APPENDIX

B

IMPLEMENTATION OF FFT USING DISCRETE WAVELET PACKET TRANSFORM (DWPT) AND IT'S APPLICATION TO SNR ESTIMATION IN OFDM SYSTEMS

B.1 Introduction

In this chapter, wavelet packet based FFT and its application to SNR estimation is reported. OFDM systems demodulate data using FFT. The proposed solution computes the exact result, and its computational complexity is same order of FFT, i.e. O ($N\log_2 N$). SNR estimation is done inside wavelet packet based FFT block unlike previous SNR estimations techniques which perform SNR estimation after FFT. Wavelet packet analyzed data is used to perform SNR estimation. The proposed estimator takes into consideration the different noise power levels over the OFDM sub-carriers. The OFDM band is divided into several sub-bands using wavelet packet and noise in each sub-band is considered white. The second-order statistics of the transmitted OFDM preamble are calculated in each sub-band and the power noise is estimated. The proposed estimator is compared with Reddy's estimator for colored noise in terms of mean squared error (MSE).

Signal-to-noise ratio (SNR) is defined as the ratio of the desired signal power to the noise power. Noise variance and hence SNR estimates of the received signal are very important parameters for the channel quality control in communication systems. The search for a good SNR estimation technique is motivated by the fact that various algorithms require knowledge of the SNR for optimal performance [1, 2, 3, 4, 5]. For instance, in OFDM systems, SNR estimation is used for power control, adaptive coding and modulation, turbo decoding etc.

SNR estimation indicates the reliability of the link between the transmitter and receiver. In adaptive system, SNR estimation is commonly used for measuring the quality of the channel and accordingly changing the system parameters [6]. For example, if the measured channel quality is low, the transmitter may add some redundancy or complexity to the information bits (more powerful coding), or reduce the modulation level (better Euclidean distance), or increase the spreading rate (longer spreading code) for lower data rate transmission. Therefore, instead of implementing fixed information rate for all levels of channel quality, variable rates of information transfer can be used to maximize system resource utilization with high quality of user experience.

Many SNR estimation algorithms have been suggested in the last ten years as discussed in chapter two of this thesis and successfully implemented in OFDM systems using the system pilot symbols. The essential requirement for an SNR estimator in OFDM system is of low computational load. This is in order to minimize hardware complexity as well as the computational time.

In many SNR estimation techniques, noise is assumed to be uncorrelated or white. But, in wireless communication systems, where noise is mainly caused by a strong interferer, noise is colored in nature.

OFDM demodulation uses discrete Fourier transform (DFT). An FFT (fast Fourier transform) is used to demodulate data. In this appendix, wavelet packet based FFT and its application to SNR estimation is presented. The proposed solution computes the exact result, and its computational complexity is same order of FFT, i.e. $O\ (N\log_2 N)$.

The proposed SNR technique performs SNR estimation inside FFT unlike previous SNR estimators. SNR estimator for the colored noise in OFDM system is proposed. The algorithm is based on the two identical halves property of time synchronization preamble used in some OFDM systems. The OFDM band is divided into several sub-bands using wavelet packet and noise in each sub-band is considered white. The second-order statistics of the transmitted OFDM preamble are calculated in each sub-band and the power noise is estimated. Therefore, the proposed approach estimates both local (within smaller sets of subcarriers) and global (over all sub-carriers) SNR values. The short term local estimates calculate the noise power variation across OFDM sub-carriers. When the noise is white, the proposed algorithm works as good as the conventional noise power estimation schemes, showing the generality of the proposed method.

The remainder of the appendix is organized as follows. In Section B.2, the proposed FFT technique is presented. Section B.3 provides the proposed SNR estimation. Section B.4 presents simulation results and discussion. Section B.5 concludes the appendix**.**

B.2. Proposed Wavelet Packet Based FFT (DWPT-FFT)

The fundamental principle that the FFT is based upon is that of decomposing the computation of the discrete Fourier transform of a sequence of length *N* into successively smaller discrete Fourier transforms of the even and odd parts. In the proposed method the even – odd separation is replaced by wavelet packet decomposition.

The block diagram of proposed DWPT-FFT is shown in Fig.1. The idea borrowed from Guo [7]. Wavelet packet based FFT first performs Wavelet Packet decomposition, followed by reduced size FFT and butterfly operation as shown in Fig.B.1. This can be extended so that WP analysis is 3 to 4 level analysis and FFT is N/4, N/8 or N/16 size FFT. Butterfly is appropriately designed.

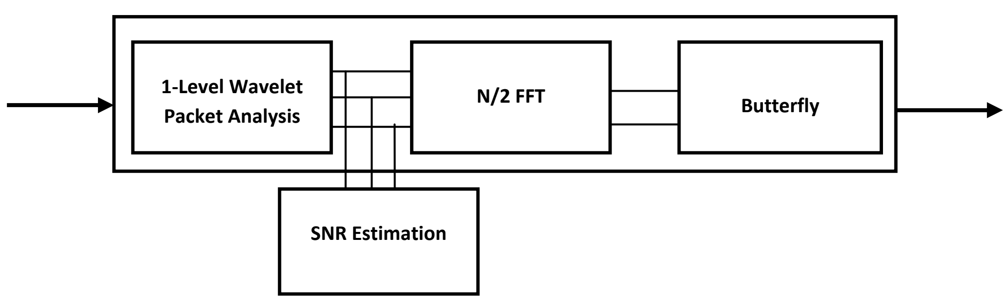
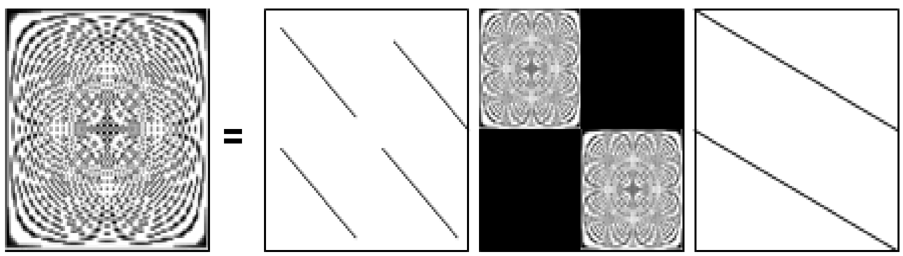


Fig.B.1 Block Diagram of DWPT-FFT

The wavelet packet based FFT (DWPT-FFT) shown in Fig B.1 is represented by eq. B.1.

$$F_N = \begin{bmatrix} A_{N/2} & B_{N/2} \\ C_{N/2} & D_{N/2} \end{bmatrix} \begin{bmatrix} F_{N/2} & 0 \\ 0 & F_{N/2} \end{bmatrix} [WP_N] \qquad (B.1)$$

where $A_{N/2}$,$B_{N/2}$,$C_{N/2}$ and $D_{N/2}$ are all diagonal matrices. In eq. B.1, the values on the diagonal of $A_{N/2}$ and $C_{N/2}$ are the length-N DFT of '*h,* ' and the values on the diagonal of $B_{N/2}$ and $D_{N/2}$ are the length-N DFT of '*g*. The factorization can be visualized as



where we image the real part of DFT matrices, and the magnitude of the matrices for butterfly operations and the one-scale DWPT using Db3 wavelets. Clearly we can see that the twiddle factors have non-unit magnitude.

The above factorization suggests a DWPT-FFT algorithm. The block Diagram of length 8 algorithm is shown in Fig.B.2. Following this, the high pass and the low pass DWPT outputs go through separate length-4 DFT, then they are combined with butterfly operations.

Same procedure in Fig.B.2 is iteratively applied to short length DFTs to get the full DWPT based FFT algorithm where the twiddle factors are the frequency wavelet filters. The detail of butterfly operations is shown in Fig.B.3 where '*i*' belongs to {0,1,….N/2-1}.

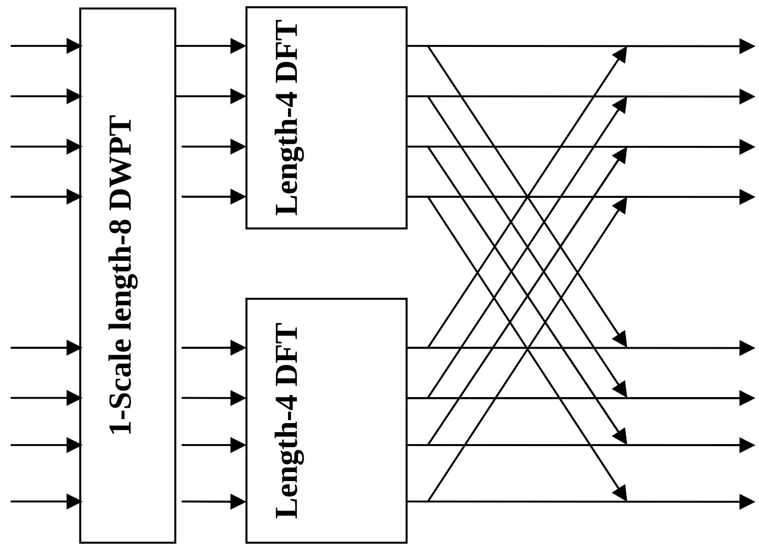


Fig.B.2. Last stage of length 8 DWPT-FFT

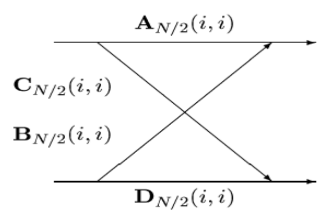
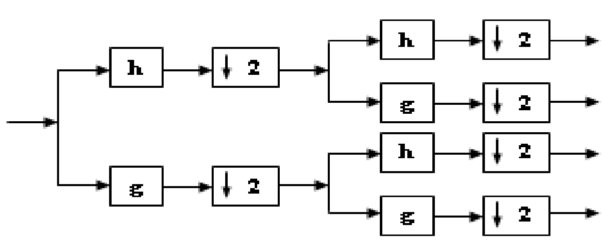


Fig.B.3.Butterfly operation in DWPT-FFT

Wavelet packet allows a finer and adjustable resolution of frequencies at high frequency. Input data are first filtered by pair of filters **h** and **g** (low pass and high pass respectively) and then down sampled. The same analysis is further iterated on both low and high frequency bands as shown in Fig.B.4.



FigB.4. Two-scale discrete wavelet packet transforms

For the DWPT based FFT algorithm, the computational complexity is also $O(N$ log, $\boldsymbol{N})$ **.** However, the constant appears before $\boldsymbol{N} \log_2 N$ depends on the wavelet filters used.

B.3. SNR Estimation inside FFT Block

As discussed in chapter 3, the preamble used for timing synchronization is derived from alternate loading of subcarriers with PN-sequence modulated constellation as follows:

$$P_{even}(k) = \begin{cases} \sqrt{2}\,.P(m) & k = 2m \qquad m = 1,2,3.........N/2 \\ 0 & otherwise \end{cases} \tag{B.2}$$

Here, $P(m)$ is the PN sequence loaded onto even subcarriers taken from IEEE802.16d. The factor $\sqrt{2}$ is related to the 3 dB boost and k shows the sub-carriers index.

In actual practice, an OFDM signal is provided with a guard band on either side of its spectrum. Accordingly the data are not loaded on the sides. For example, for a typical IEEE802.16d signal of length 256 subcarriers vide, 28 carriers on either side are null carriers as shown in Fig.B.5.

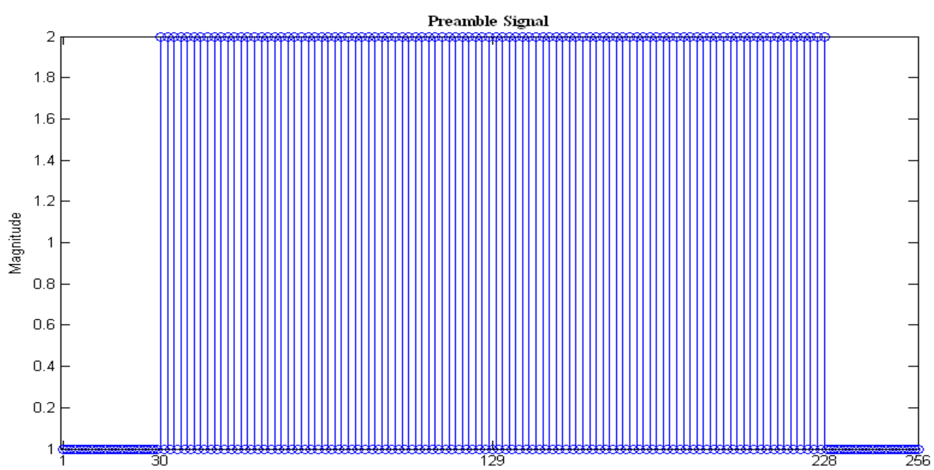


Fig.B.5 Preamble signal loaded on even subcarriers using PN sequence

Therefore for our purposes, eq.B.2 is rewritten as

$$P_{even}(k) = \begin{cases} \sqrt{2}.P(m) & k=2m \quad m=15,16,17.........(N/2-14) \\ 0 & m=1,2,3......14 \\ 0 & m=N/2-13,N/2-12,.....N/2 \end{cases} \qquad \text{(B.3)}$$

The corresponding time-domain preamble $P(n)$, is obtained by Inverse discrete Fourier transform (IDFT) of $P_{even}(k)$ as follows.

$$\begin{aligned} p(n) &= IDFT\{P_{even}(k)\} \\ &= \sum_{k=0}^{N-1} P_{even}(k).e^{j2\pi nk/N} \qquad 0 \le n \le N-1 \end{aligned} \qquad \text{(B.4)}$$

Since $P_{even}(k)$ has values only at even subcarriers, this can be seen from the properties of $e^{j2\pi nm/N/2}$ (also written as $W_{N/2}^{-nm}$, where W_N is the *N-th* root of unity).

For $k = 2m$,

$$e^{j2\pi n2m/N} = e^{j2\pi nm/N/2} \qquad \text{(B.5)}$$

So, for $n = n + N/2$,

$$\begin{aligned} e^{j2\pi(n+N/2)m/N/2} &= e^{j2\pi nm/N/2}.e^{j2\pi m.N/2/N/2} \\ &= e^{j2\pi nm/N/2} \end{aligned} \qquad \text{(B.6)}$$

In other words

$$p(n) = p(n + N/2) \tag{B.7}$$

To avoid intersymbol interference (ISI) caused by multipath fading channels, cyclic prefix (CP) of length l_{CP} is added so that the total length of OFDM data becomes $N_{total} = N + l_{CP}$. It is assumed that the signal is transmitted over Rayleigh multipath fading channel characterized by

$$h(t,\tau) = \sum_{l=1}^{L} h_l(t)\,\delta(t - \tau_l) \tag{B.8}$$

where $h_l(t)$ are the different path complex gains, τ_l are different path time delays, and L is the number of paths. $h_l(t)$ are wide-sense stationary (WSS) narrow-band complex Gaussian processes. At the receiver side, with the assumption that the guard interval duration is longer than the channel maximum excess delay, the received OFDM data can be represented by

$$y(n) = x(n) + n(n) \tag{B.9}$$

where

$x(n) = s(n) * h(n)$

$*$ = Linear convolution

$s(n) = IDFT\ \{S(K)\}$, $S(K)$ are the constellation symbols, and $S(n)$ is the transmitted signal in time-domain.

$n(n)$ = white Gaussian noise with variance σ^2.

$h(n)$ = discretized version of impulse response of the system.

B.3.1 Autocorrelation based SNR Estimator

Wavelet Packet analyzed data becomes available for SNR estimation inside FFT block as shown in Fig.1. After removing cyclic prefix at receiver, OFDM data is divided into 2^n

sub-bands using periodic wavelet packets where '*n*' shows the number of levels. The length of each sub-band is $N_{sub}=N/2^n$. It inherits the two identical halves property of synchronization preamble. The noise in each sub-band is considered white. The system's parameters and the structure of wavelet packet used for the simulations are shown in table B.1. It makes use of two identical halves property of time synchronization preamble and relies on the autocorrelation of the same. From eq. B.10, it can be shown that the autocorrelation function of the received signal, $R_{yy}(m)$, has the following relationship to the autocorrelation of the data signal, $R_{xx}(m)$ and the noise, $R_{nn}(m)$:

$$R_{yy}(m) = R_{xx}(m) + R_{nn}(m) \tag{B.10}$$

where

$$R_{yy}(m) = \sum_{n} y(n)\ y^*(n+m)$$

$$R_{xx}(m) = \sum_{n} x(n)\ x^*(n+m)$$

$$R_{nn}(m) = \sum_{n} n(n)\ n^*(n+m)$$

The noise in the channel is modeled as additive white Gaussian noise and its autocorrelation function only has a value at a delay of $m = 0$, with magnitude given by the noise variance (σ^2), expressed as

$$R_{nn}(m) = \sigma^2 \delta(m) \tag{B.11}$$

where $\delta(m)$ is the discreet delta sequence.

B.3.2 Signal Power and Noise Power Estimation

We undertake the study of OFDM signal statistics, and observe, as shown in Fig. B.6, that its power spectrum is nearly white. Hence its autocorrelation is generally given by:

$$R_{ss}(m) = P_o \delta(m) \tag{B.12}$$

where P_o is signal power.

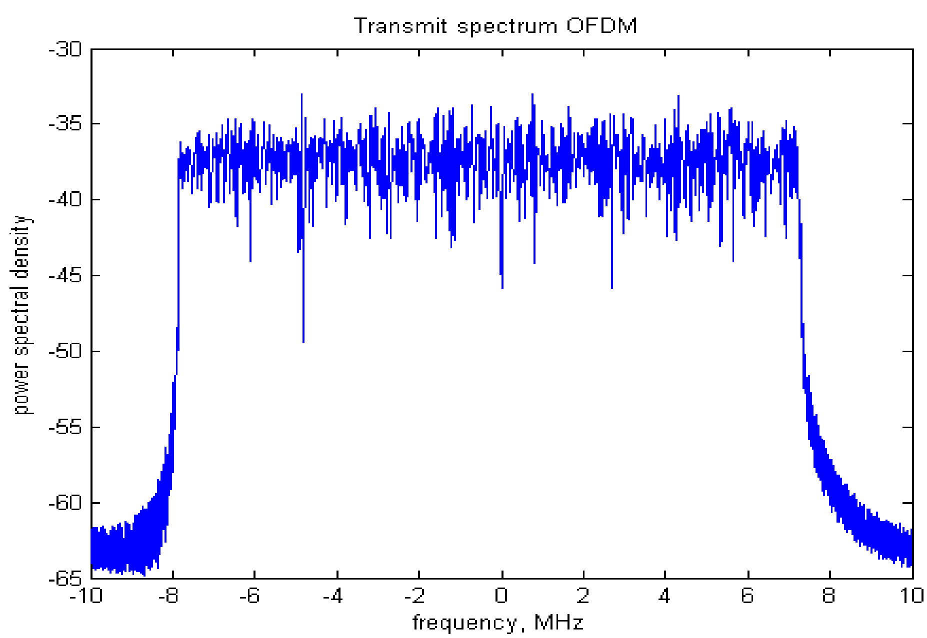


Fig.B.6 Power spectrum of an OFDM signal

Hence, at zero lag (shown at '*L*' in Fig.B.8) the autocorrelation $R_{xx}(0)$ contains both the signal power estimate and noise power estimate indistinguishable from each other as shown in eq. B.10 and eq. B.11 before. However, because of the identical halves nature of the preamble, the received signal power can be estimated from auto correlation peak at N/2 or at –N/2 as shown in Fig.B.7a. In Fig.B.7, $R_{xx}(m)$ has been sketched for $N=256$.

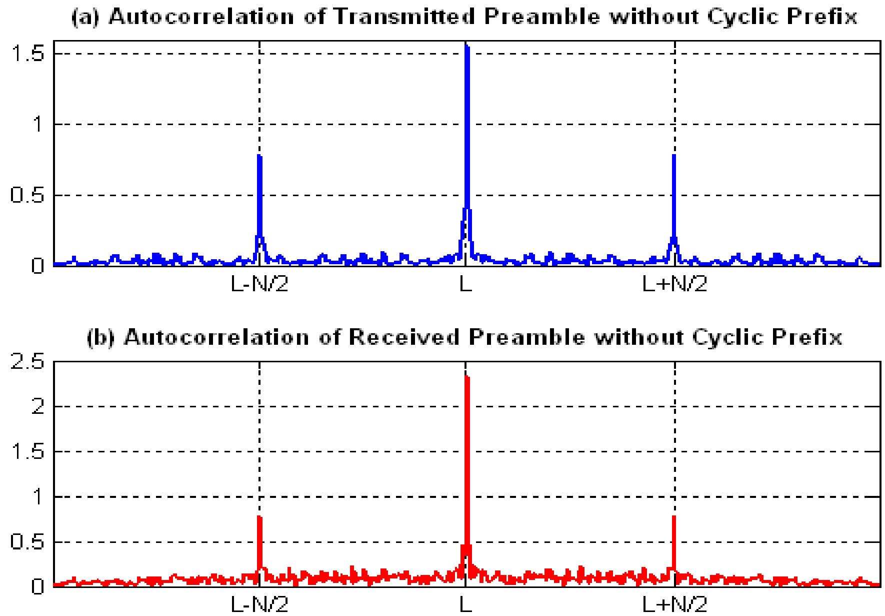


Fig.B.7 (a): Transmitted Preamble (b): Received Preamble after coming through a channel (Plots show two identical halves with no cyclic prefix).

It is clear that the autocorrelation values apart from the zero-offset are unaffected by the channel effects, so one can find the signal power from the *N/2* or *–N/2* lag autocorrelation value.

B.3.2.1 Signal Power Estimation in each Sub-band

Sub-bands inherit the two identical halves property of synchronization preamble as shown in Fig B.8. After removing CP, WP data is available inside FFT for SNR estimation; the length of data is changed. So after correlation of each sub-band, first peak rises at at L_{sub}-$N_{sub}/2$ when one half of sub-band matches with itself with energy of $\rho.(N_{sub}/2)$ and main peak at zero-lag (L_{sub}) rises when full sub-band matches with itself with energy of $\rho.(N_{sub})$

Taking into consideration the autocorrelation values for L_{sub}-$N_{sub}/2$ lag or $L_{sub}+N_{sub}/2$ as shown in Fig.B.8 , signal power is given as

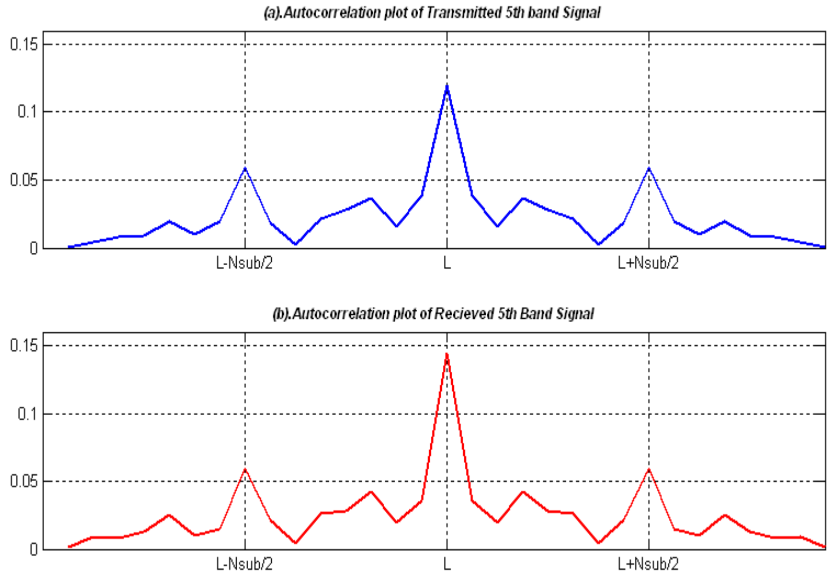


Fig.B.8.Autocorrelation plot of Transmitted (a) and Received (b) 5th band signal

$$\hat{P}_{ss} = 2R_{yy}\left((L - N_{sub} / 2)\right) \tag{B.21}$$

Or

$$\hat{P}_{ss} = 2R_{yy}\left((L + N_{sub} / 2)\right) \tag{B.22}$$

B.3.2.2 Noise Power Estimation in each Sub-band

Having obtained the power of signal in certain sub-band, noise power can be calculated using eq.B.23.

$$\hat{\sigma}^2 = R_{yy}(L_{sub}) - \hat{P}_{ss} \tag{B.23}$$

Where $\hat{\sigma}^2$ is the estimated value of noise power in each sub-band.

B.3.2.3 SNR Estimation in Each Sub-band

Finally we can find the SNR estimates in the sub-band by using equation (B.21 or B.22) and equation (B.23).

$$S\hat{N}R = \frac{\hat{P}_{ss}}{\hat{\sigma}^{2}} \tag{B.24}$$

Where $S\hat{N}R$ is the estimated value for SNR in each sub-band.

B.4. Results and Discussions

For the DWPT based FFT algorithm, the computational complexity is also $O\,(N \log_2 N)$. However, the constant appears before $N \log_2 N$ depends on the wavelet filters used. The proposed DWPT-FFT computes the exact result, for example, for $x(n) = e^{j\pi n^2 / N}$, chirp signal $X(k) = \sum_n x(n) e^{-j2\pi nk / N}$ as shown in Fig.B.9.

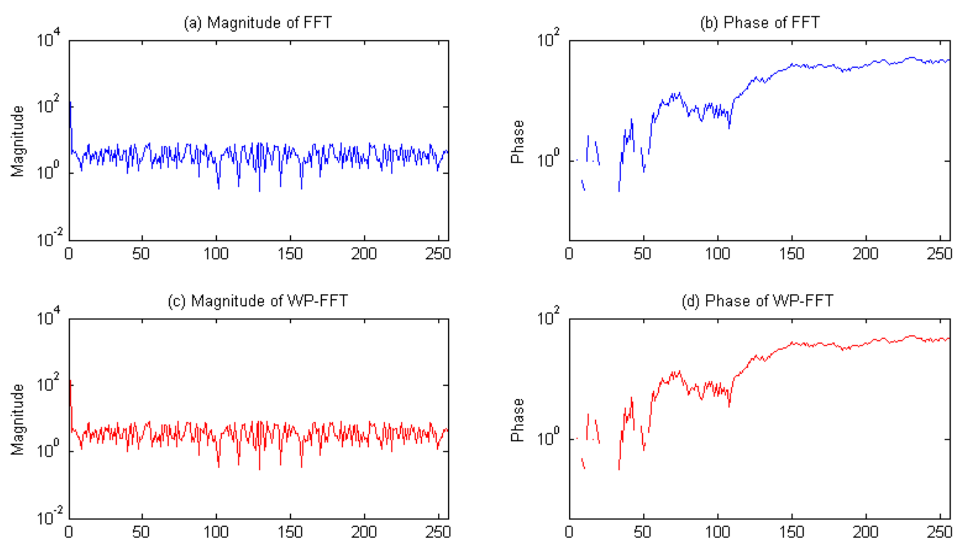


Fig.B.9. FFT result with and without wavelet packet

The proposed SNR estimator is compared with Reddy's estimator for colored noise in OFDM systems with parameters given in Table B.1. SNR is varied from 1 dB to 14 dB for each sub-band and to measure statistically accurate SNR, the mean-squared error (MSE) is obtained over 2000 samples according to the following formula

$$MSE = \frac{1}{2000} \sum_{i=1}^{2000} (\hat{SNR}(i) - SNR)^2 \tag{B.25}$$

B.4.1 Performance Comparison

From Fig.B.10 and Fig.B.11 it is clear that the proposed estimator gives better performance in SNR estimation as compared to Reddy estimator. It is observed that the proposed technique can estimate local statistics of the noise power when the noise is colored. The proposed estimator has relatively low computational complexity because it makes use of only one OFDM preamble signal to find the SNR estimates. The proposed estimator is fulfills the criteria of best SNR estimator because it is unbiased (or exhibits the smallest Bias) and has the smallest variance of SNR estimates as shown from results clearly.

Table B.1: Parameters for the simulation [9]

Ifft size	256
Sampling Frequency = F_s	$20 MHz.$
Sub Carrier Spacing= $\Delta f = F_s / Ifft$	1×10^5
Useful Symbol Time = $T_b = 1/\Delta f$	1×10^{-5}
CP Time $= T_g = G * T_b$ *where* $G = 1/4$	2.5×10^{-6}
OFDM Symbol Time $= T_s = T_b + T_g$	1.25×10^{-5}
$T_s = 5/4 * T_s$ *(Because ¼ CP makes the sampling faster by 5/4 times)*	1.56×10^{-5}
$T_{sub} = T_s / 16$	9.8×10^{-7}

Wavelet Packet Object Structure
Wavelet Decomposition Command : wpdec *Size of initial data : [1 320]* *Order= 2* *Deptth=: 4* *Terminal nodes : [15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30]* *Wavelet Name : Daubechies (db3) ,* *Entropy Name : Shannon*

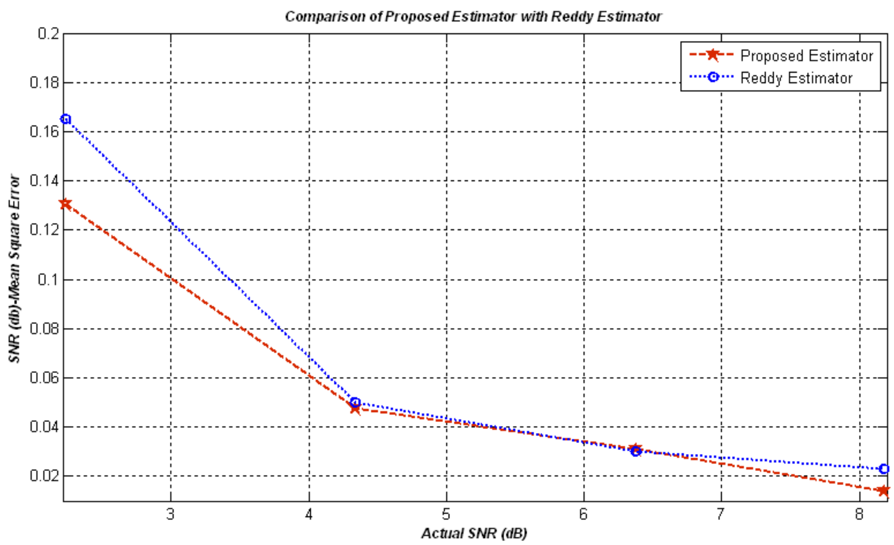


Fig.B.10. MSE performance of the proposed technique

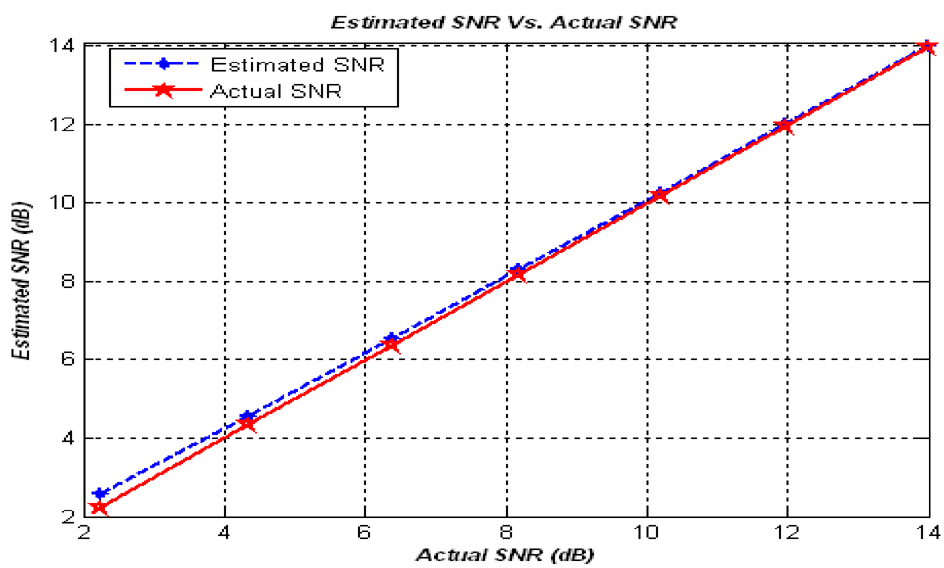


Fig.B.11. Actual SNR vs. Estimated SNR of colored noise

B.5. Summary

In this appendix, discrete wavelet packet based FFT and its application to SNR estimation inside FFT is presented. Also, variation of the noise power across OFDM sub-carriers is allowed. The second-order statistics of the transmitted OFDM preamble are calculated in each sub-band and the power noise is estimated. Therefore, the proposed approach estimates both local (within smaller sets of subcarriers) and global (over all sub-carriers) SNR values. The short term local estimates calculate the noise power variation across OFDM sub - carriers. These estimates are specifically very useful for diversity combining, adaptive modulation, and optimal soft value calculation for improving channel decoder performance. Its performance has been evaluated via computer simulations using AWGN and multipath fading channels and implemented in OFDM systems. The results show that the current estimator performs better than other conventional methods. Complexity to find SNR estimates is much lower because the current estimator makes use of only one OFDM preamble signal. The current estimator fulfills the criteria of best SNR estimator as it is unbiased and has the smallest variance of SNR estimates.

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