UNIVERSITI TEKNOLOGI PETRONAS

FINAL EXAMINATION
MAY 2024 SEMESTER

COURSE	**:**	**AAB1032 - STATICS AND DYNAMICS**
DATE	**:**	**1 AUGUST 2024 (THURSDAY)**
TIME	**:**	**9.00 AM - 11.00 AM (2 HOURS)**

INSTRUCTIONS TO CANDIDATES

1. Answer **ALL** questions in the Answer Booklet.
2. Begin **EACH** answer on a new page in the Answer Booklet.
3. Indicate clearly answers that are cancelled, if any.
4. Where applicable, show clearly steps taken in arriving at the solutions and indicate **ALL** assumptions, if any.
5. **DO NOT** open this Question Booklet until instructed.

Note :

i. There are **NINE (9)** pages in this Question Booklet including the cover page and appendix.

ii. **DOUBLE-SIDED** Question Booklet.

1. The overhanging beam shown in **FIGURE Q1** is pin-supported at point A, while the beam has a total length of 6 m to point C. At the end of the beam is 5 kN force attached to the system. The beam is loaded with uniform distributed load of 2 kN/m from A to B where a roller supports the beam. From location B, the uniform distributed load of 2 kN/m trims to form a triangular distributed load which ends at 0 kN at point C. Point D is located just to the left of the roller support at B, where the couple moment acts.

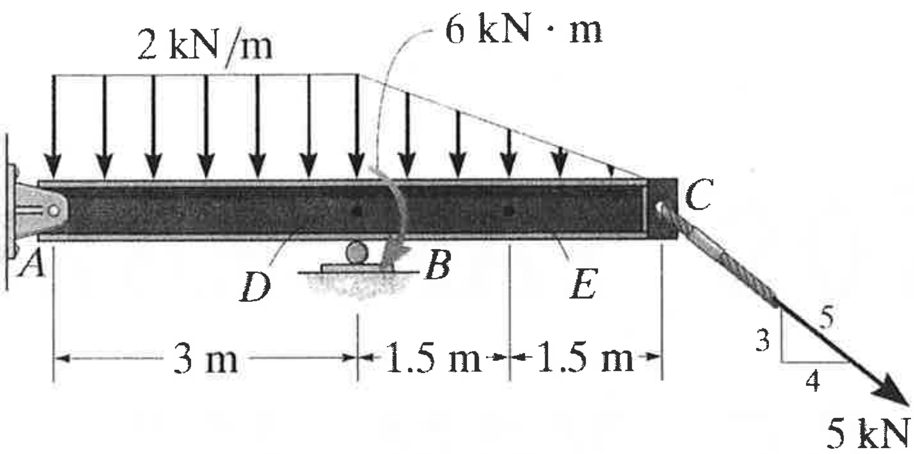


FIGURE Q1

a. Determine the internal normal force, shear force, and moment at point D showing the free-body diagram.

[13 marks]

b. Analyze the forces and moment at point D when it is moved where the final location of point D is 2 meters from support A while the distributed loads are replaced with a point load of 20 kN at 1.5 m to the left of point C. Explain the effect of stability on the system.

[12 marks]

2. Referring to **FIGURE Q2,**

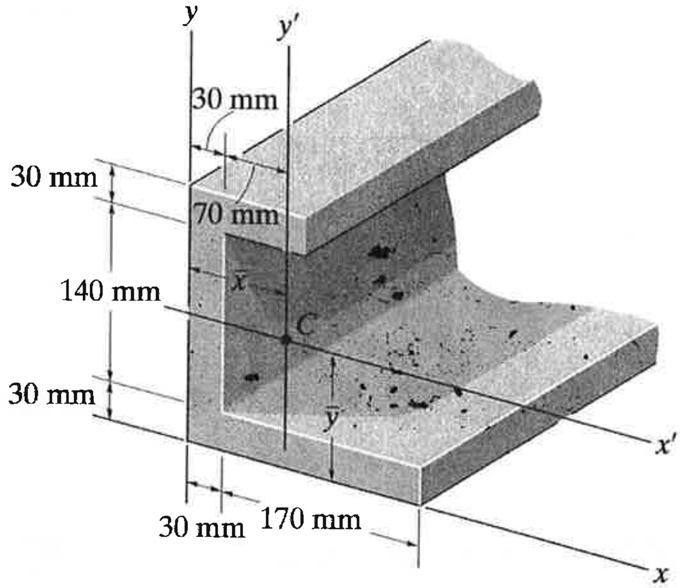


FIGURE Q2

a. Compute the moments of inertia I_x and I_y for the beam's cross-sectional area about the x and y axes.

[13 marks]

b. i. Determine the distance $\overline{y}$ to the centroid C of the beam's cross-sectional area.

[3 marks]

ii. Compute the moment of inertia $\overline{I}_{x'}$ about the x' axis.

[3 marks]

c. i. Determine the distance $\overline{x}$ to the centroid C of the beam's cross-sectional area.

[3 marks]

ii. Compute the moment of inertia $\overline{I}_{y'}$ about the y' axis.

[3 marks]

Appendices

Cartesian Vector

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

Magnitude

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Directions

$$\mathbf{u}_A = \frac{\mathbf{A}}{A} = \frac{A_x}{A}\mathbf{i} + \frac{A_y}{A}\mathbf{j} + \frac{A_z}{A}\mathbf{k}$$
$$= \cos\alpha\mathbf{i} + \cos\beta\mathbf{j} + \cos\gamma\mathbf{k}$$
$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

Dot Product

$$\mathbf{A}\cdot\mathbf{B} = AB\cos\theta = A_xB_x + A_yB_y + A_zB_z$$

Cross Product

$$\mathbf{C} = \mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Cartesian Position Vector

$$\mathbf{r} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} + (z_2 - z_1)\mathbf{k}$$

Cartesian Force Vector

$$\mathbf{F} = F\mathbf{u} = F\left(\frac{\mathbf{r}}{r}\right)$$

Moment of a Force

$$M_o = Fd$$
$$\mathbf{M}_o = \mathbf{r}\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Moment of a Force About a Specified Axis

$$M_a = \mathbf{u}\cdot\mathbf{r}\times\mathbf{F} = \begin{vmatrix} u_x & u_y & u_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Simplification of a Force and Couple System

$$\mathbf{F}_R = \Sigma\mathbf{F}$$
$$(\mathbf{M}_R)_O = \Sigma\mathbf{M} + \Sigma\mathbf{M}_O$$

Equilibrium

Particle

$$\Sigma F_x = 0,\ \Sigma F_y = 0,\ \Sigma F_z = 0$$

Rigid Body-Two Dimensions

$$\Sigma F_x = 0,\ \Sigma F_y = 0,\ \Sigma M_O = 0$$

Rigid Body-Three Dimensions

$$\Sigma F_x = 0,\ \Sigma F_y = 0,\ \Sigma F_z = 0$$
$$\Sigma M_{x'} = 0,\ \Sigma M_{y'} = 0,\ \Sigma M_{z'} = 0$$

Friction

Static (maximum) $F_s = \mu_s N$

Kinetic $F_k = \mu_k N$

Center of Gravity

Particles or Discrete Parts

$$\bar{r} = \frac{\Sigma\tilde{r}W}{\Sigma W}$$

Body

$$\bar{r} = \frac{\int \tilde{r}\,dW}{\int dW}$$

Area and Mass Moments of Inertia

$$I = \int r^2\,dA \qquad I = \int r^2\,dm$$

Parallel-Axis Theorem

$$I = \bar{I} + Ad^2 \qquad I = \bar{I} + md^2$$

Radius of Gyration

$$k = \sqrt{\frac{I}{A}} \qquad k = \sqrt{\frac{I}{m}}$$

Virtual Work

$$\delta U = 0$$

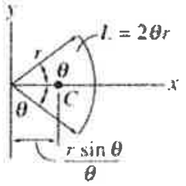
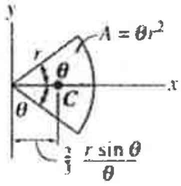
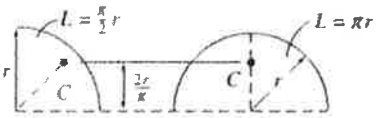
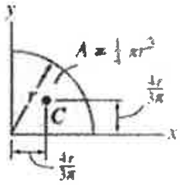
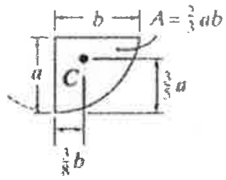
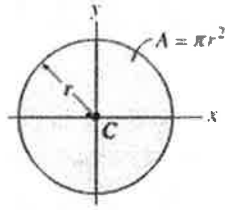
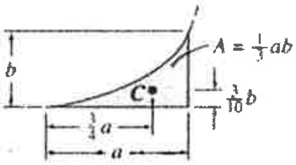
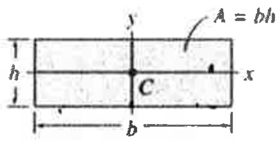
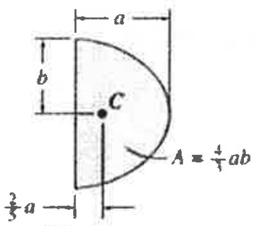
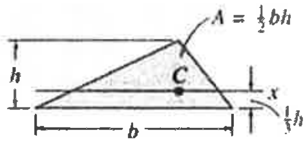
$$\bar{x} = \frac{\int \tilde{x}\,dW}{\int dW} \qquad \bar{y} = \frac{\int \tilde{y}\,dW}{\int dW} \qquad \bar{z} = \frac{\int \tilde{z}\,dW}{\int dW}$$

KINEMATICS

Particle Rectilinear Motion

Variable a	*Constant $a = a_c$*
$a = \frac{dv}{dt}$	$v = v_0 + a_c t$
$v = \frac{ds}{dt}$	$s = s_0 + v_0 t + \frac{1}{2}a_c t^2$
$v\,dv = a\,ds$	$v^2 = v_0^2 + 2a_c(s - s_0)$

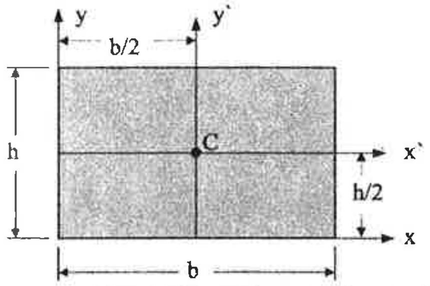
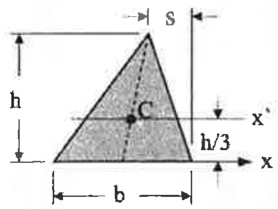
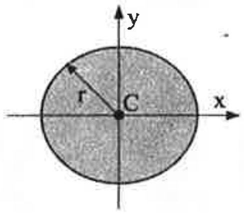
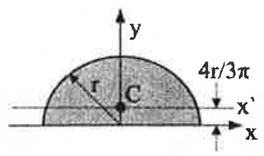
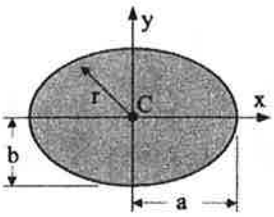
Cartesian Vector | Equilibrium

Centroid Location	Centroid Location	Area Moment of Inertia
 Circular arc segment	 Circular sector area	$I_x = \frac{1}{4} r^4(\theta - \frac{1}{2}\sin 2\theta)$ $I_y = \frac{1}{4} r^4(\theta + \frac{1}{2}\sin 2\theta)$
 Quarter and semicircle arcs	 Quarter circle area	$I_x = \frac{1}{16}\pi r^4$ $I_y = \frac{1}{16}\pi r^4$
 Semiparabolic area	 Circular area	$I_x = \frac{1}{4}\pi r^4$ $I_y = \frac{1}{4}\pi r^4$
 Exparabolic area	 Rectangular area	$I_x = \frac{1}{12}bh^3$ $I_y = \frac{1}{12}hb^3$
 Parabolic area	 Triangular area	$I_x = \frac{1}{36}bh^3$

Note: In the table below, the overbar indicates the moment of inertia is taken about an axis that passes through the centroid, denoted as '*C*'. Parallel axis theorems are:

$$I_x = \bar{I}_x + Ad^2 \qquad I_y = \bar{I}_y + Ad^2 \qquad I_{xy} = \bar{I}_{xy} + A\bar{x}\bar{y}$$

Here, *A* is the area of the shape, *d* is the distance from the centroidal axis to the desired parallel axis, and $\bar{x}\ \bar{y}$ are the *x* and *y* distances of the centroid from the origin of the desired coordinate frame.

Rectangle: $\bar{I}_{x'} = \frac{1}{12}bh^3$ $\quad I_x = \frac{1}{3}bh^3$ $\bar{I}_{y'} = \frac{1}{12}b^3h$ $\quad I_y = \frac{1}{3}b^3h$ $\bar{I}_{xy'} = 0$ $\quad Area = bh$	
Triangle: $\bar{I}_{x'} = \frac{1}{36}bh^3$ $\quad I_x = \frac{1}{12}bh^3$ $\bar{I}_{xy} = \frac{b(b-2s)h^2}{72}$ $\quad Area = \frac{1}{2}bh$	
Circle: $\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$ $\bar{I}_{xy'} = 0$ $Area = \pi r^2$	
Semi-circle: $I_x = \bar{I}_y = \frac{1}{8}\pi r^4$ $\quad \bar{I}_{x'} = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)r^4$ $\bar{I}_{xy'} = 0$ $\quad Area = \frac{\pi r^2}{2}$	
Ellipse: $\bar{I}_x = \frac{1}{4}\pi ab^3$ $\quad \bar{I}_y = \frac{1}{4}\pi a^3b$ $\bar{I}_{xy'} = 0$ $Area = \pi ab$	

Double Angle Formulas	Half Angle Formulas
$\sin 2\theta = 2\sin\theta\cos\theta$	$\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$
$\cos 2\theta = \cos^2\theta - \sin^2\theta$	$\sin\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{2}}$
$= 2\cos^2\theta - 1$	$\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$
$= 1 - 2\sin^2\theta$	$\sin\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1+\cos\theta}{2}}$
$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$	$\tan\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}}$