



UNIVERSITI
TEKNOLOGI
PETRONAS

FINAL EXAMINATION JANUARY 2025 SEMESTER

COURSE : EEB1073/EFB2023 - ELECTROMAGNETIC THEORY
DATE : 14 APRIL 2025 (MONDAY)
TIME : 2.30 PM - 5.30 PM (3 HOURS)

INSTRUCTIONS TO CANDIDATES

1. Answer **ALL** questions in the Answer Booklet.
2. Begin **EACH** answer on a new page in the Answer Booklet.
3. Indicate clearly answers that are cancelled, if any.
4. Where applicable, show clearly steps taken in arriving at the solutions and indicate **ALL** assumptions, if any.
5. **DO NOT** open this Question Booklet until instructed.

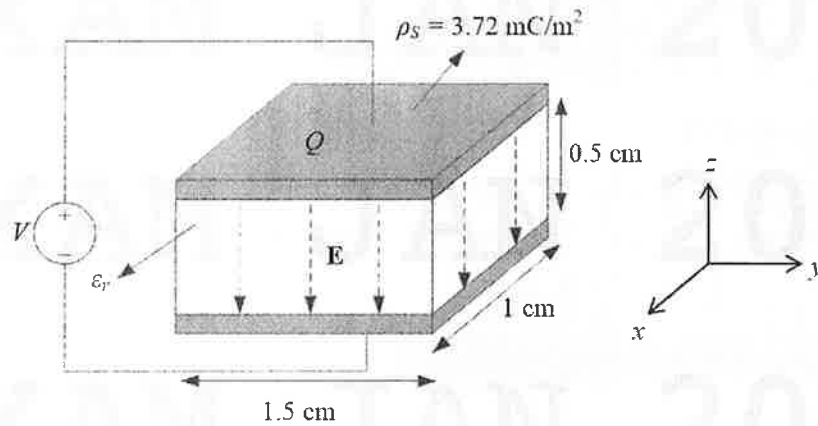
Note :

- i. There are **TWELVE (12)** pages in this Question Booklet including the cover page and appendix.
- ii. **DOUBLE-SIDED** Question Booklet.

1. a. A $4\text{ m} \times 4\text{ m}$ square lies in the xy -plane, centered at the origin, with its sides parallel to the x - and y -axes. A $20\text{ }\mu\text{C}$ point charge is placed at each of its four corners. Using Coulomb's Law, determine the electric flux density \mathbf{D} at a point located 10 m above the center of the square, along the z -axis in free space. In your solution, draw a labelled diagram showing the square, the point charges, and relevant measurements.

[10 marks]

- b. **FIGURE Q1** illustrates a parallel-plate capacitor consisting of two identical conducting plates separated by a dielectric material with relative permittivity ϵ_r . The total charge Q on the plates induces an electric field \mathbf{E} with a magnitude of $70 \times 10^6\text{ V/m}$.

**FIGURE Q1**

- Determine the charge, Q stored on the plates of the capacitor.
[3 marks]
- Obtain the relative permittivity, ϵ_r of the dielectric material and identify the most likely dielectric material used in the capacitor.
[4 marks]
- Determine the breakdown voltage, V_{br} of the capacitor based on the answer in **part (ii)**.
[4 marks]
- Calculate the capacitance, C of the capacitor. Then, briefly suggest **TWO (2)** ways to increase its capacitance.
[4 marks]

2. a. In free space, a magnetic vector potential, \mathbf{A} is given by:

$$\mathbf{A} = \hat{\mathbf{r}} 10 r^3 \cos \phi + \hat{\mathbf{z}} 30 r^2 \sin \phi \quad (\text{Wb/m})$$

- i. Determine the corresponding magnetic flux density, \mathbf{B} .

[4 marks]

- ii. Prove that \mathbf{B} obtained in **part (i)** satisfies the Gauss' Law for magnetism.

[3 marks]

- iii. Calculate the magnetic flux, Φ through a segment of a cylindrical surface defined by $r = 2$, $\frac{\pi}{3} \leq \phi \leq \frac{\pi}{2}$ and $0 \leq z \leq 3$ as shown in

FIGURE Q2a. All distances are given in meters.

[3 marks]

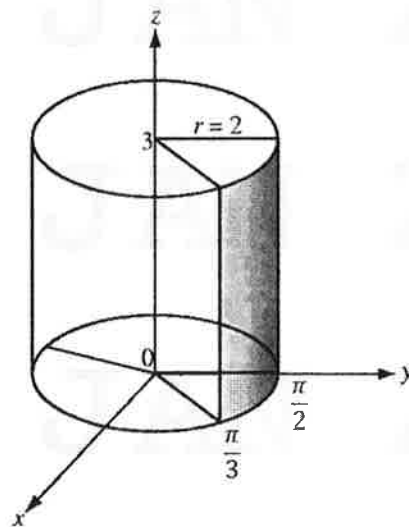


FIGURE Q2a

- b. Consider the two filamentary wires with cylindrical cross section as illustrated in **FIGURE Q2b**. The wires are part of a very large circuit. The diameter of each wire is 1 cm. The wire centred at the origin carries a current of 2 mA while the wire centred at (8 cm, 0, 0) carries the return current.

- i. Determine the corresponding current density, \mathbf{J} on the surface of the wire centered at the origin.

[4 marks]

- ii. Apply Biot-Savart's Law to determine the magnetic flux density, \mathbf{B} induced by the two infinitely long wires at point P (4 cm, 0, 0).

[7 marks]

- iii. A charged particle with charge $q = 1.6 \times 10^{-19}$ C moves with a velocity of $\mathbf{u} = \hat{x} 2 \times 10^6$ m/s through point P (4 cm, 0, 0). Using \mathbf{B} obtained in **part (ii)**, calculate the magnetic force, \mathbf{F}_m experienced by the charged particle.

[4 marks]

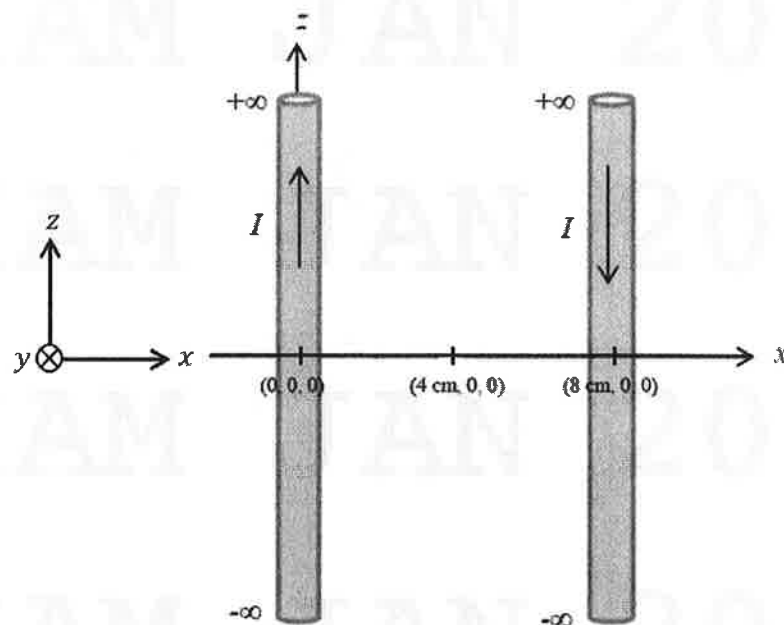


FIGURE Q2b

3. a. A time-varying electric flux density is propagating through seawater is given by:

$$\mathbf{D} = \hat{\mathbf{y}} 5.2 \omega \sin(\omega t - 0.1z) \text{ C/m}^2$$

The seawater has the following properties:

$$\sigma = 4 \text{ S/m}, \quad \epsilon_r = 80, \quad \mu_r = 1$$

- i. Using Faraday's Law, determine the corresponding magnetic field intensity, \mathbf{H} .

[8 Marks]

- ii. Calculate the conduction current density, \mathbf{J}_c and the displacement current density, \mathbf{J}_d .

[5 marks]

- b. An inductor in **FIGURE Q3** is created by winding N turns of a thin conducting wire into a circular loop of radius r , positioned in the xy -plane with its center at the origin. The loop is connected to a resistor R . The system is exposed to a time-varying magnetic field expressed as:

$$\mathbf{B} = B_0(\hat{\mathbf{x}}7 + \hat{\mathbf{y}}22 + \hat{\mathbf{z}}5) \cos \omega t \text{ T}$$

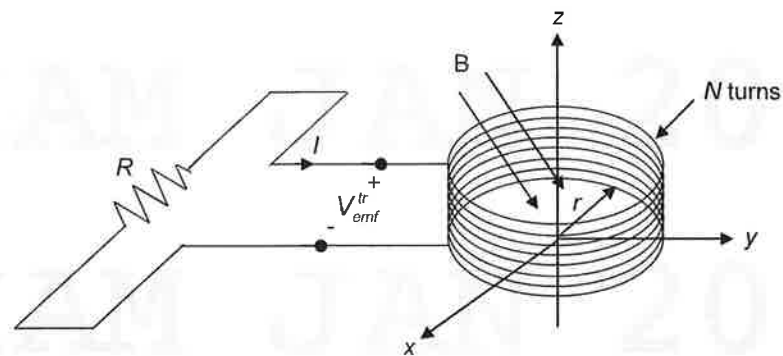


FIGURE Q3

- i. Determine the magnetic flux, Φ linking a single turn of the inductor.
[5 marks]
- ii. Obtain the transformer electromotive force, V_{emf}^{tr} given that $N = 20$,
 $B_0 = 0.6 \text{ T}$, $r = 2 \text{ cm}$ and $\omega = 10^3 \text{ rad/s}$.
[5 marks]
- iii. Calculate the induced current, I in the circuit for the loop resistance,
 $R = 5 \text{ k}\Omega$ and the internal resistance, $R_i = 17 \Omega$.
[2 marks]

4. Consider the network shown in **FIGURE Q4**. All transmission lines in the network are lossless with the characteristic impedance of 50Ω .

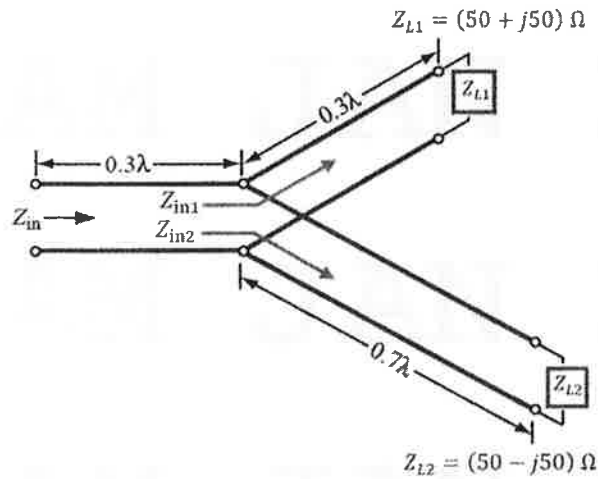


FIGURE Q4

- Determine the reflection coefficients Γ_1 and Γ_2 at each of the load impedances Z_{L1} and Z_{L2} respectively. [6 marks]
- Obtain the effective load impedance, Z'_L , at the parallel junction by combining Z_{in1} and Z_{in2} . Next, draw the equivalent circuit of the impedance transformation. [11 marks]
- Based on the equivalent circuit in **part (b)**:
 - Calculate the input impedance Z_{in} of the transmission line. [4 marks]
 - Determine the reflection coefficients Γ at the effective load impedance. [2 marks]
 - Calculate the Voltage Standing Wave Ratio (VSWR) at the input. [2 marks]

– END OF PAPER –

APPENDIX

Summary of Vector Relations

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Coordinate variables	x, y, z	r, ϕ, z	R, θ, ϕ
Vector representation $\mathbf{A} =$	$\hat{x}A_x + \hat{y}A_y + \hat{z}A_z$	$\hat{r}A_r + \hat{\phi}A_\phi + \hat{z}A_z$	$\hat{R}A_R + \hat{\theta}A_\theta + \hat{\phi}A_\phi$
Magnitude of \mathbf{A} $ \mathbf{A} =$	$\sqrt{A_x^2 + A_y^2 + A_z^2}$	$\sqrt{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt{A_R^2 + A_\theta^2 + A_\phi^2}$
Position vector $\overrightarrow{OP_1} =$	$\hat{x}x_1 + \hat{y}y_1 + \hat{z}z_1$ for $P = (x_1, y_1, z_1)$	$\hat{r}r_1 + \hat{z}z_1$ for $P = (r_1, \phi_1, z_1)$	$\hat{R}R_1$ for $P = (R_1, \theta_1, \phi_1)$
Base vectors properties	$\hat{x} \cdot \hat{x} = \hat{y} \cdot \hat{y} = \hat{z} \cdot \hat{z} = 1$ $\hat{x} \cdot \hat{y} = \hat{y} \cdot \hat{z} = \hat{z} \cdot \hat{x} = 0$ $\hat{x} \times \hat{y} = \hat{z}$ $\hat{y} \times \hat{z} = \hat{x}$ $\hat{z} \times \hat{x} = \hat{y}$	$\hat{r} \cdot \hat{r} = \hat{\phi} \cdot \hat{\phi} = \hat{z} \cdot \hat{z} = 1$ $\hat{r} \cdot \hat{\phi} = \hat{\phi} \cdot \hat{z} = \hat{z} \cdot \hat{r} = 0$ $\hat{r} \times \hat{\phi} = \hat{z}$ $\hat{\phi} \times \hat{z} = \hat{r}$ $\hat{z} \times \hat{r} = \hat{\phi}$	$\hat{R} \cdot \hat{R} = \hat{\theta} \cdot \hat{\theta} = \hat{\phi} \cdot \hat{\phi} = 1$ $\hat{R} \cdot \hat{\theta} = \hat{\theta} \cdot \hat{\phi} = \hat{\phi} \cdot \hat{R} = 0$ $\hat{R} \times \hat{\theta} = \hat{\phi}$ $\hat{\theta} \times \hat{\phi} = \hat{R}$ $\hat{\phi} \times \hat{R} = \hat{\theta}$
Dot product $\mathbf{A} \cdot \mathbf{B} =$	$A_x B_x + A_y B_y + A_z B_z$	$A_r B_r + A_\phi B_\phi + A_z B_z$	$A_R B_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product $\mathbf{A} \times \mathbf{B} =$	$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{r} & \hat{\phi} & \hat{z} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{R} & \hat{\theta} & \hat{\phi} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length $d\mathbf{l} =$	$\hat{x} dx + \hat{y} dy + \hat{z} dz$	$\hat{r} dr + \hat{\phi} r d\phi + \hat{z} dz$	$\hat{R} dR + \hat{\theta} R d\theta + \hat{\phi} R \sin \theta d\phi$
Differential surface areas	$ds_x = \hat{x} dy dz$ $ds_y = \hat{y} dx dz$ $ds_z = \hat{z} dx dy$	$ds_r = \hat{r} r d\phi dz$ $ds_\phi = \hat{\phi} dr dz$ $ds_z = \hat{z} r dr d\phi$	$ds_R = \hat{R} R^2 \sin \theta d\theta d\phi$ $ds_\theta = \hat{\theta} R \sin \theta dR d\phi$ $ds_\phi = \hat{\phi} R dR d\theta$
Differential volume $dV =$	$dx dy dz$	$r dr d\phi dz$	$R^2 \sin \theta dR d\theta d\phi$

Coordinate Transformation Relations

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to cylindrical	$r = \sqrt{x^2 + y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{r} = \hat{x} \cos \phi + \hat{y} \sin \phi$ $\hat{\phi} = -\hat{x} \sin \phi + \hat{y} \cos \phi$ $\hat{z} = \hat{z}$	$A_r = A_x \cos \phi + A_y \sin \phi$ $A_\phi = -A_x \sin \phi + A_y \cos \phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r \cos \phi$ $y = r \sin \phi$ $z = z$	$\hat{x} = \hat{r} \cos \phi - \hat{\phi} \sin \phi$ $\hat{y} = \hat{r} \sin \phi + \hat{\phi} \cos \phi$ $\hat{z} = \hat{z}$	$A_x = A_r \cos \phi - A_\phi \sin \phi$ $A_y = A_r \sin \phi + A_\phi \cos \phi$ $A_z = A_z$
Cartesian to spherical	$R = \sqrt{x^2 + y^2 + z^2}$ $\theta = \tan^{-1}[\sqrt{x^2 + y^2}/z]$ $\phi = \tan^{-1}(y/x)$	$\hat{R} = \hat{x} \sin \theta \cos \phi + \hat{y} \sin \theta \sin \phi + \hat{z} \cos \theta$ $\hat{\theta} = \hat{x} \cos \theta \cos \phi + \hat{y} \cos \theta \sin \phi - \hat{z} \sin \theta$ $\hat{\phi} = -\hat{x} \sin \phi + \hat{y} \cos \phi$	$A_R = A_x \sin \theta \cos \phi + A_y \sin \theta \sin \phi + A_z \cos \theta$ $A_\theta = A_x \cos \theta \cos \phi + A_y \cos \theta \sin \phi - A_z \sin \theta$ $A_\phi = -A_x \sin \phi + A_y \cos \phi$
Spherical to Cartesian	$x = R \sin \theta \cos \phi$ $y = R \sin \theta \sin \phi$ $z = R \cos \theta$	$\hat{x} = \hat{R} \sin \theta \cos \phi + \hat{\theta} \cos \theta \cos \phi - \hat{\phi} \sin \phi$ $\hat{y} = \hat{R} \sin \theta \sin \phi + \hat{\theta} \cos \theta \sin \phi + \hat{\phi} \cos \phi$ $\hat{z} = \hat{R} \cos \theta - \hat{\theta} \sin \theta$	$A_x = A_R \sin \theta \cos \phi + A_\theta \cos \theta \cos \phi - A_\phi \sin \phi$ $A_y = A_R \sin \theta \sin \phi + A_\theta \cos \theta \sin \phi + A_\phi \cos \phi$ $A_z = A_R \cos \theta - A_\theta \sin \theta$
Cylindrical to spherical	$R = \sqrt{r^2 + z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{R} = \hat{r} \sin \theta + \hat{z} \cos \theta$ $\hat{\theta} = \hat{r} \cos \theta - \hat{z} \sin \theta$ $\hat{\phi} = \hat{\phi}$	$A_R = A_r \sin \theta + A_z \cos \theta$ $A_\theta = A_r \cos \theta - A_z \sin \theta$ $A_\phi = A_\phi$
Spherical to cylindrical	$r = R \sin \theta$ $\phi = \phi$ $z = R \cos \theta$	$\hat{r} = \hat{R} \sin \theta + \hat{\theta} \cos \theta$ $\hat{\phi} = \hat{\phi}$ $\hat{z} = \hat{R} \cos \theta - \hat{\theta} \sin \theta$	$A_r = A_R \sin \theta + A_\theta \cos \theta$ $A_\phi = A_\phi$ $A_z = A_R \cos \theta - A_\theta \sin \theta$

Summary of Gradient, Divergence, Curl and Laplacian Operators

CARTESIAN (RECTANGULAR) COORDINATES (x, y, z)

$$\nabla V = \hat{x} \frac{\partial V}{\partial x} + \hat{y} \frac{\partial V}{\partial y} + \hat{z} \frac{\partial V}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \hat{x} \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \hat{y} \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \hat{z} \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

CYLINDRICAL COORDINATES (r, ϕ, z)

$$\nabla V = \hat{r} \frac{\partial V}{\partial r} + \hat{\phi} \frac{1}{r} \frac{\partial V}{\partial \phi} + \hat{z} \frac{\partial V}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r} \frac{\partial}{\partial r} (r A_r) + \frac{1}{r} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \frac{1}{r} \begin{vmatrix} \hat{r} & \hat{\phi} & \hat{z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & r A_\phi & A_z \end{vmatrix} = \hat{r} \left(\frac{1}{r} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right) + \hat{\phi} \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right) + \hat{z} \frac{1}{r} \left[\frac{\partial}{\partial r} (r A_\phi) - \frac{\partial A_r}{\partial \phi} \right]$$

$$\nabla^2 V = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial V}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

SPHERICAL COORDINATES (R, θ, ϕ)

$$\nabla V = \hat{R} \frac{\partial V}{\partial R} + \hat{\theta} \frac{1}{R} \frac{\partial V}{\partial \theta} + \hat{\phi} \frac{1}{R \sin \theta} \frac{\partial V}{\partial \phi}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{R^2} \frac{\partial}{\partial R} (R^2 A_R) + \frac{1}{R \sin \theta} \frac{\partial}{\partial \theta} (A_\theta \sin \theta) + \frac{1}{R \sin \theta} \frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \mathbf{A} = \frac{1}{R^2 \sin \theta} \begin{vmatrix} \hat{R} & \hat{\theta} & \hat{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & R A_\theta & (R \sin \theta) A_\phi \end{vmatrix}$$

$$= \hat{R} \frac{1}{R \sin \theta} \left[\frac{\partial}{\partial \theta} (A_\phi \sin \theta) - \frac{\partial A_\theta}{\partial \phi} \right] + \hat{\theta} \frac{1}{R} \left[\frac{1}{\sin \theta} \frac{\partial A_R}{\partial \phi} - \frac{\partial}{\partial R} (R A_\phi) \right] + \hat{\phi} \frac{1}{R} \left[\frac{\partial}{\partial R} (R A_\theta) - \frac{\partial A_R}{\partial \theta} \right]$$

$$\nabla^2 V = \frac{1}{R^2} \frac{\partial}{\partial R} \left(R^2 \frac{\partial V}{\partial R} \right) + \frac{1}{R^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial V}{\partial \theta} \right) + \frac{1}{R^2 \sin^2 \theta} \frac{\partial^2 V}{\partial \phi^2}$$

Properties of Standing Waves on A Lossless Transmission Line

Voltage Maximum	$ \tilde{V} _{\max} = V_0^+ [1 + \Gamma]$
Voltage Minimum	$ \tilde{V} _{\min} = V_0^+ [1 - \Gamma]$
Positions of voltage maxima (also positions of current minima)	$d_{\max} = \frac{\theta_r \lambda}{4\pi} + \frac{n\lambda}{2}, \quad n = 0, 1, 2, \dots$
Position of first maximum (also position of first current minimum)	$d_{\max} = \begin{cases} \frac{\theta_r \lambda}{4\pi}, & \text{if } 0 \leq \theta_r \leq \pi \\ \frac{\theta_r \lambda}{4\pi} + \frac{\lambda}{2}, & \text{if } -\pi \leq \theta_r \leq 0 \end{cases}$
Positions of voltage minima (also positions of current maxima)	$d_{\min} = \frac{\theta_r \lambda}{4\pi} + \frac{(2n+1)\lambda}{4}, \quad n = 0, 1, 2, \dots$
Position of first minimum (also position of first current maximum)	$d_{\min} = \begin{cases} d_{\max} + \lambda/4, & \text{if } d_{\max} < \lambda/4, \\ d_{\max} - \lambda/4, & \text{if } d_{\max} \geq \lambda/4. \end{cases}$
Input Impedance	$Z_{\text{in}} = Z_0 \left(\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right), \quad \beta = \frac{2\pi}{\lambda}$
Positions at which Z_{in} is real	at voltage maxima and minima
Z_{in} at voltage maxima	$Z_{\text{in}} = Z_0 \left(\frac{1 + \Gamma }{1 - \Gamma } \right)$
Z_{in} at voltage minima	$Z_{\text{in}} = Z_0 \left(\frac{1 - \Gamma }{1 + \Gamma } \right)$
Z_{in} of short-circuited line	$Z_{\text{in}}^{\text{sc}} = jZ_0 \tan \beta l$
Z_{in} of open-circuited line	$Z_{\text{in}}^{\text{oc}} = -jZ_0 \cot \beta l$
Z_{in} of line of length $l = n\lambda/2$	$Z_{\text{in}} = Z_L, \quad n = 0, 1, 2, \dots$
Z_{in} of line of length $l = \lambda/4 + n\lambda/2$	$Z_{\text{in}} = Z_0^2 / Z_L, \quad n = 0, 1, 2, \dots$
Z_{in} of matched line	$Z_{\text{in}} = Z_0$
$ V_0^+ $ = amplitude of incident wave; $\Gamma = \Gamma e^{j\theta_r}$ with $-\pi < \theta_r < \pi$; θ_r in radians; $\Gamma_l = \Gamma e^{-j2\beta l}$.	

Reflection coefficient

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Load impedance

$$Z_L = Z_0 \left[\frac{1 + \Gamma}{1 - \Gamma} \right]$$

Voltage Standing Wave Ratio

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$