UNIVERSITI
TEKNOLOGI
PETRONAS

FINAL EXAMINATION
JANUARY 2025 SEMESTER

COURSE	**:**	**EEB1063/EFB1043 - SIGNALS & SYSTEMS**
DATE	**:**	**12 APRIL 2025 (SATURDAY)**
TIME	**:**	**2.30 PM - 5.30 PM (3 HOURS)**

INSTRUCTIONS TO CANDIDATES

1. Answer **ALL** questions in the Answer Booklet.
2. Begin **EACH** answer on a new page in the Answer Booklet.
3. Indicate clearly answers that are cancelled, if any.
4. Where applicable, show clearly steps taken in arriving at the solutions and indicate **ALL** assumptions, if any.
5. **DO NOT** open this Question Booklet until instructed.

Note :

i. There are **ELEVEN (11)** pages in this Question Booklet including the cover page and appendices.

ii. **DOUBLE-SIDED** Question Booklet.

1. a. The impulse response of a linear time-invariant (LTI) system is given by $h(t) = e^{-2t}u(t-3)$. Determine the output of the system, if a periodic signal, $x(t) = 10 + 14\cos(5t - \pi/3)$ is input to the filter.

[8 marks]

b. A causal LTI system with frequency response given by

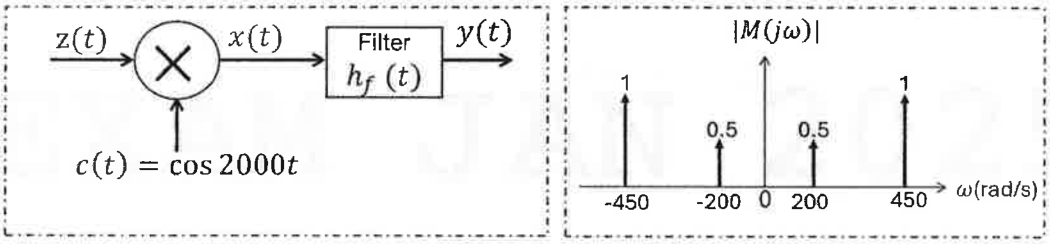
$$H(j\omega) = \frac{1}{1+j\omega} \ .$$

Determine the input to the system, $x(t)$, if the system is observed to produce the output

$$y(t) = e^{-3t}u(t) - e^{-4t}u(t).$$

[8 marks]

c. A signal $z(t)$ is to be processed by a demodulator as shown in **FIGURE Q1c(i)**. The magnitude spectrum of $m(t)$ is given in **FIGURE Q1c(ii)** and $z(t) = m(t)\cos 2000t$.



(i) (ii)

FIGURE Q1c

i. Sketch the magnitude spectrum of $z(t)$ and $x(t)$.

[7 marks]

ii. Given that the output of filter, $h_f(t)$ is $y(t) = m(t)$, sketch the magnitude response of the filter that will satisfy this requirement. In your sketch, label the filter gain and the cut-off frequency.

[3 marks]

2. a. Determine the overall impulse response of a cascaded connection of two discrete-time LTI systems, $h_1[n]$ and $h_2[n]$ given, by

$$h_1[n] = -3\delta[n+2] + \delta[n-1]$$
$$h_2[n] = 4\delta[n+2] + 2\delta[n+1] - 0.5\delta[n-1].$$

[5 marks]

b. The impulse response of an LTI system,

$$h(t) = \delta(t-1) + 3\,\delta(t-2).$$

and the plot of the input, x(t) is shown in **FIGURE Q2b**

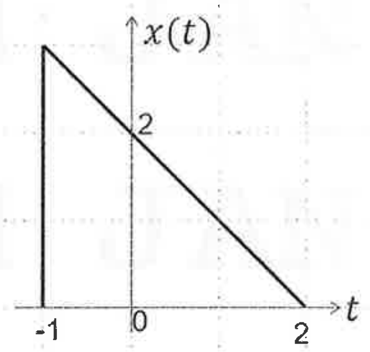


FIGURE Q2b

i. Determine and sketch the output of the system.

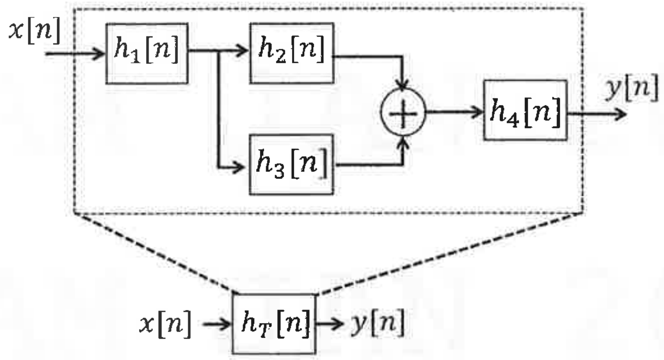
[8 marks]

ii. Evaluate whether the system is causal and/or stable.

[5 marks]

c. An interconnection of four discrete LTI systems is shown in **FIGURE Q2c**. Determine the overall impulse response, $h_T[n]$, of the system

[10 marks]



$$h_1[n] = \left(\frac{1}{3}\right)^n u[n-1], \quad h_2[n] = \delta[n-2],$$
$$h_3[n] = (n+1)u[n], \quad h_4[n] = \delta[n-4].$$

FIGURE Q2c

3. a. Given the continuous-time signal, x(t), in **FIGURE Q3a** sketch

$$m(t) = x(-0.5t + 6)$$

[10 marks]

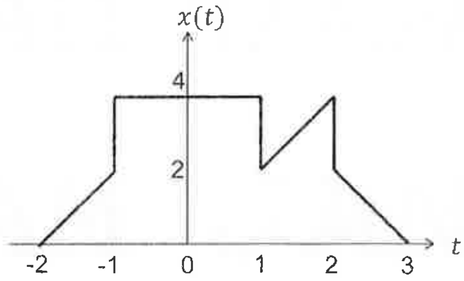


FIGURE Q3a

b. Determine whether the discrete-time signal, $x[n]$ as given below is periodic.

$$x[n] = \sin\frac{12\pi n}{5} + e^{-j13\pi n}$$

[5 marks]

c. Given $x(t)$ in **FIGURE Q3c**,

i. determine the energy value of $k(t) = x(t)[u(t+1) - u(t-1)]$,

[5 marks]

ii. sketch the odd component of $m(t) = x(t)u(t)$.

[6 marks]

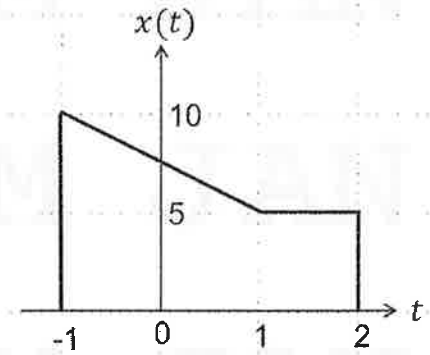


FIGURE Q3c

4. a. A continuous-time LTI system is represented by the following differential equation:

$$\frac{d^2y(t)}{dt^2} + \frac{3dy(t)}{dt} + 2y(t) = x(t)$$

i. Determine the transfer function, $H(s)$ and sketch its pole-zero plot.

[5 marks]

ii. Determine $h(t)$ if the LTI system is not stable and not causal.

[3 marks]

b. An LTI system with impulse response, $h(t) = -e^{-4t}u(t) + \delta(t-2)$ has input signal, $x(t) = -te^{-2t}u(t)$. Using Laplace transform, determine the output, $y(t)$ of the system.

[6 marks]

c. In practical applications, such as control system stability analysis, the Final Value Theorem (FVT) and Initial Value Theorem (IVT) of the Laplace Transform are essential for analyzing the short-term and long-term behavior of dynamic systems. Given the system's output

$$\mathrm{Y}(s) = \frac{(2s+5)}{(s^2+3s+2)},$$

determine its initial value and steady-state response using these theorems.

[6 marks]

– END OF PAPER –

APPENDIX I

Continuous-Time Fourier Series

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_o t}, \qquad a_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_o t}dt$$

APPENDIX II

TABLE 2. Properties of Continuous-Time Fourier Transform

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\,d\omega, \qquad X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

Property	Aperiodic Signal	Fourier transform
	$x(t)$ $y(t)$	$X(j\omega)$ $Y(j\omega)$
Linearity	$ax(t) + by(t)$	$aX(j\omega) + bY(j\omega)$
Time-shifting	$x(t - t_0)$	$e^{-j\omega t_0}X(j\omega)$
Frequency-shifting	$e^{j\omega_0 t}x(t)$	$X(j(\omega - \omega_0))$
Conjugation	$x^*(t)$	$X^*(-j\omega)$
Time-Reversal	$x(-t)$	$X(-j\omega)$
Time- and Frequency-Scaling	$x(at)$	$\frac{1}{\lvert a \rvert}X\left(\frac{j\omega}{a}\right)$
Convolution	$x(t) * y(t)$	$X(j\omega)Y(j\omega)$
Multiplication	$x(t)y(t)$	$\frac{1}{2\pi}X(j\omega) * Y(j\omega)$
Differentiation in Time	$\frac{d}{dt}x(t)$	$j\omega X(j\omega)$
Integration	$\int_{-\infty}^{t} x(t)dt$	$\frac{1}{j\omega}X(j\omega) + \pi X(0)\delta(\omega)$
Differentiation in Frequency	$tx(t)$	$j\frac{d}{d\omega}X(j\omega)$
Conjugate Symmetry for Real Signals	$x(t)$ real	$\begin{cases} X(j\omega) = X^*(-j\omega) \\ \Re e\{X(j\omega)\} = \Re e\{X(-j\omega)\} \\ \Im m\{X(j\omega)\} = -\Im m\{X(-j\omega)\} \\ \lvert X(j\omega)\rvert = \lvert X(-j\omega)\rvert \\ \sphericalangle X(j\omega) = -\sphericalangle X(-j\omega) \end{cases}$
Symmetry for Real and Even Signals	$x(t)$ real and even	$X(j\omega)$ real and even
Symmetry for Real and Odd Signals	$x(t)$ real and odd	$X(j\omega)$ purely imaginary and odd
Even-Odd Decomposition for Real Signals	$x_e(t) = \mathcal{E}v\{x(t)\}$ $[x(t)$ real$]$ $x_o(t) = \mathcal{O}d\{x(t)\}$ $[x(t)$ real$]$	$\Re e\{X(j\omega)\}$ $j\Im m\{X(j\omega)\}$

Parseval's Relation for Aperiodic Signals

$$\int_{-\infty}^{+\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |X(j\omega)|^2 d\omega$$

APPENDIX III

TABLE 3. Basic Continuous-Time Fourier Transform Pairs

Signal	Fourier transform	Fourier series coefficients (if periodic)
$\sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta(\omega - k\omega_0)$	a_k
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$	$a_1 = 1$ $a_k = 0$, otherwise
$\cos\omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$	$a_1 = a_{-1} = \frac{1}{2}$ $a_k = 0$, otherwise
$\sin\omega_0 t$	$\frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$	$a_1 = -a_{-1} = \frac{1}{2j}$ $a_k = 0$, otherwise
$x(t) = 1$	$2\pi\delta(\omega)$	$a_0 = 1, \quad a_k = 0, k \neq 0$ (this is the Fourier series representation for any choice of $T > 0$)
Periodic square wave $x(t) = \begin{cases} 1, & \lvert t\rvert < T_1 \\ 0, & T_1 < \lvert t\rvert \leq \frac{T}{2} \end{cases}$ and $x(t+T) = x(t)$	$\sum_{k=-\infty}^{+\infty} \frac{2\sin k\omega_0 T_1}{k} \delta(\omega - k\omega_0)$	$\frac{\omega_0 T_1}{\pi} \operatorname{sinc}\left(\frac{k\omega_0 T_1}{\pi}\right) = \frac{\sin k\omega_0 T_1}{k\pi}$
$\sum_{n=-\infty}^{+\infty} \delta(t - nT)$	$\frac{2\pi}{T} \sum_{k=-\infty}^{+\infty} \delta\left(\omega - \frac{2\pi k}{T}\right)$	$a_k = \frac{1}{T}$ for all k
$x(t) = \begin{cases} 1, & \lvert t\rvert < T_1 \\ 0, & \lvert t\rvert > T_1 \end{cases}$	$\frac{2\sin\omega T_1}{\omega}$	—
$\frac{\sin Wt}{\pi t}$	$X(j\omega) = \begin{cases} 1, & \lvert\omega\rvert < W \\ 0, & \lvert\omega\rvert > W \end{cases}$	—
$\delta(t)$	1	—
$u(t)$	$\frac{1}{j\omega} + \pi\delta(\omega)$	—
$\delta(t - t_0)$	$e^{-j\omega t_0}$	—
$e^{-at}u(t), \Re e\{a\} > 0$	$\frac{1}{a + j\omega}$	—
$te^{-at}u(t), \Re e\{a\} > 0$	$\frac{1}{(a + j\omega)^2}$	—
$\frac{t^{n-1}}{(n-1)!}e^{-at}u(t)$, $\Re e\{a\} > 0$	$\frac{1}{(a + j\omega)^n}$	—

APPENDIX IV

TABLE 4. Properties of Laplace Transform

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt, \qquad x(t) = \frac{1}{2\pi j}\int_{\sigma-j\omega}^{\sigma+j\omega} X(s)e^{st}ds$$

Property	Signal	Transform	ROC
	$x(t)$	$X(s)$	R
	$x_1(t)$	$X_1(s)$	R_1
	$x_2(t)$	$X_2(s)$	R_2
Linearity	$ax_1(t) + bx_2(t)$	$aX_1(s) + bX_2(s)$	At least $R_1 \cap R_2$
Time shifting	$x(t - t_0)$	$e^{-st_0}X(s)$	R
Shifting in the s-Domain	$e^{s_0 t}x(t)$	$X(s - s_0)$	Shifted version of R [i.e., s is in the ROC if $(s - s_0)$ is in R]
Time scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	"Scaled" ROC (i.e., s is in the ROC if (s/a) is in R)
Conjugation	$x^*(t)$	$X^*(s^*)$	R
Convolution	$x_1(t) * x_2(t)$	$X_1(s)X_2(s)$	At least $R_1 \cap R_2$
Differentiation in the Time Domain	$\frac{d}{dt}x(t)$	$sX(s)$	At least R
Differentiation in the s-Domain	$-tx(t)$	$\frac{d}{ds}X(s)$	R
Integration in the Time Domain	$\int_{-\infty}^{t} x(\tau)d(\tau)$	$\frac{1}{s}X(s)$	At least $R \cap \{\Re e\{s\} > 0\}$

Initial- and Final Value Theorems

If $x(t) = 0$ for $t < 0$ and $x(t)$ contains no impulses or higher-order singularities at $t = 0$, then

$$x(0^+) = \lim_{s\to\infty} sX(s)$$

If $x(t) = 0$ for $t < 0$ and $x(t)$ has a finite limit as $t \to \infty$, then

$$\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s)$$

APPENDIX V

TABLE 5. Laplace Transforms of Elementary Functions

Signal	Transform	ROC
1. $\delta(t)$	1	All s
2. $u(t)$	$\frac{1}{s}$	$\Re e\{s\} > 0$
3. $-u(-t)$	$\frac{1}{s}$	$\Re e\{s\} < 0$
4. $\frac{t^{n-1}}{(n-1)!}u(t)$	$\frac{1}{s^n}$	$\Re e\{s\} > 0$
5. $-\frac{t^{n-1}}{(n-1)!}u(-t)$	$\frac{1}{s^n}$	$\Re e\{s\} < 0$
6. $e^{-\alpha t}u(t)$	$\frac{1}{s+\alpha}$	$\Re e\{s\} > -\Re e\{\alpha\}$
7. $-e^{-\alpha t}u(-t)$	$\frac{1}{s+\alpha}$	$\Re e\{s\} < -\Re e\{\alpha\}$
8. $\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(t)$	$\frac{1}{(s+\alpha)^n}$	$\Re e\{s\} > -\Re e\{\alpha\}$
9. $-\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(-t)$	$\frac{1}{(s+\alpha)^n}$	$\Re e\{s\} < -\Re e\{\alpha\}$
10. $\delta(t-T)$	e^{-sT}	All s
11. $[\cos\omega_0 t]u(t)$	$\frac{s}{s^2+\omega_0^2}$	$\Re e\{s\} > 0$
12. $[\sin\omega_0 t]u(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$	$\Re e\{s\} > 0$
13. $[e^{-\alpha t}\cos\omega_0 t]u(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega_0^2}$	$\Re e\{s\} > -\Re e\{\alpha\}$
14. $[e^{-\alpha t}\sin\omega_0 t]u(t)$	$\frac{\omega_0}{(s+\alpha)^2+\omega_0^2}$	$\Re e\{s\} > -\Re e\{\alpha\}$
15. $u_n(t) = \frac{d^n\delta(t)}{dt^n}$	s^n	All s
16. $u_{-n}(t) = \underbrace{u(t) * \cdots * u(t)}_{n \text{ times}}$	$\frac{1}{s^n}$	$\Re e\{s\} > 0$

APPENDIX VI

TABLE 5. Useful Formula

	Continuous time	Discrete time
Energy	$\int_{-\infty}^{\infty} \lvert x(t)\rvert^{2} dt$	$\sum_{-\infty}^{\infty} \lvert x[n]\rvert^{2}$
Power	$\lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \lvert x(t)\rvert^{2} dt$	$\lim_{N\to\infty} \frac{1}{2N+1}\sum_{-N}^{N} \lvert x[n]\rvert^{2}$
Convolution	$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau) d\tau$	$x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$
Even component	$x_e(t) = \frac{x(t) + x(-t)}{2}$	
Odd component	$x_o(t) = \frac{x(t) - x(-t)}{2}$	
Trigonometry identity	$\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$	