Parametric Study on Passive Suspension of Light Rail Vehicle (LRV)

by

Mohd Izwan Hadi b. Ismail @ Mansor

Dissertation submitted in partial fulfilment of

the requirements for the

Bachelor of Engineering (Hons)

(Mechanical Engineering)

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CERTIFICATION OF APPROVAL

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Approved by,

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(Dr. Ir. Setyamartana Parman)

UNIVERSITI TEKNOLOGI PETRONAS

TRONOH, PERAK

December 2008

CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

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MOHD IZWAN HADI B. ISMAIL @ MANSOR

ABSTRACT

This research project proposes the passive suspension of light rail vehicle (LRV) to be studied parametrically. The system is modelled as a multi-body dynamic model and used passive systems suspension which is the simplest and common type of suspensions to isolate disturbance from the track to the car-body. One model of light rail vehicle (LRV) will be selected and form a mathematical model for analysis purpose. The equation of motion will be develop from the mathematical model and then from the equation of motion the matrices of the LRV systems are derived, and it will be translated it into MATLAB language. The response of the system under varying parameters of suspension will be simulated computationally by using MATLAB. In this research, the parameters are divided in 5 cases, which is for case 0 is using the nominal parameters, case 1 the nominal parameters is reduced by 20%, case 2 the nominal parameter is reduced by 50%, case 3 the nominal parameters is increased by 20% and for case 4 the nominal parameters is increased by 50%. For the results, the maximum suspension deflections and the stability of each car-body on random gradient track will be discussed.

ACKNOWLEDGEMENT

First and foremost, I would like to praise Allah the Almighty for His guidance. Though difficulties occurred, His guidance gave me the chance to still complete this final year report successfully. Here, I would like to use this special opportunity to express my heartfelt gratitude to everyone that has contributed for finish up my final year project.

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CHAPTER 1

INTRODUCTION

1.1 Background of Study

The major functions of suspension for light rail vehicles (LRV) are to support the car-body and bogie, to isolate the forces generated by the track unevenness at wheels, and to control the attitude of the car-body with respect to the track surface for providing ride comfort. Accordingly, the suspension influences vehicle ride comfort and stability, and should be designed to isolate the car-body from track roughness, and to maintain the suitable space between the track and car-body.

1.2 Problem Statement

1.2.1 Problem Identification

The stability of the car-body for LRV needs to be done cautiously in order to get a better ride comfort. This research is to simulate whether the passive suspension deflection that runs onto uneven track within the clearance between the car-body and the track or not.

1.2.2 Significance of Project

The stability of the car-body and suspension deflections within their allowable clearance when light rail vehicle (LRV) runs onto the uneven track condition can be satisfied by selection the suitable parameters of suspension of each car-body. This research will be useable in order to determine the selection of the passive suspension parameters for the LRV.

1.3   Scope of Study and Objective

Scopes of study for this project are the selection of the LRV with only three car-bodies, uneven track is define as sinusoidal wave, and the pitching motion for the carboy only be study in two-dimensional (2-D) movement which is clockwise and counter-clockwise. The project involves the derivation of equation of motion for LRV, develop a MATLAB code based on the equation of motion, and computer simulation for the dynamics of LRV under varying parameters of its passive suspension. This project will cover the simulation and analysis by using MATLAB.

The objectives of this project are:

- ➢ To study the maximum displacement of each car-body.
- ➢ To study on the car-body's stability.
- ➢ To identify the best parameters for the suspension of LRV.

CHAPTER 2
LITERATURE REVIEW

2.1 Vibration and Vibration Systems

Vibration is any motion that repeats itself after an interval of time is called vibration or oscillation. The swinging of pendulum and the motion of plucked string are type of examples of vibration. The theory of vibration deals with the study of oscillatory motions of bodies and the forces associated with them [2].

A vibratory system, in general, include a means for storing potential energy (spring or elasticity), means for storing kinetic energy (mass or inertia), and means by which energy is gradually lost (damper). The vibration of a system involves the transfer of its potential energy to kinetic energy and kinetic energy to potential energy, alternately [2].

2.2 Degree of Freedom (DOF)

The minimum number of independent coordinate required to determine completely the positions of all parts of a system at any instant of time defines the degree of freedom of the system. In mechanics, degrees of freedom (DOF) are the set of independent displacements that specify completely the displaced or deformed position of the body or system. This is a fundamental concept relating to systems of moving bodies in mechanical engineering, aeronautical engineering, robotics, automotive engineering, locomotive engineering, structural engineering, etc.

2.3 D'Alembert Principle

The equation of motion derive by Newton second law of motion $F(t) = m\ddot{x} \text{ or } M(t) = J\ddot{\theta}$ can be written as $F(t) - m\ddot{x} = 0 \text{ or } M(t) - J\ddot{\theta} = 0$. These equations can be considered equilibrium equation provided that $-m\ddot{x} \text{ and } -J\ddot{\theta}$ are treated as a force and a moment. This fictitious force or moment is known as inertia force or inertia moment and the artificial state of equilibrium is known as dynamic equilibrium. This principle is called D'Alembert's principle [2].

2.4 Bogie

2.4.1 Role of Bogie

Railcar bogies are very important in safe railway operations and perform the following functions [5]:

➢ Support railcar body firmly

➢ Run stably on both straight and curved track

➢ Ensure good ride comfort by absorbing vibration generated by track irregularities and minimizing impact of centrifugal forces when train runs on curves at high speed

➢ Minimize generation of track irregularities and rail abrasion

2.4.2 Bogie Configuration

2.4.2.1 Non-Articulated and Articulated bogies

Bogies can be classified into non-articulated and articulated types according to the suspension. Two non-articulated bogies usually support one railcar body (Figure 2.1a), but one articulated bogie supports the back end of the forward car and the front end of the rear car (Figure 2.1b) as seen in the Spanish Talgo, French TGV, and some express trains of the Odakyu Line in suburban Tokyo. Although the articulated

bogie has some disadvantages, such as a complex structure, increased axle load due to the support of one body by one bogie, and difficult maintenance, it offers various advantages including a lower centre of gravity, better ride comfort because car ends do not overhang bogies, and less effect of running noise on passengers because seats are not over bogies [5].
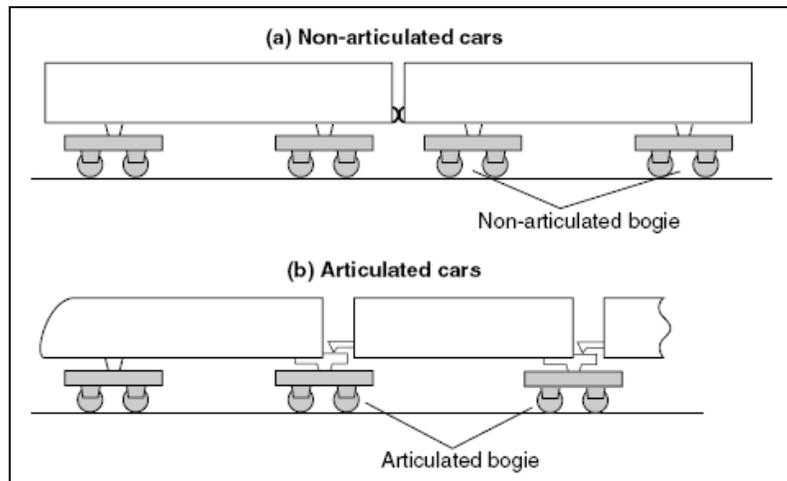

Figure 2.1 Non-articulated bogie and articulated bogie
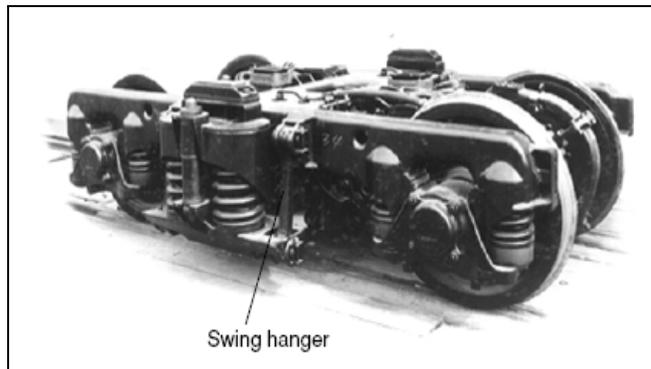
Figure 2.2 Swing hanger bogie
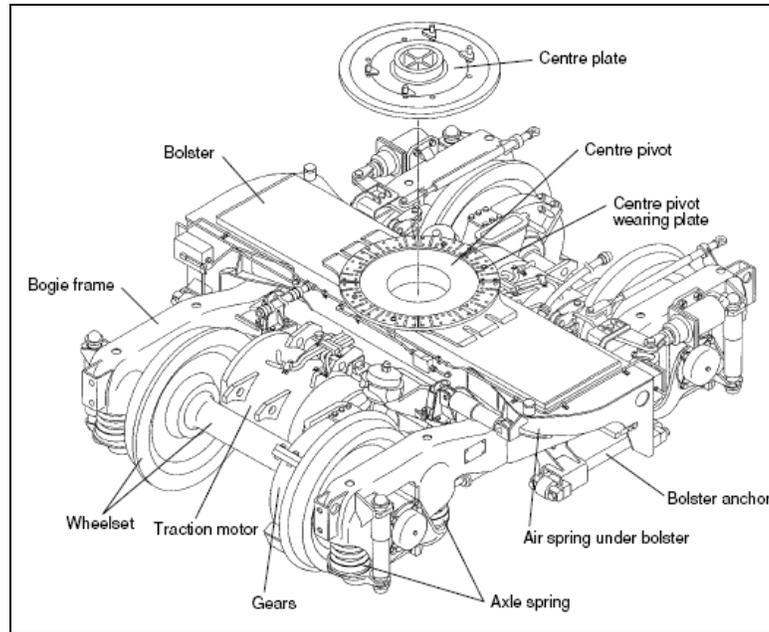
Figure 2.3 Lateral stiffness bolster bogie

2.4.2.2 Bolster and Bolsterless Bogies

Bolster and bolsterless bogies (Figure 2.4) are differentiated by their suspension gear. The bolster bogie was developed first. A fundamental characteristic of the bogie is that it must rotate relative to the body on curves, while retaining high rotational resistance during high-speed running on straight sections in order to prevent wheelset hunting (Figure 2.5) that reduces ride comfort. To achieve these characteristics, the bolster bogie has a centre pivot that serves as the centre of rotation, and side bearers that resist rotation.

In the 1980s, a bolsterless bogie was commercialized to improve performance by reducing the number of parts and the bogie weight (the bolsterless bogie for Shinkansen was commercialized in the 1990s). In recent years, most narrow-gauge and Shinkansen cars use the bolsterless bogie, which permits rotational displacement on curves through the horizontal deformation of bolster springs (also known as secondary suspension springs) on both sides of the bogie. Rotation is resisted by longitudinal anti-yawing dampers on both sides of the bogie, resulting in better rotational resistance than conventional side bearers [5].

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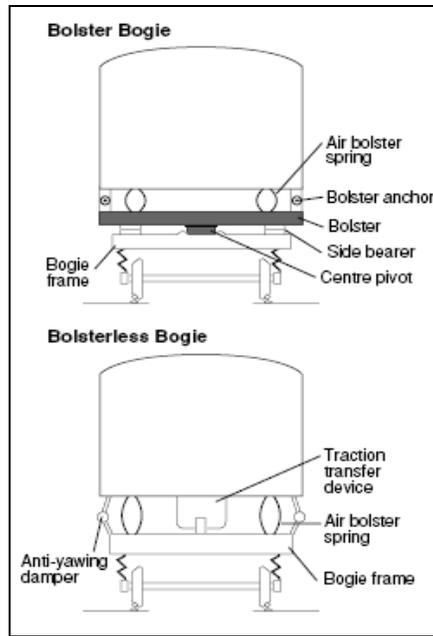
Figure 2.4 Bolster and bolsterless bogies
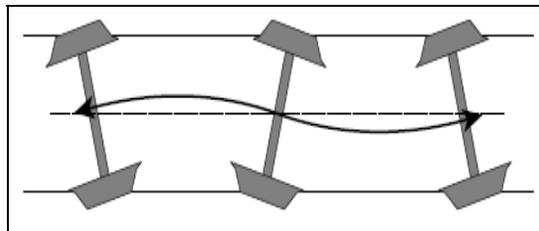
Figure 2.5 Wheelset hunting

2.4.3   Basic Bogie Elements

2.4.3.1 Suspension Gear

The suspension gear (bolster spring, traction transfer device, anti-yawing damper, and lateral damper) plays an important role in supporting the body, allowing the bogie to rotate relative to the car body on curves, isolating the body from vibration (including high harmonics) generated by the bogie, and transmitting traction force from the bogie to the body. For these purposes, the bolsterless bogie (Figure 2.6) has air springs that permit large horizontal displacement, as well as a traction transfer device (classified into permanent-link type, the Z-link type and laminated-rubber type) at the virtual rotational centre of the bogie for transmitting the tractive force to the car. In addition, the bolsterless bogie used for express trains and Shinkansen has anti-yawing dampers (Figure 2.4) at the outer side of the side beam of the body and

bogie frame (parallel to the side beam) to prevent wheelset hunting that reduces ride comfort [5].

Coil springs were originally used as bolster springs to support the body, but in the 1960s, air springs were commercialized for railcars in Japan and were used for high-speed trains. Since the 1980s, they have been used for commuter and short-distance trains in order to take advantage of their improved ride comfort and ability to maintain body height [5].
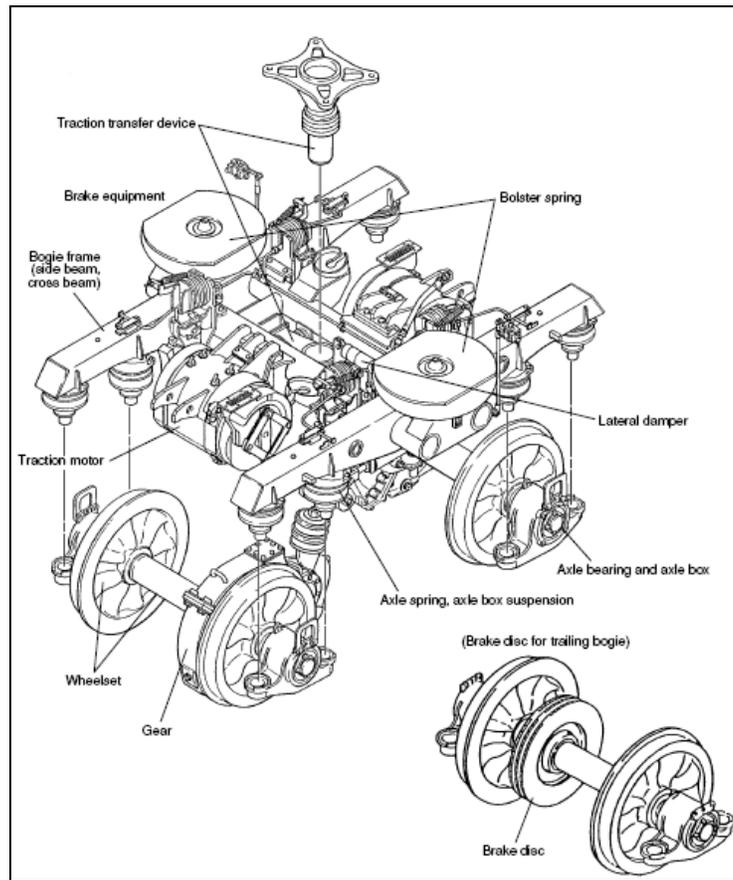
Figure 2.6 Bolsterless bogie (model DT50)

2.4.3.2 Axle Box Suspension

This device supports the axle via the bearing from the bogie frame. It is a critical component determining the running performance of the bogie, ride comfort and bogie frame construction, due to the suspension method and support rigidity. Various designs are used (Figure 2.7). The pedestal swing spring design supports the axle

box using sliders around the pedestal on the bogie frame and was used widely in a variety of railcars of the former JNR. While its construction is simple, the sliders wear with time, creating play in the suspension and cause wheelset hunting, so it is not suitable for high-speed operation. The IS type was developed for Shinkansen and its good cushion rubber stiffness effectively eliminates play between the axle and bogie frame. However, it has several disadvantages, such as increased bogie frame length and severe maintenance requirements for leaf springs. As a result, recent bolsterless bogies for shinkansen use other types, including the coil spring with cylindrical laminated rubber type. Similarly, narrow-gauge railcars are increasingly using the conical laminated rubber type, the roll rubber type (serving both as the axle box suspension and axle spring) and axle beam type in order to simplify construction and reduce manufacturing costs [5].
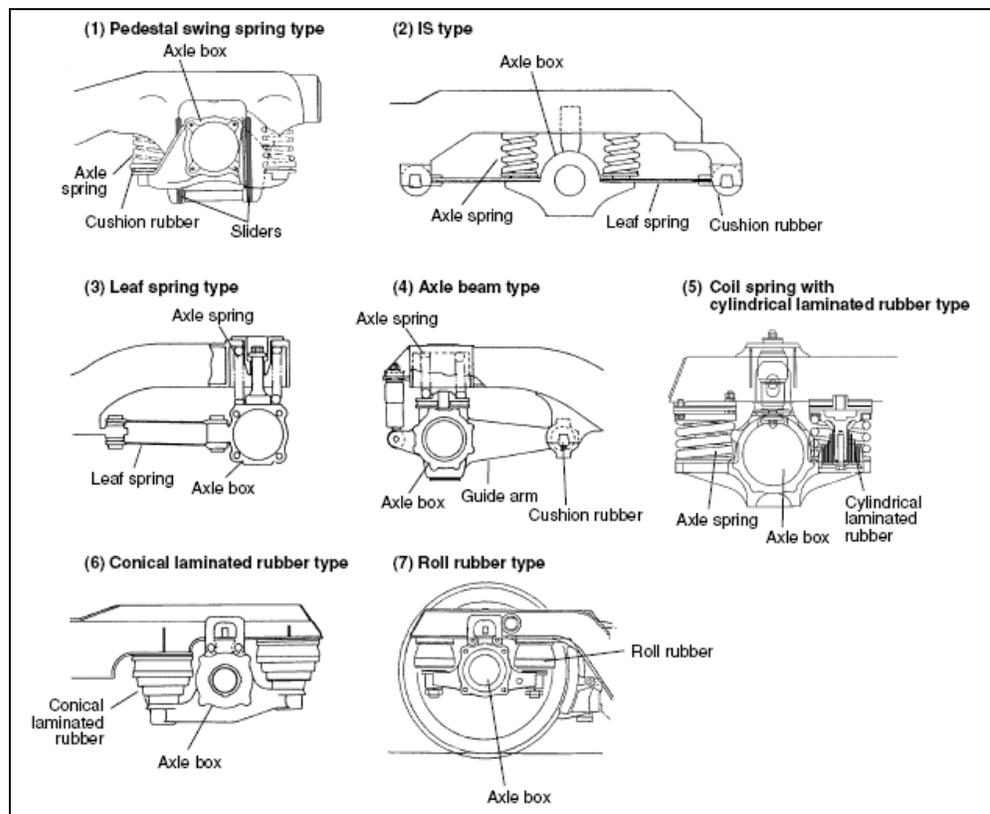
Figure 2.7 Various axle box suspensions

2.5   Tracks

Train operation and performance are dependent to a great extent on the track condition since train equipment and lading damage can result from improperly constructed or deteriorated track. The criteria for the use different weight of rail are based on maximum axle loads and tonnage moved over a particular track section. Typical railway gradients are considered for checking the suspension deflections due to the deterministic input of the track [3].

2.6   Mathematical Modeling

The purpose of mathematical modeling is to represent all the important features of the system for the purpose of deriving the mathematical (or analytical) equation governing the system's behavior. The mathematical model should include enough detail to be able to describe the system in terms of equation without making too complex. The mathematical model may be linear or non-linear, depending on the behavior of the system's component [2].

2.7   Runge-Kutta

Runge-Kutta method is used in order to solve the multi-degree of freedom system beside the method is more accurate than Euler's method. For this project, the fourth order Runge-Kutta method is used. The fourth-order Runge-Kutta method requires four evaluations of the right-hand side per step h. This will be superior to the midpoint method if at least twice as large a step is possible with for the same accuracy.

2.8   MATLAB

MATLAB (Matrix Laboratory) is a high-performance interacting data-intensive software environment for high-efficiency engineering and scientific numerical calculation. Application include heterogenous simulations and data-intensive analysis of very complex systems and signals, comprehensive matrix and array manipulations in numerical analysis, finding roots of polynomial, two and three-dimensional plotting and graphic different coordinate systems, integration and differentiation, signal processing, control, identification, symbolic calculus, optimization, etc. the goal of Matlab is to enable the user to solve a wide spectrum of analytical and numerical problem using matrix-based methods, attain excellent interfacing and interactive capabilities, compile with high-level programming languages, ensure robustness in data-intensive analysis and heterogenous simulations, provide easy access to and straightforward implementation of state-of-the-art numerical algorithms, guarantee graphic features, etc [4].

CHAPTER 3

METHODOLOGY / PROJECT WORK

3.1   Procedure Identification

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┌─────────────────────────────────────────────┐
│  Information gathering and literature review  │
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                      │
                      ▼
┌─────────────────────────────────────────────┐
│            Selecting the LRV model            │
└─────────────────────────────────────────────┘
                      │
                      ▼
┌─────────────────────────────────────────────┐
│         Derive equation of motion from        │
│             mathematical modelling            │
└─────────────────────────────────────────────┘
                      │
                      ▼
┌─────────────────────────────────────────────┐
│        Develop and writing MATLAB code for    │
│                 LRV's dynamics                │
└─────────────────────────────────────────────┘
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                      ▼
┌─────────────────────────────────────────────┐
│      Simulate computationally the dynamics of │
│         LRV under varying parameters of its   │
│                passive suspension             │
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│       Analysing result and data interpretation│
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│      Documentation of report and presentation │
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3.2 Analysis

3.2.1 LRV Configuration

The LRV considered is composed of two power-car-bodies with two power-bogies (each equipped with two traction motors), one trailer-car-body with one trailer-bogie, as shown in Figure 12. Each end car-body is connected with one power-bogie by the use of a bolster, while the middle car-body is directly connected with one bolsterless trailer-bogie. An articulation is used to connect the power-car-body and the trailer-carbody. The low floor area is about 70% of the total floor area. The track gauge is 1067 mm. The primary suspension consists of rubber chevrons and the secondary suspension comprises coil springs and dampers. Full load mass for power car-body and trailer car-body is 10820 kg and 4470 kg respectively.
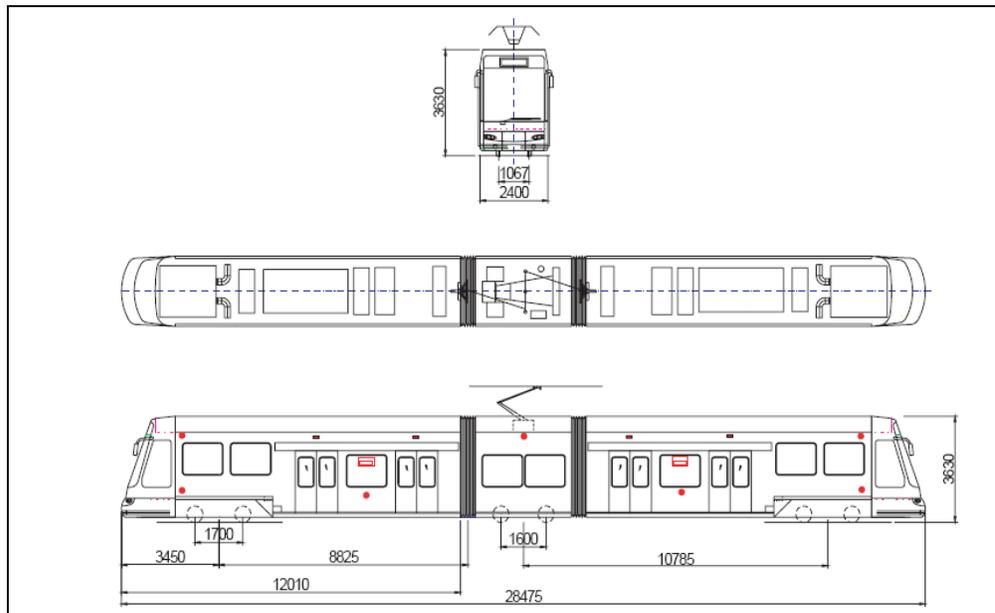

Figure 3.1 Layout of LRV [1]

3.2.2 LRV Mathematical Modeling

Based on LRV configuration in section 3.2.1, Figure 3.2 below is the mathematical model for the LRV: (see the Appendix 1 for the unknown variables reference)
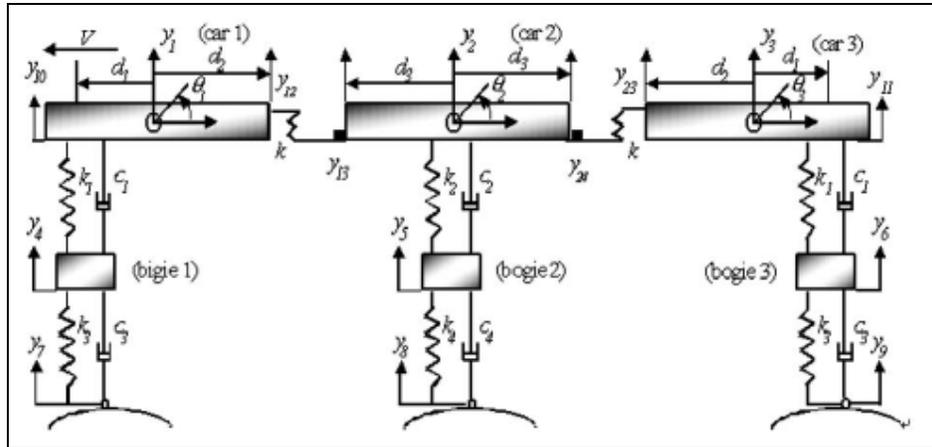

Figure 3.2 Multi-body dynamic model of LRV [1]

3.2.3 LRV Equation of Motion

From the mathematical model in figure 3.2, there are 2 degree of freedoms for each car-body which is bounce and pitch motions, and 1 degree of freedom for each bogie which is bounce motion only. Overall, 9 degree of freedoms will be developed in this mathematical model. The force in secondary suspension between car-body and bogie is due to the relative displacement of suspension mounting. Also the force in articulation between car-bodies is due to relative displacement of articulation. The articulation is modelled as a spring with vertical deflection capacity. Using the d' Alembert force method [2], the equation of motion derived as follows from each of free body diagrams:

The free-body-diagram of car-body 1 is shown in figure 3.3.

Figure 3.3 FBD for car-body 1

By seeing this figure, the equation for bounce motion (Figure 3.3) cab be written as folows:

$$m_p\ddot{y}_1 + c_1\dot{y}_1 - c_1 d_1\dot{\theta}_1 - c_1\dot{y}_4 + (k_1 + k)y_1 - ky_2 + (kd_2 - k_1 d_1)\theta_1 + kd_3\theta_2 - k_2 y_4 = 0 \qquad (3.1)$$

and equation for the pitch motion (Figure 3.3), its equation is:

$$I_p\ddot{\theta}_1 - c_1 d_1\dot{y}_1 + c_1 d_1^2\dot{\theta}_1 + c_1 d_1\dot{y}_4 + (kd_2 - k_1 d_1)y_1 - kd_2 y_2 + (k_1 d_1^2 + kd_2^2)\theta_1 + kd_2 d_3\theta_2 + k_1 d_1 y_4 = 0 \qquad (3.2)$$

Figure 3.4 FBD for car-body 2

Figure 3.4 shows the FBD of car-body 2. Based on this figure, the equation for bounce motion is as follows:

$$m_t \ddot{y}_2 + c_2 \dot{y}_2 - c_2 \dot{y}_5 + (k_2 + 2k)y_2 - ky_1 - ky_3 - kd_2\theta_1 + kd_2\theta_3 - k_2 y_5 = 0 \tag{3.3}$$

and the equation for pitch motion (Figure 3.4) can be expressed as:

$$I_t \ddot{\theta}_2 + kd_3 y_1 - kd_3 y_2 + kd_2 d_3 \theta_1 + 2kd_3^2 \theta_1 + kd_2 d_3 \theta_3 = 0 \tag{3.4}$$
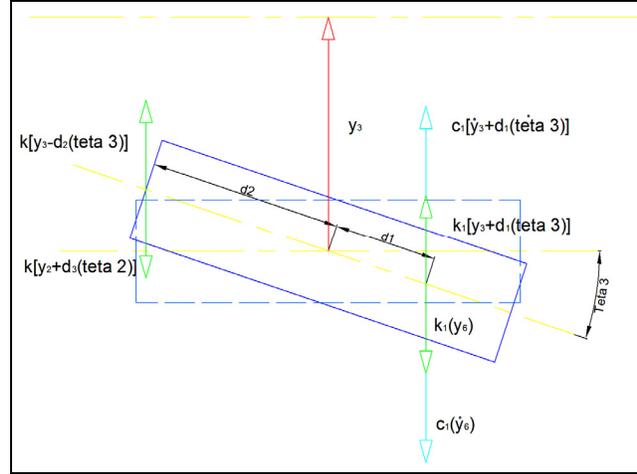

Figure 3.5 FBD for car-body 3

The FBD od car-body 3 can be seen in figure 3.5. Then, the equation for bounce motion can be written as:

$$m_p \ddot{y}_3 + c_1 \dot{y}_3 + c_1 d_1 \dot{\theta}_3 - c_1 \dot{y}_6 + (k_1 + k)y_3 - ky_2 + (k_1 d_1 - kd_2)\theta_3 - kd_3\theta_2 - k_1 y_6 = 0 \tag{3.5}$$

and for the pitch motion , its equation is as follows:

$$I_p \ddot{\theta}_3 + c_1 d_1 \dot{y}_3 + c_1 d_1^2 \dot{\theta}_3 - c_1 d_1 \dot{y}_6 + (k_1 d_1 - kd_2)y_1 + kd_2 y_2 + (k_1 d_1^2 + kd_2^2)\theta_3 + kd_2 d_3 \theta_2 - k_1 d_1 y_6 = 0 \tag{3.6}$$

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Figure 3.6 FBD for bogies; from left bogie 1, bogie 2 and bogie 3

The FBD for the three bogies are shown in figure 3.6, the equations for bounce motion are as follows:

1) $m_{pb}\ddot{y}_4 - c_1\dot{y}_1 + c_1 d_1\dot{\vartheta}_1 + (c_1 + c_3)\dot{y}_4 - k_1 y_1 + k_1 d_1\theta_1 + (k_1 + k_3)y_4 = k_3 y_7 + c_3\dot{y}_7$ (3.7)

2) $m_{tb}\ddot{y}_5 - c_2\dot{y}_2 + (c_2 + c_4)\dot{y}_5 - k_2 y_2 + (k_2 + k_4)y_5 = k_4 y_8 + c_4\dot{y}_8$ (3.8)

3) $m_{pb}\ddot{y}_6 - c_1\dot{y}_3 - c_1 d_1\dot{\vartheta}_3 + (c_1 + c_3)\dot{y}_6 - k_1 y_3 - k_1 d_1\theta_3 + (k_1 + k_3)y_6 = k_3 y_9 + c_3\dot{y}_9$ (3.9)

Below are overall equations of motion in LRV passive suspension system.

1) $m_p \ddot{y}_1 + c_1 \dot{y}_1 - c_1 d_1 \dot{\theta}_1 - c_1 \dot{y}_4 + (k_1 + k)y_1 - ky_2 + (kd_2 - k_1 d_1)\theta_1 + kd_3\theta_2 - k_2 y_4 = 0$

2) $m_t \ddot{y}_2 + c_2 \dot{y}_2 - c_2 \dot{y}_5 + (k_2 + 2k)y_2 - ky_1 - ky_3 - kd_2\theta_1 + kd_2\theta_3 - k_2 y_5 = 0$

3) $m_p \ddot{y}_3 + c_1 \dot{y}_3 + c_1 d_1 \dot{\theta}_3 - c_1 \dot{y}_6 + (k_1 + k)y_3 - ky_2 + (k_1 d_1 - kd_2)\theta_3 - kd_3\theta_2 - k_1 y_6 = 0$

4) $I_p \ddot{\theta}_1 - c_1 d_1 \dot{y}_1 + c_1 d_1^2 \dot{\theta}_1 + c_1 d_1 \dot{y}_4 + (kd_2 - k_1 d_1)y_1 - kd_2 y_2 + (k_1 d_1^2 + kd_2^2)\theta_1 + kd_2 d_3 \theta_2 + k_1 d_1 y_4 = 0$

5) $I_t \ddot{\theta}_2 + kd_3 y_1 - kd_3 y_2 + kd_2 d_3 \theta_1 + 2kd_3^2 \theta_1 + kd_2 d_3 \theta_3 = 0$

6) $I_p \ddot{\theta}_3 + c_1 d_1 \dot{y}_3 + c_1 d_1^2 \dot{\theta}_3 - c_1 d_1 \dot{y}_6 + (k_1 d_1 - kd_2)y_1 + kd_2 y_2 + (k_1 d_1^2 + kd_2^2)\theta_3 + kd_2 d_3 \theta_2 - k_1 d_1 y_6 = 0$

7) $m_{pb} \ddot{y}_4 - c_1 \dot{y}_1 + c_1 d_1 \dot{\theta}_1 + (c_1 + c_3)\dot{y}_4 - k_1 y_1 + k_1 d_1 \theta_1 + (k_1 + k_3)y_4 = k_3 y_7 + c_3 \dot{y}_7$

8) $m_{tb} \ddot{y}_5 - c_2 \dot{y}_2 + (c_2 + c_4)\dot{y}_5 - k_2 y_2 + (k_2 + k_4)y_5 = k_4 y_8 + c_4 \dot{y}_8$

9) $m_{pb} \ddot{y}_6 - c_1 \dot{y}_3 - c_1 d_1 \dot{\theta}_3 + (c_1 + c_3)\dot{y}_6 - k_1 y_3 - k_1 d_1 \theta_3 + (k_1 + k_3)y_6 = k_3 y_9 + c_3 \dot{y}_9$

The equations show that the motion of the each mass will influence the motion of mass next to it. Therefore the equations can be written in matrix form as:

$$[m]\{\ddot{y}\} + [c]\{\dot{y}\} + [k]\{y\} = \{F\} \qquad (3.10)$$

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where [m], [c], [k], and {F} are called the mass, damping, stiffness, and force vectors matrices respectively, and {y} is generalized coordinate vector define as follows:

$$[m] = \begin{bmatrix} m_p & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_t & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_p & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_p & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_t & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I_p & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m_{pb} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{tb} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{pb} \end{bmatrix}$$

$$[c] = \begin{bmatrix} c_1 & 0 & 0 & -c_1 d_1 & 0 & 0 & -c_1 & 0 & 0 \\ 0 & c_2 & 0 & 0 & 0 & 0 & 0 & c_2 & 0 \\ 0 & 0 & c_1 & 0 & 0 & c_1 d_1 & 0 & 0 & -c_1 \\ -c_1 d_1 & 0 & 0 & c_1 d_1^2 & 0 & 0 & c_1 d_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_1 d_1 & 0 & 0 & c_1 d_1^2 & 0 & 0 & -c_1 d_1 \\ -c_1 & 0 & 0 & c_1 d_1 & 0 & 0 & c_1 + c_3 & 0 & 0 \\ 0 & -c_2 & 0 & 0 & 0 & 0 & 0 & c_2 + c_4 & 0 \\ 0 & 0 & -c_1 & 0 & 0 & -c_1 d_1 & 0 & 0 & c_1 + c_3 \end{bmatrix}$$

$$[k] = \begin{bmatrix} k+k_1 & -k & 0 & -k_1 d_1 + k d_2 & k d_3 & 0 & -k_1 & 0 & 0 \\ -k & k_2 + 2k & -k & -k d_2 & 0 & k d_2 & 0 & -k_2 & 0 \\ 0 & -k & k_1 + k & 0 & -k d_3 & k_1 d_1 - k d_2 & 0 & 0 & -k_1 \\ -k_1 d_1 + k d_2 & -k d_2 & 0 & k_1 d_1^2 + k d_2^2 & k d_2 d_3 & 0 & k_1 d_1 & 0 & 0 \\ k d_3 & 0 & -k d_3 & k d_2 d_3 & 2 k d_3^2 & k d_2 d_3 & 0 & 0 & 0 \\ 0 & k d_2 & k_1 d_1 - k d_2 & 0 & k d_2 d_3 & k_1 d_1^2 + k d_2^2 & 0 & 0 & -k_1 d_1 \\ -k_1 & 0 & 0 & k_1 d_1 & 0 & 0 & k_1 + k_3 & 0 & 0 \\ 0 & -k_2 & 0 & 0 & 0 & 0 & 0 & k_2 + k_4 & 0 \\ 0 & 0 & -k_1 & 0 & 0 & -k_1 d_1 & 0 & 0 & k_1 + k_3 \end{bmatrix}$$

$$\{F\} = [0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ k_3 y_7 + c_3 \dot{y}_7\ \ k_4 y_8 + c_4 \dot{y}_8\ \ k_3 y_9 + c_3 \dot{y}_9]^T$$

$$\{y\} = [y_1\ \ y_2\ \ y_3\ \ \theta_1\ \ \theta_3\ \ \theta_3\ \ y_4\ \ y_5\ \ y_6]^T$$

3.2.4 Runge-Kutta Form

In order to solve the second order linear differential equations of the system observed, the fourth Runge-Kutta methods is selected. Based on these equations of motion, the acceleration vector can be written as

$$\ddot{\vec{y}}(t) = [m]^{-1}(\vec{F}(t) - [c]\dot{\vec{y}}(t) - [k]\vec{y}(t)) \tag{3.11}$$

Assuming the displacements and velocities as unknowns, a new vector is defined as $\vec{Y}(t) = \left\{ \begin{array}{c} \vec{y} \\ \dot{\vec{y}} \end{array} \right\}$ so that

$$\dot{\vec{Y}}(t) = \left\{ \begin{array}{c} \dot{\vec{y}} \\ \ddot{\vec{y}} \end{array} \right\} = \left\{ \begin{array}{c} \dot{\vec{y}} \\ [m]^{-1}(\vec{F} - [c]\dot{\vec{y}} - [k]\vec{y}) \end{array} \right\} \tag{3.12}$$

Rearrange the equation above to obtain

$$\dot{\vec{Y}}(t) = \begin{bmatrix} [0] & [I] \\ -[m]^{-1}[k] & -[m]^{-1}[c] \end{bmatrix} \left\{ \begin{array}{c} \vec{y}(t) \\ \dot{\vec{y}}(t) \end{array} \right\} + \left\{ \begin{array}{c} 0 \\ [m]^{-1}\vec{F}(t) \end{array} \right\} \tag{3.13}$$

That is,

$$\dot{\vec{Y}}(t) = \vec{f}(\vec{Y}, t) \tag{3.14}$$

where,

$$\vec{f}(\vec{Y}, t) = [A]\vec{Y}(t) + \vec{F}(t)$$

$$[A] = \begin{bmatrix} [0] & [I] \\ -[m]^{-1}[k] & -[m]^{-1}[c] \end{bmatrix} \tag{3.15}$$

$$\vec{F}(t) = \left\{ \begin{array}{c} 0 \\ [m]^{-1}\vec{F}(t) \end{array} \right\}$$

With this, the recurrence formula to evaluate $\dot{\vec{Y}}(t)$ at different grid points t_i according to the fourth order Runge-kutta method becomes

$$\vec{Y}_{i+1} = Y_i + \frac{1}{6}[\vec{K}_1 + 2\vec{K}_2 + 2\vec{K}_3 + \vec{K}_4] \tag{3.16}$$

where,

$$\vec{K}_1 = h\vec{f}(\vec{Y}_i, t_i)$$

$$\vec{K}_2 = h\vec{f}(\vec{Y}_i + \frac{\vec{K}_1}{2}, t_i + \frac{h}{2})$$

$$\vec{K}_3 = h\vec{f}(\vec{Y}_i + \frac{\vec{K}_2}{2}, t_i + \frac{h}{2})$$

$$\vec{K}_4 = h\vec{f}(\vec{Y}_i + \vec{K}_3, t_{i+1})$$

MATLAB program in order to solve the differential equation of motion of the system can be seen in Appendix 2.

3.3 Tools / Equipment Required

- MATLAB – For programming, simulation and plotting the result and outcome.
- AutoCAD – For drawing the free body diagram (FBD) of the LRV.

CHAPTER 4
RESULTS AND DISCUSSION

4.1   Case 0

In this study, it will be simulated the dynamics of the LRV in several cases of configuration. In case 0, the nominal parameters of the suspension are used as shown in Table 4.1. The duration of MATLAB simulation is set to be 10 second. The distance between the centre of mass of sequence of car-bodies is, $l = 9$ m. The external force working on the system is the result of the uneven track. In this study, the track is assumed to be sinusoidal with amplitude 5 cm and wavelength 10 m. The velocity of LRV travel is 72 km/h (or 20 m/s), therefore the period of the sinusoidal track will be 0.5 s. The track can be expresses as

$$\lambda = 10 \text{ m}, v = 20 \text{ m/s}$$
$$\lambda = vT \tag{4.1}$$

then,

$$T = 10/20$$
$$= 0.5 \text{ s}$$

where,

$$\omega_n = 2\pi/T \tag{4.2}$$
$$\omega_n = 2(3.1416)/0.5$$
$$= 12.57 \text{ rad/s}$$

therefore,

$$y_7 (t) = 0.05 \sin \omega_n t \tag{4.3}$$
$$y_8 (t) = 0.05 \sin \omega_n (t - l/v) \tag{4.4}$$
$$y_9 (t) = 0.05 \sin \omega_n (t - 2l/v) \tag{4.5}$$

The results show in figure 4.1 to 4.3.

Table 4.1 Suspension parameters for case 0

Items	Parameters
Secondary spring of car 1 and 3 (k1)	560000 N/m
Secondary damper of car 1 and 3 (c1)	29584 Ns/m
Secondary spring of car 2 (k2)	1092000 N/m
Secondary damper of car 2 (c2)	50205 Ns/m
Primary spring of car 1 and 3 (k3)	2400000 N/m
Primary damper of car 1 and 3 (c3)	11883 Ns/m
Primary spring of car 2 (k4)	3864000 N/m
Primary damper of car 2 (c4)	176673 Ns/m
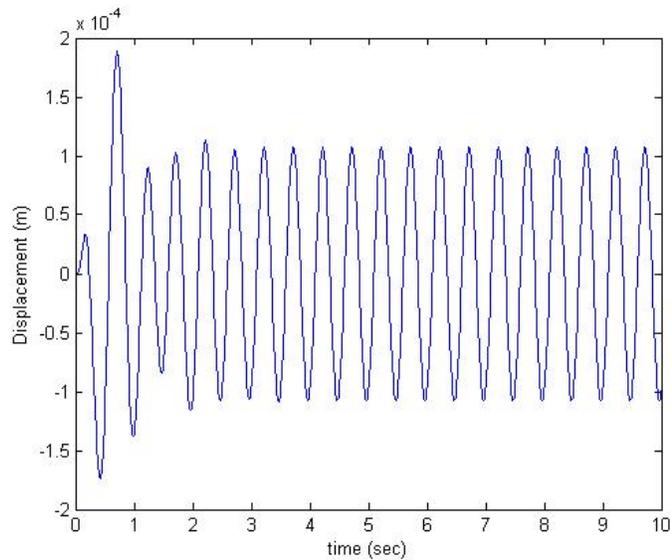
Figure 4.1 Car-body 1 in case 0

Figure 4.1 shows the maximum displacement that the car-body 1 experienced is 0.18 mm and the time taken for the car-body to stable is about 2.6 second before it run below 0.1 mm of displacement.
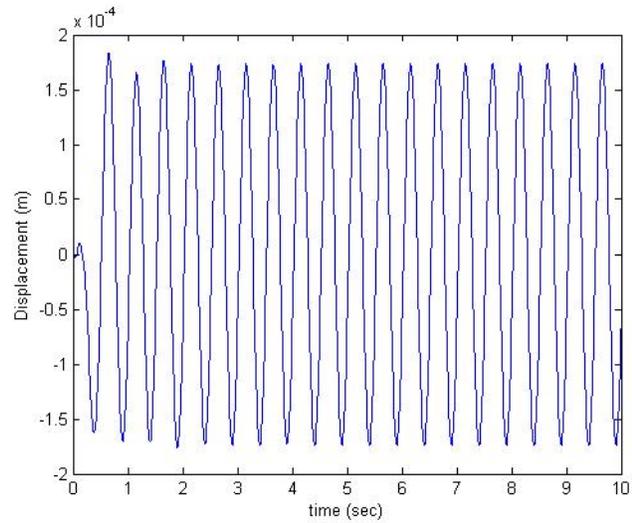
Figure 4.2 Car-body 2 in case 0

Figure 4.2 shows the maximum displacement that the car-body 2 experienced is 0.18 mm and the time taken for the car-body to stable is about 1.9 second before it run below 0.14 mm of displacement.
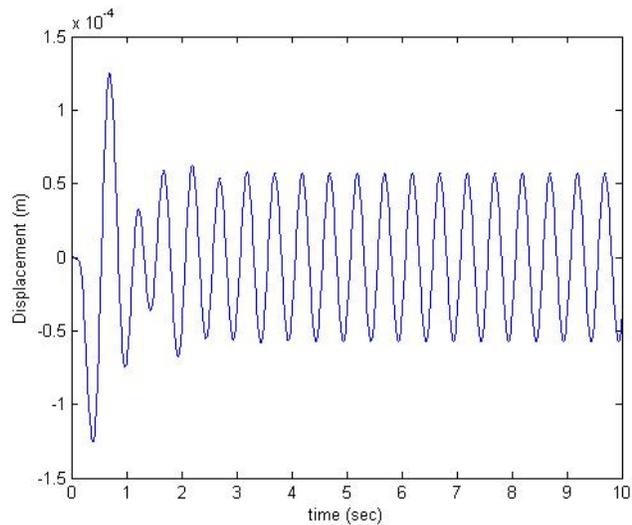

Figure 4.3 Car-body 3 in case 0

Figure 4.3 shows the maximum displacement that the car-body 3 experienced is 0.13 mm and the time taken for the car-body to stable is about 2.5 second before it run below 0.05 mm of displacement.

4.2   Case 1

In case 1, the parameters of the suspension are reduced to 80% of its nominal parameter as shown in Table 4.2. The duration of MATLAB simulation is set to be 10 second. The parameters of the uneven track are the same as in the case 0. The results show in figure 4.4 to 4.6.

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Table 4.2 Suspension parameters for case 1

Items	Parameters
Secondary spring of car 1 and 3 (k1)	448000 N/m
Secondary damper of car 1 and 3 (c1)	23667 Ns/m
Secondary spring of car 2 (k2)	873600 N/m
Secondary damper of car 2 (c2)	40164 Ns/m
Primary spring of car 1 and 3 (k3)	1920000 N/m
Primary damper of car 1 and 3 (c3)	9506 Ns/m
Primary spring of car 2 (k4)	3091200 N/m
Primary damper of car 2 (c4)	141338 Ns/m
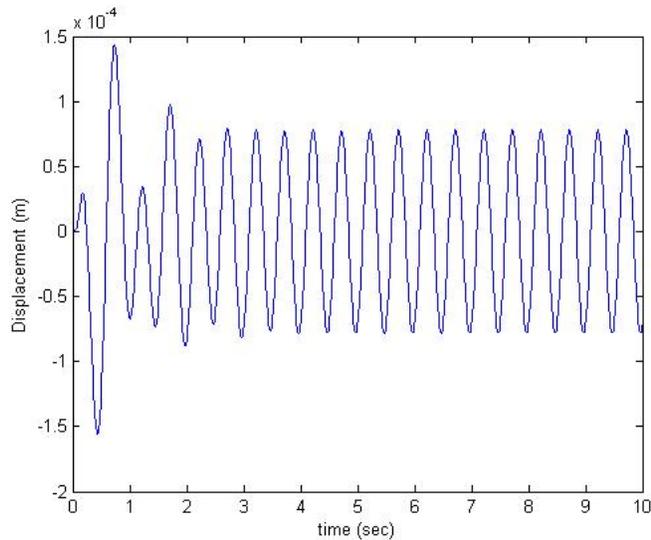
Figure 4.4 Car-body 1 in case 1

Figure 4.4 shows the maximum displacement that the car-body 1 experienced is 0.14 mm and the time taken for the car-body to stable is about 3 second before it run below 0.1 mm of displacement.
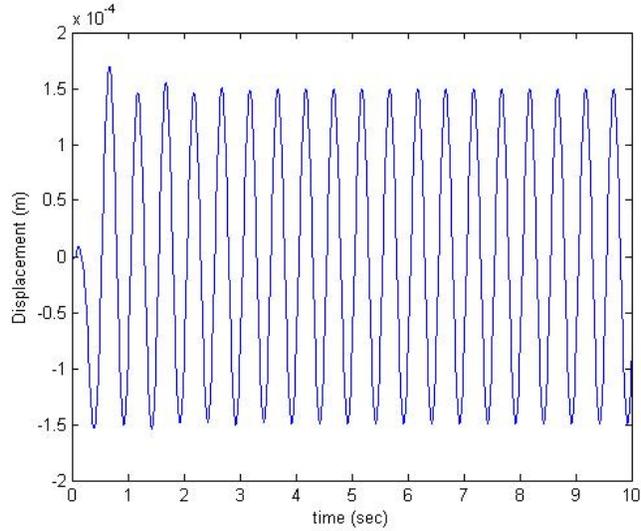

Figure 4.5 Car-body 2 in case 1

Figure 4.5 shows the maximum displacement that the car-body 2 experienced is 0.16 mm and the time taken for the car-body to stable is about 2.2 second before it run below 0.15 mm of displacement.
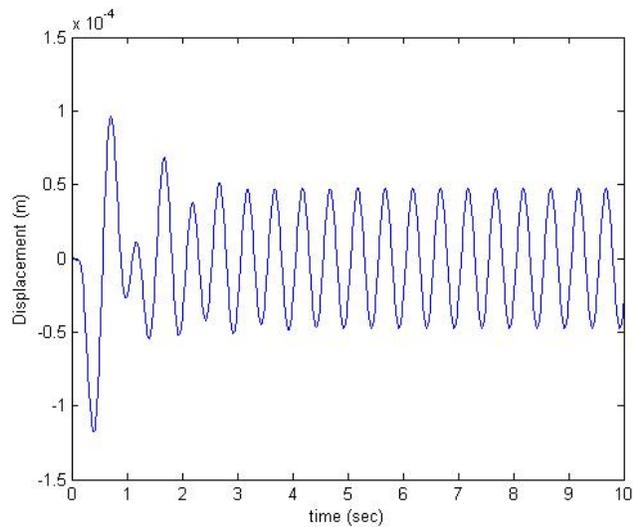

Figure 4.6 Car-body 3 in case 1

Figure 4.6 shows the maximum displacement that the car-body 3 experienced is 0.12 mm and the time taken for the car-body to stable is about 3.5 second before it run below 0.1 mm of displacement.

4.3   Case 2

In case 2, the parameters of the suspension are reduced to 50% of its nominal parameter as shown in Table 4.3. The duration of MATLAB simulation is set to be 10 second. The parameters of the uneven track are the same as in the case 0. The results show in figure 4.7 to 4.9.

Table 4.3 Suspension parameters for case 2

| Items	Parameters
Secondary spring of car 1 and 3 (k1)	280000 N/m
Secondary damper of car 1 and 3 (c1)	14792 Ns/m
Secondary spring of car 2 (k2)	546000 N/m
Secondary damper of car 2 (c2)	25103 Ns/m
Primary spring of car 1 and 3 (k3)	1200000 N/m
Primary damper of car 1 and 3 (c3)	5942 Ns/m
Primary spring of car 2 (k4)	1932000 N/m
Primary damper of car 2 (c4)	88337 Ns/m
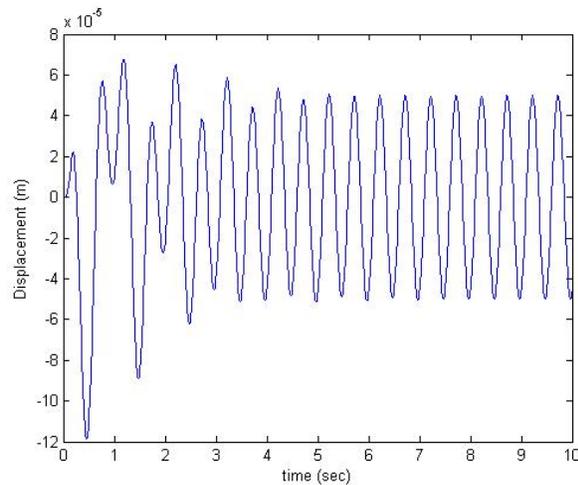
Figure 4.7 Car-body 1 in case 2

Figure 4.7 shows the maximum displacement that the car-body 1 experienced is 0.12 mm and the time taken for the car-body to stable is about 4.8 second before it run below 0.06 mm of displacement.
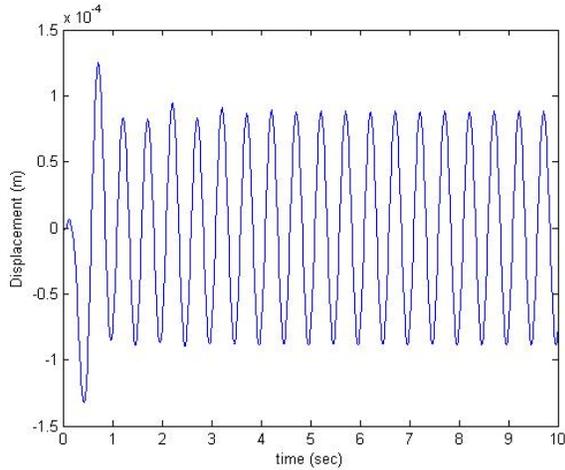
Figure 4.8 Car-body 2 in case 2

Figure 4.8 shows the maximum displacement that the car-body 2 experienced is 0.13 mm and the time taken for the car-body to stable is about 4.2 second before it run below 0.1 mm of displacement.
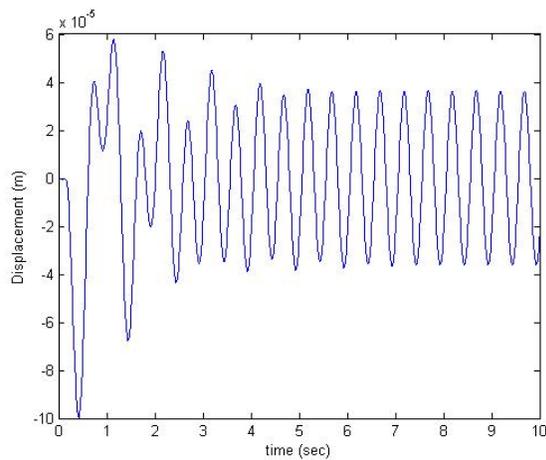

Figure 4.9 Car-body 3 in case 2

Figure 4.9 shows the maximum displacement that the car-body 3 experienced is 0.10 mm and the time taken for the car-body to stable is about 5 second before it run below 0.04 mm of displacement.

4.4   Case 3

In case 3, the parameters of the suspension are increased to 120% of its nominal parameter as shown in Table 4.4. The duration of MATLAB simulation is set to be 10 second. The parameters of the uneven track are the same as in the case 0. The results show in figure 4.10 to 4.12.

Table 4.4 Suspension parameters for case 3

Items	Parameters
Secondary spring of car 1 and 3 (k1)	672000 N/m
Secondary damper of car 1 and 3 (c1)	35501 Ns/m
Secondary spring of car 2 (k2)	1310400 N/m
Secondary damper of car 2 (c2)	60246 Ns/m
Primary spring of car 1 and 3 (k3)	2880000 N/m
Primary damper of car 1 and 3 (c3)	14260 Ns/m
Primary spring of car 2 (k4)	4636800 N/m
Primary damper of car 2 (c4)	212008 Ns/m
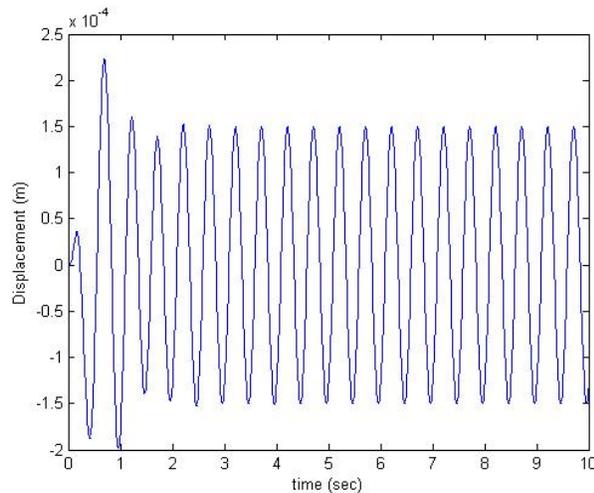
Figure 4.10 Car-body 1 in case 3

Figure 4.10 shows the maximum displacement that the car-body 1 experienced is 0.23 mm and the time taken for the car-body to stable is about 2 second before it run below 0.15 mm of displacement.
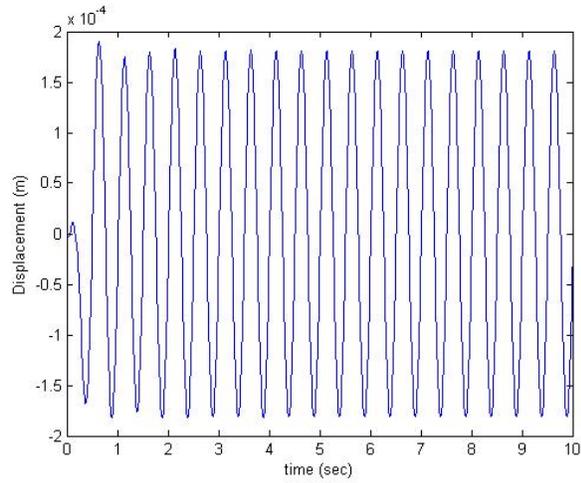
Figure 4.11 Car-body 2 in case 3

Figure 4.11 shows the maximum displacement that the car-body 2 experienced is 0.18 mm and the time taken for the car-body to stable is about 1.5 second before it run below 0.13 mm of displacement.
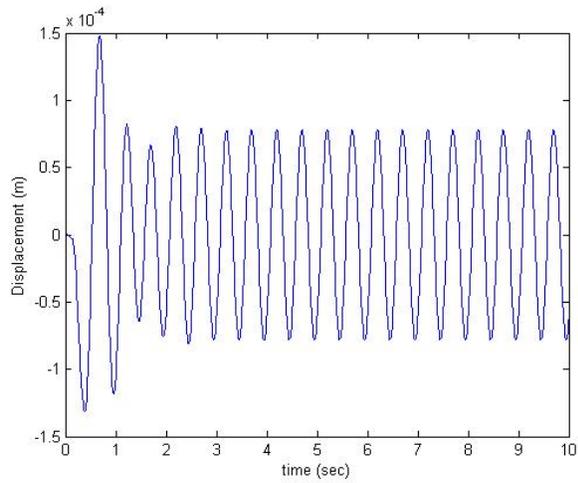
Figure 4.12 Car-body 3 in case 3

Figure 4.12 shows the maximum displacement that the car-body 3 experienced is 0.15 mm and the time taken for the car-body to stable is about 1.9 second before it run below 0.08 mm of displacement.

4.5 Case 4

In case 4, the parameters of the suspension are increased to 150% of its nominal parameter as shown in Table 4.5. The duration of MATLAB simulation is set to be 10 second. The parameters of the uneven track are the same as in the case 0. The results show in figure 4.13 to 4.15.

Table 4.5 Suspension parameters for case 4

| Items	Parameters
Secondary spring of car 1 and 3 (k1)	840000 N/m
Secondary damper of car 1 and 3 (c1)	44376 Ns/m
Secondary spring of car 2 (k2)	1638000 N/m
Secondary damper of car 2 (c2)	75308 Ns/m
Primary spring of car 1 and 3 (k3)	3600000 N/m
Primary damper of car 1 and 3 (c3)	17825 Ns/m
Primary spring of car 2 (k4)	5796000 N/m
Primary damper of car 2 (c4)	265010 Ns/m
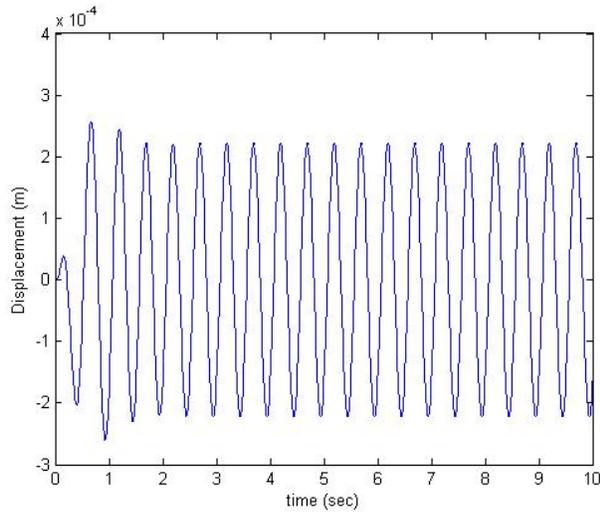
Figure 4.13 Car-body 1 in case 4

Figure 4.13 shows the maximum displacement that the car-body 1 experienced is 0.28 mm and the time taken for the car-body to stable is about 1.4 second before it run below 0.22 mm of displacement.
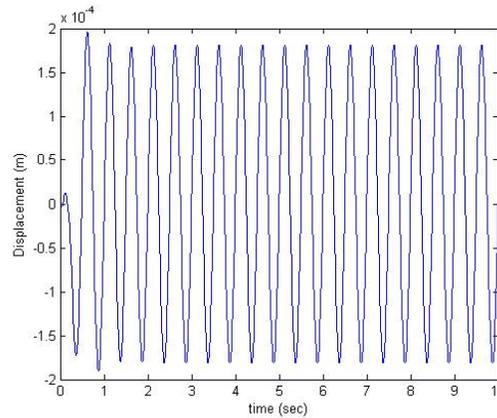
Figure 4.14 Car-body 2 in case 4

Figure 4.14 shows the maximum displacement that the car-body 2 experienced is 0.2 mm and the time taken for the car-body to stable is about 0.9 second before it run below 0.17 mm of displacement.
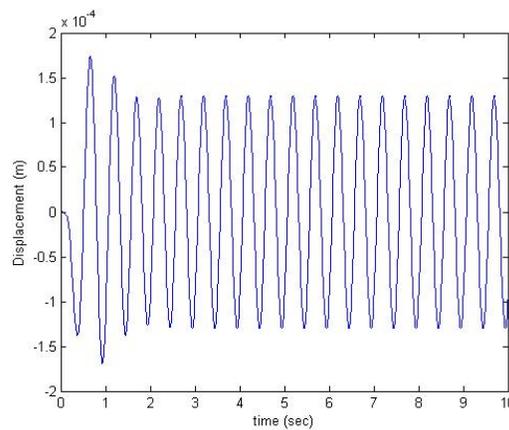

Figure 4.15 Car-body 3 in case 4

Figure 4.15 shows the maximum displacement that the car-body 3 experienced is 0.17 mm and the time taken for the car-body to stable is about 1.9 second before it run below 0.14 mm of displacement.

4.6   Summary of the Results

Figure 4.16and 4.17 shows the summary of the results obtained to compared between each cases. For figure 4.16 is the summary of the maximum displacement of each car-bodies experienced, from the figure shows that the least of displacement experienced by the car-bodies occur in case 2 where the parameters is reduce to 50% from its nominal parameters. In figure 4.17 shows the time taken for the car-bodies to stabilize from the impact of the uneven track condition.
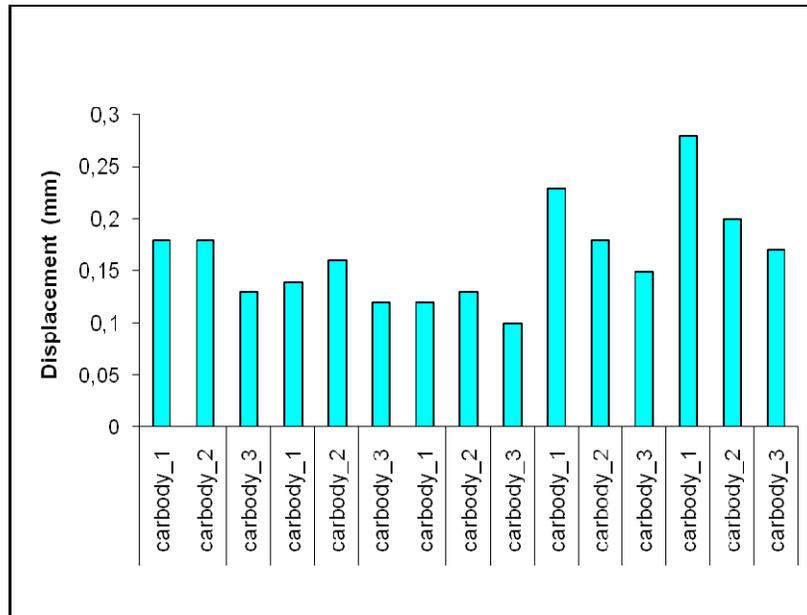
Figure 4.16 Summary of car-bodies' maximum displacement
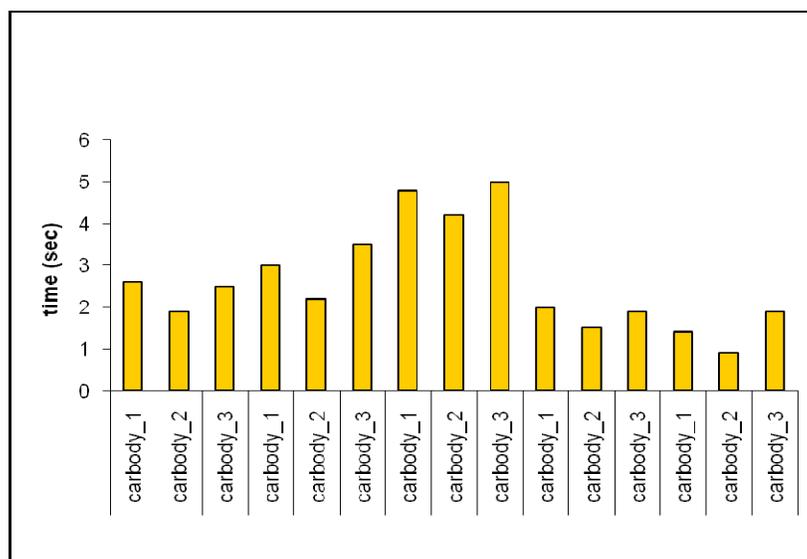
Figure 4.17 Summary of time taken for car-bodies to stabilize

The relationship between the maximum displacement and time taken for the car-bodies to stable are shown in figure 4.18.
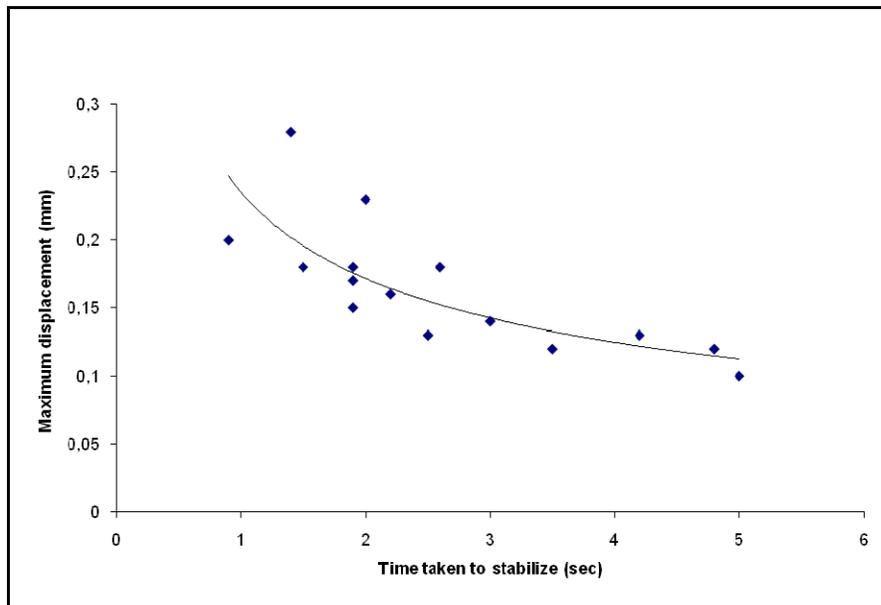
Figure 4.18 Maximum displacement vs. time to stabilize

CHAPTER 5

CONCLUSION AND RECOMMENDATION

5.1 Conclusion

From the result obtained shows that there is a relationship between the maximum displacement and the time taken for the car-body to stabilize. In this research there are 5 cases been studied which are case 0 to case 4. If these cases are rearrange back in terms of maximum displacement that the car-bodies experienced, it would be case 2, case 1, case 0, case 3 and case 4, where case 2 is the least and case 4 is the highest of the displacement. All the parameters are met the clearance between the track and car-body. In terms of time taken for the car-body to stable, the arrangement is the same as the maximum displacement but it is opposite in the value, where case 2 is the highest and case 4 is the lowest value of the time. This relationship between the maximum displacement and the time taken for car-body to stable can be represented in the graph shown in figure 4.18. The figure shows that the maximum displacement is inversely proportional to the time taken for the car-bodies to stabilize.

In order to identify the best parameters for the selected LRV, the most important key factor here is the comfortable of the passenger of LRV. Comfort in this case is how the passenger can adapt the shock caused by the uneven track condition. All human body will not feel uneasy with any forces toward their bodies in very short time. Human body need some times to adapt the forces so that the human body will feel comfortable enough to neglect the forces that react with their bodies. But the time is also must not be too long, that can cause the human bodies to feel uncomfortable again. So by understanding the relationship between the maximum displacement and time taken to stabilize the car-bodies as shown in figure 4.18, the selection of the

parameter can be done easily. For this research the most acceptable parameters for the selected LRV is in case 2, where the maximum displacement is the lowest compared to other cases and also the time for the car-bodies to stable is not too short.

5.2   Recommendation

- Identify the parameters of the suspension by using numerous inputs of tracks.
- Using the real track equation to define the uneven track condition (this research is using sinusoidal wave as the uneven track condition).
- Select the LRV with different number of the car-body.

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APPENDICES

Appendix 1

Unknown variables reference for the mathematical model (boundary condition for the equation of motion):

y_1 vertical displacement of the centre of gravity (c.g.) position of first power-carbody (1 dof)

y_2 vertical displacement of the c.g. position of trailer-carbody (2 dof)

y_3 vertical displacement of the c.g. position of second power-carbody (3 dof)

θ_1 pitch angle of the c.g. position of first power-carbody (4 dof)

θ_2 pitch angle of the c.g. position of trailer-carbody (5 dof)

θ_3 pitch angle of the c.g. position of second power-carbody (6 dof)

y_4 vertical displacement of the c.g. position of first power-bogie (7 dof)

y_5 vertical displacement of the c.g. position of trailer-bogie (8 dof)

y_6 vertical displacement of the c.g. position of second power-bogie (9 dof)

y_7 track vertical profile (track irregularity) for first power-bogie (base motion input)

y_8 track vertical profile (track irregularity) for trailer-bogie (base motion input)

y_9 track vertical profile (track irregularity) for second power-bogie (base motion input)

y_{10} vertical displacement of the suspension position of first power-carbody

y_{12} vertical displacement of the rear position of first power-carbody

y_{13} vertical displacement of the front position of trailer-carbody (connecting first power-carbody)

y_{24} vertical displacement of the rear position of trailer-carbody (connecting second power-carbody)

y_{11} vertical displacement of the suspension position of second power-carbody

y_{23} vertical displacement of the rear position of second power-carbody

m_p power-carbody mass (10820 kg)

I_p power-carbody pitch inertia (71000 kg m^2)

m_t trailer-carbody mass (4470 kg)

I_t trailer-carbody pitch inertia (6000 kg m^2)

m_{pb} power-bogie mass (2940 kg)

m_{tb} trailer-bogie mass (1150 kg)

d_1 distance between c.g. and suspension positions of power-carbody (2.825 m)

d_2 distance between c.g. and rear positions of power-carbody (6 m)

d_3 distance between c.g. and end positions of trailer-carbody (1.9625 m)

k_1, c_1 spring and damping constants of secondary suspension of power-carbody

k_2, c_2 spring and damping constants of secondary suspension of trailer-carbody

k_3, c_3 spring and damping constants of primary suspension of power-carbody

k_4, c_4 spring and damping constants of primary suspension of trailer-carbody

k spring constant of articulation (1630000 N/m)

Appendix 2

Main program:

```
clear all;
mp=10820;mt=4470;mpb=2940;mtb=1150;Ip=71000;It=6000;
d1=2.825;d2=6;d3=1.965;
c1=44376;c2=75308;c3=17825;c4=265010;
k0=1630000;k1=840000;k2=1638000;k3=3600000;k4=5796000;

m=zeros(9,9);
m(1,1)=mp;m(2,2)=mt;m(3,3)=m(1,1);m(4,4)=Ip;m(5,5)=It;
m(6,6)=m(4,4);m(7,7)=mpb;m(8,8)=mtb;m(9,9)=m(7,7);

c=zeros(9,9);
c(1,1)=c1;c(1,4)=-c1*d1;c(1,7)=-c1;c(2,2)=c2;c(2,8)=c2;
c(3,3)=c1;c(3,6)=c1*d1;c(3,9)=-c1;c(4,1)=-c1*d1;c(4,4)=c1*d1*d1;
c(4,7)=c1*d1;c(6,3)=c1*d1;c(6,6)=c1*d1*d1;c(6,9)=-c1*d1;
c(7,1)=-c1;c(7,4)=c1*d1;c(7,7)=c1+c3;c(8,2)=-c2;c(8,8)=c2+c4;
c(9,3)=-c1;c(9,6)=-c1*d1;c(9,9)=c1+c3;

k=zeros(9,9);
k(1,1)=k0+k1;k(1,2)=-k0;k(1,4)=-k1*d1+k0*d2;k(1,5)=k0*d3;
k(1,7)=-k1;k(2,1)=-k0;k(2,2)=k2+2*k0;k(2,3)=-k0;k(2,4)=-k0*d2;
k(2,6)=k0*d2;k(2,8)=-k2;k(3,2)=-k0;k(3,3)=k1+k0;k(3,5)=-k0*d3;
k(3,6)=k1*d1-k0*d2;k(3,9)=-k1;k(4,1)=-k1*d1+k0*d2;k(4,2)=-k0*d2;
k(4,4)=k1*d1*d1+k0*d2*d2;k(4,5)=k0*d2*d3;k(4,7)=k1*d1;
k(5,1)=k0*d3;k(5,3)=-k0*d3;k(5,4)=k0*d2*d3;k(5,5)=2*k0*d3*d3;
k(5,6)=k0*d2*d3;k(6,2)=k0*d2;k(6,3)=k1*d1-k0*d2;k(6,5)=k0*d2*d3;
k(6,6)=k1*d1*d1+k0*d2*d2;k(6,9)=-k1*d1;k(7,1)=-k1;k(7,4)=k1*d1;
k(7,7)=k1+k3;k(8,2)=-k2;k(8,8)=k2+k4;k(9,3)=-k1;k(9,6)=-k1*d1;
k(9,9)=k1+k3;

tstart=0;
tend=10;
h=0.01;
nloop=(tend-tstart)/h;
n=1;
t=tstart;
y1=0;y2=0;y3=0;y4=0;y5=0;y6=0;
teta1=0;teta2=0;teta3=0;
y1dot=0;y2dot=0;y3dot=0;y4dot=0;y5dot=0;y6dot=0;
teta1dot=0;teta2dot=0;teta3dot=0;

Y0(1:18,1:1)=0;
Y1(1:18,1:1)=0;
Y2(1:18,1:1)=0;
Y3(1:18,1:1)=0;
Y4(1:18,1:1)=0;

Y0(1,1)=y1;Y0(2,1)=y2;Y0(3,1)=y3;Y0(4,1)=teta1;Y0(5,1)=teta2;
Y0(6,1)=teta3;Y0(7,1)=y4;Y0(8,1)=y5;Y0(9,1)=y6;
Y0(10,1)=y1dot;Y0(11,1)=y2dot;Y0(12,1)=y3dot;Y0(13,1)=teta1dot;Y0(14
,1)=teta2dot;
Y0(15,1)=teta3dot;Y0(16,1)=y4dot;Y0(17,1)=y5dot;Y0(18,1)=y6dot;

tn(1:nloop)=0;
```

```
y1n(1:nloop)=0;y2n(1:nloop)=0;y3n(1:nloop)=0;teta1n(1:nloop)=0;
teta2n(1:nloop)=0;teta3n(1:nloop)=0;y4n(1:nloop)=0;y5n(1:nloop)=0;
y6n(1:nloop)=0;
y1dotn(1:nloop)=0;y2dotn(1:nloop)=0;y3dotn(1:nloop)=0;teta1dotn(1:nl
oop)=0;
teta2dotn(1:nloop)=0;teta3dotn(1:nloop)=0;y4dotn(1:nloop)=0;y5dotn(1
:nloop)=0;
y6dotn(1:nloop)=0;

itime=1;
tn(itime)=t;
y1n(itime)=y1;y2n(itime)=y2;y3n(itime)=y3;teta1n(itime)=teta1;
teta2n(itime)=teta2;teta3n(itime)=teta3;y4n(itime)=y4;
y5n(itime)=y5;y6n(itime)=y6;
y1dotn(itime)=y1dot;y2dotn(itime)=y2dot;y3dotn(itime)=y3dot;teta1dot
n(itime)=teta1dot;
teta2dotn(itime)=teta2dot;teta3dotn(itime)=teta3dot;y4dotn(itime)=y4
dot;
y5dotn(itime)=y5dot;y6dotn(itime)=y6dot;

%integration
for n = 1:1:nloop
Y1=Y0;
[Ft]=F1(t,k3,k4,c3,c4,m);
[FYT]=fy(t,Y1,Ft,k,m,c);
K1=h*FYT;

t2=t+h/2;
Y2=Y0+0.5*K1;
[Ft]=F1(t2,k3,k4,c3,c4,m);
[FYT]=fy(t2,Y2,Ft,k,m,c);
K2=h*FYT;

t3=t+h/2;
Y3=Y0+0.5*K2;
[Ft]=F1(t3,k3,k4,c3,c4,m);
[FYT]=fy(t3,Y3,Ft,k,m,c);
K3=h*FYT;

t4=t+h;
Y4=Y0+K3;
[Ft]=F1(t4,k3,k4,c3,c4,m);
[FYT]=fy(t4,Y4,Ft,k,m,c);
K4=h*FYT;

t=t+h;
Y0=Y0+(K1+2*K2+2*K3+K4)/6;

itime=itime+1;
tn(itime)=t;
y1n(itime)=Y0(1,1);y2n(itime)=Y0(2,1);y3n(itime)=Y0(3,1);
teta1n(itime)=Y0(4,1);teta2n(itime)=Y0(5,1);teta3n(itime)=Y0(6,1);
y4n(itime)=Y0(7,1);y5n(itime)=Y0(8,1);y6n(itime)=Y0(9,1);
y1dotn(itime)=Y0(10,1);y2dotn(itime)=Y0(11,1);y3dotn(itime)=Y0(12,1)
;teta1dotn(itime)=Y0(13,1);
teta2dotn(itime)=Y0(14,1);teta3dotn(itime)=Y0(15,1);y4dotn(itime)=Y0
(16,1);
y5dotn(itime)=Y0(17,1);y6dotn(itime)=Y0(18,1);
n=n+1;
end
```

```
plot(tn,y1n)
%plot(tn,y2n)
%plot(tn,y3n)
```

Function program:

```
function [FYT]=fy(t,Y,Ft,k,m,c)
FYT(1:18,1:1)=0;
A(1:18,1:18)=0;
A(1:9,1:9)=zeros(9,9);
A(1:9,10:18)=eye(9,9);
A(10:18,1:9)=-k/m;
A(10:18,10:18)=-c/m;
B(1:18,1:18)=0;
B(10:18,10:18)=inv(m);
Ft1=Ft;
FYT=A*Y+B*Ft1;
```

Input program:

```
function [Ft]=F1(tt,k3,k4,c3,c4,m)
YA=0.05;l=9;
v=20;lambda=10;
t=lambda/v;
wn=2*3.1416/t;
y7=YA*cos(wn*tt)   ;                 %input
y7dot=-wn*YA*sin(wn*tt);             %input
y8=YA*cos(wn*(tt-(l/v)))  ;          %input
y8dot=-wn*YA*sin(wn*(tt-(l/v)));     %input
y9=YA*cos(wn*(tt-(2*l/v)))  ;        %input
y9dot=-wn*YA*sin(wn*(tt-(2*l/v))); %input
C(1:18,1:18)=0;
C(10:18,10:18)=inv(m);
D(1:18,1:1)=0;
D(16,1)=k3*y7+c3*y7dot;
D(17,1)=k4*y8+c4*y8dot;
D(18,1)=k3*y9+c3*y9dot;
Ft=C*D;
```

Gantt Chart – First Semester

No.	Detail/ Week	1	2	3	4	5	6	7	Mid - Semester Break	8	9	10	11	12	13	14
1	Selection of Project Topic	▓														
2	Preliminary Research Work		▓	▓	▓											
	i) Research and literature review		▓	▓	▓											
3	Submission of Preliminary Report				▤											
4	Seminar 1 (optional)					▓	▓	▓								
5	Project Work					▓	▓	▓								
	i) Derive equation of motion of LRV									▤						
6	Submission of Progress Report															
7	Seminar 2 (compulsory)										▓	▓	▓	▓		
8	Project work continues									▓	▓	▓	▓	▓		
9	Submission of Interim Report Final Draft														▤	
10	Oral Presentation															▤

▤ Important Event
▓ Complete

Gantt Chart – Second Semester

No.	Detail/Week	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	Project Work Continue	▨	▨	▨												
2	Submission of Progress Report 1				▦											
3	Project Work Continue					▨	▨	▨								
4	Submission of Progress Report 2								▦							
5	Seminar (compulsory)								▦							
6	Project work continue									▨		▨	▨			
7	Poster Exhibition											▦				
8	Submission of Dissertation (soft bound)													▦		
9	Oral Presentation														▦	
10	Submission of Project Dissertation (Hard Bound)															▦

Mid - Semester Break (Weeks 9–10)

Legend:
▦ Important
▤ Process
▨ Complete