Reducing Sway of Gantry Crane Load

by

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CERTIFICATION OF APPROVAL

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Approved by,

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UNIVERSITI TEKNOLOGI PETRONAS

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CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

_____
(NURFARIZA BT MOHD ZUKI)

ABSTRACT

Cranes are widely used for transportation of heavy material in factories, warehouse, shipping yards, building construction and nuclear facilities. This project is focused on gantry crane. There are several factors that will contribute to the crane swinging such as wind, crane design and operations. Hence, the objective of this project is to reduce and minimize the sway in order to reduce the operational time and cost. To achieve the objective, study on gantry cranes had been done to gather the design parameter as well as the data of the gantry crane. A study and analysis of the theoretical model of gantry crane system had been done to identify factors that influence the sway of the load. ADAMS software is used to develop a simple model of gantry crane. By the end, with this model the load will be able to transfer from point to point and, at the same time, the load swing is kept small during the transfer process and completely reaches at the load destination.

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CHAPTER 1

INTRODUCTION

1.1. Background of Study

Crane is a mechanical device used to lift and lower materials and also to move them horizontally. In addition, it has been designed to move loads beyond the normal capability of a human. Cranes are usually employed in many types of industry such as transport, construction and manufacturing. In order to lift heavy payloads in these types of industry, cranes usually have very strong structures. There are many different kinds of cranes available, for example, mobile crane, gantry crane, tower crane, hammerhead crane, floating crane, and jib crane. These cranes have similar functions but differ in design and also used at difference places.

This project is focused on gantry crane. There are two major types of gantry crane used in shipping yard: rubber-tired gantry crane as shown in Figure 1.1 and ship-to-shore gantry crane shown in Figure 1.2. A gantry crane has a hoist system suspended from a trolley which can move horizontally along the gantry rails, usually fitted under a beam. It has a similar mechanism supported between uprights which themselves have wheels so that the whole crane can move at right angles to the direction of the gantry rails. Basically, there are different sizes of gantry crane and they can move very heavy and large loads for examples used in shipyards or industrial installations.

Figure 1.1: Rubber-tired gantry crane

Figure 1.2: Ship-to-shore gantry crane

There are two major considerations that should take into account in designing of gantry crane. Firstly, the gantry crane must be able to lift a container of a specified load and secondly, it must remain stable and not fall down when the container is lifted and moved to another location. The major task of this project is on how to reduce the swinging of gantry crane when it lifted and laid the container. There are several methods and ways to overcome this problem have been found and they are covered in the literature review.

1.2.   Problem Statement

Nowadays, most logistic players in the world are using gantry crane to load and unload their container ships from one place to another place. Previously, all the cranes were manually operated means that the cranes operated hundred percent using human skills. But manual operation became difficult when cranes became larger, faster and higher. Due to this, efficient controllers are incorporated into the cranes system to guarantee fast turnover time and also to meet safety requirement.

During the operation of a crane system, for example in the container yard, it is necessary to control the crane trolley position so that the swing of the hanging container is minimized. In fact, it is impossible to ensure that the crane is not swinging while doing work because there are many factors that impact on it such as wind, crane design, and the crane operator skill level.

The efficiency and safety of gantry crane operations depends highly on the skill of the crane operator. An efficient gantry crane would minimize the time it takes to transfer a load from one point to another and also will reduce the cost.

1.3. Objectives of the Project

From the problems that are stated before, this project objective is to reduce and minimize the swinging of the gantry crane load. When the sway is reduced, the operational time and cost are also reduced.

In order to complete the project within the time limit and the scope given, several objectives for this project have been identified and listed such as below:

1. To conduct a study of load sway of a gantry crane using an analytical model.
2. To identify and analyze factors that influences the sway of gantry crane load.
3. To develop a simple model of gantry crane motion using ADAMS software and run simulation on the model.

1.4. Scope of Study

The scopes for this project are:

- Study and analyze the analytical model of the gantry crane system
- Develop a simple model of gantry crane by ADAMS software
- Simulate a gantry crane model by using ADAMS software.

CHAPTER 2

LITERATURE REVIEW

This chapter consists of some information about gantry crane system and also an overview of the work that has been published in relation to minimize sway of the gantry crane load.

2.1.   Gantry Crane Operation

Rubber-tired and ship-to-shore gantry crane consists of three main components which are trolley, bridge, and gantry. Figure 2.1 shows a typical gantry crane. Trolley with a movable or fixed hoisting mechanism is the load lifting component. It moves on and parallel to a bridge which is rigidly attached to a supporting structure called gantry. The gantry extends downward from the bridge to the ground where it can be mobilized on wheels or set of tracks.

Figure 2.1: Picture of a gantry crane at a shipping dock

Gantry crane is one of the structures which are very flexible in nature. The gantry crane load can oscillate because of the internal motion of the bridge or trolley or the external disturbances such as wind. There are large amounts of research pertaining to control the unwanted oscillations. Engineers have sought to improve the ease-of-use, increase operational efficiency, and mitigate safety concerns by addressing three primary aspects of crane systems [1]:

1. motion-induced oscillation;
2. disturbance-induced oscillation; and
3. positioning capability

When a human operator attempts to maneuver loads using an overhead gantry crane, the oscillations induced into the load by the motion of the trolley is important. These oscillations make it difficult to control the load quickly and with positioning accuracy. Furthermore, when the workspace is cluttered with obstacles, the oscillations can create significant safety issues, especially when the load or obstacles are of hazardous nature [2]. Therefore, it is important to minimize the load sway so that handling time can be minimized while chances of misalignments and collisions in crane operations are reduced.

2.2. Literature Review of the Gantry Crane Controller

This section is a review of the literatures that address various aspects the different of the gantry crane control system.

2.2.1. Open Loop Techniques

The most advanced and practical crane controllers today are controllers based on an open loop approach. It is designed to automate and shorten the cycle time for the gantry cranes operating along a pre-defined path. The open loop approach is intended not to excite the swing motion. It is simple to implement but cannot suppress swing motion caused by external disturbance such as wind. The most widely used of the open loop control techniques is input shaping [3].

Input shaping is one of the successful approach uses to reduce the crane sway. It is used to minimize the motion-induced oscillation and plus, it does not require the feedback mechanisms of closed-loop controllers. In addition, it is also easier to derive and implement than time-optimal control schemes and does not require the feedback mechanisms of closed-loop and adaptive controllers. Input shaping is implemented in real time by convolving the command signal with an impulse sequence (an input shaper) [4].

This technique is illustrated in Figure 2.2 with a pulse input and an input shaper containing three positive impulses. Note that the shaped input that results from the convolution has a rise time that is longer than the unshaped input by an amount equal to the duration, Δ, of the input shaper [2].
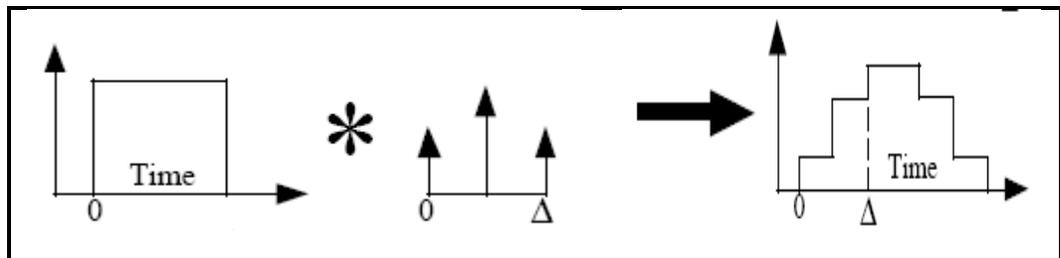
Figure 2.2: Illustration of input shaping technique

Figure 2.3 shows an example situation when a gantry crane is used to maneuver a payload through an environment filled with obstacles. The projected position of the trolley does not encounter any obstacles, but the oscillating payload does contact with the obstacles [2].
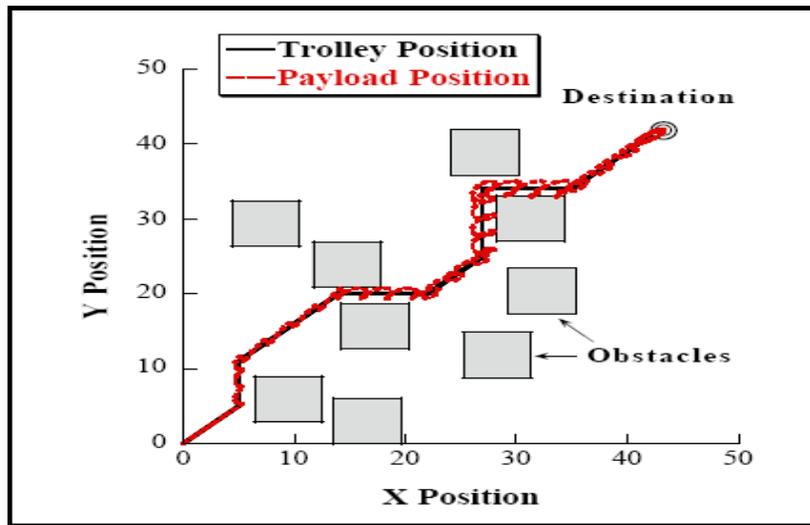
Figure 2.3: Example of payload oscillation

Figure 2.4 shows the response of an overhead gantry crane to identical operator commands as those used in Figure 2.2; however, the commands have been filtered with an input shaper. The payload now tracks the trolley position very closely and there are no collisions with obstacles [2].
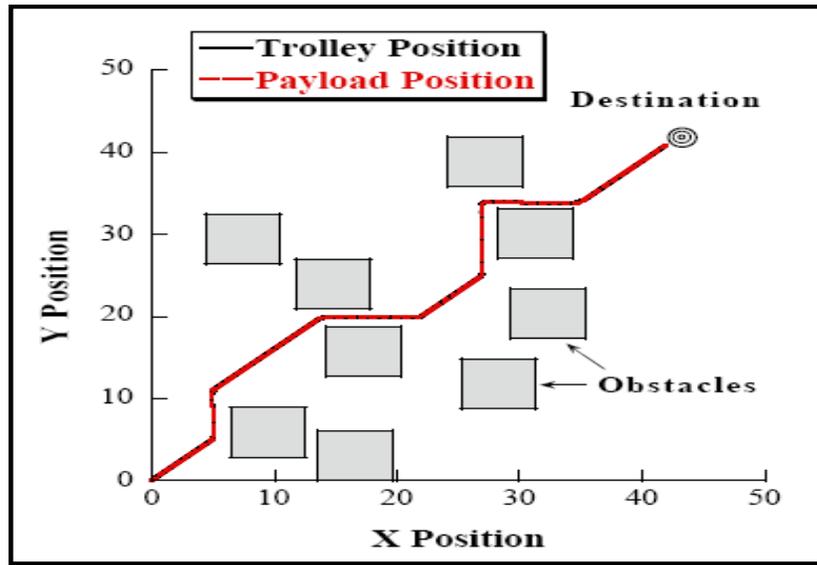
Figure 2.4: Example of response when commands are shaped

But, input-shaping techniques are very sensitive to changes in system parameters, time delays, external disturbances, and they require "highly accurate values of the system parameters" to achieve satisfactory system response. Robustness of input-shaping techniques could be improved to frequency variations resulting from changes in the cable length [5].

Singer et al. proposed robust input shaping techniques to reduce motion-induced oscillations of a 15-ton bridge crane. Fang et al. proposed to control final trolley position and motion-induced oscillation through proportional-derivative (PD) control in which the coupling between the cable angle and the motion of the trolley was artificially increased [1].

2.2.2. Closed Loop Techniques

The closed loop approach of swing motion provides the ability to eliminate existing swing motion and rejects disturbance at the cost of extra sensors. There are different types of closed loop approach that developed to control crane system such as linear control, adaptive control, fuzzy logic control and non-linear control.

The controller developed by the Khalid et al. has been designed with positioning and oscillation suppression properties by merging feedback control with input shaping [1].

In a nonlinear controller is proposed for the trolley crane systems using Lyapunov method and a modified version of sliding-surface control is then utilized to achieve cart position control. However, the sway angle dynamics has not been considered for stability analysis [6].

2.3. Mathematical Model of the Crane

In order to study the motion of the crane, equations of motion of the payload need to be established. The solution to these equations should describe the motion of a load suspended by a cable to a moving trolley. The trolley is moving horizontally and the cable is assumed to be rigid. Figure 2.5 defines the parameters of the load and trolley in x-z directions.

Lagrangian approach is used to derive the equation of motion. Based on Figure 2.5, the kinetic and potential energies in term of generalized coordinate can be expressed.
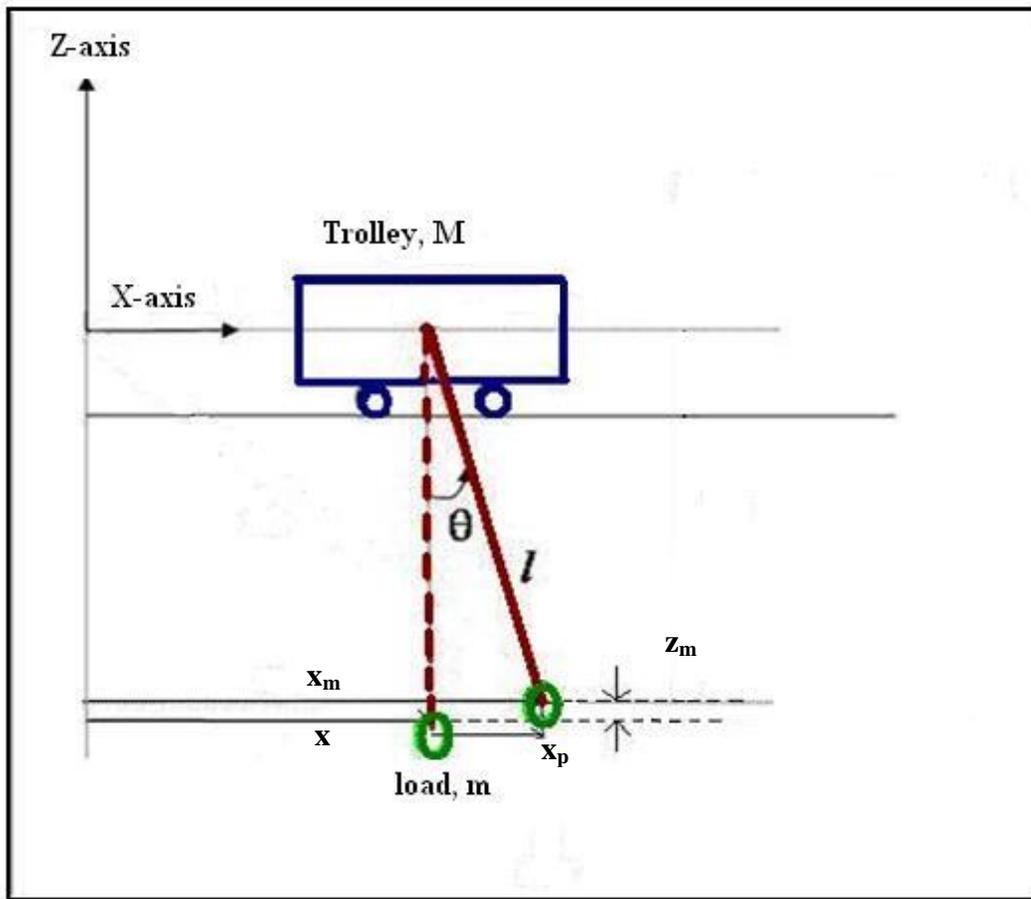
Figure 2.5: Position of the trolley and the load before and after displaced

2.3.1. Kinetic Energy

The kinetic energy of the system T_s the sum of the kinetic energies of the trolley and the load is given by the equation below:

$$T_s = T_{trolley} + T_{load}$$

$$= \frac{1}{2} M v_T^2 + \frac{1}{2} m v_P^2 \tag{1}$$

where the first term refers to the mass (M) and velocity (v_T) of the trolley and the second term (m and v_P) refers to those of the load.

The velocity of the trolley is

$$v_T = \dot{x} \tag{2}$$

While the velocity of the load is

$$v_P = \sqrt{\dot{x}_m^{\,2} + \dot{z}_m^{\,2}}$$

To get the expression of v_P in terms of θ and x,

$$x_m = x + x_P$$
$$\dot{x}_m = \dot{x} + \dot{x}_P$$

$$x_P = l\sin\theta$$
$$\dot{x}_P = l\dot{\theta}\cos\theta$$

$$z_m = l(1 - \cos\theta)$$
$$\dot{z}_m = l\dot{\theta}\sin\theta$$

where x_m is the absolute displacement of the load in the x-axis, x is the absolute displacement of the trolley in the x-axis and x_P is the displacement of the load relative to the trolley in the x-axis. z_m is the absolute displacement of the load in the z-axis, l is the cable length and θ is the sway angle of the load.

Thus, the v_P can be expressed in terms of θ and x is given by the following derivation

$$v_P^{\,2} = \dot{x}_m^{\,2} + \dot{z}_m^{\,2}$$

$$= (\dot{x} + \dot{x}_P)^2 + \dot{z}_m^{\,2}$$
$$= \dot{x}^2 + 2\dot{x}l\dot{\theta}\cos\theta + l^2\dot{\theta}^2\cos^2\theta + l^2\dot{\theta}^2\sin^2\theta \tag{3}$$
$$= \dot{x}^2 + l^2\dot{\theta}^2 + 2\dot{x}l\dot{\theta}\cos\theta$$

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By substituting (2) and (3) into (1), yields:

$$T = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m\left(\dot{x}^2 + l^2\dot{\theta}^2 + 2\dot{x}l\dot{\theta}\cos\theta\right) \tag{4}$$

2.3.2. Potential Energy

The total potential energy of the system, U can only come from the load because the trolley does not move vertically. It is given by the equation below:

$$U = mgz_m$$
$$= mgl(1 - \cos\theta) \tag{5}$$

2.3.3. Lagrange's Equation

The angle θ is selected as the generalized parameter of the Lagrange's equation. From equation (4), the term is to be differentiated against θ and $\dot{\theta}$ respectively and yields,

$$\frac{\partial T}{\partial \theta} = \frac{1}{2}m\left(-2\dot{x}l\dot{\theta}\sin\theta\right)$$
$$= -m\dot{x}l\dot{\theta}\sin\theta \tag{6}$$

$$\frac{\partial T}{\partial \dot{\theta}} = \frac{1}{2}m\left(2l^2\dot{\theta} + 2\dot{x}l\cos\theta\right)$$
$$= ml\left(l\dot{\theta} + \dot{x}\cos\theta\right)$$

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From equation (5), the term is differentiated against θ,

$$\frac{\partial U}{\partial \theta} = mgl\sin\theta \qquad (7)$$

Applying the Lagrange's equation on θ,

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) - \frac{\partial T}{\partial \theta} + \frac{\partial U}{\partial \theta} = 0$$

(8)

where:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) = ml\left(l\ddot{\theta} + \ddot{x}\cos\theta - \dot{x}\dot{\theta}\sin\theta\right) \qquad (9)$$

Therefore, by substituting all the derivatives (6), (7), and (9) into (8) gives:

$$ml\left(l\ddot{\theta} + \ddot{x}\cos\theta - \dot{x}\dot{\theta}\sin\theta\right) - \left(-m\dot{x}l\dot{\theta}\sin\theta\right) + mgl\sin\theta = 0$$
$$ml(l\ddot{\theta} + \ddot{x}\cos\theta) + mgl\sin\theta = 0$$

Simplifying the above equation by dividing with ml^2, gives:

$$\ddot{\theta} + \frac{\ddot{x}}{l}\cos\theta + \frac{g}{l}\sin\theta = 0 \qquad (10)$$

The system is of second-order control type if the trolley acceleration is constant. Therefore, the system is to be analyzed when the trolley acceleration is constant.

$$\ddot{x} = K$$

where K is a constant.

The trigonometric terms can be simplified by using Taylor's expansion method which is accurate if θ is small. The expansions and their magnitudes of error are listed in the table below where θ is in radian:

Table 2.1: Trigonometric terms using Taylor's expansion method

Approximation	Difference	$\theta \cong 5°$	$\theta \cong 10°$	$\theta \cong 20°$
$\sin\theta \cong \theta$	$\sin\theta - \theta$	-0.0001	-0.0009	-0.0070
$\cos\theta \cong 1$	$\cos\theta - 1$	-0.0038	-0.0152	-0.0603
$\cos\theta \cong 1 - \theta^2/2$	$\cos\theta - (1 - \theta^2/2)$	0.0000	0.0000	0.0006

Applying the simpler approximations of the sine and cosine terms (i.e. $\sin\theta \cong \theta$ and $\cos\theta \cong 1$), equation (10) is now becomes:

$$\ddot{\theta} + \frac{g}{l}\theta + \frac{K}{l} = 0 \qquad (11)$$

2.3.4. Laplace Transform

Applying Laplace Transform with $\theta(t)$ transformed into $\Phi(s)$, equation (11) becomes:

$$s^2\Phi - s\theta(0) - \dot{\theta}(0) + \frac{g}{l}\Phi + \frac{K}{ls} = 0$$

$$\left(s^2 + \frac{g}{l}\right)\Phi = s\theta(0) + \dot{\theta}(0) - \frac{K}{ls}$$

$$\Phi = \frac{s\theta(0) + \dot{\theta}(0) - \dfrac{K}{ls}}{s^2 + \dfrac{g}{l}}$$

Defining the natural frequency, ω as:

$$\omega = \sqrt{\frac{g}{l}}$$

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Then,

$$\Phi = \frac{s\theta(0) + \dot{\theta}(0) - \dfrac{K}{ls}}{s^2 + \omega^2}$$

$$= \frac{s\theta(0)}{s^2 + \omega^2} + \frac{\dot{\theta}(0)}{s^2 + \omega^2} - \frac{K}{ls\left(s^2 + \omega^2\right)}$$

$$= \frac{s}{\left(s^2 + \omega^2\right)}\theta(0) + \frac{\omega}{\left(s^2 + \omega^2\right)}\frac{\dot{\theta}(0)}{\omega} - \left(\frac{1}{s} - \frac{s}{\left(s^2 + \omega^2\right)}\right)\frac{K}{l\omega^2}$$

$$= \frac{s}{\left(s^2 + \omega^2\right)}\left(\theta(0) + \frac{K}{l\omega^2}\right) + \frac{\omega}{\left(s^2 + \omega^2\right)}\frac{\dot{\theta}(0)}{\omega} - \frac{1}{s}\frac{K}{l\omega^2}$$

Inverting the Laplace transform into the time domain will give the sway angle of the load and the velocity of the load with respect to time respectively.

$$\theta(t) = \left(\theta(0) + \frac{K}{l\omega^2}\right)\cos\omega t + \frac{\dot{\theta}(0)}{\omega}\sin\omega t - \frac{K}{l\omega^2} \tag{12}$$

$$\dot{\theta}(t) = -\left(\omega\theta(0) + \frac{K}{l\omega}\right)\sin\omega t + \dot{\theta}(0)\cos\omega t \tag{13}$$

Where:

$\theta(0)$ = initial displacement of the load

$\dot{\theta}(0)$ = initial velocity of the load

l = cable length

K = acceleration of the trolley

CHAPTER 3

PROJECT METHODOLOGY

3.1. Procedure Identification

This project will be implemented according to the following project methodology. This methodology was designed to have a basic overview of the project tasks and as preparation to complete this project. Figure 3.1 below shows the steps that have been followed in carrying out this project.

```
                    ( Project Assigned )
                            |
                            v
              +-------------------------------+
              |  Define and analyze the problems  |
              +-------------------------------+
                            |
                            v
              +-------------------------------+
              |  Research and review work done   |
              |       by other researchers       |
              +-------------------------------+
                            |
                            v
              +-------------------------------+
              |  Establish the design parameters  |
              |    for the gantry crane model     |
              +-------------------------------+
                            |
                            v
    +---------------------------+      +---------------------------+
    |  Derive the mathematical   |----->|  Gather data to simulate   |
    |    model of the crane      |      |  and solve the equations   |
    +---------------------------+      +---------------------------+
                                                   |
                                                   v
    +---------------------------+      +---------------------------+
    |  Build and simulate a      |<-----|  Determine conditions that |
    |  simple model of gantry    |      |  reduce the load sway       |
    |  crane using ADAMS         |      +---------------------------+
    +---------------------------+                  |
               |                                   v
               v                       +---------------------------+
    ( Project Completed )<-------------|  Propose suitable trolley   |
                                       |  motion profiles for the    |
                                       |  minimum sway               |
                                       +---------------------------+
```

Figure 3.1: Project design flow chart

3.2. Software Required

In completing this project, there are two tools required. They are excel spreadsheet and ADAMS software.

3.2.1. Excel Spreadsheet

A common spreadsheet is sufficient to solve the analytical model. By using the spreadsheet, the suitable trolley motion profiles can be proposed by producing the graph of trolley velocity and sway angle of the load.

3.2.2. ADAMS Software

ADAMS/View is a powerful modeling and simulation environment. It enables to design, visualize and improve the mechanical system model prior to building a physical prototype. Besides, ADAMS/View can be used to submit simulations to ADAMS/Solver to compute the force and motion behavior of the system and also to write that information to output files.

CHAPTER 4

RESULTS AND DISCUSSION

4.1.   Results

This project has adopted a simple velocity profile of the trolley as shown in Figure 4.1 and uses the values of certain trolley motion parameters as input in a mathematical model of the motion which was derived in the Chapter 2. A simple approximation of the trolley velocity profile is a constant acceleration from rest to a velocity which remains constant for a certain period of time and then followed by a constant deceleration before stopping. With a few assumptions, a linear model on the sway magnitude of the load as a function of the trolley motion parameters can be used.
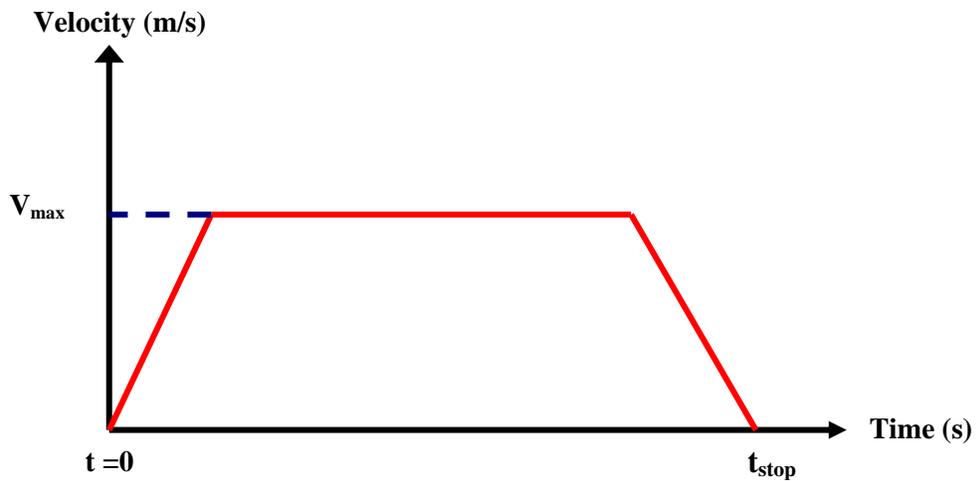
Figure 4.1: Velocity profile

<u>Assumptions and Limitations</u>

There are a few assumptions made to simplify the process of system modeling. The assumptions are listed below:

a) The load size and shape do not affect the motion and thus can be considered as point of mass.

b) The cable is rigid and no change in tension.

c) The trolley and the load are assumed to move in two dimensional only which is x-z plane.

4.1.1. Analytical Solution

Equation (12) and (13) from Chapter 2 are used to analyze the motion of the load suspended to the trolley. It is important to recognize the different phases of the trolley motion as defined in Table 4.1 before use both equations.

Sway angle of the load: $\theta(t) = \left(\theta(0) + \dfrac{K}{l\omega^2} \right) \cos \omega t + \dfrac{\dot{\theta}(0)}{\omega} \sin \omega t - \dfrac{K}{l\omega^2}$

Velocity of the load: $\dot{\theta}(t) = -\left(\omega\, \theta(0) + \dfrac{K}{l\omega} \right) \sin \omega t + \dot{\theta}(0) \cos \omega t$

Table 4.1: Phases of the trolley motion

Phase	Trolley Motion Status
P-1	Acceleration from 0 to V_{max}
P-2	Constant velocity, V
P-3	Deceleration from V_{max} to 0
P-4	Stationary

The both equations has been solved using the excel spreadsheet. The example detail of excel spreadsheet has been attached in the Appendix. The results are described by the plot of two parameters against time. They are the trolley velocity and the sway angle of the load. Figure 4.2 and Figure 4.3 show the examples of the trolley velocity and the result for sway angle of the load which have been divided into four phases.
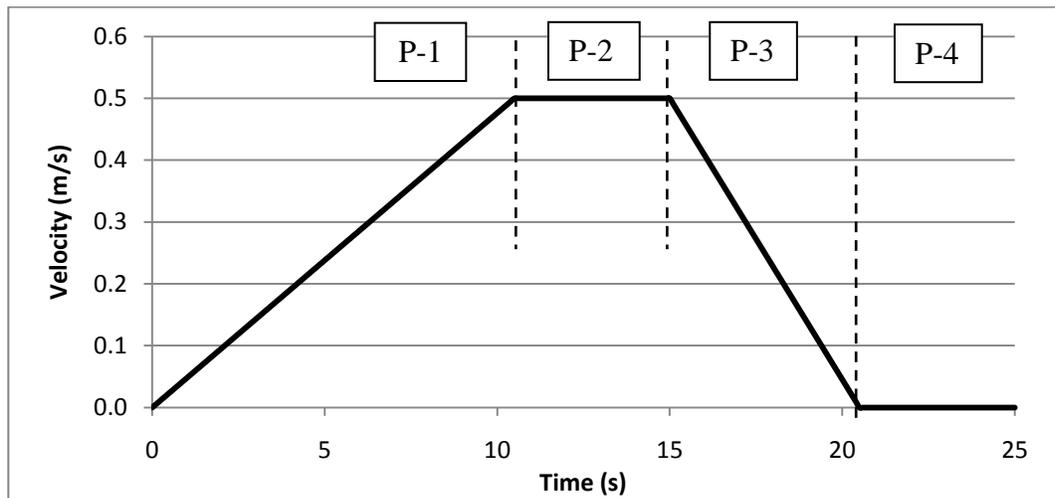
Figure 4.2: A simple velocity profile of the trolley
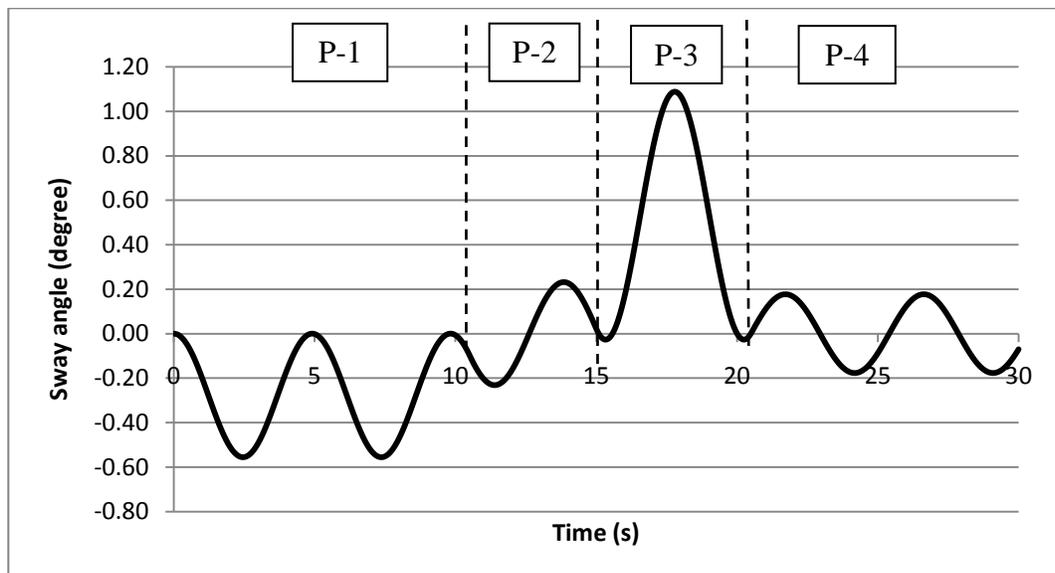
Figure 4.3: The sway angle of the load in the four phases of trolley motion

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There are a few sets of different trolley velocity profiles are made and shown in this Chapter. The data used are tabulated in the table for each set. Every set of trolley velocity profiles have variation values of maximum velocity, acceleration and also deceleration.

Table 4.2 and Table 4.3 show that the value for maximum velocity of the trolley is same which is 0.5 m/s but the values for acceleration and deceleration are differ. Figure 4.4 and Figure 4.6 are the velocity profile while Figure 4.5 and Figure 4.7 are the sway angle of the load for each profile respectively.

Table 4.2: Data used as input in both equations

Cable length, l (m)	6
Gravity, g (m/s^2)	9.81
Natural frequency, ω (rad/s)	1.28
Maximum velocity, V_{max} (m/s)	0.5
Acceleration, K (m/s^2)	0.048
Deceleration, K (m/s^2)	0.091
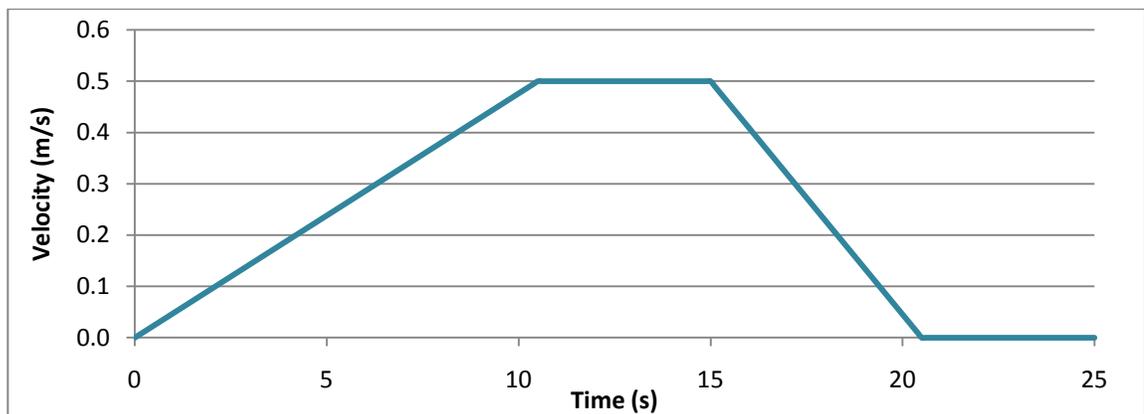
Figure 4.4: Velocity profile of the trolley A
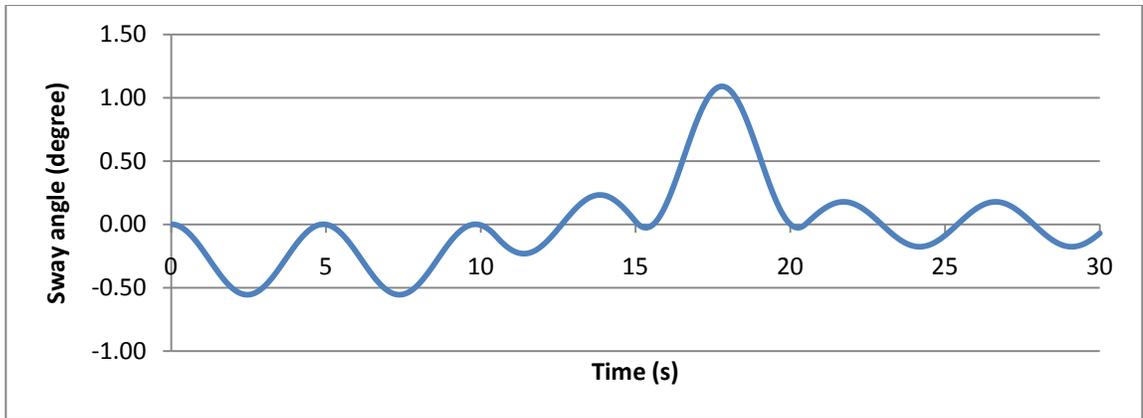
Figure 4.5: The sway angle of the load A

Table 4.3: Data used as input in both equations

Cable length, l (m)	6
Gravity, g (m/s²)	9.81
Natural frequency, ω (rad/s)	1.28
Maximum velocity, V_{max} (m/s)	0.5
Acceleration, K (m/s²)	0.050
Deceleration, K (m/s²)	0.100
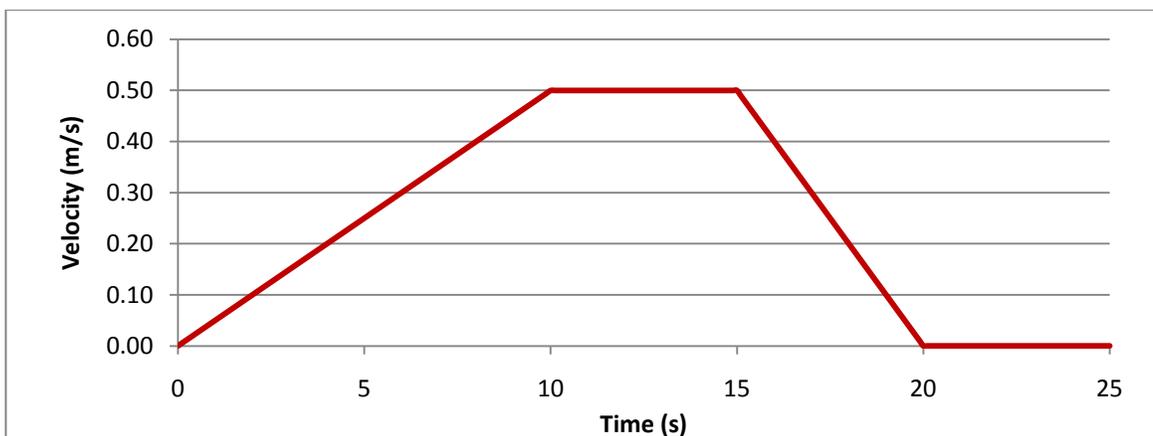
Figure 4.6: Velocity profile of the trolley B

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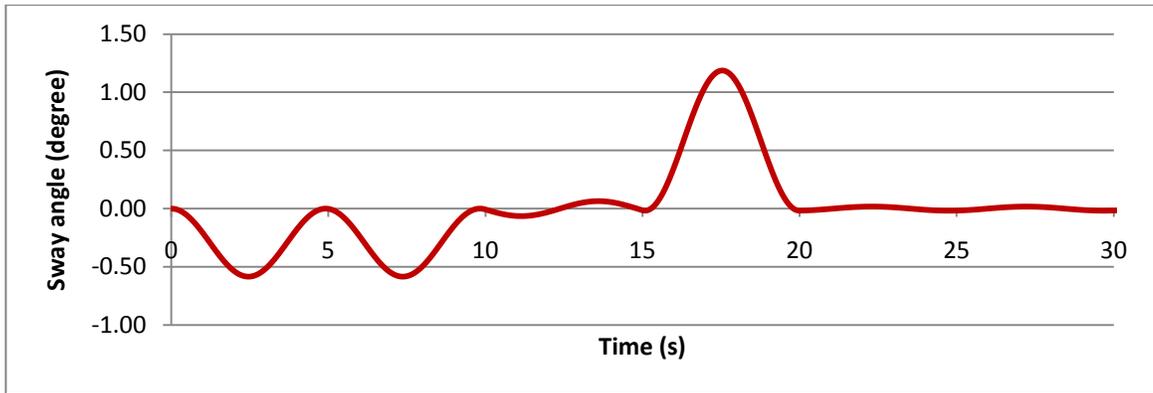
Figure 4.7: The sway angle of the load B

Figure 4.8 and Figure 4.9 show the comparison of velocity profile for trolley A and trolley B and the sway angle of load A and load B respectively.
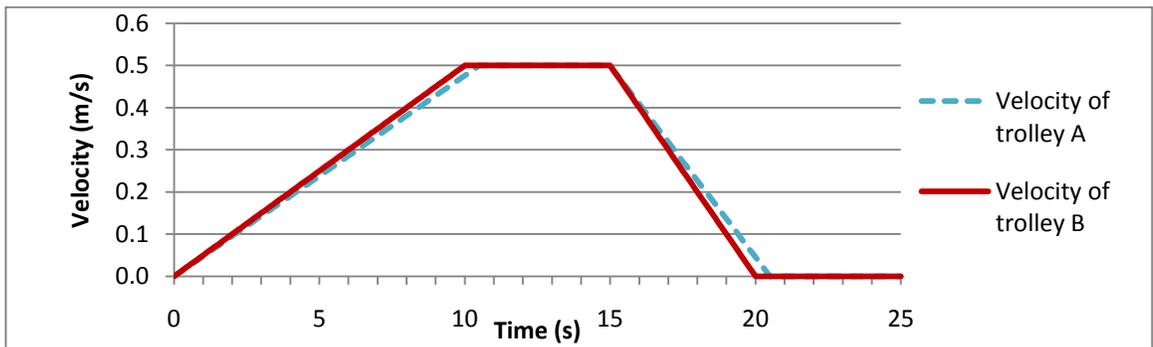
Figure 4.8: Comparison of velocity profile for the trolley A and trolley B
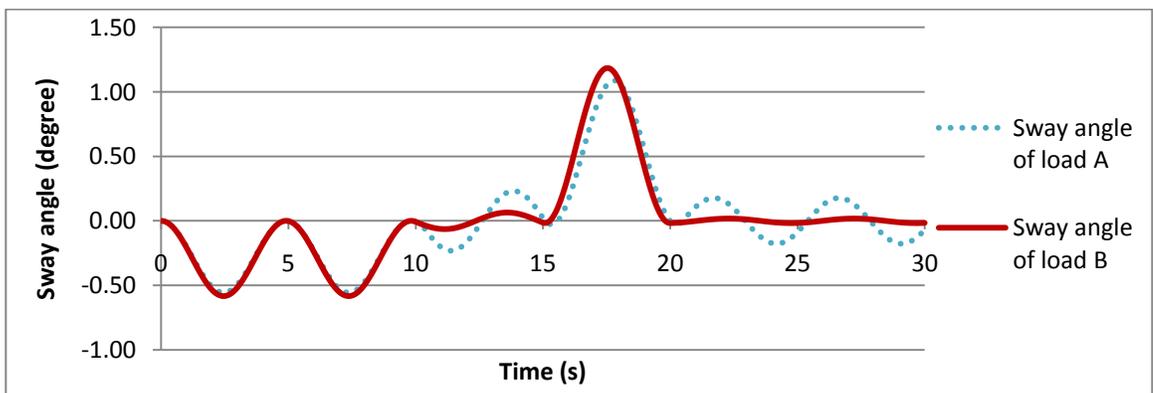
Figure 4.9: Comparison sway angle of load A with load B

Table 4.4 and Table 4.5 show that the value for maximum velocity of the trolley is same which is 1.0 m/s but the values for acceleration and deceleration are differ. Figure 4.10 and Figure 4.12 are the velocity profile while Figure 4.11 and Figure 4.13 are the sway angle of the load for each profile respectively.

Table 4.4: Data used as input in both equations

Cable length, l (m)	6
Gravity, g (m/s^2)	9.81
Natural frequency, ω (rad/s)	1.28
Maximum velocity, V_{max} (m/s)	1.0
Acceleration, K (m/s^2)	0.181
Deceleration, K (m/s^2)	0.227
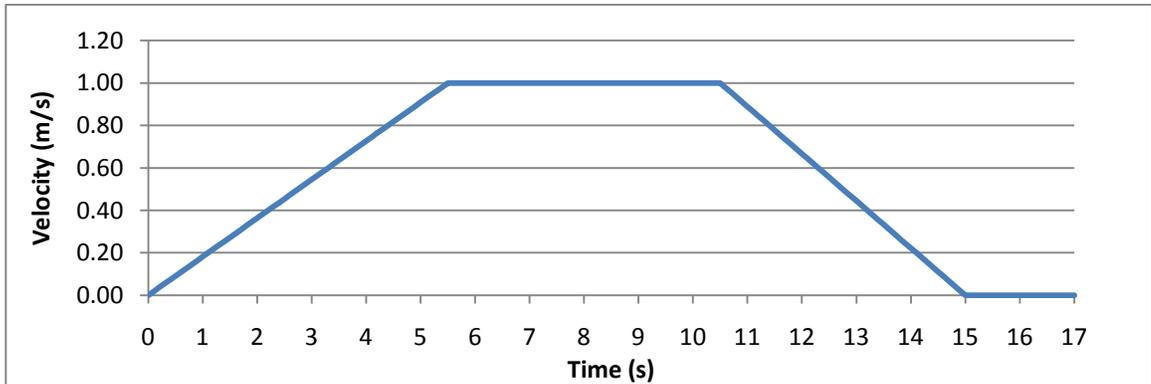
Figure 4.10: Velocity profile of the trolley A
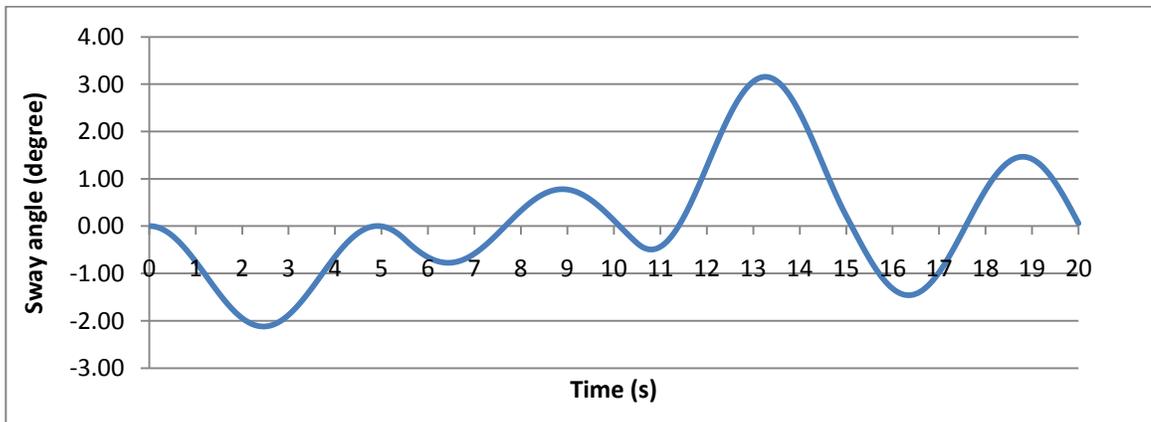
Figure 4.11: The sway angle of the load A

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Table 4.5: Data used as input in both equations

Cable length, l (m)	6
Gravity, g (m/s^2)	9.81
Natural frequency, ω (rad/s)	1.28
Maximum velocity, V_{max} (m/s)	1.0
Acceleration, K (m/s^2)	0.200
Deceleration, K (m/s^2)	0.200
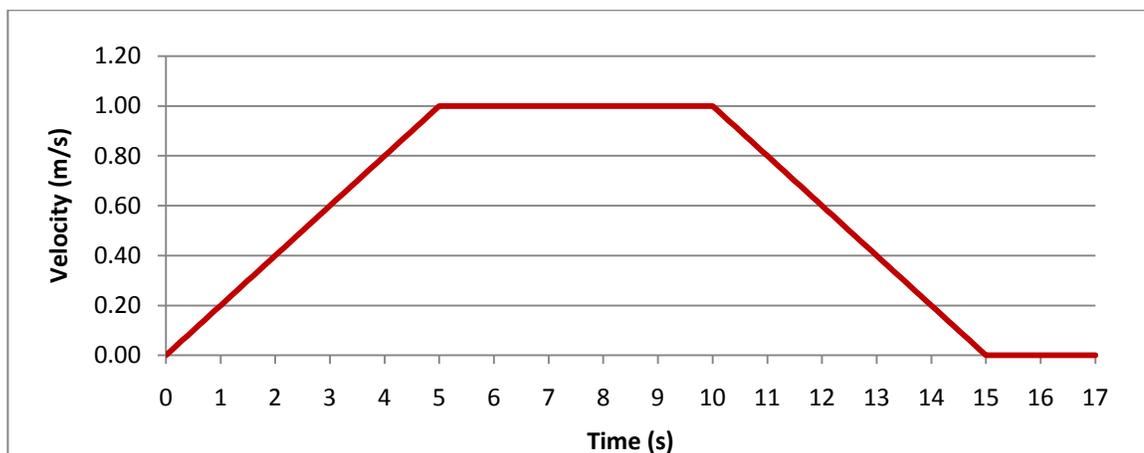
Figure 4.12: Velocity profile of the trolley B
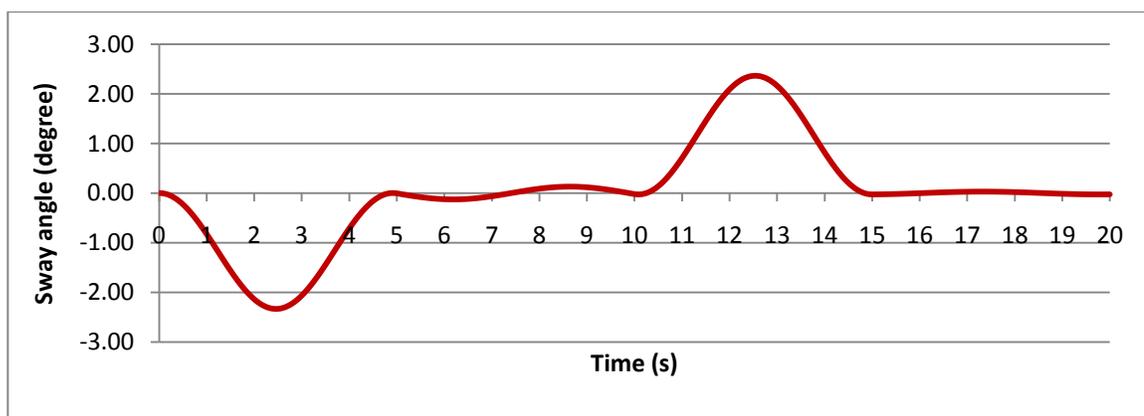
Figure 4.13: The sway angle of the load B

Figure 4.14 and Figure 4.15 show the comparison of velocity profile for trolley A and trolley B and the sway angle of load A and load B respectively.
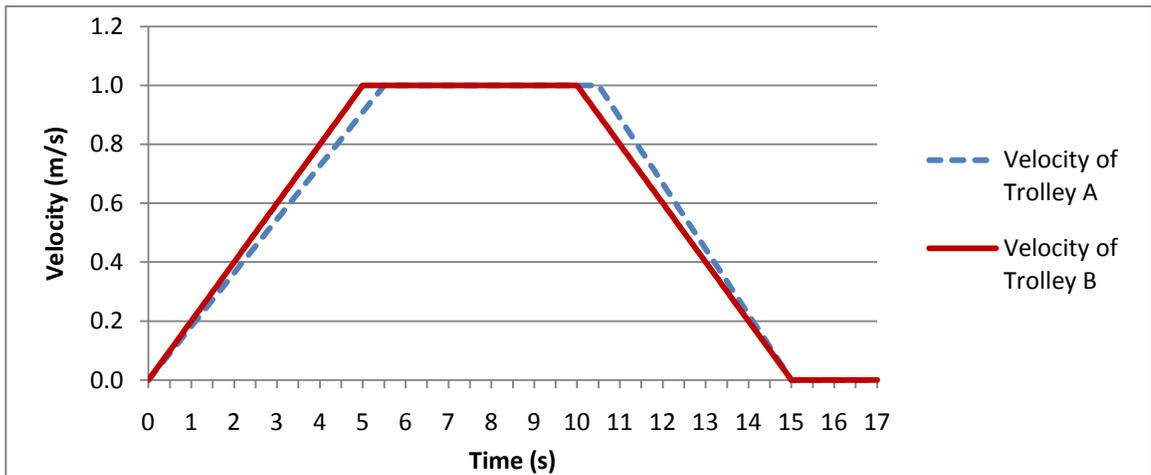
Figure 4.14: Comparison of velocity profile for the trolley A and trolley B
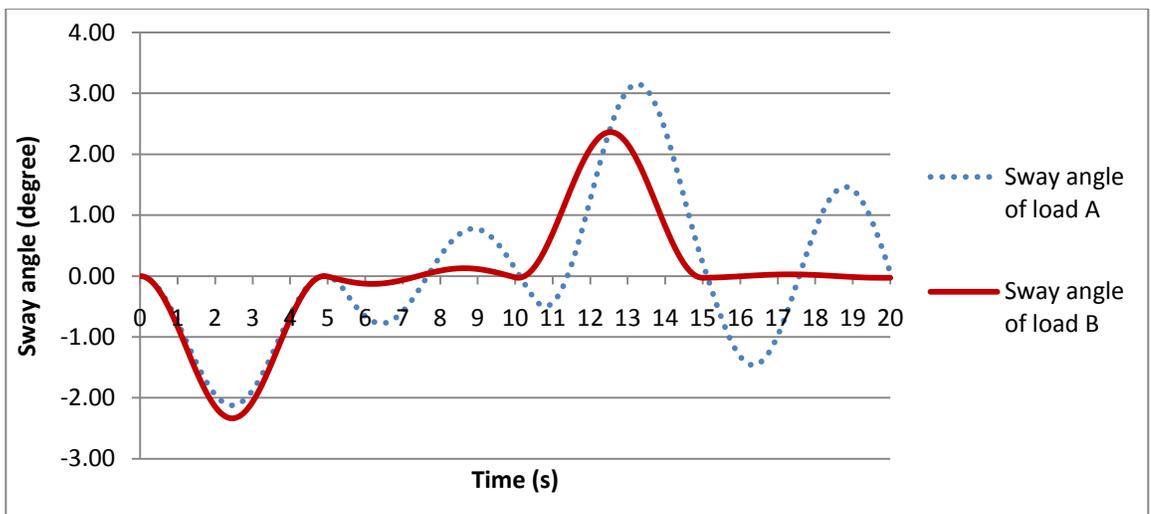
Figure 4.15: Comparison sway angle of load A with load B

Table 4.6 and Table 4.7 show that the value for maximum velocity of the trolley is same which is 1.5 m/s but the values for acceleration and deceleration are differ. Figure 4.16 and Figure 4.18 are the velocity profile while Figure 4.17 and Figure 4.19 are the sway angle of the load for each profile respectively.

Table 4.6: Data used as input in both equations

Cable length, l (m)	6
Gravity, g (m/s^2)	9.81
Natural frequency, ω (rad/s)	1.28
Maximum velocity, V_{max} (m/s)	1.5
Acceleration, K (m/s^2)	0.375
Deceleration, K (m/s^2)	0.250
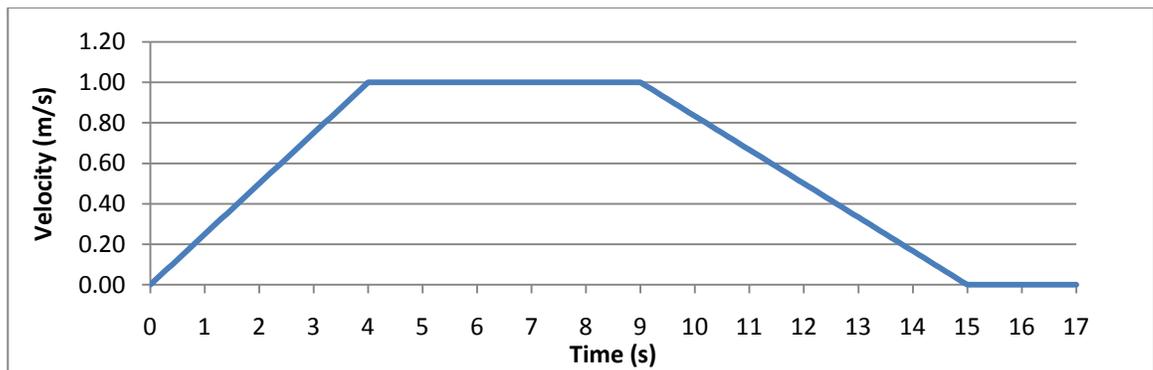
Figure 4.16: Velocity profile of the trolley A
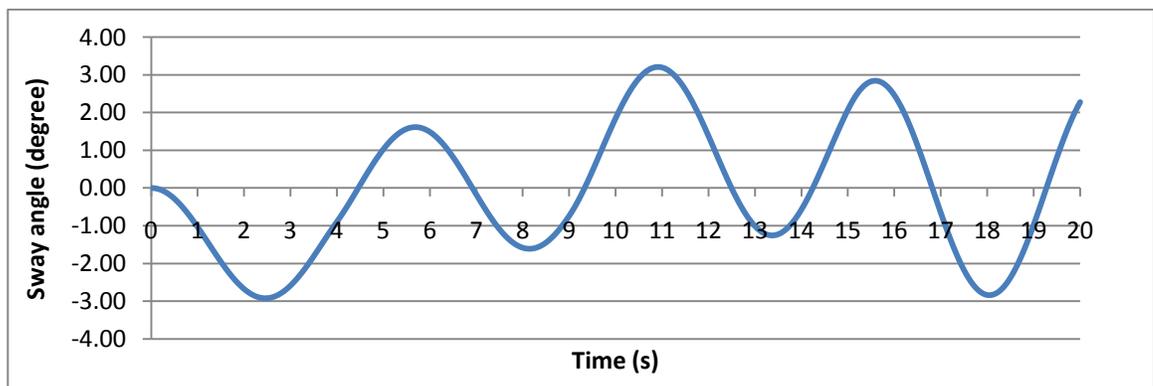
Figure 4.17: The sway angle of the load A

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Table 4.7: Data used as input in both equations

Cable length, l (m)	6
Gravity, g (m/s^2)	9.81
Natural frequency, ω (rad/s)	1.28
Maximum velocity, V_{max} (m/s)	1.5
Acceleration, K (m/s^2)	0.349
Deceleration, K (m/s^2)	0.263
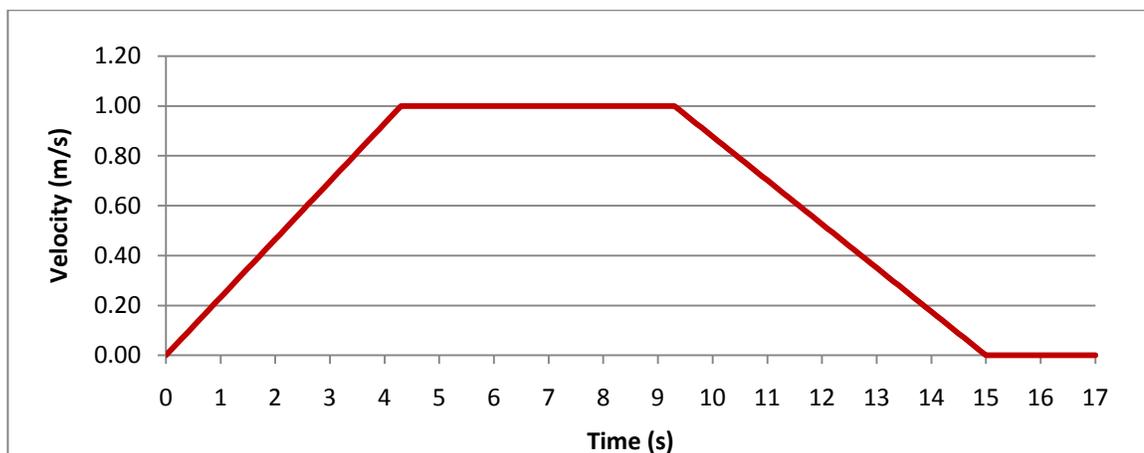
Figure 4.18: Velocity profile of the trolley B
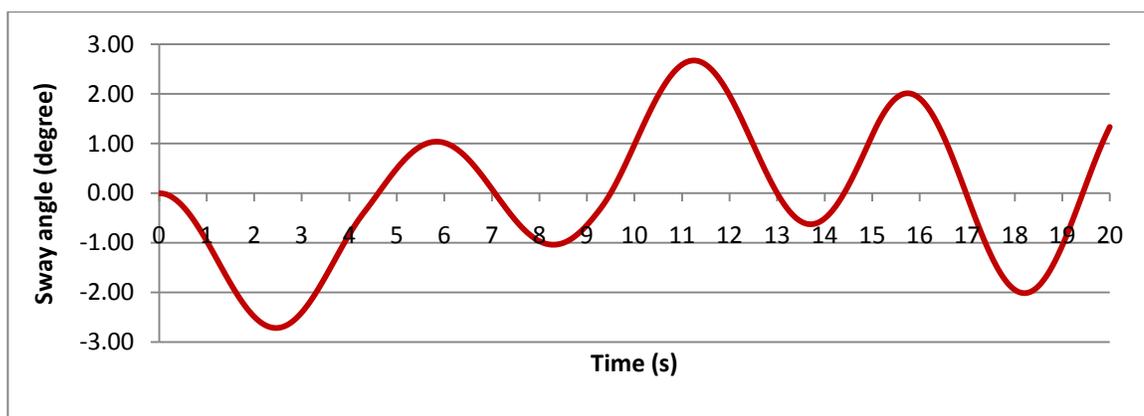
Figure 4.19: The sway angle of the load B

Figure 4.20 and Figure 4.21 show the comparison of velocity profile for trolley A and trolley B and the sway angle of load A and load B respectively.
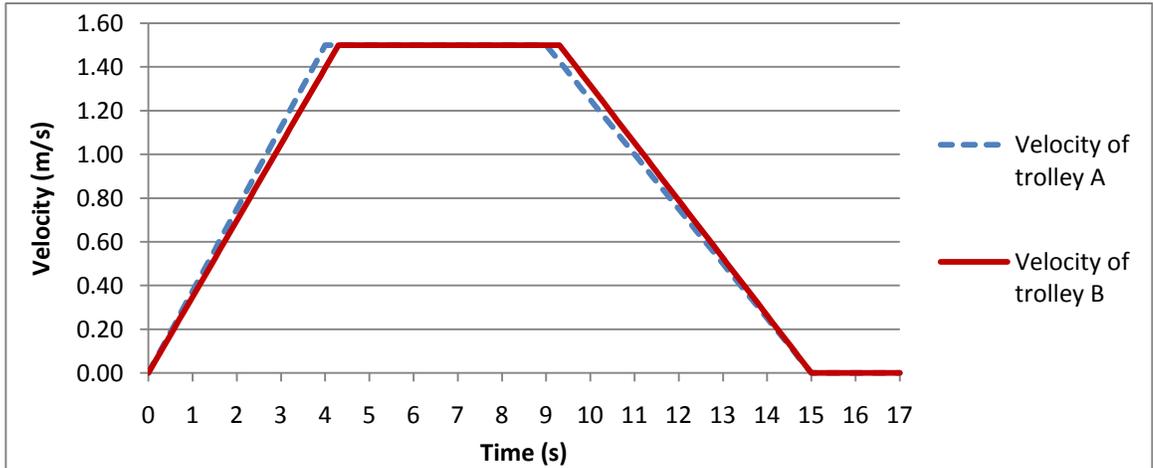
Figure 4.20: Comparison of velocity profile for the trolley A and trolley B
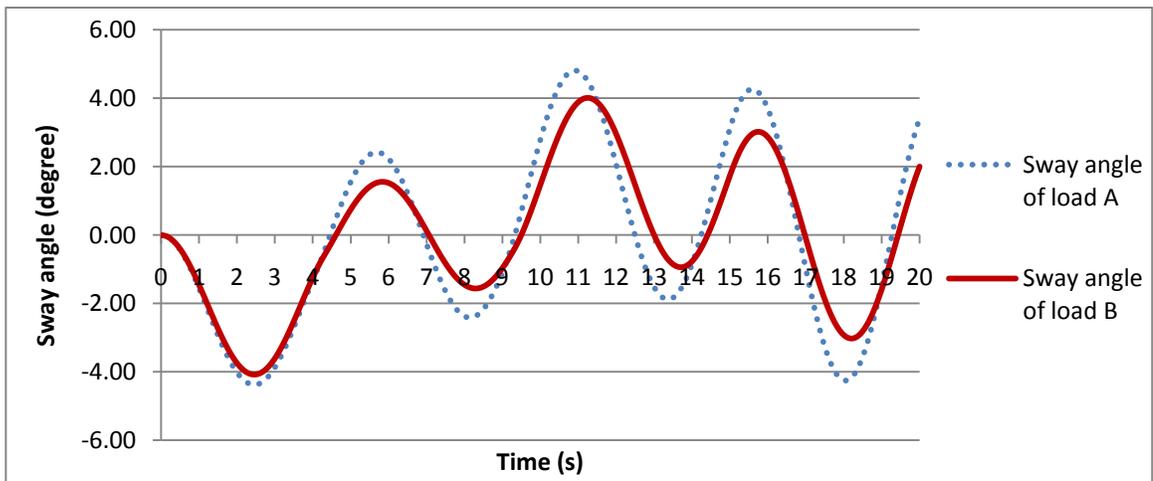
Figure 4.21: Comparison sway angle of load A with load B

4.2. Discussion

Based on observation, the sway angle of the load for each set of trolley velocity profiles has been reduced. It was found that the minimum sway angle can be achieved by proper timing of the acceleration and deceleration of the trolley. From the variation of maximum trolley velocity which are 0.5 m/s, 1.0 m/s and 1.5 m/s, the result for sway angle of the load are vary.

Figures 4.8, 4.14 and 4.20 show the comparison between velocity profile after and before modification. Trolley velocity profile A is the original profile while trolley velocity profile B is after modification.

From Figure 4.8, velocity of trolley B which is 0.5 m/s has been modified to reduce the load sway by increasing the acceleration rate of the trolley so that it ends 0.5 s earlier and decelerating rate is made higher begins at the same time as velocity A and stop 0.5 s earlier. The result in Figure 4.9 shows the sway angle of load B has been reduced.

Figure 4.14 shows two trolleys velocity profiles, A and B with maximum value of trolley velocity 1.0 m/s. Profile B has been modified to reduce the sway angle of the load for profile A. The acceleration rate of the trolley is made higher and the deceleration is lowered so that eventhough the trolley begins deceleration 0.5 s earlier, it stops at the same time as the original motion (Profile A). Figure 4.15 shows the load sway has been minimized.

Figure 4.20 shows the velocity profile with maximum velocity is 1.5 m/s. The acceleration of trolley has been reduced from 0.375 m/s^2 to 0.349 m/s^2 and the deceleration of trolley is increased from 0.250 m/s^2 to 0.263 m/s^2. Eventhough the sway angle of the load has been reduced, but this set of trolley velocity profiles did not shows a major improvement in the load motion performance.

Figure 4.22 below shows the comparison between three of trolley velocity profiles of each maximum trolley velocity that have been modified. The result for the sway angle of the load is shown in Figure 4.23.

From observation, the velocity profiles of trolley A and B are significance. They produced very low sway angle compare with the velocity profile of trolley C.
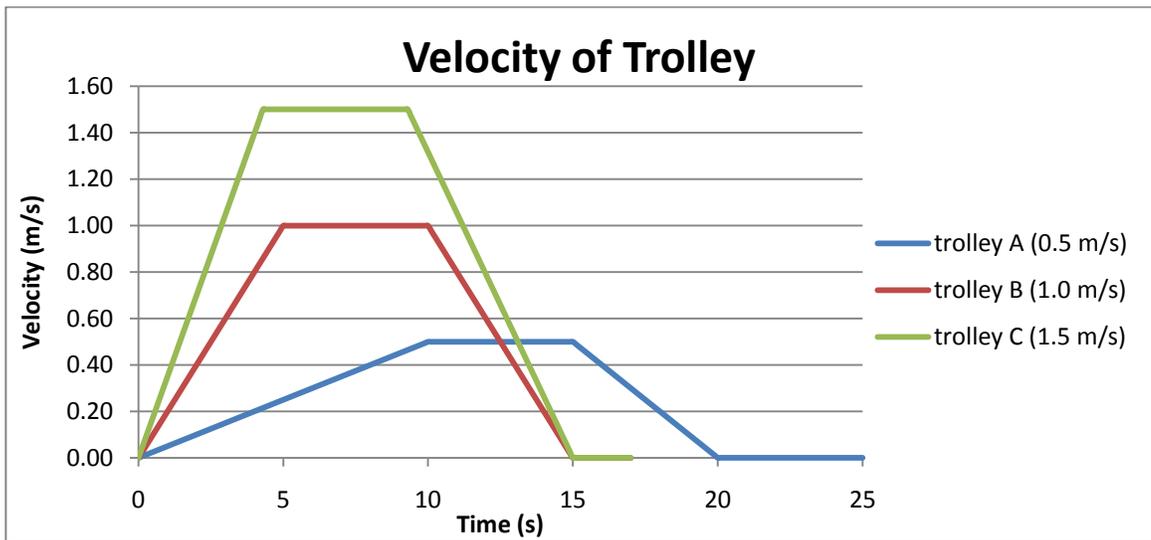
Figure 4.22: Comparison between three velocity profiles
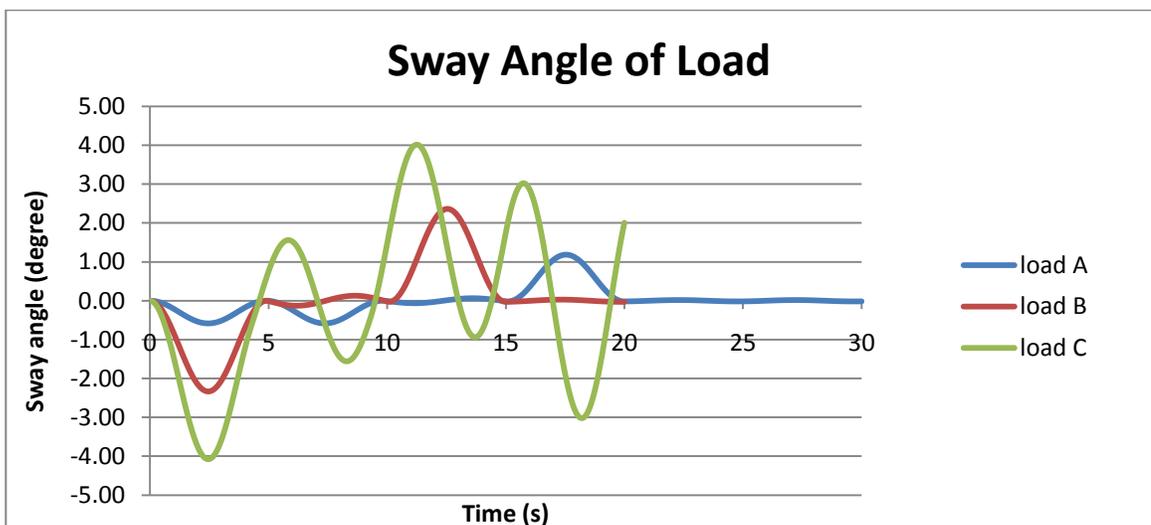
Figure 4.23: Comparison of the sway angle of load A, B, and C

4.2.1. Effect of the variables on the gantry crane load

Based on the study, there are a few variables could influence the sway of gantry crane load. The analysis on how these variables affect the sway angle of the load was done. The variables are:

1. Cable length
2. The acceleration of the trolley
3. The initial sway angle of the load
4. The initial velocity of the load

The analysis was done one by one for all the variables. The graphs are plotted for the velocity profile of trolley A and B and analysis was done base on it.

4.2.1.1. Cable Length

This project is used the cable length of 6 m. This is because the other values of cable length will increase the value sway angle of the load.

Figures 4.24, 4.25, and 4.26 show the sway angle of the load with maximum velocity 0.5 m/s when are using the cable length of 5, 7, and 10 m respectively
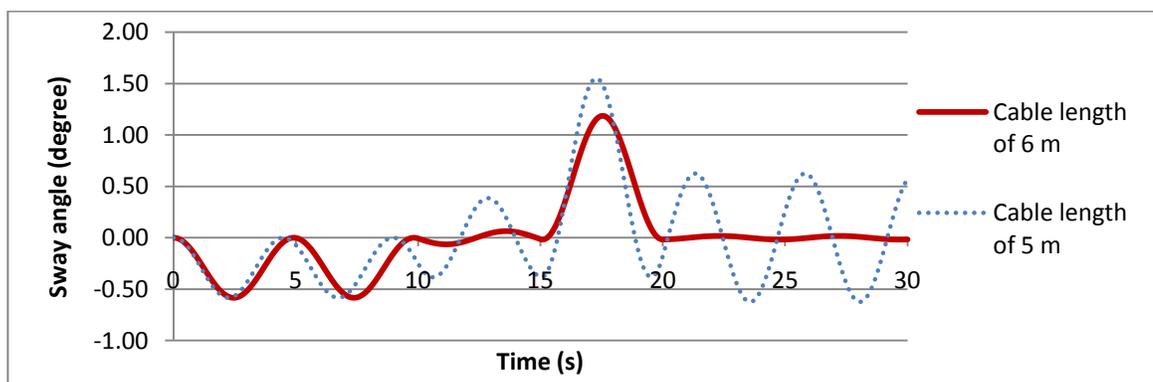
Figure 4.24: Comparison sway angle of load when using 5 m and 6 m cable length
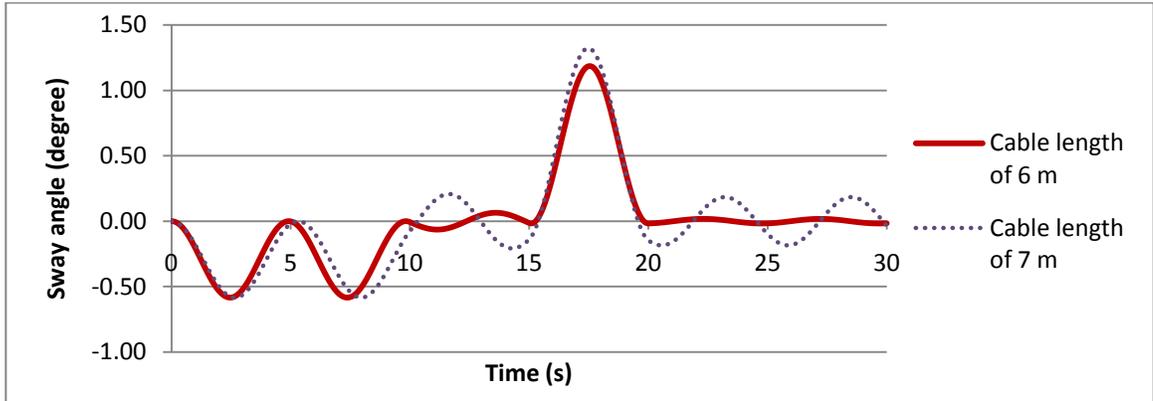
Figure 4.25: Comparison sway angle of load when using 6 m and 7 m cable length
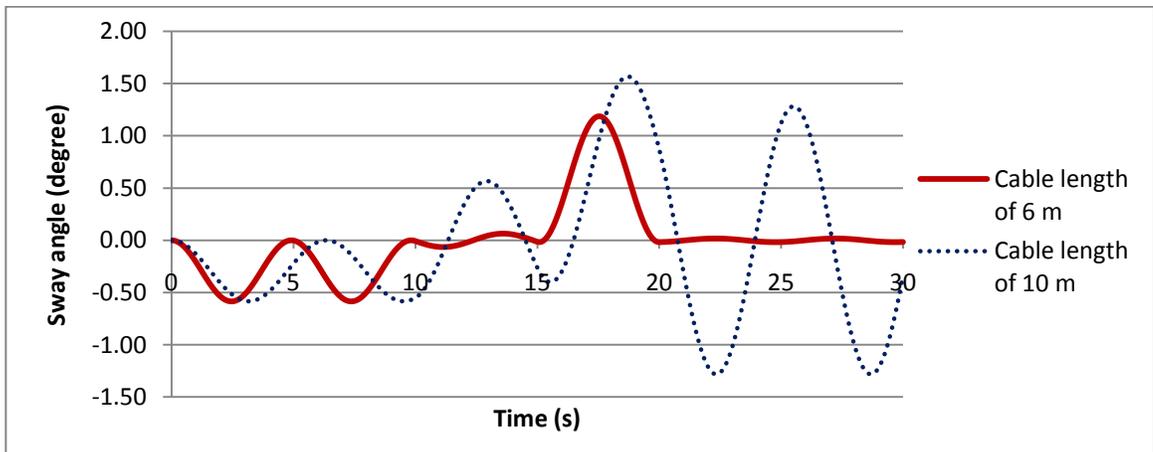
Figure 4.26: Comparison sway angle of load when using 6 m and 10 m cable length

Figures 4.27, 4.28, and 4.29 show the sway angle of the load with maximum velocity 1.0 m/s when are using the cable length of 5, 7, and 10 m respectively.
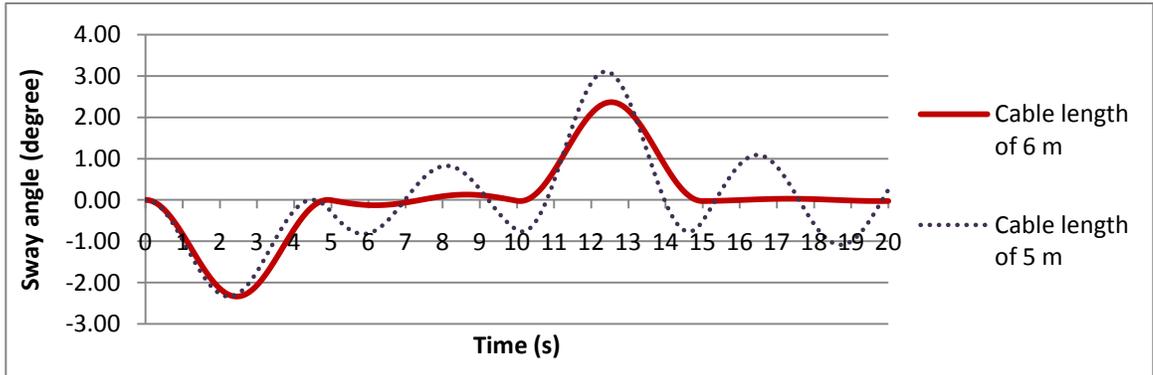
Figure 4.27: Comparison sway angle of load when using 5 m and 6 m cable length
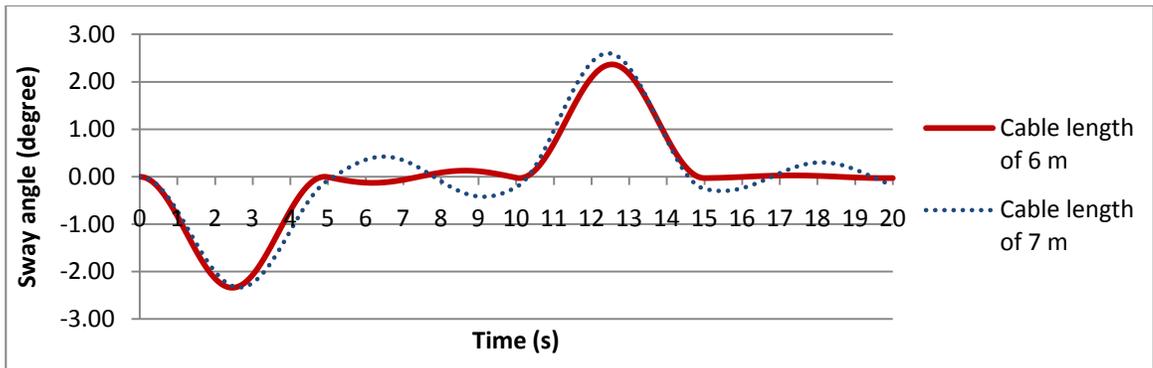
Figure 4.28: Comparison sway angle of load when using 6 m and 7 m cable length
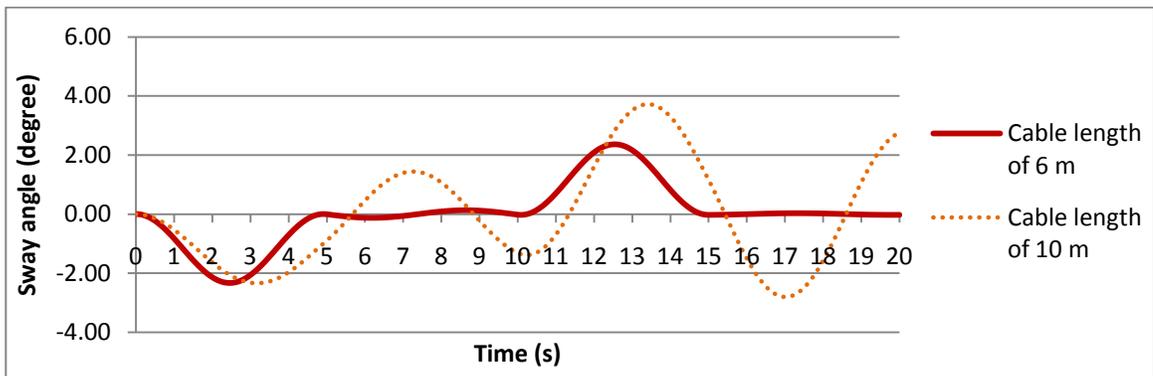
Figure 4.29: Comparison sway angle of load when using 6 m and 10 m cable length

4.2.1.2. Acceleration of the trolley

From the graph below, by varying the trolley velocity while other variables such as the time and cable length are kept constant, there are changes in the sway angle of the load. As the trolley velocity higher, sway angle of the load increase. This shows that the lower trolley velocity, the lower the sway angle if the load will be.
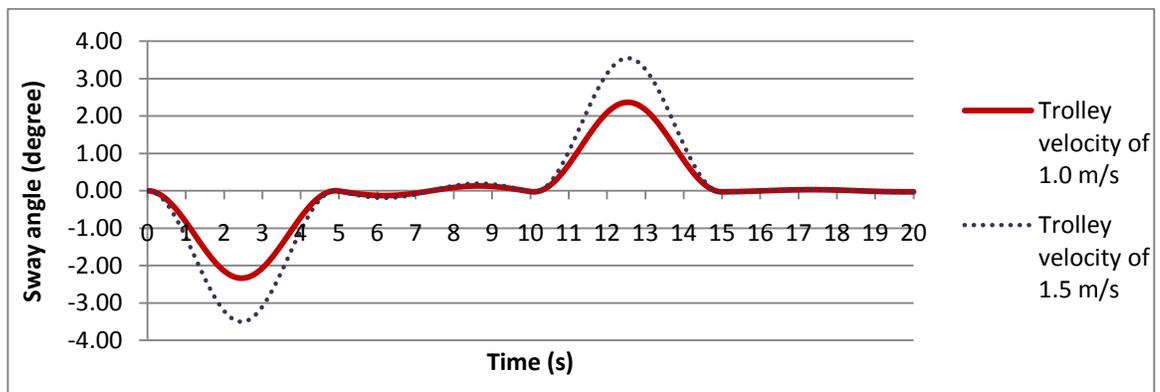
Figure 4.30: Comparison sway angle of load when trolley velocity 1.0 m/s and 1.5 m/s
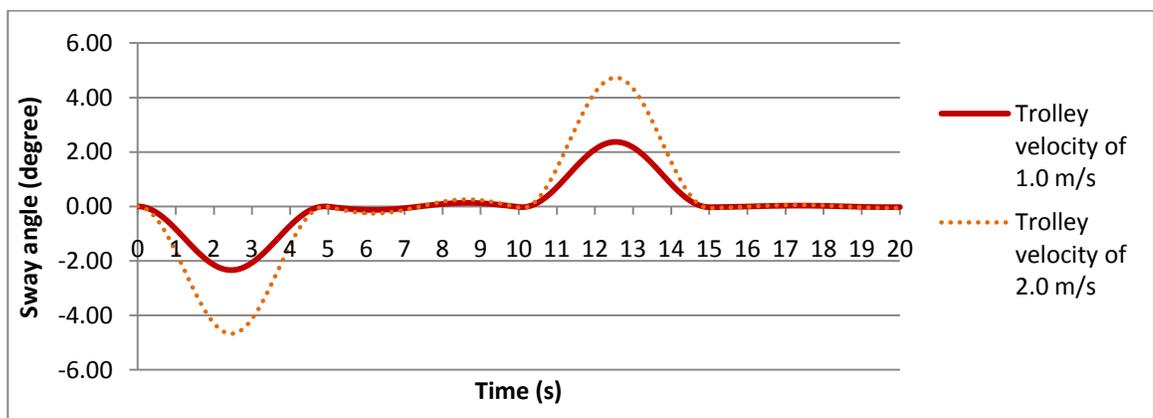
Figure 4.31: Comparison sway angle of load when trolley velocity 1.0 m/s and 2.0 m/s

4.2.1.3. Initial Sway Angle and Initial Velocity of the Load

These two variables, initial sway angle and initial velocity of the load will change if the time set for every phases change. The time is varied while other variables such as trolley velocity and cable length are kept constant. Figure 4.32 below shows that the P-1, P-2 and P-4 of trolley velocity profile A are same with trolley velocity of profile A'. But, the P-3 for both profile is differ. The difference will affect the value of initial sway angle and initial velocity of the load therefore will either increase or decrease the load sway angle.
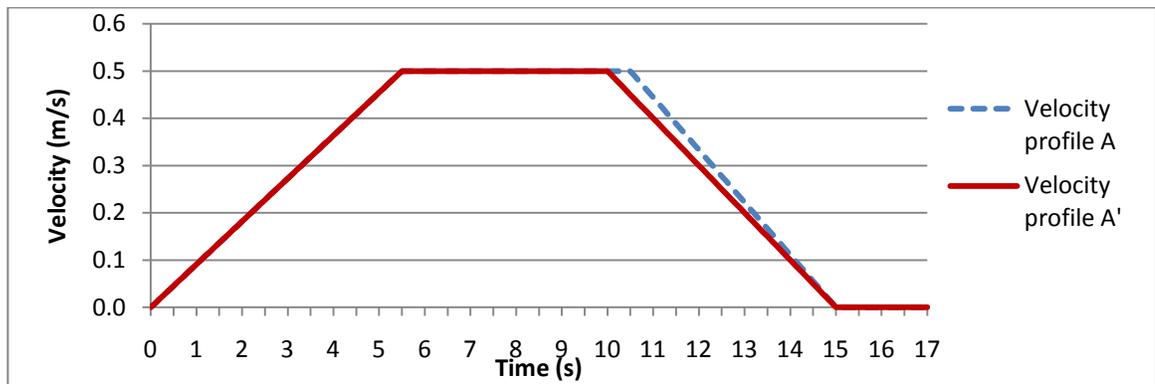
Figure 4.32: Comparison of velocity profile A and A'
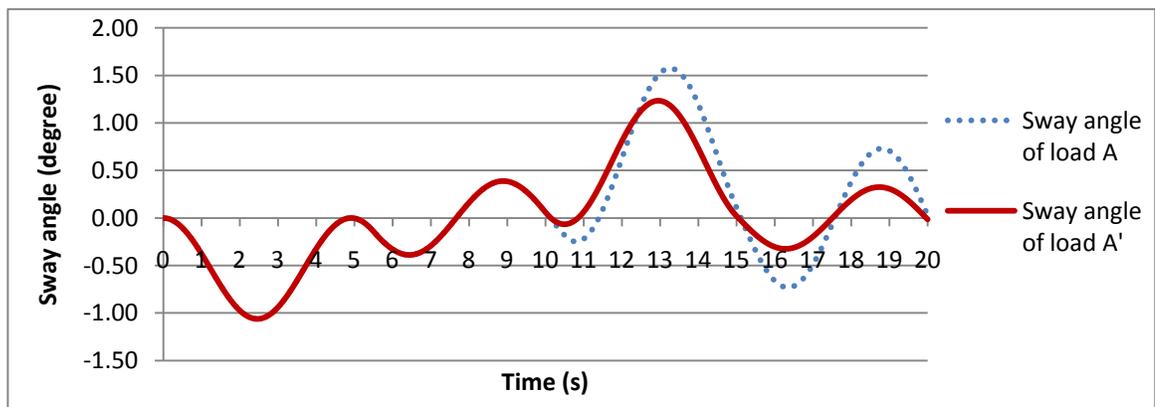
Figure 4.33: Comparison of sway angle of load A and A'

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4.2.2. Model of Gantry Crane using ADAMS software

Model of gantry crane has been developed by using the ADAMS software. The model is used to run the simulation. Figure 4.35 is shown the result after the simulation using ADAMS software.
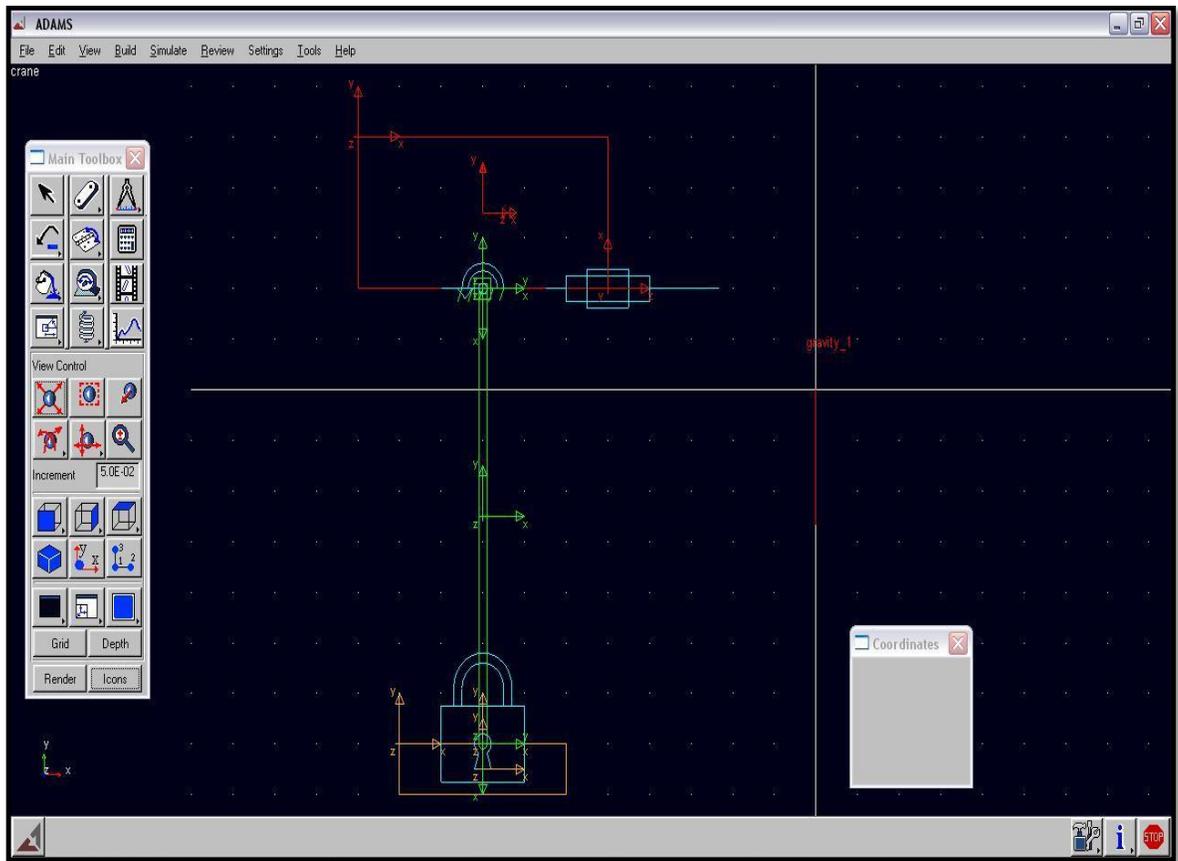
Figure 4.34: Model of gantry crane using ADAMS
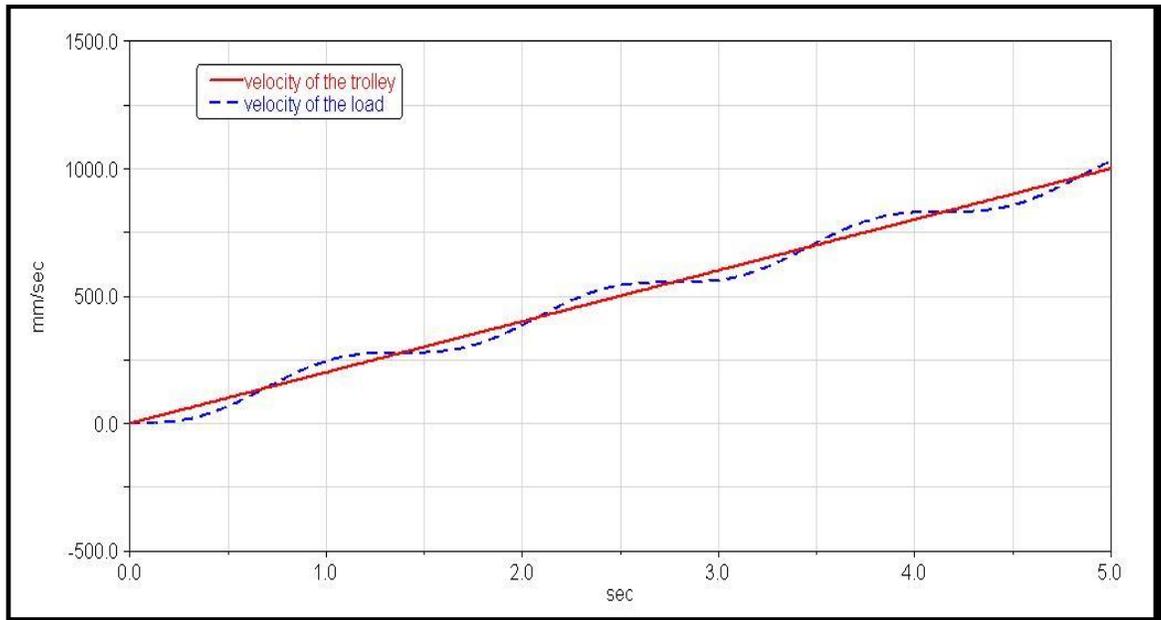
Figure 4.35: Result from the simulation using ADAMS software

CHAPTER 5

CONCLUSION AND RECOMMENDATION

Based on the result obtained, we can see that the simplified linearised model is sufficient to predict a velocity profile of a trolley to generate minimum sway of the gantry crane load. The result also proved that by increasing the acceleration rate and lowering the deceleration rate, we can actually get a major improvement in the load motion performance. By using the mathematical model to find the sway angle of the gantry crane load, a few factors that influenced the sway can be predict such as cable length, acceleration, initial sway angle of the load and the initial velocity of the load.

There are many ways to reduce the sway of gantry crane load that is available right now but further study is needed to get the most appropriate system for the gantry crane system.

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[12]   Crane data <http://www.cranemgt.com/cranedata.html>

APPENDICES

Table A.1: Sample of calculation using spreadsheet for phase 1 (P-1)

phase	time, t (s)	trolley velocity, V (m/s)	trolley acceleration, K (m/s^2)	natural frequency, ω (rad/s)	initial sway angle, $\theta(0)$ (rad)	initial load velocity	sway angle, $\theta(t)$ (rad)	$\theta(t)$ (degree)	velocity of load
1	0.0	0.00	0.2000	1.27867	0	0	0.000000	0.000000	0.000000
	0.1	0.02	0.2000	1.27867	0	0	-0.000166	-0.009536	-0.003324
	0.2	0.04	0.2000	1.27867	0	0	-0.000663	-0.037989	-0.006594
	0.3	0.06	0.2000	1.27867	0	0	-0.001482	-0.084895	-0.009757
	0.4	0.08	0.2000	1.27867	0	0	-0.002609	-0.149487	-0.012760
	0.5	0.10	0.2000	1.27867	0	0	-0.004027	-0.230711	-0.015554
	0.6	0.12	0.2000	1.27867	0	0	-0.005711	-0.327240	-0.018095
	0.7	0.14	0.2000	1.27867	0	0	-0.007636	-0.437499	-0.020340
	0.8	0.16	0.2000	1.27867	0	0	-0.009768	-0.559687	-0.022253
	0.9	0.18	0.2000	1.27867	0	0	-0.012074	-0.691809	-0.023803
	1.0	0.20	0.2000	1.27867	0	0	-0.014516	-0.831708	-0.024964
	1.1	0.22	0.2000	1.27867	0	0	-0.017054	-0.977100	-0.025718
	1.2	0.24	0.2000	1.27867	0	0	-0.019646	-1.125610	-0.026051
	1.3	0.26	0.2000	1.27867	0	0	-0.022250	-1.274815	-0.025960
	1.4	0.28	0.2000	1.27867	0	0	-0.024823	-1.422277	-0.025444
	1.5	0.30	0.2000	1.27867	0	0	-0.027325	-1.565589	-0.024513
	1.6	0.32	0.2000	1.27867	0	0	-0.029713	-1.702412	-0.023182
	1.7	0.34	0.2000	1.27867	0	0	-0.031948	-1.830510	-0.021472
	1.8	0.36	0.2000	1.27867	0	0	-0.033995	-1.947793	-0.019412
	1.9	0.38	0.2000	1.27867	0	0	-0.035820	-2.052345	-0.017034
	2.0	0.40	0.2000	1.27867	0	0	-0.037393	-2.142460	-0.014379
	2.1	0.42	0.2000	1.27867	0	0	-0.038688	-2.216666	-0.011489
	2.2	0.44	0.2000	1.27867	0	0	-0.039684	-2.273751	-0.008411
	2.3	0.46	0.2000	1.27867	0	0	-0.040366	-2.312784	-0.005196

phase	time, t (s)	trolley velocity, V (m/s)	trolley acceleration, K (m/s^2)	natural frequency, ω (rad/s)	initial sway angle, $\theta(0)$ (rad)	initial load velocity	sway angle, $\theta(t)$ (rad)	$\theta(t)$ (degree)	velocity of load
1	2.4	0.48	0.2000	1.27867	0	0	-0.040721	-2.333127	-0.001896
	2.5	0.50	0.2000	1.27867	0	0	-0.040744	-2.334448	0.001435
	2.6	0.52	0.2000	1.27867	0	0	-0.040434	-2.316725	0.004743
	2.7	0.54	0.2000	1.27867	0	0	-0.039798	-2.280247	0.007973
	2.8	0.56	0.2000	1.27867	0	0	-0.038844	-2.225611	0.011073
	2.9	0.58	0.2000	1.27867	0	0	-0.037589	-2.153709	0.013992
	3.0	0.60	0.2000	1.27867	0	0	-0.036054	-2.065714	0.016682
	3.1	0.62	0.2000	1.27867	0	0	-0.034262	-1.963063	0.019101
	3.2	0.64	0.2000	1.27867	0	0	-0.032244	-1.847432	0.021207
	3.3	0.66	0.2000	1.27867	0	0	-0.030032	-1.720709	0.022967
	3.4	0.68	0.2000	1.27867	0	0	-0.027663	-1.584964	0.024352
	3.5	0.70	0.2000	1.27867	0	0	-0.025175	-1.442413	0.025340
	3.6	0.72	0.2000	1.27867	0	0	-0.022609	-1.295382	0.025914
	3.7	0.74	0.2000	1.27867	0	0	-0.020006	-1.146274	0.026064
	3.8	0.76	0.2000	1.27867	0	0	-0.017410	-0.997522	0.025789
	3.9	0.78	0.2000	1.27867	0	0	-0.014862	-0.851555	0.025093
	4.0	0.80	0.2000	1.27867	0	0	-0.012405	-0.710758	0.023988
	4.1	0.82	0.2000	1.27867	0	0	-0.010078	-0.577427	0.022490
	4.2	0.84	0.2000	1.27867	0	0	-0.007919	-0.453741	0.020626
	4.3	0.86	0.2000	1.27867	0	0	-0.005964	-0.341719	0.018424
	4.4	0.88	0.2000	1.27867	0	0	-0.004244	-0.243191	0.015922
	4.5	0.90	0.2000	1.27867	0	0	-0.002788	-0.159764	0.013160
	4.6	0.92	0.2000	1.27867	0	0	-0.001620	-0.092801	0.010183
	4.7	0.94	0.2000	1.27867	0	0	-0.000757	-0.043395	0.007040
	4.8	0.96	0.2000	1.27867	0	0	-0.000216	-0.012354	0.003781
	4.9	0.98	0.2000	1.27867	0	0	-0.000003	-0.000183	0.000461
	5.0	1.00	0.2000	1.27867	0	0	-0.000124	-0.007082	-0.002866

Table A.2: Sample of calculation using spreadsheet for phase 2 (P-2)

phase	time, t (s)	trolley velocity, V (m/s)	trolley acceleration, K (m/s²)	natural frequency, ω (rad/s)	initial sway angle, $\theta(0)$ (rad)	initial load velocity	sway angle, $\theta(t)$ (rad)	$\theta(t)$ (degree)	velocity of load
2	5.1	1.00	0.0000	1.27867	-0.000124	-0.002866	-0.000408	-0.023401	-0.002823
	5.2	1.00	0.0000	1.27867	-0.000124	-0.002866	-0.000687	-0.039339	-0.002733
	5.3	1.00	0.0000	1.27867	-0.000124	-0.002866	-0.000954	-0.054634	-0.002599
	5.4	1.00	0.0000	1.27867	-0.000124	-0.002866	-0.001205	-0.069037	-0.002422
	5.5	1.00	0.0000	1.27867	-0.000124	-0.002866	-0.001437	-0.082313	-0.002206
	5.6	1.00	0.0000	1.27867	-0.000124	-0.002866	-0.001645	-0.094245	-0.001954
	5.7	1.00	0.0000	1.27867	-0.000124	-0.002866	-0.001826	-0.104638	-0.001669
	5.8	1.00	0.0000	1.27867	-0.000124	-0.002866	-0.001978	-0.113323	-0.001358
	5.9	1.00	0.0000	1.27867	-0.000124	-0.002866	-0.002097	-0.120157	-0.001024
	6.0	1.00	0.0000	1.27867	-0.000124	-0.002866	-0.002182	-0.125030	-0.000674
	6.1	1.00	0.0000	1.27867	-0.000124	-0.002866	-0.002232	-0.127861	-0.000313
	6.2	1.00	0.0000	1.27867	-0.000124	-0.002866	-0.002245	-0.128604	0.000054
	6.3	1.00	0.0000	1.27867	-0.000124	-0.002866	-0.002221	-0.127247	0.000419
	6.4	1.00	0.0000	1.27867	-0.000124	-0.002866	-0.002161	-0.123813	0.000778
	6.5	1.00	0.0000	1.27867	-0.000124	-0.002866	-0.002066	-0.118357	0.001124
	6.6	1.00	0.0000	1.27867	-0.000124	-0.002866	-0.001937	-0.110969	0.001452
	6.7	1.00	0.0000	1.27867	-0.000124	-0.002866	-0.001776	-0.101769	0.001756
	6.8	1.00	0.0000	1.27867	-0.000124	-0.002866	-0.001587	-0.090907	0.002031
	6.9	1.00	0.0000	1.27867	-0.000124	-0.002866	-0.001371	-0.078561	0.002273
	7.0	1.00	0.0000	1.27867	-0.000124	-0.002866	-0.001133	-0.064932	0.002478
	7.1	1.00	0.0000	1.27867	-0.000124	-0.002866	-0.000877	-0.050243	0.002643
	7.2	1.00	0.0000	1.27867	-0.000124	-0.002866	-0.000606	-0.034734	0.002764
	7.3	1.00	0.0000	1.27867	-0.000124	-0.002866	-0.000326	-0.018657	0.002840
	7.4	1.00	0.0000	1.27867	-0.000124	-0.002866	-0.000040	-0.002276	0.002870
	7.5	1.00	0.0000	1.27867	-0.000124	-0.002866	0.000247	0.014142	0.002853
	7.6	1.00	0.0000	1.27867	-0.000124	-0.002866	0.000529	0.030330	0.002790
	7.7	1.00	0.0000	1.27867	-0.000124	-0.002866	0.000803	0.046022	0.002681

phase	time, t (s)	trolley velocity, V (m/s)	trolley acceleration, K (m/s^2)	natural frequency, ω (rad/s)	initial sway angle, $\theta(0)$ (rad)	initial load velocity	sway angle, $\theta(t)$ (rad)	$\theta(t)$ (degree)	velocity of load
2	7.8	1.00	0.0000	1.27867	-0.000124	-0.002866	0.001064	0.060963	0.002528
	7.9	1.00	0.0000	1.27867	-0.000124	-0.002866	0.001307	0.074908	0.002334
	8.0	1.00	0.0000	1.27867	-0.000124	-0.002866	0.001529	0.087630	0.002101
	8.1	1.00	0.0000	1.27867	-0.000124	-0.002866	0.001727	0.098922	0.001835
	8.2	1.00	0.0000	1.27867	-0.000124	-0.002866	0.001895	0.108598	0.001538
	8.3	1.00	0.0000	1.27867	-0.000124	-0.002866	0.002033	0.116501	0.001217
	8.4	1.00	0.0000	1.27867	-0.000124	-0.002866	0.002138	0.122502	0.000875
	8.5	1.00	0.0000	1.27867	-0.000124	-0.002866	0.002208	0.126503	0.000519
	8.6	1.00	0.0000	1.27867	-0.000124	-0.002866	0.002242	0.128438	0.000155
	8.7	1.00	0.0000	1.27867	-0.000124	-0.002866	0.002239	0.128276	-0.000212
	8.8	1.00	0.0000	1.27867	-0.000124	-0.002866	0.002199	0.126020	-0.000575
	8.9	1.00	0.0000	1.27867	-0.000124	-0.002866	0.002124	0.121706	-0.000929
	9.0	1.00	0.0000	1.27867	-0.000124	-0.002866	0.002014	0.115405	-0.001268
	9.1	1.00	0.0000	1.27867	-0.000124	-0.002866	0.001871	0.107220	-0.001586
	9.2	1.00	0.0000	1.27867	-0.000124	-0.002866	0.001698	0.097284	-0.001878
	9.3	1.00	0.0000	1.27867	-0.000124	-0.002866	0.001497	0.085759	-0.002139
	9.4	1.00	0.0000	1.27867	-0.000124	-0.002866	0.001271	0.072834	-0.002366
	9.5	1.00	0.0000	1.27867	-0.000124	-0.002866	0.001025	0.058720	-0.002554
	9.6	1.00	0.0000	1.27867	-0.000124	-0.002866	0.000762	0.043648	-0.002700
	9.7	1.00	0.0000	1.27867	-0.000124	-0.002866	0.000486	0.027862	-0.002802
	9.8	1.00	0.0000	1.27867	-0.000124	-0.002866	0.000203	0.011622	-0.002859
	9.9	1.00	0.0000	1.27867	-0.000124	-0.002866	-0.000084	-0.004808	-0.002869
	10.0	1.00	0.0000	1.27867	-0.000124	-0.002866	-0.000369	-0.021160	-0.002831

Table A.3: Sample of calculation using spreadsheet for phase 3 (P-3)

phase	time, t (s)	trolley velocity, V (m/s)	trolley acceleration, K (m/s²)	natural frequency, ω (rad/s)	initial sway angle, θ(0) (rad)	initial load velocity	sway angle, θ(t) (rad)	θ(t) (degree)	velocity of load
3	10.1	0.98	-0.2000	1.27867	-0.000369	-0.002831	-0.000482	-0.027629	0.000576
	10.2	0.96	-0.2000	1.27867	-0.000369	-0.002831	-0.000254	-0.014576	0.003974
	10.3	0.94	-0.2000	1.27867	-0.000369	-0.002831	0.000310	0.017789	0.007308
	10.4	0.92	-0.2000	1.27867	-0.000369	-0.002831	0.001203	0.068936	0.010522
	10.5	0.90	-0.2000	1.27867	-0.000369	-0.002831	0.002409	0.138029	0.013564
	10.6	0.88	-0.2000	1.27867	-0.000369	-0.002831	0.003909	0.223942	0.016384
	10.7	0.86	-0.2000	1.27867	-0.000369	-0.002831	0.005677	0.325270	0.018938
	10.8	0.84	-0.2000	1.27867	-0.000369	-0.002831	0.007686	0.440361	0.021182
	10.9	0.82	-0.2000	1.27867	-0.000369	-0.002831	0.009902	0.567334	0.023080
	11.0	0.80	-0.2000	1.27867	-0.000369	-0.002831	0.012289	0.704116	0.024601
	11.1	0.78	-0.2000	1.27867	-0.000369	-0.002831	0.014809	0.848474	0.025721
	11.2	0.76	-0.2000	1.27867	-0.000369	-0.002831	0.017419	0.998051	0.026420
	11.3	0.74	-0.2000	1.27867	-0.000369	-0.002831	0.020078	1.150404	0.026689
	11.4	0.72	-0.2000	1.27867	-0.000369	-0.002831	0.022742	1.303047	0.026521
	11.5	0.70	-0.2000	1.27867	-0.000369	-0.002831	0.025368	1.453487	0.025921
	11.6	0.68	-0.2000	1.27867	-0.000369	-0.002831	0.027912	1.599267	0.024897
	11.7	0.66	-0.2000	1.27867	-0.000369	-0.002831	0.030334	1.738007	0.023467
	11.8	0.64	-0.2000	1.27867	-0.000369	-0.002831	0.032593	1.867442	0.021653
	11.9	0.62	-0.2000	1.27867	-0.000369	-0.002831	0.034653	1.985458	0.019486
	12.0	0.60	-0.2000	1.27867	-0.000369	-0.002831	0.036480	2.090130	0.017001
	12.1	0.58	-0.2000	1.27867	-0.000369	-0.002831	0.038044	2.179746	0.014238
	12.2	0.56	-0.2000	1.27867	-0.000369	-0.002831	0.039320	2.252845	0.011243
	12.3	0.54	-0.2000	1.27867	-0.000369	-0.002831	0.040286	2.308233	0.008064
	12.4	0.52	-0.2000	1.27867	-0.000369	-0.002831	0.040928	2.345005	0.004754
	12.5	0.50	-0.2000	1.27867	-0.000369	-0.002831	0.041234	2.362560	0.001366
	12.6	0.48	-0.2000	1.27867	-0.000369	-0.002831	0.041200	2.360614	-0.002044
	12.7	0.46	-0.2000	1.27867	-0.000369	-0.002831	0.040827	2.339196	-0.005421

phase	time, t (s)	trolley velocity, V (m/s)	trolley acceleration, K (m/s²)	natural frequency, ω (rad/s)	initial sway angle, θ(0) (rad)	initial load velocity	sway angle, θ(t) (rad)	θ(t) (degree)	velocity of load
3	12.8	0.44	-0.2000	1.27867	-0.000369	-0.002831	0.040119	2.298657	-0.008710
	12.9	0.42	-0.2000	1.27867	-0.000369	-0.002831	0.039089	2.239659	-0.011856
	13.0	0.40	-0.2000	1.27867	-0.000369	-0.002831	0.037754	2.163165	-0.014809
	13.1	0.38	-0.2000	1.27867	-0.000369	-0.002831	0.036136	2.070424	-0.017520
	13.2	0.36	-0.2000	1.27867	-0.000369	-0.002831	0.034260	1.962950	-0.019945
	13.3	0.34	-0.2000	1.27867	-0.000369	-0.002831	0.032158	1.842498	-0.022044
	13.4	0.32	-0.2000	1.27867	-0.000369	-0.002831	0.029863	1.711035	-0.023783
	13.5	0.30	-0.2000	1.27867	-0.000369	-0.002831	0.027414	1.570708	-0.025134
	13.6	0.28	-0.2000	1.27867	-0.000369	-0.002831	0.024850	1.423806	-0.026074
	13.7	0.26	-0.2000	1.27867	-0.000369	-0.002831	0.022213	1.272730	-0.026589
	13.8	0.24	-0.2000	1.27867	-0.000369	-0.002831	0.019547	1.119946	-0.026670
	13.9	0.22	-0.2000	1.27867	-0.000369	-0.002831	0.016894	0.967948	-0.026315
	14.0	0.20	-0.2000	1.27867	-0.000369	-0.002831	0.014298	0.819218	-0.025531
	14.1	0.18	-0.2000	1.27867	-0.000369	-0.002831	0.011802	0.676185	-0.024329
	14.2	0.16	-0.2000	1.27867	-0.000369	-0.002831	0.009445	0.541183	-0.022731
	14.3	0.14	-0.2000	1.27867	-0.000369	-0.002831	0.007268	0.416418	-0.020761
	14.4	0.12	-0.2000	1.27867	-0.000369	-0.002831	0.005305	0.303927	-0.018452
	14.5	0.10	-0.2000	1.27867	-0.000369	-0.002831	0.003587	0.205546	-0.015842
	14.6	0.08	-0.2000	1.27867	-0.000369	-0.002831	0.002145	0.122881	-0.012974
	14.7	0.06	-0.2000	1.27867	-0.000369	-0.002831	0.001000	0.057282	-0.009893
	14.8	0.04	-0.2000	1.27867	-0.000369	-0.002831	0.000171	0.009821	-0.006651
	14.9	0.02	-0.2000	1.27867	-0.000369	-0.002831	-0.000327	-0.018728	-0.003301
	15.0	0.00	-0.2000	1.27867	-0.000369	-0.002831	-0.000487	-0.027899	0.000104

Table A.4: Sample of calculation using spreadsheet for phase 4 (P-4)

phase	time, t (s)	trolley velocity, V (m/s)	trolley acceleration, K (m/s^2)	natural frequency, ω (rad/s)	initial sway angle, $\theta(0)$ (rad)	initial load velocity	sway angle, $\theta(t)$ (rad)	$\theta(t)$ (degree)	velocity of load
4	15.1	0.00	0.0000	1.27867	-0.000487	0.000104	-0.000473	-0.027078	0.000182
	15.2	0.00	0.0000	1.27867	-0.000487	0.000104	-0.000451	-0.025815	0.000258
	15.3	0.00	0.0000	1.27867	-0.000487	0.000104	-0.000421	-0.024130	0.000329
	15.4	0.00	0.0000	1.27867	-0.000487	0.000104	-0.000385	-0.022051	0.000395
	15.5	0.00	0.0000	1.27867	-0.000487	0.000104	-0.000342	-0.019613	0.000455
	15.6	0.00	0.0000	1.27867	-0.000487	0.000104	-0.000294	-0.016854	0.000507
	15.7	0.00	0.0000	1.27867	-0.000487	0.000104	-0.000241	-0.013819	0.000551
	15.8	0.00	0.0000	1.27867	-0.000487	0.000104	-0.000184	-0.010560	0.000586
	15.9	0.00	0.0000	1.27867	-0.000487	0.000104	-0.000124	-0.007127	0.000611
	16.0	0.00	0.0000	1.27867	-0.000487	0.000104	-0.000062	-0.003579	0.000626
	16.1	0.00	0.0000	1.27867	-0.000487	0.000104	0.000000	0.000028	0.000631
	16.2	0.00	0.0000	1.27867	-0.000487	0.000104	0.000063	0.003635	0.000626
	16.3	0.00	0.0000	1.27867	-0.000487	0.000104	0.000125	0.007182	0.000611
	16.4	0.00	0.0000	1.27867	-0.000487	0.000104	0.000185	0.010612	0.000585
	16.5	0.00	0.0000	1.27867	-0.000487	0.000104	0.000242	0.013869	0.000550
	16.6	0.00	0.0000	1.27867	-0.000487	0.000104	0.000295	0.016899	0.000506
	16.7	0.00	0.0000	1.27867	-0.000487	0.000104	0.000343	0.019653	0.000454
	16.8	0.00	0.0000	1.27867	-0.000487	0.000104	0.000385	0.022087	0.000394
	16.9	0.00	0.0000	1.27867	-0.000487	0.000104	0.000422	0.024159	0.000328
	17.0	0.00	0.0000	1.27867	-0.000487	0.000104	0.000451	0.025838	0.000257
	17.1	0.00	0.0000	1.27867	-0.000487	0.000104	0.000473	0.027094	0.000181
	17.2	0.00	0.0000	1.27867	-0.000487	0.000104	0.000487	0.027908	0.000103
	17.3	0.00	0.0000	1.27867	-0.000487	0.000104	0.000493	0.028267	0.000022
	17.4	0.00	0.0000	1.27867	-0.000487	0.000104	0.000492	0.028164	-0.000058
	17.5	0.00	0.0000	1.27867	-0.000487	0.000104	0.000482	0.027601	-0.000138
	17.6	0.00	0.0000	1.27867	-0.000487	0.000104	0.000464	0.026587	-0.000215
	17.7	0.00	0.0000	1.27867	-0.000487	0.000104	0.000439	0.025139	-0.000289

phase	time, t (s)	trolley velocity, V (m/s)	trolley acceleration, K (m/s^2)	natural frequency, ω (rad/s)	initial sway angle, $\theta(0)$ (rad)	initial load velocity	sway angle, $\theta(t)$ (rad)	$\theta(t)$ (degree)	velocity of load
4	17.8	0.00	0.0000	1.27867	-0.000487	0.000104	0.000406	0.023281	-0.000358
	17.9	0.00	0.0000	1.27867	-0.000487	0.000104	0.000367	0.021043	-0.000422
	18.0	0.00	0.0000	1.27867	-0.000487	0.000104	0.000322	0.018461	-0.000478
	18.1	0.00	0.0000	1.27867	-0.000487	0.000104	0.000272	0.015577	-0.000527
	18.2	0.00	0.0000	1.27867	-0.000487	0.000104	0.000217	0.012440	-0.000567
	18.3	0.00	0.0000	1.27867	-0.000487	0.000104	0.000159	0.009099	-0.000598
	18.4	0.00	0.0000	1.27867	-0.000487	0.000104	0.000098	0.005610	-0.000619
	18.5	0.00	0.0000	1.27867	-0.000487	0.000104	0.000035	0.002029	-0.000630
	18.6	0.00	0.0000	1.27867	-0.000487	0.000104	-0.000028	-0.001585	-0.000630
	18.7	0.00	0.0000	1.27867	-0.000487	0.000104	-0.000090	-0.005174	-0.000621
	18.8	0.00	0.0000	1.27867	-0.000487	0.000104	-0.000151	-0.008677	-0.000601
	18.9	0.00	0.0000	1.27867	-0.000487	0.000104	-0.000210	-0.012039	-0.000571
	19.0	0.00	0.0000	1.27867	-0.000487	0.000104	-0.000265	-0.015205	-0.000532
	19.1	0.00	0.0000	1.27867	-0.000487	0.000104	-0.000316	-0.018122	-0.000485
	19.2	0.00	0.0000	1.27867	-0.000487	0.000104	-0.000362	-0.020743	-0.000429
	19.3	0.00	0.0000	1.27867	-0.000487	0.000104	-0.000402	-0.023026	-0.000367
	19.4	0.00	0.0000	1.27867	-0.000487	0.000104	-0.000435	-0.024933	-0.000298
	19.5	0.00	0.0000	1.27867	-0.000487	0.000104	-0.000461	-0.026432	-0.000225
	19.6	0.00	0.0000	1.27867	-0.000487	0.000104	-0.000480	-0.027500	-0.000148
	19.7	0.00	0.0000	1.27867	-0.000487	0.000104	-0.000491	-0.028119	-0.000068
	19.8	0.00	0.0000	1.27867	-0.000487	0.000104	-0.000494	-0.028279	0.000012
	19.9	0.00	0.0000	1.27867	-0.000487	0.000104	-0.000488	-0.027977	0.000093
	20.0	0.00	0.0000	1.27867	-0.000487	0.000104	-0.000475	-0.027218	0.000172