UNIVERSITI TEKNOLOGI PETRONAS

FINAL EXAMINATION
JANUARY 2025 SEMESTER

COURSE	**:**	**EDB3603/RBB2042 - LINEAR ALGEBRA AND MATRIX METHODS**
DATE	**:**	**14 APRIL 2025 (MONDAY)**
TIME	**:**	**2.30 PM - 5.30 PM (3 HOURS)**

INSTRUCTIONS TO CANDIDATES

1. Answer **ALL** questions in the Answer Booklet.
2. Begin **EACH** answer on a new page in the Answer Booklet.
3. Indicate clearly answers that are cancelled, if any.
4. Where applicable, show clearly steps taken in arriving at the solutions and indicate **ALL** assumptions, if any.
5. **DO NOT** open this Question Booklet until instructed.

Note :

i. There are **SEVEN (7)** pages in this Question Booklet including the cover page .

ii. **DOUBLE-SIDED** Question Booklet.

1. a. Solving systems of linear equations is a fundamental concept in algebra, with applications spanning various fields such as engineering, physics, economics, and computer science. One of the most efficient and structured methods for solving such systems involves the use of matrices. Using matrices to solve a system of linear equations is essentially a structured approach that aligns with the principles of the elimination method. Refer to the linear system provided below for further analysis.

$$2x_1 + 2x_2 + x_3 - 4 = 0$$
$$3x_2 = -3x_1 - x_3 + 8$$
$$2x_1 + 4x_2 + x_3 = 5$$

i. Write the system in the matrix form $Ax = b$.

[2 marks]

ii. Compute the inverse of matrix A, obtained in **part (a)(i)**, by augmenting it with the identity matrix, $\boldsymbol{I}$ to form $(A \mid I)$. Next, apply elementary row operations to transform this augmented matrix into the form $(I \mid A^{-1})$ and solve the provided system of linear equations.

[10 marks]

b. $\boldsymbol{LU}$-factorization is a modern and efficient algorithm used to solve linear systems of the form $Ax = b$. By decomposing the square matrix, $\boldsymbol{A}$ into the product of a lower triangular matrix $\boldsymbol{L}$, and an upper triangular matrix $\boldsymbol{U}$, $\boldsymbol{A} = \boldsymbol{LU}$, this method simplifies solving linear systems.

$$\begin{aligned} x_1 - 3x_2 &= -5 \\ x_2 + 3x_3 &= -1 \\ 2x_1 - 10x_2 + 2x_3 &= -20 \end{aligned}$$

i. Determine the $\boldsymbol{LU}$-factorization of the coefficient matrix $\boldsymbol{A}$ from the provided linear system by applying the method of elementary matrices.

[8 marks]

ii. Solve the lower triangular system $\boldsymbol{Ly} = \boldsymbol{b}$ for $\boldsymbol{y}$ and the upper triangular system $\boldsymbol{Ux} = \boldsymbol{y}$ for $\boldsymbol{x}$.

[8 marks]

2. a. Cramer's Rule offers an elegant and precise approach to solving systems of linear equations, particularly when the number of equations matches the number of variables. By relying on determinants, this method finds applications in diverse fields such as computer graphics, robotics, and electrical engineering, where analyzing and optimizing networks is essential. For the system of linear equations below, verify the existence of a unique solution using the determinant of the coefficient matrix. Then, employ Cramer's Rule to find the values of x_1, x_2, and x_3.

$$3x_1 + 3x_2 - x_3 = 11$$
$$2x_1 - x_2 + 2x_3 = 9$$
$$4x_1 + 3x_2 - 2x_3 = 25$$

[10 marks]

b. In analytic geometry, determinants serve as a versatile mathematical tool with key applications. They enable the derivation of plane equations from given points and provide a method to verify the coplanarity of points in three-dimensional space, making them essential for solving geometric and spatial problems. Using determinant, solve the following problems.

i. Formulate the equation of the plane passing through points (-2, 2, 4), (5, -1, 2), and (0, 0, 1).

[8 marks]

ii. If the plane passes through another point (2, 0, 4), determine and justify whether the **FOUR (4)** given points are coplanar.

[4 marks]

3. a. In linear algebra, understanding the properties of a set of vectors is fundamental to analyzing vector spaces. Consider the set of vectors $\mathbf{S} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ for $\boldsymbol{R}^3$ where $\mathbf{v}_1 = (1, 2, 3)$, $\mathbf{v}_2 = (2, -1, 4)$, and $\mathbf{v}_3 = (0, 1, 1)$.

i. Solve the vector $x = (5, 0, 7)$ as a linear combination of the given vectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$.

[6 marks]

ii. Determine whether the vectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ form a basis for $\boldsymbol{R}^3$. Justify your answer.

[3 marks]

b. In computer graphics, homogeneous systems are used to represent and solve problems involving linear transformations, such as rotations, scaling, and translations. The null space of a transformation matrix helps identify fixed points or invariant directions. Consider the homogeneous system as shown by the linear equations below.

$$x_1 + 2x_2 - x_3 = 0$$
$$2x_1 + 4x_2 - 2x_3 = 0$$
$$3x_1 + 6x_2 - 3x_3 = 0$$

Determine the basis and dimension of the solution space of the given homogeneous system.

[8 marks]

c. Within the study of vector spaces, understanding how vectors relate to a given basis is essential for analyzing subspaces and solving related problems. Let $\mathbf{V}$ be a subspace of R^3 with basis $\mathbf{B} = \{(1, 2, 1), (2, 1, 0)\}$ and let $\mathbf{x} = (4, 5, 3)$ be a vector in the subspace $\mathbf{V}$.

i. Write $\mathbf{x}$ as a linear combination of the vectors in $\mathbf{B}$.

[4 marks]

ii. Transform $\mathbf{B}$ into the orthonormal set (basis) $\mathbf{B''}$, by applying the Gram-Schmidt orthonormalization process. Then, evaluate $\mathbf{x}$ as a linear combination of the vectors in the orthonormal set $\mathbf{B''}$.

[9 marks]

4. In robotics, eigenvalues and eigenvectors are used to analyze the stability and controllability of robotic systems. For example, the Jacobian matrix of a robotic manipulator is analyzed using eigenvalues to determine singular configurations. Consider the matrix system below.

$$\text{Let } A = \begin{bmatrix} 3 & 1 & -1 \\ 0 & 2 & 0 \\ -1 & 1 & 3 \end{bmatrix},$$

a. Determine the eigenvalues of A and the corresponding eigenvectors.

[12 marks]

b. Show the basis for the eigenspace obtained in **part (a)**.

[3 marks]

c. Show that A is diagonalizable. Then, using A and a matrix $\boldsymbol{P}$ that diagonalizes A, evaluate the diagonal matrix $\boldsymbol{D}$.

[5 marks]

-END OF PAPER-