UNIVERSITI TEKNOLOGI PETRONAS

FINAL EXAMINATION
JANUARY 2025 SEMESTER

COURSE : EEB4213 - MODERN CONTROL ENGINEERING

DATE : 12 APRIL 2025 (SATURDAY)

TIME : 2.30 PM - 5.30 PM (3 HOURS)

INSTRUCTIONS TO CANDIDATES

1. Answer **ALL** questions in the Answer Booklet.
2. Begin **EACH** answer on a new page in the Answer Booklet.
3. Indicate clearly answers that are cancelled, if any.
4. Where applicable, show clearly steps taken in arriving at the solutions and indicate **ALL** assumptions, if any.
5. **DO NOT** open this Question Booklet until instructed.

Note :

i. There are **TEN (10)** pages in this Question Booklet including the cover page and appendices.

ii. **DOUBLE-SIDED** Question Booklet.

1. a. The Truck Support Model shown in **FIGURE Q1a** consists of a two-degree-of-freedom system with a truck vehicle mass (m_v) supported by a suspension system (k_1, b_1) and a tire (k_2, b_2), modelled as another mass (m_t). The system is subjected to an external force $F(t)$, affecting the displacement $x_1(t)$ of the truck body.

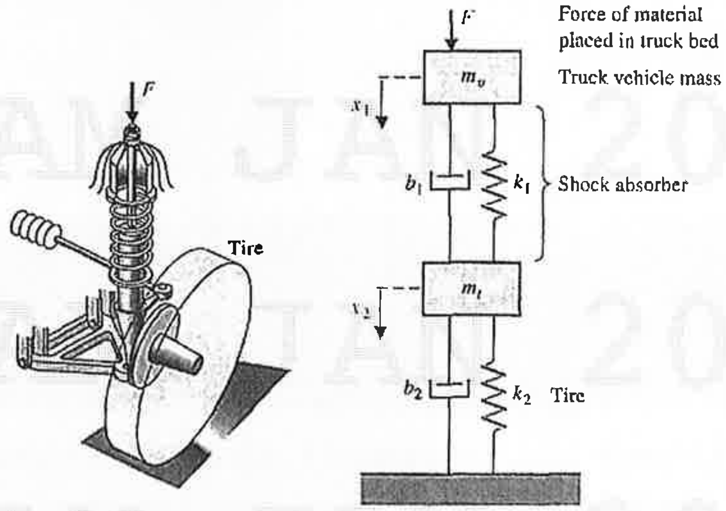

FIGURE Q1a: Truck Support Model

Develop the state-space representation of the translational mechanical system where the input to the system is the external force $F(t)$ and the output of the system is $x_1(t)$.

[10 marks]

b. An aircraft arresting gear is a vital component of an aircraft carrier, designed to decelerate and safely stop incoming aircraft rapidly. It comprises several interconnected subsystems, including the extender, which controls the deployment and retraction of the arresting cable. The dynamic behaviour of the extender can be mathematically modelled using the following transfer function:

$$G(s) = \frac{Y(s)}{R(s)} = \frac{14(s+4)}{s^3 + 10s^2 + 31s + 16}$$

Develop the state-space representation of the system in any **TWO (2)** different forms. Also, construct the signal flow diagram for each form.

[10 marks]

c. Consider the single-input, single-output (SISO) system described by

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t) + Du(t)$$

where

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} -2 & 1 \end{bmatrix}$$

$$D = 5$$

Determine the transfer function representation of the system.

[5 marks]

2. a. A robotic arm equipped with a camera can be used for fruit picking, as illustrated in **FIGURE Q2a**. The camera provides visual feedback to a microcomputer, which processes the information and controls the arm's movements to ensure precise operation.

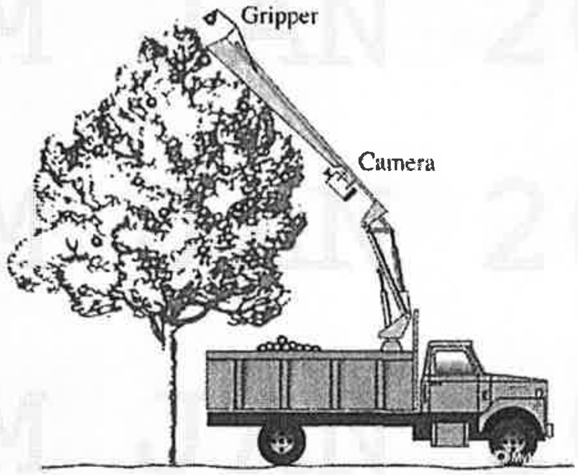

FIGURE Q2a: Automated Fruit-picking System

The following state-space model represents the dynamic behaviour of the system:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \\ -4 & 3 & -2 \end{bmatrix} x(t) + \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 2 & 0 \end{bmatrix} x(t)$$

As a control engineer, analyse the system's stability, controllability, and observability.

[10 marks]

b. The electromagnetic suspension system in **FIGURE Q2b** uses an upper-mounted electromagnet to suspend an iron ball. Due to its inherent instability, feedback control is essential. An eddy-current induction probe beneath the ball provides real-time position feedback for stabilisation.

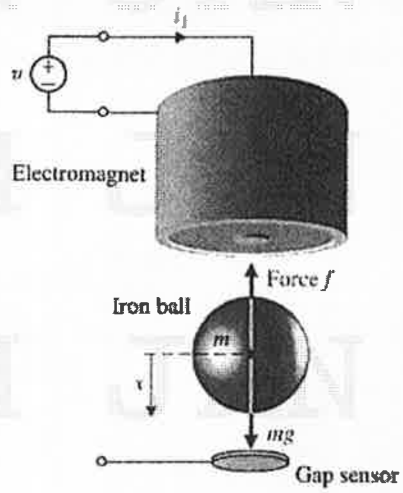

FIGURE Q2b: Electromagnetic Suspension System

The state-space model of the suspension system is as follows:

$$\dot{x}(t) = \begin{bmatrix} -2 & 0 \\ -1 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} x(t)$$

i. Determine the state transition matrix of the system $\Phi(t)$

[5 marks]

ii. Utilizing the properties of state-transition matrices, find $[\Phi(t)]^3$.

[3 marks]

iii. Solve the state and output equations for $y(t)$ using the following formula, where $u(t)$ is the unit step input. The initial condition of the system is $x(0) = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$

$$x(t) = \Phi(t)x(0) + \int_0^t \Phi(t-\tau)Bu(\tau)d\tau$$

[7 marks]

3. a. To enable exploration of the far side of the Moon, studies have examined the feasibility of positioning a communication satellite near the translunar equilibrium point in the Earth-Sun-Moon system. The satellite follows a halo orbit, as illustrated in **FIGURE Q3a**. The controller's objective is to maintain the satellite on this trajectory, ensuring continuous visibility from Earth and an uninterrupted communication link. Signals are transmitted from Earth to the satellite and then relayed to the far side of the Moon.

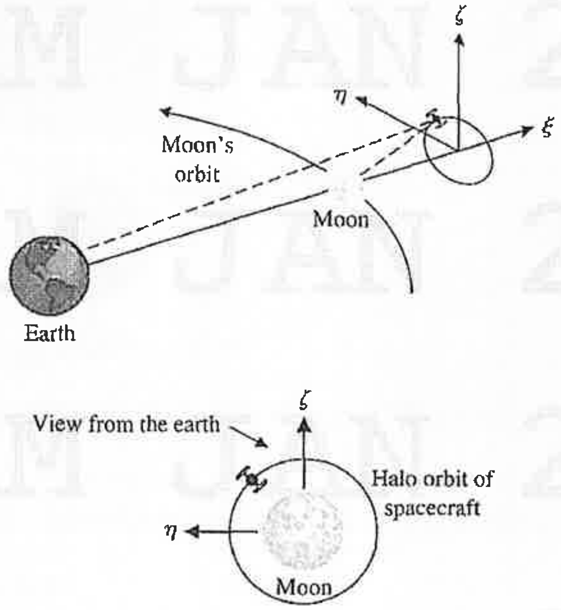

FIGURE Q3a: Translunar Satellite Halo Orbit

The state-space model of the satellite around the translunar equilibrium point is given as follows:

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4.3 & -1.7 & -6.7 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 0.35 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} x + [0]u$$

By using the state-feedback control $u = -Kx$ it is desired to have the closed-loop poles at $-1.4 \pm j1.4$ and -2.

Solve for the state-feedback gain matrix K and draw the signal flow graph of the system with the designed state feedback controller.

[13 marks]

b. Consider an inverted pendulum mounted on a frictionless motor, as shown in **FIGURE Q3b**. The pendulum is attached to the horizontal shaft of a servomotor equipped with a tacho-generator, providing velocity feedback but no position signal. When unpowered, the pendulum hangs vertically and oscillates when disturbed. At the top of its arc, it remains inherently unstable.

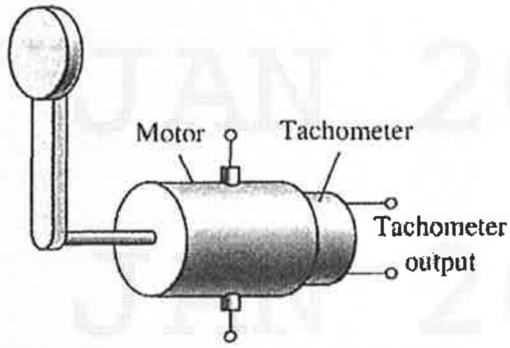

FIGURE Q3b: Inverted Pendulum Mounted on Motor

The third-order state-space model of the system is defined by

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -8 & -3 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} u$$

$$y = \begin{bmatrix} 2 & -9 & 2 \end{bmatrix} x + [0] u$$

Design a full-order state observer for the system by placing the desired closed-loop system poles at $-30, -12 \pm j2$ and draw the signal flow graph of the system with the designed full-order state observer.

[12 marks]

4. a. A Linear-time invariant system is given by,

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

Assume the control signal to be $u(t) = -Kx(t)$. Determine the optimal feedback gain matrix K such that the following performance index J is minimised.

$$J = \int_0^{\infty} (x^T Qx + u^T u)dt\,,\, Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The Ricatti equation with respect to K is $A^T P + PA - PBR^{-1}B^T P + Q = 0$

[13 marks]

b. Consider the closed-loop system shown in **FIGURE Q4b**.

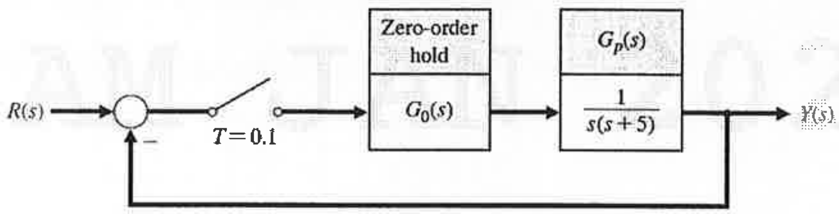

FIGURE Q4b: Closed-loop Sampled System

i. Derive the closed-loop transfer function of the sampled system $T(z) = \frac{Y(z)}{R(z)}$.

[9 marks]

ii. Evaluate the stability of the system when $T = 1$ second.

[3 marks]

- END OF PAPER -

APPENDIX I: LAPLACE and z-TRANSFORMS

$x(t)$	$X(s)$	$X(z)$
$\delta(t) = \begin{cases} \frac{1}{\epsilon}, & t < \epsilon, \epsilon \to 0 \\ 0 & \text{otherwise} \end{cases}$	1	—
$\delta(t-a) = \begin{cases} \frac{1}{\epsilon}, & a < t < a+\epsilon, \epsilon \to 0 \\ 0 & \text{otherwise} \end{cases}$	e^{-as}	—
$\delta_0(t) = \begin{cases} 1 & t = 0, \\ 0 & t = kT, k \neq 0 \end{cases}$	—	1
$\delta_0(t-kT) = \begin{cases} 1 & t = kT, \\ 0 & t \neq kT \end{cases}$	—	z^{-k}
$u(t)$, unit step	$1/s$	$\frac{z}{z-1}$
t	$1/s^2$	$\frac{Tz}{(z-1)^2}$
e^{-at}	$\frac{1}{s+a}$	$\frac{z}{z-e^{-aT}}$
$1-e^{-at}$	$\frac{1}{s(s+a)}$	$\frac{(1-e^{-aT})z}{(z-1)(z-e^{-aT})}$
$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$	$\frac{z\sin(\omega T)}{z^2-2z\cos(\omega T)+1}$
$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$	$\frac{z(z-\cos(\omega T))}{z^2-2z\cos(\omega T)+1}$
$e^{-at}\sin(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}$	$\frac{(ze^{-aT}\sin(\omega T))}{z^2-2ze^{-aT}\cos(\omega T)+e^{-2aT}}$
$e^{-at}\cos(\omega t)$	$\frac{s+a}{(s+a)^2+\omega^2}$	$\frac{z^2-ze^{-aT}\cos(\omega T)}{z^2-2ze^{-aT}\cos(\omega T)+e^{-2aT}}$

APPENDIX II: PROPERTIES OF z-TRANSFORMS

	$x(t)$	$X(z)$
1.	$kx(t)$	$kX(z)$
2.	$x_1(t) + x_2(t)$	$X_1(z) + X_2(z)$
3.	$x(t + T)$	$zX(z) - zx(0)$
4.	$tx(t)$	$-Tz\dfrac{dX(z)}{dz}$
5.	$e^{-at}x(t)$	$X(ze^{aT})$
6.	$x(0)$, initial value	$\lim_{z\to\infty} X(z)$ if the limit exists
7.	$x(\infty)$, final value	$\lim_{z\to 1}(z - 1)X(z)$ if the limit exists and the system is stable; that is, if all poles of $(z - 1)X(z)$ are inside the unit circle $\lvert z\rvert = 1$ on z-plane.

APPENDIX III: ACKERMANN'S FORMULA

Controller Design

The Ackermann's formula to calculate state feedback controller gains K is

$$K = [0 \quad 0 \quad 1][B \quad AB \quad A^2B]^{-1}\phi(A)$$

$$\phi(A) = A^3 + \alpha_1 A^2 + \alpha_2 A + \alpha_3 I$$

where α_i are the coefficients of the desired characteristic polynomial.

Observer Design

The Ackermann's formula to calculate observer gains K_e is

$$K_e = \phi(A)\begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\phi(A) = A^3 + \alpha_1 A^2 + \alpha_2 A + \alpha_3 I$$

where α_i are the coefficients of the desired characteristic polynomial.