CHAPTER 2

BACKGROUND

2.1 Introduction

In Chapter 1, motivations of SNR estimation in OFDM systems and types of SNR estimators were introduced. Multipath is a phenomenon that occurs in wireless channel when multiple reflected versions of the signal arrive at the receiver at slightly different time and with various amplitudes and phases. The two channel impairments caused by multipath are multipath fading and intersymbol interference (ISI).

The key enabling technology to overcome these channel impairments is OFDM, introduced in this chapter. The principles of OFDM technology will be explored. Following this, literature review on SNR estimation and SNR estimation algorithms used later for comparison with the purposed technique will be presented. It will be followed by applications that may benefit from SNR. It is shown that in order to fully harness the power of OFDM technology, accurate SNR estimates must be obtained at the receiver. This establishes the importance of SNR estimation in an OFDM system.

2.2 Wireless Channel

Before introducing both OFDM and SNR estimation, we introduce the wireless channel itself and the 2 main impairments of multipath wireless channel, ISI and multipath fading. We undertake this in order to appreciate how OFDM technology is able to overcome these impairments. The following literature is borrowed from the Rappaport' s book [Rappaport, 2002].

The wireless channel is the single most important factor that limits the performance of wireless communication system. It is extremely hostile to communication. The transmission path from the transmitter to the receiver can vary from a simple LOS to one that is heavily obstructed by buildings, mountains and foliage. The signal from the transmitter is reflected, diffracted and scattered by various objects in the surrounding before reaching the receiver. When the signal impinges on an object that is very large compared to its wavelength, it will be reflected. Examples of reflecting objects are the earth's surface, buildings and walls. Some of the signal's energy will be transmitted through refraction while the rest will be reflected.

On the other hand, diffraction happens when the signal seemingly bends around obstacles that obstruct the path between the transmitter and receiver. Although one would expect the signal strength to be zero behind the obstacle, this is actually not the case. There is signal energy behind and beyond the obstacle. Due to this, although there is no LOS between the transmitter and receiver, the latter can still receive the signal. This phenomenon can be explained using Huygen's principle, which states that all points on a wavefront can be considered as point sources for the production of secondary wavelets and these wavelets combine to produce a new wavefront in the direction of propagation. When the signal impinges on objects that are small compared to its wavelength, scattering occurs. These objects usually have rough surfaces. Foliage, street signs and lamp posts are some objects that produce scattering.

The three signal propagation mechanisms (reflection, diffraction and scattering) affect the received signal strength. Naturally, the signal strength will be lower the farther the receiver is from the transmitter due to the spread of the signal in all direction through free space. But reflection, diffraction and scattering cause the signal strength to decrease faster as the distance increases. There are models that predict the mean received signal strength given the distance of the receiver from the transmitter. Due to the randomness of the wireless channel, these models are statistically based and they are known as large-scale propagation models.

Making matters worse, a signal from the transmitter can take multiple different paths before reaching the receiver as shown in Fig. 2.1. This can be caused by reflecting or even scattering objects. The multiple reflected versions of the signal will arrive at the receiver at slightly different time and with various amplitudes and phases. This phenomenon is called multipath. The multipath signals add up vectorially. The received signal strength will be amplified if the signals reinforce each other or are in phase with one another. The received signal strength can also drop drastically to a level that impedes any communication if the signals cancel each other or are out of phase. This is also known as multipath fading and is detrimental to wireless communications. Due to multipath, the signal strength at the receiver can change drastically and rapidly in a short period of time and over short distances. This is called small scale fading. Although the transmitters and receivers in a FBWA system are stationary, movement of other objects

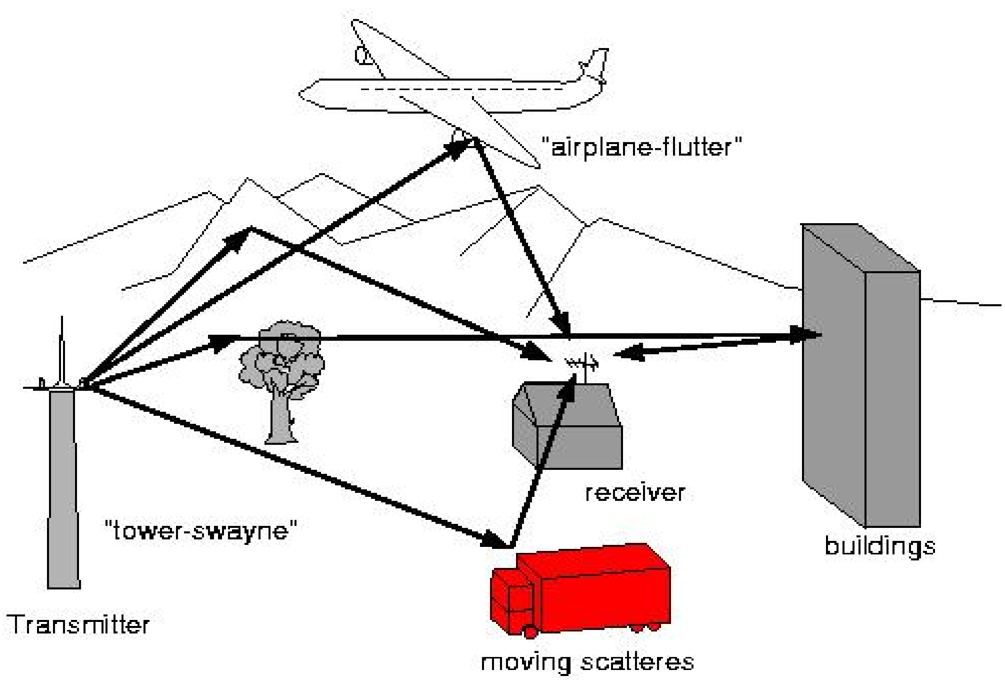

Figure 2.1: Multipath propagation

in the surrounding environment still causes fading to occur. It has been reported in [Sampath, Talwar, Tellado, Erceg & Paulraj, 2002] that FBWA system experiences fades that are as bad as mobile wireless communication systems when the measurements are done over a large timescale of the order of tens of hours.

In order for communication links to work even in deep fades, the transmit power required for a given link reliability should be much higher, depending on the severity of the fades expected. This extra power is known as fade margin. If effective techniques are used to mitigate the fades, the extra power will not be needed, making the BS and CPEs cheaper. On the other hand, the extra power can be used to increase the reliability or extend the range of the system to cover a larger geographical area.

Multipath can also cause signal distortion. When different copies of the transmitted signal arrive at slightly different times, it can overlap across different symbol times. The delayed previous symbol interferes with the current symbol causing what is known as intersymbol interference (ISI) as shown in Fig2.2. When the symbol rate increases, ISI will be worse because the delayed previous symbol can interfere with many symbols after it. This means that ISI becomes a major problem with high data rate wireless communication systems like FBWA. This signal distortion needs to be corrected using various equalization techniques [Sklar, 1997].

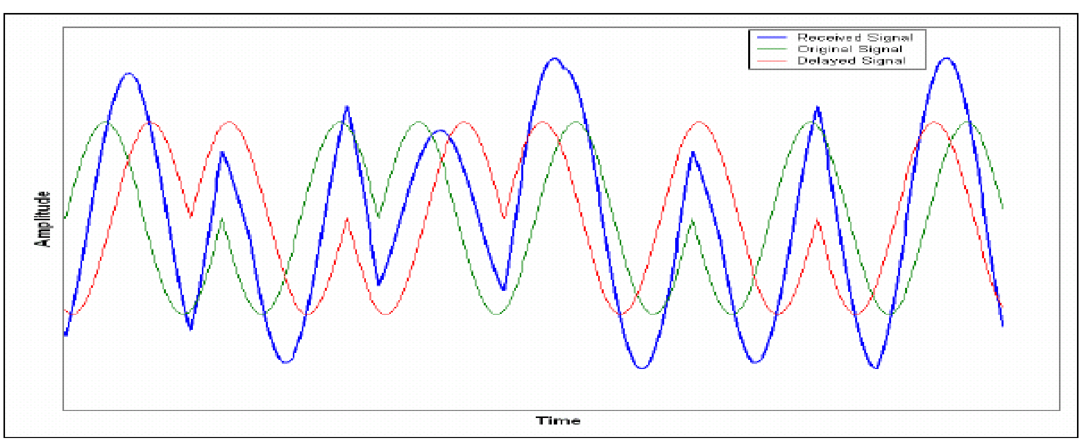

Fig.2.2 Intersymbol interference

In the frequency domain, ISI manifests itself as frequency selective fading where the gain and phase shift are different for different frequencies. Therefore the 2 most important challenges presented by the wireless channel to a FBWA system are multipath fading and ISI.

2.3 Orthogonal Frequency Division Multiplexing (OFDM)

In high rate communication systems, OFDM, as a technique, proves very effective in overcoming ISI in frequency selective fading channels. It is a special form of multicarrier modulation. In OFDM, a single high rate data stream is transmitted over a number of lower rate data streams. For example, in a packet based system, N data bits of a single high rate data stream are mapped to 1 bit on each of the parallel data streams. So, the symbol period of each of the data stream is N times the symbol period of the single data stream. The symbol duration of each of the lower rate data streams are now N times longer. ISI now only causes an overlap into the next symbol by an amount that is reduced by a factor of N. The relative dispersion in time due to ISI is therefore reduced considerably because of this. In fact, ISI is completely eliminated by introducing a guard time in each OFDM symbol. In the guard time, the OFDM symbol is cyclically extended to avoid inter-carrier interference (ICI). This cyclic extension is called cyclic prefix and the reasons for it will be further explained in the Section 2.3.1.

In OFDM, the broad bandwidth of the signal has been divided into N narrower sub-channels. If the OFDM system is designed properly, each of these narrow sub-channels will undergo flat fading as opposed to frequency selective fading when using single carrier systems. Therefore in OFDM, equalization is very much simplified and involves just a complex multiplication for each of the subcarrier.

The concept of using parallel data transmission and frequency division multiplexing (FDM) is not new. In FDM, the total frequency band is divided into N non-overlapping frequency sub-channels in order to avoid inter-channel interference. There would also be guard bands between the sub-channels so that each subcarrier can be filtered and demodulated. This is shown in Fig.2.3.

However, this leads to inefficient use of the available spectrum. In order to avoid this spectrum wastage, ideas were proposed that made use of overlapping sub-channels instead. This is shown in Fig.2.4. Comparing with Fig.2.3, one can see that there is significant savings in bandwidth in OFDM compared with conventional FDM.

The question remains as to how the subcarriers can be separated and demodulated without suffering interference from other subcarriers. A technique, that allows the sub-channels to overlap and yet not interfere with one another, is to have the subcarriers mathematically orthogonal to each other [Van Nee, 2000]. A set of signals $\phi_j(t)$ is said to be orthogonal over an interval (*a,b*) if it satisfies the following: [Soliman & Srinath, 1990]

$$\int_a^b \phi_i(t)\,\phi_k(t)\,dt = \begin{cases} C & for \quad i = k \\ 0 & for \quad i \neq k \end{cases} \tag{2.1}$$

where *C* is a constant.

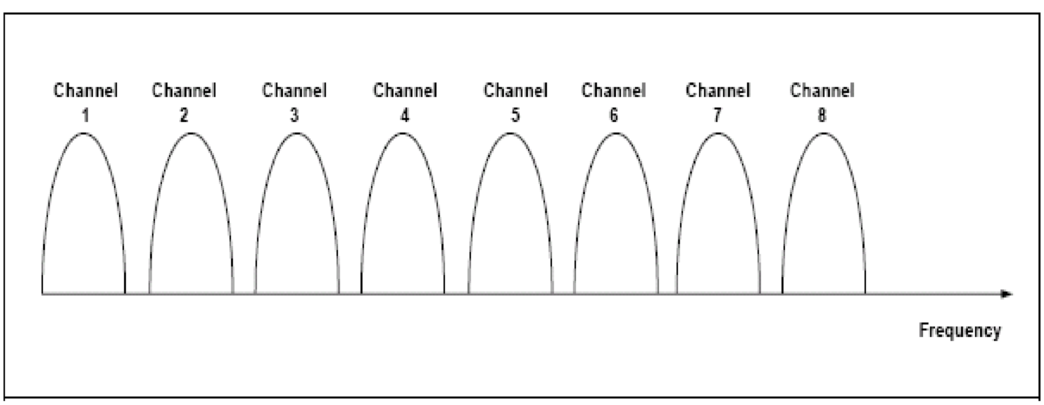

Fig.2.3 Frequency division multiplexing

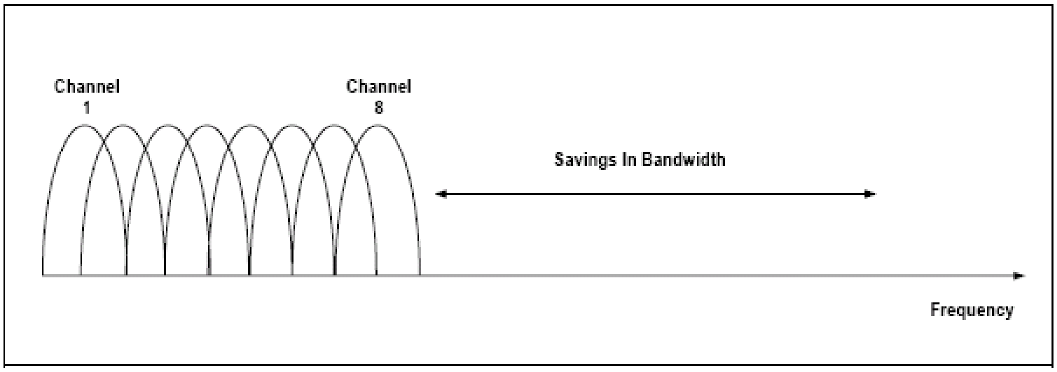

Fig.2.4 Orthogonal Frequency division multiplexing

Fig.2.5 shows the spectrum of a single sub-channel and Fig.2.6 shows the spectrum of an 7 subcarrier OFDM signal. From Fig.2.6, it can be seen that sub-channels overlap but the peak of any sub-channel is at the nulls of other sub-channels. This means that there is no interference and ICI between the sub-channels at the centre frequency of each sub-channel.

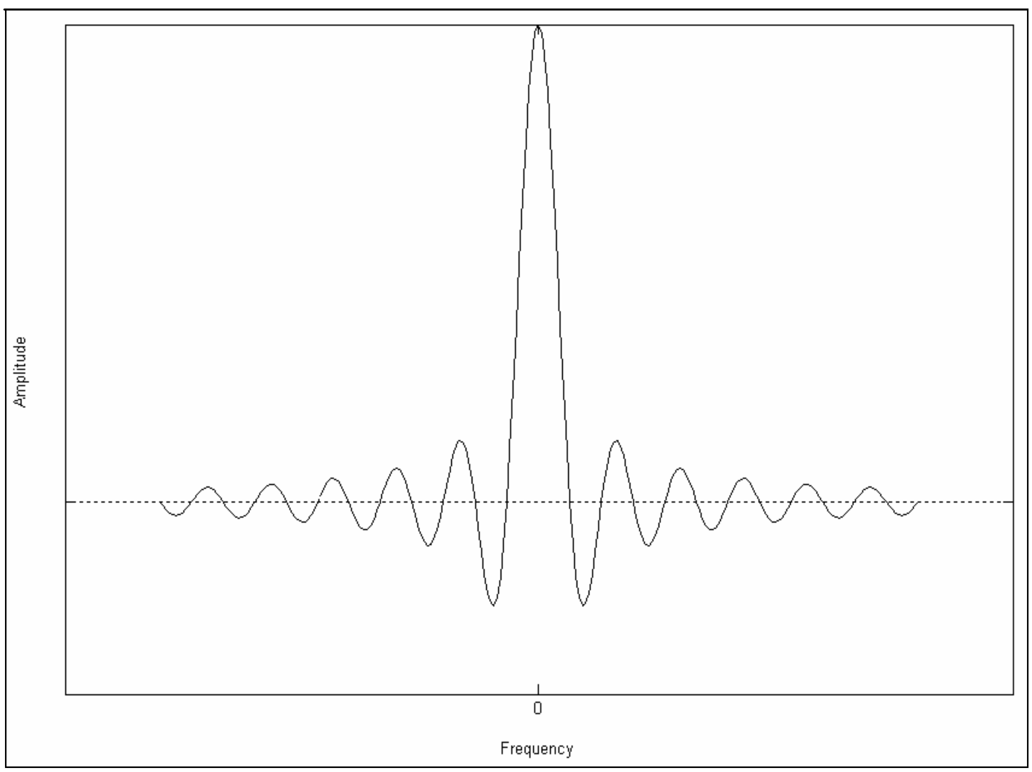

Figure 2.5: Spectrum of a single subchannel

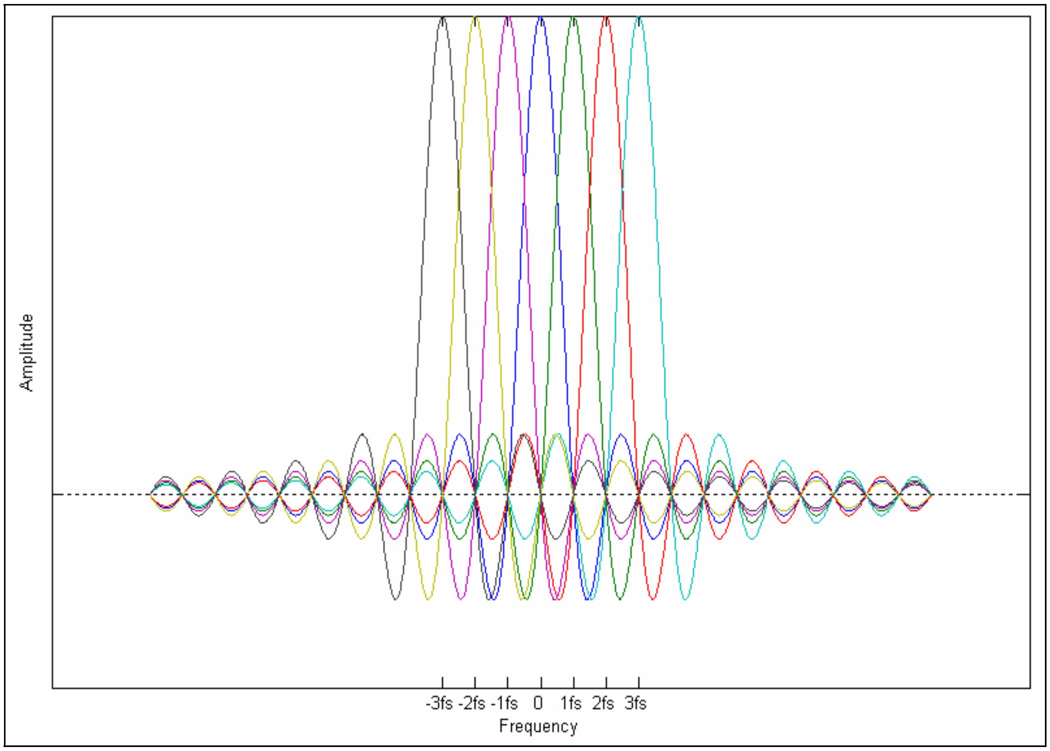

Figure 2.6: Spectrum of an OFDM signal

The next challenge is to have an efficient implementation of the OFDM. Initially, the OFDM was implemented using banks of subcarrier oscillators and coherent demodulators. Today, the use of discrete Fourier transform (DFT) and inverse discrete Fourier transform (IDFT) and their efficient implementation of fast Fourier transform (FFT) and inverse fast Fourier transform (IFFT) makes OFDM a practical and viable technique.

2.3.1 OFDM Principles

An OFDM signal is made up of many subcarriers that are independently modulated using either phase shift keying (PSK) or quadrature amplitude modulation (QAM).

Mathematically, one OFDM symbol $s(t)$ starting at time $t = t_s$ is written as [Van Nee, 2000]

$$s(t) = \mathrm{Re}\left\{ \sum_{i=-\frac{N_s}{2}}^{\frac{N_s}{2}-1} d_{i+N_s/2} \exp\left(j2\pi \left(f_c + \frac{i+0.5}{T} \right)(t - t_s) \right) \right\}, \qquad t_s \le t \le t_s + T$$

$$s(t) = 0, \qquad t < t_s \ \& \ t > t_s + T \tag{2.2}$$

and its equivalent complex baseband notation

$$\hat{s}(t) = \mathrm{Re}\left\{ \sum_{i=-\frac{N_s}{2}}^{\frac{N_s}{2}-1} d_{i+\frac{N_s}{2}} \exp\left(j2\pi \frac{i}{T}(t - t_s) \right) \right\}, \qquad t_s \le t \le t_s + T$$

$$\hat{s}(t) = 0, \qquad t < t_s \ \& \ t > t_s + T \tag{2.3}$$

In the complex baseband representation, the real and imaginary parts are the in-phase and quadrature phase parts of the OFDM signal. Both parts need to be multiplied by a cosine and sine of the carrier frequency respectively to produce the final OFDM signal.

Fig.2.7 shows the baseband OFDM modulator block diagram and Fig.2.8 shows the baseband OFDM demodulator block diagram.

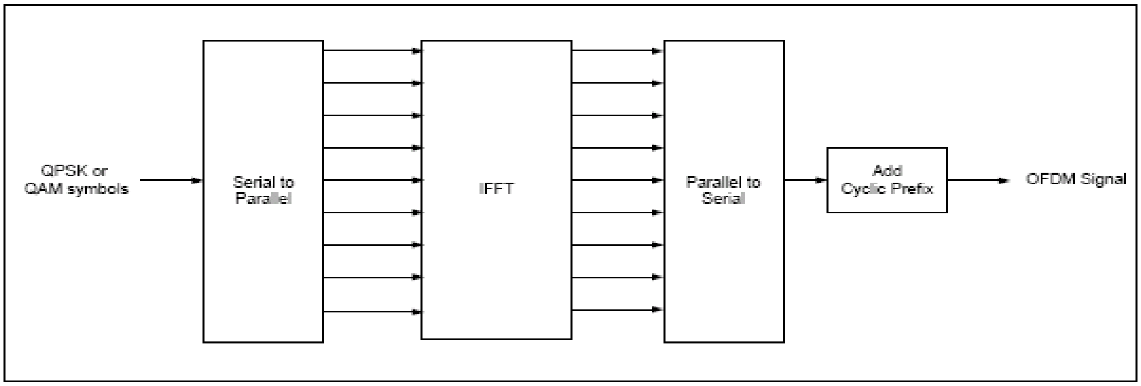

Figure 2.7 OFDM modulator

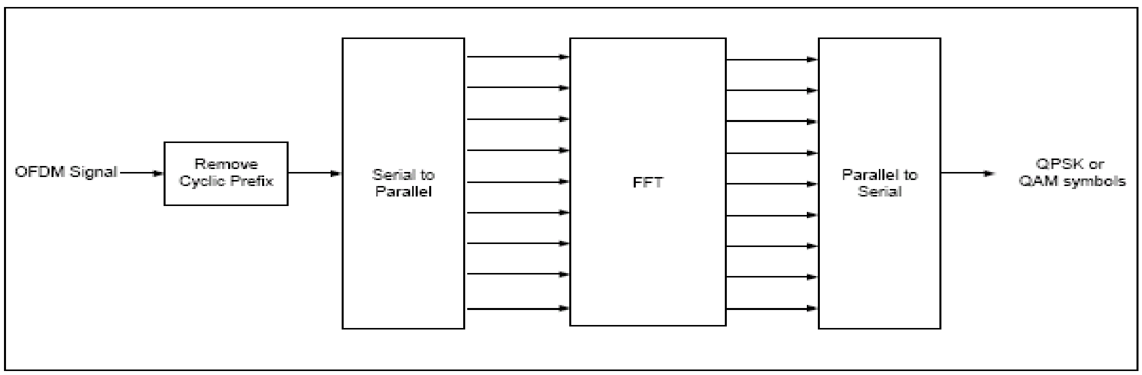

Fig2.8 OFDM demodulator

It is important to note that each of the subcarrier has a frequency that is an integer multiple of *1/T* where *T* is the OFDM useful symbol period. This makes the sinc function spectrum of each of the subcarriers to be *1/T* spaced from each other resulting in the OFDM spectrum as shown in Fig.2.6. This property makes the subcarriers orthogonal to each other because the peak of any subcarrier corresponds to the nulls of all other subcarriers.

In order to completely eliminate ISI in OFDM, a guard time is introduced at the beginning of each OFDM symbol. The guard time is chosen to be larger than the maximum expected delay spread of the channel so that multipath from one symbol will not interfere with the next symbol. Fig.2.9 shows how ISI is eliminated in OFDM by introducing a guard time. Note in the figure that the delayed symbol 1 does not spill over to symbol 2 due to the guard time. Notice also that if the delay is longer than the guard time, ISI will occur.

This guard time could consist of no signal at all as in the case of Fig.2.9 but this will introduce ICI as shown in Fig.2.10, causing the loss of orthogonality among the subcarriers. Fig.2.10 shows subcarrier *i* and a delayed subcarrier *j*. During the demodulation of subcarrier *i*, there would be some interference from subcarrier *j* because there is no integer number of cycles of subcarrier *j* within the FFT integration interval and vice versa.

To eliminate this ICI, the OFDM symbol is cyclically extended in the guard time. This ensures that delayed replicas of the OFDM symbol always have an integer number of cycles within the FFT interval, as long as the delay spread is smaller than the guard time. Fig.2.11 shows that there is no ICI when cyclic prefix is used in the guard time.

In order to exploit the advantages and maximized the performance of OFDM technology; using high data rates, channel state information is necessary. Key role in the channel state information has Signal-to-Noise Ratio (SNR).

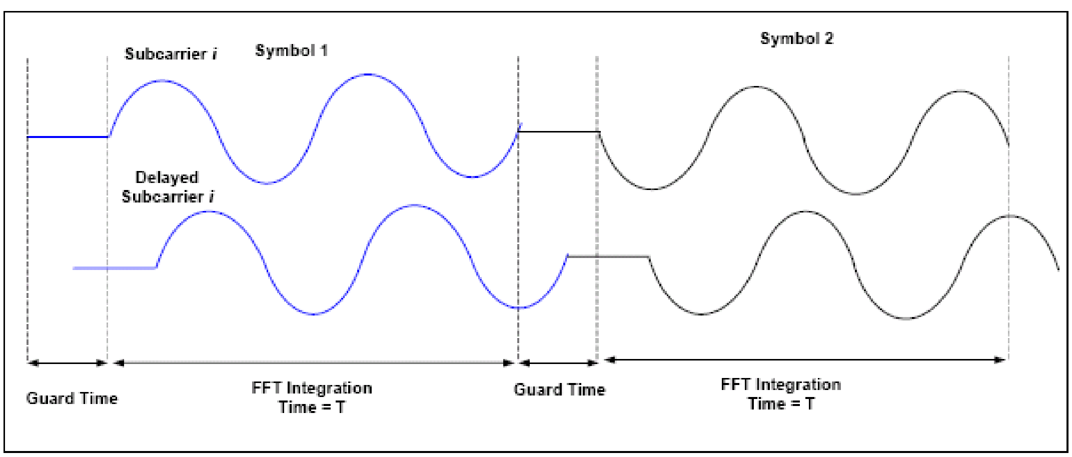

Fig.2.9 No ISI with introduction of guard time

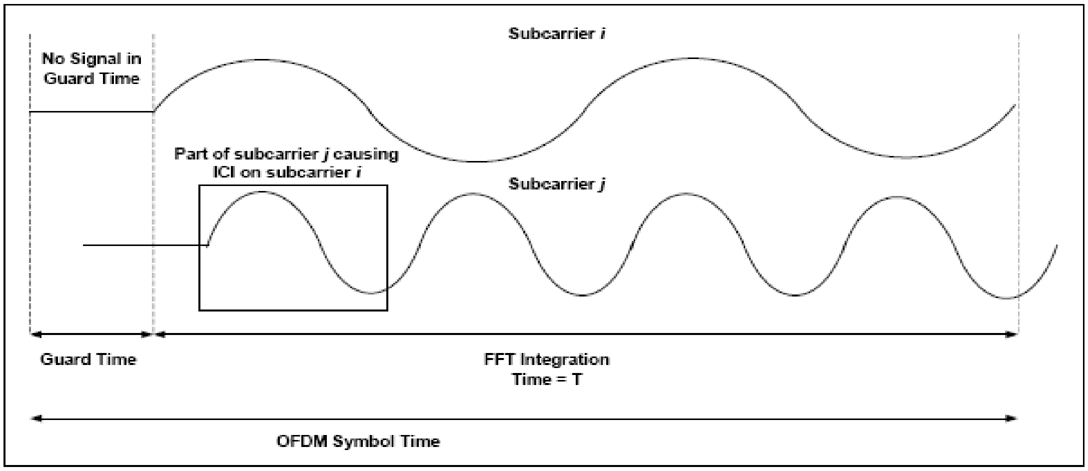

Fig 2.10 Effect of ICI with no signal in the guard time

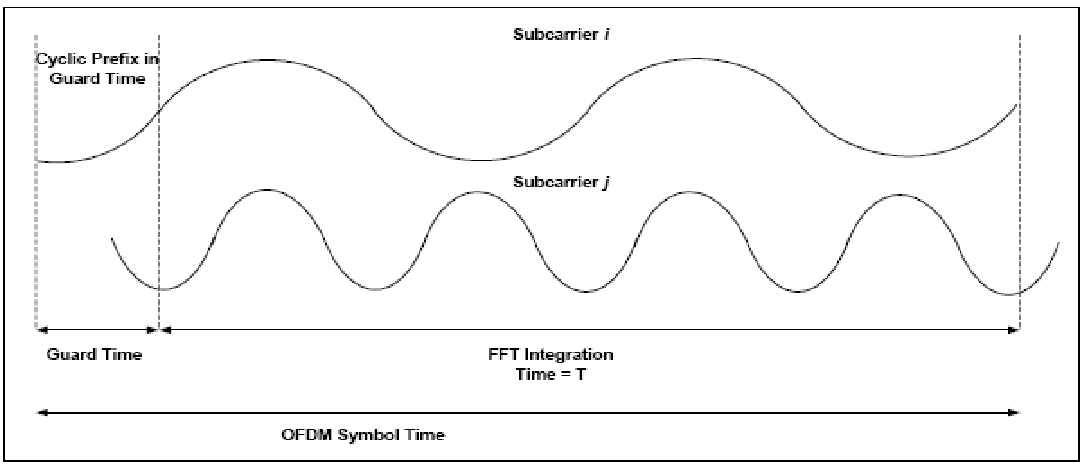

Fig.2.11 No ICI with cyclic prefix in the guard time

2.4 Literature Review

2.4.1 Overview of SNR Estimation

The signal-to-noise ratio (SNR) is broadly defined as the ratio of the desired signal power to the noise power and has been accepted as a standard measure of signal quality for communication system. The SNR estimators reported in the literature fall in two categories called data-aided (DA) and non-data-aided (NDA) estimators.

SNR estimators which extract the SNR from a single channel on which both a desired signal and noise are present using knowledge of the transmitted message sequence are referred to as data-aided (DA) estimators. A DA SNR estimator performs best when the message sequence used by the estimator is identical to the true transmitted message sequence. An example of this type of SNR estimator may be implemented using the correlation between the noisy signal and the known transmitted signal.

Non-data-aided SNR estimators generate estimates of the SNR assuming knowledge only of the statistics of the signal and channel. These estimation techniques are usually moment-based methods. Since no knowledge of the transmitted symbols is required, these techniques can derive SNR estimates from the information-bearing portion of the received signal and so are classified as in-service estimators.

- Interest in techniques to generate estimates of the SNR began in the mid- to late-1960's. The earliest recorded work on SNR estimation that could be found is a university report written by Nahi and Gagliardi [Nahi et al, 1964]. A subset of this work was published by Nahi and Gagliardi [Nahi et al, 1967]. The estimators described by Nahi and Gagliardi form estimates of the SNR by measuring the power of a hard limited, received (noisy) signal at the output of a filter. Both the signal and noise are assumed to be Gaussian stochastic processes with correlation functions of known shape. An expression is given by Nahi and Gigliardi showing the output power as a function of the filter transfer function and the SNR. The expression is not easily inverted so that, if the output power is known (measured), the SNR must be found implicitly using iterative techniques or a lookup table.

- Other early work on SNR estimation includes that of Benedict and Soong [Benedict et al, 1967]. The authors present three different methods to estimate separately the carrier strength and the noise level based on a finite number of samples. A maximum likelihood (ML) estimator, an estimator based on amplitude moments, and an estimator based on square-law moments are presented along with plots of the bias and root mean square (RMS) error of the simulated signal and noise estimates for the three estimation techniques.

- Benedict and Soong refer to work done by [Middleton 1962] which predates that of Nahi [Nahi et al, 1967]. However, the estimation method developed by Middleton assumes that the noise level is known and so is not applicable to this study. The maximum likelihood (ML) estimator derived by Benedict and Soong is complicated. The ML SNR estimation problem was formulated in a different manner by Kerr [Kerr et al, 1966], [Gagliardi et al, 1968], [Thomas et al, 1967] and [Gilchriest et al, 1966] to yield much simpler expressions.

- Kerr proposes two different variations of a maximum likelihood SNR estimator where antipodal signaling in AWGN is assumed [Kerr et al, 1966]. The estimators derived by Kerr can be manipulated into the form of the SNR estimator derived by Gagliardi and Thomas. The probability density function (pdf) of the ML SNR estimator and analytical expressions for the bias and variance are offered by Gagliardi [Gagliardi et al, 1968].

- In the Jet Propulsion Laboratory report by Gilchriest [Gilchriest et al, 1966], a simple, intuitive SNR estimator is proposed based on the absolute mean and variance of an antipodal (BPSK) signal corrupted by AWGN. An analysis of the pdf of this estimator is presented along with confidence intervals. This work was extended by Layland to study the performance of this SNR estimator at low levels of SNR [Layland et al, 1967]. It is indicated in this work that this intuitive SNR estimator is a type of ML estimator. An analog method for determining the SNR of BPSK signals in additive white Guassian noise (AWGN) was published by Edbauer [Edbauer et al, 1977]. The method is based on the processing of the in-phase and quadrature branches of a Costas demodulator.

- Simon and Mileant introduced an SNR estimator called the split symbol moments estimator (SSME) which is designed for BPSK signals in wideband AWGN channels [Simon et al, 1986]. Shah and Hinedi study the SSME in narrowband channels and provide plots of the means and inverse normalized variances of theoretical and simulated SSME estimates [Shah et al, 1990]. In a Jet Propulsion Laboratory memo, Shah and Holmes discuss a modification of the SSME designed to improve performance in narrowband channels. The channel models of this work assume that all of the filtering occurs after noise is added.

- Matzner presented an SNR estimator whose structure was first introduced by Benedict and Soong in 1967 as the "square-law method of carrier strength and noise level estimation [Matzner et al, 1993]. Matzner evaluates the performance of the SNR estimater as opposed to the performances of the separate estimators of carrier strength and noise level as treated by Benedict and Soong. Matzner also provides more derivation details. The derivation assumes complex baseband signals in complex AWGN, but the estimator structures developed are also applicable, with relatively minor modifications, to real baseband signals in real AWGN. The mean square error (MSE) of the logarithm (dB) of simulated SNR estimates is plotted by Matzner as a function of the block length and as a function of the true SNR.

- A hardware implementation is described by Matzner, Engleberger, and Sietvert [Matzner et al, 1997]. The complex form of this SNR estimator may be modified to be used as a more general signal-to-interference ration (SIR) estimator in fading channels with co-channel interference (CCI) and AWGN. The "signal-to-variation ratio (SVR) estimator proposed by Brand is an SIR estimator used to measure the quality of signals in channels corrupted by multipath, CCI, and AWGN [Brand et al,1996]. This estimator may be modified to be used as an SNR estimator for complex signals in complex AWGN, or for real signals in real AWGN. The SVR estimator is identified as being of the "in-service type which is a term sometimes used to refer to an estimator that forms estimates from the information-bearing received signal, thus avoiding the need to perform SNR measurements off-line.

- Plots provided by Brand showing the theoretical and simulated SVR estimates as a function of signal power for three different fading channels [Brand et al, 1994]. [Yoshida et al, 1991] and [Yoshida et al, 1992] describe an in-service estimator that, like the SVR estimator, also reflects the multipath spread and level of CCI in a wireless channel.

- Pauluzzi et al in 2000 compares different SNR estimation techniques for AWGN channel [Pauluzzi et al, 2000]. Ramesh proposed SNR estimation in generalized fading channels [Ramesh et al, 2001]. Wiesel proposed non data aided (NDA) SNR estimation for PSK signals in AWGN [Wiesel at al, 2002]. Xu extend the work done by Wiesel and proposed NDA-SNR estimation for QAM signals [Xu et al, 2004]. NDA-SNR estimation is derived based on a statistical ratio of observables over a block of data. This estimator performs well with only large block of data.

- The entire estimator presented above derived the SNR estimates solely from the received signal at the output of the matched filter (MF). The perfect carrier and intersymbol interference (ISI) is assumed. Nidal proposed an SNR estimation technique for AWGN channel [Nidal et al, 2007], which can operate on data collected at the front-end of the receiver without any restriction on ISI.

2.5 SNR Estimation in OFDM Systems

All the estimators presented in the literature so far handle the issue considering single carrier transmission and presume ISI free reception of the signal. There is not that much work found for the SNR estimation in OFDM systems. OFDM is a multicarrier transmission scheme as described before in section 2.3 and one can received ISI free signal at the receiver using cyclic prefix (CP) of proper guard band length.

- Shousheng proposed SNR estimator for OFDM systems but the wireless channel used is flat fading [Shousheng et al, 1998]. Additionally no quantitative results are given to connect the estimated and actual SNR values but only constellation diagram demonstrating the received vector scatter in the complex plain.

- Arslan & Reddy presented a method to estimate the noise power and SNR for OFDM systems [Arslan et al, 2003]. In this work noise power estimation, which takes into account the color and variation of noise statistics over OFDM sub-carriers, is considered. Instead of averaging the instantaneous noise samples estimates over all of the OFDM sub-carriers, dividing the total number of sub-carriers into several sub-groups and averaging the sub-carriers separately within each sub-group is proposed. It is observed that this estimator gives better results when the number of OFDM symbols is large. Due to averaging each sub-carrier separately complexity is increased.

- Xiaodoung reported a subspace based noise variance and SNR estimation technique for OFDM systems which is based on eigenvector decomposition of the estimated channel correlation matrix [Xiaodoung et al, 2005]. Subspace based estimator gives accurate measurements of the noise variance and SNR after an observation interval of about 20 OFDM symbols for various fading channels. Due to large number of OFDM symbols complexity of the system is high.

- Yucek and Arslan proposed MMSE noise power and SNR estimation for OFDM systems in which noise variance, and hence SNR, is calculated by using two cascaded filter in time and frequency directions whose coefficients are calculated using the statistics of noise/interference variance [Yucek et al, 2005]. Averaging is used over 20 OFDM symbols and considers estimation over subcarriers only.

- Doukas reported SNR estimation for low bit rate OFDM systems in AWGN channel [Doukas et al, 2006]. Doukas introduced two new methods to estimate SNR, SNV-SNR estimator and MMSE-SNR estimator in AWGN. Satisfactory SNR estimation can be achieved using these estimators with 1000 OFDM samples. Due to large number of samples used to find the SNR estimate, complexity of the system increased.

- According to the best knowledge of author, all the SNR estimators for OFDM systems discussed above gives SNR estimate at the back-end of the receiver. Unlike all the SNR estimators for OFDM systems which perform SNR estimation at the back-end of the receiver, the SNR estimator proposed in this thesis gives SNR estimates at the front-end of the receiver. The proposed method based on one OFDM preamble used for timing synchronization.

- In contrast to other SNR estimators, the proposed technique operates on data collected at the front-end of the receiver, imposing no restriction on ISI. This will improve the SNR estimates in severe ISI channels and also help extending the implementation of SNR estimators towards systems that require SNR estimates at the input of the receiver. One such application is antenna diversity combining, where at least two antenna signal paths are communicably connected to a receiver. The combiner can use the SNR estimates obtained for each antenna signal to respectively weight each signal and thereby generate a combined output signal.

Reddy's SNR estimator and subspace based SNR estimator are selected to comparing them with our purposed SNR estimation technique and discussed in the next section.

2.6 Previous SNR Estimation Algorithms for OFDM Systems

The two back-end SNR estimators, the Reddy's estimator [Reddy et al, 2003], and subspace based estimator [Xiaodong et al, 2005], used later for comparison are discussed below.

2.6.1 Reddy's SNR Estimation

An OFDM based system model is used. Time domain samples of an OFDM symbol can be obtained from frequency domain symbols as

$$x_m(n) = IDFT\ \{X_m(k)\} = \sum_{k=0}^{N-1} X_m(k)\ e^{j2\pi nk/N} \qquad 0 \le n \le N-1 \tag{2.4}$$

where $X_m(k)$ is the symbol that is transmitted on k-th subcarrier of the m-th OFDM symbol, and N is the number of sub-carriers. After the addition of cyclic prefix and D/A conversion, the signal is passed through the mobile radio channel. Assuming a wide-sense stationary and uncorrelated scattering (WSSUS) channel, the channel $H(f,t)$ can be characterized for all time and all frequencies by the two-dimensional spaced-frequency, spaced-time correlation function

$$\phi(\Delta f, \Delta t) = E\left\{H^*(f,t)H(f+\Delta f),(t+\Delta t)\right\} \tag{2.5}$$

In this technique, it is assume that the channel remains constant over an OFDM symbol, but time- varying across OFDM symbols, which is a reasonable assumption for low and medium mobility.

At the receiver, the signal is received together with the noise. The noise power is assumed to be varying across OFDM sub-carriers as well as in time. After down converting, synchronization and removing the cyclic prefix, the simplified received baseband model of the samples can be formulated as

$$Y_m(n) = \sum_{l=0}^{L-1} x_m(n-l)\ h_m(l) + n_m(n) \tag{2.6}$$

where L is the number of channel taps, $n_m(n)$ is the noise sample which is combination of white Gaussian noise and colored interference source. The channel is assumed quasi static, therefore the time domain channel impulse response (CIR), $h_m(l)$, over an OFDM symbol is given as time-invariant linear filter. After taking DFT of the OFDM symbols, the received samples in frequency domain can be shown as

$$\begin{aligned} Y_m(k) &= DFT\{y_m(n)\} = \sum_n y_m(n)\, e^{-j2\pi nk/N} \\ &= S_m(k)\, H_m(k) + N_m(k) \end{aligned} \tag{2.7}$$

where $H_m(k)$ and $N_m(k)$ are DFT of $h_m(l)$ and $n_m(l)$, respectively.

Note that, in this technique, the noise is not assumed to be white. In practical wireless communication systems, often the received signal is impaired by dominant interference sources. For example, in cellular systems, the dominant interference source can be a co-channel or an adjacent channel interferer.

2.6.1.1 Estimation of Noise Power and Signal Power

The noise power estimation is performed in frequency domain. The instantaneous noise estimate for each OFDM carrier is calculated by finding difference between noisy received sample and the best hypothesis of the noiseless received signal. The variance of the noise estimator is thus given by

$$\sigma^2_{N(m)}(k) = \left| Y_m(k) - \hat{X}_m(k).\, \hat{H}_m(k) \right|^2 \tag{2.8}$$

where $\hat{X}_m(k)$ is the best hypothesis of the received symbol and $\hat{H}_m(k)$ is the channel estimate for the k-th carrier of the m-th OFDM symbol. For noiseless channel estimates and for correctly detected symbols, the above equation will provide the absolute square of the exact instantaneous noise samples. However, the channel estimation error and incorrect decisions will bias the noise estimates. The problem with incorrect symbol estimates can be resolved by estimating noise samples using only known data (training symbols), or by finding the error after decoding and re-encoding the detected symbols.

Using the decoded information improves performance as the decoder corrects the incorrect decisions.

2.6.1.2 Signal Power Estimation

Signal power can be estimated from the knowledge of channel estimates. The channel estimates in frequency domain can be obtained using OFDM training symbols, or by transmitting regularly spaced pilot symbols in between the data symbols and by employing frequency domain interpolation. In this technique, transmission of training OFDM pilots is assumed. Using the knowledge of the training pilots, channel frequency response can be estimated as

$$\hat{H}_m(k) = \frac{Y_m(k)}{S_m(k)}$$
$$\hat{H}_m(k) = H_m(k) + w_m(k) \tag{2.9}$$

where $Y_m(k)$ is the received pilot value and $w_m(k)$ is the channel estimation error. It is to be noted that interpolation is critical for the pilot based estimation because pilots are inserted at fixed places in an ODFM symbols. For example in IEEE 802.16d std. (WiMAX) eight pilots are used in 256 bit longer frame. Pilots are used in each frame unlike preamble which is used at the front of OFDM data symbols only once.

2.6.1.3 Noise Power Estimation in Sub-bands

In conventional noise power estimation algorithms, the absolute square of the instantaneous noise samples are averaged over all OFDM sub-carriers, providing an averaged noise power estimate. The conventional approaches assume the noise to be white and of Gaussian distribution and estimate a single noise variance (power) for all the OFDM sub-carriers. Therefore, these approaches do not provide any information about the variation of noise within the transmission bandwidth.

In this technique, noise power estimates for overall OFDM subcarriers as well as noise power estimates for noise varying within the transmission bandwidth (colored noise) are derived. For the noise power estimation of colored noise, the whole OFDM data is divided (i.e. the total number of sub-carriers) into sub-bands (i.e. to a set of subcarriers). If the number of sub-carriers in each sub-band is *k*, then the number of sub-bands will be *N/k*. Then, the absolute square of the instantaneous noise estimates in each sub-band are averaged,

$$\hat{\sigma}^2_{N_m}(j) = \frac{1}{k}\sum_{l=1}^{k}\hat{\sigma}^2_{N_m}(l) \qquad 1 \le j \le N/k \tag{2.10}$$

where $\hat{\sigma}^2_{N_m}(j)$ is the estimated noise power in the *j-th* sub-band.

2.6.1.4 Signal Power Estimation in Each Sub-band

Using the knowledge of channel estimates, signal power over each sub-band is estimated as

$$\hat{P}_s(j) = \frac{1}{k}\sum_{l=1}^{k}\left|Y_m(l)\right|^2 \tag{2.11}$$

where $\hat{P}_s(j)$ is the estimated signal power in the *j-th* sub-band.

2.6.1.5 SNR Estimation in Each Sub-band

Having knowledge of noise power estimates and signal power estimates in each sub-band, the SNR is computed as

$$S\hat{N}R(j) = \frac{\hat{P}_s(j)}{\hat{\sigma}^2_{N_m}(j)} \tag{2.12}$$

where $S\hat{N}R(j)$ is the estimated value of actual SNR in the *j-th* sub-band.

The size of k depends on the color of the noise. If the noise is completely white, then it is desired that averaging be done across all the available OFDM sub-carriers, i.e. to have k equal to N. Therefore, increasing the number of samples over which averaging is done yields a lower mean-squared-error in the case of white noise, but the same does not apply for colored noise. The averaged noise estimates over each sub-band are further averaged across several OFDM symbols to get global SNR estimates (Over-all SNR values).

The methodology and parameters to perform simulation for this technique is discussed in the chapter 3 and results of this method are used for comparison with the proposed front-end SNR estimator are shown in chapter 4.

2.6.2 Subspace Based SNR Estimation

In this technique, an OFDM system that consists of N subcarriers among which N_t subcarriers at the central spectrum are used for transmission and the other subcarriers at both edges form the guard bands. A cyclic prefix is also added as guard interval for every OFDM symbol to avoid intersymbol interference caused by multipath fading channels. Each transmission subcarrier is modulated by a data symbol $X_{i,n}$, where i represent the OFDM symbol number and n represents the subcarrier number. It is assumed that the signal is transmitted over a multipath Rayleigh fading channel characterized by

$$h(t,\tau) = \sum_{l=1}^{L} h_l(t)\delta(t-\tau_l) \tag{2.13}$$

where $h_l(t)$ are the different path complex gains, τ_l are different path time delays, and L is the number of paths. $h_l(t)$ are wide-sense stationary (WSS) narrow-band complex Gaussian processes and the different path gains are uncorrelated with respect to each other where total channel energy is normalized to one. At the receiver side, with the assumption that the channel is quasi-stationary (in other words, the guard interval duration is longer than the channel maximum excess delay and the channel does not change within one OFDM symbol duration), then the n^{th} subcarrier output during the i^{th} OFDM symbol can be represented by

$$Y_{i,n} = X_{i,n} . H_{i,n} + N_{i,n} \tag{2.14}$$

where $N_{i,n}$ is a white Gaussian noise with variance, $H_{i,n}$ is the channel frequency response given by

$$H_{i,n} = \sum_{l=1}^{L} h_i(i, T_s) . e^{-j 2 \pi \frac{n \tau_l}{NT}} \tag{2.15}$$

where $h_i(i, T_s)$ denotes the channel *l-th* path gain during the *i-th* OFDM symbol and *T* is the sampling time interval of the OFDM signal. In this technique the SNR during the *i-th* OFDM symbol is defined as the ratio of the channel power to noise power and may be written as

$$S\hat{N}R = \frac{\sum_{i=1}^{N} \left| h_l(iT_s) \right|^2}{\sigma_N^2} \tag{2.16}$$

2.6.2.1 Estimation of Subspace Based SNR

Consider an OFDM system that uses *M* pilot subcarriers to estimate channel. It is assumed that *M* (*M* > *L*) pilots are evenly inserted in OFDM symbols. Let *P* denote the set that contains the position indexes of *M* pilots sub-carriers. At the pilot position we have

$$X_{i,p(m)} = \gamma_m \qquad m = 0,1,........ M - 1 \tag{2.17}$$

where $\{\gamma_m\}$ are the pilot subcarrier symbols with unit amplitude. Then the channel frequency response at the pilot subcarriers during the *i-th* OFDM symbol can be expressed as

$$\begin{aligned}\hat{H}_{i,p(m)} &= \frac{Y_{i,p(m)}}{\gamma_m} \\ &= \sum_{l=1}^{L} h_l(i.T_s).e^{-j2\pi\frac{p(m)\tau_l}{NT}} + \frac{N_{i,p(m)}}{\gamma_m}\end{aligned} \tag{2.18}$$

Eq.2.18 can also be written in matrix as

$$\hat{H}_{i,p} = W_p h_i + N_p \tag{2.19}$$

where $\hat{H}_{i,p}$ is $M*1$ column vector and N_p is $M*1$ noise column vector, W_p is the $M*L$ matrix with *m-th* Row given by

$$\left[e^{-j2\pi\frac{p(m)\tau_l}{NT}} \ldots\ldots\ldots\, e^{-j2\pi\frac{p(m)\tau_l}{NT}} \right] \tag{2.20}$$

here h_i is $L*1$ column vector representing the path complex gains. It should be noted that in mobile communications the multipath time delays are slowly varying in time. In contrast, the amplitude and relative phase of each path are relatively fast varying [Yang et al, 2001]. So we can regard W_p is unchanged during the K continuous OFDM symbols and h_i varies from symbol to symbol.

Because the noise vector N_p is zero mean and independent of the h_i, it follows that the correlation matrix of $\hat{H}_{i,p}$, is given by

$$R = \psi + \sigma_N^2 .I \qquad (2.21)$$

Where I is the identity matrix and

$$\psi = \mathrm{W}_p \, E(\mathrm{h}_l \cdot \mathrm{h}_l^H)\, \mathrm{W}_p^H \qquad (2.22)$$

where

W_p = FFT matrix of pilot symbols

h_l = channel impulse response for l-th multipath

$(.)^H$ = Hermitian conjugate matrix transpose

P = pilot symbol index $\quad p \in \{1, 2, \ldots\ldots, M)$

M = number of pilot symbols

E = expectation operator

Because the matrix W_p is of full column rank and the correlation matrix of h_i is nonsingular, it follows that the rank of Ψ is L, or equivalently, the $M - L$ smallest eigenvalues of Ψ are equal to zero. Denoting the eigenvalues of R by $\lambda_1 \geq \lambda_2 \ldots\ldots \geq \lambda_n$ it follows, therefore, that the smallest $M - L$ eigenvalues of R are all equal to σ_N^2 , i.e.,

$$\lambda_{L+1} = \lambda_{L+2} \ldots\ldots \lambda_m = \sigma_N^2 \qquad (2.23)$$

And we also have

$$\mathrm{Trace}(R) = M\,(\sigma_S^2 + \sigma_N^2) = \sum_{i=1}^{M} \lambda_i \qquad (2.24)$$

Where $\sigma_s^2 = E(\mathrm{h}_l \cdot \mathrm{h}_l^H)$ denotes the channel power. This implies that the observation space can be partitioned into a signal subspace spanned by the columns of W_p and a noise subspace. Now if we get the estimate of the channel correlation matrix R and the multipath number L (also is the dimension of the signal subspace), the noise variance and then the SNR can be derived.

Since we want to track time variations of the SNR, we form a moving average of the correlation matrix from the K most recent observation vectors. Let m denote the m^{th} OFDM symbol, then we have

$$\hat{R}(m) = \frac{1}{k} \sum_{i=m-K+1}^{m} H_{i,p} . H_{i,p}^{H} \tag{2.25}$$

The estimate of L can be decided by the well-known Minimum Descriptive Length (MDL) criterion presented by [Wax et al, 1985] as,

$$MDL(k) = -K(M-k)\log\left(\frac{\prod_{i=k+1}^{M} \hat{\lambda}_i^{\,1/(M-K)}}{\frac{1}{M-K}\sum_{i=k+1}^{M} \hat{\lambda}_i}\right) + \frac{1}{2}k(2M-k)\log(K) \tag{2.26}$$

The number of multipath is estimated as

$$\hat{L} = \arg\min_{k} MDL(k) \qquad k \in \{0,1,2....,M-1\} \tag{2.27}$$

From equations (2.23, 2.24, 2.26, 2.27), we can obtain subspace based SNR estimator.

2.6.2.2 Subspace based SNR estimator

1. Make an Eigen vector decomposition of the correlation matrix $\hat{R}$.

2. Estimate the current dimensions of the signal subspace $\hat{L}$ using equations (3.26 & 3.27).

3. According to eq.3.23 and eq.3.24 , estimate the noise power as

$$\hat{\sigma}_N^2 = \frac{1}{M - \hat{L}} \sum_{i=\hat{L}+1}^{M} \hat{\lambda}_i \tag{2.28}$$

And channel power as

$$\hat{\sigma}_s^2 = \frac{1}{M} \left(\sum_{i=1}^{\hat{L}} \hat{\lambda}_i - \hat{L} \ .\hat{\sigma}_N^2 \right) \tag{2.29}$$

4. The estimate of the SNR is then obtained as

$$S\hat{N}R = \frac{\hat{\sigma}_s^2}{\hat{\sigma}_N^2} \tag{2.30}$$

where $S\hat{N}R$ is the estimated value of actual SNR.

The methodology and parameters to perform simulation for this technique is discussed in the chapter 4 and results of this method are used for comparison with the proposed front-end SNR estimator are shown in chapter 5.

2.7 Applications Which May Benefit From Knowledge of the SNR

In many applications, the total received power is estimated for simplicity rather than the SNR. Goldsmith [Goldsmith et al, 1994] discusses power measurement for time-varying cellular channels, and point out that real-time measurement of the received power is required for operations such as power control, handoff, and dynamic channel allocation. Examples of applications that use total power or received signal strength estimates are described by Holtzman [J. M. Holtzman1992], Vijayan [R. Vijayan et al,1992], Whitehead [J. F. Whitehead et al,1993], Zhang [N. Zhang et al,1994]. In fact, the use of SNR estimates can improve the performances of the signal-strength-based algorithms used in these applications. Some references are listed below which describe applications that ideally require knowledge of the SNR.

2.7.1 Resource Management Algorithms

Measurement of the SNR is of great interest today as wireless service providers are finding that co-channel interference (CCI) is the greatest factor limiting the extent to which cell sizes can be reduced in an effort to increase frequency reuse and system capacity. For this reason, methods to measure the SNR (where the impairment, in this case, is mainly CCI are attracting much attention for use in resource management algorithms such as those used for handoff, dynamic channel allocation, and power control.

2.7.2 Power Control

Zender [J. Zander,1992] indicates that power control is important "to adjust the power of each transmitter for a given channel allocation such that the interference levels at the receiver locations are minimized”. He points out that, in practice, power control algorithms typically keep the total received power at a constant level; however, he adds that keeping the signal-to-interference power ratio constant instead could improve system capacity. This view is supported by Jalali [Jalali et al, 1994]. Zander admits that a practical implementation of the power control algorithm would be difficult since the path gains of the desired signal and interferers are, in general, unknown. A means to estimate the SNR would facilitate the practical implementation of this power control algorithm.

2.7.3 Diversity Combining

The classic pre-detection maximal-ratio combiner is described by Brennan [D.G.Brennan,1959], Lee [W.C.Lee, 1982] and Jakes [W.C.Jakes,1974]. These combiners form the weighted sum of two or more diversity branches where the weights are proportional to the amplitude of the signal, and inversely proportional to the noise variance. The weights for this diversity scheme are often implemented using the signal-plus-noise envelope as opposed to the signal amplitude to-noise variance ratio described by Adachi [F.Adachi et al.,1967]. If an algorithm were available to estimate the signal and noise powers separately, the desired weightings for each of the branches of the maximal-ratio combiner could be formed trivially as the ratio of the square root of the signal power to the noise power. If the SNR is determined as an inseparable parameter, $\rho = S/N$, then the signal power and noise power could be computed by also estimating the total received power, $P = S + N$, so that simultaneous equations for 'ρ' and P may be solved for S and N to yield $N = P/(I+ \rho)$ and $S = P - N$. The branch weights are then formed trivially as $\sqrt{S}/N$.

Adachi [F.Adachi, 1993] presents an optimal post detection diversity combiner that weights the differentially-detected symbols of each of the branches based on a formula which depends explicitly on both the SNR and the signal-to-interference ratio. The implementation of this formula actually requires two separate estimators-one to measure the SNR, the other to measure the signal-to-interference ratio.

A postdetection selection diversity combiner is described by Hladik [Hladik et al, 1992] where the SNR of each diversity branch is measured on a symbol-by- symbol basis. Each symbol interval, the differentially-detected symbol corresponding to the branch with the largest instantaneous SNR is the one that is presented to the decision device. The specific SNR estimator is not given there but is simply described as an approximation to the maximum-likelihood (ML) estimate of the SNR.

2.7.4 Equalization

Balaban [Balaban et al, 1991] describe an equalizer for frequency-selective fading channels. The tap update algorithm of the equalizer requires both an estimate of the channel impulse response and an estimate of the SNR, thus illustrating another application requiring some means to estimate the SNR.

2.7.5 Synchronization

A maximum likelihood estimator of the bit timing is presented by Wintz [Wintz et al, 1992] which is a function of the noise variance. For additive white Gaussian noise (AWGN), the noise variance drops from the estimator expression as shown by equation (11) of Wintz work. However, in time-varying channels, the noise variance cannot be assumed to be constant so it, or the SNR, must be estimated for optimal performance. Though a noise power estimator is required here, an SNR estimator could be used together with a total received power estimator to derive the noise power, as described in Section 2.5.3. Chennakeshu [Chennakeshu et al, 1993] present a method to achieve timing and frequency synchronization by maximizing the SIR with respect to the timing and frequency offset.

2.7.6 Adaptive Arrays

Adaptive arrays are used in wireless communications systems to cancel interference and mitigate fading effects by appropriately weighting and combining the output of two or more antennas. The optimal weight equation is given by Winters [Winters et al, 1993] or, equivalently, by Winters [Winters at al, 1984], and is found to be a function of the noise variance. Again, using a technique such as that described in Section 2.5.3 an estimate of the noise power may be found from estimates of the SNR and the total received signal power.

2.7.7 Viterbi Equalization and Decoding

The path metric used in Viterbi equalization and decoding is shown by Hagenauer [Hagenauer et al, 1989] to depend on what the authors call the 'instantaneous SNR, $E_s(k)/N_o$, where E_s is the energy per symbol, N_o is the noise power spectral density, and k is the time index. The time dependence arises as a result of the time-varying nature of the multipath channel assumed in this equalization and decoding.

2.8 Summary

In this chapter, background of the for FBWA-OFDM systems is discussed. There are two main impairments in wireless channels that are ISI and multipath fading. OFDM is highly effective in mitigating these channel impairments. The principles of OFDM technology are discussed and it is shown that how guard interval makes the OFDM system ISI free. Following this, literature review of SNR estimation and formulation of SNR estimation algorithms used later for comparing them with proposed methods are discussed and applications which may be benefited from SNR are discussed. It is shown that in order to fully harness the power of OFDM technology, accurate SNR estimates must be obtained at the receiver. This establishes the importance of SNR estimation in an FBWA-OFDM system.