UNIVERSITI TEKNOLOGI PETRONAS

FINAL EXAMINATION
JANUARY 2025 SEMESTER

COURSE : EEB4033/EFB4023 - DIGITAL SIGNAL PROCESSING

DATE : 15 APRIL 2025 (TUESDAY)

TIME : 2.30 PM - 5.30 PM (3 HOURS)

INSTRUCTIONS TO CANDIDATES

1. Answer **ALL** questions in the Answer Booklet.
2. Begin **EACH** answer on a new page in the Answer Booklet.
3. Indicate clearly answers that are cancelled, if any.
4. Where applicable, show clearly steps taken in arriving at the solutions and indicate **ALL** assumptions, if any.
5. **DO NOT** open this Question Booklet until instructed.

Note :

i. There are **NINE (9)** pages in this Question Booklet including the cover page and appendices.

ii. **DOUBLE-SIDED** Question Booklet.

1. a. The MATLAB code in **FIGURE Q1a(i)** illustrates the frequency-shifting property of the Discrete-Time Fourier Transform (DTFT). Additionally, the magnitude spectrum of a discrete-time signal, x1[n] is presented in **FIGURE Q1a(ii)**. Here, h = freqz(b,a,w) returns the frequency response vector h evaluated at the normalized frequencies supplied in w. Here, b is the numerator and a is the denominator of the transfer function,

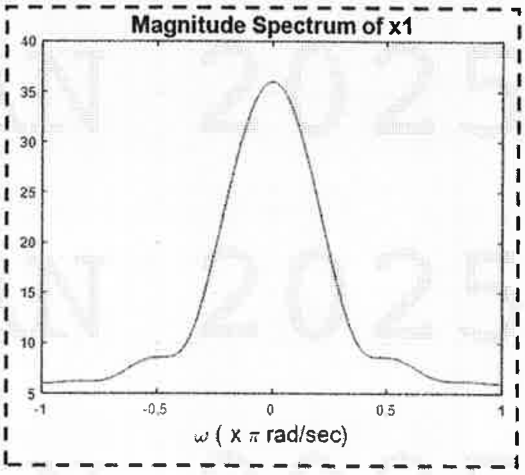
$$H(z) = \frac{B(z)}{A(z)} = \frac{b_1 + b_2 z^{-1} \cdots + b_n z^{-(n-1)} + b_{n+1} z^{-n}}{a_1 + a_2 z^{-1} \cdots + a_m z^{-(m-1)} + a_{m+1} z^{-m}}$$

```
clc;close all
w = -pi:2*pi/255:pi; wo = 0.4*pi;
x1 = [1 3 5 7 9 11];

L = length(x1);  h1 = freqz(x1, 1, w);
n = 0:L-1;  x2 = exp(wo*1i*n) .* x1;
h2 = freqz(x2, 1, w);

figure; plot(w/pi, abs(h1)); grid
title('Magnitude Spectrum of x1')
figure; plot(w/pi, abs(h2)); grid
title('Magnitude Spectrum of x2')
```

(i)



(ii)

FIGURE Q1a

i. Determine the DTFT of x1[n] and x2[n].

[6 marks]

ii. Sketch the magnitude spectrum of x2[n] and explain the frequency-shifting property of the DTFT.

[6 marks]

b. Consider a causal and stable linear time-invariant (LTI) system that produces an output $y[n]$ in response to an input $x[n]$. If the input and output signals are given as follows,

$$y[n] = n\left(\frac{2}{3}\right)^n u[n] \quad and \quad x[n] = \left(\frac{4}{5}\right)^n u[n],$$

determine the frequency response and the difference equation governing its operation.

[8 marks]

c. A low-pass signal with a bandwidth of 1.5 kHz, as illustrated in **FIGURE Q1c**, is sampled at a rate of 2000 samples per second. Sketch the spectrum of the sampled signal in the normalized frequency domain using the Discrete-Time Fourier Transform (DTFT) (i.e., in radians). Additionally, analyze the challenges associated with reconstructing the original signal.

[6 marks]

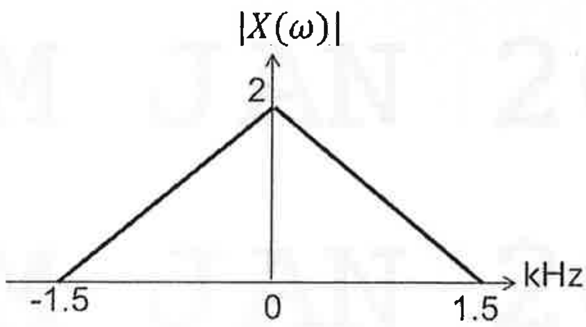


FIGURE Q1c

2. A discrete LTI system with zero initial conditions is described by the following difference equation.

$$y[n] - y[n-1] + \frac{3}{16}y[n-2] = x[n] - \frac{1}{2}x[n-1]$$

a. Determine the impulse response of the system if the ROC is $\frac{1}{4} < |z| < \frac{3}{4}$ and based on the impulse response, discuss the causality and stability of the system.

[10 marks]

b. Determine the output response if the input signal is

$$x[n] = \left(\frac{1}{2}\right)^n u[n-4].$$

[6 marks]

3. a. Signal $x[n]$ is given by

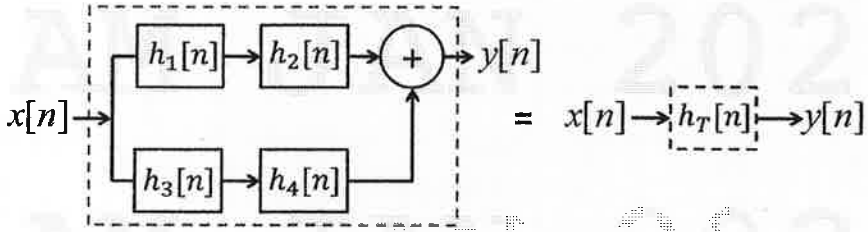
$$x[n] = \{2, 1, 3, \underline{-2}, 1, -1, 0.5, 6\}$$

where the underline indicates the origin of the sequence. Determine $k[n] = x[-2n-2]$ and $m[n] = k[n](\delta[n+1] + 2\delta[n-2])$.

[10 marks]

b. An interconnection of four LTI systems is shown in **FIGURE Q3**. Find the overall impulse response of the system, $h_T[n]$.

[10 marks]

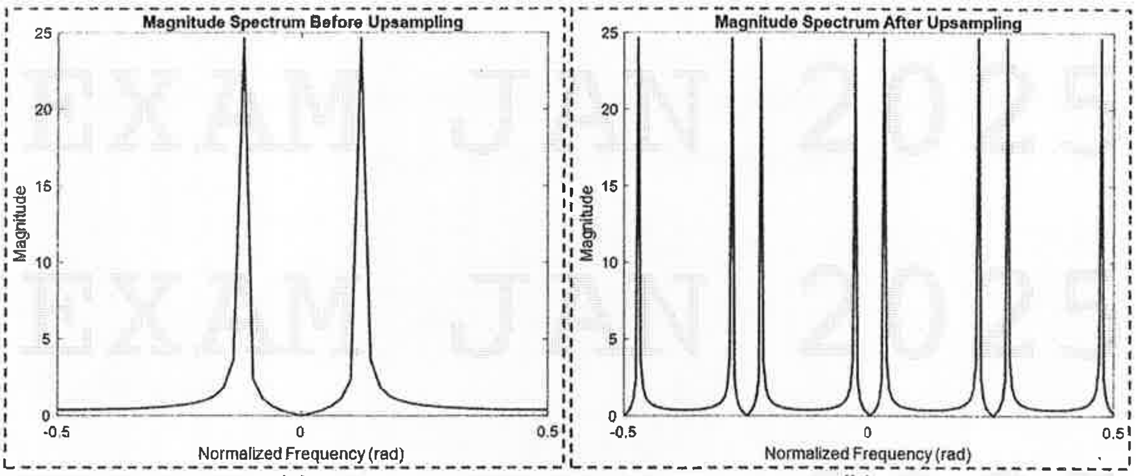


$$h_1[n] = \left(\tfrac{1}{5}\right)^n u[n-5],\ h_2[n] = 0.3\delta[n-7],$$

$$h_3[n] = u[n-4],\ h_4[n] = 3\delta[n+1] - \delta[n-1].$$

FIGURE Q3

4. In a digital audio application, the signal $x[n]$ must be upsampled to ensure compatibility with the varying sampling rates of different audio storage media. The plot in **FIGURE Q4(a)** represents the magnitude spectrum of a discrete-time signal $x[n]$, while **FIGURE Q4(b)** shows the magnitude spectrum of $x[n]$ after undergoing upsampling by a factor of L.



(a) (b)

FIGURE Q4

a. What is the upsampling factor, L and justify your answer.

[2 marks]

b. Based on **FIGURE Q4(b),** discuss the problem of upsampling a discrete-time signal.

[2 marks]

c. Based on **part (b)**, propose a system to mitigate these problems, providing the block diagram of the proposed system. Additionally, specify the parameters of the filter used in the system.

[4 marks]

d. If $x[n]$ is to be downsampled by a factor of 3, sketch the magnitude spectrum of the downsampled signal. Clearly label relevant frequency points in your sketch.

[6 marks]

5. A causal digital band-pass biquad filter designed for an audio processing application has the zero-pole plot, magnitude response and direct-form II implementation as shown in **FIGURE Q5 (a), (b) and (c)**, respectively.

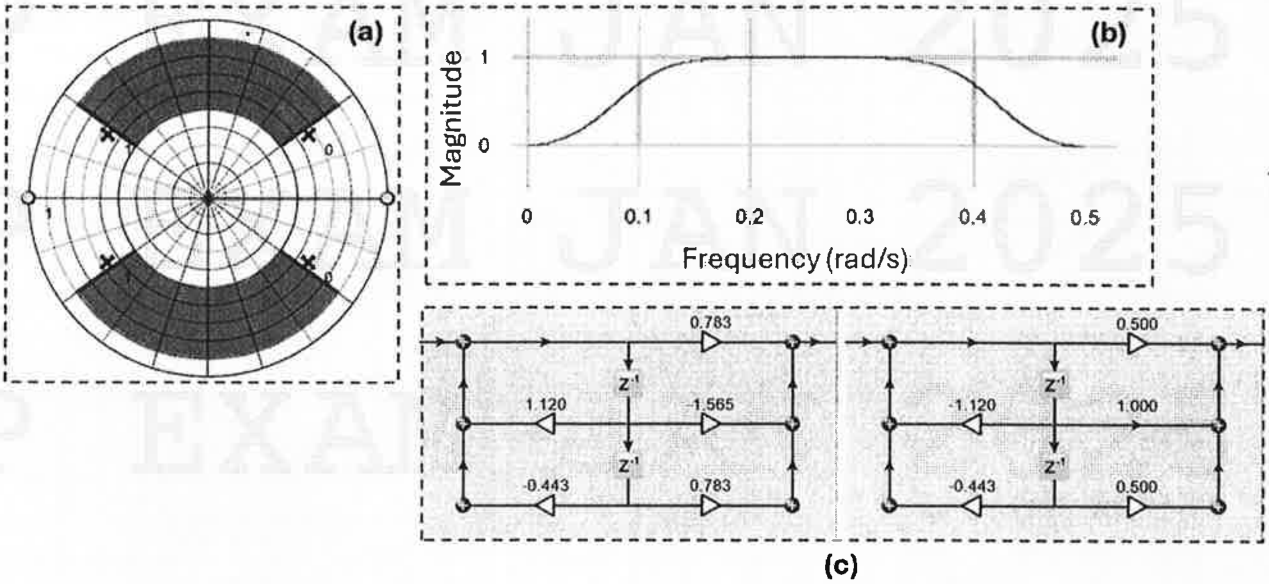


FIGURE Q5

a. With the aid of a diagram and based on the zero-pole plot of the filter, determine the filter's magnitude response at $\omega = \pi/10$ rad/s.

[6 marks]

b. Based on the position of the poles, discuss the stability of the filter, and explain how to guarantee the stability of the designed filter.

[6 marks]

c. Based on the direct-form II implementation, derive the transfer function of the filter.

[6 marks]

d. In another audio processing application, you are required to design a band-stop filter without changing the structure of the filter. Provide the zero-pole plot of your proposed design and justify your answer.

[6 marks]

– END OF PAPER –

APPENDIX I

TABLE AI.1 Common DTFT Pairs

Sequence	DTFT
$\delta[n]$	1
$\delta[n-n_0]$	$e^{-jn_0\omega}$
1	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega+2\pi k)$
$a^n u[n] \quad ,\lvert a\rvert<1$	$\frac{1}{1-ae^{-j\omega}}$
$-a^n u[-n-1] \quad ,\lvert a\rvert>1$	$\frac{1}{1-ae^{-j\omega}}$
$u[n]$	$\frac{1}{1-e^{-j\omega}}+\sum_{k=-\infty}^{\infty}\pi\delta(\omega+2\pi k)$
$(n+1)a^n u[n] \quad ,\lvert a\rvert<1$	$\frac{1}{(1-ae^{-j\omega})^2}$
$\frac{r^n \sin\omega_p(n+1)}{\sin\omega_p}u[n] \ ,\lvert r\rvert<1$	$\frac{1}{1-2r\cos\omega_p e^{-j\omega}+r^2e^{-j2\omega}}$
$\frac{\sin\omega_c}{\pi n}$	$X(e^{j\omega})=\begin{cases}1, & \lvert\omega\rvert<\omega_c\\ 0, & \omega_c<\lvert\omega\rvert\le\pi\end{cases}$
$x[n]=\begin{cases}1, & 0\le n\le M\\ 0, & otherwise\end{cases}$	$\frac{\sin[\omega(M+1)/2]}{\sin(\omega/2)}e^{-j\omega M/2}$
$e^{jn\omega_0}$	$\sum_{k=-\infty}^{\infty}2\pi\delta(\omega-\omega_0+2\pi k)$
$\cos(\omega_0 n+\varphi)$	$\sum_{k=-\infty}^{\infty}[\pi e^{j\varphi}\delta(\omega-\omega_0+2\pi k)+\pi e^{-j\varphi}\delta(\omega+\omega_0+2\pi k)$

TABLE AI.2 Properties of DTFT

Property	Sequence	DTFT
Linearity	$ax[n]+by[n]$	$aX(e^{j\omega})+bY(e^{j\omega})$
Time-shift	$x[n-n_0]$	$e^{-jn_0\omega}X(e^{j\omega})$
Time-reversal	$x[-n]$	$X(e^{-j\omega})$
Modulation	$e^{jn\omega_0}x[n]$	$X(e^{j(\omega-\omega_0)})$
Convolution	$x[n]*y[n]$	$X(e^{j\omega})Y(e^{j\omega})$
Conjugation	$x^*[n]$	$X^*(e^{-j\omega})$
Derivative	$nx[n]$	$j\frac{dX(e^{j\omega})}{d\omega}$
Multiplication	$x[n]y[n]$	$\frac{1}{2\pi}\int_{-\pi}^{\pi}X(e^{j\theta})\,Y(e^{j(\omega-\omega_0)})d\theta$

APPENDIX II

TABLE AII.1 Common z-Transform Pairs

Sequence	z-Transform	ROC
$\delta[n]$	1	$All\ z$
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\|z\| > \|a\|$
$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\|z\| < \|a\|$
$u[n]$	$\frac{1}{1-z^{-1}}$	$\|z\| > 1$
$-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\|z\| < 1$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| > \|a\|$
$-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| < \|a\|$
$(\cos\omega_0 n)u[n]$	$\frac{1-(\cos\omega_0)z^{-1}}{1-(2\cos\omega_0)z^{-1}+z^{-2}}$	$\|z\| > 1$
$(\sin\omega_0 n)u[n]$	$\frac{(\sin\omega_0)z^{-1}}{1-(2\cos\omega_0)z^{-1}+z^{-2}}$	$\|z\| > 1$
$(r^n\cos\omega_0 n)u[n]$	$\frac{1-(r\cos\omega_0)z^{-1}}{1-(2r\cos\omega_0)z^{-1}+r^2z^{-2}}$	$\|z\| > r$
$(r^n\sin\omega_0 n)u[n]$	$\frac{(r\sin\omega_0)z^{-1}}{1-(2r\cos\omega_0)z^{-1}+r^2z^{-2}}$	$\|z\| > r$
$x(n) = \begin{cases} a^n, 0 \le n \le N-1 \\ 0, otherwise \end{cases}$	$\frac{1-a^N z^{-N}}{1-z^{-1}}$	$\|z\| > 0$

TABLE AII.2 Properties of z-Transform

Property	Sequence	z-transform	ROC
Linearity	$ax[n]+by[n]$	$aX(z)+bY(z)$	$Contains\ R_x \cap R_y$
Time-shift	$x[n-n_0]$	$z^{-n_0}X(z)$	R_x
Time-reversal	$x[-n]$	$X(z^{-1})$	$1/R_x$
Exponentiation	$\alpha^n x[n]$	$X(\alpha^{-1}z)$	$\|\alpha\|R_x$
Convolution	$x[n]*y[n]$	$X(z)Y(z)$	$Contains\ R_x \cap R_y$
Conjugation	$x^*[n]$	$X^*(z^*)$	R_x
Derivative	$nx[n]$	$-z\frac{dX(z)}{dz}$	R_x

Convolution, $x[n]*h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$