Study on Influences of Rear-Wheel Turn Angle to Performance of Four Wheel Steering (4WS) Vehicle

By

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Dissertation submitted in partial fulfillment of
the requirements for the
Bachelor of Engineering (Hons)
(Mechanical Engineering)

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CERTIFICATION OF APPROVAL

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A project dissertation submitted to the
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(MECHANICAL ENGINEERING)

Approved by,

____
(Dr. Setyamartana Parman)

UNIVERSITI TEKNOLOGI PETRONAS

TRONOH, PERAK

June 2010

CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

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(ALIF SYAHRIZAD BIN RAMLI)

ABSTRACT

The main objective of this project is to study on influences of rear-wheel turn angle to the performances of 4-wheel-steering (4WS) vehicle. As to determine either the evaluated performances of 4WS is either an advantage or disadvantage, thus, a datum is required. Therefore 2-wheel-steering (2WS) system is also included in the study since the 2WS system does not utilize the rear-wheel turn angle (zero rear-wheel turn angles).

The core problem statement implies in this project is what are the advantages and disadvantages of 4WS vehicle; is the influences of rear-wheel turn angle to 4WS does affecting in good or bad way?

Thus, the first scope done in this project is the configuration of the 4WS system shall be established to correlate how the rear-wheel shall turn accordingly with respect to the steering-wheel and the front-wheel turn angle. For this study, Honda's 4WS system is used. K.N Spentzas and I. Alkhazali stated that to study 4WS vehicle, simplification shall be made and one of it is the used of two-degree-of-freedom bicycle model. This model is very useful for analyzing the kinematics of the vehicle (i.e. turning radius, vehicle side-slip). However, when it comes to dynamic parameters (i.e. wheels side-slip angle, lateral velocity, yaw velocity), three-degree-of-freedom is desired since certain elements is essential to get a qualified data.

By referring to recent literatures made related to this project, the quality of the data, methodology and the procedures to produce them are also been checked beforehand to ensure that the project is done professionally.

The study includes few critical parameters such as turning radius, yaw velocity, lateral velocity, and side-slip angles that are necessary to evaluate how the rear-wheel turn angle affects the vehicle's performances by comparing those findings between 4WS and 2WS vehicles.

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In the name of Allah, the Most Gracious, the Most Merciful. Thanks to Dear Lord for giving me opportunities to complete my Final Year Project.

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CHAPTER 1
INTRODUCTION

1.1 BACKGROUND OF THE STUDY

Four-wheel-steering system (4WS) or also known as all-wheel steering system is a system applied on a vehicle to control both front and rear wheel. The typical steering system applied on the vehicle is two wheel steering system; 2WS (a.k.a. front wheel steering system, normal steering system) where the system does not utilize the rear-wheel turn angle; only the front-wheel. Four wheel steering system generally applied on large vehicles and also on some recent cars manufactured by Honda, Toyota, Mazda, Nissan and so on [2]. As mentioned, four wheel steering utilize the rear-wheel turn angle.

This study is made to determine the influences of the rear wheel turn angle to the performance of the applied 4WS vehicles. The rear wheel turn angle shall move accordingly with respect to the front-wheel turn angle to ensure the mobility of the vehicle is optimized. Comparison between the performance of normal steering system (2WS) and four wheel steering system are also made in this study.

Few claims has been made that four wheel steering system (4WS) provide a better steering response, improve vehicle stability while maneuvering at high speed, and also decrease turning radius at low speed. However, there are also some negative feedbacks for the users, such as during long corners at high speed, the user can feel the 4WS getting rid of body roll, and also when taking shaper corners at the same high speed, its makes the driver to feels the back end moving sideways [15][16].

Thus this study is to prove these claims and to provide data and figures by using the engineering software (MATLAB) to analyze the influences of rear-wheel turn angle to the 4WS vehicle. The data gained will then be compared to other studies made to test the quality of the data.

*References made, [1], could be refer to the *List of References* on No.1 references; and so with other [x] on the x-th references.

1.2 Problem Statement

For 2WS vehicle, the vehicle is possessing risk of oversteer while making turn at high speed due to the inertia of the vehicle and the rear-wheel side-slipping. Thus, considering that the rear wheel turn angle on 4WS System may actually reduce the risky-oversteer condition where the rear-wheel turn in the same direction with front steer angle with a crab-like movement thus reducing the lateral forces acted on the tires.

There are claims by the users saying the vehicle is enhancing stability of the car at high speed and during lane-change maneuvering. However, there are still some negative feedbacks from the current user of 4WS vehicle that their vehicle tends to feel like swinging away when having a sharp turn with medium speed.

So, is it true:-

- 4WS is better than 2WS vehicle?
- Is 4WS vehicle really possessed a better maneuverability and stability?
- Is it safer since a few studies claims that 4WS could reduce lateral force acted on the wheels; which reduce the possibility of the 4WS car having oversteer?
- Why the users feel the vehicle tends to swinging away? Etc.

Therefore this project is significantly to study the influences of the rear wheel turn angle on the performances of 4WS vehicle where some advantages been claimed for the 4WS system, and even so its disadvantages, which will then be included in this study.

1.3 OBJECTIVE

- To verify the corresponding rear-wheel turn angle with respect to front-wheel turn angle for 4WS vehicle
- To provide data and figures on the outputs of the vehicle performances due to the rear-wheel turn angle using engineering software (i.e. MATLAB).
- To study the effects of rear wheel turn angle to the performance of four wheel steering (4WS).
- To compare the performance of 4WS and 2WS system

1.4 SCOPE OF THE STUDY

- **Derive the formula** which will be used for this study
- Proper **assumptions and modeling** shall be made to ease the project's studies and at the same time, optimize the data integrity
- Analyze the **corresponding rear wheel turn angle** w.r.t. the front wheel turn angle
- Provide **data and figures on** the 4WS vehicle's performances influenced by rear-wheel turn angle by using engineering software(s)
- Present **relevant data comparisons** within the study and also with other studies made.

1.5 RELEVANCY OF THE PROJECT

This project is generally involving engineering software to determine the data which is the product of this project. Since all the software are available in UTP, hence, the availability of the tools are present, thus making this project feasible to be completed.

Conversely, this study shall not be taken lightly since it is important as to establish the findings on influences of vehicle's system applied (which is rear-wheel turn angle) to the vehicle's performances. Through this study, we could justify on the safety precautions and improvements that to be applied on vehicles; providing that this project is important and significant for automobile engineering field.

CHAPTER 2
THEORY

2.1 LITERATURE REVIEW

Formulas and relations stated in this section can be referred to *Interim Report* under the same title provided earlier for the detailed derivations and elaboration.

2.2 Symbols and Abbreviations used

a, b	Distance of the front & rear axle to the centre of gravity of the sprung mass (absolute value)
A_x, A_y	Longitudinal and lateral acceleration / deceleration respectively
C_f, C_r	Aligning stiffness of a single front and of a single rear wheel respectively
D_{roll}	Total roll damping of the suspension
F_{kj}	Force acting along the *k* direction on the *j*th wheel (k = x,y,z and j = fo,fi,ro,ri)
F_{yx}	Projection on the vehicle's longitudinal axis of the resultant of all the forces that are contained in the tire-road plane and are applied on the wheels
g	Acceleration of the gravity
h_{ra}	Roll axis distance from the centre of gravity of the sprung mass
I_{xs}	Roll moment of inertia of the sprung mass
I_z	Yaw moment of inertia of the vehicle
K_f, K_r	Roll stiffness of the front and rear suspension respectively
K_{roll}	Total roll stiffness of the suspension
k	Ratio of the rear steer angle to the front steer angle
ℓ	Wheelbase of the vehicle
m	Mass
n	Transmission ratio of the steering box
p	Roll angular velocity of the sprung mass
q	Pitch angular velocity of the sprung mass
r	Yaw angular velocity of the vehicle
t	Time
2_{tf}, 2_{tr}	Absolute value of the front and of the rear track width respectively
U	Forward velocity of the vehicle
V	Lateral velocity of the vehicle
W_L	Longitudinal weight transfer
W_{bf}, W_{br}	Front and rear transversal weight transfer due to body roll
W_{rf}, W_{rr}	Front and rear transversal weight transfer due to roll centre height
W_{Tf}, W_{Tr}	Front and rear transversal weight transfer
W_{uf}, W_{ur}	Front and rear transversal weight transfer due to the height of the un-sprung mass
α_j	Sideslip angle of the *j*th wheel of the vehicle
β	Sideslip angle of the vehicle
δ_j	Steer angle of the *j*th wheel of the vehicle
θ_s	Rotation angle of the steering wheel
φ	Roll angle of the sprung mass
ψ	Heading angle of the vehicle

Subscripts

f	Front wheel
G	Centre of mass
i	Inner wheel of the negotiated curve
J	*J*th wheel of the vehicle (*j* = f for front wheels, r for rear wheels; or 1 for front left wheel, 2 for front right, 3 for rear left, and 4 for rear right
o	Outer wheel of the negotiated curve
r	Rear wheel
s	Sprung
u	Un-sprung
w	Wheel
x, y, z	Along the longitudinal, transversal and vertical axis of the vehicle / wheel

*Symbols / Abbreviations used in this report are as per above, unless stated otherwise

2.3 Relevant Literature Studies Made

Refer to ***List of References*** for the full list of 42 references made for this study.

2.4 Forces and Moments on the Wheel

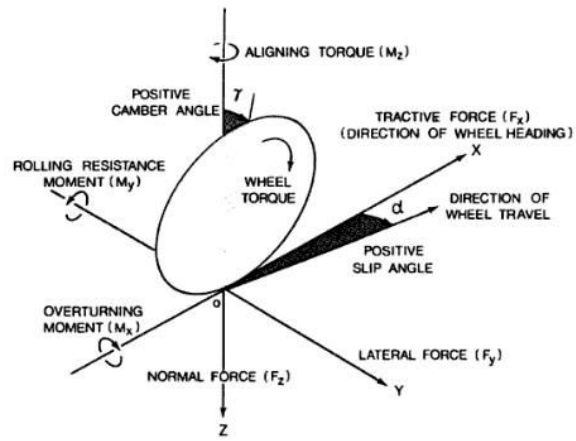


Figure 2.1: Tire Axis System

As the basic principle, there are three forces acting on the wheel (as shown in Figure 2.1 above) [3], which are normal or vertical force (F_z), tractive or longitudinal force (F_x) and lateral force (F_y). The normal force (F_z) is due to the weight of the vehicle and its vertical inertial force. Longitudinal or tractive force (F_x) is due to the resultant force exerted on the tire by the road and vehicle's inertial force, whereas the lateral force (F_y) is due to the vehicle's centrifugal forces [3]. For this study, we assume the car is properly aligned, where camber angle γ is neglected.

2.5 Currently applied 4WS Technology [2][8][10][17][30]

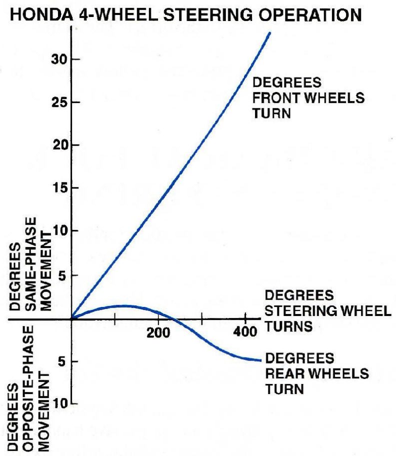


Figure 2.2: In Honda mechanical system, steering wheel angle alone determines the direction and degree that the rear wheels steer

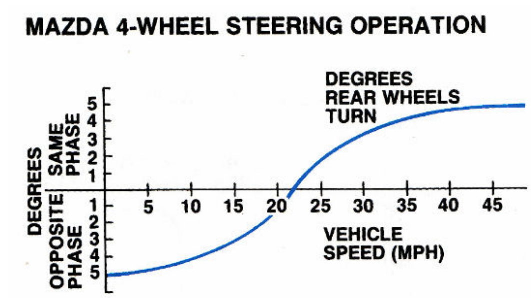


Figure 2.3: Mazda 4WS operate in the opposite phase mode when vehicle speed is below 22mph (35km/h), and in the same-phase mode at higher road speeds. Steering wheels affects how many degrees the rear wheels steer.

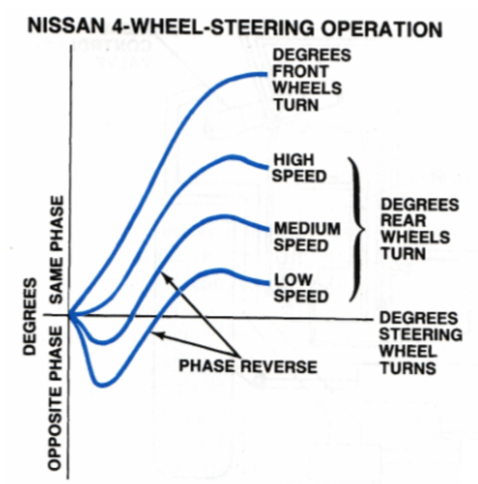


Figure 2.4: Depending on vehicle speed, the same steering wheel angle and rate of turn produce different responses at the rear wheels

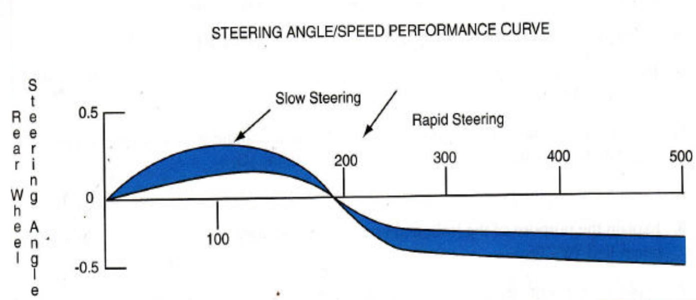


Figure 2.5: Rear steering angle in relation to vehicle speed and steering wheel rotation

2.6 Turning Radius

In Figure 2.6 below, 2WS and 4WS vehicle been modeled as a two-wheel bicycle model. This model is applied by most recent studies made on 4WS [1][4][5][11][12][13][25][31][32] and assumed to be valid mostly for the kinematical analysis of the vehicle with few assumptions applied.

To model the steering behavior of the vehicle, the so-called bicycle model is used with two degrees of freedom. The body of the vehicle is assumed to be a rigid beam, and the left and right tires are combined into a single one. Furthermore, it is assumed that all motions occur in the plane of the vehicle model.

This model describes the vehicle in-plane behavior sufficiently well for high speeds; however, it is not able to consider the effect of load transfer between the left and right side wheels.

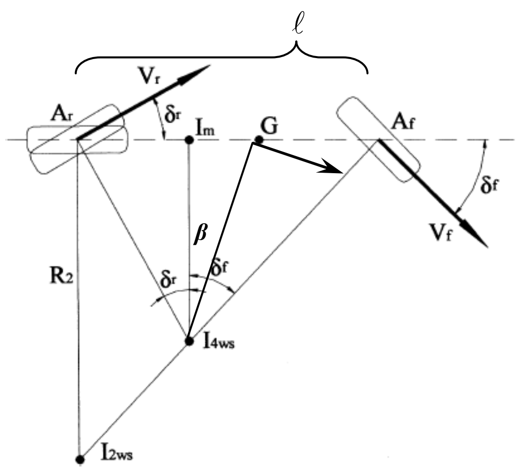


Figure 2.6: Two-wheel (Bicycle) model of 2WS and 4WS vehicle

Turning radius for 2WS R_{2WS} is shown as;

$$\text{2WS turning radius, } R_{2WS} = (A_r A_f)/\sin\delta_f = \ell/\sin\delta_f \quad (1)$$

As for 4WS turning radius, considering the triangle $A_rI_{4WS}I_m$ and $A_fI_{4WS}I_m$;

$$\ell = I_m I_{4WS}\left(-\tan\delta_r + \tan\delta_f\right) \rightarrow I_m I_{4WS} = \frac{\ell}{\left(-\tan\delta_r + \tan\delta_f\right)}$$

*The negative sign on "tan δ_r" is to consider the direction of the I_m either forward or backward.

and $$R_{4WS} = \frac{I_m I_{4WS}}{\cos\delta_f}$$

Hence, 4WS turning radius, $$R_{4WS} = \frac{\ell}{\cos\delta_f\left(-\tan\delta_r + \tan\delta_f\right)} \qquad (2)$$

Noted that the term turning radius R as defined in certain sources [2][17][30], is the distance from the turning center to the most outside wheels (front-outside wheel). Therefore, using above formula, adjustments are made where the width of the car is considered.

Comparison of the turning radii given by equation (1) and (2) gives following results:

$R_{4ws} < R_{2ws}$: when the rear wheels turn in opposite direction than the front ones

$R_{4ws} > R_{2ws}$: when the rear wheels turn in the same direction than the front ones

Accordingly, a 4WS vehicle presents a maneuvering advantage over 2WS vehicle only if its rear wheels can turn in the opposite direction than its front wheel; giving smaller turning radius.

Maneuverability is important in urban areas, where typically a speed limit of 50 km/h or lower applies and the sideslip angle of the wheels is negligible.

Consequently, the steering system of a 4WS vehicle must have the ability (at low speed) to turn the rear steering wheels in the opposite direction than the front steering wheels.

As the vehicle's speeds increases above that limit, the turning radius of the vehicle should be higher, allowing the rear wheels turn in the same direction of the front wheels in order to improve the stability of the vehicle's movement.

2.7 Sideslip Angles [1][4][5][11][12][13]

2.7.1 Vehicle's Sideslip Angle, β

Referring to Figure 5, earlier derivation made (Eqn 1), and let say that;

$$IG = A$$

$$IA_f = B$$

$$I_m I_{4WS} = C$$

$$IA_r = D$$

$$GA_f = E$$

Hence,

$$\tan\delta_r = \frac{D}{C} = D \times \frac{(-\tan\delta_r + \tan\delta_f)}{\ell}$$

$$D = \frac{\ell \tan\delta_r}{(-\tan\delta_r + \tan\delta_f)} \tag{3}$$

The value of δ_r and δ_f are the wheel-turn angles, hence, the value can be obtained from the steering-wheel input and the car's 4WS operation (refer Section 5.3). The value of ℓ (the wheelbase) and E (car geometry; E or a = distance of c.g. to the front axle) is known and defined as constant.

Knowing the value of D, hence;

$$\ell - D - E = A$$

$$\tan\beta = \frac{A}{C}$$

$$\beta = \tan^{-1}\left(\frac{A}{C}\right)$$

Therefore the vehicle sideslip angle, β

$$\beta = \tan^{-1}\left(A \times \frac{(-\tan\delta_r + \tan\delta_f)}{\ell}\right) \tag{4}$$

2.7.2 Wheel-sideslip Angles, δ_f and δ_f

2.7.2.1 4WS vehicle, sideslip angles not neglected

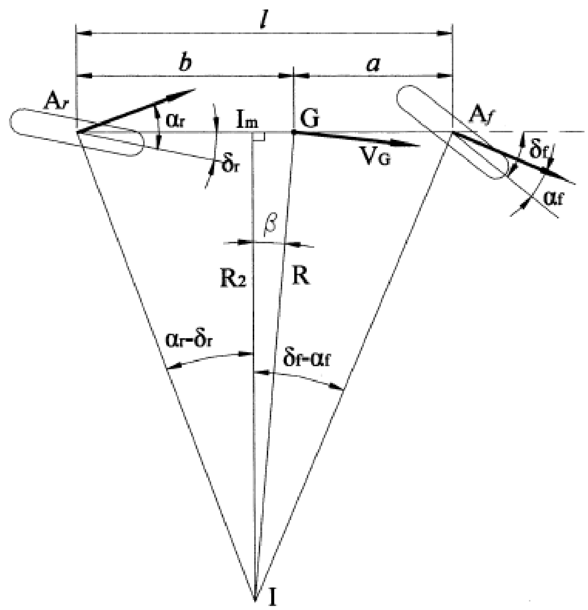


Figure 2.7: Two-wheel (Bicycle) model of a 4WS vehicle

(The side-slip angles of the tires are not neglected)

Considering the triangles II_mA_f & II_mA_r in Fig. 2.7, and using following contractions:

$$a = (GA_f)$$
$$b = (GA_r)$$
$$e = (GI_m)$$

Hence,

$$\begin{aligned} (I_m A_f) &= |a| + |e| = R_2 \tan(\delta_f - \alpha_f) \\ (A_r I_m) &= |b| + |e| = R_2 \tan(\alpha_r - \delta_r) \end{aligned} \tag{5}$$

By adding above two equations and taking into consideration on the obvious relation:

$$\ell = (A_r I_m) + (I_m A_f) = |a| + |b|$$

Thus, obtain

$$\tan(\delta_f - \alpha_f) + \tan(\alpha_r - \delta_r) = \ell / R\cos\beta$$

Above equation can also be written as

$$\tan(\delta_f - \alpha_f) - \tan(\delta_r - \alpha_r) = \ell / R\cos\beta \tag{6}$$

By considering the relations of

$$R_2 = R\cos\beta$$
$$e = R\sin\beta$$

We can write

$$|a| + |e| = |a| + R\sin\beta$$
$$|b| - |e| = |b| - R\sin\beta$$

Consequently obtaining

$$\begin{aligned}\tan(\delta_f - \alpha_f) &= (|a| + R\sin\beta)/R\cos\beta \\ &= |a|/(R\cos\beta) + \tan\beta\end{aligned} \tag{7}$$

$$\begin{aligned}\tan(\delta_r - \alpha_r) &= -[(|b| - R\sin\beta)/R\cos\beta] \\ &= -[|b|/(R\cos\beta) - \tan\beta]\end{aligned} \tag{8}$$

and finally have

$$\delta_f = \arctan[|a|/(R\cos\beta) + \tan\beta] + \alpha_f$$
$$\delta_r = -\arctan[|b|/(R\cos\beta) - \tan\beta] + \alpha_r$$

$$\delta_f + \delta_r = \arctan[|a|/(R\cos\beta) + \tan\beta] - \arctan[|b|/(R\cos\beta) - \tan\beta] + \alpha_r + \alpha_f$$

By applying ratio of

$$k = \delta_r / \delta_f$$

Substituting into the equation, we can also write

$$\begin{aligned}\delta_f = {} & [1/(1+k)]\{\arctan[|a|/(R\cos\beta) + \tan\beta] - \arctan[|b|/(R\cos\beta) - \tan\beta]\} \\ & + \alpha_f + \alpha_r\end{aligned} \tag{9}$$

$$\begin{aligned}\delta_r = {} & [k/(1+k)]\{\arctan[|a|/(R\cos\beta) + \tan\beta] - \arctan[|b|/(R\cos\beta) - \tan\beta]\} \\ & + \alpha_f + \alpha_r\end{aligned} \tag{10}$$

2.7.2.2 4WS vehicle, sideslip angles neglected

When the vehicle is moving a low speed, the impact of the sideslip angles can be neglected. Therefore, we can make the assumption

$$\alpha_f = \alpha_r = 0$$

Introducing these values in equations (5), (6), (7), (8), (9), and (10), we obtain

$$(\tan\delta_f - \tan\delta_r) = \ell / R\cos\beta$$

$$\tan\delta_f = (|a| + R\sin\beta)/R\cos\beta$$
$$= |a|/(R\cos\beta) + \tan\beta$$
$$\tan\delta_r = -[(|b| - R\sin\beta)/R\cos\beta]$$
$$= -[|b|/(R\cos\beta) - \tan\beta]$$

$$\delta_f = \arctan[|a|/(R\cos\beta) + \tan\beta]$$
$$\delta_r = -\arctan[|b|/(R\cos\beta) - \tan\beta]$$

$$\delta_f + \delta_r = \arctan[|a|/(R\cos\beta) + \tan\beta] - \arctan[|b|/(R\cos\beta) - \tan\beta] \tag{11}$$

$$\delta_f = [1/(1+k)]\{\arctan[|a|/(R\cos\beta) + \tan\beta] - \arctan[|b|/(R\cos\beta) - \tan\beta]\} \tag{12}$$
$$\delta_r = [k/(1+k)]\{\arctan[|a|/(R\cos\beta) + \tan\beta] - \arctan[|b|/(R\cos\beta) - \tan\beta]\} \tag{13}$$

2.7.2.3 2WS vehicle, side-slip angles not neglected

By taking that the rear wheel steering angle is zero

$$\delta_r = 0$$

Derive from the equation (5), the formulae that are valid for a 2WS vehicle with non-negligible sideslip angles:

$$\tan(\delta_f - \alpha_f) + \tan\alpha_r = \ell / R\cos\beta$$

$$\tan(\delta_f - \alpha_f) = \ell / R\cos\beta - \tan\alpha_r \tag{14}$$

Above equation gives also the value of δ_f

$$\delta_f = \arctan[\ell / R\cos\beta - \tan\alpha_r] + \alpha_f \tag{15}$$

2.7.2.4 2WS vehicle, sideslip angles neglected

Assuming as before that,

$$\alpha_f = \alpha_r = 0$$

By simplifying equation (14) and (15):

$$\tan\delta_f = \ell / R\cos\beta = \ell / |b|$$

$$\delta_f = \arctan(\ell / R\cos\beta) = \arctan(\ell / |b|) \tag{16}$$

2.7.2.5 Simplification on Turning Radius Analysis

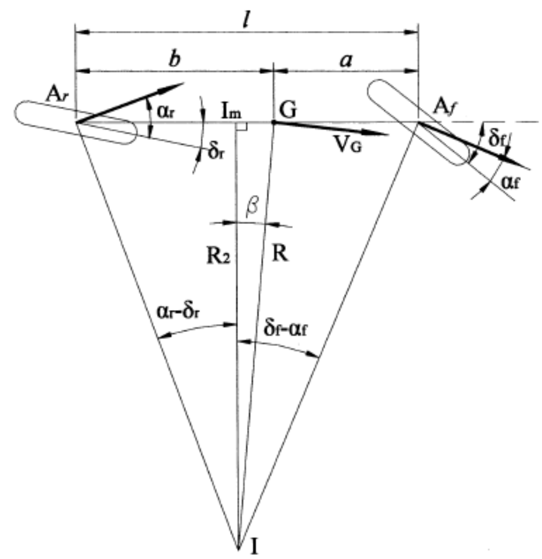


Figure 2.8: Two-wheel (Bicycle) model of a 4WS vehicle – Turning radius

As stated above, the wheel-turn angles (δ_f and δ_f), the car's geometry (*a* and *b*), turning radius (R), vehicle's sideslip (β), and the wheel's sideslip (α_f and α_r) are related by such equations:

$$\delta_f = \arctan\left[|a|/(R\cos\beta) + \tan\beta\right] + \alpha_f \tag{17}$$

$$\delta_r = -\arctan\left[|b|/(R\cos\beta) - \tan\beta\right] + \alpha_r \tag{18}$$

In earlier section, the value of the wheel-turn angles (δ_f and δ_f), turning radius (R), and the vehicle's sideslip (β) can be gained using the mentioned equation (Eqn. 7, 8, 9, and 10), while the car's geometry (*a* and *b*) is known and constant. Therefore, the wheel-sideslip angles are calculated as:

$$\alpha_f = \delta_f - \arctan\left[|a|/(R\cos\beta) + \tan\beta\right] \tag{19}$$

$$\alpha_r = \delta_r + \arctan\left[|b|/(R\cos\beta) - \tan\beta\right] \tag{20}$$

2.8 Dynamics of Four-Wheel Steering Vehicle

Considerable efforts and researches have been invested recently in the development of four-wheel-steering (4WS) vehicles, aiming to improve the maneuvering ability and the safety of operation of vehicles.

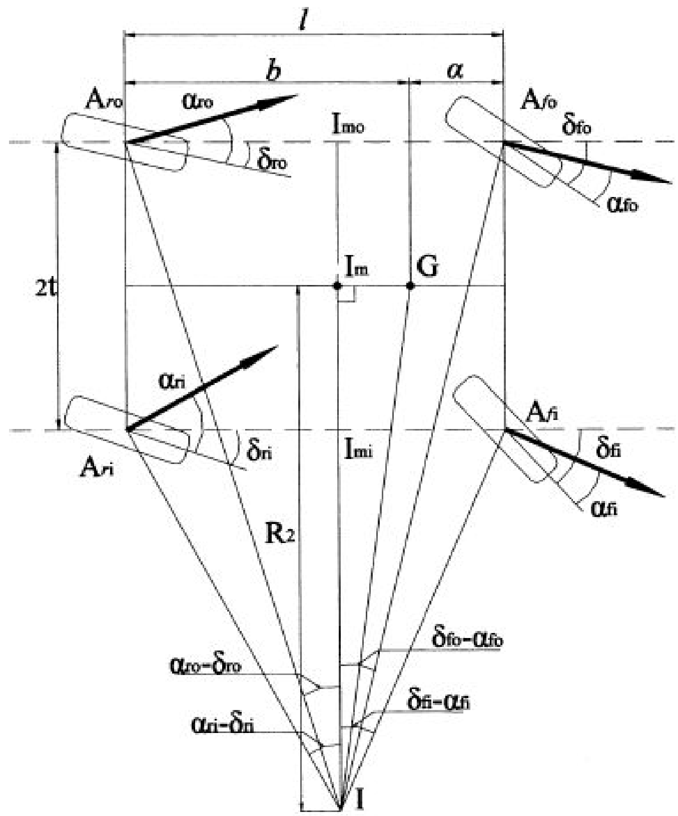


Figure 2.9: Four-wheel model of a 4WS vehicle [4][5]

(The side-slip angles of the tires are not neglected)

2.8.1 4WS Dynamic Model [5][11][12][13]

The dynamic model is elaborated on the basis of following general principles & assumptions:

i. The dynamic model of a 4WS vehicle must be able to take into consideration all the physical phenomena that are related to the motion of the vehicle, included the phenomenon of the lateral and the longitudinal weight transfer.
ii. The dynamic model must be as simple as possible, retaining no more degrees of freedom than it is necessary. This is favorable to as fast solution of the EOM.
iii. The chassis of the vehicle is considered to be rigid.
iv. The wheels of the vehicle are considered to remain all time in contact with the ground.
v. Without sacrificing the accuracy of the model, the body of the vehicle is considered kinetically equivalent to the sprung mass and the two un-sprung masses of the front and rear suspensions. These three masses are interconnected by the roll axis of the vehicle, the later being defined as the axis that joins the front roll centre to the rear roll centre.
vi. The centre of mass G of the vehicle is coincident with the origin of the axis system O; the vehicle is symmetric about the x-z plane. Thus, the inertia product $I_{xy} = 0$.
vii. The vehicle is moving on a smooth road. Therefore, the heave velocity and the pitch velocity are zero, $W = q = 0$, and also $\Sigma[F_z] = \Sigma[M_y] = 0$.
viii. The total mass of the vehicle is the sum of the sprung mass m_s and the un-sprung mass m_u. The coordinate of the centre of the sprung mass are $(0,0,h_s)$ and the rolling velocity p and the pitching velocity q have a meaning only for the sprung mass.

2.8.2 Equations of Motion

Referring to the "List of Symbols and Abbreviations", in time domain, the equations of motion for model III are as follows:

$$m \cdot (dU/dt - V \cdot r) + m_s \cdot h_s \cdot p \cdot r = \sum [F_x] \tag{21}$$

$$m \cdot (dV/dt + U \cdot r) + m_s \cdot h_{ra} \cdot (dp/dt) = \sum [F_y] \tag{22}$$

$$I_{xs} \cdot (dp/dt) + m_s \cdot h_{ra} \cdot (dV/dt + U \cdot r) = \sum [M_x] \tag{23}$$

$$I_z \cdot r = \sum [M_z] \tag{24}$$

This system of equations is derived from general equations of motion by applying above assumptions.

Given the fact that the driver of the vehicle chooses the value of the forward velocity U (U is not an unknown). Hence, the unknowns in the system of equations of motion are reduced to three; the lateral velocity V, the rolling velocity p, and the yaw velocity r.

2.8.3 Total Forces and Total Moments

The total forces and total moments of the equations of motion are computed by using following relations:

$$\sum [F_x] = \sum [F_j \cdot \sin \delta_j]$$

$$\sum [F_y] = \sum [F_j \cdot \cos \delta_j]$$

$$\sum [M_x] = (m_s \cdot g \cdot h_{ra} - K_{roll}) \cdot \varphi - D_{roll} \cdot p$$

$$\begin{aligned} \sum [M_z] = \quad & a \cdot (F_1 \cdot \cos \delta_1 + F_2 \cdot \cos \delta_2) \\ & - b \cdot (F_3 \cdot \cos \delta_3 + F_4 \cdot \cos \delta_4) \\ & + t_f \cdot (F_2 \cdot \sin \delta_2 + F_4 \cdot \sin \delta_4) \\ & - t_r \cdot (F_1 \cdot \sin \delta_1 + F_3 \cdot \sin \delta_3) \end{aligned}$$

2.8.4 Sideslip Angles of the Wheels

The sideslip angle of each wheel is considered to be a function of its steer angle:

$$\alpha_1 = \delta_1 - \arctan[(V + a \cdot r)/(U + t_f \cdot r)]$$

$$\alpha_2 = \delta_2 - \arctan[(V + a \cdot r)/(U - t_f \cdot r)]$$

$$\alpha_3 = \delta_3 - \arctan[(b \cdot r - V)/(U + t_r \cdot r)]$$

$$\alpha_4 = \delta_4 - \arctan[(b \cdot r - V)/(U - t_r \cdot r)]$$

Given the definition of the model, assumptions used:

$$\delta_1 = \delta_2 = \delta_f$$

$$\delta_3 = \delta_4 = \delta_r$$

2.9 Tire Model

For this study, the magic formula tire model will be used [3].

$$y(x) = D \sin\{C \arctan[Bx - E(Bx - \arctan Bx)]\}$$
$$Y(X) = y(x) + S_v$$

and
$$x = X + S_h$$

Where;

Y(X)	Cornering force, self-aligning torque, or braking effort
X	Slip angle or skid
B	Stiffness factor
C	Shape factor
D	Peak factor
E	Curvature Factor
S_h	Horizontal shift
S_v	Vertical shift

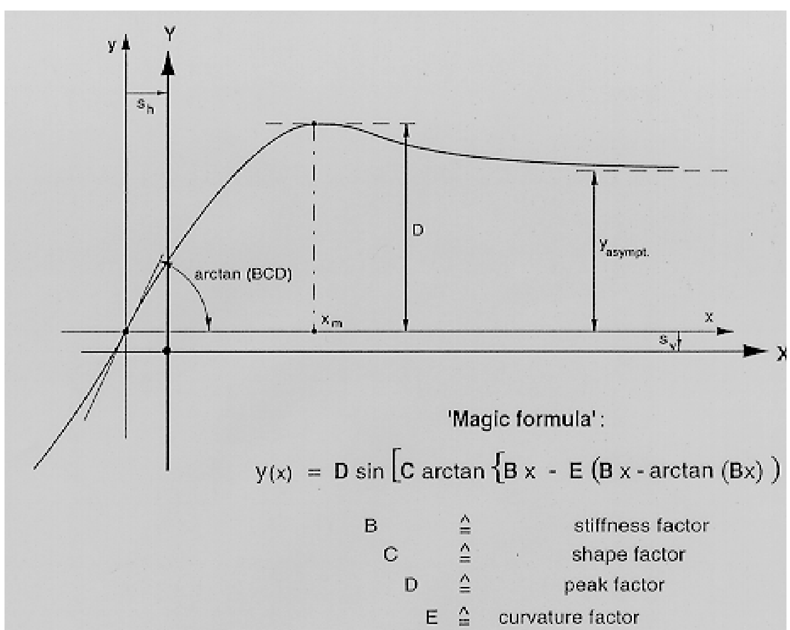


Figure 2.10: Characteristics of the Magic Formula for fitting tire test data [14]

2.10 Weight Transfers

The longitudinal and transversal weight transfers are taken into account in the computation of the normal reaction forces of the road on a wheel.

The longitudinal weight transfer is due to the acceleration or deceleration of the vehicle and is given by:

$$W_L = \left(m_s \cdot g \cdot A_x \cdot h + m_{uf} \cdot g \cdot A_x \cdot h_{uf} + M_{ur} \cdot g \cdot A_x \cdot h_{ur}\right)/2L$$

The value of A_x in above equation must be entered as:

- Positive for accelerating vehicle,
- Negative for decelerating vehicle.

The transversal weight transfer consists of three components;

i. The transversal weight transfer due to body roll W_b,

ii. The transversal weight transfer due to the height of the roll centre W_r

iii. The transversal weight transfer due to the height of the un-sprung mass W_u.

Hence, the front and rear transversal weight transfers are:

$$W_{Tf} = W_{bf} + W_{rf} + W_{uf}$$

$$W_{Tr} = W_{br} + W_{rr} + W_{ur}$$

Each one of the terms figuring in the last two equations can be computed from following relations:

$$W_{bf} = m_s\left(dV/dt + U \cdot r\right) \cdot h_{ra} \cdot \cos\varphi \times \left(K_f / K_{roll}\right) \cdot \left(1/2t_f\right)$$

$$W_{br} = m_s\left(dV/dt + U \cdot r\right) \cdot h_{ra} \cdot \cos\varphi \times \left(K_f / K_{roll}\right) \cdot \left(1/2t_r\right)$$

$$W_{rf} = m_s\left(dV/dt + U \cdot r\right) \cdot \left(b/\ell\right) \cdot \left(h_f / 2t_f\right)$$

$$W_{rr} = m_s\left(dV/dt + U \cdot r\right) \cdot \left(a/\ell\right) \cdot \left(h_r / 2t_r\right)$$

$$W_{uf} = m_{uf}\left(dV/dt + U \cdot r\right) \cdot \left(h_f / 2t_f\right)$$

$$W_{ur} = m_{ur}\left(dV/dt + U \cdot r\right) \cdot \left(h_r / 2t_r\right)$$

2.11 Normal Reaction Forces on the Wheels

Given the computation of the weight transfers earlier, the normal reaction forces on the wheels can be determined as follows:

$$F_{zfo} = m \cdot g \cdot b / 2\ell + W_L + W_{Tf}$$

$$F_{zfi} = m \cdot g \cdot b / 2\ell + W_L - W_{Tf}$$

$$F_{zro} = m \cdot g \cdot a / 2\ell - W_L + W_{Tr}$$

$$F_{zri} = m \cdot g \cdot a / 2\ell - W_L - W_{Tr}$$

2.12 Steer Angles of the Wheels

The steering wheel angles are considered to be linear functions not only of the steering wheel angle θ_s, but also of $d\theta_s/dt$;

$$\delta_f = k_{1f} \cdot \theta_s + k_{2f} \cdot (d\theta_s / dt)$$

$$\delta_r = k_{1r} \cdot \theta_s + k_{2r} \cdot (d\theta_s / dt)$$

In usual method of steering a vehicle by a steering wheel, the driver decides on the front steer angles ratio; values of k_{1f} and k_{2f}, whereas the designer of the 4WS vehicle decides on the values of the rear wheel steer angles ratio; values of k_{1r} and k_{2r}, either by giving a fix value to them or by providing a adequate algorithm (materialized by steering system controller – computer) that provides the optimum values for k_{1r} and k_{2r} as functions of k_{1f} and k_{2f}.

CHAPTER 3

METHODOLOGY

3.1 METHODOLOGY

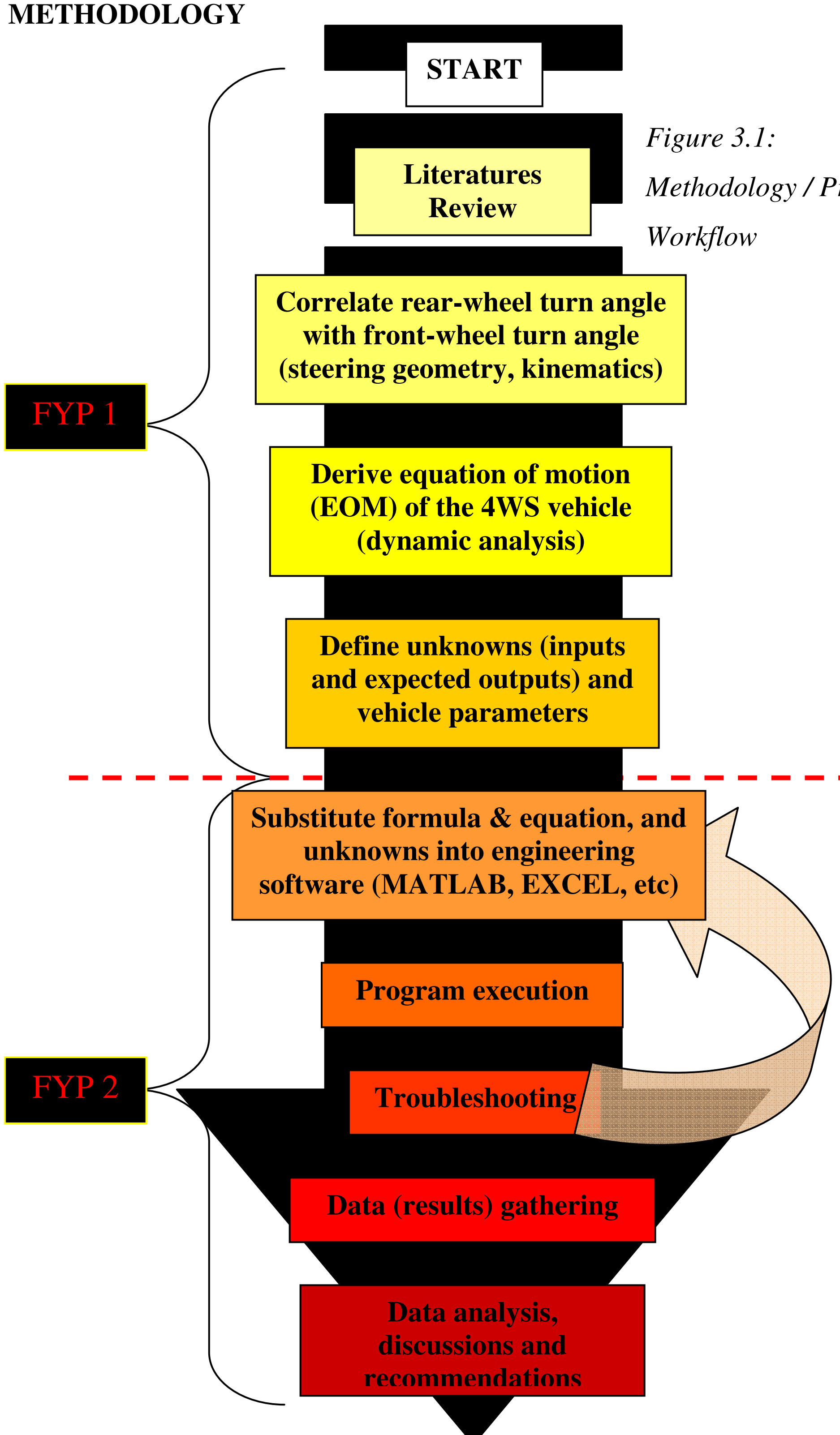


Figure 3.1: Methodology / Project Workflow

3.2 Methodology / Project Activities

3.2.1 Start

- Planning the project
- Submit project proposal including objectives, scopes, etc

3.2.2 Literature Review

- Review literatures and recent studies made related to the project
- Do further references to enhanced knowledge on the project matters
- Review EOM, kinematics, dynamics, and steering geometry of 4WS

3.2.3 Correlate rear-wheel turn angle with front-wheel turn angle (Kinematics Analysis)

- Define relation of the rear wheel turn angle with front wheel turn angle
- In this analysis, the correlation of the rear wheel turn angle with respect to the front wheel turn angle been determined.
- Includes the steering geometry of 4WS vehicle
- Analyzing the equations for turning radius, car geometry, vehicle side-slip angle, etc.

3.2.4 Derive Equation of Motion (EOM) of 4WS vehicle (Dynamic Analysis)

- Equation of motion of the model been produced and proper derivations been made to suit the assumptions and principles applied on the model (as included in the *Literature Review Section*)
- Forces acting on the vehicle and its wheel, reaction forces acted on the wheel, and weight transfer equation been analyzed.
- Analyzing the equations for yaw velocity, lateral velocity, wheels side-slip angles, etc.

3.2.5 Define unknowns and 4WS vehicle parameters

- After all derivation and formulation been done, the unknowns and the known variable been defined.
- Control variable and the response (expected outcome's variable) been clarified.
- The parameters of the studied 4WS vehicle also been identified.

3.2.6 Substitute formula & equation, and unknowns into engineering software (MATLAB, EXCEL, etc)

- Substitute the derivations and formula produced into the engineering software (MATLAB & Excel)
- The control variables (vehicle's forward speed, steering-wheel angle, vehicle's geometry and other vehicle's parameter) been keyed into the software.

3.2.7 Program Execution & Troubleshooting

- The software will then be executed to produces data.
- Any programming bugs or trouble encountered during the execution will be troubleshot to ensure proper data execution been done
- Errors or improper values produced by the software will also been re-evaluate during this phase.

3.2.8 Data (Results) Gathering

- By gathering all the data produced by the software, proper results and data compilation will be done.
- Compilation includes preparation on visual aids of the data such as tables, graphs, and so on.
- Correlation between the inputs and the outputs of the data will be seen in this phase.

3.2.9 Analysis and Discussions

- Discussions will be made to justify the outputs produced by the engineering software MATLAB (i.e. graphs)
- Proper comparisons between the data will be done.
- Any recommendations and errors will also be included.
- The data gained will also been compared with other journal provided by other researches to get a better comparisons and to check on the data integrity.

3.3 Key Milestone

Excellent data gathering and analysis on the rear-wheel turn angle to the vehicle's performances been gained. Relevant plot, graph, and patterns of the outputs with respect to the input been provided.

3.4 Gantt Chart / Milestones

No.	Detail / Week	1	2	3	4	5	6	7	8	9		10	11	12	13	14
1	Selection & Review of Project Topic										Mid- Semester Break					
2	Review Literatures & Recent Studies															
3	Submission of Preliminary Report															
4	Correlate rear-wheel with front-wheel turn angles (kinematics analysis)															
5	Submission of Progress Rep. & Seminar															
6	Derive formula on dynamics of 4WS Vehicle (dynamic analysis)															
7	Define unknowns & vehicle's parameters															
8	Submission of Interim Report Final Draft															
9	Oral Presentation															

Table 3.1: Scheduling for Final Year Project 1

No.	Detail / Week	1	2	3	4	5	6	7	8	9		10	11	12	13	14
1	Substituting equations & unknowns into engineering software (MATLAB, Excel)										Mid- Semester Break					
2	Submission of Progress Report 1															
3	Data Execution and Troubleshooting															
4	Submission of Progress Report 2															
5	Seminar															
6	Data Gathering, Analysis & Discussions															
7	Poster Exhibition															
8	Submission of Dissertation Final Draft															
9	Oral Presentation															
10	Submission of Dissertation (Hard Bound)															

Table 3.2: Scheduling for Final Year Project 2

3.5 Tool(s) used: MATLAB and Microsoft Excel

CHAPTER 4
RESULTS AND DISCUSSION

As the results through the references and relevant studies made so far, below are the summarized components that to be included for the data construction

4.1 Data Gathering - Vehicle's Parameters

Below are the 4WS vehicle's parameters used in the study [15] [19] [20][24].

Size	Length x Width x Height:	4.36 x 1.69 x 1.385 m^3	
Wheelbase:		2.5 m	
a	1.1 m (c.g. to front axle)	b	1.4 m (c.g. to rear axle)
m	1120 kg	C_f	55 000 N/rad
m_s	920 kg	C_r	45 000 N/rad
m_{uf}	125 kg	$C_{Øf}$	1 100 Nms/rad
m_{ur}	75 kg	$C_{Ør}$	1 000 Nms/rad
G	9.81 m/s^2	$C_Ø$	2 100 Nms/rad
V	60 km/h	$k_{Øf}$	15 450 Nms/rad
h_s	0.55 m	$k_{Ør}$	15 450 Nms/rad
d_f	1.0 m	$k_Ø$	65 690 Nms/rad
d_r	1.5 m	J_{zz}	2 130 kgm^2
d_t	0.65 m	J_{xx}	500 kgm^2
B_f	0.122 s	J_{xz}	4 750 kgm^2
C_{sf}	10 Nms/rad	R_f	-0.17
C_{sr}	13 Nms/rad	R_r	0.15
k_{sf}	82 Nm/rad	R_t	0.33 m
k_{sr}	80 Nm/rad	b_r	0.1625 s
J_t	0.07 kg m^2	γ	5 deg
α_1	2 321.7	α_2	1.6929
α_3	0.2292	α_4	0.225
α_5	0.0911	α_6	3 317

Table 4.1: Typical Passenger Cars' Parameters (Proton Wira 1.6 416GLXi)

4.2 Relations of front-wheel turn angle to rear-wheel turn angle

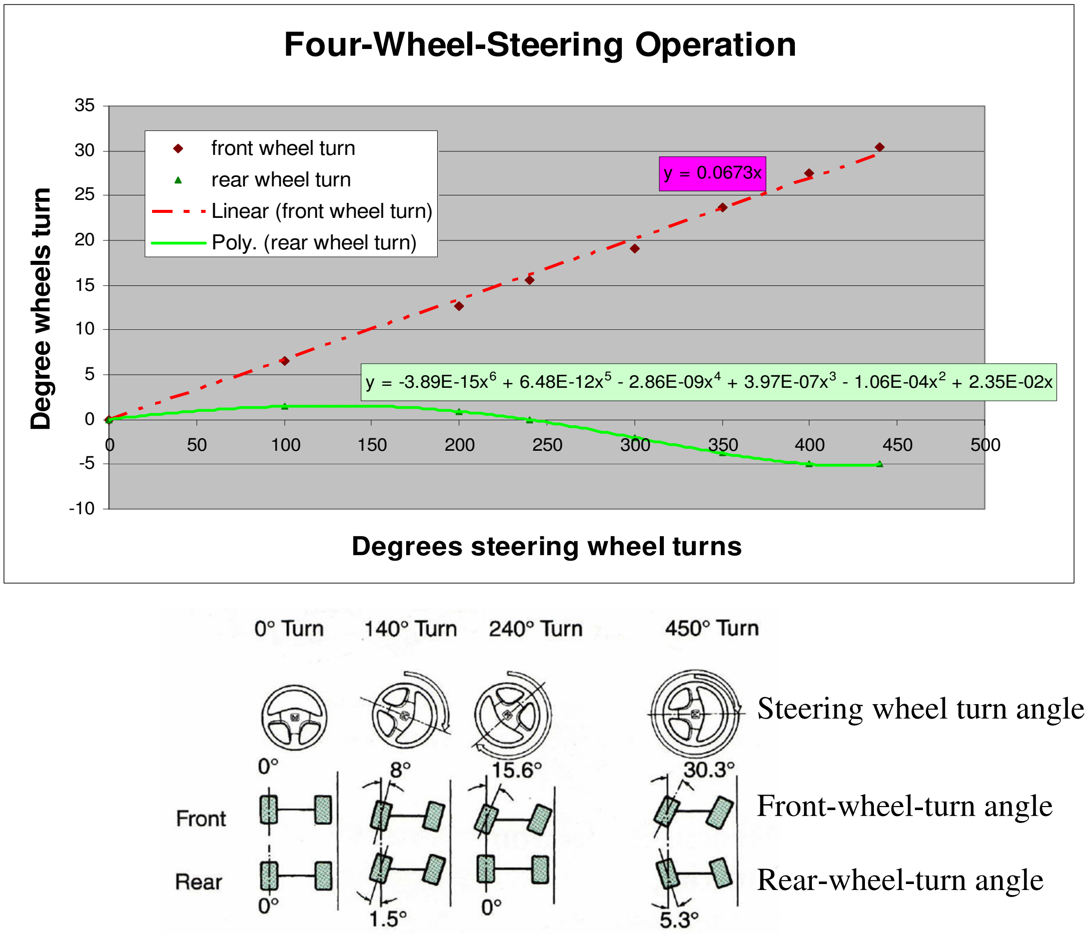


Figure 4.1: Correlation of front- and rear-wheels-turn angle with the steering wheel turn

Figure above shows the relations of rear-wheel turn angle (full line) with respect to the front wheel turn angle (mixed line); where the steering-wheel as the x-axis. Above graph is produced by extracting the data and visualized it into graph's figure as to produce the relations of front and rear-wheel turn angles [17].

Note that the above data is applied by the infamous car manufacturer; Honda for their 4WS car's product (i.e. Honda Prelude and Honda Accord). This 4WS operation used in Honda mechanical system, steering wheel angle alone determines the direction and degree that the rear wheels steer; which means the vehicle's velocity and the steering-wheel turn rate does not affect the front and rear-wheel turn angle.

4.3 Turning Radius

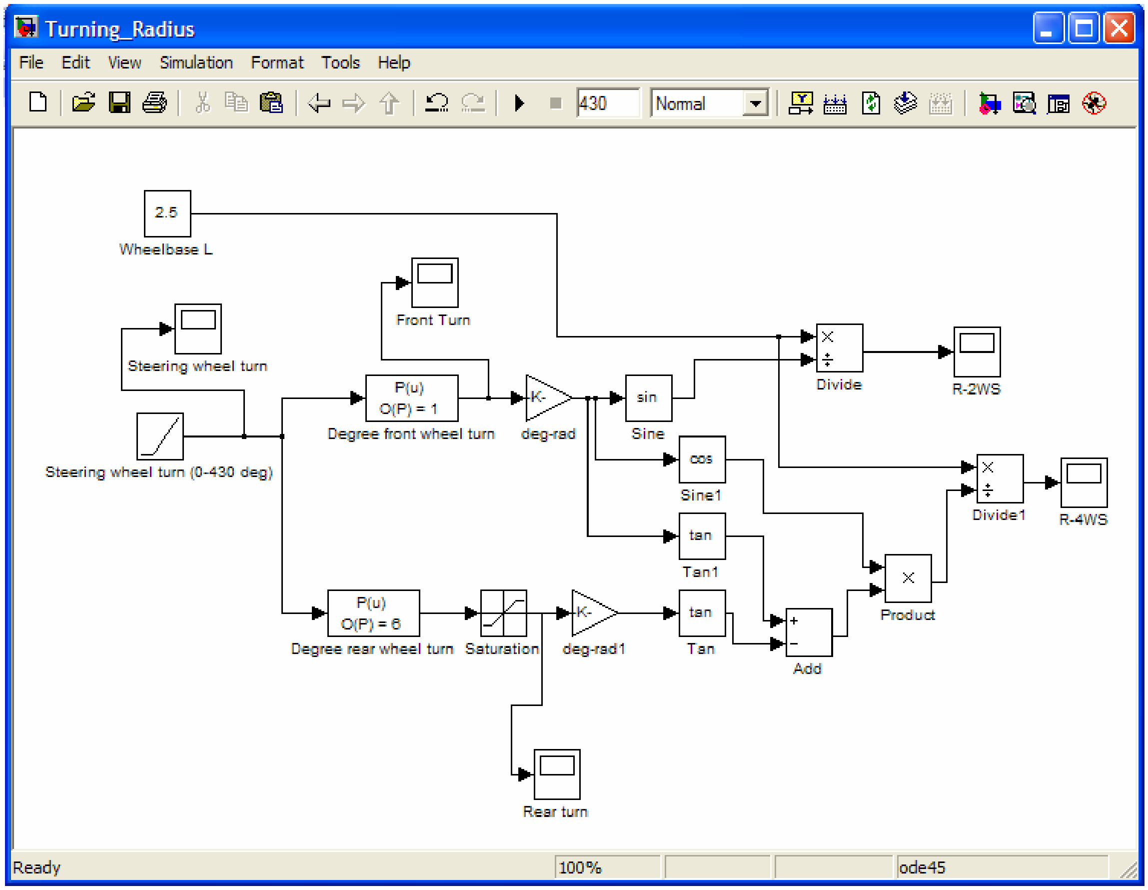
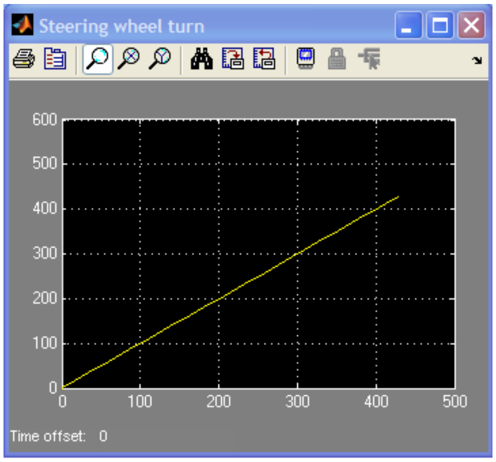


Figure 4.2: MATLAB Simulation modeling for turning radius of 2WS and 4WS vehicles

Using **MATLAB Simulation Modeling** (as shown above), the relations of the front and rear wheel-turn angle are determined, referring to equation (1) and (2). The simulation run on basis of the steering wheel turns from 0 to 430 degree. The steering wheel turns produce certain value of front-wheel turn angle and rear-wheel turn angle for 4WS. The graphs produced are as per below:



Steering ratio = 0.0673

*1 degree of steering-wheel been steered will turns 0.0673 degrees of front-wheel angle

Figure 4.3: Steering wheel turns from 0 to 430 degree (input)

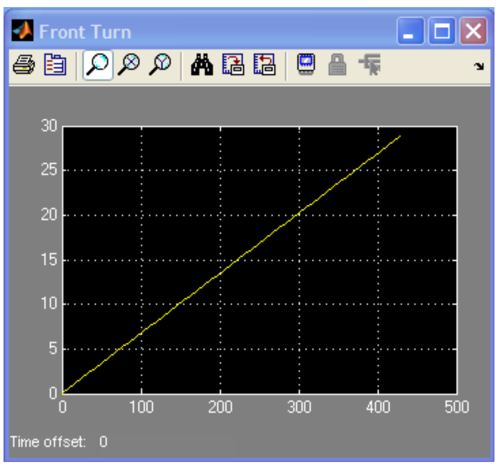


Figure 4.4: Front-wheel turn angle

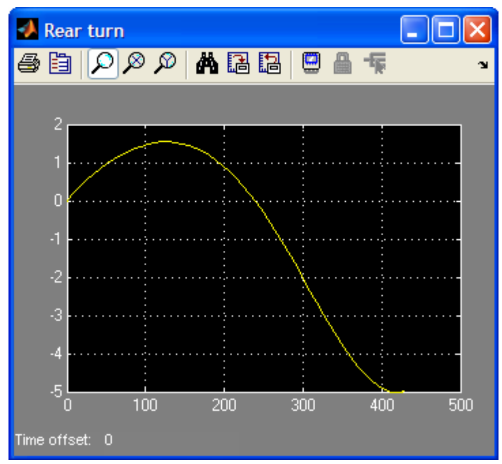


Figure 4.5: Rear-wheel turn angle

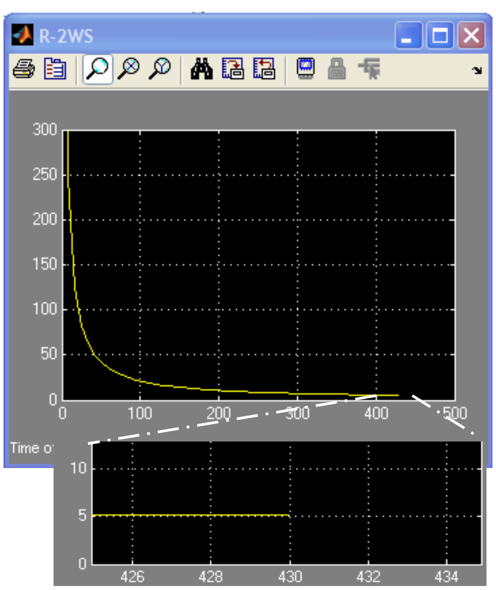


Figure 4.6: Turning radius of 2WS vehicle

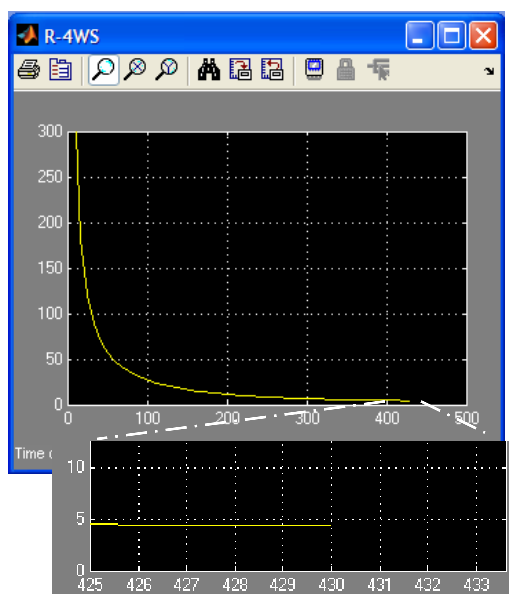


Figure 4.7: Turning radius of 4WS vehicle

* For all above graph, the x-axis represents the steering-wheel turn angle (driver input)

As shown in Figure 4.4 and Figure 4.5, basically, it is the same line from Figure 2.2. It is the front-wheel turn angle for 2WS, and including the rear wheel turn angle for 4WS vehicle. Figure 4.4, 4.5, 4.6 and 4.7 are corresponding to the steering-wheel turns (x-axis) as the driver input.

In Figure 4.4, the front wheel turn angle is directly proportional to the steering-wheel turns (driver input) from 0^o (at 0^o steering-wheel turn) to 29^o (at 430^o steering-wheel turn).

In Figure 4.5, as for the rear-wheel turn angle, it is in a polynomial form; in this case, assumed to be in polynomial of 6 (refer Figure 4.1 for detail). The maximum rear-wheel turn degree in same-phase movement (parallel to the front wheel) is 1.55^o, and for opposite-phase movement (opposite direction to the front wheel) is 5^o.

In Figure 4.6, at maximum steering-wheel turns (430^o), the turning radius for 2WS is 4.96 m (9.92 m in diameter), while in Figure 4.7, for 4WS is 4.35 m (8.71 m in diameter).

However, considering the vehicle is not a plane (as per two-wheel bicycle model), therefore the width of the vehicle are to be considered. Hence, at maximum steering-wheel turns, the real turning radius for 2WS is 5.81 m (11.61 m in diameter), and for 4WS is 5.20 m (10.39 m in diameter).

The turning radius for 4WS is less than 2WS with around 0.61 m (1.22 m in diameter) lesser; means higher maneuverability.

4.4 Car's Conditions

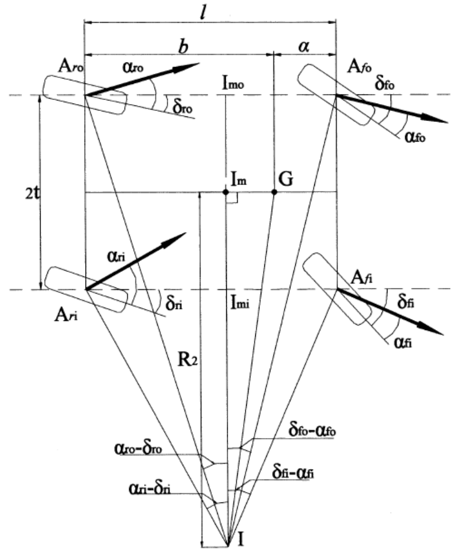


Figure 4.8: 4-Wheel-Steering Vehicle Model

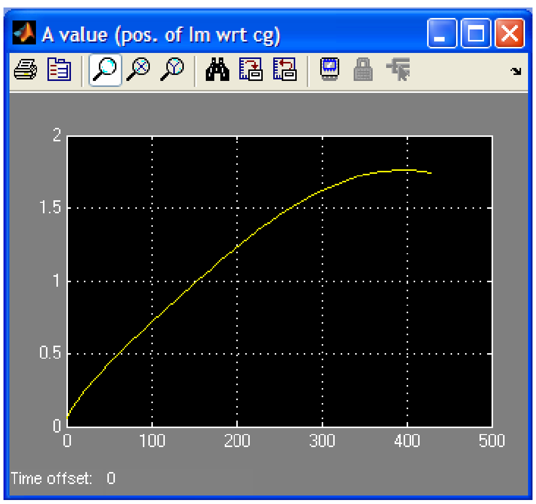


Figure 4.9: Distance of I_m from the center of gravity of the car vs steering wheel turns for 4WS vehicle

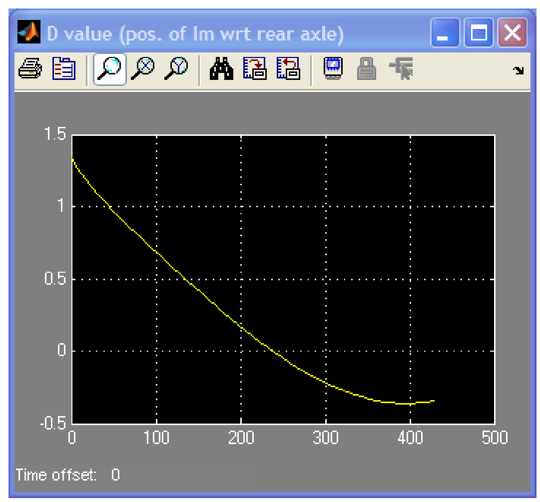


Figure 4.10: Distance of I_m from rear axle of the car vs steering wheel turns for 4WS vehicle

In Figure 4.9, it shows that for 4WS, the smallest value is 0.06 m (6 cm) from the centre of gravity and the value increases as the steering wheel been turned. It means that the location of I_m is getting further. Whereas in Figure 4.10, it shows that the location of I_m become negative when the steering wheel turns more than approximately 240°. This negative sign shows that when the steering-wheel been turned more than 240°, the position of I_m is in behind the rear axle, hence giving a much greater turning radius to the 4WS vehicle.

4.5 Yaw Velocity *r*, and Lateral Velocity *V*

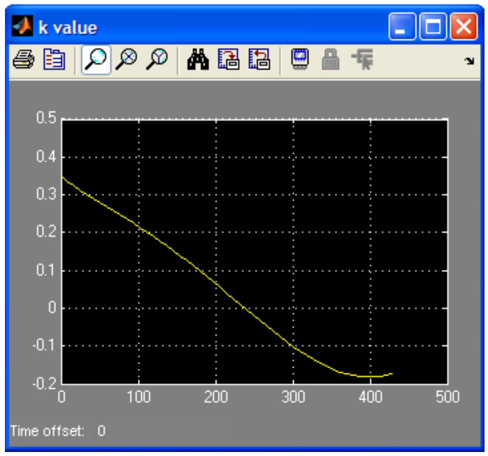


Figure 4.11: Ratio of front-wheel and rear-wheel turns k, versus steering-wheel turns.

$$k = \frac{\textit{Rear-wheel-turn angle}}{\textit{Front-wheel-turn angle}}$$

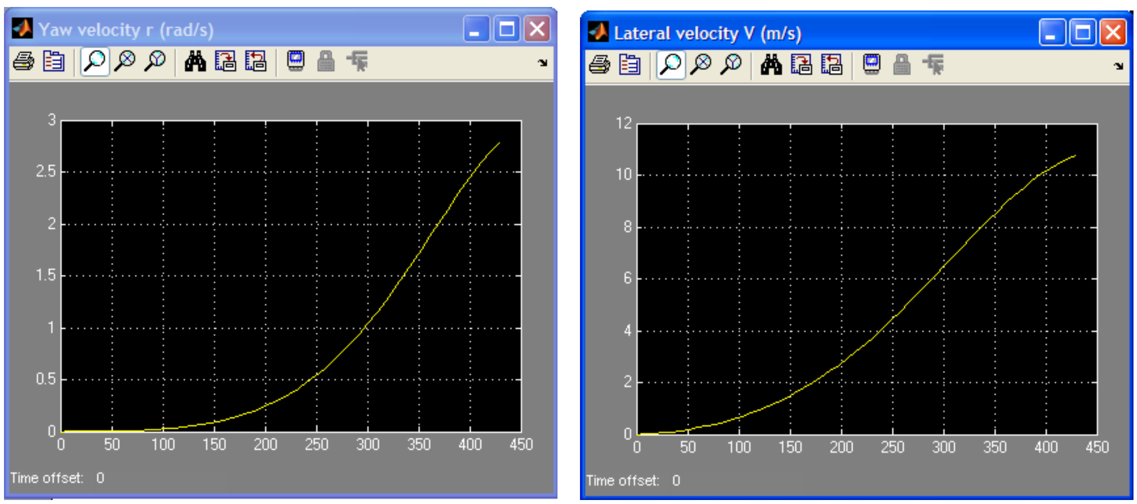


Figure 4.12: Yaw velocity r and Lateral velocity V of 4WS vehicle at speed U= 90km/hr

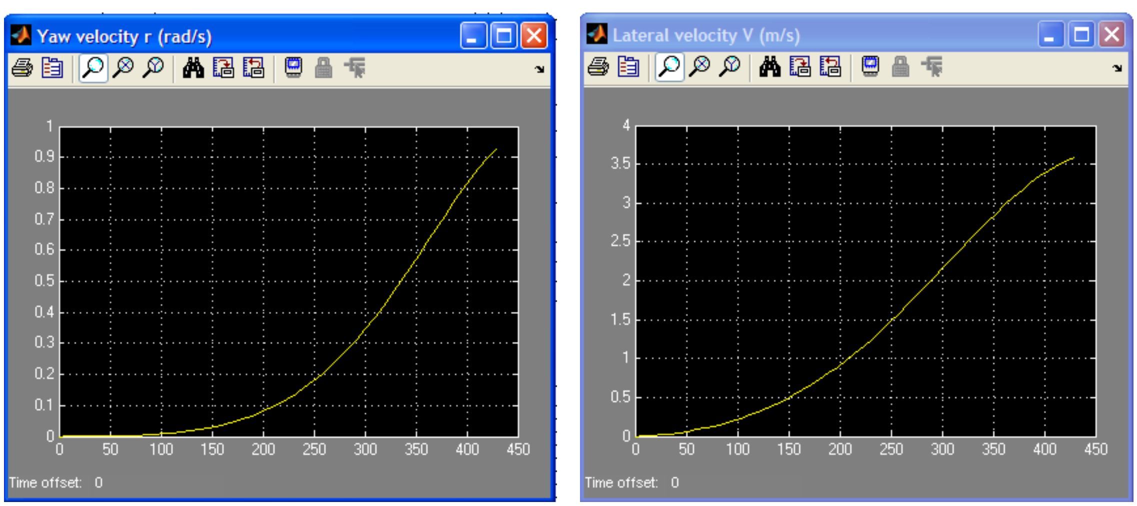


Figure 4.13: Yaw velocity r and Lateral velocity V of 4WS vehicle at speed U= 30km/hr

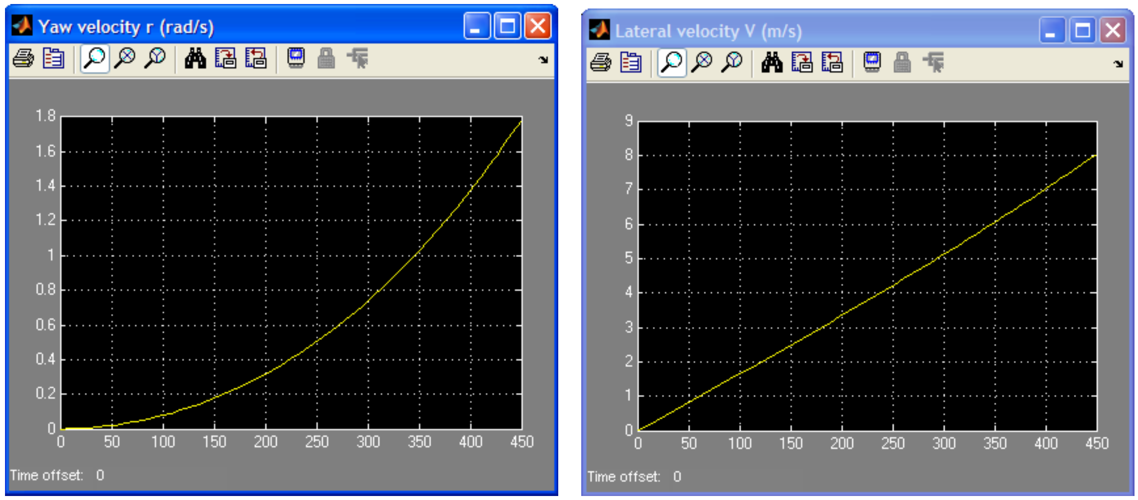


Figure 4.14: Yaw velocity r and Lateral velocity V of 2WS vehicle at speed U= 90km/hr

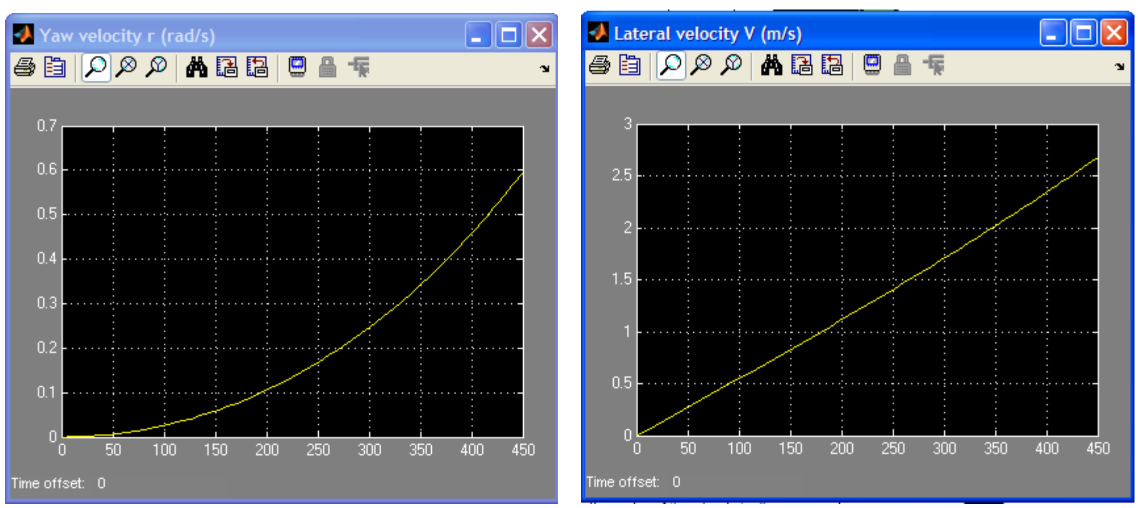


Figure 4.15: Yaw velocity r and Lateral velocity V of 2WS vehicle at speed U= 30km/hr

By comparing the 4WS and 2WS yaw velocity and lateral velocity (as shown in Figure 4.12, 4.13, 4.14, and 4.15), the pattern is different to each other.

One of the major patterns that can be referred to above figures concerning the performance of 4WS compared to 2WS vehicle is that, at below 240^o steering-wheel turn, both yaw and lateral velocities of 4WS is lower than 2WS. Thus, this shows that 4WS is having a much stable condition compared to 2WS when the driver is handling low than 240^o steering-wheel. Lower yaw and lateral velocities of 4WS also means that the vehicle is having less tendency to roll (which may cause accidents – higher risk to the driver) when handling the vehicle at high speed considering that the driver steered below than 240^o steering wheel turns.

This is applicable during high speed since generally the driver will steered the wheel certainly below 240^o turn. Whereas for more than 240^o steering-wheel been turned, the 4WS is having a higher yaw and lateral velocities compared to 2WS vehicle. This is applicable during low speed where the driver will steered usually almost full steering turn; which means more than 240^o when making a U-turn or making a side-parking where the car steering response is desired during the time.

The conditions of which 4WS having higher yaw and lateral velocity at more than 240^o steering-wheel turn shows that the 4WS is turning at higher rate than 2WS.

However, considering if the driver is handling the vehicle at relatively high speed at a sharp corner, this would cause the 4WS vehicle a higher tendency to roll. This support the statements made by 4WS users [15][19] (mentioned in *Section 1.2: Problem Statements*) where while driving 4WS vehicle at relatively high speed having a sharp turn, the driver feels that the vehicle have a tendency to swinging away. This condition can be referred to, as for in this study, when the steering-wheel been turned more than 240^o while having a sharp turn at relatively high speed.

4.6 Sideslip Angles

As refer to earlier earlier, certain components are essential to gained the sideslip angles for the vehicle and its wheel, i.e. the position of the point I_m with respect to the front and rear axle, and to the car's center of gravity. Therefore, by using MATLAB, these components are considered for the sideslip angles calculations.

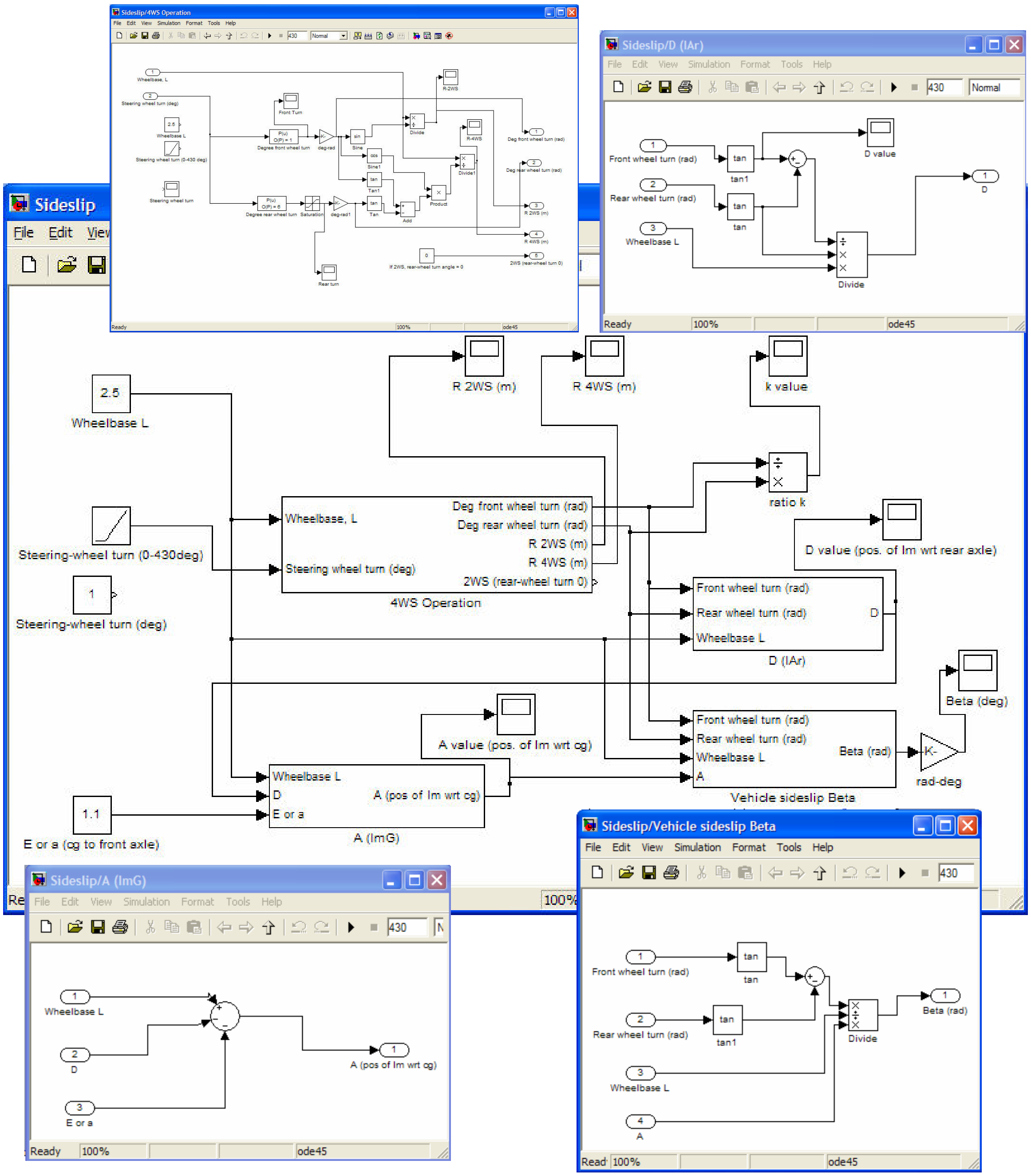


Figure 4.16: MATLAB Simulation Modeling for sideslip angles

4.6.1 Vehicle Sideslip Angle, β

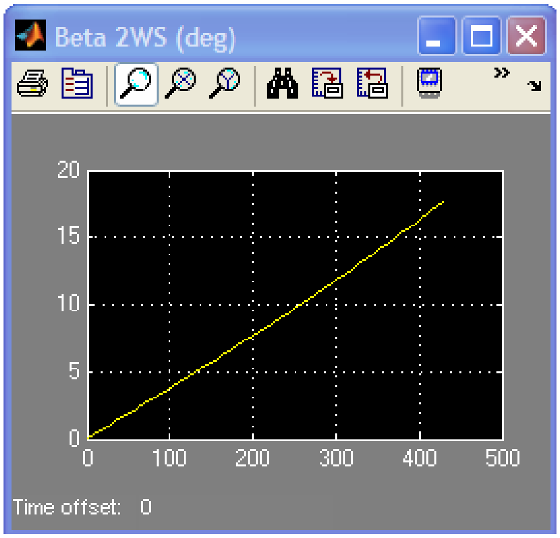


Figure 4.17: 2WS vehicle sideslip angle versus steering-wheel turn degree

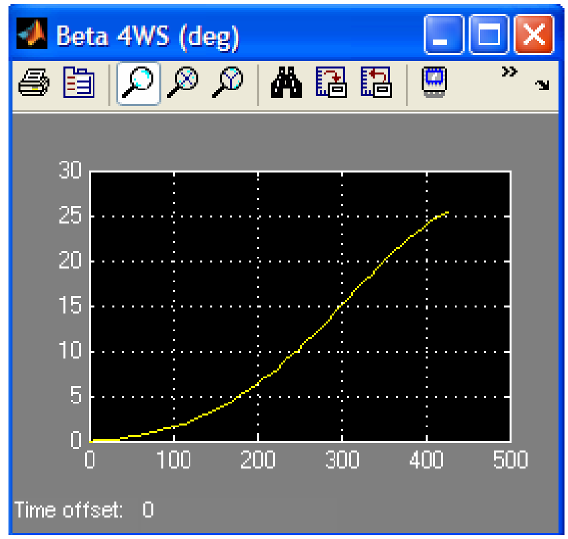


Figure 4.18: 4WS vehicle sideslip angle versus steering-wheel turn degree

As shown in above Figure 4.17, for 2WS system, the vehicle sideslip angle is directly proportional to the steering-wheel turning degree; starting from 0^o sideslip (at 0^o steering-wheel turns) up until 17.7^o sideslip (at 430^o steering-wheel turns)

Whereas for 4WS system (Figure 4.18), the relations of the steering-wheel turns and the vehicle sideslip angle is in a polynomial relations, where from 0^o sideslip (at 0^o steering-wheel turns) up until approx. 13^o sideslip (at 280^o steering-wheel turns), the vehicle sideslip is increase with an increasing rate, then, from 13^o until 25.5^o sideslip (at 430^o steering-wheel turns), the vehicle sideslip increase with decreasing rate.

At 238^o steering-wheel turns, both 2WS and 4WS having the same value of 9.2^o vehicle sideslip angle. This is due to that at 238^o steering-wheel turns, the 4WS system is having a 0^o rear-wheel turn angle, which is assumed to be in that instance, the 4WS is the same as 2WS system.

Thus, below 238^o steering-wheel, the 4WS is having less sideslip than 2WS, which means less tendency for the vehicle to turns and vice versa.

This is a good characteristic for 4WS where below 238^o steering-wheel, it is less likely to loss stability at high speed (since driver rarely need to turn the steering wheel too much when driving at high speed on road), but at low speed, considering when making a U-turn or parking, where the steering wheel turned more than 238^o, the 4WS vehicle having a higher tendency to turns and giving a better response.

4.6.2 Wheel Sideslip Angles, α_f and α_r

4.6.2.1 2WS

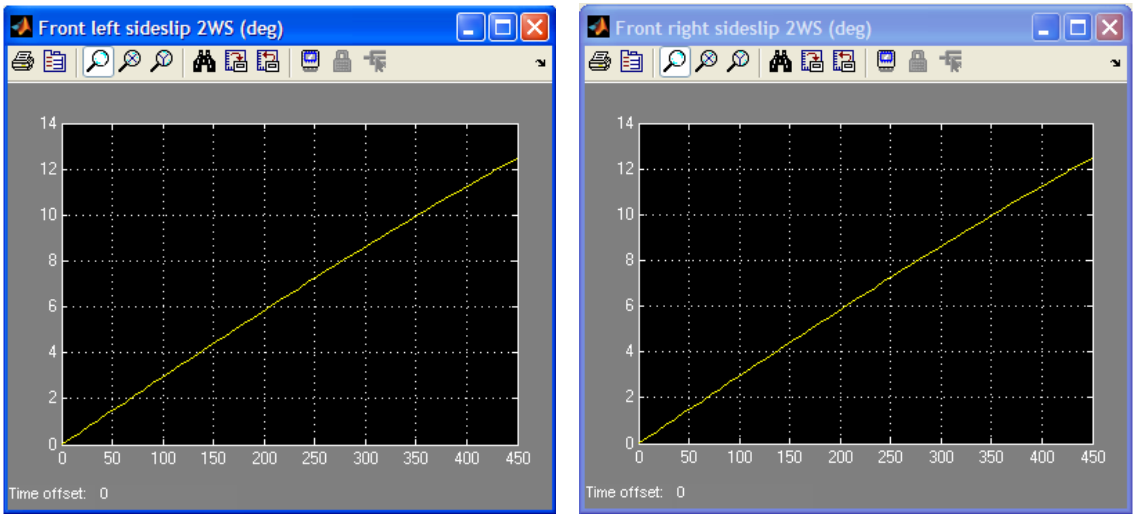


Figure 4.19: Front left and right wheels side-slip angles (deg) of 2WS vehicle

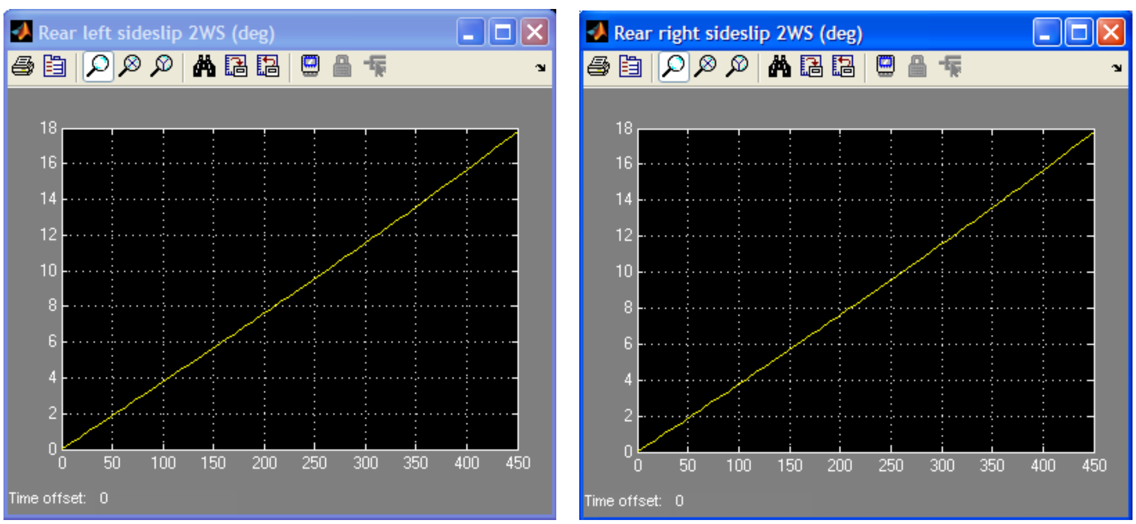


Figure 4.20: Rear left and right wheels side-slip angles (deg) of 2WS vehicle

4.6.2.2 4WS

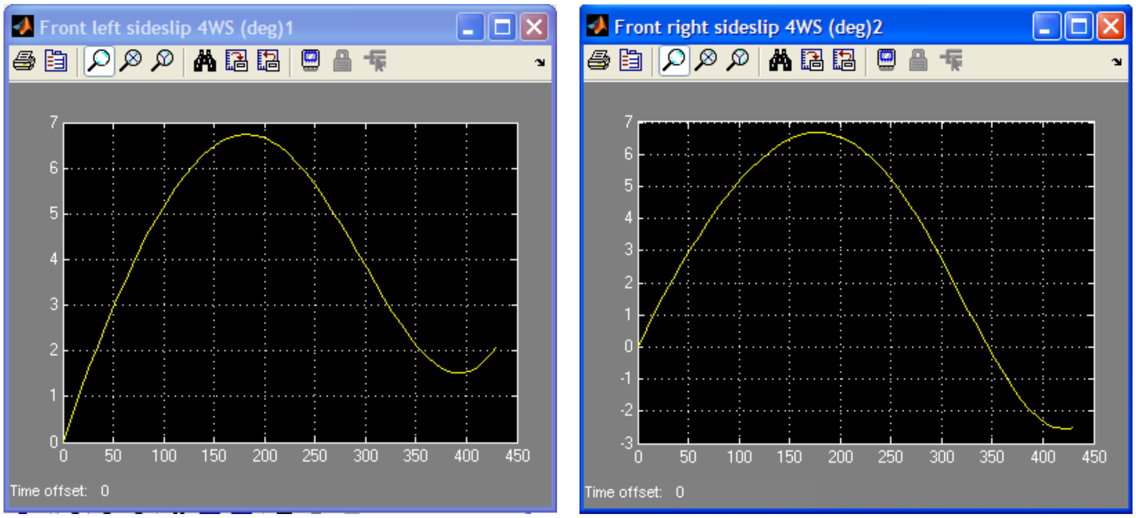


Figure 4.21: Front left and right wheels side-slip angles (deg) of 4WS vehicle

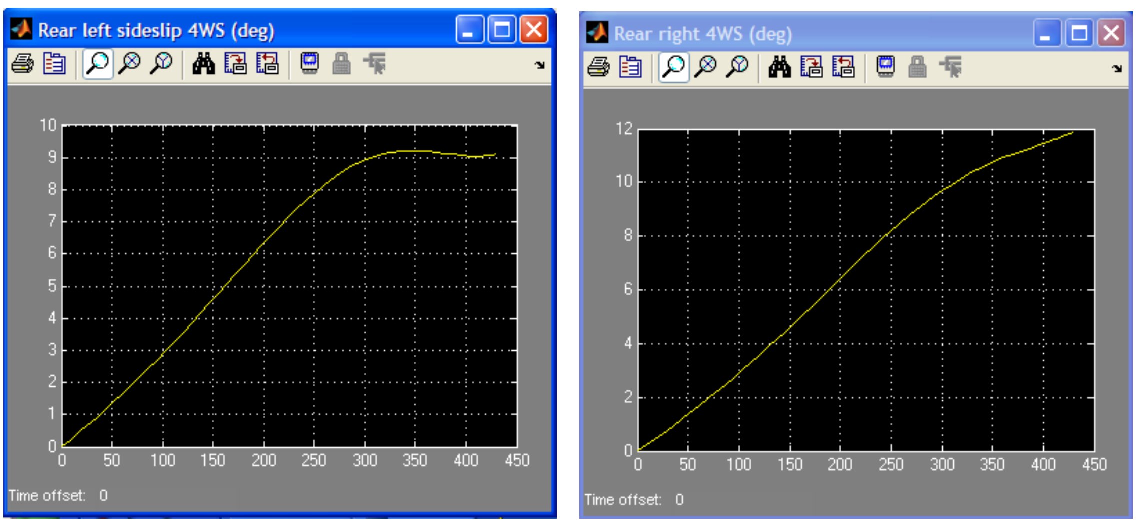


Figure 4.22: Rear left and right wheels side-slip angles (deg) of 4WS vehicle

In Figure 4.19, 4.20, 4.21, and 4.22, it show the wheels side-slip angles for both 2WS and 4WS. The pattern for 2WS is an almost straight line while for 4WS, the pattern differs where the front wheels show sinusoidal correlations and the rear wheel increase with different rates.

In Figure 4.21 where the side-slip angles of the front wheels is peak in range of 150^o to 200^o steering-wheel turns is because at that instant, the 4WS vehicle is reaching the same conditions as 2WS vehicle possess; the 4WS's rear-wheel turn angle reaching zero. That is why the having a curve such that. Then, the value decreases afterward because the 4WS's rear-wheel turn is reaching to its maximum utilization; where the rear-wheel turn angle reaching 5^o (maximum turn angle), affecting the effect of side-slip angle of the front-wheels.

The point of interest here is that the maximum side-slip angles for both 2WS and 4WS. As for 2WS, the wheels side-slip angles can be as high as 12.2^o for front wheels and 18^o for rear wheels. For 4WS, the maximum wheels side-slip angles only reach around 6.8^o for front wheels and 9^o to 12^o degrees for rear wheels.

This data shows that 4WS wheels are having lower side-slip angles range compared to the 2WS wheels. Lower side-slip angles means that the 4WS wheels is having lower tire wear rate [3][8][9], giving the wheels for 4WS a higher time period before the tire completely wear off if compared to 2WS vehicle's wheels - desirable.

CHAPTER 5
CONCLUSION AND RECOMMENDATION

5.1 CONCLUSION

As a conclusion, this project has successfully achieved the objectives of this study.

The reason why both 2WS and 4WS vehicle been studied in this project is to study the effect of rear-wheel turn angle to 4WS vehicle's performances. Since the project need a datum to refer to for comparing either the performance is an advantage or disadvantage, thus, 2WS is included in this study since 2WS does not utilize the rear-wheel turn angles (zero rear-wheel turn angles)

The corresponding rear-wheel-turn angle with respect to the front-wheel-turn angle for 4WS vehicle has been determined. This relations is referred to the infamous car's manufacturers; Honda [2][8][10][17][30] (Figure 2.2).

Data and figures that shows the performances of 4WS that influenced by the rear-wheel-turn angle are also provided in *Chapter 4* where all the results are generated by using MATLAB. The gained data are discussed and justified by through the references made with latest literature reviews. The outputs been studied on how exactly that the rear-wheel-turn angle for 4WS actually affect the vehicle's performances.

The differences of 2WS and 4WS vehicles actually show the advantages and disadvantages of the rear-wheel influences onto the 4WS vehicle by using 2WS as the datum.

In this project, by using the car model defined in *Table 4.1*, when the steering-wheel is turned lower than 240^o, the 4WS vehicle is having lower turning radius, yaw velocity, lateral velocity, and vehicle side-slip angle compared to 2WS vehicle. This shows 4WS vehicle is having higher stability than 2WS vehicle when steering-wheel angle is less than 240^o, but lower response. This condition can be applied at high speed where driver's handling for steering-wheel at high speed generally lower than 240^o. Thus, 4WS is possessing higher stability than 2WS when driving at high speed – desirable.

Whereas when the steering-wheel angle is more than 240^o, the 4WS vehicle is having higher turning radius, yaw velocity, lateral velocity, and vehicle side-slip angle compared to 2WS vehicle. This shows that 4WS vehicle is having higher maneuverability and higher response compared to 2WS vehicle when the steering-wheel angle is more than 240^o, but lower stability. This condition can be applied at low speed where usually the steering-wheel is required to be turned hugely usually only when the vehicle is making a U-turn or having a side parking. This features of higher response and maneuverability for 4WS when the steering-wheel turned more than 240^o is highly desirable at usually both done at low speed – desirable. However, considering that if the car is making a turn that required the driver to turn more than (in this case) 240^o at relatively higher speed, the driver might feel the vehicle tends to swinging away.

Hence, these conclude the results and findings made for this project that successfully answered all the problem statements made.

Therefore, this project has successfully achieved the objectives of this study.

5.2 RECOMMENDATIONS

Suggested future work for expansion and continuation of this project that will improve the quality is this project are:

i. Applying **more dynamic analysis** referring to the performances of 4WS that influenced by the rear-wheel-turn angles (i.e. lateral forces, weight transfer, inertia, etc). This will give better view on the comparison that can be made between 2WS and 4WS vehicle.

ii. Study 4WS performances at **various** roads' **condition** (i.e. slope, rough road, smooth road, bumpy road, etc) **and** various **situations** of the 4WS vehicle (i.e. making a lane-changing, overtaking other vehicle, etc).

iii. Provide a **higher degree-of-freedom** for the 4WS vehicle model. This will surely improve the data accuracy on how exactly the rear-wheel turn angle affect the 4WS performances.

iv. Providing **more than one vehicle model** (i.e. used two or more vehicle that having different parameters such as mass, track, width, etc). By having more than one vehicle model, the characteristics of the 4WS vehicle can be studied where, such as, determining the critical features of the vehicle to be optimize where it may critically affect the performances of the 4WS vehicle; optimize the advantage effects and minimize the disadvantage effects.

v. Use **recent** established **4WS system software** to do a much complex and detailed studies. By using well-developed software that capable to analyze 4-wheel-steering system, a much complex and detailed studies may possibly be done since MATLAB is hardly to be used for very complex relations. However, MATLAB does have a better basic-understanding. Therefore, make sure that the software shall be understood properly (its basic principle of analysis ability).

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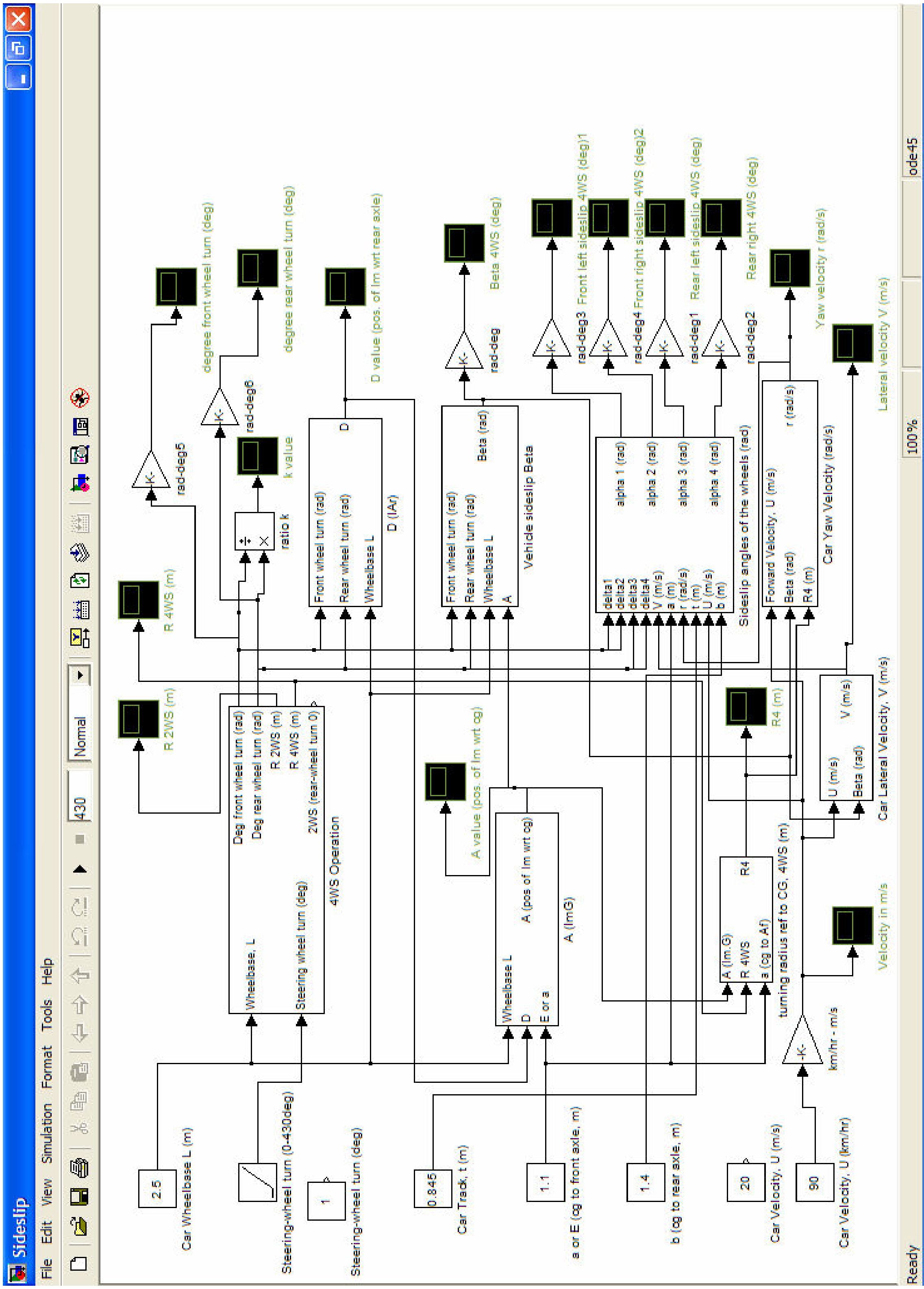
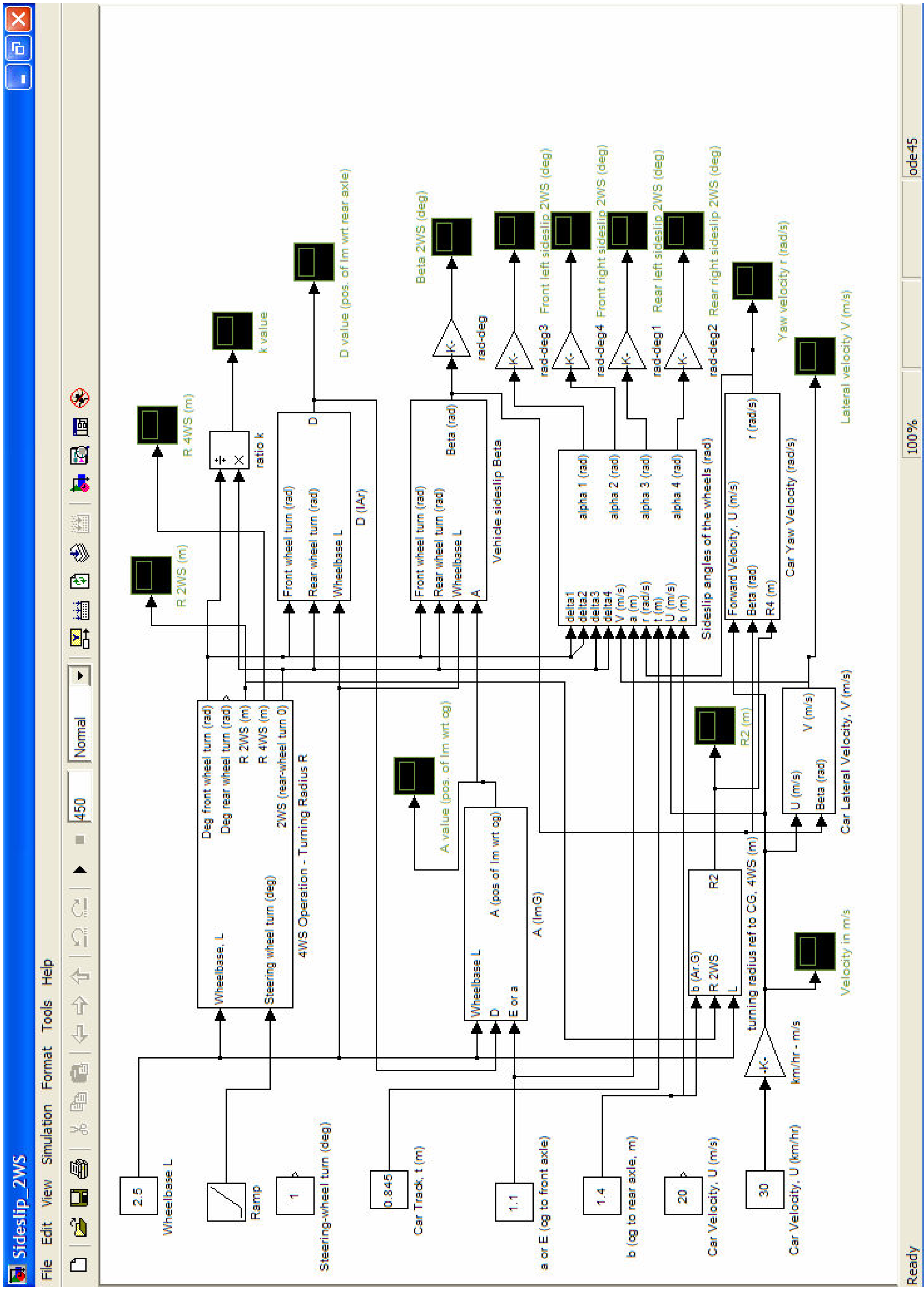
APPENDICES

4WS Full MATLAB Simulation Modeling – Main System

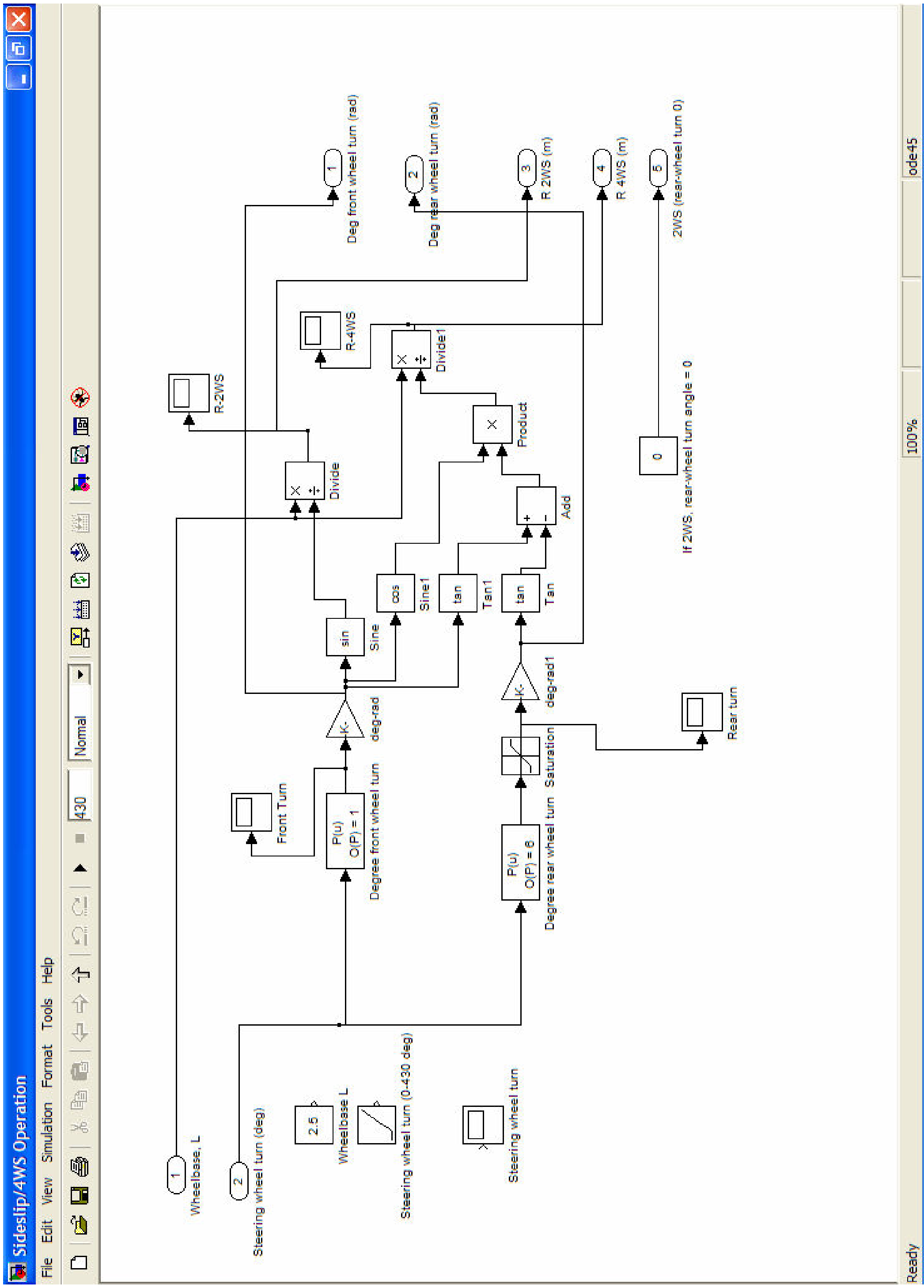


2WS Full MATLAB Simulation Modeling – Main System

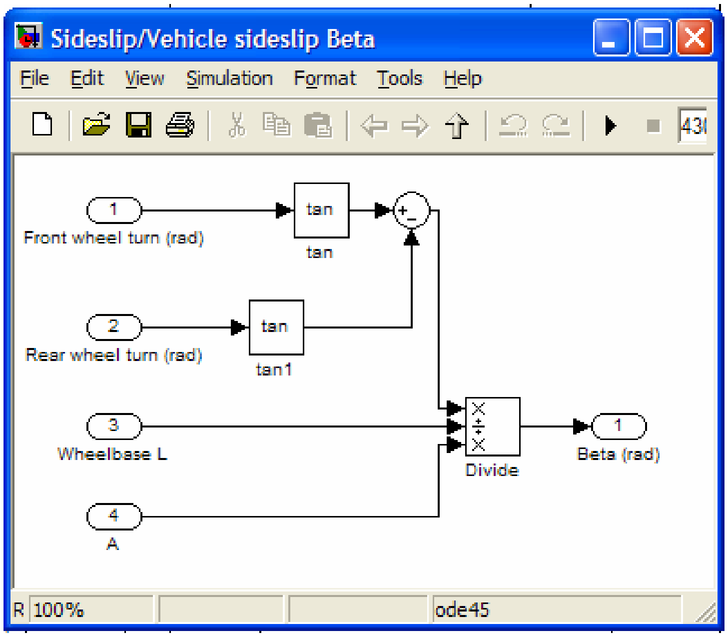


Subsystem – 4WS / 2WS Operation

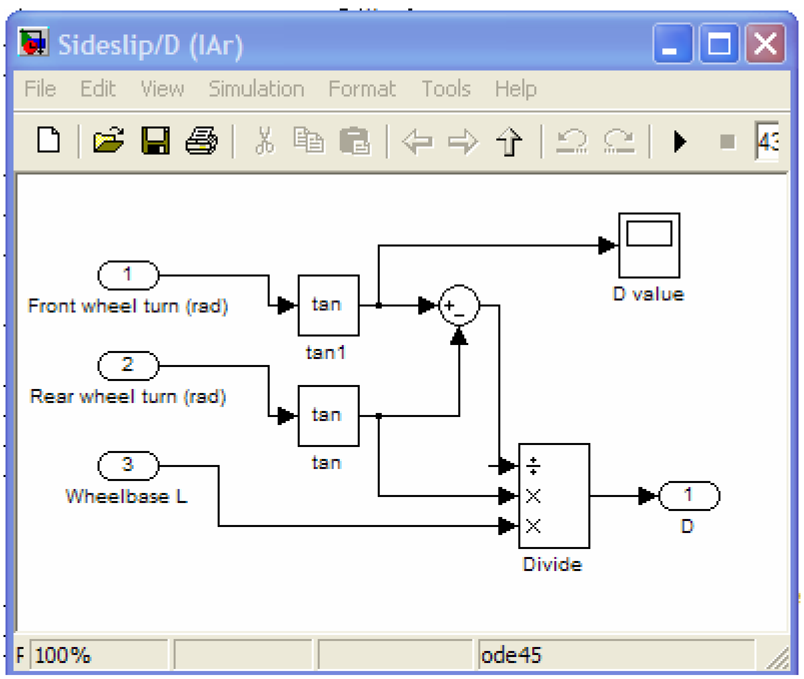
(Determine turning radius, R and wheel turning angle, δ)



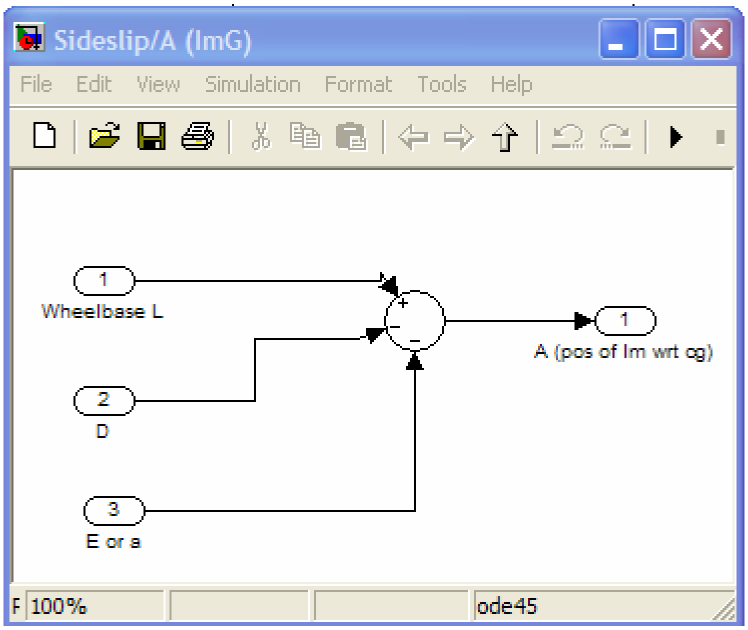
Subsystem – Vehicle sideslip Beta, β



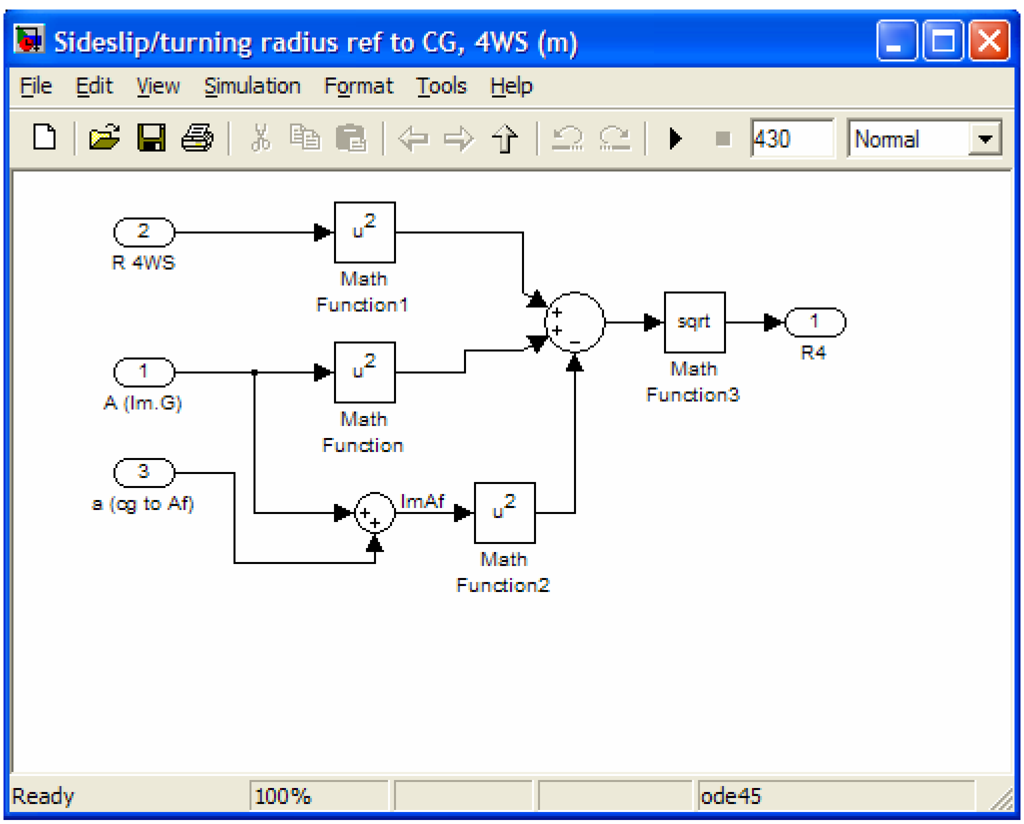
Subsystem – D (I.A$_r$) (Determine the position of I_m w.r.t. the rear axle)



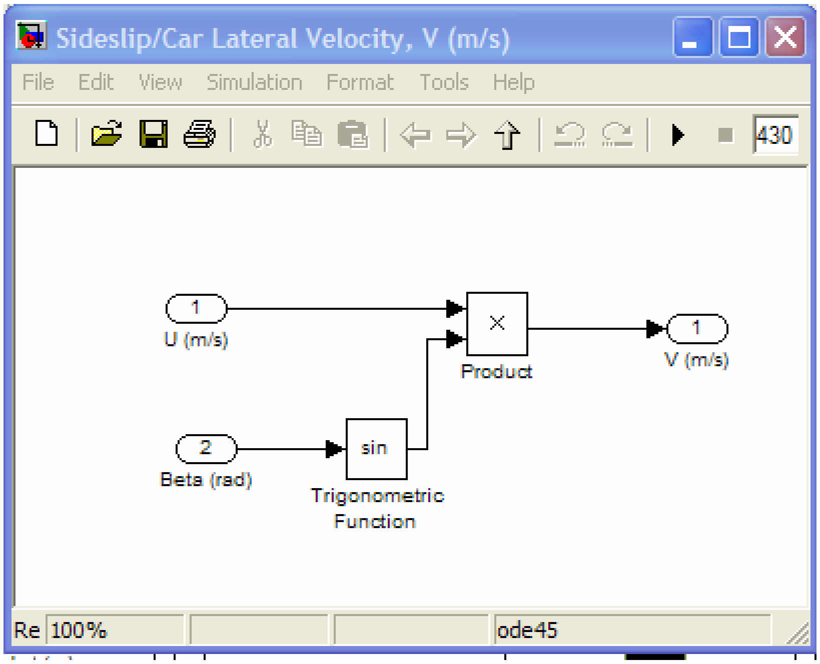
Subsystem – A (I_m.G) (Determine position of I_m w.r.t. the centre-of-gravity)



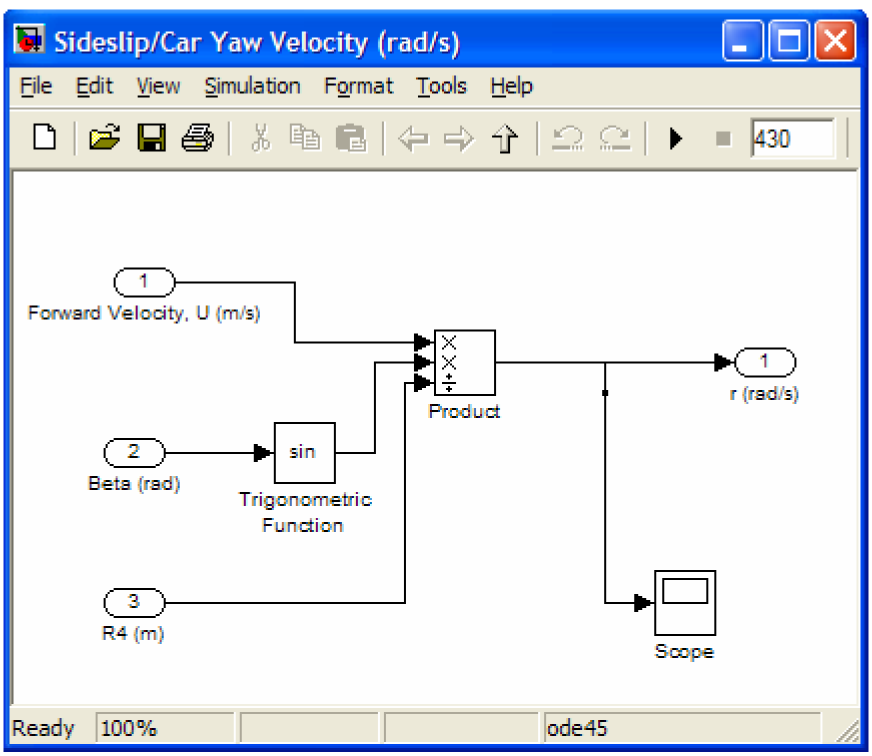
Subsystem – Turning radius, reference to centre-of-gravity



Subsystem – Car lateral velocity, *V* (m/s)



Subsystem – Car yaw velocity, *r* (rad/s)



Subsystem – Sideslip angles of the wheels, α (rad)