggunakan kaedah perbezaan terhingga daripada versi terurai, kaedah anggaran kecerunan dicadangkan untuk mengurangkan keperluan memori untuk mengemaskini pembolehubah – pembolehubah.

Kaedah pembelajaran yang dicadangkan di dalam tesis ini adalah berasal daripada Rangkaian Fungsi Asas Radial bersaiz tetap dan ianya dibandingkan dengan “*Extreme Learning Machine*”. Kaedah yang dicadangkan didalam tesis ini diuji menggunakan masalah tidak linier dan dinilai untuk sistem yang tak tersusun sampel waktu atau dikenali sebagai “*lost packets*”. Bagaimanapun kaedah perbezaan terhingga berdasarkan anggaran kecerunan dicadangkan didalam tesis ini hanya sesuai digunakan sebagai penyelesaian kepada sistem dengan tak tersusun sampel waktu.

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## LIST OF ABBREVIATIONS

ANN	Artificial Neural Networks
RBF	Radial Basis Function
MLP	Multi Layer Preceptor
RBF Network	Radial Basis Function Neural Networks
SISO	Single Input Single Output
ELM	Extreme Learning Machine
RPE	Recursive Prediction Error
PDP	Probability Drop Pocket
MRAN	Minimal Recourse Allocation Network
RAN	Recourse Allocation Network
ELM-Kmeans	Extreme Learning Machine with K means clustering initialization
ELM-Fuzzy-C means	Extreme Learning Machine with Fuzzy C-means clustering initialization
FD-RPE-Rand	Finite Difference RPE with Random Initialization
FD-dRPE-Rand	Finite Difference RPE with Random Initialization on decomposed RBF



FD-RPE-Kmeans	Finite Difference RPE with K-means clustering Initialization
FD-dRPE-Kmeans	Finite Difference RPE with K-means clustering Initialization on decomposed RBF
FD-RPE-Fuzzy-C means	Finite Difference RPE with Fuzzy C-means clustering Initialization
FD-dRPE-Fuzzy-C means	Finite Difference RPE with Fuzzy C-means clustering Initialization on decomposed RBF
MSE	Mean Square Error

## NOMENCLATURE

$\forall$	Universal quantification ( for all )
$\exists$	Existential quantification (there exist)
$\rightarrow$	Proportional logic for material implication ( if ..then)
$x$	Input data
$c$	Center (nonlinear learning parameter)
$\sigma$	Width (nonlinear learning parameter)
$\Theta$	Vector of nonlinear learning parameters $[c, \sigma]$
$\beta$	Height (linear Learning parameter)
$G(c_i, \sigma_i, x)$	Gaussian (the activation function)
$\hat{y}(t, c, \sigma, \beta, x)$	Prediction of output
$e$	error
$u(k)$	Input of the system
$y(k)$	Output of the system
$P$	Covariance matrix
$U$	Matrix Unitary
$\  \ $	magnitude

$\nabla c$	Gradient for output prediction subject to center ( $c$ )
$\nabla \sigma$	Gradient for output prediction subject to width ( $\sigma$ )
$\nabla \hat{y}(\Theta)$	Gradient for output prediction subject to center ( $c$ ) and width ( $\sigma$ )
$\nabla P$	Search direction for RPE algorithm
$F(\Theta)$	Objective function of the optimization process (learning process)
$H(\Theta)$	Hessian matrix of the objective function subject to $\Theta$
$\gamma_m$	Momentum
$\gamma_g$	Adaptive gain
$h$	Step size of the finite difference
$N(\Theta^*, r)$	Neighborhood $N$ with center $\Theta^*$ , radius $r$
	$\forall x \in \mathfrak{R}, \exists \Theta^*, \exists r, \&  x - \Theta^*  < r \rightarrow x \in N(\Theta^*, r)$
$\mathfrak{R}$	Set of real number