UNIVERSITI
TEKNOLOGI
PETRONAS

FINAL EXAMINATION
MAY 2024 SEMESTER

COURSE	**:**	**PDB3023/PEB2053/PFB2043 - RESERVOIR ENGINEERING II**
DATE	**:**	**13 AUGUST 2024 (TUESDAY)**
TIME	**:**	**2:30 PM - 5:30 PM (3 HOURS)**

INSTRUCTIONS TO CANDIDATES

1. Answer **ALL** questions in the Answer Booklet.
2. Begin **EACH** answer on a new page in the Answer Booklet.
3. Indicate clearly answers that are cancelled, if any.
4. Where applicable, show clearly steps taken in arriving at the solutions and indicate **ALL** assumptions, if any.
5. **DO NOT** open this Question Booklet until instructed.
6. **Distribute the necessary appendix page separately and tie it with the answer booklet.**

Note :

i. There are **TWENTY (20)** pages in this Question Booklet including the cover page and appendices.

ii. **DOUBLE-SIDED** Question Booklet.

1. An oil well is producing at a constant flow rate of 300 STB/day. The properties of the reservoir's rock and fluids are listed in **TABLE Q1**.

TABLE Q1: Reservoir Rock and Fluid Properties

Properties	Value	Unit
Oil formation volume factor, B_o	1.25	bbl/STB
Oil viscosity, μ_o	1.50	cP
Total compressibility, c_t	12×10^{-6}	psi^{-1}
Permeability of reservoir, k_o	80	mD
Reservoir thickness, h	15	ft
Initial reservoir pressure, P_i	4000	psia
Porosity, ϕ	0.20	
Wellbore radius, r_w	0.25	ft
Drainage radius, r_e	3000	ft

Determine the bottomhole flowing pressure 700 hours after the production starts. Round the pressure to the nearest 0.001 psi.

[20 marks]

2. a. A gas well, featuring a wellbore radius of 0.3 ft, is currently producing gas at a steady rate of 20,000 scf/day during transient flow condition. The initial reservoir pressure (shut-in pressure) stands at 3,200 psia at a temperature of 140°F. The reservoir exhibits permeability and thickness values of 65 mD and 15 ft, respectively, with a recorded porosity of 0.20. For reference, the pertinent properties of gas are provided in **TABLE Q2**.

TABLE Q2: Gas properties at different pressure

Pressure, P (psi)	Gas viscosity, μ_g (cP)	Compressibility factor, z
0	0.01270	1.000
800	0.01390	0.882
1600	0.01680	0.794
2400	0.02010	0.763
3200	0.02340	0.797

Determine the real gas pseudopressure ($m(p)$) at the respective pressure. Assuming that the $c_t = 3 \times 10^{-4}$ psi^{-1}, calculate the p_{wf} after 4 hours.

[12 marks]

b. The underground withdrawal of fluid at times t and $t + \Delta t$ can be expressed mathematically as

- $F_t = \int_0^t [Q_o B_o + Q_w B_w + (Q_g - Q_o R_s - Q_w R_w) B_g] dt$

and

- $F_{t+\Delta t} = \int_0^{t+\Delta t} [Q_o B_o + Q_w B_w + (Q_g - Q_o R_s - Q_w R_w) B_g] dt$

respectively.

Explain the term $(Q_g - Q_o R_s - Q_w R_w) B_g$ that appears in both equations above.

[8 marks]

3. a. The material balance method is a simple tool in estimating original hydrocarbon in place and understanding reservoir dynamics. The classical material balance method is based upon law of conservation of mass and a homogeneous reservoir. Material balance method can also be represented graphically using the straight-line equation of Havlena and Odeh.

Iris Field has been producing for four years. The reservoir pressure is still above bubble point and the field is located in shallow water. There is a small aquifer underneath the field.

i. Express the straight-line material balance equation for this field. Please state any assumptions when writing the equation.

[8 marks]

ii. Sketch the plot of the straight-line equation and show how it can be used to estimate the original oil in place.

[7 marks]

b. The change in water volume in a reservoir and the change in grain volume can be expressed mathematically as shown in Equation (1) and Equation (2), respectively.

$$\begin{bmatrix}\text{Change in}\\ \text{water volume}\end{bmatrix} = -W_e + B_w W_p - W C_w \Delta\bar{P} \qquad \text{Equation (1)}$$

$$\begin{bmatrix}\text{Change in}\\ \text{grain volume}\end{bmatrix} = -V_f c_f \Delta\bar{P} \qquad \text{Equation (2)}$$

Show that the change in water and rock volume can be written as:

$$\begin{bmatrix}\text{Change in water and}\\ \text{rock volume}\end{bmatrix} = -W_e + B_w W_p - (1+m) N B_{oi} \left[\frac{c_w S_{wi} + c_f}{1 - S_{wi}}\right] \Delta\bar{P}.$$

[15 marks]

4. a. The production history of an oil well is tabulated in **TABLE Q4**.

TABLE Q4: Production History of the Oil Well

Time, t (year)	Rate, Q_o (STB/day)	Cumulative Oil Production, N_p (STB)
0	750.0	0
1	265.5	155,776.3
2	161.3	230,530.1
3	115.8	280,192.3
4	90.4	317,438.3
5	74.1	347,251.1
6	62.8	372,109.7
7	54.4	393,428.3
8	48.1	412,090.6

Let 1 year = 365 days

The plot of log rate versus cumulative oil production is shown in **FIGURE Q4a**, and the plot of log rate versus log time is shown in **FIGURE Q4b**.

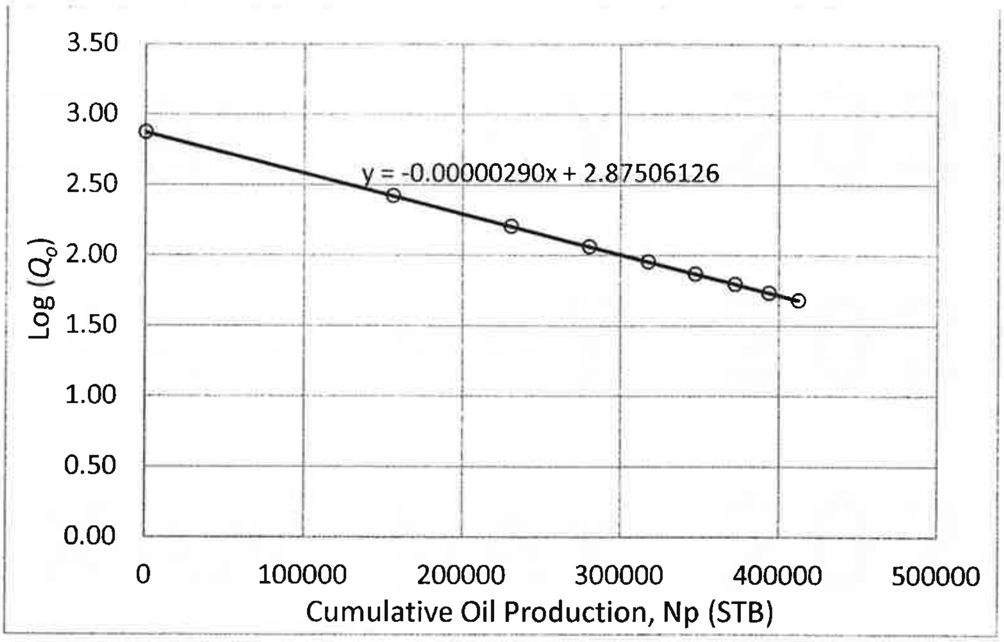

FIGURE Q4a: The plot of log rate versus cumulative oil production (rate in STB/day and oil production in STB)

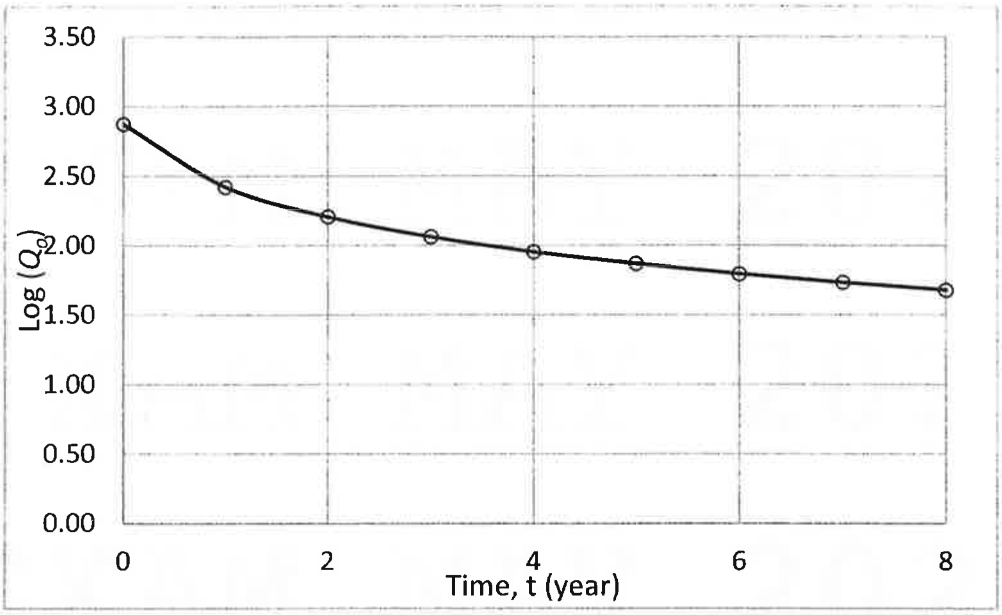

FIGURE Q4b: The plot of log rate versus log time (rate in STB/day and time in years)

i. Identify the best decline curve type to represent the decline. Justify your answer by deriving and verifying the appropriate equation.

[10 marks]

ii. Estimate the oil production rate and cumulative oil production for years 9 and 10.

[11 marks]

b. Reserves and resources can be classified according to their uncertainty and risk. Differentiate between proven reserves, probable reserves and possible reserves.

[9 marks]

- END OF PAPER –

Table 1: Values of the exponential integral, $-E_i(-x)$

$-E_i(-x)$, 0.000 < 0.209, interval = 0.001										
x	0	1	2	3	4	5	6	7	8	9
0.00	$+\infty$	6.332	5.639	5.235	4.948	4.726	4.545	4.392	4.259	4.142
0.01	4.038	3.944	3.858	3.779	3.705	3.637	3.574	3.514	3.458	3.405
0.02	3.355	3.307	3.261	3.218	3.176	3.137	3.098	3.062	3.026	2.992
0.03	2.959	2.927	2.897	2.867	2.838	2.810	2.783	2.756	2.731	2.706
0.04	2.681	2.658	2.634	2.612	2.590	2.568	2.547	2.527	2.507	2.487
0.05	2.468	2.449	2.431	2.413	2.395	2.377	2.360	2.344	2.327	2.311
0.06	2.295	2.279	2.264	2.249	2.235	2.220	2.206	2.192	2.178	2.164
0.07	2.151	2.138	2.125	2.112	2.099	2.087	2.074	2.062	2.050	2.039
0.08	2.027	2.015	2.004	1.993	1.982	1.971	1.960	1.950	1.939	1.929
0.09	1.919	1.909	1.899	1.889	1.879	1.869	1.860	1.850	1.841	1.832
0.10	1.823	1.814	1.805	1.796	1.788	1.779	1.770	1.762	1.754	1.745
0.11	1.737	1.729	1.721	1.713	1.705	1.697	1.689	1.682	1.674	1.667
0.12	1.660	1.652	1.645	1.638	1.631	1.623	1.616	1.609	1.603	1.596
0.13	1.589	1.582	1.576	1.569	1.562	1.556	1.549	1.543	1.537	1.530
0.14	1.524	1.518	1.512	1.506	1.500	1.494	1.488	1.482	1.476	1.470
0.15	1.464	1.459	1.453	1.447	1.442	1.436	1.431	1.425	1.420	1.415
0.16	1.409	1.404	1.399	1.393	1.388	1.383	1.378	1.373	1.368	1.363
0.17	1.358	1.353	1.348	1.343	1.338	1.333	1.329	1.324	1.319	1.314
0.18	1.310	1.305	1.301	1.296	1.291	1.287	1.282	1.278	1.274	1.269
0.19	1.265	1.261	1.256	1.252	1.248	1.243	1.239	1.235	1.231	1.227
0.20	1.223	1.219	1.215	1.210	1.206	1.202	1.202	1.195	1.191	1.187

$-E_i(-x)$, 0.000 < 2.09, interval = 0.01

x	0	1	2	3	4	5	6	7	8	9
0.0	$+\infty$	4.038	3.335	2.959	2.681	2.468	2.295	2.151	2.027	1.919
0.1	1.823	1.737	1.660	1.589	1.524	1.464	1.409	1.358	1.309	1.265
0.2	1.223	1.183	1.145	1.110	1.076	1.044	1.014	0.985	0.957	0.931
0.3	0.906	0.882	0.858	0.836	0.815	0.794	0.774	0.755	0.737	0.719
0.4	0.702	0.686	0.670	0.655	0.640	0.625	0.611	0.598	0.585	0.572
0.5	0.560	0.548	0.536	0.525	0.514	0.503	0.493	0.483	0.473	0.464
0.6	0.454	0.445	0.437	0.428	0.420	0.412	0.404	0.396	0.388	0.381
0.7	0.374	0.367	0.360	0.353	0.347	0.340	0.334	0.328	0.322	0.316
0.8	0.311	0.305	0.300	0.295	0.289	0.284	0.279	0.274	0.269	0.265
0.9	0.260	0.256	0.251	0.247	0.243	0.239	0.235	0.231	0.227	0.223
1.0	0.219	0.216	0.212	0.209	0.205	0.202	0.198	0.195	0.192	0.189
1.1	0.186	0.183	0.180	0.177	0.174	0.172	0.169	0.166	0.164	0.161
1.2	0.158	0.156	0.153	0.151	0.149	0.146	0.144	0.142	0.140	0.138
1.3	0.135	0.133	0.131	0.129	0.127	0.125	0.124	0.122	0.120	0.118
1.4	0.116	0.114	0.113	0.111	0.109	0.108	0.106	0.105	0.103	0.102
1.5	0.1000	0.0985	0.0971	0.0957	0.0943	0.0929	0.0915	0.0902	0.0889	0.0876
1.6	0.0863	0.0851	0.0838	0.0826	0.0814	0.0802	0.0791	0.0780	0.0768	0.0757
1.7	0.0747	0.0736	0.0725	0.0715	0.0705	0.0695	0.0685	0.0675	0.0666	0.0656
1.8	0.0647	0.0638	0.0629	0.0620	0.0612	0.0603	0.0595	0.0586	0.0578	0.0570
1.9	0.0562	0.0554	0.0546	0.0539	0.0531	0.0524	0.0517	0.0510	0.0503	0.0496
2.0	0.0489	0.0482	0.0476	0.0469	0.0463	0.0456	0.0450	0.0444	0.0438	0.0432

$2.0 < x < 10.9$, interval $= 0.1$										
x	0	1	2	3	4	5	6	7	8	9
2	4.89×10^{-2}	4.26×10^{-2}	3.72×10^{-2}	3.25×10^{-2}	2.84×10^{-2}	2.49×10^{-2}	2.19×10^{-2}	1.92×10^{-2}	1.69×10^{-2}	1.48×10^{-2}
3	1.30×10^{-2}	1.15×10^{-2}	1.01×10^{-2}	8.94×10^{-3}	7.89×10^{-3}	6.87×10^{-3}	6.16×10^{-3}	5.45×10^{-3}	4.82×10^{-3}	4.27×10^{-3}
4	3.78×10^{-3}	3.35×10^{-3}	2.97×10^{-3}	2.64×10^{-3}	2.34×10^{-3}	2.07×10^{-3}	1.84×10^{-3}	1.64×10^{-3}	1.45×10^{-3}	1.29×10^{-3}
5	1.15×10^{-3}	1.02×10^{-3}	9.08×10^{-4}	8.09×10^{-4}	7.19×10^{-4}	6.41×10^{-4}	5.71×10^{-4}	5.09×10^{-4}	4.53×10^{-4}	4.04×10^{-4}
6	3.60×10^{-4}	3.21×10^{-4}	2.86×10^{-4}	2.55×10^{-4}	2.28×10^{-4}	2.03×10^{-4}	1.82×10^{-4}	1.62×10^{-4}	1.45×10^{-4}	1.29×10^{-4}
7	1.15×10^{-4}	1.03×10^{-4}	9.22×10^{-5}	8.24×10^{-5}	7.36×10^{-5}	6.58×10^{-5}	5.89×10^{-5}	5.26×10^{-5}	4.71×10^{-5}	4.21×10^{-5}
8	3.77×10^{-5}	3.37×10^{-5}	3.02×10^{-5}	2.70×10^{-5}	2.42×10^{-5}	2.16×10^{-5}	1.94×10^{-5}	1.73×10^{-5}	1.55×10^{-5}	1.39×10^{-5}
9	1.24×10^{-5}	1.11×10^{-5}	9.99×10^{-6}	8.95×10^{-6}	8.02×10^{-6}	7.18×10^{-6}	6.44×10^{-6}	5.77×10^{-6}	5.17×10^{-6}	4.64×10^{-6}
10	4.15×10^{-6}	3.73×10^{-6}	3.34×10^{-6}	3.00×10^{-6}	2.68×10^{-6}	2.41×10^{-6}	2.16×10^{-6}	1.94×10^{-6}	1.74×10^{-6}	1.56×10^{-6}

- $\frac{1}{r}\frac{\partial}{\partial r}(r[v\rho]) = \frac{\partial}{\partial t}(\phi\rho)$
- $\frac{1}{r}\frac{\partial}{\partial r}\left(r\left[\frac{k}{\mu}\frac{\partial p}{\partial r}\rho\right]\right) = \rho\phi c_f\frac{\partial p}{\partial t} + \phi\frac{\partial \rho}{\partial t}$
- $\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = \frac{\phi c_t \mu}{k}\frac{\partial p}{\partial t}$
- $\nabla \cdot (\rho \boldsymbol{u}) = -\frac{\partial}{\partial t}(\phi\rho)$
- $v = \frac{k}{\mu}\frac{\partial p}{\partial r}$
- $c = \frac{1}{\rho}\frac{\partial \rho}{\partial p}$
- $v = 0.001127\frac{k}{\mu}\frac{\partial p}{\partial r}$
  - Where,
    - ϕ = porosity
    - ρ = density, lb/ft^3
    - v = fluid velocity, ft/day also known as Darcy velocity
    - v = Volumetric flow rate, bbl/day
- $\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = \frac{\phi c_t \mu}{0.000264k}\frac{\partial p}{\partial t}$
  - where,
    - k is permeability in mD
    - r is radial position in ft
    - p is pressure in psia
    - c_t is total isothermal compressibility in psi^{-1}
    - t is time in hours (if in days, replace 0.000264 with 0.006328)
    - ϕ is porosity in fraction
    - μ is viscosity in Cp
- $p(r,t) = p_i + \left[\frac{70.6 Q_o \mu_o B_o}{kh}\right] E_i\left[-\frac{948\phi\mu_o c_t r^2}{kt}\right]$
    - $p(r,t)$ = pressure at radius r in ft from well after t hours,
    - Q_o = flow rate in STB/day
- $E_i(-x) = -\int_x^\infty \frac{e^{-u}du}{u}$
- $E_i(-x) = \left[\ln x - \frac{x}{1!} + \frac{x^2}{2(2!)} - \frac{x^3}{3(3!)} + \cdots\right]$
- $E_i(-x) \approx \ln(1.781x)$

- $Q_o = 0.00708 \frac{kh(p_e - p_{wf})}{\mu_o B_o \ln\left(\frac{r_e}{r_w}\right)}$
- $P_D = \frac{p_e - p_{wf}}{\left(\frac{Q_o \mu_o B_o}{0.00708kh}\right)}$
- $r_{eD} = \frac{r_e}{r_w}$
- $t_D = \frac{0.000264kt}{\phi \mu c_t r_w^2}$
- $t_{DA} = \frac{0.000264kt}{\phi \mu c_t A} = t_D \left(\frac{r_w^2}{A}\right)$
- $r_D = \frac{r}{r_w}$
- $\frac{\partial^2 p_D}{\partial r_D^2} + \frac{1}{r_D}\frac{\partial p_D}{\partial r_D} = \frac{\partial p_D}{\partial t_D}$
- Harmonic DCA
    - $q_t = \frac{q_i}{1 + D_i t}$
    - $N_p = \frac{q_i}{D_i} \ln\left(\frac{q_i}{q_t}\right)$
- Hyperbolic DCA
    - $q_t = \frac{q_i}{(1 + bD_i t)^{\frac{1}{b}}}$
    - $N_p = \left[\frac{q_i}{(1-b)D_i}\right]\left[1 - \left(\frac{q_t}{q_i}\right)^{1-b}\right]$
- Exponential DCA
    - $q_t = q_i e^{-D_i t}$
    - $N_p(t) = \frac{q_i - q_t}{D_i}$
- $NB_{oi} + NmB_{oi} - N[B_o + (R_{si} - R_s)B_g] + N_p[B_o + (R_{si} - R_s)B_g] + N_p B_g (R_p - R_{si}) - \frac{NmB_{oi}B_g}{B_{gi}} = W_e - B_w W_p + (1+m)NB_{oi}\left[\frac{c_w S_{wi} + c_f}{1 - S_{wi}}\right]\Delta\bar{P}$
- $B_t = B_o + (R_{sb} - R_s)B_g$
- $F = N(E_o + E_{f,w} + mE_g) + W_e$
    - $F = N_p[B_t + (R_p - R_{si})B_g] + B_w W_p$
    - $E_o = (B_o - B_{oi}) + (R_p - R_{si})B_g$
    - $E_g = B_{oi}\left(\frac{B_g}{B_{gi}} - 1\right)$
    - $E_{f,w} = (1+m)B_{oi}\left[\frac{c_w S_{wi} + C_f}{1 - S_{wi}}\right]\Delta\bar{P}$

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p_D vs. t_D—Infinite-Radial System, Constant-Rate at the Inner Boundary (*After Lee, J., Well Testing, SPE Textbook Series.*) (*Permission to publish by the SPE, copyright SPE, 1982*)

t_D	p_D	t_D	p_D	t_D	p_D
0	0	0.15	0.3750	60.0	2.4758
0.0005	0.0250	0.2	0.4241	70.0	2.5501
0.001	0.0352	0.3	0.5024	80.0	2.6147
0.002	0.0495	0.4	0.5645	90.0	2.6718
0.003	0.0603	0.5	0.6167	100.0	2.7233
0.004	0.0694	0.6	0.6622	150.0	2.9212
0.005	0.0774	0.7	0.7024	200.0	3.0636
0.006	0.0845	0.8	0.7387	250.0	3.1726
0.007	0.0911	0.9	0.7716	300.0	3.2630
0.008	0.0971	1.0	0.8019	350.0	3.3394
0.009	0.1028	1.2	0.8672	400.0	3.4057
0.01	0.1081	1.4	0.9160	450.0	3.4641
0.015	0.1312	2.0	1.0195	500.0	3.5164
0.02	0.1503	3.0	1.1665	550.0	3.5643
0.025	0.1669	4.0	1.2750	600.0	3.6076
0.03	0.1818	5.0	1.3625	650.0	3.6476
0.04	0.2077	6.0	1.4362	700.0	3.6842
0.05	0.2301	7.0	1.4997	750.0	3.7184
0.06	0.2500	8.0	1.5557	800.0	3.7505
0.07	0.2680	9.0	1.6057	850.0	3.7805
0.08	0.2845	10.0	1.6509	900.0	3.8088
0.09	0.2999	15.0	1.8294	950.0	3.8355
0.1	0.3144	20.0	1.9601	1,000.0	3.8584
		30.0	2.1470		
		40.0	2.2824		
		50.0	2.3884		

Notes: For $t_D < 0.01$, $p_D \cong 2$ Zt_D/x.
For $100 < t_D < 0.25\, r_{eD}^2$, $p_D \cong 0.5\,(\ln t_D + 0.80907)$.

- For $tD < 0.01$: $p_D = 2\sqrt{\frac{t_D}{\pi}}$
- For $tD > 100$: $p_D = 0.5[\ln(t_D) + 0.80907]$
- For $0.02 < tD < 1000$:
  - $p_D = a_1 + a_2 \ln(tD) + a_3[\ln(tD)]2 \ + a_4[\ln(tD)]3 \ + a_5(tD) + a_6(tD)^2 + a_6(tD)^2 + a_7(tD)^3 + \frac{a_8}{(tD)}$

where

$a_1 = 0.8085064$ $a_2 = 0.29302022$ $a_3 = 3.5264177(10^{-2})$

$a_4 = -1.4036304(10^{-3})$ $a_5 = -4.7722225(10^{-4})$ $a_6 = 5.1240532(10^{-7})$

$a_7 = -2.3033017(10^{-10})$ $a_8 = -2.6723117(10^{-3})$

APPENDIX 1

p_D vs. t_D—Finite-Radial System, Constant-Rate at the Inner Boundary
(After Lee, J., Well Testing, SPE Textbook Series.)
(Permission to publish by the SPE, copyright SPE, 1982)

$r_{eD} = 1.5$		$r_{eD} = 2.0$		$r_{eD} = 2.5$		$r_{eD} = 3.0$		$r_{eD} = 3.5$		$r_{eD} = 4.0$	
t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D
0.06	0.251	0.22	0.443	0.40	0.565	0.52	0.627	1.0	0.802	1.5	0.927
0.08	0.288	0.24	0.459	0.42	0.576	0.54	0.636	1.1	0.830	1.6	0.948
0.10	0.322	0.26	0.476	0.44	0.587	0.56	0.645	1.2	0.857	1.7	0.968
0.12	0.355	0.28	0.492	0.46	0.598	0.60	0.662	1.3	0.882	1.8	0.988
0.14	0.387	0.30	0.507	0.48	0.608	0.65	0.683	1.4	0.906	1.9	1.007
0.16	0.420	0.32	0.522	0.50	0.618	0.70	0.703	1.5	0.929	2.0	1.025
0.18	0.452	0.34	0.536	0.52	0.628	0.75	0.721	1.6	0.951	2.2	1.059
0.20	0.484	0.36	0.551	0.54	0.638	0.80	0.740	1.7	0.973	2.4	1.092
0.22	0.516	0.38	0.565	0.56	0.647	0.85	0.758	1.8	0.994	2.6	1.123
0.24	0.548	0.40	0.579	0.58	0.657	0.90	0.776	1.9	1.014	2.8	1.154
0.26	0.580	0.42	0.593	0.60	0.666	0.95	0.791	2.0	1.034	3.0	1.184
0.28	0.612	0.44	0.607	0.65	0.688	1.0	0.806	2.25	1.083	3.5	1.255
0.30	0.644	0.46	0.621	0.70	0.710	1.2	0.865	2.50	1.130	4.0	1.324
0.35	0.724	0.48	0.634	0.75	0.731	1.4	0.920	2.75	1.176	4.5	1.392
0.40	0.804	0.50	0.648	0.80	0.752	1.6	0.973	3.0	1.221	5.0	1.460
0.45	0.884	0.60	0.715	0.85	0.772	2.0	1.076	4.0	1.401	5.5	1.527
0.50	0.964	0.70	0.782	0.90	0.792	3.0	1.328	5.0	1.579	6.0	1.594
0.55	1.044	0.80	0.849	0.95	0.812	4.0	1.578	6.0	1.757	6.5	1.660
0.60	1.124	0.90	0.915	1.0	0.832	5.0	1.828			7.0	1.727
0.65	1.204	1.0	0.982	2.0	1.215					8.0	1.861
0.70	1.284	2.0	1.649	3.0	1.506					9.0	1.994
0.75	1.364	3.0	2.316	4.0	1.977					10.0	2.127
0.80	1.444	5.0	3.649	5.0	2.398						

$r_{eD} = 4.5$		$r_{eD} = 5.0$		$r_{eD} = 6.0$		$r_{eD} = 7.0$		$r_{eD} = 8.0$		$r_{eD} = 9.0$		$r_{eD} = 10.0$	
t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D
2.0	1.023	3.0	1.167	4.0	1.275	6.0	1.436	8.0	1.556	10.0	1.651	12.0	1.732
2.1	1.040	3.1	1.180	4.5	1.322	6.5	1.470	8.5	1.582	10.5	1.673	12.5	1.750
2.2	1.056	3.2	1.192	5.0	1.364	7.0	1.501	9.0	1.607	11.0	1.693	13.0	1.768
2.3	1.702	3.3	1.204	5.5	1.404	7.5	1.531	9.5	1.631	11.5	1.713	13.5	1.784
2.4	1.087	3.4	1.215	6.0	1.441	8.0	1.559	10.0	1.653	12.0	1.732	14.0	1.801
2.5	1.102	3.5	1.227	6.5	1.477	8.5	1.586	10.5	1.675	12.5	1.750	14.5	1.817
2.6	1.116	3.6	1.238	7.0	1.511	9.0	1.613	11.0	1.697	13.0	1.768	15.0	1.832
2.7	1.130	3.7	1.249	7.5	1.544	9.5	1.638	11.5	1.717	13.5	1.786	15.5	1.847
2.8	1.144	3.8	1.259	8.0	1.576	10.0	1.663	12.0	1.737	14.0	1.803	16.0	1.862
2.9	1.158	3.9	1.270	8.5	1.607	11.0	1.711	12.5	1.757	14.5	1.819	17.0	1.890
3.0	1.171	4.0	1.281	9.0	1.638	12.0	1.757	13.0	1.776	15.0	1.835	18.0	1.917

$r_{eD} = 4.5$		$r_{eD} = 5.0$		$r_{eD} = 6.0$		$r_{eD} = 7.0$		$r_{eD} = 8.0$		$r_{eD} = 9.0$		$r_{eD} = 10.0$	
t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D
3.2	1.197	4.2	1.301	9.5	1.668	13.0	1.810	13.5	1.795	15.5	1.851	19.0	1.943
3.4	1.222	4.4	1.321	10.0	1.698	14.0	1.845	14.0	1.813	16.0	1.867	20.0	1.968
3.6	1.246	4.6	1.340	11.0	1.757	15.0	1.888	14.5	1.831	17.0	1.897	22.0	2.017
3.8	1.269	4.8	1.360	12.0	1.815	16.0	1.931	15.0	1.849	18.0	1.926	24.0	2.063
4.0	1.292	5.0	1.378	13.0	1.873	17.0	1.974	17.0	1.919	19.0	1.955	26.0	2.108
4.5	1.349	5.5	1.424	14.0	1.931	18.0	2.016	19.0	1.986	20.0	1.983	28.0	2.151
5.0	1.403	6.0	1.469	15.0	1.988	19.0	2.058	21.0	2.051	22.0	2.037	30.0	2.194
5.5	1.457	6.5	1.513	16.0	2.045	20.0	2.100	23.0	2.116	24.0	2.906	32.0	2.236
6.0	1.510	7.0	1.556	17.0	2.103	22.0	2.184	25.0	2.180	26.0	2.142	34.0	2.278
7.0	1.615	7.5	1.598	18.0	2.160	24.0	2.267	30.0	2.340	28.0	2.193	36.0	2.319
8.0	1.719	8.0	1.641	19.0	2.217	26.0	2.351	35.0	2.499	30.0	2.244	38.0	2.360
9.0	1.823	9.0	1.725	20.0	2.274	28.0	2.434	40.0	2.658	34.0	2.345	40.0	2.401
10.0	1.927	10.0	1.808	25.0	2.560	30.0	2.517	45.0	2.817	38.0	2.446	50.0	2.604
11.0	2.031	11.0	1.892	30.0	2.846					40.0	2.496	60.0	2.806
12.0	2.135	12.0	1.975							45.0	2.621	70.0	3.008
13.0	2.239	13.0	2.059							50.0	2.746	80.0	3.210
14.0	2.343	14.0	2.142							60.0	2.996	90.0	3.412
15.0	2.447	15.0	2.225							70.0	3.246	100.0	3.614

Notes: For t_D smaller than values listed in this table for a given r_{eD}, reservoir is infinite acting.
Find p_D in Table 6-2.
For $25 < t_D$ and t_D larger than values in table.

$$p_D \cong \frac{(\frac{1}{2} + 2t_D)}{(r_{eD}^2 - 1)} - \frac{3r_{eD}^4 - 4r_{eD}^4 \ln r_{eD} - 2r_{eD}^2 - 1}{4(r_{eD}^2 - 1)^2}$$

For wells in rebounded reservoirs with $r_{eD}^2 \gg 1$

$$p_D \cong \frac{2t_D}{r_{eD}^2} + \ln r_{eD} - \frac{3}{4}.$$

- **For $25 < t_D$ and $0.25 r_{eD}^2 < t_D$**
    - $p_D = \frac{0.5 + 2t_D}{r_{eD}^2 - 1} - \frac{r_{eD}^4 (3 - 4\ln(r_{eD}) - 2r_{eD}^2 - 1)}{4(r_{eD}^2 - 1)^2}$
- **When $r_{eD}^2 \gg 1$, $p_D = \frac{2t_D}{r_{eD}^2} + \ln(r_{eD}) - 0.75$**
- $\frac{dP}{dt} = -\frac{0.23396q}{c_t \pi r_e^2 h \phi}$
- $\bar{P}_r = \frac{\sum_i \bar{P}_{r,i} V_i}{\sum_i V_i}$
- $\bar{P}_r = \frac{\sum_i \bar{P}_{r,i} q_i}{\sum_i q_i}$
- $\bar{P}_r = \frac{\sum_j \frac{(\bar{P}q)_j}{\left(\frac{\partial \bar{P}}{\partial t}\right)_j}}{\sum_j \frac{q_j}{\left(\frac{\partial \bar{P}}{\partial t}\right)_j}}$
- $\bar{P}_r = \frac{\sum_j \frac{\bar{P}_j \Delta(F)_j}{\Delta \bar{P}_j}}{\sum_j \frac{\Delta(F)_j}{\Delta \bar{P}_j}}$
- $\Delta(F) = F_{t+\Delta t} - F_t$
- $\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} = -\frac{887.22 q\mu}{Ahk}$

- $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial P}{\partial r}\right) = -\frac{887.22q\mu}{\pi r_e^2 hk}$
- $Q = \frac{0.00708kh(\bar{P}_r - P_{wf})}{\mu B\left(\ln\left(\frac{r_e}{r_w}\right) - \frac{3}{4}\right)}$
- $P_{wf} = \bar{P}_r - \frac{162.6QB\mu}{kh}\log\left[\frac{4A}{1.781C_A r_w^2}\right]$
- $\bar{P}_r = P_i - \frac{0.23396qt}{c_t Ah\phi}$
- $\frac{\partial^2 m(P)}{\partial r^2} + \frac{1}{r}\frac{\partial m(P)}{\partial r} = \frac{\phi\mu c_t}{0.000264}\frac{\partial m(P)}{\partial t}$
- $Q_g = \frac{kh[m(\bar{P}_r) - m(P_{wf})]}{1422T\left[\ln\left(\frac{r_e}{r_w}\right) - 0.75\right]}$
- Where,
  - Q_g = Gas flow rate in Mscf/day
  - T = Temperature in ºR
  - k = permeability in mD
- $Q_g = \frac{kh\left[\bar{P}_r^2 - P_{wf}^2\right]}{1422T\bar{\mu}\bar{z}\left[\ln\left(\frac{r_e}{r_w}\right) - 0.75\right]}$
- $\bar{P} = \sqrt{\frac{\bar{P}_r^2 + P_{wf}^2}{2}}$

Shape Factors for Various Single-Well Drainage Areas
(*After Earlougher, R.,* Advances in Well Test Analysis,
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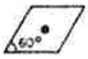
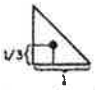
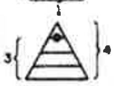
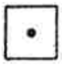
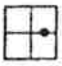
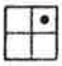
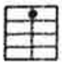
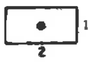
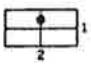
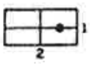
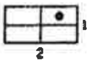
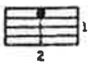
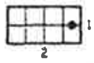
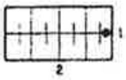
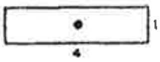
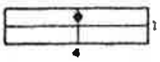
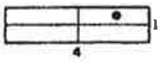
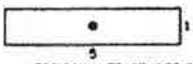
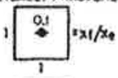
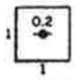
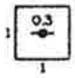
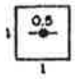
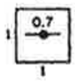
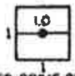
In Bounded Reservoirs	C_A	$\ln C_A$	$\frac{1}{2}\ln\left(\frac{2.2458}{C_A}\right)$	Exact for t_{DA} >	Less Than 1% Error For t_{DA} >	Use Infinite System Solution with Less Than 1% Error for t_{DA} <
	31.62	3.4538	−1.3224	0.1	0.06	0.10
	31.6	3.4532	−1.3220	0.1	0.06	0.10
	27.6	3.3178	−1.2544	0.2	0.07	0.09
	27.1	3.2995	−1.2452	0.2	0.07	0.09
	21.9	3.0865	−1.1387	0.4	0.12	0.08
	0.098	−2.3227	+1.5659	0.9	0.60	0.015
	30.8828	3.4302	−1.3106	0.1	0.05	0.09
	12.9851	2.5638	−0.8774	0.7	0.25	0.03
	4.5132	1.5070	−0.3490	0.6	0.30	0.025
	3.3351	1.2045	−0.1977	0.7	0.25	0.01
	21.8369	3.0836	−1.1373	0.3	0.15	0.025
	10.8374	2.3830	−0.7870	0.4	0.15	0.025
	4.5141	1.5072	−0.3491	1.5	0.50	0.06
	2.0769	0.7309	−0.0391	1.7	0.50	0.02
	3.1573	1.1497	−0.1703	0.4	0.15	0.005

In Bounded Reservoirs	C_A	$\ln C_A$	$\frac{1}{2}\ln\left(\frac{2.2458}{C_A}\right)$	Exact for t_{DA} >	Less Than 1% Error For t_{DA} >	Use Infinite System Solution with Less Than 1% Error for t_{DA} <
	0.5813	−0.5425	+0.6758	2.0	0.60	0.02
	0.1109	−2.1991	+1.5041	3.0	0.60	0.005
	5.3790	1.6825	−0.4367	0.8	0.30	0.01
	2.6896	0.9894	−0.0902	0.8	0.30	0.01
	0.2318	−1.4619	+1.1355	4.0	2.00	0.03
	0.1155	−2.1585	+1.4838	4.0	2.00	0.01
	2.3606	0.8589	−0.0249	1.0	0.40	0.025
IN VERTICALLY FRACTURED RESERVOIRS	Use $(x_e/x_f)^2$ in place of A/r_w^2 for fractured systems					
	2.6541	0.9761	−0.0835	0.175	0.08	cannot use
	2.0348	0.7104	+0.0493	0.175	0.09	cannot use
	1.9986	0.6924	+0.0583	0.175	0.09	cannot use
	1.6620	0.5080	+0.1505	0.175	0.09	cannot use
	1.3127	0.2721	+0.2685	0.175	0.09	cannot use
	0.7887	−0.2374	+0.5232	0.175	0.09	cannot use
IN WATER-DRIVE RESERVOIRS						
	19.1	2.95	−1.07	—	—	—
IN RESERVOIRS OF UNKNOWN PRODUCTION CHARACTER						
	25.0	3.22	−1.20	—	—	—