Optimization of
Upstream Offshore Oilfield Production Planning under Uncertainty
and
Downstream Crude Oil Scheduling at Refinery Front-End

by

Tan Yin Keong

Dissertation submitted in partial fulfilment of
the requirements for the
Bachelor of Engineering (Hons)
(Chemical Engineering)

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CERTIFICATION OF APPROVAL

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Approved by,

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(KHOR CHENG SEONG)

UNIVERSITI TEKNOLOGI PETRONAS

TRONOH, PERAK

January 2009

CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

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(TAN YIN KEONG)

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I would like to take this opportunity to acknowledge the people who had given their support and help throughout the completion of this project.

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ABSTRACT

In this work, we have attempted to solve two problems concerning the planning and scheduling of crude oil operations: first, on the upstream production planning of crude oil from offshore sources and second, on the scheduling of downstream processing of crude oil at the refinery front-end.

The first part is on the offshore oilfield infrastructures planning under both exogenous uncertainty and endogeneous decision-dependent uncertainty. A model representative of the oilfield that is able to select the best routes to obtain the desired objective function is considered. The methodology used is by firstly developing a deterministic model and modeling it with GAMS, followed by a stochastic one. The results obtained show a high accuracy representation in which the uncertainties in both the exogenous and endogeneous uncertainties in planning are accounted for. The stochastic model is a more thorough representation of the problem because it considers all the uncertainties along with the associated probabilities. Having validated the model formulation and solution obtained, we believe that the model can be a useful basic tool to assist upper-level management in deciding on an optimal plan for crude oil production from an offshore operation.

The second part is on the scheduling of crude oil operations at a refinery front-end. A technique for obtaining globally optimal schedules for the flow of crude is developed. A continuous time model based on transfer events is used to represent the scheduling problem and this model is a nonconvex MINLP model which presents multiple local optima. We implement a branch-and-contract algorithm that aims at reducing the size of the search region. In order to obtain a global optimum solution of the problem, an outer-approximation algorithm is proposed, whereby lower and upper bounds on the global optimum are generated, which are converged to a specified tolerance. The solution obtained from the LB–MILP model, i.e., the decision variables (binary variables), was used to obtain a feasible solution for model UB–NLP. This solution is the upper bound solution. The application of the proposed algorithm shows significant reduction in the computational effort involved in solving the problem. Slack variables are introduced to overcome the integer infeasibility

problem. The optimization model is developed using GAMS and an optimal solution is found with no logical constraints conflicts or error.

The main contribution on this work in the first part is to conduct an extensive study on the implementation of the model formulation in Iyer et al. (1998). As well, in the second part, we are focused on investigating effective implementation strategies of the model formulation and solution strategy in Karuppiah et al. (2008) using our choice of the modeling platform GAMS and the best numerical solvers that are available. Hence, most of the exposition on the model formulation and solution algorithms are taken directly from the original papers so as to provide the readers with the most accurate information possible.

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CHAPTER A1
INTRODUCTION

A1.1 INTRODUCTION TO STOCHASTIC PROGRAMMING

Stochastic programming is a framework for modeling optimization problems that involve uncertainty. Whereas deterministic optimization problems are formulated with known parameters, real world problems almost invariably include some unknown parameters. When the parameters are known only within certain bounds, one approach to tackling such problems is called robust optimization (Beasley, **2002**). Here the goal is to find a solution which is feasible for all such data and optimal in some sense. Stochastic programming models are similar in style but take advantage of the fact that probability distributions governing the data are known or can be estimated.

The most widely applied and studied stochastic programming models are two-stage linear programs. Here the decision maker takes some action in the first stage, after which a random event occurs affecting the outcome of the first-stage decision. A recourse decision can then be made in the second stage that compensates for any bad effects that might have been experienced as a result of the first-stage decision. The optimal policy from such a model is a single first-stage policy and a collection of recourse decisions (a decision rule) defining which second-stage action should be taken in response to each random outcome.

In summary, stochastic programming, as the name implies, is mathematical (i.e. linear, integer, mixed-integer, nonlinear) programming but with a stochastic element present in the data.

In deterministic mathematical programming the data (coefficients) are known numbers. In stochastic programming these numbers are unknown; instead we may have a probability distribution present. Stochastic programming therefore deals with situations where we have uncertainty present.

A1.2 PROBLEM STATEMENT

The design of an oil production system is a complex task that requires the consideration of many factors (Heever and Grossmann, **2000**). We need to develop and implement a systematic and automated approach to efficiently and rigorously integrate the elaborate interactions between the design decision variables selecting the best among a set of possible solutions, using sound engineering judgment.

Comment [C.S.Khor1]: The idea in this sentence is slightly unclear (might need (more) elaboration).

An oilfield layout consists of a number of fields, each containing one or more reservoirs, and each reservoir contains one or more well sites. After the decision has been made to produce oil from a given well site, it is drilled from a well platform (WP) using drilling rigs. A network of pipelines connects the wells to the WPs and the WPs to the production platforms (PP). The location of production platforms (PP), well platforms (WP), and allocation of well (W) to them is a complex optimization problem, as the costs of drilling are affected by the lengths of pipe required (Iyer et. al., **1998**).

A1.2.1 Model Assumptions

The major assumptions in the model are as follows:

- the productivity index (PI) of each well is constant across the planning horizon;
- operating conditions are constant during a given time period;
- each reservoir contains a single homogeneous mixture at the same pressure;
- multiple wells in a reservoir produce independently on the basis of each well's PI and the reservoir's average pressure;
- pressure drop in pipes and wells is linear in the oil flow rate and gas flow rate (in reality, flow rate is proportional to the square root of pressure drop);
- the well oil flow rate is proportional to the pressure difference between the well head and well bore;

- each well has a single completion in a single reservoir. This implies that a drilled well is capped after extracting oil from the well. Therefore, it is not reused to drill from this well to other well locations in the reservoir;
- the location of potential wells and platforms as well as their potential interconnections to wells.

Further, we consider that each well may be allocated to two or more WPs. However, this will clearly increase the computational effort significantly. A more practical option might be to consider allocating each well to up to two WPs. In this case, the two different allocations are treated as two choices of identical wells, of which only one can be selected (Heever and Grossmann, 2000).

Note that wells from a reservoir may be potentially allocated to more than one well platform and one WP may produce wells from more than one reservoir. However, once a selection is made, each well is assigned to a single WP; Iyer et al., 1998).

A1.3 RESEARCH OBJECTIVES

The questions that we are interested to answer in this research project relates to the planning decisions of oilfield production in the specified time periods, in which the decision variables include:

- whether or not to install a production platform or well platform/which well platforms should be built and when?;
- whether or not to drill each well (at a specified given location and allocation);
- the production profile (of oil) for each well;
- when and where should the drilling rigs be moved?;
- how much is the investment for WP, PP, and drilling rigs?

The superstructure is as shown in Figure A1.1 below:

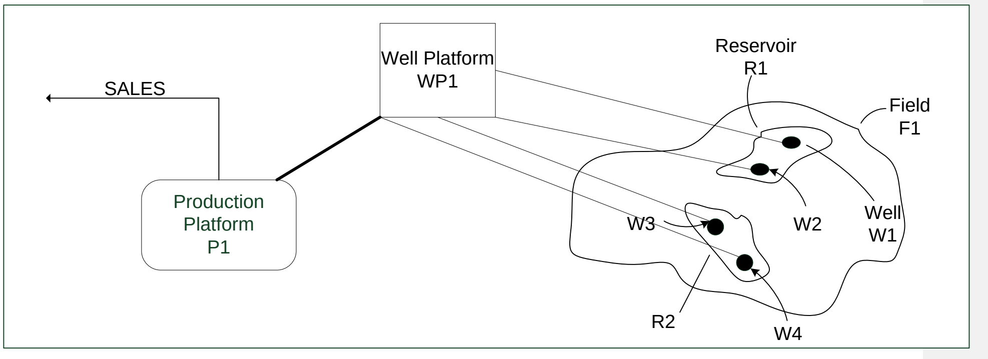


FIGURE A1.1: Superstructure of an oil field

CHAPTER A2
LITERATURE REVIEW

A2.1 PREVIOUS WORK

Previous work in the optimization of offshore field development may be classified into two broad categories, which are location-allocation problems and production planning and scheduling problems.

A2.1.1 Location-Allocation Problem

One of the earliest works by Devine and Lesso (1972) solves the problem of determining the continuous two-dimensional location of production platforms and the allocation of wells to well platforms. The trade-offs associated with the costs involve the capacity of the platforms and the cost of piping from wells to the platforms. They proposed an iterative two-stage algorithm that first fixes the allocation of wells to the platforms and then determines the location of platforms. In the next stage, the location determined in the first stage is fixed and the allocation problem is solved to get a new set of well allocation variables. The problem, however, does not include the scheduling of well drilling or production planning of wells.

Garcia-Diaz et al. (1996) presented a network representation for the same problem and proposed a Lagrangean relaxation solution method.

A2.1.2 Production Planning and Scheduling

Lee and Aranofsky (1958) formulated a production planning problem that expressed the performance of reservoirs linearly as a function of time. Sullivan (1982) used a nonlinear reservoir performance equation and approximated it by using piecewise linear interpolation using integer variables. Both these works did not include the scheduling of drilling of wells and surface pressure constraints.

Aranofsky and Williams (1962) proposed an LP model for scheduling of well drilling, assuming a preset production profile. In addition, the model had the drawback of fractional solutions for the number of wells drilled in a time period. Bohannon (1970) proposed a mixed-integer linear programming (MILP) model for development of multireservoir systems, assuming a predetermined linear decline of production rate with cumulative oil produced. Frair and Devine (1973) proposed a model that simultaneously included the location-allocation of wells, scheduling of facility operation, and production rates for different time periods. However, the model did not include the reservoir performance equations and assumed a linear production decline curve for each reservoir.

Costa (1975) and Dogru (1975) proposed models for optimal platform location and scheduling of well drilling, but they did not include any production planning. Harding et al. (1996) tested sequential quadratic programming, simulated annealing, and a genetic algorithm for planning the multiple field development. They assumed a predetermined target production rate and prescheduled the capital investment within a fixed time window.

Iyer and Grossmann (1998) proposed multiperiod mixed-integer linear programming (MILP) model formulation for the planning and scheduling of investment and operation in offshore oil field facilities. The formulation employs a general objective function that optimizes a selected economic indicator (e.g., net present value). For a given planning horizon, the decision variables in the model are the choice of reservoirs to develop, selection from among candidate well sites, the well drilling and platform installation schedule, among others. A sequential decomposition strategy using aggregation of time periods and wells, followed by successive disaggregation, is proposed. However, they did not include the uncertainty in reserves.

Van den Heever and Grossmann (2000) proposed a multiperiod mixed-integer nonlinear programming (MINLP) model for offshore oilfield infrastructure planning where nonlinear reservoir behavior is incorporated directly into the formulation. Discrete decisions include the selection of production platforms, well platforms and wells to be installed/drilled, and the drilling schedule for the wells over the planning horizon. Continuous decisions include the capacities of the platforms and the production profile for each well in each time period. For the solution of this model, an iterative aggregation/disaggregation algorithm is proposed in which logic-based methods, a bilevel decomposition technique, the use of convex envelopes, and aggregation of time periods are integrated.

CHAPTER A3
MODEL FORMULATION

A3.1 DETERMINISTIC MODEL FORMULATION FOR OILFIELD PRODUCTION PLANNING

The model is configured hierarchically in terms of sets to represent the physical configuration in a general form. The following sets, indices, variable, and parameters are defined.

A3.1.1 Nomenclature

(a) Sets and Indices

PP	set of production platforms
p	production platform $p \in$ PP
WP(p)	set of well platforms connected to platform p
n	well platform $n \in$ WP(p)
F	set of fields
f	field $f \in F$
$R(f)$	set of reservoirs associated with field f
r	reservoir $r \in R(f)$
$W_{\mathrm{WP}}(n)$	set of wells connected to well platform n
$W_R(r)$	set of wells connected to reservoir r
$W_{\mathrm{WP},R}(r,n)$	set of wells associated with reservoir r and well platform n
w	well $w \in W_{(.)}(.)$
t	time periods
τ	aggregated time periods
D	set of drilling rigs

k	drilling rigs $k \in D$
$J(r)$	linear interpolation pieces used for reservoir r
j	index of piece used in piecewise linear interpolation $j \in J$

(b) Continuous Variables

x_t	oil/gas flow in period t
$\hat{x}_t$	cumulative oil/gas flow amount up to period t
l_t	oil flow (mass) in period t
g_t	gas flow (volumetric) in period t
ϕ_t	gas-to-oil ratio in period t
v_t	pressure in period t
δ_t	pressure drop at choke in period t
d_t	design variable in period t
e_t	design expansion variable in period t
$Z_{k,t}$	number of times drilling rig k is moved in period t
$Z_{k,t}^{p}$	number of times drilling rig k is moved in period t
$\lambda_{j,\mathrm{t}}$	interpolation variable in period t

(c) Binary Variables

z_t	= 1 if facility (well or platforms) is drilled/installed in period t
$zd_{k,t}$	= 1 if drilling rig k is located on facility in period t
$zf_{k,t}$	= 1 if the kth rig is located on well platform n first in period t
$zl_{k,t}$	= 1 if the kth rig is located on well platform n last in period t
$s_{k,t}$	= 1 (a slack) when only the nth platform is used for drilling using the kth rig in period t
$ZT_{k,t}$	= 1 if drilling rig k is moved after period t
$y_{j,t}$	= 1 if piece j used for linear interpolation in period t
	= otherwise

(d) Parameters

ρ	productivity index of well
$P_{\max}$	maximum pressure drop from well bore to well head
$\mathrm{GOR}_{\max}$	maximum GOR (gas-to-oil ratio)
t_{a}	number of periods in an aggregated time period τ
T_{a}	number of aggregated time periods, $\tau = 1 \ldots Ta$
M_{w}	maximum number of wells drilled by a rig in a time period
Δt	length of time period t
Ω^{u}	upper bound parameter (defined by the respective equation)
α	pressure drop coefficient for oil flow rate
β	pressure drop coefficient for GOR
c_{1t}	discounted revenue price coefficient for oil sales
c_{2t}	discounted fixed cost coefficient for capital investment
c_{3t}	discounted variable cost coefficient for capital investment
c_{4t}	discounted cost coefficient for moving rigs

(e) Superscripts

(w,n,p)	variables associated with well $w \in W$, with well platform n and production platform p
(n,p)	variables associated with well platform n and production platform p
(p)	variables associated with production platform p
(r)	variables associated with reservoir r

(f) Other Notations

(.)	formulation with fixed value of variable
<(.)>	aggregation with respect to well variables and parameter

A3.1.2 Model Equations

(a) Material Balances

The sum of flow of oil/gas from all wells $w \in W_{\text{WP}}(n)$ associated with a well platform $n \in \text{WP}(p)$ is the total flow of oil/gas at the well platform.

$$\sum_{w \in W_{\text{WP}}(n)} x_{w,n,p,t} = x_{n,p,t} \qquad \forall n \in \text{WP}(p), p \in \text{PP}, t \in T \qquad \textbf{(A1)}$$

Similarly, flows from well platforms are added to determine the flow at the production platforms.

$$\sum_{n \in \text{WP}(p)} x_{n,p,t} = x_{p,t} \qquad \forall p \in \text{PP} \qquad \textbf{(A2)}$$

The total flow at period t is the sum of flows from all production platforms.

$$\sum_{p \in \text{PP}} x_t^p = x_t^{total} \qquad \textbf{(A3)}$$

for $t = 1 \ldots T$

(b) Pressure Balances at Well Platforms and Production Platforms

The pressure at the well platform $n \in \text{WP}(p)$ is the pressure at the wells $w \in W_{\text{WP}}(n)$ associated with well platform n minus the pressure drop in the corresponding pipe. In reality, the flowrate of oil/gas is equal to square root of pressure drop. Here, pressure drop is expressed as a linear function of the oil and gas flow rate for simplicity. Here, δ is defined as the pressure drop across the pressure chokes on the lines.

$$v_{n,p,t} = v_{w,n,p,t} - \alpha x_{w,n,p,t} - \beta g_{w,n,p,t} - \delta_{w,n,p,t} \qquad \forall w \in W_{\text{WP}}(n),\ n \in \text{WP}(p),\ p \in \text{PP} \ \textbf{(A4)}$$

The pressure at the production platform $p \in \text{PP}$ is the pressure at the well platform $n \in \text{WP}(p)$ associated with production platform p minus the pressure drop in the corresponding pipe.

$$v_p = v_{n,p} - \alpha x_{n,p} - \beta g_{n,p} - \delta_{n,p}, \qquad \forall n \in \text{WP}(p),\ p \in \text{PP} \qquad \textbf{(A5)}$$

for $t = 1 \ldots T$

(c) Flow Constraints in Wells

The oil/gas flow in period t is related to the maximum flow of oil ($l_{w,n,p,t}$) and the maximum gas flow ($g_{w,n,p,t}$):

$$x_{w,n,p,t} = l_{w,n,p,t} + g_{w,n,p,t} \qquad \textbf{(A6)}$$

The maximum flow of oil is related to the PI of the well (ρ) and the allowable pressure drop ($P_{\max}$):

$$l_{w,n,p,t} \le \rho_{w,n,p,t} P_{\max} \qquad \textbf{(A7)}$$

The maximum gas flow is limited by the maximum flow of oil ($l_{w,n,p,t}$) and maximum allowable GOR ($\mathrm{GOR}_{\max}$):

$$g_{w,n,p,t} \le l_{w,n,p,t} \mathrm{GOR}_{\max} \qquad \forall w \in W_{\mathrm{WP}}(n), n \in \mathrm{WP}(p), p \in \mathrm{PP} \qquad \textbf{(A8)}$$

for $t = 1 \ldots T$

(d) Cumulative Flow Amount from Wells up to a Certain Period, Θ

The cumulative amount of oil from a well is calculated from the sum of the amount of oil from all periods up to time period, θ. Thus,

$$\hat{x}_{w,n,p,\theta} = \sum_{t=1}^{\theta-1} x_{w,n,p,t} \Delta t \qquad \forall w \in W_{\mathrm{WP}}(n), n \in \mathrm{WP}(p), p \in \mathrm{PP} \qquad \textbf{(A9)}$$

for $\theta = 1, \ldots, T$

For instance, when $\theta = 1$: $\hat{x}_{w,\pi,p,\theta} = 0$;

when $\theta = 5$: $\hat{x}_{w,\pi,p,\theta} = \sum_{t=1}^{4} x_{w,n,p,t} \Delta t$

The cumulative oil amount from a reservoir is obtained by summing the cumulative amounts from all wells associated with the reservoir. We define a new set as follows:

$$W_{F,R}\left(f,r\right)\left\{(w,n,p)\mid w\in W_{\mathrm{WP}}(n)\cap W_{R}(r), n\in \mathrm{WP}(p), p\in \mathrm{PP}\right\} \qquad \forall r\in R(f), f\in F$$

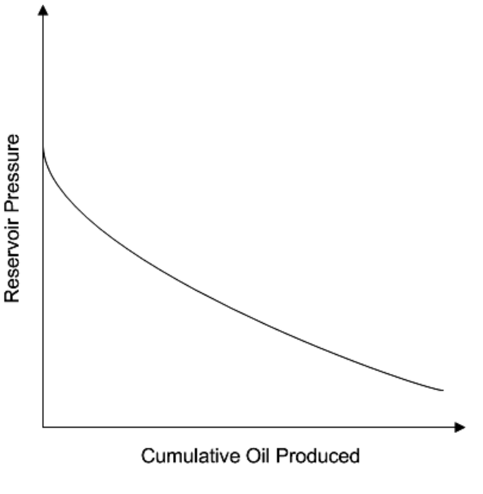
Thus, the cumulative amount of oil from a well

$$\hat{x}_{r,f,\theta}=\sum_{(w,n,p)\in W_{F,R}(f,r)}\hat{x}_{w,n,p,\theta} \qquad \forall r\in R, f\in F \qquad \textbf{(A10)}$$

$$\text{for } \theta = 1, \ldots, T$$

(e) Piecewise Linear Interpolation at Well and Reservoir Level

The pressure and GOR in a reservoir is a nonlinear function of the cumulative oil produced, as can be seen in Figure A3.1 below:



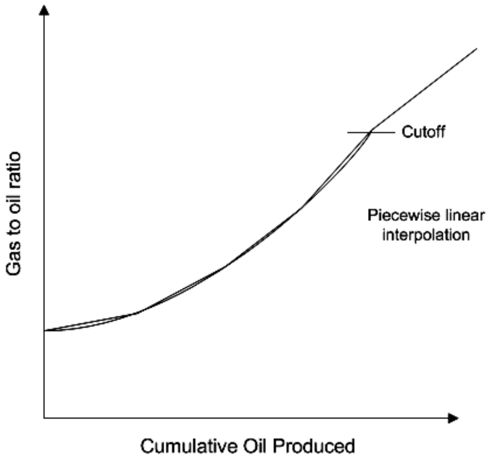


FIGURE A3.1: Reservoir performance characteristics

The actual relation is taken to be a function of the initial pressure, the oil's PVT properties, and the initial oil volume. The pressure and GOR are then calculated using piecewise linear interpolation.

The pressure in a reservoir $r \in R(f)$ is as shown below:

$$v_{r,f,t} = h_1(\hat{x}_{r,f,t}, y_{r,f,j,t}, \lambda_{r,f,j,t}, \tilde{v}_{r,f,t}) \qquad \forall r \in R(f),\ f \in F,\ t = 1,\cdots,T \quad \textbf{(A11)}$$

The oil and gas flow rates from individual wells in a reservoir are also calculated using piecewise linear interpolation:

$$x_t^{w,\pi,p} = h_1(\hat{x}_t^{w,\pi,p}, y_{j,t}^{w,\pi,p}, \lambda_{j,t}^{w,\pi,p}, \tilde{x}_t^{w,\pi,p}) \quad \textbf{(A12)}$$

The GOR in period *t* is as shown below:

$$\phi_{w,n,p,t} = h_1(\hat{x}_{w,n,p,t}, y_{w,n,p,j,t}, \lambda_{w,n,p,j,t}, \tilde{w}_{w,n,p,t})$$
$$\forall w \in W_{\mathrm{WP}}(n),\ n \in \mathrm{WP}(p),\ p \in \mathrm{PP},\ t = 1,\cdots,T \quad \textbf{(A13)}$$

It is vital to model the reservoir performance for us to capture the dynamic conditions in the reservoir. The reservoir pressure and GOR is a function of the cumulative amount of oil removed (see Figure A3.1). This performance curve is nonlinear. In this work, it will be approximated using piecewise linear interpolation. The choice of the linear approximation piece used for interpolation is made using binary variables $y_{r,f,j,t}$.

The value of the interpolated variable as a convex combination of $\tilde{v}_{r,f,j,t}$ and $\tilde{v}_{r,f,j-1,t}$ are determined by:

$$v_{r,f,t} = \sum_{j \in J(r)} \lambda_{r,f,j,t} \tilde{v}_{r,f,j,t} \qquad \textbf{(A14)}$$

The constraint on the interpolation variable $\lambda_{r,f,j,t}$ is as shown below:

$$\sum_{j \in J(r)} \lambda_{r,f,j,t} = 1 \qquad \textbf{(A15)}$$

From the following equation, $\lambda_{r,f,j,t}$ can be non-zero for only two consecutive *j*. Thus, the corresponding *j*th piece is used for linear interpolation as all other $\lambda_{r,f,j,t} = 0$.

$$\lambda_{r,f,j,t} \le y_{r,f,j,t} + y_{r,f,j-1,t} \qquad \textbf{(A16)}$$

The following equation allows only one index j for which $y_{r,f,j,t}$ = 1. Note that the variables y may be treated as special ordered sets (SOS) variables over the set *j*.

$$\sum_{j} y_{r,f,j,t} = 1 \qquad \textbf{(A17)}$$

(f) Logical Constraints for Installation and Flow from Facilities

The wells may be drilled only once in one of the time periods.

$$\sum_{t=1}^{T} z_{w,n,p,t} \leq 1 \qquad \forall w \in W_{\mathrm{WP}}(n), n \in \mathrm{WP}(p), p \in \mathrm{PP} \qquad \textbf{(A18)}$$

The well platforms may be installed only in one of the time periods.

$$\sum_{t=1}^{T} z_{n,p,t} \leq 1 \qquad \forall n \in \mathrm{WP}(p), p \in \mathrm{PP} \qquad \textbf{(A19)}$$

The production platforms may be installed only in one of the time periods.

$$\sum_{t=1}^{T} z_{p,t} \leq 1 \qquad \forall p \in \mathrm{PP} \qquad \textbf{(A20)}$$

for t = 1 … T

Note that the variables z_t may be treated as special ordered sets (SOS) by introducing a dummy period t_d such that

$$\sum_{t=1}^{T} z_t + z_{t_d} = 1 \qquad \textbf{(A21)}$$

By treating the investment binaries as SOS variables, one can potentially reduce the number of nodes enumerated in a branch and bound tree.

The flow of oil and gas from a facility can be nonzero only after it is installed.

For the flow from a well w to a well platform n:

$$x_{w,n,p,\theta} \leq \Omega^u \sum_{t=1}^{\theta} z_{w,n,p,t} \qquad \forall w \in W_{\mathrm{WP}}(n), n \in \mathrm{WP}(p), p \in \mathrm{PP} \qquad \textbf{(A22)}$$

For the flow from a well platform n to a production platform p:

$$x_{n,p,\theta} \le \Omega^u \sum_{t=1}^{\theta} z_{n,p,t} \qquad \forall n \in \mathrm{WP}(p),\, p \in \mathrm{PP} \qquad \textbf{(A23)}$$

For the flow from a production platform *p* to shore for sales:

$$x_{p,\theta} \le \Omega^u \sum_{t=1}^{\theta} z_{p,t} \qquad \forall p \in \mathrm{PP} \qquad \textbf{(A24)}$$

for *t* = 1 … *T*

Note that the upper bound Ω^u is defined accordingly based on the variables in the equation. In eq 22, Ω^u is equal to the maximum well oil flow rate $x_{w,n,p,\theta}$. The well platform associated with a well must be installed before drilling that well.

$$z_{w,n,p,\theta} \le \sum_{t=1}^{\theta} z_{n,p,t} \qquad \forall w \in W_{\mathrm{WP}}(n), n \in \mathrm{WP}(p),\, p \in \mathrm{PP} \qquad \textbf{(A25)}$$

Similarly, production platforms must be installed before the associated well platforms.

$$z_{n,p,\theta} \le \sum_{t=1}^{\theta} z_{p,t} \qquad \forall \pi \in \mathrm{WP}(p),\, p \in \mathrm{PP} \qquad \textbf{(A26)}$$

for *t* = 1 … *T*

(g) Design of Facility

The design capacity of a facility is determined by the maximum flow for all periods *t* = 1, …, *T*. The maximum flow at well platform is:

$$x_{n,p,t} \le d_{n,p,t} \qquad \forall n \in \mathrm{WP}(p),\, p \in \mathrm{PP} \qquad \textbf{(A27)}$$

Similarly, for production platform:

$$x_{p,t} \leq d_{p,t} \qquad \forall p \in \mathrm{PP} \qquad \textbf{(A28)}$$

To model the variable design cost using linear terms, a design *expansion* variable is used, which is equal to the design capacity for the time period when the facility is installed. The design variable for well platform is:

$$d_{n,p,t} = d_{n,p,t-1} + e_{n,p,t} \qquad \forall n \in \mathrm{WP}(p), p \in \mathrm{PP} \qquad \textbf{(A29)}$$

Similarly, for production platform:

$$d_{p,t} = \quad d_{p,t-1} + e_{p,t} \qquad \forall p \in \mathrm{PP} \qquad \textbf{(A30)}$$

The expansion variable for well platform is:

$$e_{n,p,t} \leq \Omega^u z_{n,p,t} \qquad \forall n \in \mathrm{WP}(p), p \in \mathrm{PP} \qquad \textbf{(A31)}$$

Similarly, for production platform:

$$e_{p,t} \leq \Omega^u z_{p,t} \qquad \forall p \in \mathrm{PP} \qquad \textbf{(A32)}$$

Note that the expansion variable *e* may be nonzero only in one time period (from eqs 19 and 31 for well platforms, and from eqs 20 and 32 for production platforms), which represents the period when the facility is installed.

(h) Number of Wells Drilled in Time Period t

The maximum number of wells that can be drilled in a time period (M_{W}) is dependent on the length of the time period (t), the time taken by a drilling rig to drill a well, and the number of available drilling rigs.

$$\sum_{w \in W_{\mathrm{WP}}(n)} z_{w,n,p,t} \leq h_2(Z_{p,k,t}, ZT_{n,p,k,t}, M_{\mathrm{W}}) \qquad \forall n \in \mathrm{WP}(p), p \in \mathrm{PP} \qquad \textbf{(A33)}$$

The following equations are used to model the resource constraints associated with drilling rigs. The main issues involved in the resource constraints are summarized below. Wells are drilled from the associated well platforms using drilling rigs.

However, there are only a limited number of drilling rigs (usually less than the number of well platforms) available in any time period for drilling a well. Besides the availability of the rig, the physical location of the rig on the well platform also determines which wells may be drilled in any time period. Thus, the scheduling of the movement of rigs between well platforms is essential to ensure its availability. The movement of drilling rigs across well platforms also leads to a fixed charge and a loss of time in the availability of the rig.

This loss of time is accounted for in eq 32, where the number of wells drilled in a time period is a function of the number of moves of the rigs across well platforms.

If any well associated with well platform n is drilled in period *t*, then $\sum_k zd_{n,p,k,t}$ must be 1 in that time period, implying at least one drilling rig is located on that well platform:

$$z_{w,n,p,t} \leq \sum_{k \in D} zd_{n,p,k,t} \qquad \forall w \in W_{\mathrm{WP}}(n), n \in \mathrm{WP}(p), p \in \mathrm{PP} \qquad \textbf{(A34)}$$

If in any period, if drilling rig *k* is located first on well platform *n*, then = 1.
The sum of *zf* and *zl* must be less than 1 when the rig is located on more than one well platform in the same time period. However, if all wells are drilled from the same well platform in a time period, then both *zf* and *zl* will be 1. To model both possibilities, a slack binary variable $s_{p,k,t}$ is used.

The *k*th rig is located on the *n*th platform either first or last in that time period *t* (when $s_{p,k,t}$ = 0). The location of the rigs on the well platforms is modeled by the following equations:

$$zf_{n,p,k,t} + zl_{n,p,k,t} \le 1 + s_{p,k,t} \qquad \forall w \in W_{\mathrm{WP}}(n), n \in \mathrm{WP}(p), p \in \mathrm{PP} \qquad \textbf{(A35)}$$

The slack variable takes a non-zero value if all wells are drilled from a single nth platform in that time period. Thus, $s_{p,k,t}$ takes a value of 1 (a slack) when only the *n*th platform is used for drilling using the *k*th rig in period *t*. If both *zf* and *zl* are 1, then the slack is non-zero (= 1):

$$\Big(\sum_{n \in \mathrm{WP}(p)} zd_{n,p,k,t} - 1\Big) \le \Omega^{\mathrm{u}}(1 - s_{p,k,t}) \qquad \forall n \in \mathrm{WP}(p), p \in \mathrm{PP} \qquad \textbf{(A36)}$$

The binary variables *zf* and *zl* are non-zero only if a well is drilled from well platform *n*. Thus, *zf* and *zl* are zero when the *k*th drilling rig is not used to drill from platform *n*:

$$zf_{n,p,k,t} \le zd_{n,p,k,t} \qquad \forall n \in \mathrm{WP}(p), p \in \mathrm{PP} \qquad \textbf{(A37)}$$

$$zl_{n,p,k,t} \le zd_{n,p,k,t} \qquad \forall n \in \mathrm{WP}(p), p \in \mathrm{PP} \qquad \textbf{(A38)}$$

Any move of rig *k* from period *t – 1* to t is calculated from the value of zf_t and zl_{t-1}. The minimum number of moves required within a time period is then calculated from the number of well platforms on which rig *k* is located in that period.

The movement of the *k*th rig from platform *n* across time periods based on whether the rig is located first in period *t* on *n* and last in period *t – 1* is determined by:

$$zf_{n,p,k,t} - zl_{n,p,k,t-1} \le ZT_{n,p,k,t} \qquad \forall n \in \mathrm{WP}(p), p \in \mathrm{PP}$$

$$\left(zf_{n,p,k,t}, zl_{n,p,k,t-1}, ZT_{n,p,k,t}\right) = (0,0,0),(0,0,1),(1,0,1),(0,1,0),(0,1,1),(1,1,0),(1,1,1)$$

(A39)

The movement of the rigs within a time period from the number of well platforms used for drilling in that period is determined by:

$$(\sum_n zd_{n,p,k,t} - 1) \le Z_{p,k,t} \qquad \forall n \in \mathrm{WP}(p),\, p \in \mathrm{PP}$$

(A40)

$$zd_{\mathrm{W1},p,k,t}, zd_{\mathrm{W2},p,k,t} = (0\ ,\ 0)(1,0),(0,1),(1,1)$$

(i) Drilling Rig Scheduling Constraints

The number of times a drilling rig is moved depends on the allocation of drilling rigs to well platforms for drilling wells.

$$Z_{p,k,t} = h_3(z_{w,n,p,t}) \qquad \forall w \in W_{\mathrm{WP}}(n), n \in \mathrm{WP}(p),\, p \in \mathrm{PP} \tag{A41}$$

(j) Flow Profile Constraints

An additional operational constraint is that the flows should be a non-increasing function of time in order to ensure a smooth flow profile.

$$x_{w,n,p,t} \ge x_{w,n,p,t+1} - \Omega^{\mathrm{u}}(1 - \sum_{t'=1}^{t} z_{w,n,p,t'}) \qquad \forall w \in W_{\mathrm{WP}}(n), n \in \mathrm{WP}(p),\, p \in \mathrm{PP} \tag{A42}$$

(k) Objective Function

$$\begin{aligned}
\max\ \Psi = & \underbrace{\sum_t c_{1,t} x_{\mathrm{total},t}}_{\text{income from sales}} - \underbrace{\sum_t \sum_{p \in \mathrm{PP}} c_{p,2,t} z_{p,t} + c_{p,3,t} e_{p,t}}_{\text{investment costs of the production platform}} \\
& - \underbrace{\sum_t \sum_{p \in \mathrm{PP}} \sum_{n \in \mathrm{WP}(p)} c_{n,p,2,t} z_{n,p,t} + c_{n,p,3,t} e_{n,p,t}}_{\text{investment costs of the well platform}} \\
& - \underbrace{\sum_t \sum_{p \in \mathrm{PP}} \sum_{n \in \mathrm{WP}(p)} \sum_{n \in W_{\mathrm{WP}}(p)} c_{w,n,p,2,t} z_{w,n,p,t}}_{\text{drilling costs of wells}} \\
& - \underbrace{\left(\sum_t c_{4,t} \sum_{k \in D} \sum_{p \in \mathrm{PP}} z_{p.k.t} + \sum_t c_{4,t} \sum_{k \in D} \sum_{p \in \mathrm{PP}} \sum_{n \in \mathrm{WP}(p)} ZT_{n,p,k,t} \right)}_{\text{costs for moving the drilling rigs}}
\end{aligned} \tag{A43}$$

In the equation above, the first sum corresponds to the income from sales, the second and third terms are the investment costs of the production and well platforms, respectively, the fourth term represents the drilling costs of wells, and the last term is the sum of costs for moving the drilling rigs.

For our computation, we consider an oil field with 4 wells (W1, W2, W3, W4), 1 well platform (N1), and 1 production platforms (P1). We consider 6 time periods for our computation (T1 to T6). The data for computation is referred to Iyer, **1998** paper, and it is as shown in Table A3.1 below:

TABLE A3.1: Data for production planning computation

Description	Unit	Symbol	Facility	T1	T2	T3	T4	T5	T6
Productivity index of well	NA	ρ	W1	47.6	TBD	TBD	TBD	TBD	TBD
			W2	83.8	TBD	TBD	TBD	TBD	TBD
			W3	158.4	TBD	TBD	TBD	TBD	TBD
			W4	210.2	TBD	TBD	TBD	TBD	TBD
Discounted revenue price coefficient for oil sales	$/barrel	$c_{1,t}$	N/A	22	22	22	22	22	22
Discounted fixed cost coefficient for capital investment	million $	$c_{p,2,t}$	P1	70	69	68	67	66	65
		$c_{n,p,2,t}$	N1	11	10	9	8	7	6
		$c_{w,n,,p,2,t}$	W1	6.69	6.69	6.69	6.69	6.69	6.69
			W2	6.34	6.34	6.34	6.34	6.34	6.34
			W3	5.76	5.76	5.76	5.76	5.76	5.76
			W4	5.83	5.83	5.83	5.83	5.83	5.83
Discounted variable cost coefficient for capital investment	million $	$c_{p,3,t}$	P1	0.47	0.46	0.45	0.44	0.43	0.42
		$c_{n,p,3,t}$	N1	0.19	0.18	0.17	0.16	0.15	0.14
Discounted cost coefficient for moving rigs	million $	$c_{4,t}$	N/A	50	49	48	47	46	45

*TBD = to be determined (Using piecewise linear interpolation)

A3.2 STOCHASTIC MODEL FORMULATION FOR OILFIELD PRODUCTION PLANNING

A3.2.1. Exogeneous uncertainty – price

Price is a type of exogeneous uncertainty since it is beyond our control. The price of oil will not be affected by our operations.

A mixed integer programming model for optimal development of an oil field under uncertain future oil prices was developed earlier (Jonsbraten, **1998**). A finite set of oil price scenarios with associated probabilities was given, and the scenario and policy aggregation technique developed by Rockafellar and wets was used to solve the problem.

A near replica of earlier work will be attempted here, by taking into account the exogenous uncertainty of price. A set of scenarios are generated, namely 1, 2, and 3. These scenarios represent the price scenarios, each with their own probability values.

The prices of oil for each scenario with their probabilities are shown in Table A3.2 below:

TABLE A3.2: Oil price scenarios

Scenario, S	Oil price, $/barrel	Probability
1	17	0.4
2	19	0.3
3	22	0.3

The values for the oil prices are not in accordance to the current price (oil in year 2009 is around USD50 per barrel). This is because the values are taken from the Iyer's 1998 paper.

The formulation for the objective function has to take into account the oil price for different scenarios, with their own values of probability, by adding another subscript to discounted revenue price coefficient for oil sales, c_{1t}.

A3.2.2. Endogeneous uncertainty – reserves

Reserves are considered to be a type of endogeneous uncertainty. This is due to the fact that the accuracy of our predictions of reserves values can only be assessed after we have begun our operations.

Optimal investment and operational planning of gas field developments under uncertainty in gas reserves has been considered by Goel, **2004**. A novel stochastic programming model that incorporates the decision – dependence of the scenario tree is presented. In the paper, they take into account the uncertainty of size and initial deliverability in the fields of the gas fields. The problem has been formulated as a multistage stochastic program.

The problem of optimal investment and operational planning for development of gas fields under uncertainty in gas reserves have been attempted (Goel, **2006**). This takes the same type of uncertainty as the Goel, **2004** paper, which is size and initial deliverabilities of the gas fields. A Lagrangean duality based branch and bound algorithm to solve the stochastic programming model was presented.

The same approach will be taken in our consideration of endogeneous uncertainty of reserves, which are the size and initial deliverabilities. The difference here is that we are attempting on an oil well platform, whereas previous works were on gas fields.

(a) Model equations

The oil reserves of a field are characterized by the “size” and “deliverability” of the field or well platform. The size of a well platform refers to the total amount of oil that can be recovered from the wells connected to it, while the deliverability of a well platform at any time is the maximum rate of oil production that can be obtained from the wells connected to it. The deliverability of a well platform is highest (initial deliverability) when no oil has been recovered from the wells connected to it, and decreases with increase in cumulative production from the wells. When the cumulative production from the wells equals the size of the well platform, the deliverability reduces to zero and hence, no more oil can be produced. In this paper, we assume that the deliverability of a well decreases linearly with the increase in cumulative production from the well platform. Thus, we assume a linear reservoir model, as shown in Figure A3.2 below:

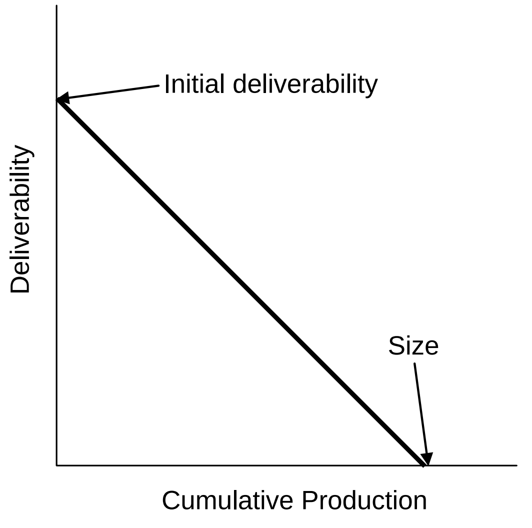


FIGURE A3.2: Linear reservoir model for an oil well platform

Note that in reality, reservoir behavior is characterized by a complex system of partial differential equations. In the planning process, this behavior is frequently approximated by simplified algebraic models. For the sake of simplicity, we have used a linear model to approximate the reservoir behavior.

We assume discrete probability distributions for the sizes and initial deliverabilities of all wells/well platforms. Let $\theta_{\mathrm{Size},w,n,p,t,s}$ and $\theta_{\mathrm{ID},w,n,p,t,s}$ represent the set of possible realizations in the distributions for the size and initial deliverability, respectively, of well *w*. the overall set of possibilities is represented by a set of “scenarios”, where each scenario is one possible combination of values for the sizes and initial deliverabilities of all fields, and has a given probability. We assume that the set of scenarios consists of every possible combination of realizations for the sizes and initial deliverabilities of various wells. Figure A3.3 shows the nine possible reservoir models for a well with three realizations each for its size and its initial deliverability.

For a problem where decisions have to be made only for this field, each of the nine reservoir models shown in Figure A3.3 corresponds to a scenario.

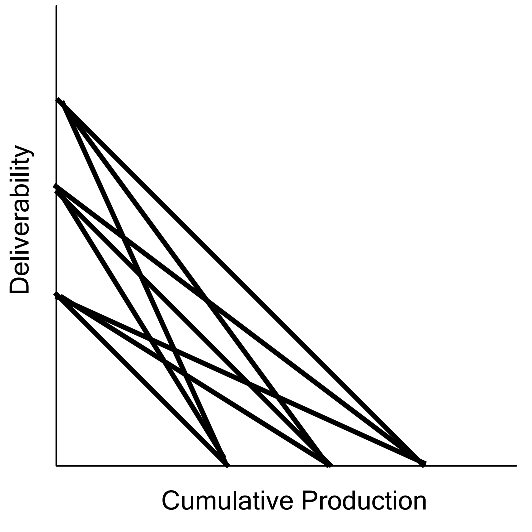


FIGURE A3.3: Nine scenarios arising from uncertainty in size and initial deliverability of well

The scenarios with their respective probability values (in brackets) are shown in Figure A3.4 below:

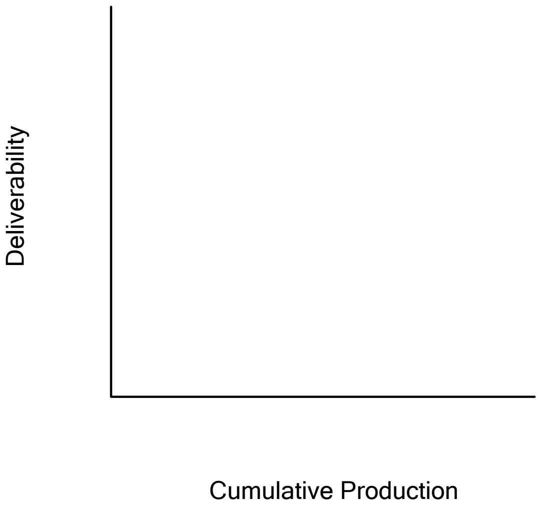


FIGURE A3.4: Nine scenarios with their probability values

The scenarios, along with their values for size, initial deliverability, and probability, is shown in Table A3.3 below:

TABLE A3.3: Size, initial deliverability and probability for each scenario

Scenario	Size	Initial deliverability	Probability
1	300	50	0.09
2	600	50	0.12
3	900	50	0.09
4	300	100	0.12
5	600	100	0.16
6	900	100	0.12
7	300	150	0.09
8	600	150	0.12
9	900	150	0.09

To model the scenarios, a new equation is introduced that takes relates the deliverability of oil from a well, i.e. the maximum oil flow from a well, $x_{D,w,n,p,t,s}$, the cumulative oil flow up to period t, $\hat{x}_{w,n,p,t,s}$, the size of the well, $\theta_{Size,w,n,p,t,s}$, and the initial deliverability of the well, $\theta_{ID,w,n,p,t,s}$. This equation is derived from Figure

A3.2, where deliverability represents oil flow from a well. The steps are as shown below:

- Slope of the line, $m = -\frac{\theta_{\text{ID},w,n,p,t,s}}{\theta_{\text{Size},w,n,p,t,s}}$
- Put it in the form y = mx +c:

$$x_{\text{D},w,n,p,t,s} = -\frac{\theta_{\text{ID},w,n,p,t,s}}{\theta_{\text{Size},w,n,p,t,s}} \hat{x}_{w,n,p,t,s} + c \qquad (1)$$

- Taking the coordinate of $\hat{x}_{w,n,p,t,s} = 0$, and $x_{\text{D},w,n,p,t,s}$ = initial deliverability = $\theta_{\text{ID},w,n,p,t,s}$, substituting into (1), and solving:

$$\theta_{\text{ID},w,n,p,t,s} = -\frac{\theta_{\text{ID},w,n,p,t,s}}{\theta_{\text{Size},w,n,p,t,s}}(0) + c$$

$$c = \theta_{\text{ID},w,n,p,t,s}$$

- Substituting values of m and c into (1), and rearranging:

$$x_{\text{D},w,n,p,t,s} = -\frac{\theta_{\text{ID},w,n,p,t,s}}{\theta_{\text{Size},w,n,p,t,s}} \hat{x}_{w,n,p,t,s} + \theta_{\text{ID},w,n,p,t,s}$$

$$x_{\text{D},w,n,p,t,s} + \frac{\theta_{\text{ID},w,n,p,t,s}}{\theta_{\text{Size},w,n,p,t,s}} \hat{x}_{w,n,p,t,s} = \theta_{\text{ID},w,n,p,t,s}$$

- Dividing both LHS and RHS by $\theta_{\text{ID},w,n,p,t,s}$:

$$\frac{x_{\text{D},w,n,p,t,s}}{\theta_{\text{ID},w,n,p,t,s}} + \frac{\hat{x}_{w,n,p,t,s}}{\theta_{\text{Size},w,n,p,t,s}} = 1 \qquad \textbf{(A44)}$$

Deliverability is the maximum oil flow from a well, so the oil flow from a well must not be greater than the deliverability of the well at that time and scenario:

$$x_{w,n,p,t,s} \le x_{\mathrm{D},w,n,p,t,s} \tag{A45}$$

The cumulative oil production from a well at most can be equal to the size of the well:

$$\hat{x}_{w,n,p,t,s} \le \theta_{\mathrm{Size},w,n,p,t,s} \tag{A46}$$

The objective function, Ψ, has to account for all the scenarios. It is to maximize the expected profit, which is the probability weighted average of the profits over all scenarios:

$$\max \psi = \sum_{s} \psi_s p_s \tag{A47}$$

(b) Non-anticipativity constraints

Decisions for different scenarios are linked to each other by non-anticipativity constraints. These constraints ensure that decisions at any time are based only on information available at that time, and not on foresight. Scenarios *s*, *s'* are said to be indistinguishable at some time *t* if *s*, *s'* are identical in realizations for all parameters in which uncertainty has been resolved up till that time. The non-anticipativity rule states that if two scenarios are indistinguishable at some time, then decisions at that time should be the same in the two scenarios.

The decision variables involved in our model are $z_{w,n,p,t,s}$, $z_{n,p,t,s}$, $z_{p,t,s}$, $zd_{n,p,k,t,s}$, $ZT_{w,n,p,t,s}$, $x_{w,n,p,t,s}$, $e_{n,p,t,s}$ and $e_{n,p,t,s}$,. The non-anticipativity constraints are as shown below:

$$z_{w,n,p,t,s} = z_{w,n,p,t,s'} \qquad \forall w \in W_{\mathrm{WP}}(n), n \in \mathrm{WP}(p), p \in \mathrm{PP}, s < s' \tag{A48}$$

$$z_{n,p,t,s} = z_{n,p,t,s'} \qquad \forall n \in \mathrm{WP}(p), p \in \mathrm{PP}, s < s' \tag{A49}$$

$$z_{p,t,s} = z_{p,t,s'} \quad \forall p \in \text{PP}, s < s' \tag{A50}$$

$$zd_{n,p,k,t,s} = zd_{n,p,k,t,s'} \quad \forall n \in \text{WP}(p), p \in \text{PP}, s < s' \tag{A51}$$

$$ZT_{w,n,p,t,s} = ZT_{w,n,p,t,s'} \quad \forall w \in W_{\text{WP}}(n), n \in \text{WP}(p), p \in \text{PP}, s < s' \tag{A52}$$

$$x_{w,n,p,t,s} = x_{w,n,p,t,s'} \quad \forall w \in W_{\text{WP}}(n), n \in \text{WP}(p), p \in \text{PP}, s < s' \tag{A53}$$

$$e_{n,p,t,s} = e_{n,p,t,s'} \quad \forall n \in \text{WP}(p), p \in \text{PP}, s < s' \tag{A54}$$

$$e_{p,t,s} = e_{p,t,s'} \quad \forall p \in \text{PP}, s < s' \tag{A55}$$

Note that non-anticipativity constraints are imposed only on the decision variables. All other variables are "state variable" that can be eliminated from the model. Hence, non-anticipativity is not imposed on those variables. Also, to avoid duplication of non-anticipativity constraints for a pair of scenarios, constraints **(A48) – (A55)** are applied for (*s*,*s'*) only if $s < s'$.

The domains for the variables are specified as below:

$z_{w,n,p,t,s} \in \{0,1\} \quad \forall w \in W_{\text{WP}}(n), n \in \text{WP}(p), p \in \text{PP}, s$;

$z_{n,p,t,s} \in \{0,1\} \quad \forall n \in \text{WP}(p), p \in \text{PP}, s$;

$z_{p,t,s} \in \{0,1\} \quad \forall p \in \text{PP}, s$;

$zd_{n,p,k,t,s} \in \{0,1\} \quad \forall n \in \text{WP}(p), p \in \text{PP}, s$;

$ZT_{w,n,p,t,s} \in \{0,1\} \quad \forall w \in W_{\text{WP}}(n), n \in \text{WP}(p), p \in \text{PP}, s$;

$x_{w,n,p,t,s} \geq 0 \quad \forall w \in W_{\text{WP}}(n), n \in \text{WP}(p), p \in \text{PP}, s$;

$e_{n,p,t,s} \geq 0 \quad \forall n \in \text{WP}(p), p \in \text{PP}, s$;

$e_{p,t,s} \geq 0 \quad \forall p \in \text{PP}, s$

A3.2.3 Scenario Generation

Given two random variables X and Y, the joint distribution of X and Y defines the probability of events defined in terms of both X and Y. The sum of all joint probabilities must be equal to 1. If discrete random variables X and Y are independent of each other, the joint distribution, i.e. the joint probability, is the probability of event Y occurring at the same time event X occurs, are formulated as below:

$$P(X = x \, and \, Y = y) = P(X = x).P(Y = y) \qquad \textbf{(A55)}$$

Since we have two separate uncertainties for our model, i.e., price and reserves, they need to be linked to each other to generate practical scenarios. Joint probability distribution will be used to link between the scenarios, since they are discrete random variables independent of each other. The calculation for joint probability is shown in Table A3.4 below:

TABLE A3.4: Joint probability distribution calculations for scenario generation

Reserves			Price ($/barrel)			Total Probability
			17	**19**	**22**	
Size	**Initial Deliverability**	**Probability**	0.4	0.3	0.3	1
300	**50**	0.09	0.036	0.027	0.027	0.09
600	**50**	0.12	0.048	0.036	0.036	0.12
900	**50**	0.09	0.036	0.027	0.027	0.09
300	**100**	0.12	0.048	0.036	0.036	0.12
600	**100**	0.16	0.064	0.048	0.048	0.16
900	**100**	0.12	0.048	0.036	0.036	0.12
300	**150**	0.09	0.036	0.027	0.027	0.09
600	**150**	0.12	0.048	0.036	0.036	0.12
900	**150**	0.09	0.036	0.027	0.027	0.09
Total Probability		1	0.4	0.3	0.3	1

CHAPTER A4
COMPUTATIONAL EXPERIMENTS AND DISCUSSIONS ON NUMERICAL RESULTS

A4.1 COMPUTATIONAL RESULTS

The formulated logical constraints are coded on GAMS. Please refer to Appendix II and IV for the GAMS input file code and Appendix III and V for the output file containing the result generated by GAMS/CPLEX 10.

Numerical studies and computational experiments of the proposed model in this paper is implemented on GAMS 22.3 for Windows XP platform. The model is solved using the branch-and-cut algorithms available in GAMS/CPLEX 10 on the computing facilities with attributes listed in Table A4.1 below:

TABLE A4.1: Attributes of computing facilities for computational experiments

Computer Type	Laptop (Lenovo)
Processor Type	Intel Centrino Duo
Processor Speed	1.73 GHz
RAM	512 MB

The computational statistics including the CPU times on the mentioned machine are as reported in Table A4.2 below:

TABLE A4.2: Computational statistics for the deterministic and stochastic models

Solver	Model	Number of single continuous variables	Number of discrete (binary 0–1) variables	Number of single equations	Resource usage/ CPU time (s)	Number of iterations
CPLEX 10	Deterministic	386	114	446	0.468	67
	Stochastic	3619	1026	5125	0.500	245

The result of the deterministic model is shown in Table A4.3 below:

TABLE A4.3: Deterministic model results

Well/Time	T1	T2	T3	T4	T5	T6
W1	0	0	0	0	0	1
W2	1	0	0	0	0	0
W3	0	0	1	0	0	0
W4	0	1	0	0	0	0

The result of the stochastic model is shown in Table A4.4 below:

TABLE A4.4: Stochastic model results

Time		T1				T2				T3			
Well		W1	W2	W3	W4	W1	W2	W3	W4	W1	W2	W3	W4
Scenario	S1	0	1	0	0	0	0	1	0	1	0	0	0
	S2	0	1	0	0	0	0	1	0	1	0	0	0
	S3	0	1	0	0	0	0	1	0	1	0	0	0
	S4	0	1	0	0	0	0	1	0	1	0	0	0
	S5	0	1	0	0	0	0	1	0	1	0	0	0
	S6	0	1	0	0	0	0	1	0	1	0	0	0
	S7	0	1	0	0	0	0	1	0	1	0	0	0
	S8	0	1	0	0	0	0	1	0	1	0	0	0
	S9	0	1	0	0	0	0	1	0	1	0	0	0

Time		T4				T5				T6			
Well		W1	W2	W3	W4	W1	W2	W3	W4	W1	W2	W3	W4
Scenario	S1	0	0	0	0	0	0	0	1	0	0	0	0
	S2	0	0	0	0	0	0	0	1	0	0	0	0
	S3	0	0	0	0	0	0	0	1	0	0	0	0
	S4	0	0	0	0	0	0	0	1	0	0	0	0
	S5	0	0	0	0	0	0	0	1	0	0	0	0
	S6	0	0	0	0	0	0	0	1	0	0	0	0
	S7	0	0	0	0	0	0	0	1	0	0	0	0
	S8	0	0	0	0	0	0	0	1	0	0	0	0
	S9	0	0	0	0	0	0	0	1	0	0	0	0

A summary of the computational results obtained from the deterministic and stochastic models are shown in Table A4.5 below:

TABLE A4.5: Computational results for the deterministic and stochastic models

No.	Description	Symbol	Unit	Deterministic Model	Stochastic Model
1	Objective function (Profit)	Z	Million dollars	331.08	145.58
2	Wells drilled	Z1	-	W2 (T1) W4 (T2) W3 (T3) W1 (T6)	W2 (T1) W3 (T2) W1 (T3) W4 (T5)
3	Well platforms installed	Z2	-	N1 (in T1, T2, T3, T6)	N1 (in T1, T2, T3, T5)
4	Production platforms installed	Z3	-	P1 (in T1, T2, T3, T6)	P1 (in T1, T2, T3, T5)
5	Oil flow for wells	X1	Million barrel	5MMbls per well	5MMbls per well
6	Oil flow for well platforms	X2	Million barrel	5MMbls from N1 (in T1, T2, T3, T6)	5MMbls from N1 (in T1, T2, T3, T5)
7	Oil flow for production platforms	X3	Million barrel	5MMbls from P1 (in T1, T2, T3, T6)	5MMbls from P1 (in T1, T2, T3, T5)

A4.2 DISCUSSION

From the results obtained, it can be seen not only the costs alone are taken into account to choose which well to be drilled. Other factors include the productivity index of the well, the size, and the initial deliverability of the wells. In order to maximize the profit, which is the objective function of the model, it will select the facilities with lower cost, and also taking into account the productivity of oil from the wells. As can be seen from the deterministic model, well 2 (W1) is chosen to be drilled at time period 1 (T1). This can be attributed to the fact that it has higher productivity index compared to W1, and also a lower cost of drilling. Besides that, it is chosen to be drilled at T1 because the profit for sales of oil at T1 is the highest.

Also, it can be seen that, the oil flows have non-zero values only for wells, well and productions platforms that are chosen to be installed. This is in accordance to the fact that, oil can only flow from wells, well or productions platforms that are either drilled or installed, enforced by the constraints in Chapter A3.

An observation on the results of the stochastic model shows that the non-anticipativity constraints are enforced, since all the scenarios are the same for the same time period T, as can be seen from Figure A4.2. This non-anticipativity constraint is an important part of our model because it ensures that decisions at any time are based only on information available at that time, and not on foresight. Our results are in accordance to the non-anticipativity rule, which states that if two scenarios are indistinguishable at some time, then decisions at that time should be the same in the two scenarios, as the case with our results.

The objective function obtained from the stochastic model is 145.58 million dollars profit, lower than the value obtained from the deterministic model (331.08 million dollars). In this case, the stochastic model is a better representation of the problem. This is because, it takes into account the various scenarios associated with uncertainties in price, size and initial deliverability of the well. Each of the scenarios has their own unique probability values. The representation of the problem is more thorough since the objective function has to account for all the scenarios by taking the weighted average of the profits over all scenarios.

CHAPTER A5
CONCLUSIONS AND RECOMMENDATIONS

A5.1 CONCLUSIONS

An optimization model on the infrastructures planning for an offshore oil field was developed with the objective function of maximizing profit, subject to constraints of mass balance, pressure balance, and flow constraints, among others. Two models were proposed, namely the deterministic model and the stochastic model. The deterministic model does not possess uncertain parameters, whereas the stochastic model takes into account the uncertainties in price, size and initial deliverabilities. Since this is a maximization problem, the model selected the facility that returns the highest profit. Thus, the objective function has been achieved. The mathematical model was developed using GAMS and an optimal solution was found with no logical constraints conflicts or error. Although the stochastic model obtained an objective value lower than the deterministic model, it is a more representative model as it takes into account the uncertainties along with their associated probabilities. The model can be used as a basic tool for upper-management to decide on refinery technologies to be applied in a grass-root refinery.

A5.2. RECOMMENDATIONS

It is recommended that real data are taken for the data input into the model formulation. This will enable a better representation of the model, and also make the manner of comparison easier, because real data is being used. By comparing between the values in reality and what is modeled will give us a clearer picture on the accuracy of the model.

SECTION B

AN OPTIMIZATION-BASED COMPUTATIONAL FRAMEWORK FOR DOWNSTREAM CRUDE OIL SCHEDULING AT REFINERY FRONT-END

CHAPTER B1
INTRODUCTION

B1.1 BACKGROUND OF STUDY

Scheduling and planning of the flow of crude oil is an important issue in petroleum refineries. This is due to the potential realization of large savings in cost and improved feeds. Linear programming (LP) models have previously been used for scheduling and planning problems because of their ease of modeling, and also since they are relatively easy to solve. However, it is usually difficult to model refinery operations, since they involve units operating in both batch and continuous modes with multiple grades of crude oil and products. Furthermore, detailed scheduling models usually require a continuous time representation that normally includes nonlinear (MINLP) models. These models provide additional flexibility to the problem by allowing the modeling of discrete decisions and constraints.

Two major approaches for modeling scheduling problems are discrete time formulations and continuous time formulations. In discrete time models, it is relatively easy to model the material balances and the flow constraints. However, the number of time intervals required for an accurate representation of the system is usually large. Continuous time models, on the other hand, are smaller in comparison, and they allow for a complete utilization of the time domain, although it is difficult to synchronize the material balances and time sequencing constraints in such a representation. Furthermore, it is often not trivial to determine a priori the number of time points or events that are needed.

In this work, the continuous time formulation method is adopted for short-term scheduling of crude oil at the front-end of a refinery and to solve the corresponding MINLP to global optimality. This scheduling problem involves crude oil unloading from a crude supply source to the crude storage tanks, transfer of crude oil from these tanks to the charging tanks, and charging the crude distillation units (CDUs) continuously over a time horizon with crude-mixes from the charging tank.

B1.2 PROBLEM STATEMENT

The front-end of a refinery is a network consisting of crude oil supply streams, storage tanks, charging tanks, and crude distillation units (CDUs), which structure is shown in Figure B1.1 the crude supply streams are connected to the storage tanks, which are connected to the charging tanks, which in turn, are connected to the CDUs. The crude supply streams, which are vessels that carry crude, deliver crude oil to the storage tanks (intermediate tanks), which transfer the crude to the charging tanks. Different qualities of crude get blended into various crude mixtures inside the charging tanks, which are then charged directly to the distillation units.

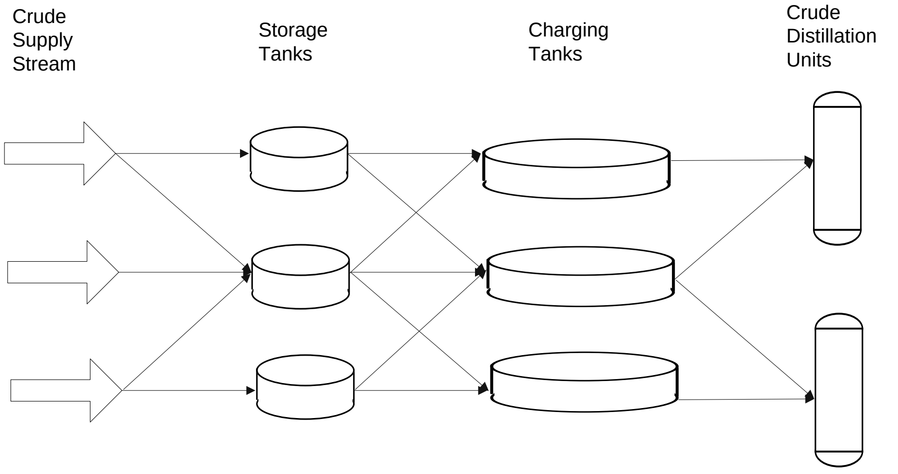


FIGURE B1.1: Superstructure representation of the crude oil scheduling operations at refinery front-end

B1.2.1 Model Information

The major assumptions in the model are as follows:The following information is given:

- the maximum and minimum inventory levels for a tank (capacity limitations);

- the initial total and component inventories in a tank;
- upper and lower bounds on the fraction of key components in the crude inside a tank (crude quality limitations);
- times of arrival of crude oil in the crude supply streams;
- amount of crude arriving in the crude supply streams;
- fractions of various components in the crude supply streams;
- demand of crude-mix to be charged from a charging tank;
- bounds on the flowrates of the streams in the network;
- time horizon for scheduling;
- cost coefficients for calculating the various costs involved;

B1.2.2 Model Constraints

Given the following operating constraints in the network:

- simultaneous inputs into and outputs from a tank cannot be allowed. This is done to allow settling of the crude-mix in a tank;
- each distillation unit may be charged by at most one charging tank over a period of time. This is another operational norm followed in many refineries;
- each charging tank may charge at most one distillation unit at a point of time;
- each charging tank has to discharge a specified amount of crude-mix to the various distillation units within the given time horizon;
- all the distillation units have to be operated continuously throughout the entire time horizon;

B1.2.3 Model Assumptions

We model the optimization of the network as a nonconvex MINLP problem. Certain assumptions are made prior to modeling the system:

- perfect mixing takes place in each tank;
- negligible change in specific gravities on mixing;
- the crude flows into and from a tank need not be continuous;
- changeover times for CDU charging are neglected;
- continuous time model;

B1.3 RESEARCH OBJECTIVES

The questions that we are interested to answer in this research project relates to the optimum values of the following items in the system in order to minimize the total operating cost of the network:

- the total inventory levels and the component inventory levels in the tanks at various instances of time;
- the total flow volume and the component flow volume from one unit to another in a certain time interval;
- start and end times of the flows in each stream present in the network;

CHAPTER B2

LITERATURE REVIEW

B2.1 PREVIOUS WORK

Karuppiah, et al (2008) presented an outer-approximation algorithm to obtain the global optimum of a nonconvex mixed-integer nonlinear programming (MINLP) model that is used to represent the scheduling of crude oil movement at the front-end of a petroleum refinery. The model relies on a continuous time representation making use of transfer events. They proposed an algorithm which focuses on effectively solving a mixed0integer linear programming (MILP) relaxation of the nonconvex MINLP to obtain a rigorous lower bound (LB) on the global optimum. Cutting planes derived by spatially decomposing the network are added to the MILP relaxation of the original nonconvex MINLP in order to reduce the solution time for the MILP relaxation. The solution of this relaxation is used as a heuristic to obtain a feasible solution to the MINLP which serves as an upper bound (UB). The lower and upper bounds are made to converge to within a specified tolerance in the proposed outer-approximation algorithm. As result of applying the proposed technique to test examples, significant savings are realized in the computational effort required to obtain provably global optimal solutions.

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The paper by Lee et al (1996) addressed the problem of inventory management of a refinery that imports several types of crude oil which are delivered by different vessels. This problem involves optimal operation of crude oil unloading, is transfer from storage tanks to charging tanks, and the charging schedule for each crude oil distillation unit. A mixed-integer optimization model is developed which relies on time discretization. The problem involves bilinear equations due to mixing operations. The LP-based branch and bound method is applied to solve the model, and several techniques, such as priority branching and bounding, and special ordered sets are implemented to reduce the computation time.

Reddy et al (2004) presented a mixed-integer linear programming (MILP)-based solution approach for optimizing crude oil unloading, storage, and processing operations in a multi-CDU refinery receiving crude from multiparcel Very Large Crude Carriers (VLCCs) Through a high-volume, single-bouy mooring (SBM) pipeline and/or single-parcel tankers through multiple jetties. Their primarily discrete-time model allows multiple sequential crude transfers to occur within a time slot. As a result, an interesting approach to this problem is presented.

Rocha et al (2008) described how mathematical programming is being used to solve the Petroleum Allocation problem and they show the effectiveness of a Local Branching method to solve real industrial problems. They proposed a MILP formulation of the problem that relies on a time/space discretization network. An algorithm was implemented based on a heuristic to find a feasible solution and on a local search procedure by optimization to improve it. As a result, solutions are found for most of the case studies within 10% of optimality in less than 5 hours.

A generalized model is proposed by Furman et al (2007) for the continuous time scheduling problem of fluid transfer in tanks. Their model generally and more robustly handles the synchronization of time events with material balances. A novel method for representing the flow to and from a tank is developed with the potential for significant reduction in the number of necessary time events required for continuous time scheduling formulations. An efficient MINLP formulation is developed based on continuous representation of time domain under the assumption of no simultaneous input and output flow to a tank for fluid streams comprised of multiple components.

Karuppiah and Grossmann (2007) presented a global optimization algorithm for solving a class of large-scale nonconvex optimization models that have a decomposable structure. Such models are very expensive to solve to global optimality. They are frequently encountered in two-stage stochastic programming problems, engineering design, and also in planning and scheduling. They proposed a specialized deterministic branch-and-cut algorithm to solve these models to global optimality, wherein bounds on the global optimum are obtained by solving convex relaxations of these models with certain cuts added to them in order to tighten the relaxations. Numerical examples are presented to illustrate the effectiveness of the proposed method compared to available commercial global optimization solvers that are based on branch and bound methods.

CHAPTER B3
MODEL FORMULATION AND SOLUTION STRATEGY

B3.1 FORMULATION

The mathematical model for the scheduling problem has largely been taken from Furman et al (2007). This is a continuous time model for scheduling for which a number of transfer events are postulated for the transfer of material between the units in the network over a given time horizon.

B3.1.1 Nomenclature

(a) Indices

A	tank input source
B	crude tank
C	tank output destination
D	distillation unit
G	charging tank
I	component
M	source unit of split pipeline
K	destination unit of split pipeline
U	crude supply stream
Y	storage tank
L	transfer event
PP	set of production platforms

PP = set of production platforms*n*allocated to*n*

(b) Sets

A	tank input sources
A_b	inputs to tank *b*
A_s	inputs to storage tank y
B	tanks
B_{Q1}	tanks belonging to sub-structure Q1
B_{Q2}	tanks belonging to sub-structure Q2
C	tank output destinations
C_b	outputs from tank *b*
C_y	outputs from storage tank *y*
C_g	outputs from charging tank *g*
D	distillation units
D_{Q1}	distillation units present in sub-structure Q1
D_{Q2}	distillation units present in sub-structure Q2
D_g	distillation units that can be charged by charging tank *g*
$G(B)$	charging tanks
G_d	charging tanks that charge distillation unit d
I	components
M	source units of the split pipelines
K_m	destination units of split pipelines with source *m*
U	crude supply streams
U_{Q1}	crude supply streams present in sub-structure Q1
U_{Q2}	crude supply streams present in sub-structure Q2
$Y(B)$	storage tanks
Y_u	storage tanks connected to crude supply stream *u*
L	transfer events

(c) Parameters

$Cinv_b$ inventory maintenance cost for tank *b*

Csea waiting cost for crude supply streams

Cset changeover cost for charged oil switch

Cunload unloading cost for crude supply streams

DM_g demand of crude-mix to be charged from charging tank *g*

$f_{i,b}^{L}$ lower bound on fraction of component *i* inside tank *b*

$f_{i,b}^{U}$ upper bound on fraction of component *i* inside tank *b*

$f_{i,u}^{supply}$ fraction of component i in crude supply stream u

$F_{a,b}^{L}$ lower bound on flowrate from *a* to *b*

$F_{a,b}^{U}$ upper bound on flowrate from *a* to *b*

H time horizon for scheduling

$I_b^{init\text{-}tot}$ initial total inventory of tank *b*

$I_{i,b}^{init}$ initial inventory of component *i* in tank *b*

I_b^{L} lower bound on total inventory in *a* tank *b*

I_b^{U} upper bound on total inventory in *a* tank *b*

ND number of distillation units in the network

ND_{Q1} number of distillation units in sub-structure Q1

ND_{Q2} number of distillation units in sub-structure Q2

NE number of transfer events

$T_u^{arrival}$ arrival time of crude in crude supply stream *u*

V_u^{supply} total volume of crude oil arriving in crude supply stream *u*

$V_{a,b}^{L}$ lower bound on flow volume from *a* to *b*

$V_{b,c}^{L}$ lower bound on flow volume from *b* to *c*

$V_{a,b}^{U}$ upper bound on flow volume from *a* to *b*

$V_{b,c}^{U}$ upper bound on flow volume from *b* to *c*

$\lambda_{m,k,l}^{T1}$ Lagrange multiplier

$\lambda_{m,k,l}^{T2}$ Lagrange multiplier

$\lambda^{V\mathrm{tot}}_{m,k,l}$ Lagrange multiplier

$\lambda^{VC}_{m,k,l}$ Lagrange multiplier

$\lambda^{w}_{m,k,l}$ Lagrange multiplier

(d) Continuous Variables

$I_{\mathrm{tot},b,l}$ total inventory of tank *b* at the end of transfer event *l*

$I_{i,b,l}$ inventory of component *i* in tank *b* at the end of transfer event *l*

$T_{\mathrm{start},a,b,l}$ starting time of a transfer from *a* to *b* in transfer event *l*

$T_{\mathrm{start},b,c,t}$ starting time of a transfer from *b* to *c* in transfer event *l*

$T_{\mathrm{end},a,b,l}$ ending time of a transfer from *a* to *b* in transfer event *l*

$T_{\mathrm{end},b,c,t}$ ending time of a transfer from *b* to *c* in transfer event *l*

$T_{\mathrm{end},u}$ overall ending time of crude transfer from crude supply stream *u*

$T_{\mathrm{start},u}$ initial starting time of crude transfer from crude supply stream *u*

$V_{\mathrm{tot},a,b,l}$ total flow from *a* to *b* in transfer event *l*

$V_{\mathrm{tot},b,c,l}$ total flow from *b* to *c* in transfer event *l*

$V_{\mathrm{tot},i,a,b,l}$ flow of component *i* from *a* to *b* in transfer event *l*

$V_{\mathrm{tot},i,b,c,l}$ flow of component *i* from *b* to *c* in transfer event *l*

(e) Binary variables

$w_{a,b,l}$ = 1 if there is a flow from *a* to *b* in transfer event *l*

$w_{b,c,l}$ = 1 if there is a flow from *b* to *c* in transfer event *l*

= 0 otherwise

B3.1.2 Model Equations

(a) Tank Constraints

The majority of the constraints in the model pertain to the crude tanks in the network.

The representation of a crude tank $b \in B$ with crude input source a and a crude output destination c is shown in Figure B3.1

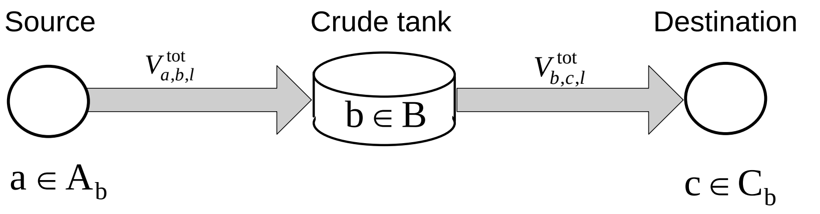

FIGURE B3.1: Crude tank representation

(i) Big-M logical constraints for flow transfers:

$$V_{tot,a,b,t} \le V^{U}_{a,b} w_{a,b,l} \qquad \forall a \in A_b,\ \forall b \in B,\ \forall l \in L \qquad \textbf{(B1)}$$

$$V_{tot,b,c,t} \le V^{U}_{b,c} w_{b,c,l} \qquad \forall c \in C_b,\ \forall b \in B,\ \forall l \in L \qquad \textbf{(B2)}$$

These constraints force the total flow in a stream $V_{tot,a,b,t}$ from a source tank a to any destination b in a particular transfer event l to zero if the binary variable, $w_{a,b,t}$, which enforces the existence of flow in that stream in transfer event l, takes a value of zero. Note that the first subscript in the total flow variable denotes the source from where the flow is taking place, while the second subscript denotes the destination to where the flow is going. The third and final subscript denotes the transfer event when the particular flow occurs.

The binary variable, $w_{a,b,t}$, represents the existence of flow between source a and tank b in transfer event l. The same is true for binary variable, $w_{b,c,t}$, which takes on a value of 1 or 0, respectively, depending on whether or not there is flow between tank b and a destination unit c in transfer event l.

(ii) Duration constraints

Duration constraints for flows: (i) between source and tank and (ii) between tank and destination

$$F^{\mathrm{U}}_{a,b}\left(T_{\mathrm{end},a,b,l}-T_{\mathrm{start},a,b,l}\right)+F^{\mathrm{U}}_{a,b}H\left(1-w_{a,b,l}\right)\geq V_{\mathrm{tot},a,b,l} \quad \forall a\in A_b,\ \forall b\in B,\ \forall l\in L \quad \textbf{(B3.i)}$$

$$F^{\mathrm{U}}_{b,c}\left(T_{\mathrm{end},a,b,l}-T_{\mathrm{start},a,b,l}\right)+F^{\mathrm{U}}_{b,c}H\left(1-w_{b,c,l}\right)\geq V_{\mathrm{tot},b,c,l} \quad \forall c\in C_b,\ \forall b\in B,\ \forall l\in L \quad \textbf{(B3.ii)}$$

For a flow between source *a* and tank *b*, the timing variables $T_{\mathrm{start},a,b,l}$ and $T_{\mathrm{end},a,b,l}$ correspond to the start and end times of flow in a stream from *a* to *b* in transfer event *l*.

The timing variables $T_{\mathrm{start},b,c,l}$ and $T_{\mathrm{end},b,c,l}$ are similarly defined for a flow between tank *b* and a destination *c* in transfer event *l*. *H* is the overall time horizon of operation. These constraints are relaxed, i.e., they are redundant when $w_{a,b,l}$ = 0. Thus, the timing variables can take on any value if there is no flow in a certain transfer event.

The above is expressed through big-*M* logical constraints, which state that if there is a flow in a stream in the network in transfer event *l*, then the product of the upper bound on the flowrate of the crude stream with the duration of flow in the transfer event gives an upper bound on the total flow volume in that transfer event.

$$\begin{aligned}
&w_{a,b,t}=0:\\
&\qquad F^{\mathrm{U}}_{a,b}\left(T_{2,a,b,t}-T_{1,a,b,t}\right)+F^{\mathrm{U}}_{a,b}H\geq V_{\mathrm{tot},a,b,t}\Rightarrow \text{the constraint is relaxed}\\
&w_{a,b,t}=1:\\
&\qquad F^{\mathrm{U}}_{a,b}\left(T_{2,a,b,t}-T_{1,a,b,t}\right)\geq V_{\mathrm{tot},a,b,t}
\end{aligned}$$

Similarly, as given in equations (B4.i) and (B4.ii), if there is a flow in transfer event *l* into or from a tank, the lower bound on the volume of a flow is obtained by multiplying the fluid flowrate lower bound with the duration of flow:

$$F_{a,y}^{\mathrm{L}}\left(T_{\mathrm{end},a,y,l}-T_{\mathrm{start},a,y,l}\right)-F_{a,s}^{\mathrm{L}}H\left(1-w_{a,y,l}\right)\le V_{\mathrm{tot},a,y,l}\quad \forall a\in A_b,\ \forall y\in Y,\ \forall l\in L \qquad \textbf{(B4.i)}$$

$$F_{y,c}^{\mathrm{L}}\left(T_{\mathrm{end},y,c,l}-T_{\mathrm{start},y,c,l}\right)-F_{y,c}^{\mathrm{L}}H\left(1-w_{y,c,l}\right)\le V_{\mathrm{tot},y,c,l}\quad \forall a\in C_y,\ \forall y\in Y,\ \forall l\in L \qquad \textbf{(B4.ii)}$$

$$F_{g,c}^{\mathrm{L}}\left(T_{\mathrm{end},g,c,l}-T_{\mathrm{start},g,c,l}\right)\le V_{\mathrm{tot},g,c,l}\quad \forall c\in C_g,\ \forall g\in G,\ \forall l\in L \qquad \textbf{(B4.iii)}$$

We should note that for the charging tanks, the start and end times have to coincide if there is no flow in a particular time event (B4.iii). This enforces the continuity of operation of the CDUs under the condition that only one charging tank can charge a CDU in a certain transfer event.

(iii) Simple sequencing constraints

A flow into or from a tank *b* in transfer event *l* has to take place before the same flow in event (*l* + 1). Equations (B5) – (B10) correspond to this necessary condition.

For flows between source and tank and between tank and destination:

$$T_{\mathrm{start},a,b,(l+1)}\ge T_{\mathrm{end},a,b,l}-H\left(1-w_{a,b,l}\right)\qquad \forall a\in A_b,\ \forall b\in B,\ \forall l\in L,\ l<\left|L\right| \qquad \textbf{(B5)}$$

$$T_{\mathrm{start},a,b,(l+1)}\ge T_{\mathrm{start},a,b,l}\qquad \forall a\in A_b,\ \forall b\in B,\ \forall l\in L,\ l<\left|L\right| \qquad \textbf{(B6)}$$

$$T_{\mathrm{end},a,b,(l+1)}\ge T_{\mathrm{end},a,b,l}\qquad \forall a\in A_b,\ \forall b\in B,\ \forall l\in L,\ l<\left|L\right| \qquad \textbf{(B7)}$$

For flows into or from a tank:

$$T_{\mathrm{start},b,c,(l+1)}\ge T_{\mathrm{end},b,c,l}-H\left(1-w_{b,c,l}\right)\qquad \forall c\in C_b,\ \forall b\in B,\ \forall l\in L,\ l<\left|L\right| \qquad \textbf{(B8)}$$

$$T_{\mathrm{start},b,c,(l+1)}\ge T_{\mathrm{start},b,c,l}\qquad \forall c\in C_b,\ \forall b\in B,\ \forall l\in L,\ l<\left|L\right| \qquad \textbf{(B9)}$$

$$T_{\mathrm{end},b,c,(l+1)}\ge T_{\mathrm{end},b,c,l}\qquad \forall c\in C_b,\ \forall b\in B,\ \forall l\in L,\ l<\left|L\right| \qquad \textbf{(B10)}$$

If no flow exists between *a* and *b* in transfer event *l* (that is, $w_{a,b,l} = 0$), then the big-M inequality (B5) is relaxed. This is because the constraint simply becomes redundant (as $T_{\text{start},a,b,(l+1)}$ is definitely greater than $T_{\text{end},a,b,l}$ and so subtracting with H would make the RHS value even smaller compared to the LHS)

$$
\begin{aligned}
&T_{\text{start},a,b,(l+1)} \geq T_{\text{end},a,b,l} - H\left(1 - w_{a,b,l}\right)\\
&w_{a,b,l} = 0:\\
&T_{\text{start},a,b,(l+1)} \geq T_{\text{end},a,b,l} - H
\end{aligned}
$$

Similarly, if there is no flow between *b* and *c* in transfer event *l* (that is, $w_{b,c,l} = 0$), then the big-M inequality (B8) is relaxed. Essentially, it means that if there is no flow in a stream in a transfer event *l*, then the values taken by the variables pertaining to the start and end times of flow in transfer event *l* are meaningless and do not affect the flow times in the next transfer event when there is flow.

(iv) Input and output restraints for the entire horizon

A set of constraints has to enforce the condition that any inputs or outputs of the current transfer event *t* must occur after the inputs and outputs of the preceding transfer event. The inclusion of these time constraints, which are expressed as big-M constraints, enforces the material balances to be calculated properly across all tanks in the same transfer event.

$$T_{\text{start},a,b,(l+1)} \geq T_{\text{end},a',b,l} - H\left(1 - w_{a',b,l}\right) \qquad \forall a, a' \in A_b, a \neq a', \forall b \in B, \forall l \in L, l < |L| \quad \textbf{(B11.i)}$$

$$T_{\text{start},a,b,(l+1)} \geq T_{\text{end},b,c,l} - H\left(1 - w_{b,c,l}\right) \qquad \forall a \in A_b, \forall c \in C_b, \forall b \in B, \forall l \in L, l < |L| \quad \textbf{(B11.ii)}$$

$$T_{\text{start},b,c,(l+1)} \geq T_{\text{end},a,b,l} - H\left(1 - w_{a,b,l}\right) \qquad \forall a \in A_b, \forall c \in C_b, \forall b \in B, \forall l \in L, l < |L| \quad \textbf{(B11.iii)}$$

$$T_{\text{start},b,c,(l+1)} \geq T_{\text{end},b,c',l} - H\left(1 - w_{b,c',l}\right) \qquad \forall c, c' \in C_b, c \neq c', \forall b \in B, \forall l \in L, l < |L| \quad \textbf{(B11.iv)}$$

Also, since all inputs into a tank *b* are required to finish before any output starts from that tank *b* in any transfer event, we need the following constraint:

$$T_{\text{end},a,b,l} - H\left(1 - w_{a,b,l}\right) \le T_{\text{start},b,c,l} + H\left(1 - w_{b,c,l}\right) \qquad \forall a \in A_b, \forall c \in C_b, \forall b \in B, \forall l \in L \quad \textbf{(B12)}$$

What do we want to enforce here through this constraint?

When $w_{b,c,l}$ = 1 (i.e., output from tank b starts),

must have $w_{a,b,l}$ = 0

We do not want the condition:

$$T_{\text{end},a,b,l} - H\left(1 - w_{a,b,l}\right) \le T_{\text{start},b,c,l} + H\left(1 - w_{b,c,l}\right) \qquad \forall a \in A_b, \forall c \in C_b, \forall b \in B, \forall l \in L$$

$w_{a,b,l} = 1 (\Rightarrow$ must have $w_{b,c,l} = 0)$

$$T_{\text{end},a,b,l} \le T_{\text{start},b,c,l} + H\left(1 - w_{b,c,l}\right)$$

If $w_{b,c,l} = 1$ also, which it is not supposed to be, then obtain constraint:

$T_{\text{end},a,b,l} \le T_{\text{start},b,c,l}$

this is redundant: $T_{\text{end},a,b,l} \le T_{\text{start},b,c,l} + H$, which means that when $w_{b,c,l} = 1$, we do not need the condition $w_{b,c,l} = 0$ because it naturally holds

$$T_{\text{end},a,b,l} - H\left(1 - w_{a,b,l}\right) \le T_{\text{start},b,c,l} + H\left(1 - w_{b,c,l}\right) \qquad \forall a \in A_b, \forall c \in C_b, \forall b \in B, \forall l \in L$$

$w_{a,b,l} = 1 \Rightarrow w_{b,c,l} = 0$ and the following constraint is enforced:

$$T_{\text{end},a,b,l} \le T_{\text{start},b,c,l} + H$$

$w_{a,b,l} = 0 \Rightarrow w_{b,c,l} = 1$ and the following constraint is enforced:

$$T_{\text{end},a,b,l} - H \le T_{\text{start},b,c,l}$$

This helps in upholding material balances in the transfer event *l* and prevents the situation in which output could occur before any input into a tank.

(v) Mass Balances

For each tank $b \in B$ in the network, we have an overall inventory balance (B13 and B14), individual inventory balances (B15) for each component $i \in I$ and the total flow balances (B17 and B18).

The inventory balances imply that the inventory in tank *b* at the end of a transfer event *l* is equal to the inventory at the end of transfer event (*l* – 1) plus the volume flow into the tank from any input source *a* in transfer event *l*, minus the flow to any output destination *c* in the transfer event *l*. The variables $I_{\text{tot},b,l}$ and $I_{i,b,l}$ correspond to the total inventory and the individual component inventory in a tank b at the end of transfer event l, respectively.

$$I_{\text{tot},b,(l-1)} + \sum_{a \in A_b} V_{\text{tot},a,b,l} = I_{\text{tot},b,l} + \sum_{c \in C_b} V_{\text{tot},b,c,l} \quad \forall b \in B, \forall l \in L \qquad \textbf{(B13)}$$

$$I_{\text{tot},b,0} = I_b^{\text{init-tot}} \quad \forall b \in B \qquad \textbf{(B14)}$$

$$I_{i,b,(l-1)} + \sum_{a \in A_b} V_{i,a,b,l} = I_{i,b,l} + \sum_{c \in C_b} V_{i,b,c,l} \quad \forall i \in I, \forall b \in B, \forall l \in L \qquad \textbf{(B15)}$$

The volume flow balances imply that the total flow into or out from a tank equals the sum of the individual component flows. $V_{tot,a,b,l}$ stands for the total volume flow from any source *a* to tank *b* in transfer event *l*, while $V_{tot,b,c,l}$ represents the flow from tank *b* to a destination *c* to which this tank is connected. $V_{i,a,b,l}$ and $V_{i,b,c,l}$ are the respective component flows.

$$I_{i,b,0} = I_{i,b}^{\text{init}} \quad \forall i \in I, \forall b \in B \qquad \textbf{(B16)}$$

$$V_{\text{tot},a,b,l} = \sum_{i \in I} V_{i,a,b,l} \quad \forall a \in A_b, \forall b \in B, \forall l \in L \qquad \textbf{(B17)}$$

$$V_{\text{tot},b,c,l} = \sum_{i \in I} V_{i,b,c,l} \quad \forall c \in C_b, \forall b \in B, \forall l \in L \qquad \textbf{(B18)}$$

(vi) Component Balances

On assuming perfect mixing in a tank, the fraction of a component i in the output flow from a tank should be equal to the fraction of that component present inside the tank. In the same time event, any input flows must occur before any output flows. The final inventory of the tank from the previous time event is used, rather than the final inventory of the tank for a particular time event, to avoid the numerical irregularitires that can result when the tank is completely emptied. This constraint is formulated as follows, with bilinear terms, which give rise to be the nonconvexities of the model:

$$\left(I_{\text{tot},b,(l-1)} + \sum_{a \in A_b} V_{\text{tot},a,b,l} \right) V_{i,b,c,l} = \left(I_{i,b,(l-1)} + \sum_{a \in A_b} V_{i,a,b,l} \right) V_{\text{tot},b,c,l} \quad \forall b \in B, \forall c \in C_b, \forall i \in I, \forall l \in L$$

(B19)

(vii) Inventory Bounds

Because both the input and output of material may occur in the same time event, simple bounds on the total tank inventory variable $I_{tot,b,l}$ are not sufficient. The following constraint must hold in order to ensure that the total inventory in any transfer event does not exceed the upper bound of the inventory since both inputs and outputs can occur in the same transfer event.

$$I_{\text{tot},b,(l-1)} + \sum_{a \in A_b} V_{\text{tot},a,b,l} \le I_b^{\text{U}} \qquad \forall b \in B, \forall l \in L \qquad \textbf{(B20)}$$

The sum $\left(I_{\text{tot},b,(l-1)} + \sum_{a \in A_b} V_{\text{tot},a,b,l} \right)$ is the total inventory in a tank b in transfer event l before any output flow starts to occur from the tank in the same transfer event.

(viii) Bounds on component fractions inside a tank

The fraction of a component in the crude inside any tank should lie between given bounds. This is enforced by the following constraints:

$$f^{\mathrm{L}}_{i,b} I_{\mathrm{tot},b,l} \le I_{i,b,l} \le f^{\mathrm{U}}_{i,b} I_{\mathrm{tot},b,l} \qquad \forall i \in I, \forall b \in B, \forall l \in L \tag{B21}$$

$$f^{\mathrm{L}}_{i,b} V_{\mathrm{tot},b,c,l} \le V_{i,b,c,l} \le f^{\mathrm{U}}_{i,b} V_{\mathrm{tot},b,c,l} \qquad \forall i \in I,\ \forall b \in B,\ \forall c \in C_b, \forall l \in L \tag{B22}$$

(ix) Crude-mix demand constraints

Each charging tank $g \in \mathrm{G}$ must charge a specified amount of crude-mix over the entire scheduling horizon. This volume of crude mix is distributed to the different CDUs in the network.

$$\sum_{d \in D_g} \sum_{l} V_{\mathrm{tot},g,d,l} = \mathrm{DM}_g \qquad \forall g \in G \tag{B23}$$

(x) Bound strengthening cuts

I. The following constraints are added to the model in an attempt to tighten the relaxation of the MINLP model so as to accelerate the convergence to compute the optimal solution. These are derived using a reformulation and linearization technique given in Sherali and Alameddine (1992). In this we take Eq. (B19) and expend it to get the following equation:

$$I_{\mathrm{tot},b,(l-1)} V_{i,b,c,l} + \sum_{a \in A_b} V_{\mathrm{tot},a,b,l} V_{i,b,c,l} = I_{i,b,(l-1)} V_{\mathrm{tot},b,c,l} + \sum_{a \in A_b} V_{i,a,b,t} V_{\mathrm{tot},b,c,l} \qquad \forall b \in B, \forall c \in C_b, \forall i \in I, \forall l \in L$$

$$\begin{aligned}
&\sum_{i \in I} I_{\mathrm{tot},b,(l-1)} V_{i,b,c,l} = I_{\mathrm{tot},b,(l-1)} V_{\mathrm{tot},b,c,l} && \forall b \in B, \forall c \in C_b, \forall l \in L \\
&\sum_{i \in I} V_{\mathrm{tot},a,b,l} V_{j,b,c,l} = V_{\mathrm{tot},a,b,l} V_{\mathrm{tot},b,c,l} && \forall a \in A_b, \forall b \in B, \forall c \in C_b, \forall l \in L \\
&\sum_{i \in I} I_{l,b,(l-1)} V_{\mathrm{tot},b,c,l} = I_{\mathrm{tot},b,(l-1)} V_{\mathrm{tot},b,c,l}, && \forall b \in B, \forall c \in C_b, \forall l \in L \\
&\sum_{i \in I} V_{l,a,b,l} V_{\mathrm{tot},b,c,l} = V_{\mathrm{tot},a,b,l} V_{\mathrm{tot},b,c,l} && \forall a \in A_b, \forall b \in B, \forall c \in C_b, \forall l \in L
\end{aligned}$$

(B24)

(b) Distillation Unit Constraints

Each distillation unit $d \in D$ is modeled with the following set of constraints:

(i) Allocation constraints

The condition that each distillation unit can be charged by at most one charging tank in a transfer event is enforced by the following equation:

$$\sum_{g \in G_d} w_{g,d,l} \le 1 \qquad \forall d \in D, \forall l \in L \qquad \textbf{(B25)}$$

The condition that at most one CDU can be charged by a single charging tank in a transfer event is enforced by equation (26):

$$\sum_{d \in D_g} w_{g,d,l} \le 1 \qquad \forall g \in G, \forall l \in L \qquad \textbf{(B26)}$$

(ii) Continuous operation constraints

Each crude distillation unit must be operated continuously and the total time of operation of each CDU must be equal to the time horizon *H*. To ensure that the crude distillation unit (CDU) has a continuous feed within certain specifications and meeting demand:

$$\sum_{l} \sum_{g \in G_d} \left[T_{\text{end},g,d,l} - T_{\text{start},g,d,l} \right] \qquad \forall d \in D \qquad \textbf{(B27)}$$

This equation ensures that a CDU has a crude oil flow into it for the entire duration of the time horizon without a gap in time.

Because of the continuity required in the duration of operation, and the requirement that only one charging tank can charge a CDU over a period of time, for a CDU which is charged in transfer event *l*, the next charge (in transfer event (*l*+1)) will start at the ending time of the current transfer event *l*. This is enforced by equations (28) and (29).

$$T_{\text{start},g,d,(l+1)} \geq T_{\text{end},g',d,l} - H(1 - w_{g',d,l}) \qquad \forall g, g' \in G_d,\ g \neq g',\ \forall d \in D, \forall l \in L,\ l < |L|$$

(B28)

$$T_{\text{start},g,d,(l+1)} \leq T_{\text{end},g',d,l} + H(1 - w_{g',d,l}) \qquad \forall g, g' \in G_d,\ g \neq g',\ \forall d \in D, \forall l \in L,\ l < |L|$$

(B29)

(c) Crude Supply Stream Constraints

The crude supply streams have to follow certain mass balance and timing constraints:

(i) Timing constraints

All the flows from a crude supply stream *u* to storage tank *y* in any transfer event must start after a particular time ($T_{\text{start},u}$) and end before a certain time ($T_{\text{end},u}$):

$$T_{\text{start},u} \leq T_{\text{start},u,y,l} + H(1 - w_{u,y,l}) \qquad \forall u \in U,\ \forall y \in Y_u, \forall l \in L \qquad \textbf{(B30a)}$$

$$T_{\text{end},u} \geq T_{\text{end},u,y,l} - H(1 - w_{u,y,l}) \qquad \forall u \in U,\ \forall y \in Y_u, \forall l \in L \qquad \textbf{(B30b)}$$

It is to be noted that the flow from a crude supply stream can be split such that one or more storage tanks are simultaneously fed by a single crude supply stream. Also, two or more supply streams can feed the same storage tank at the same time.

(ii) Overall mass balances

The total amount of crude oil arriving in a crude supply stream *u* (given by V_u^{supply}), must be completely transferred to the storage tanks over the set of all transfer events in the horizon.

$$\sum_{l \in L} \sum_{y \in Y_u} V_{\text{tot},u,y,l} = V_u^{\text{supply}} \qquad \forall u \in U \tag{B31}$$

(iii) Component balances

The component flow from a crude supply stream *u* to a tank *y* (storage tank) in a transfer event *l* is equal to the product of the total flow from that crude supply stream to the tank and the fraction of the component in the crude supply stream, which is known.

$$V_{i,u,y,l} = f_{i,u}^{\text{supply}} V_{\text{to},u,y,l} \qquad \forall i \in I,\ \forall y \in Y_u, \forall u \in U,\ \forall l \in L \tag{B32}$$

(d) Variable Bounds

All the continuous variables must lie between specified bounds and the discrete variables can be either 0 or 1.

$$\begin{aligned}
&0 \le I_{i,b,l} \le I_b^{\text{U}} && \forall i \in I,\ \forall b \in B,\ \forall l \in L \\
&I_b^{\text{L}} \le I_{\text{tot},b,l} \le I_b^{\text{U}} && \forall b \in B,\ \forall l \in L \\
&0 \le V_{i,a,b,l} \le V_{a,b}^{\text{U}} && \forall i \in I,\ \forall a \in A_b,\ \forall b \in B,\ \forall l \in L \\
&0 \le V_{i,b,c,l} \le V_{b,c}^{\text{U}} && \forall i \in I,\ \forall c \in C_b,\ \forall b \in B,\ \forall l \in L \\
&0 \le V_{\text{tot},a,b,l} \le V_{a,b}^{\text{U}} && \forall a \in A_b,\ \forall b \in B,\ \forall l \in L \\
&0 \le V_{\text{tot},a,b,l} \le V_{b,c}^{\text{U}} && \forall c \in C_b,\ \forall b \in B,\ \forall l \in L \\
&0 \le T_{\text{start},a,b,l} \le H && \forall a \in A_b,\ \forall b \in B,\ \forall l \in L \\
&0 \le T_{\text{end},a,b,l} \le H && \forall a \in A_b,\ \forall b \in B,\ \forall l \in L \\
&0 \le T_{\text{start},b,c,l} \le H && \forall c \in C_b,\ \forall b \in B,\ \forall l \in L \\
&0 \le T_{\text{end},b,c,l} \le H && \forall c \in C_b,\ \forall b \in B,\ \forall l \in L \\
&T_u^{\text{arrival}} \le T_{\text{start},u} \le H && \forall u \in U \\
&T_u^{\text{arrival}} \le T_{\text{end},u} \le H && \forall u \in U \\
&w_{a,b,l}, w_{b,c,l} \in \{0,1\}
\end{aligned} \tag{B33}$$

(e) Objective Function

$$\min z = \underbrace{\text{Csea}\sum_{u\in U}\left(T_{\text{start},u} - T_u^{\text{arrival}}\right)}_{\text{waiting cost for a crude supply stream}} + \underbrace{\text{Cunload}\sum_{u\in U}\left(T_{\text{start},u} - T_u^{\text{arrival}}\right)}_{\text{unloading cost of crude for a crude supply stream}}$$

$$+\underbrace{\frac{H\times\sum_{b\in B}\text{Cinv}_b\times\left[\left(\sum_{b\in B}\sum_{l} I_{\text{tot},b,l}\right)+\left(\sum_{b\in B}\sum_{l}\sum_{a\in A_b} V_{\text{tot},a,b,l}\right)+\left(\sum_{b\in B}\sum_{l\in|L|} I_{\text{tot},b,l}\right)+\left(\sum_{b\in B} 2I_b^{\text{init-tot}}\right)\right]}{\left(2\times\text{NE}+1\right)}}_{\text{total inventory maintenance cost of all tanks in the system}}$$

$$+\underbrace{\text{Cset}\left(\sum_{d\in D_g}\sum_{g\in G_d}\sum_{l} w_{g,d,l} - \text{ND}\right)}_{\text{setup cost of charging the CDUs with different crude-mixes}}$$

$$\max\ \Psi = \underbrace{\sum_t c_{1,t}x_{\text{total},t}}_{\text{income from sales}} - \underbrace{\sum_t\sum_{p\in\text{PP}} c_{p,2,t}z_{p,t} + c_{p,3,t}e_{p,t}}_{\text{investment costs of the production platform}}$$

$$-\underbrace{\sum_t\sum_{p\in\text{PP}}\sum_{n\in\text{WP}(p)} c_{n,p,2,t}z_{n,p,t} + c_{n,p,3,t}e_{n,p,t}}_{\text{investment costs of the well platform}}$$

$$-\underbrace{\sum_t\sum_{p\in\text{PP}}\sum_{n\in\text{WP}(p)}\sum_{n\in W_{\text{WP}}(p)} c_{w,n,p,2,t}z_{w,n,p,t}}_{\text{drilling costs of wells}} \qquad \textbf{(B434)}$$

$$-\underbrace{\left(\sum_t c_{4,t}\sum_{k\in D}\sum_{p\in\text{PP}} z_{p.k.t} + \sum_t c_{4,t}\sum_{k\in D}\sum_{p\in\text{PP}}\sum_{n\in\text{WP}(p)} ZT_{n,p,k,t}\right)}_{\text{costs for moving the drilling rigs}}$$

In the equation above, the first sum is a waiting cost for a crude supply stream while the second term represents the unloading cost of crude for a crude supply stream. The total inventory maintenance cost of all the tanks in the system is given by the third term. This term represents the cost due to an approximate average inventory level in each of the tanks in a transfer event, that is computed by considering the inventory levels at the boundaries of the transfer event, and at the middle point between the end of inputs into a particular tank and start of outputs from that tank in the transfer event. That final term corresponds to the setup cost of charging the 'ND' CDUs with different crude-mixes.

Equations (B1) – (B23) and (B25) – (B34) comprise the MINLP model (*P*) which is to be optimized. The equation that involves bilinearities and is responsible for the nonconvexity of the model if equation (B19).

B3.2 SOLUTION STRATEGY

Large-scale MINLPs such as problem (*P*) require specialized solution algorithms. Here, a specialized outer-approximation algorithm is proposed for solving the nonconvex model (*P*) to global optimality within a specified tolerance. In the proposed technique, lower and upper bounds are generated on the global optimum of (*P*) over a search region, which are then converged in the proposed algorithm.

B3.2.1 PREPROCESSING

The bounds of the variables in the model are determined by physical inspection of the network structure and using the numerical data given for the tanks, crude supply streams and the distillation units. Also, in this step, the original nonconvex MINLP may be locally optimized to obtain an initial overall upper bound (OUB) for the objective function.

B3.2.2 LOWER BOUNDING PROBLEM

The cross-product constraint described in Eq. (B19) is used to properly calculate component fractions and has nonconvex nonlinearity in the form of bilinear terms. A tight convex relaxation for Eq. (B19) may be stated as a set of linear constraints known as McCormick estimators. This set of constraints could be added to a formulation to generally tighten it. They are also useful as a replacement for Eq. (19) to relax the problem and reduce it from the exact nonconvex MINLP to a convex MILP relaxation. New artificial variables would be needed to replace the bilinear terms in Eq. (B19).

The expanded form of Eq. (B19) is shown here:

$$\underbrace{\left(I_{\text{tot},b,(l-1)} + \sum_{a \in A_b} V_{\text{tot},a,b,l}\right) V_{i,b,c,l} = \left(I_{i,b,(l-1)} + \sum_{a \in A_b} V_{i,a,b,l}\right) V_{\text{tot},b,c,l} \quad \forall b \in B, \forall c \in C_b, \forall i \in I, \forall l \in L}$$

$$I_{\text{tot},b,(l-1)} V_{i,b,c,l} + \sum_{a \in A_b} V_{\text{tot},a,b,l} V_{i,b,c,l} = I_{i,b,(l-1)} V_{\text{tot},b,c,l} + \sum_{a \in A_b} V_{i,a,b,l} V_{\text{tot},b,c,l} \qquad \forall b \in B, \forall c \in C_b, \forall i \in I, \forall l \in L$$

(B19)(expanded)

These new variables are defined as follows:

$$I^{VJ}_{i,b,c,l} = I_{\text{tot},b,(l-1)} V_{i,b,c,l} \qquad \forall b \in B, \forall c \in C_b, \forall i \in I, \forall l \in L$$

$$I^{VT}_{i,b,c,l} = I_{i,b,(l-1)} V_{\text{tot},b,c,l} \qquad \forall b \in B, \forall c \in C_b, \forall i \in I, \forall l \in L$$

$$V^{VJ}_{i,a,b,c,l} = V_{\text{tot},a,b,l} V_{i,b,c,l} \qquad \forall a \in A_b, \forall b \in B, \forall c \in C_b, \forall i \in I, \forall l \in L$$

$$V^{VT}_{i,a,b,c,l} = V_{i,a,b,l} V_{\text{tot},b,c,l} \qquad \forall a \in A_b, \forall b \in B, \forall c \in C_b, \forall i \in I, \forall l \in L$$

Replacing the bilinear terms in constraint (B19) with the new variables, constraint (B19) can be converted to constraint (B35):

$$I^{VJ}_{i,b,c,l} + \sum_{a \in A_b} V^{VJ}_{i,a,b,c,l} = I^{VT}_{i,b,c,l} + \sum_{a \in A_b} V^{VT}_{i,a,b,c,l} \qquad \forall i \in I,\ \forall l \in L,\ \forall c \in C_b,\ \forall b \in B \qquad \textbf{(B35)}$$

As shown by McCormick, considering a bilinear term $z = xy$, with the upper and lower bounds on x and y,

$$x^{\mathrm{L}} \le x \le x^{\mathrm{U}}$$
$$y^{\mathrm{L}} \le y \le y^{\mathrm{U}}$$

the valid overestimator and underestimator for z take the following form :

$$z \ge x^{\mathrm{L}}y + xy^{\mathrm{L}} - x^{\mathrm{L}}y^{\mathrm{L}}$$
$$z \ge x^{\mathrm{U}}y + xy^{\mathrm{U}} - x^{\mathrm{U}}y^{\mathrm{U}}$$
$$z \le x^{\mathrm{U}}y + xy^{\mathrm{L}} - x^{\mathrm{U}}y^{\mathrm{L}}$$
$$z \le x^{\mathrm{L}}y + xy^{\mathrm{U}} - x^{\mathrm{L}}y^{\mathrm{U}}$$

Similarly, the convex envelope constraints for bilinear terms $I^{VJ}_{i,b,c,l}$, $I^{VT}_{i,b,c,l}$, $V^{VJ}_{i,a,b,c,l}$, and $V^{VT}_{i,a,b,c,l}$ can be written as in constraints (36) – (39) as follows:

$$
\begin{aligned}
&I^{VJ}_{i,b,c,l} \geq I^{\mathrm{L}}_{b} V_{i,b,c,l} + V^{\mathrm{L}}_{b,c} I_{\mathrm{tot},b,(l-1)} - I^{\mathrm{L}}_{b} V^{\mathrm{L}}_{b,c} && \forall i,\ \forall c \in C_b,\ \forall l \in L,\ \forall b \in B \\
&I^{VJ}_{i,b,c,l} \geq I^{\mathrm{U}}_{b} V_{i,b,c,l} + V^{\mathrm{U}}_{b,c} I_{\mathrm{tot},b,(l-1)} - I^{\mathrm{U}}_{b} V^{\mathrm{U}}_{b,c} && \forall i,\ \forall c \in C_b,\ \forall l \in L,\ \forall b \in B \\
&I^{VJ}_{i,b,c,l} \leq I^{\mathrm{L}}_{b} V_{i,b,c,l} + V^{\mathrm{U}}_{b,c} I_{\mathrm{tot},b,(l-1)} - I^{\mathrm{L}}_{b} V^{\mathrm{U}}_{b,c} && \forall i,\ \forall c \in C_b,\ \forall l \in L,\ \forall b \in B \\
&I^{VJ}_{i,b,c,l} \leq I^{\mathrm{U}}_{b} V_{i,b,c,l} + V^{\mathrm{L}}_{b,c} I_{\mathrm{tot},b,(l-1)} - I^{\mathrm{U}}_{b} V^{\mathrm{L}}_{b,c} && \forall i,\ \forall c \in C_b,\ \forall l \in L,\ \forall b \in B
\end{aligned}
\qquad \textbf{(B36)}
$$

$$
\begin{aligned}
&I^{VT}_{i,b,c,l} \geq I^{\mathrm{L}}_{b} V_{\mathrm{tot},b,c,l} + V^{\mathrm{L}}_{b,c} I_{i,b,(l-1)} - I^{\mathrm{L}}_{b} V^{\mathrm{L}}_{b,c} && \forall i,\ \forall c \in C_b,\ \forall l \in L,\ \forall b \in B \\
&I^{VT}_{i,b,c,l} \geq I^{\mathrm{U}}_{b} V_{\mathrm{tot},b,c,l} + V^{\mathrm{U}}_{b,c} I_{i,b,(l-1)} - I^{\mathrm{U}}_{b} V^{\mathrm{U}}_{b,c} && \forall i,\ \forall c \in C_b,\ \forall l \in L,\ \forall b \in B \\
&I^{VT}_{i,b,c,l} \leq I^{\mathrm{L}}_{b} V_{\mathrm{tot},b,c,l} + V^{\mathrm{U}}_{b,c} I_{i,b,(l-1)} - I^{\mathrm{L}}_{b} V^{\mathrm{U}}_{b,c} && \forall i,\ \forall c \in C_b,\ \forall l \in L,\ \forall b \in B \\
&I^{VT}_{i,b,c,l} \leq I^{\mathrm{U}}_{b} V_{\mathrm{tot},b,c,l} + V^{\mathrm{L}}_{b,c} I_{i,b,(l-1)} - I^{\mathrm{U}}_{b} V^{\mathrm{L}}_{b,c} && \forall i,\ \forall c \in C_b,\ \forall l \in L,\ \forall b \in B
\end{aligned}
\qquad \textbf{(B37)}
$$

$$
\begin{aligned}
&V^{VJ}_{i,a,b,c,l} \geq V^{\mathrm{L}}_{a,b} V_{i,b,c,l} + V^{\mathrm{L}}_{b,c} V_{\mathrm{tot},a,b,l} - V^{\mathrm{L}}_{a,b} V^{\mathrm{L}}_{b,c} && \forall i,\ \forall a \in A_b,\ \forall c \in C_b,\ \forall l \in L,\ \forall b \in B \\
&V^{VJ}_{i,a,b,c,l} \geq V^{\mathrm{U}}_{a,b} V_{i,b,c,l} + V^{\mathrm{U}}_{b,c} V_{\mathrm{tot},a,b,l} - V^{\mathrm{U}}_{a,b} V^{\mathrm{U}}_{b,c} && \forall i,\ \forall a \in A_b,\ \forall c \in C_b,\ \forall l \in L,\ \forall b \in B \\
&V^{VJ}_{i,a,b,c,l} \leq V^{\mathrm{L}}_{a,b} V_{i,b,c,l} + V^{\mathrm{U}}_{b,c} V_{\mathrm{tot},a,b,l} - V^{\mathrm{L}}_{a,b} V^{\mathrm{U}}_{b,c} && \forall i,\ \forall a \in A_b,\ \forall c \in C_b,\ \forall l \in L,\ \forall b \in B \\
&V^{VJ}_{i,a,b,c,l} \leq V^{\mathrm{U}}_{a,b} V_{i,b,c,l} + V^{\mathrm{L}}_{b,c} V_{\mathrm{tot},a,b,l} - V^{\mathrm{U}}_{a,b} V^{\mathrm{L}}_{b,c} && \forall i,\ \forall a \in A_b,\ \forall c \in C_b,\ \forall l \in L,\ \forall b \in B
\end{aligned}
\qquad \textbf{(B38)}
$$

$$
\begin{aligned}
&V^{VT}_{i,a,b,c,l} \geq V^{\mathrm{L}}_{a,b} V_{\mathrm{tot},b,c,l} + V^{\mathrm{L}}_{b,c} V_{i,a,b,l} - V^{\mathrm{L}}_{a,b} V^{\mathrm{L}}_{b,c} && \forall i,\ \forall a \in A_b,\ \forall c \in C_b,\ \forall l \in L,\ \forall b \in B \\
&V^{VT}_{i,a,b,c,l} \geq V^{\mathrm{U}}_{a,b} V_{\mathrm{tot},b,c,l} + V^{\mathrm{U}}_{b,c} V_{i,a,b,l} - V^{\mathrm{U}}_{a,b} V^{\mathrm{U}}_{b,c} && \forall i,\ \forall a \in A_b,\ \forall c \in C_b,\ \forall l \in L,\ \forall b \in B \\
&V^{VT}_{i,a,b,c,l} \leq V^{\mathrm{L}}_{a,b} V_{\mathrm{tot},b,c,l} + V^{\mathrm{U}}_{b,c} V_{i,a,b,l} - V^{\mathrm{L}}_{a,b} V^{\mathrm{U}}_{b,c} && \forall i,\ \forall a \in A_b,\ \forall c \in C_b,\ \forall l \in L,\ \forall b \in B \\
&V^{VT}_{i,a,b,c,l} \leq V^{\mathrm{U}}_{a,b} V_{\mathrm{tot},b,c,l} + V^{\mathrm{L}}_{b,c} V_{i,a,b,l} - V^{\mathrm{U}}_{a,b} V^{\mathrm{L}}_{b,c} && \forall i,\ \forall a \in A_b,\ \forall c \in C_b,\ \forall l \in L,\ \forall b \in B
\end{aligned}
\qquad \textbf{(B39)}
$$

A rigorous lower bound on the global optimum of problem (P) can be obtained by solving an MILP relaxation of the original nonconvex MINLP model (P). This relaxation can be constructed by replacing the nonlinear Eq. (B19) with Eq. (B35) and using convex envelopes. Eqs. (B36) – (B39) for the bilinear terms appearing in Eq. (B19).

The relaxed MILP problem (R) consists of Eqs. (B1) – (B18), (B20) – (B23) and (B25) – (B39). The MILP relaxation (R) is often very large in size and therefore needs significant computational effort to solve. To reduce the computational effort in solving this problem, cutting planes are added to model (R) which are derived using a technique similar to that given in Karuppiah and Grossmann (2007).

The description of the derivation of these cutting planes is as follows. The network is split into separate decoupled structures, as shown in Figure B3.2, following the concept of spatial decomposition. Here the network is split into two decoupled sub-structures, although more sub-structures are possible. The sub-structure to the left of the dotted line in Figure 3.B is called Q1, while the sub-structure on the right is termed Q2. Physically, such a split can be interpreted as cutting the pipelines between some of the units in the network.

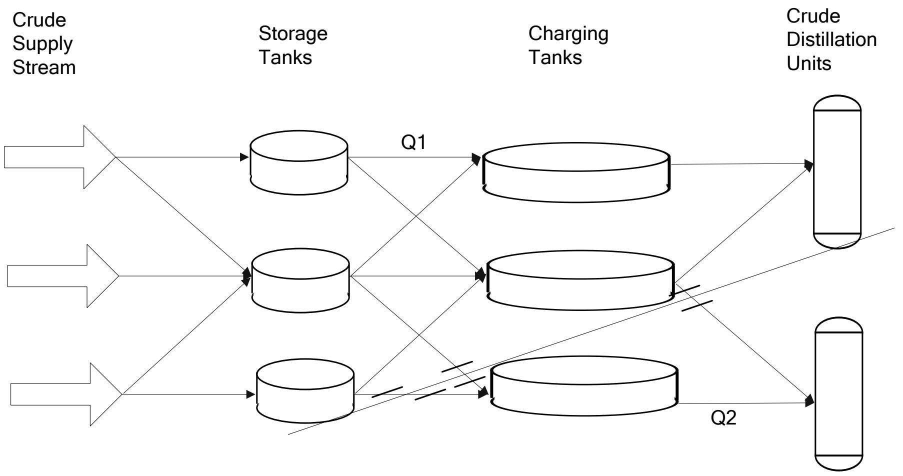

FIGURE B3.2: Spatial decomposition of network structure

The variables pertaining to the flow existence (binary variables $w_{a,b,l}$ and $w_{b,c,l}$), total flow, component flows, and start and end times of flow are duplicated for all the connections in the network that have been split. The results are two sets of duplicate variables $\left\{V^1_{\text{to},m,k,l}, V^1_{i,m,k,l}\left(\forall i\right), T^1_{\text{start},m,k,l}, T^1_{\text{end},m,k,l}, w^1_{m,k,l}\right\} \forall m \in M,\ \forall k \in K_m,\ \forall l$ and $\left\{V^2_{\text{to},m,k,l}, V^2_{i,m,k,l}\left(\forall i\right), T^2_{\text{start},m,k,l}, T^2_{\text{end},m,k,l}, w^2_{m,k,l}\right\} \forall m \in M,\ \forall k \in K_m,\ \forall l$, one set for each decomposed problem, and replace the variables

$\left\{V_{\text{tot},m,k,l}, V_{i,m,k,l}\left(\forall i\right), T_{\text{start},m,k,l}, T_{\text{end},m,k,l}, w_{m,k,l}\right\} \forall m \in M,\ \forall k \in K_m,\ \forall l$ with these newly created variables in model (*P*). The subscript *m* stands for the source of the pipeline that has been split, while the subscript *k* stands for the destination of a pipe that has been split. The variables $\left\{V_{\text{tot},m,k,l}, V_{i,m,k,l}\left(\forall i\right), T_{\text{start},m,k,l}, T_{\text{end},m,k,l}, w_{m,k,l}\right\} \forall m \in M,\ \forall k \in K_m,\ \forall l$ are said to be the linking variables since they link the different sub-structures. The remaining variables in model (*P*) are called non-linking variables since they are separate for both sub-structures Q1 and Q2. Due to the introduction of the duplicate variables, the equations involving the split pipelines get duplicated and are written in terms of the variables $\left\{V^1_{\text{tot},m,k,l}, V^1_{i,m,k,l}\left(\forall i\right), T^1_{\text{start},m,k,l}, T^1_{\text{end},m,k,l}, w^1_{m,k,l}\right\} \forall m \in M,\ \forall k \in K_m,\ \forall l$ (equations corresponding to Q1) and $\left\{V^2_{\text{tot},m,k,l}, V^2_{i,m,k,l}\left(\forall i\right), T^2_{\text{start},m,k,l}, T^2_{\text{end},m,k,l}, w^2_{m,k,l}\right\} \forall m \in M,\ \forall k \in K_m,\ \forall l$ (equations corresponding to Q2). Further, since these newly formed variables are duplicates of the variables present in the original model, they are related by the following equality constraints which are added to model (*P*):

$$V^1_{\text{tot},m,k,l} - V^2_{\text{tot},m,k,l} = 0 \qquad \forall m \in M,\ \forall k \in K_m,\ \forall l \tag{B40}$$

$$V^1_{i,m,k,l} - V^2_{i,m,k,l} = 0 \qquad \forall i \in I,\ \forall m \in M,\ \forall k \in K_m,\ \forall l \tag{B41}$$

$$T^1_{\text{start},m,k,l} - T^2_{\text{start},m,k,l} = 0 \qquad \forall m \in M,\ \forall k \in K_m,\ \forall l \tag{B42}$$

$$T^1_{\text{end},m,k,l} - T^2_{\text{end},m,k,l} = 0 \qquad \forall m \in M,\ \forall k \in K_m,\ \forall l \tag{B43}$$

$$w^1_{m,k,l} - w^2_{m,k,l} = 0 \qquad \forall m \in M,\ \forall k \in K_m,\ \forall l \tag{B44}$$

Equations (40) – (44) are then dualized, that is, they are multiplied by the fixed values of Lagrange multipliers $\lambda^{V\text{tot}}_{m,k,l},\ \lambda^{VC}_{i,m,k,l}\left(\forall i\right),\ \lambda^{T\text{start}}_{m,k,l},\ \lambda^{T\text{end}}_{m,k,l}, \lambda^{w}_{m,k,l}\ \ \forall m \in M,\ \forall k \in K_m,\ \forall l$, respectively, and transferred to the objective function. The initial values of the

Lagrange multipliers are chosen arbitrarily and updated using the method given in Section 3.3. Dualizing Eqs. (40) – (44) yields a Lagrangean relaxation of the original problem, which is denoted by (LRP), and is decomposable into smaller sub-problems corresponding to Q1 and Q2, which are easier to solve.

The model (LRP) is decomposed into two smaller sub-problems (LQ1) and (LQ2) such that model (LQ1) includes equations and variables pertaining to structure Q1, while model (LQ2) includes equations and variables corresponding to the structure Q2. The bounds of all the non-linking variables in both the sub-problems are the same as in the original full space problem (*P*). For the case of the duplicate variables, their bounds are the same as the bounds of the corresponding linking variables in the original problem. The two models (LQ1) and (LQ2) are as follows:

$$
\begin{aligned}
\min z^{\mathrm{LQ1}} = {} & \mathrm{Csea} \sum_{u \in U_{\mathrm{Q1}}} \left(T_{\mathrm{start},u} - T_u^{\mathrm{arrival}} \right) + \mathrm{Cunload} \sum_{u \in U_{\mathrm{Q1}}} \left(T_{\mathrm{start},u} - T_u^{\mathrm{arrival}} \right) \\
& + \frac{H \times \sum_{b \in B_{\mathrm{Q1}}} \mathrm{Cinv}_b \times \left[\left(\sum_{b \in B_{\mathrm{Q1}}} \sum_{l} I_{\mathrm{tot},b,l} \right) + \left(\sum_{b \in B_{\mathrm{Q1}}} \sum_{l} \sum_{a \in A_b} V_{\mathrm{tot},a,b,l} \right) + \left(\sum_{b \in B_{\mathrm{Q1}}} \sum_{l \in |L|} I_{\mathrm{tot},b,l} \right) + \left(\sum_{b \in B_{\mathrm{Q1}}} 2 I_b^{\mathrm{init\text{-}tot}} \right) \right]}{(2 \times \mathrm{NE} + 1)} \\
& + \mathrm{Cset} \left(\sum_{d \in D_{\mathrm{Q1}}} \sum_{g \in G_d} \sum_{l} w_{g,d,l} - \mathrm{ND}_{\mathrm{Q1}} \right) + \sum_{m \in M} \sum_{k \in K_m} \sum_{l} \lambda_{m,k,l}^{V\mathrm{tot}} V_{\mathrm{tot},m,k,l}^{1} + \sum_{i} \sum_{m \in M} \sum_{k \in K_m} \sum_{l} \lambda_{i,m,k,l}^{VC} V_{i,m,k,l}^{1} \\
& + \sum_{m \in M} \sum_{k \in K_m} \sum_{l} \lambda_{m,k,l}^{T\mathrm{start}} T_{\mathrm{start},m,k,l}^{1} + \sum_{m \in M} \sum_{k \in K_m} \sum_{l} \lambda_{m,k,l}^{T\mathrm{end}} T_{\mathrm{end},m,k,l}^{1} + \sum_{m \in M} \sum_{k \in K_m} \sum_{l} \lambda_{m,k,l}^{w} w_{m,k,l}^{1}
\end{aligned}
$$

s.t. constraints corresponding to units and connections in Q1 **(LQ1)**

$$
\begin{aligned}
\min z^{\mathrm{LQ2}} = {} & \mathrm{Csea} \sum_{u \in U_{\mathrm{Q2}}} \left(T_{\mathrm{start},u} - T_u^{\mathrm{arrival}} \right) + \mathrm{Cunload} \sum_{u \in U_{\mathrm{Q2}}} \left(T_{\mathrm{start},u} - T_u^{\mathrm{arrival}} \right) \\
& + \frac{H \times \sum_{b \in B_{\mathrm{Q2}}} \mathrm{Cinv}_b \times \left[\left(\sum_{b \in B_{\mathrm{Q2}}} \sum_{l} I_{\mathrm{tot},b,l} \right) + \left(\sum_{b \in B_{\mathrm{Q2}}} \sum_{l} \sum_{a \in A_b} V_{\mathrm{tot},a,b,l} \right) + \left(\sum_{b \in B_{\mathrm{Q2}}} \sum_{l \in |L|} I_{\mathrm{tot},b,l} \right) + \left(\sum_{b \in B_{\mathrm{Q2}}} 2 I_b^{\mathrm{init\text{-}tot}} \right) \right]}{(2 \times \mathrm{NE} + 1)} \\
& + \mathrm{Cset} \left(\sum_{d \in D_{\mathrm{Q2}}} \sum_{g \in G_d} \sum_{l} w_{g,d,l} - \mathrm{ND}_{\mathrm{Q2}} \right) + \sum_{m \in M} \sum_{k \in K_m} \sum_{l} \lambda_{m,k,l}^{V\mathrm{tot}} V_{\mathrm{tot},m,k,l}^{2} + \sum_{i} \sum_{m \in M} \sum_{k \in K_m} \sum_{l} \lambda_{i,m,k,l}^{VC} V_{i,m,k,l}^{2} \\
& + \sum_{m \in M} \sum_{k \in K_m} \sum_{l} \lambda_{m,k,l}^{T\mathrm{start}} T_{\mathrm{start},m,k,l}^{2} + \sum_{m \in M} \sum_{k \in K_m} \sum_{l} \lambda_{m,k,l}^{T\mathrm{end}} T_{\mathrm{end},m,k,l}^{2} + \sum_{m \in M} \sum_{k \in K_m} \sum_{l} \lambda_{m,k,l}^{w} w_{m,k,l}^{2}
\end{aligned}
$$

s.t. constraints corresponding to units and connections in Q2 **(LQ2)**

The MILP relaxations of models (LQ1) and (LQ2), termed (LQ1-R) and (LQ2-R), respectively, are constructed by replacing the nonlinear terms in these models by convex envelopes. Models (LQ1-R) and (LQ2-R) are solved to obtain their optimal objective values z_{Q1}^{*} and z_{Q2}^{*}, respectively. Using these solutions, the following valid linear cuts are generated in the full space of the original problem, which are given by Eqs. (45) and (46):

$$
\begin{aligned}
z_{Q1}^{*} \le\ & \mathrm{Csea}\sum_{u\in U_{Q1}}\left(T_{\mathrm{start},u}-T_{u}^{\mathrm{arrival}}\right)+\mathrm{Cunload}\sum_{u\in U_{Q1}}\left(T_{\mathrm{start},u}-T_{u}^{\mathrm{arrival}}\right)\\
& +\frac{H\times\sum_{b\in B_{Q1}}\mathrm{Cinv}_{b}\times\left[\left(\sum_{b\in B_{Q1}}\sum_{l}I_{\mathrm{tot},b,l}\right)+\left(\sum_{b\in B_{Q1}}\sum_{l}\sum_{a\in A_b}V_{\mathrm{tot},a,b,l}\right)+\left(\sum_{b\in B_{Q1}}\sum_{l\in|L|}I_{\mathrm{tot},b,l}\right)+\left(\sum_{b\in B_{Q1}}2I_{b}^{\mathrm{init\text{-}tot}}\right)\right]}{\left(2\times\mathrm{NE}+1\right)}\\
& +\mathrm{Cset}\left(\sum_{d\in D_{Q1}}\sum_{g\in G_d}\sum_{l}w_{g,d,l}-\mathrm{ND}_{Q1}\right)+\sum_{m\in M}\sum_{k\in K_m}\sum_{l}\lambda_{m,k,l}^{V\mathrm{tot}}V_{\mathrm{tot},m,k,l}+\sum_{i}\sum_{m\in M}\sum_{k\in K_m}\sum_{l}\lambda_{i,m,k,l}^{VC}V_{i,m,k,l}\\
& +\sum_{m\in M}\sum_{k\in K_m}\sum_{l}\lambda_{m,k,l}^{T\mathrm{start}}T_{\mathrm{start},m,k,l}+\sum_{m\in M}\sum_{k\in K_m}\sum_{l}\lambda_{m,k,l}^{T\mathrm{end}}T_{\mathrm{end},m,k,l}+\sum_{m\in M}\sum_{k\in K_m}\sum_{l}\lambda_{m,k,l}^{w}w_{m,k,l}
\end{aligned}
$$

(B45)

$$
\begin{aligned}
z_{Q2}^{*} \le\ & \mathrm{Csea}\sum_{u\in U_{Q2}}\left(T_{\mathrm{start},u}-T_{u}^{\mathrm{arrival}}\right)+\mathrm{Cunload}\sum_{u\in U_{Q2}}\left(T_{\mathrm{start},u}-T_{u}^{\mathrm{arrival}}\right)\\
& +\frac{H\times\sum_{b\in B_{Q2}}\mathrm{Cinv}_{b}\times\left[\left(\sum_{b\in B_{Q2}}\sum_{l}I_{\mathrm{tot},b,l}\right)+\left(\sum_{b\in B_{Q2}}\sum_{l}\sum_{a\in A_b}V_{\mathrm{tot},a,b,l}\right)+\left(\sum_{b\in B_{Q2}}\sum_{l\in|L|}I_{\mathrm{tot},b,l}\right)+\left(\sum_{b\in B_{Q2}}2I_{b}^{\mathrm{init\text{-}tot}}\right)\right]}{\left(2\times\mathrm{NE}+1\right)}\\
& +\mathrm{Cset}\left(\sum_{d\in D_{Q2}}\sum_{g\in G_d}\sum_{l}w_{g,d,l}-\mathrm{ND}_{Q1}\right)+\sum_{m\in M}\sum_{k\in K_m}\sum_{l}\lambda_{m,k,l}^{V\mathrm{tot}}V_{\mathrm{tot},m,k,l}+\sum_{i}\sum_{m\in M}\sum_{k\in K_m}\sum_{l}\lambda_{i,m,k,l}^{VC}V_{i,m,k,l}\\
& +\sum_{m\in M}\sum_{k\in K_m}\sum_{l}\lambda_{m,k,l}^{T\mathrm{start}}T_{\mathrm{start},m,k,l}+\sum_{m\in M}\sum_{k\in K_m}\sum_{l}\lambda_{m,k,l}^{T\mathrm{end}}T_{\mathrm{end},m,k,l}+\sum_{m\in M}\sum_{k\in K_m}\sum_{l}\lambda_{m,k,l}^{w}w_{m,k,l}
\end{aligned}
$$

(B46)

Theoretical properties of such cuts are given in Karuppiah and Grossmann (2007). Namely, the cuts are valid when added to the original problem, and the inclusion of the cuts into the relaxation (*R*) produces a lower bound at least as strong as the lower bound obtained from Lagrangean decomposition and the one obtained by solving (*R*) without any cuts. The Lagrange multipliers used in these cuts can be updated using a procedure given in Section 3.3, and additional cuts can be derived as described above. This procedure of updating the multipliers and adding cuts can be performed for any number of times. It is important to note that the performance of these cuts in reducing the solution time of the relaxation strongly depends on the values of the Lagrange multipliers. The cuts (Eqs. (45) and/or (46)) are then added to (*R*) which is the MILP relaxation of model (*P*) to get a modified MILP model (*RP*). On solving (*RP*), a valid lower bound on the solution of (*P*) is obtained.

B3.2.3 Upper Bounding Subproblem

The binary variables in problem (*P*) are fixed to the optimal values of the corresponding binary variables obtained from the solution of (*RP*) and obtain a nonconvex NLP (P-NLP), which is solved to global optimality with any standard method. To solve (P-NLP), the optimal values of the continuous variables obtained from the solution of (RP) are used as an initial point for obtaining an upper bound with the NLP solver. Solving model (P-NLP) yields an upper bound on the solution of *(P*). In case the model (P-NLP) is found to be infeasible, an upper bound can still be obtained heuristically by first finding a sub-optimal integer solution to (*RP*), and then using this integer solution to fix the binaries in (*P*)) and re-solving the model (P-NLP) so obtained.

B3.2.4 Outer-Approximation Algorithm

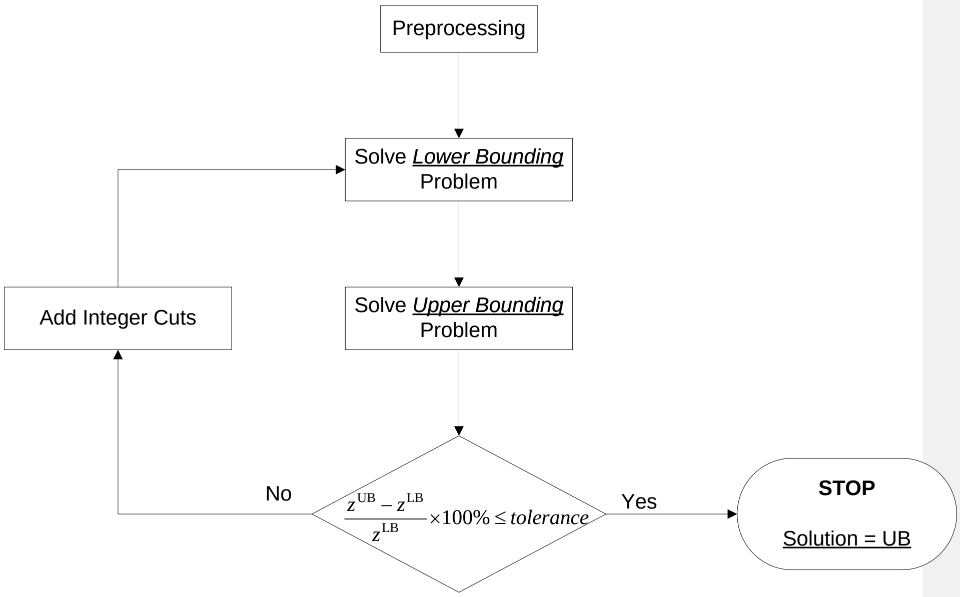

FIGURE B3.3: Proposed outer-approximation algorithm

The proposed outer-approximation algorithm is shown in flowchart in Figure B3.3. The algorithm is along the lines of the techniques proposed by Duran and Grossmann (1986), Kesavan et al. (2004), and Wei et al. (2005) and is outlined as follows:

(a) Preprocessing

The bounds of the variables in the model are determined by physical inspection of the network structure and using the numerical data given for the tanks, crude supply streams and the distillation units. Also, in this step, the original nonconvex MINLP may be locally optimized to obtain an initial overall upper bound (OUB) for the objective function.

(b) Lower bound generation

Generate a valid lower bound on the solution of the nonconvex MINLP following the technique outlined in Section B3.2.1.

(c) Upper bound generation

Generate an upper bound using the method given in Section B3.2.2 and update the OUB if the current upper bound is found to be better than the existing OUB.

(d) Integer cuts

Using the integer solution obtained from solving (*RP*), add an integer cut to model (*RP*) to exclude this particular combination of binary variables. It is important to note that if the model (P-NLP) is not globally optimized in step (c), adding these integer cuts to the relaxation in the net iteration could potentially cut off the global optimum.

(e) Termination

Iterate between solving models (*RP*) and (P-NLP) until the optimality gap between the lower and upper bounds is less than a specified tolerance. The optimality gap is similar to that in Rocha et al. (2008). In this case, our tolerance is 5%. Convergence to the global optimum is not guaranteed if a local NLP solver is used in step (c) above.

B3.3 UPDATING LAGRANGE MULTIPLIERS

Fisher (1981) proposed a sub-gradient method to update Lagrange multipliers, to be used in solving a Lagrangean relaxation of a given MILP, starting from an initial arbitrary value of the multipliers. This technique is tailored to suit our problem in order to obtain updated values of Lagrange multipliers starting with random initial values. These Lagrange multipliers starting with random initial values are then used to derive cuts to be added to the MILP. A sequence of the multipliers is generated as follows:

$$\left[\lambda_{m,k,l}^{V\mathrm{tot}}\right]^{o+1}=\left[\lambda_{m,k,l}^{V\mathrm{tot}}\right]^{o}+ts^{k}\left[\left(V_{\mathrm{tot},m,k,l}^{1}\right)^{*o}-\left(V_{\mathrm{tot},m,k,l}^{2}\right)^{*o}\right]\ \forall m\in M,\ \forall k\in K_m,\ \forall l \quad \textbf{(B46)}$$

$$\left[\lambda_{i,m,k,l}^{VC}\right]^{o+1}=\left[\lambda_{i,m,k,l}^{VC}\right]^{o}+ts^{o}\left[\left(V_{i,m,k,l}^{1}\right)^{*o}-\left(V_{i,m,k,l}^{2}\right)^{*o}\right]\ \forall i\in I,\ \forall m\in M,\ \forall k\in K_m,\ \forall l$$

(B47)

$$\left[\lambda_{m,k,l}^{T\mathrm{start}}\right]^{o+1}=\left[\lambda_{i,m,k,l}^{T\mathrm{start}}\right]^{o}+ts^{o}\left[\left(T_{\mathrm{start},m,k,l}^{1}\right)^{*o}-\left(T_{\mathrm{start},m,k,l}^{2}\right)^{*o}\right]\ \forall m\in M,\ \forall k\in K_m,\ \forall l \quad \textbf{(B48)}$$

$$\left[\lambda_{m,k,l}^{T\mathrm{end}}\right]^{o+1}=\left[\lambda_{i,m,k,l}^{T\mathrm{end}}\right]^{o}+ts^{o}\left[\left(T_{\mathrm{end},m,k,l}^{1}\right)^{*o}-\left(T_{\mathrm{end},m,k,l}^{2}\right)^{*o}\right]\ \forall m\in M,\ \forall k\in K_m,\ \forall l \quad \textbf{(B49)}$$

$$\left[\lambda_{i,m,k,l}^{w}\right]^{o+1}=\left[\lambda_{i,m,k,l}^{w}\right]^{o}+ts^{o}\left[\left(w_{m,k,l}^{1}\right)^{*o}-\left(w_{m,k,l}^{2}\right)^{*o}\right]\ \forall m\in M,\ \forall k\in K_m,\ \forall l \quad \textbf{(B50)}$$

Scalar step size, ts^o, is calculated at every iteration *o*:

$$ts^{o}=\frac{\alpha^{o}\left(z^{\mathrm{U}}-z^{\mathrm{L}}\left(\lambda^{o}\right)\right)}{\sum_{m\in M}\sum_{k\in K_m}\sum_{l}\left[\left(\left(V_{\mathrm{tot},m,k,l}^{1}\right)^{*o}-\left(V_{\mathrm{tot},m,k,l}^{2}\right)^{*o}\right)+\sum_{i}\left(\left(V_{i,m,k,l}^{1}\right)^{*o}-\left(V_{i,m,k,l}^{2}\right)^{*o}\right)+\left(\left(T_{\mathrm{start},m,k,l}^{1}\right)^{*o}-\left(T_{\mathrm{start},m,k,l}^{2}\right)^{*o}\right)+\left(\left(T_{\mathrm{end},m,k,l}^{1}\right)^{*o}-\left(T_{\mathrm{end},m,k,l}^{2}\right)^{*o}\right)+\left(\left(w_{m,k,l}^{1}\right)-\left(w_{m,k,l}^{2}\right)\right)\right]}$$

(B51)

Optimal values of the duplicates variables, obtained from the solution of the sub-problems (LQ1-R) and (LQ2-R), at the *o*th iteration, are represented by $\left\{\left(V_{\mathrm{tot},m,k,l}^{1}\right)^{*o},\left(V_{i,m,k,l}^{1}\right)^{*o}\forall i,\left(T_{\mathrm{start},m,k,l}^{1}\right)^{*o},\left(T_{\mathrm{end},m,k,l}^{1}\right)^{*o},\left(w_{m,k,l}^{1}\right)^{*o}\right\}$ and $\left\{\left(V_{\mathrm{tot},m,k,l}^{2}\right)^{*o},\left(V_{i,m,k,l}^{2}\right)^{*o}\forall i,\left(T_{\mathrm{start},m,k,l}^{2}\right)^{*o},\left(T_{\mathrm{end},m,k,l}^{2}\right)^{*o},\left(w_{m,k,l}^{2}\right)^{*o}\right\}$, respectively. α^o is a scalar chosen between 0 and 2, z^{L} (λ^{o}) is the sum of the objectives of the sub-models (LQ1-R) and (LQ2-R), when the multiplers are set to λ^o, and z^{U} is the value of the best found feasible solution of (*R*). The multiplier update is carried out only once in this work.

B3.4 BOUNDS CONTRACTION ALGORITHM

The branch-and-contract algorithm proposed by Zamora and Grossmann (1999) aims at reducing the size of the search region by eliminating portions of the domain in which the objective function takes only values above a known upper bound. The solution of contraction subproblems at selected branch-and-bound nodes is performed within a finite contraction operation that helps reducing the total number of nodes in the branch and bound solution tree.

A summary of the steps involved in performing the branch and contract algorithm is as follows:

I. Define set of nonconvex or complicating variables as the subset of variables that appear in the nonconvex functions or terms in the model. The complicating variables are characterized by the index set *CV*.

II. Specify minimum value, SP_{min}, for a successful contraction step. Specify the parameter ε_x, which determines the ε_x-closeness perperty. Specify the maximum number of contraction steps to be performed, NC_{max}. specify the maximum number of unsuccessful contraction steps, NUC_{max}. specify the index set $BLUE_0 \in CV$ that determines the subset of complicating variables over which contraction is to be performed (contraction variables). Initialize the control sets BLUE := $BLUE_0$, and RED := ∅. Specify the maximum fraction of contraction variables, F_{CV}, allowed in the RED set. Initialize counters NC = 0, NUC = 0.

III. For $i \in BLUE$, compute the lower focal distance, $\Delta_i^{f,L}(\Omega)$, and upper focal distance, $\Delta_i^{f,U}(\Omega)$, using the formula below:

$$\Delta_i^{f,L}(\Omega) = \frac{x_i^b - x_i^L}{x_i^{U,in} - x_i^{L,in}}$$

$$\Delta_i^{f,U}(\Omega) = \frac{x_i^U - x_i^b}{x_i^{U,in} - x_i^{L,in}}$$

Label all these focal distances as unmarked. If the focal point at a branch and bound node, x^b is ε_x-close to x_i^L, i.e. $x_i^b \le x_i^L(1+\varepsilon_x)$, mark $\Delta_i^{f,L}(\Omega)$. If x^b is ε_x-close to x_i^U, i.e. $x_i^b \ge x_i^U(1-\varepsilon_x)$, then mark $\Delta_i^{f,U}(\Omega)$.

IV. Determine

$$\Delta_{\max}^f(\Omega) = \underset{i \in BLUE}{\text{Max}}\left[\Delta_i^{f,L}(\Omega),\ \Delta_i^{f,U}(\Omega)\right]$$

for $\Delta_i^{f,L}(\Omega)$, $\Delta_i^{f,U}(\Omega)$, unmarked

Then, select a complicating variable x_t, with $t \in$ BLUE, such that

$$\Delta_i^{f,L}(\Omega) \text{ is unmarked and } \Delta_i^{f,L}(\Omega) = \Delta_{\max}^f(\Omega)$$

or

$$\Delta_i^{f,U}(\Omega) \text{ is unmarked and } \Delta_i^{f,U}(\Omega) = \Delta_{\max}^f(\Omega)$$

V. Here, we perform a contraction step, i.e. the process of computing and updating a bound, x_i^L or x_i^U, through the solution of the following equation:

$$\underset{(x,y,z)}{\text{Min or Max}}\ x_i$$

subject to

$$\hat{f}(x,y,z) \le OUB$$
$$(x,y,z) \in M(\Omega) \subset R^n \times R^{n_1} \times R^{n_2}$$

The inequality $\hat{f}(x, y, z) \leq OUB$ will be called the OUB constraint, and the solution to the problem will be donetod as $(\tilde{x}, \tilde{y}, \tilde{z})_{\Omega}$. The optimization direction, i.e. Min or Max, is selected depending upon which of the bounds, x_i^L or x_i^U, is to be contracted.

If $\Delta_i^{f,L}(\Omega)$ is unmarked and $\Delta_i^{f,L}(\Omega) = \Delta_{\max}^{f}(\Omega)$ perform a contraction step with a Min direction to contract x_t^L. Otherwise, perform a contraction step with a Max direction to contract x_t^U. Set $NC := NC + 1$.

VI. If the contraction subproblem in step V is feasible, then:

Determine SP using the formula below:

$$SP = \begin{cases} \left(\dfrac{\tilde{x}_i - x_i^L}{x_i^U - x_i^L} \right) & \text{if the optimization direction = Min} \\ \left(\dfrac{x_i^U - \tilde{x}_i}{x_i^U - x_i^L} \right) & \text{if the optimization direction = Max} \end{cases}$$

where $\tilde{x}_i$ is the minimum (maximum) valjue of the variable x_i, obtained by solving the contraction step with a Min (Max) direction.

If x_t^L was contracted and SP < SP_{min}, then mark $\Delta_i^{f,L}(\Omega)$ and set $NUC := NUC + 1$.

If x_t^L was contracted and $x_t^b < x_t^L$, then set $x_t^b := x_t^L$.

If x_t^U was contracted and SP < SP_{min}, then mark $\Delta_i^{f,U}(\Omega)$ and set $NUC := NUC + 1$.

If x_t^U was contracted and $x_t^b > x_t^U$, then set $x_t^b := x_t^U$.

Compute $\varepsilon(\Omega)$.

VII. If the contraction subproblem in step V is infeasible or $\varepsilon(\Omega) \geq \varepsilon_t$, then terminate the contraction operation.

Terminate the contraction operation if any of the following conditions is met: (i) |RED| ≥ F_{CV}|$BLUE_0$|; (ii) $NC = NC_{max}$; (iii) $NUC = NUC_{max}$. Otherwise, return to step IV.

Bounds contraction will be applied to equation (B19), which is shown below:

$$\left(I_{tot,b,(l-1)} + \sum_{a\in A_b} V_{tot,a,b,l}\right) V_{i,b,c,l} = \left(I_{i,b,(l-1)} + \sum_{a\in A_b} V_{i,a,b,l}\right) V_{tot,b,c,l} \quad \forall b\in B, \forall c \in C_b, \forall i\in I, \forall l \in L$$

This constraint contains bilinear terms (variable multiplied by another variable) that give rise to be the nonconvexities of the model. The complicating variables that appear in equation (B19) and are evaluated individually using the branch and contract algorithm are:

- total inventory of tank *b* at the end of transfer event *l* ($I_{tot,b,l}$);
- inventory of component *i* in tank *b* at the end of transfer event *l* ($I_{i,b,l}$);
- total flow from *a* to *b* in transfer event *l* ($V_{tot,a,b,l}$);
- total flow from *b* to *c* in transfer event *l* ($V_{tot,a,b,l}$);
- flow of component *i* from *a* to *b* in transfer event *l* ($V_{i,a,b,l}$);
- flow of component *i* from *a* to *b* in transfer event *l* ($V_{i,b,c,l}$);

B3.5 SLACK VARIABLES

Slack variables are used in optimization to turn constraints stated as inequalities into equalities. This is required to turn an inequality into an equality where a linear combination of variables is less than or equal to a given constant in the former.

The model developed was found to have integer infeasibilities, i.e. there is no assignment of integer values to variables that can satisfy all of the constraints. The problematic constraints have been identified, namely equations (B3.i), (B13), (B15), (B17), (B18), (B21), (B25), (B26), (B36.iii), (B36.iv), (B37.i), (B37.iii), (B38.iv), and (B39.i).

In order to overcome the integer infeasibility problem, slack variables, s, are introduced to these equations, as follows:

$$F^{\mathrm{U}}_{a,b}\left(T_{\mathrm{end},a,b,l} - T_{\mathrm{start},a,b,l}\right) + F^{\mathrm{U}}_{a,b}H\left(1 - w_{a,b,l}\right) \geq V_{\mathrm{tot},a,b,l} + s^{3.i}_{a,b,l} \qquad \forall a \in A_b,\ \forall b \in B,\ \forall l \in L \quad \textbf{(B3.i)}$$

$$I_{\mathrm{tot},b,(l-1)} + \sum_{a \in A_b} V_{\mathrm{tot},a,b,l} = I_{\mathrm{tot},b,l} + \sum_{c \in C_b} V_{\mathrm{tot},b,c,l} + s^{13}_{b,c,l} \qquad \forall b \in B, \forall l \in L \qquad \textbf{(B13)}$$

$$I_{i,b,(l-1)} + \sum_{a \in A_b} V_{i,a,b,l} = I_{i,b,l} + \sum_{c \in C_b} V_{i,b,c,l} + s^{15}_{i,b,l} \qquad \forall i \in I, \forall b \in B, \forall l \in L \qquad \textbf{(B15)}$$

$$V_{\mathrm{tot},a,b,l} = \sum_{i \in I} V_{i,a,b,l} + s^{17}_{a,b,l} \qquad \forall a \in A_b, \forall b \in B, \forall l \in L \qquad \textbf{(B17)}$$

$$V_{\mathrm{tot},b,c,l} = \sum_{i \in I} V_{i,b,c,l} + s^{18}_{b,c,l} \qquad \forall c \in C_b, \forall b \in B, \forall l \in L \qquad \textbf{(B18)}$$

$$f^{\mathrm{L}}_{i,b} I_{\mathrm{tot},b,l} \leq I_{i,b,l} + s^{27}_{i,b,l} \qquad \forall i \in I, \forall b \in B, \forall l \in L \qquad \textbf{(B21)}$$

$$\sum_{g \in G_d} w_{g,d,l} \leq 1 + s^{25}_{d,l} \qquad \forall d \in D, \forall l \in L \qquad \textbf{(B25)}$$

$$\sum_{d \in D_g} w_{g,d,l} \leq 1 + s^{26}_{g,l} \qquad \forall g \in G, \forall l \in L \qquad \textbf{(B26)}$$

$$I^{VJ}_{i,b,c,l} \leq I^{\mathrm{L}}_b V_{i,b,c,l} + V^{\mathrm{U}}_{b,c} I_{\mathrm{tot},b,(l-1)} - I^{\mathrm{L}}_b V^{\mathrm{U}}_{b,c} + s^{36.iii}_{i,b,c,l} \qquad \forall i,\ \forall c \in C_b,\ \forall l \in L,\ \forall b \in B \qquad \textbf{(B36.iii)}$$

$$I^{VJ}_{i,b,c,l} \leq I^{\mathrm{U}}_b V_{i,b,c,l} + V^{\mathrm{L}}_{b,c} I_{\mathrm{tot},b,(l-1)} - I^{\mathrm{U}}_b V^{\mathrm{L}}_{b,c} + s^{36.iv}_{i,b,c,l} \qquad \forall i,\ \forall c \in C_b,\ \forall l \in L,\ \forall b \in B \qquad \textbf{(B36.iv)}$$

$$I^{VT}_{i,b,c,l} \geq I^{\mathrm{L}}_{b} V_{\mathrm{to},b,c,l} + V^{\mathrm{L}}_{b,c} I_{i,b,(l-1)} - I^{\mathrm{L}}_{b} V^{\mathrm{L}}_{b,c} + s^{37.i}_{i,b,c,l} \qquad \forall i,\ \forall c \in C_b,\ \forall l \in L,\ \forall b \in B$$

(B37.i)

$$I^{VT}_{i,b,c,l} \leq I^{\mathrm{L}}_{b} V_{\mathrm{to},b,c,l} + V^{\mathrm{U}}_{b,c} I_{i,b,(l-1)} - I^{\mathrm{L}}_{b} V^{\mathrm{U}}_{b,c} + s^{37.iii}_{i,b,c,l} \qquad \forall i,\ \forall c \in C_b,\ \forall l \in L,\ \forall b \in B$$

(B37.iii)

$$V^{VJ}_{i,a,b,c,l} \leq V^{\mathrm{U}}_{a,b} V_{i,b,c,l} + V^{\mathrm{L}}_{b,c} V_{\mathrm{to},a,b,l} - V^{\mathrm{U}}_{a,b} V^{\mathrm{L}}_{b,c} + s^{38.iv}_{i,a,b,c,l} \qquad \forall i,\ \forall a \in A_b,\ \forall c \in C_b,\ \forall l \in L,\ \forall b \in B$$

(B38.iv)

$$V^{VT}_{i,a,b,c,l} \geq V^{\mathrm{L}}_{a,b} V_{\mathrm{to},b,c,l} + V^{\mathrm{L}}_{b,c} V_{i,a,b,l} - V^{\mathrm{L}}_{a,b} V^{\mathrm{L}}_{b,c} + s^{39.i}_{i,a,b,c,l} \qquad \forall i,\ \forall a \in A_b,\ \forall c \in C_b,\ \forall l \in L,\ \forall b \in B$$

(B39.i)

B3.6 MODEL DATA FOR NUMERICAL EXAMPLE

The values of the parameters are obtained from Lee et al. (1996). The units of some of the parameters are not specified in order to be consistent with these authors.

The problem is a network consisting of 3 crude supply streams, 3 storage tanks, 3 charging tanks, and 2 distillation units, whose structure is shown in Figure B3.4 below.

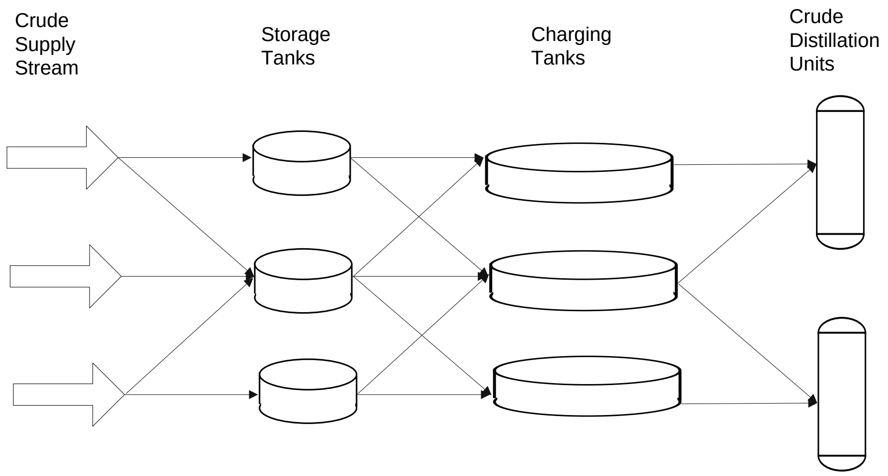

FIGURE B3.4: Connections in a refinery network

The crude oil in this example contains one key component and all the other components are combined into a bulk component, thus effectively making the given crude a two component system. The crude movement has to be scheduled over a time horizon of 12 h.

The values are shown in Table B3.1 below:

TABLE B3.1: Data for crude oil scheduling computation

Scheduling horizon (*H*): 12 h

Number of crude supply streams: 3

Crude supply stream (*u*)	Arrival time (T_u^{arrival})	Incoming volume of crude (V_u^{supply})	Fraction of key component ($f_{i,u}^{\text{supply}}$)
U1	1	50	0.01
U2	5	50	0.085
U3	9	50	0.06

Number of storage tanks: 3

Storage tank (*y*)	Capacity (I_b^{U})	Initial oil inventory ($I_b^{\text{init-tot}}$)	Initial fraction of key component (min-max) [$I_{i,b}^{\text{init}}$ ($f_{i,b}^{\text{L}} - f_{i,b}^{\text{U}}$)]
ST1	100	20	0.02 (0.01-0.03)
ST2	100	20	0.05 (0.04-0.06)
ST3	100	20	0.08 (0.07-0.09)

Number of charging tanks: 3

Charging tank (*g*)	Capacity (I_b^{U})	Initial oil inventory ($I_b^{\text{init-tot}}$)	Initial fraction of key component (min-max) [$I_{i,b}^{\text{init}}$ ($f_{i,b}^{\text{L}} - f_{i,b}^{\text{U}}$)]
CT1	100	30	0.03 (0.025-0.035)
CT2	100	50	0.05 (0.045-0.065)
CT3	1000	30	0.08 (0.075-0.085)

Number of CDUs (ND): 2

Unloading cost for crude supply streams (Cunload): 10

Changeover cost for charged oil switch (Cset): 50

Waiting cost for crude supply streams (Csea): 5

Tank inventory costs (Cinv_b): 0.04 (storage tanks); 0.08 (charging tanks)

Demand of mixed oils by CDUs (DM_g): 50 (oil mix from CT1); 50 (oil mix from CT2); 50 (oil mix from CT3)

Upper bounds on flowrates in the streams ($F_{a,b}^{\text{U}}$): 40

The initial values of Lagrange multipliers are shown in Table B3.2 below:

TABLE B3.2: Lagrange multiplier information

Lagrange multipliers	Initial values
$\lambda_{m,k,l}^{Vtot}$	0
$\lambda_{i,m,k,l}^{VC}$	0
$\lambda_{m,k,l}^{\text{Tstart}}$	0
$\lambda_{m,k,l}^{\text{Tend}}$	0
$\lambda_{m,k,l}^{w}$	0

The values from Table B3.1 and B3.2 are input into the GAMS model, and allowed to run, to determine the value of the objective function, which is to minimize cost.

CHAPTER B4
COMPUTATIONAL EXPERIMENTS AND DISCUSSIONS ON NUMERICAL RESULTS

B4.1 COMPUTATIONAL RESULTS

The formulated logical constraints are coded on GAMS. Numerical studies and computational experiments of the proposed model in this paper is implemented on GAMS 22.3 for Windows XP platform. The computing facilities along with its attributes are listed in Table B4.1 below:

TABLE B4.1: Attributes of computing facilities for computational experiments

Computer Type	Laptop (Lenovo)
Processor Type	Intel Centrino Duo
Processor Speed	1.73 GHz
RAM	512 MB

The computational statistics including the CPU times on the mentioned machine are as reported in Table 4.2 below:

TABLE B4.2: Computational statistics for all models

Solver	Model	Number of single continuous variables	Number of discrete (binary 0–1) variables	Number of single equations	Resource usage/CPU time (s)	Number of iterations
CPLEX 10	LQ1	1615	155	3423	0.516	207
CPLEX 10	LQ2	1561	161	3405	0.109	173
CPLEX 10	LB - MILP	1963	155	3797	0.14	214
CONOPT	UB - NLP	752	0	2097	0.41	0

The computational result is divided into two parts, which are bound contraction and the overall results.

B4.1.1 Results for Bounds Contraction Procedure

Bounds contraction strategy as discussed in section B3.4 earlier was implemented to deal with the bilinear terms, i.e. the complicating variables, in our model, which are:

- total inventory of tank *b* at the end of transfer event *l* ($I_{tot,b,l}$);
- inventory of component *i* in tank *b* at the end of transfer event *l* ($I_{i,b,l}$);
- total flow from *a* to *b* in transfer event *l* ($V_{tot,a,b,l}$);
- total flow from *b* to *c* in transfer event *l* ($V_{tot,a,b,l}$);
- flow of component *i* from *a* to *b* in transfer event *l* ($V_{i,a,b,l}$);
- flow of component *i* from *a* to *b* in transfer event *l* ($V_{i,b,c,l}$);

Bounds contraction techniques have been carried out for each of the complicating variables individually, thus resulting in six separate GAMS codes.

The results obtained shows that the there are no apparent improvements in the bounds after carrying out the bounds contraction technique, as all of the codes produced only ONE iteration step. An example of the branch contraction model is shown in Appendix VI for total inventory of tank *b* at the end of transfer event *l* ($I_{tot,b,l}$).

B4.1.2 Overall Results

After applying the bounds contraction technique, we move on to solve the model using the proposed outer-approximation algorithm by solving four different models, that is, models LQ1, LQ2, LB–MILP, and UB–NLP. The intermediate result for LB – MILP is as shown in Figure B4.1. Then, Figure B4.2 shows the optimal network structure for the crude oil flow for this numerical example as provided by the optimal solution of model UB–NLP.

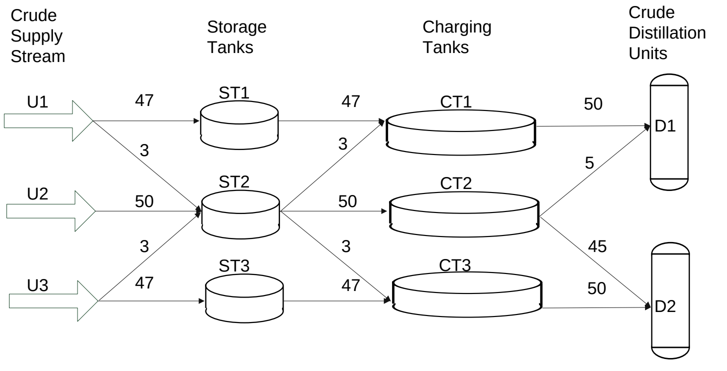

FIGURE B4.1: Flow of crude oil at refinery front-end for model LB–MILP

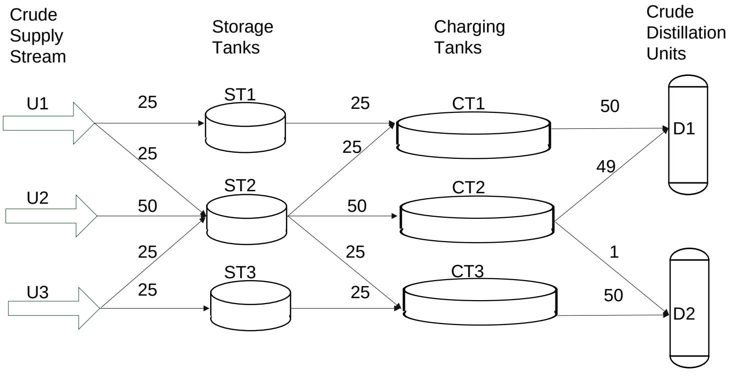

FIGURE B4.2: Optimal network structure for crude oil flow at refinery front-end

The optimal crude schedule is shown in Figure B4.3. The numbers on top of the crude transfer line segments in the figure are the actual flow volumes. The inventory profiles of the tanks are not given for this example and for the subsequent numerical examples since the model includes only the times when the crude transfers begin and end, and there is no explicit information int eh model pertaining to the start and end times of flow from a tank.

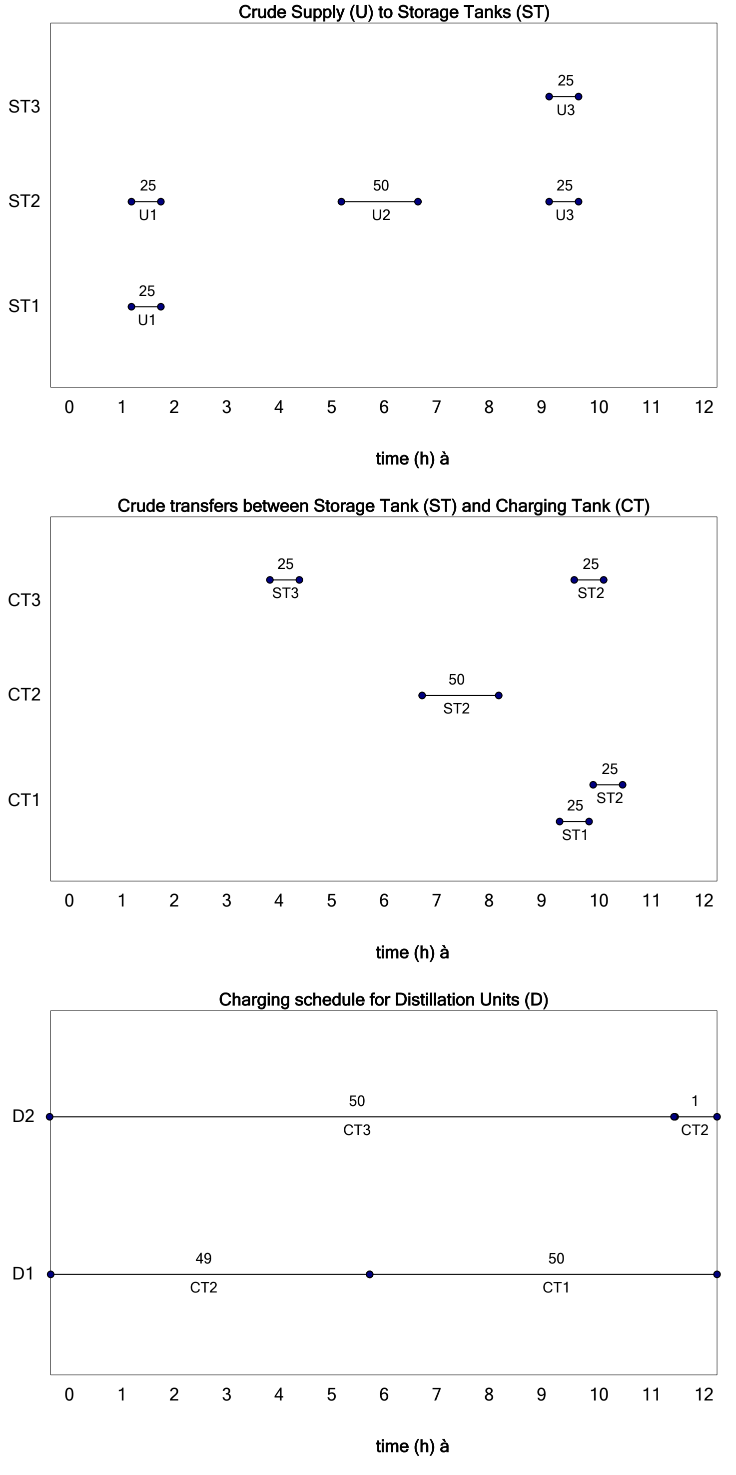

FIGURE B4.3: Gantt chart of the optimal crude oil schedule

A summary of the computational results obtained from all the models are shown in Table B4.3 below:

TABLE B4.3: Computational results for all models

(a) Binary variables (=1 if there is flow from source to destination)

No.	Source	Destination	Binary variables for model			
			LQ1	LQ2	LB - MILP	UB - NLP
1	U1	ST1	1	1	1	1
		ST2	1	1	1	1
		ST3	0	0	0	0
2	U2	ST1	1	1	1	1
		ST2	0	0	0	0
		ST3	0	0	0	0
3	U3	ST1	1	1	1	1
		ST2	1	1	1	1
		ST3	0	0	0	0
4	ST1	CT1	1	1	1	1
		CT2	0	0	0	0
		CT3	0	0	0	0
5	ST2	CT1	1	1	1	1
		CT2	1	1	1	1
		CT3	1	1	1	1
6	ST3	CT1	1	1	1	1
		CT2	0	0	0	0
		CT3	0	0	0	0
7	CT1	D1	1	1	1	1
		D2	0	0	0	0
8	CT2	D1	1	1	1	1
		D2	1	1	1	1
9	CT3	D1	1	1	1	1
		D2	0	0	0	0

(b) Total flow of oil

No.	Source	Destination	Total flow of oil for model			
			LQ1	LQ2	LB - MILP	UB - NLP
1	U1	ST1	47	47	47	25
		ST2	3	3	3	25
		ST3	0	0	0	0
2	U2	ST1	50	50	50	50
		ST2	0	0	0	0
		ST3	0	0	0	0
3	U3	ST1	3	3	3	25
		ST2	47	47	47	25
		ST3	0	0	0	0
4	ST1	CT1	47	47	47	25
		CT2	0	0	0	0
		CT3	0	0	0	0
5	ST2	CT1	3	3	3	25
		CT2	50	50	50	50
		CT3	3	3	3	25
6	ST3	CT1	47	47	47	25
		CT2	0	0	0	0
		CT3	0	0	0	0
7	CT1	D1	50	50	50	50
		D2	0	0	0	0
8	CT2	D1	3	47	5	49
		D2	47	3	45	1
9	CT3	D1	50	50	50	50
		D2	0	0	0	0

(c) Objection function (Cost)

Model	Objective function (Cost)
LQ1	538.6
LQ2	8.0114
LB - MILP	1980.44
UB - NLP	2148.105

B4.3 DISCUSSIONS

From the Gantt chart of the optimal crude oil schedule in Figure B4.3, we can see that the distillation columns are charged continuously. Besides that, the time taken for the transfers between the crude supply streams and the tanks, and for inter-tank transfers are small fractions of the overall scheduling horizon.

From the results obtained, it can be seen that the crude oil flows have non-zero values only for binary variables = 1, i.e. there is flow from source to destination. This is in accordance to the fact that, crude oil can only flow from chosen sets of source to their respective destinations.

Besides that, the results shows a consistent pattern for the total flow of crude oil from source to destination, where the values for one model shows very small difference to another model. For instance, all the flows from source to destination for models LQ1, LQ2 and LB – MILP are identical, except for the flow from CT2 to D1, and CT2 to D2. Although model UB – NLP shows a marked difference in total flow values, the decision variables, i.e. the binary variables that determines whether there is existence of flow from source to destination, are identical for all the models. This means that the decisions of the models are consistent to one another. This is logical since the parameters used are the same for all the models.

Results show that values of slack variables are non-zero for the equations that we have integer infeasibility problems. This shows that there is no assignment of integer values to variables that can satisfy all of the constraints. The use of slack variables managed to overcome the integer infeasibility problem by turning the constraints stated as inequalities into equalities.

The optimal objective function values from models LQ1 and LQ2 are used to generate the valid linear cuts in model LB–MILP by providing the values of z^*_{Q1} and z^*_{Q2}, respectively in constraints **(B45)** and **(B46)**. The binary variables for model UB – NLP are obtained from the solution from LB – MILP. Model UB – NLP uses the original equations, i.e. the solution of UB – NLP will be a global optimum (maximum), whereas model LB – MILP only generates lower bound on the global optimum (maximum), so the objective function from LB – MILP will be less than that of UB – NLP. This is shown from the results obtained, whereby the objective function for UB – NLP (2148.105) is higher than LB – MILP (1980.44).

CHAPTER B5
CONCLUSIONS AND RECOMMENDATIONS

B5.1 CONCLUSIONS

In this work, we have developed a technique for obtaining globally optimal schedules for the flow of crude oil at the front-end of a refinery. A continuous time model based on transfer events is used to represent the scheduling problem and this model is a nonconvex MINLP model which has multiple local optima. We implemented branch and contract algorithm that aims at reducing the size of the search region by eliminating portions of the domain in which the objective function takes only values above a known upper bound. In order to obtain a global optimum solution to the problem, an outer-approximation algorithm is proposed. In this approach, we generated lower and upper bounds on the global optimum, which were converged to a specified tolerance. A rigorous lower bound on the global optimum was obtained by solving a MILP relaxation of the original problem. To reduce the computational effort required in solving this MILP relaxation, cutting planes derived from a spatial decomposition of the network were added to the model LB – MILP. The solution obtained from the LB – MILP model, i.e. the decision variables (binary variables), was used to obtain a feasible solution for model UB – NLP. This solution is the upper bound solution. The application of the proposed algorithm shows significant reduction in the computational effort involved in solving the problem. Slack variables were introduced to overcome the integer infeasibility problem. The mathematical model was developed using GAMS and an optimal solution was found with no logical constraints conflicts or error.

B5.2 RECOMMENDATIONS

It is recommended that penalty should be imposed on the slack variables in the objective function. These penalty terms will give the slack variables' values significance in the fact that it will be accounted for when determining the optimal objective function value.

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APPENDICES

APPENDIX I: (A) GANTT CHART (1ST SEMESTER)

No.	Detail	Week													
		1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	Selection of Project Topic														
2	Preliminary Research Work • Literature Review • Possible process routes														
3	Superstructure construction														
4	Model formulation														
5	Modeling in GAMS														
6	Interim Report Preparation														

APPENDIX II: (B) GANTT CHART (2ND SEMESTER)

No.	Detail/ Week	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	Project Work Continue														
2	Submission of Progress Report 1														
3	Project Work Continue														
4	Submission of Progress Report 2														
5	Seminar (compulsory)														
5	Project work continue														
6	Poster Exhibition														
7	Submission of Dissertation (soft bound)														
8	Oral Presentation														
9	Submission of Project Dissertation (Hard														

APPENDIX II: COMPUTER CODE FOR GAMS INPUT FILE ON DETERMINISTIC MODEL

```
$TITLE: OIL PRODUCTION PLANNING
(DETERMINISTIC MODEL)

FILE RESPLAN2 / UPSTREAM2.res /

PUT RESPLAN2;

SETS

T                 SET OF TIME /T1*T6/
W                 SET OF WELLS /W1*W4/
N                 SET OF WELL PLATFORMS
/N1/
P                 SET OF PRODUCTION
PLATFORMS /P1/
R                 SET OF RESERVOIRS
/R1*R2/
F                 SET OF FIELDS /F1/
J                 SET OF INDEX OF PIECE
IN PIECEWISE LINEAR INTERPOLATION /J1/
D                 SET OF DRILLING RIGS
/D1/
N1(N,P)           SET OF WELL PLATFORMS
N ASSOCIATED WITH PLATFORM P /N1.P1/
W1(W,N)           SET OF WELLS W
ASSOCIATED WITH WELL PLATFORM N
/(W1*W4).N1/
W2(W,R)           SET OF WELLS W
ASSOCIATED WITH RESERVOIR R
/W1.R1,W2.R1,W3.R2,W4.R2/
W3(W,R,N)         SET OF WELLS W
ASSOCIATED WITH RESERVOIR R AND WELL
PLATFORM N
/(W1,W2).R1.N1,(W3,W4).R2.N1/
W4(W,R,F)         SET OF WELLS W IN
FIELD F ASSOCIATED WITH RESERVOIR R
/W1.R1.F1,W2.R1.F1,W3.R2.F1,W4.R2.F1/
R1(R,F)           SET OF RESERVOIRS
ASSOCIATED WITH FIELD F /R1.F1/
J1(J,R)           SET OF LINEAR
INTERPOLATION PIECES USED FOR
RESERVOIR R /J1.R1/
SP                SET OF SCENARIOS FOR
PRICE UNCERTAINTY /L,M,H/
S                 SET OF SCENARIOS FOR
UNCERTAINTY IN SIZE AND INITIAL
DELIVERABILITY OF WELL /S1*S9/
;


SCALARS

ALPHAN            PRESSURE DROP
COEFFICIENT FOR OIL FLOW RATE FOR WELL
PLATFORM /20/
ALPHAP            PRESSURE DROP
COEFFICIENT FOR OIL FLOW RATE FOR
PRODUCTION PLATFORM /20/
BETAN             PRESSURE DROP
COEFFICIENT FOR GOR FOR PRODUCTION
PLATFORM /18/
BETAP             PRESSURE DROP
COEFFICIENT FOR GOR FOR PRODUCTION
PLATFORM /18/
PMAX              MAXIMUM PRESSURE DROP
FROM WELL BORE TO WELL HEAD /8/
GORMAX            MAXIMUM GAS-TO-OIL
RATIO /11/
DELTAT            LENGTH OF TIME PERIOD
T (YEAR) /4/
ITERATION
;

ITERATION = 1;


PARAMETERS

RO(W,N,P)           PRODUCTIVITY INDEX
OF WELL W CONNECTED TO WELL PLATFORM N
CONNECTED TO PRODUCTION PLATFORM P IN
TIME T DIMENSIONLESS
/
         W1.N1.P1        47.6
         W2.N1.P1        83.8
         W3.N1.P1        158.4
         W4.N1.P1        210.2
/

C1(T,SP)           DISCOUNTED REVENUE
PRICE COEFFICIENT FOR OIL SALES
/
         (T1*T6).L     19
         (T1*T6).M     22
         (T1*T6).H     25
/

C2P(P,T)          DISCOUNTED FIXED COST
COEFFICIENT FOR INSTALLATION OF
PRODUCTION PLATFORM IN MILLIONS OF
DOLLARS
/
         P1.T1 70
         P1.T2 69
         P1.T3 68
         P1.T4 67
         P1.T5 66
         P1.T6 65
/
C2NP(N,P,T)       DISCOUNTED FIXED COST
COEFFICIENT FOR INSTALLATION OF WELL
PLATFORM IN MILLIONS OF DOLLARS
/
         N1.P1.T1 11
         N1.P1.T2 10
         N1.P1.T3 9
         N1.P1.T4 8
         N1.P1.T5 7
         N1.P1.T6 6
/
C2WNP(W,N,P,T)   DISCOUNTED FIXED COST
COEFFICIENT FOR DRILLING THE WELL IN
MILLIONS OF DOLLARS
/
         W1.N1.P1.(T1*T6) 6.69
         W2.N1.P1.(T1*T6) 6.34
         W3.N1.P1.(T1*T6) 5.76
         W4.N1.P1.(T1*T6) 5.83
/
C3P(P,T)          DISCOUNTED VARIABLE
COST OEFFICIENT FOR INSTALLATION OF
PRODUCTION PLATFORM IN MILLIONS OF
DOLLARS
/
         P1.T1 0.47
         P1.T2 0.46
         P1.T3 0.45
         P1.T4 0.44
         P1.T5 0.43
         P1.T6 0.42
/
C3NP(N,P,T)       DISCOUNTED VARIABLE
COST OEFFICIENT FOR INSTALLATION OF
WELL PLATFORM IN MILLIONS OF DOLLARS
```

```
/
        N1.P1.T1 0.19
        N1.P1.T2 0.18
        N1.P1.T3 0.17
        N1.P1.T4 0.16
        N1.P1.T5 0.15
        N1.P1.T6 0.14
/
C4(T)           DISCOUNTED COST
COEFFICIENT FOR MOVING RIGS IN
MILLIONS OF DOLLARS
/
        T1 50
        T2 49
        T3 48
        T4 47
        T5 46
        T6 45
/
V5(R,F,J,T)      INDEX OF PIECE J USED
TO CALCULATE PRESSURE IN RESERVOIR R
IN FIELD F AT TIME T IN BAR
(PRESPECIFIED FROM RESERVOIR
PERFORMANCE CURVE)
/
        R1.F1.J1.(T1*T6)     10
/


PROB(SP)
/
        L              0.3
        M              0.4
        H              0.3
/

PROBA(S)
/
        S1             0.09
        S2             0.12
        S3             0.09
        S4             0.12
        S5             0.16
        S6             0.12
        S7             0.09
        S8             0.12
        S9             0.09
/

TABLE

UID(W,N,P,T,S)     SCENARIOS FOR
UNCERTAINTY IN INITIAL
DELIVERABILITIES
                           S1      S2
S3      S4      S5      S6      S7
S8      S9
(W1*W4).N1.P1.(T1*T6)      4       4
4       5       5       5       6
6       6
;


TABLE

UIS(W,N,P,T,S)     SCENARIOS FOR
UNCERTAINTY IN SIZE
                           S1      S2
S3      S4      S5      S6      S7
S8      S9
(W1*W4).N1.P1.(T1*T6)      30      60
90      30      60      90      30
60      90
;

SCALAR

OMEGAU1            UPPER BOUND FOR OIL
FLOW RATE IN MILLION BARREL /5/
OMEGAU2            UPPER BOUND FOR OIL
FLOW RATE IN MILLION BARREL /5/
OMEGAU3            UPPER BOUND FOR OIL
FLOW RATE IN MILLION BARREL /5/
OMEGAU4            UPPER BOUND FOR
EXPANSION VARIABLE FOR WELL PLATFORM
/5/
OMEGAU5            UPPER BOUND FOR
EXPANSION VARIABLE FOR PRODUCTION
PLATFORM /5/
OMEGAU6            UPPER BOUND FOR
DRILLING RIG LOCATION /5/
OMEGAU7            UPPER BOUND FOR
FLOW PROFILE CONSTRAINT /5/
;

ALIAS

(T,THETA,TD,T1,TDASH)
;

FREE VARIABLES

Z                        TOTAL
PRODUCTION PROFIT FOR ALL SCENARIOS
AND PROBABILITIES IN DOLLARS
;

BINARY VARIABLES

Z1(W,N,P,T,S)        1 IF WELL W
CONNECTED TO WELL PLATFORM N TO
PRODUCTION PLATFORM P IS DRILLED IN
TIME T
Z2(N,P,T,S)          1 IF WELL
PLATFORM N CONNECTED TO PRODUCTION
PLATFORM P IS INSTALLED IN TIME T
Z3(P,T,S)            1 IF PRODUCTION
PLATFORM P IS INSTALLED IN TIME T
Z4(W,N,P,TD,S)       DUMMY PERIOD
VALUES AT PERIOD T
Z5(N,P,TD,S)         DUMMY PERIOD
VALUES AT PERIOD T
Z6(P,TD,S)           DUMMY PERIOD
VALUES AT PERIOD T
YD(R,F,J,T,S)        1 IF PIECE J USED
FOR LINEAR INTERPOLATION IN PERIOD T
ZD(N,P,D,T,S)        1 IF DRILLING RIG
K I LOCATED ON FACILITY IN PERIOD T
ZF(N,P,D,T,S)        1 IF THE K-TH RIG
IS LOCATED ON WELL PLATFORM N FIRST IN
PERIOD T
ZL(N,P,D,T,S)        1 IF THE K-TH RIG
IS LOCATED ON WELL PLATFORM N LAST IN
PERIOD T
SL(P,D,T,S)          1 WHEN ONLY THE
N-TH PLATFORM IS USED FOR DRILLING
USING THE K-TH RIG IN PERIOD T
ZT(N,P,D,T,S)        1 IF DRILLING RIG
D IS MOVED AFTER PERIOD T
;

POSITIVE VARIABLES

Z7(P,D,T,S)          NUMBER OF TIMES
DRILLING RIG D IS MOVED IN PERIOD T
X1(W,N,P,T,S)        AMOUNT OF GAS OR
OIL TO TRANSFER BETWEEN WELL W TO WELL
PLATFORM N TO PRODUCTION PLATFORM P IN
TIME T IN KG
X2(N,P,T,S)          AMOUNT OF GAS OR
OIL TO TRANSFER BETWEEN WELL PLATFORM
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N TO PRODUCTION PLATFORM P IN TIME T
IN KG
X3(P,T,S)              AMOUNT OF GAS OR
OIL FLOWING AT PRODUCTION PLATFORM P
IN TIME T IN KG
X4(T,S)                SUM OF ALL GAS OR
OIL FLOWING AT PRODUCTION PLATFORM P
IN TIME T IN KG
V1(W,N,P,T,S)          PRESSURE FOR FLOW
BETWEEN WELL W TO WELL PLATFORM N TO
PRODUCTION PLATFORM P IN TIME T IN BAR
V2(N,P,T,S)            PRESSURE FOR FLOW
BETWEEN WELL PLATFORM N TO PRODUCTION
PLATFORM P IN TIME T IN BAR
V3(P,T,S)              PRESSURE FOR FLOW
AT PRODUCTION PLATFORM P IN TIME T IN
BAR
GF1(W,N,P,T,S)         GAS FLOW
(VOLUMETRIC) BETWEEN WELL W TO WELL
PLATFORM N TO PRODUCTION PLATFORM P IN
TIME T IN M3
GF2(N,P,T,S)           GAS FLOW
(VOLUMETRIC) BETWEEN WELL PLATFORM N
TO PRODUCTION PLATFORM P IN TIME T IN
M3
DELTA1(W,N,P,T,S)      PRESSURE DROP AT
CHOKE FOR FLOW BETWEEN WELL W TO WELL
PLATFORM N TO PRODUCTION PLATFORM P IN
TIME T IN BAR
DELTA2(N,P,T,S)        PRESSURE DROP AT
CHOKE FOR FLOW BETWEEN WELL PLATFORM N
TO PRODUCTION PLATFORM P IN TIME T IN
BAR
LF(W,N,P,T,S)          OIL FLOW FROM
WELL W TO WELL PLATFORM N TO
PRODUCTION PLATFORM P IN TIME T IN KG
GF(W,N,P,T,S)          GAS FLOW FROM
WELL W TO WELL PLATFORM N TO
PRODUCTION PLATFORM P IN TIME T IN M3
X5(W,N,P,THETA,S)      THE CUMULATIVE
AMOUNT OF OIL TO TRANSFER BETWEEN WELL
W TO WELL PLATFORM N TO PRODUCTION
PLATFORM P IN TIME THETA IN KG
X6(R,F,THETA,S)        THE CUMULATIVE
AMOUNT OF OIL FROM RESERVOIR R IN
FIELD F IN TIME THETA IN KG
V4(R,F,T,S)            PRESSURE IN
RESERVOIR R IN FIELD F AT TIME T IN
BAR
LAMBDA(R,F,J,T,S)      INTERPOLATION
VARIABLE AT PERIOD T
X7(N,P,THETA,S)        THE CUMULATIVE
AMOUNT OF OIL TO TRANSFER BETWEEN WELL
PLATFORM N TO PRODUCTION PLATFORM P IN
TIME THETA IN KG
X8(P,THETA,S)          THE CUMULATIVE
AMOUNT OF OIL TO FLOWING FROM
PRODUCTION PLATFORM P IN TIME THETA IN
KG
DV1(N,P,T,S)           DESIGN VARIABLE
FOR WELL PLATFORM
DV2(P,T,S)             DESIGN VARIABLE
FOR PRODUCTION PLATFORM
E1(N,P,T,S)            DESIGN EXPANSION
VARIABLE FOR WELL PLATFORM
E2(P,T,S)              DESIGN EXPANSION
VARIABLE FOR PRODUCTION PLATFORM
XD(W,N,P,T,S)          DELIVERABILITY OF
OIL FROM WELL
;

EQUATIONS

PROFIT                 OBJECTIVE
FUNCTION IN MILLION DOLLAR
XTOTAL1(N,P,T,S)       SUM OF ALL GAS OR
OIL RELATED TO WELL PLATFORM N AND
PRODUCTION PLATFORM P IN TIME T IN KG
XTOTAL2(P,T,S)         SUM OF ALL GAS OR
OIL RELATED TO PRODUCTION PLATFORM P
IN TIME T IN KG
XTOTAL3(T,S)           SUM OF ALL GAS OR
OIL IN TIME T IN KG
VTOTAL1(W,N,P,T,S)     SUM OF ALL
PRESSURE RELATED TO WELL PLATFORM N1
AND PRODUCTION PLATFORM P1 IN TIME T1
IN BAR
VTOTAL2(N,P,T,S)       SUM OF ALL
PRESSURE RELATED TO PRODUCTION
PLATFORM P IN TIME T IN BAR
XFLOW1(W,N,P,T,S)      FLOW OF GAS OR
OIL FROM WELL W TO WELL PLATFORM N TO
PRODUCTION PLATFORM P IN TIME T IN KG
XFLOW2(W,N,P,T,S)      FLOW OF OIL FROM
WELL W TO WELL PLATFORM N TO
PRODUCTION PLATFORM P IN TIME T IN KG
XFLOW3(W,N,P,T,S)      FLOW OF GAS FROM
WELL W TO WELL PLATFORM N TO
PRODUCTION PLATFORM P IN TIME T IN KG
XCUM1(W,N,P,THETA,S) SUM OF THE AMOUNT
OF OIL FROM ALL PERIODS UP TO TIME
PERIOD THETA IN KG
XCUM2(R,F,THETA,S)     SUM OF THE AMOUNT
OF OIL FROM ALL PERIODS UP TO TIME
PERIOD THETA IN KG
PIECE1(R,F,T,S)        VALUE OF
INTERPOLATED VARIABLE FOR EACH
RESERVOIR R IN FIELD F IN TIME T
PIECE2(R,F,T,S)        CONSTRAINT FOR
THE INTERPOLATION VARIABLE AT PERIOD T
PIECE3(R,F,J,T,S)      CONSTRAINT FOR
GAMMA TO BE USED IN LINEAR
INTERPOLATION
PIECE4(R,F,T,S)        CONSTRAINT FOR
BINARY VARIABLE Y1=1 IF J USED FOR
LINEAR INTERPOLATION IN PERIOD T
INSTALL1(W,N,P,T,S)  Z=1 IF WELL W
CONNECTED TO WELL PLATFORM N TO
PRODUCTION PLATFORM P IS DRILLED IN
TIME T
INSTALL2(N,P,T,S)      Z=1 IF WELL
PLATFORM N CONNECTED TO PRODUCTION
PLATFORM P IS INSTALLED IN TIME T
INSTALL3(P,T,S)        Z=1 IF PRODUCTION
PLATFORM P IS INSTALLED IN TIME T
DUMMY1(W,N,P,TD,S)     TO REDUCE THE
NUMBER OF NODES ENUMERATED IN A BRANCH
AND BOUND TREE
DUMMY2(N,P,TD,S)       TO REDUCE THE
NUMBER OF NODES ENUMERATED IN A BRANCH
AND BOUND TREE
DUMMY3(P,TD,S)         TO REDUCE THE
NUMBER OF NODES ENUMERATED IN A BRANCH
AND BOUND TREE
XTOT1(W,N,P,THETA,S) THE FLOW OF OIL
FROM WELL W TO WELL PLATFORM N TO
PRODUCTION PLATFORM P IN PERIOD THETA
IN KG
XTOT2(N,P,THETA,S)     THE FLOW OF OIL
FROM WELL PLATFORM N TO PRODUCTION
PLATFORM P IN PERIOD THETA IN KG
XTOT3(P,THETA,S)       THE FLOW OF OIL
IN PRODUCTION PLATFORM P IN PERIOD
THETA IN KG
ZCON1(W,N,P,THETA,S) THE WELL PLATFORM
ASSOCIATED WITH A WELL MUST BE
INSTALLED BEFORE DRILLING THAT WELL
ZCON2(N,P,THETA,S)     THE PRODUCTION
PLATFORMS MUST BE INSTALLED BEFORE
ASSOCIATED WELL PLATFORMS
```

```
DES1(N,P,T,S)        THE FLOW FROM
WELL PLATFORM N TO PRODUCTION PLATFORM
P IN TIME T
DES2(N,P,T,S)        DESIGN VARIABLE
FOR WELL PLATFORM N TO PRODUCTION
PLATFORM P IN TIME T
DES3(N,P,T,S)        DESIGN EXPANSION
VARIABLE FOR WELL PLATFORM N TO
PRODUCTION PLATFORM P IN TIME T
DES4(P,T,S)          THE FLOW FROM
PRODUCTION PLATFORM P IN TIME T
DES5(P,T,S)          DESIGN VARIABLE
FOR PRODUCTION PLATFORM P IN TIME T
DES6(P,T,S)          DESIGN EXPANSION
VARIABLE FOR PRODUCTION PLATFORM P IN
TIME T
DRSC1(W,N,P,T,S)     IF ANY WELL
ASSOCIATED WITH WELL PLATFORM N IS
DRILLED IN PERIOD T _ SUMMATION OF Z(N
P K T) MUST BE 1 IN THAT TIME PERIOD
DRSC2(N,P,D,T,S)     THE K-TH RIG IS
LOCATED ON THE N-TH PLATFORM IN THAT
TIME PERIOD (SEE DRSC3)
DRSC3(P,D,T,S)       THE SLACK
VARIABLE TAKES A NON-ZERO VALUE IF ALL
WELLS ARE DRILLED FROM A SINGLE N-TH
PLATFORM IN THAT TIME PERIOD
DRSC4(N,P,D,T,S)     CONSTRAINT FOR
WELL DRILLED FROM WELL PLATFORM
DRSC5(N,P,D,T,S)     CONSTRAINT FOR
WELL DRILLED FROM WELL PLATFORM
DRSC6(N,P,D,T,S)     MOVEMENT OF RIG
CONSTRAINT
DRSC7(P,D,T,S)       MOVEMENT OF RIG
CONSTRAINT
FLOWPC(W,N,P,T,S)    FLOW PROFILE
CONSTRAINT
ENDO1(W,N,P,T,S)     ENDOGENEOUS
UNCERTAINTY IN SIZE AND INITIAL
DELIVERABILITY OF WELL
ENDO2(W,N,P,T,S)     OIL FLOW
CONSTRAINTS FOR ENDOGENEOUS
UNCERTAINTY
ENDO3(W,N,P,T,S)     CUMULATIVE OIL
CONSTRAINTS FOR ENDOGENEOUS
UNCERTAINTY
NAC1                 NON-
ANTICIPATIVITY CONSTRAINT
NAC2                 NON-
ANTICIPATIVITY CONSTRAINT
NAC3                 NON-
ANTICIPATIVITY CONSTRAINT
NAC4                 NON-
ANTICIPATIVITY CONSTRAINT
NAC5                 NON-
ANTICIPATIVITY CONSTRAINT
NAC6                 NON-
ANTICIPATIVITY CONSTRAINT
NAC7                 NON-
ANTICIPATIVITY CONSTRAINT
;

PROFIT..
Z =E=
SUM((S,SP,T),PROBA(S)*PROB(SP)*C1(T,SP
)*X4(T,S)) - SUM(S,
SUM(T,SUM(P,C2P(P,T)*Z3(P,T,S) +
C3P(P,T)*E2(P,T,S))) -
SUM(T,SUM(P,SUM(N$N1(N,P),(C2NP(N,P,T)
*Z2(N,P,T,S) +
C3NP(N,P,T)*E1(N,P,T,S))))) -
SUM(T,SUM(P,SUM(N,SUM(W$W1(W,N),C2WNP(
W,N,P,T)*Z1(W,N,P,T,S))))) -
(SUM(T,SUM(P,SUM(D,SUM(N$N1(N,P),ZT(N,
P,D,T,S)))))));

*1.MASS BALANCE

XTOTAL1(N,P,T,S)$N1(N,P)..
X2(N,P,T,S) =E=
SUM(W$W1(W,N),X1(W,N,P,T,S))$N1(N,P);
XTOTAL2(P,T,S)..
SUM(N$N1(N,P),X2(N,P,T,S)) =E=
X3(P,T,S);
XTOTAL3(T,S)..
SUM(P,X3(P,T,S)) =E= X4(T,S);

*2.PRESSURE BALANCE

VTOTAL1(W,N,P,T,S)$(N1(N,P)$W1(W,N))..
V1(W,N,P,T,S) - ALPHAN*X1(W,N,P,T,S) -
BETAN*GF1(W,N,P,T,S) -
DELTA1(W,N,P,T,S) =E=
V2(N,P,T,S)$(W1(W,N)$(N1(N,P)));
VTOTAL2(N,P,T,S)$N1(N,P)..
V2(N,P,T,S) - ALPHAP*X2(N,P,T,S) -
BETAP*GF2(N,P,T,S) - DELTA2(N,P,T,S)
=E= V3(P,T,S)$N1(N,P);

*3.FLOW CONSTRAINTS IN WELLS

XFLOW1(W,N,P,T,S)$(N1(N,P)$W1(W,N))..
X1(W,N,P,T,S) =E= LF(W,N,P,T,S) +
GF(W,N,P,T,S)$(W1(W,N)$(N1(N,P)));
XFLOW2(W,N,P,T,S)$(N1(N,P)$W1(W,N))..
LF(W,N,P,T,S) =L=
RO(W,N,P)*PMAX$(W1(W,N)$(N1(N,P)));
XFLOW3(W,N,P,T,S)$(N1(N,P)$W1(W,N))..
GF(W,N,P,T,S) =L=
LF(W,N,P,T,S)*GORMAX$(W1(W,N)$(N1(N,P)
));

*4.CUMULATIVE FLOW AMOUNT FROM WELLS
UP TO PERIOD T

XCUM1(W,N,P,THETA,S)$(N1(N,P)$W1(W,N))
..        X5(W,N,P,THETA,S) =E=
SUM(T$(ORD(T) LE (ORD(THETA)-1)),
X1(W,N,P,T,S)*DELTAT);
XCUM2(R,F,THETA,S)$R1(R,F)..
X6(R,F,THETA,S) =E=
SUM((W,N,P)$(W1(W,N)$(N1(N,P)$(W4(W,R,
F)))),X5(W,N,P,THETA,S));

*5.PIECEWISE LINEAR INTERPOLATION AT
WELL AND RESERVOIR LEVEL

PIECE1(R,F,T,S)..
SUM(J$J1(J,R),LAMBDA(R,F,J,T,S)*V5(R,F
,J,T)) =E= V4(R,F,T,S)$R1(R,F);
PIECE2(R,F,T,S)..
SUM(J$J1(J,R),LAMBDA(R,F,J,T,S)) =E=
1$R1(R,F);
PIECE3(R,F,J,T,S)..
LAMBDA(R,F,J,T,S) =L= YD(R,F,J,T,S) +
YD(R,F,J-1,T,S)$R1(R,F);
PIECE4(R,F,T,S)..
SUM(J,YD(R,F,J,T,S)) =E= 1$R1(R,F);

*6.LOGICAL CONSTRAINTS FOR
INSTALLATION AND FLOW FROM FACILITIES

INSTALL1(W,N,P,T,S)$(W1(W,N)$(N1(N,P))
)..       Z1(W,N,P,T,S) =L= 1;
INSTALL2(N,P,T,S)..
Z2(N,P,T,S) =L= 1$N1(N,P);
INSTALL3(P,T,S)..
Z3(P,T,S) =L= 1;
DUMMY1(W,N,P,TD,S)..
SUM(T,Z1(W,N,P,T,S)) + Z4(W,N,P,TD,S)
=E= 1$(N1(N,P)$W1(W,N));
```

```
DUMMY2(N,P,TD,S)..
SUM(T,Z2(N,P,T,S)) + Z5(N,P,TD,S) =E=
1;
DUMMY3(P,TD,S)..
SUM(T,Z3(P,T,S)) + Z6(P,TD,S) =E= 1;
XTOT1(W,N,P,T,S)$(N1(N,P)$W1(W,N))..
X1(W,N,P,T,S) =L=
OMEGAU1*Z1(W,N,P,T,S);
XTOT2(N,P,THETA,S)$N1(N,P)..
X2(N,P,THETA,S) =L=
OMEGAU2*SUM(T$((ORD(T)LE
ORD(THETA))),Z2(N,P,T,S));
XTOT3(P,THETA,S)..
X3(P,THETA,S) =L=
OMEGAU3*SUM(T$((ORD(T) LE
ORD(THETA))),Z3(P,T,S));
ZCON1(W,N,P,THETA,S)$(N1(N,P)$W1(W,N))
..        Z1(W,N,P,THETA,S) =L=
SUM(T$(ORD(T) LE
ORD(THETA)),Z2(N,P,T,S));
ZCON2(N,P,THETA,S)$N1(N,P)..
Z2(N,P,THETA,S) =L= SUM(T$(ORD(T) LE
ORD(THETA)),Z3(P,T,S));

*7.DESIGN OF FACILITY

DES1(N,P,T,S)$N1(N,P)..
X2(N,P,T,S) =L= DV1(N,P,T,S);
DES2(N,P,T,S)$N1(N,P)..
DV1(N,P,T,S) =E= DV1(N,P,T-1,S) +
E1(N,P,T,S);
DES3(N,P,T,S)$N1(N,P)..
E1(N,P,T,S) =L= OMEGAU4*Z2(N,P,T,S);
DES4(P,T,S)..
X3(P,T,S) =L= DV2(P,T,S);
DES5(P,T,S)..
DV2(P,T,S) =E= DV2(P,T-1,S) +
E2(P,T,S);
DES6(P,T,S)..
E2(P,T,S) =L= OMEGAU5*Z3(P,T,S);

*8.NUMBER OF WELLS DRILLED IN TIME
PERIOD T

DRSC1(W,N,P,T,S)$(N1(N,P)$W1(W,N))..
Z1(W,N,P,T,S) =L=
SUM(D,ZD(N,P,D,T,S));
DRSC2(N,P,D,T,S)$N1(N,P)..
ZF(N,P,D,T,S) + ZL(N,P,D,T,S) =L= 1 +
SL(P,D,T,S);
DRSC3(P,D,T,S)..
SUM(N$(N1(N,P)),ZD(N,P,D,T,S)) - 1 =L=
OMEGAU6*(1 - SL(P,D,T,S));
DRSC4(N,P,D,T,S)$N1(N,P)..
ZF(N,P,D,T,S) =L= ZD(N,P,D,T,S);
DRSC5(N,P,D,T,S)$N1(N,P)..
ZL(N,P,D,T,S) =L= ZD(N,P,D,T,S);
DRSC6(N,P,D,T,S)$N1(N,P)..
ZF(N,P,D,T,S) - ZL(N,P,D,T-1,S) =L=
ZT(N,P,D,T,S);
DRSC7(P,D,T,S)..
SUM(N$(N1(N,P)), ZD(N,P,D,T,S)) - 1
=L= Z7(P,D,T,S);

*9.DRILLING RIG SCHEDULING CONSTRAINTS

*10.FLOW PROFILE CONSTRAINTS

FLOWPC(W,N,P,T,S)$(N1(N,P)$W1(W,N))..
X1(W,N,P,T,S) =G= X1(W,N,P,T+1,S) -
OMEGAU7*(1 - SUM(TDASH$(ORD(TDASH) LE
ORD(T)),Z1(W,N,P,TDASH,S)));

*11.ENDOGENEOUS UNCERTAINTY IN SIZE
AND INITIAL DELIVERABILITY OF WELL

ENDO1(W,N,P,T,S)$(N1(N,P)$W1(W,N))..
(XD(W,N,P,T,S)/UID(W,N,P,T,S)) +
(X5(W,N,P,T,S)/UIS(W,N,P,T,S)) =E= 1;
ENDO2(W,N,P,T,S)$(N1(N,P)$W1(W,N))..
X1(W,N,P,T,S) =L= XD(W,N,P,T,S);
ENDO3(W,N,P,T,S)$(N1(N,P)$W1(W,N))..
X5(W,N,P,T,S) =L= UIS(W,N,P,T,S);

*12.NONANTICIPATIVITY CONSTRAINTS

NAC1(W,N,P,T,S)$((ORD(S) NE
CARD(S))$(N1(N,P)$W1(W,N)))..
Z1(W,N,P,T,S) =E= Z1(W,N,P,T,S+1);
NAC2(N,P,T,S)$((ORD(S) LT
9)$(N1(N,P)))..
Z2(N,P,T,S) =E= Z2(N,P,T,S+1);
NAC3(P,T,S)$((ORD(S) LT 9))..
Z3(P,T,S) =E= Z3(P,T,S+1);
NAC4(N,P,D,T,S)$((ORD(S) LT
9)$(N1(N,P)))..
ZD(N,P,D,T,S) =E= ZD(N,P,D,T,S+1);
NAC5(N,P,D,T,S)$((ORD(S) LT 9))..
ZT(N,P,D,T,S) =E= ZT(N,P,D,T,S+1);
NAC6(N,P,T,S)$((ORD(S) LT
9)$N1(N,P))..
E1(N,P,T,S) =E= E1(N,P,T,S+1);
NAC7(P,T,S)$((ORD(S) LT 9))..
E2(P,T,S) =E= E2(P,T,S+1);

*VARIABLE BOUNDS

X1.UP(W,N,P,T,S) = 1000;

MODEL OILPLAN /ALL/
;

OPTION LIMROW = 100;
OPTION LIMCOL = 100;
OPTION OPTCR = 0.0;

SOLVE OILPLAN USING MIP MAXIMIZING Z;
```

APPENDIX III: GAMS OUTPUT FILE ON DETERMINISTIC MODEL

```
GAMS Rev 149 x86/MS Windows
04/27/07 17:03:40 Page 6
: OIL PRODUCTION PLANNING
(DETERMINISTIC MODEL)
E x e c u t i o n


---- 286 VARIABLE Z.L
= 331.080 TOTAL PROFIT IN MILLI

ONS OF DOLLARS

---- 286 VARIABLE Z1.L 1 IF WELL W
CONNECTED TO WELL PLATFORM N TO
PRODUCTION PLATFORM P IS DRILLED IN
TIME T

INDEX 1 = W1

               T6

N1.P1       1.000

INDEX 1 = W2

               T1

N1.P1       1.000

INDEX 1 = W3

               T3

N1.P1       1.000

INDEX 1 = W4

               T2

N1.P1       1.000


---- 286 VARIABLE Z2.L 1 IF WELL
PLATFORM N CONNECTED TO PRODUCTION
PLATFORM P IS INSTALLED IN TIME T

               T1

N1.P1       1.000


---- 286 VARIABLE Z3.L 1 IF
PRODUCTION PLATFORM P IS INSTALLED IN
TIME T

            T1

P1       1.000


---- 286 VARIABLE X1.L AMOUNT OF
GAS OR OIL TO TRANSFER BETWEEN WELL W
TO WELL PLATFORM N TO PRODUCTION
PLATFORM P IN TIME T IN KG


INDEX 1 = W1

               T6

N1.P1       5.000

INDEX 1 = W2

               T1

N1.P1       5.000

INDEX 1 = W3

               T3

N1.P1       5.000

INDEX 1 = W4

               T2

N1.P1       5.000


---- 286 VARIABLE X2.L AMOUNT OF
GAS OR OIL TO TRANSFER BETWEEN WELL
PLATFORM N TO PRODUCTION PLATFORM P IN
TIME T IN KG

               T1                    T2
T3          T6

N1.P1            5.000            5.000
5.000       5.000


---- 286 VARIABLE X3.L AMOUNT OF
GAS OR OIL FLOWING AT PRODUCTION
PLATFORM PIN TIME T IN KG

            T1          T2           T3
T6

P1       5.000       5.000       5.000
5.000


---- 286 VARIABLE zaa.L
= 440.000

**** REPORT FILE SUMMARY

RESPLAN C:\Documents and
Settings\Kippi\My
Documents\gamsdir\projdir\UPSTREAM.re
        s


EXECUTION TIME = 0.063
SECONDS 3 Mb WIN226-149 Dec 19,
2007


USER: CS/IE 635, Spring 2007
G061206/0001AS-WIN
      Prof.                    Ferris
DC2937
      License for teaching and
research at degree granting
institutions $TITLE: OIL PRODUCTION
PLANNING (STOCHASTIC MODEL)

FILE RESPLAN2 / UPSTREAM2.res /

PUT RESPLAN2;
```

APPENDIX IV: COMPUTER CODE FOR GAMS INPUT FILE ON STOCHASTIC MODEL

```
 SETS

T                 SET OF TIME /T1*T6/
W                 SET OF WELLS /W1*W4/
N                 SET OF WELL PLATFORMS
/N1/
P                 SET OF PRODUCTION
PLATFORMS /P1/
R                 SET OF RESERVOIRS
/R1*R2/
F                 SET OF FIELDS /F1/
J                 SET OF INDEX OF PIECE
IN PIECEWISE LINEAR INTERPOLATION /J1/
D                 SET OF DRILLING RIGS
/D1/
N1(N,P)           SET OF WELL PLATFORMS
N ASSOCIATED WITH PLATFORM P /N1.P1/
W1(W,N)           SET OF WELLS W
ASSOCIATED WITH WELL PLATFORM N
/(W1*W4).N1/
W2(W,R)           SET OF WELLS W
ASSOCIATED WITH RESERVOIR R
/W1.R1,W2.R1,W3.R2,W4.R2/
W3(W,R,N)         SET OF WELLS W
ASSOCIATED WITH RESERVOIR R AND WELL
PLATFORM N
/(W1,W2).R1.N1,(W3,W4).R2.N1/
W4(W,R,F)         SET OF WELLS W IN
FIELD F ASSOCIATED WITH RESERVOIR R
/W1.R1.F1,W2.R1.F1,W3.R2.F1,W4.R2.F1/
R1(R,F)           SET OF RESERVOIRS
ASSOCIATED WITH FIELD F /R1.F1/
J1(J,R)           SET OF LINEAR
INTERPOLATION PIECES USED FOR
RESERVOIR R /J1.R1/
SP                SET OF SCENARIOS FOR
PRICE UNCERTAINTY /L,M,H/
S                 SET OF SCENARIOS FOR
UNCERTAINTY IN SIZE AND INITIAL
DELIVERABILITY OF WELL /S1*S9/
;


SCALARS

ALPHAN            PRESSURE DROP
COEFFICIENT FOR OIL FLOW RATE FOR WELL
PLATFORM /20/
ALPHAP            PRESSURE DROP
COEFFICIENT FOR OIL FLOW RATE FOR
PRODUCTION PLATFORM /20/
BETAN             PRESSURE DROP
COEFFICIENT FOR GOR FOR PRODUCTION
PLATFORM /18/
BETAP             PRESSURE DROP
COEFFICIENT FOR GOR FOR PRODUCTION
PLATFORM /18/
PMAX              MAXIMUM PRESSURE DROP
FROM WELL BORE TO WELL HEAD /8/
GORMAX            MAXIMUM GAS-TO-OIL
RATIO /11/




DELTAT            LENGTH OF TIME PERIOD
T (YEAR) /4/
ITERATION
;

ITERATION = 1;


PARAMETERS

RO(W,N,P)           PRODUCTIVITY INDEX
OF WELL W CONNECTED TO WELL PLATFORM N
CONNECTED TO PRODUCTION PLATFORM P IN
TIME T DIMENSIONLESS
/
         W1.N1.P1        47.6
         W2.N1.P1        83.8
         W3.N1.P1        158.4
         W4.N1.P1        210.2
/

C1(T,SP)           DISCOUNTED REVENUE
PRICE COEFFICIENT FOR OIL SALES
/
         (T1*T6).L     19
         (T1*T6).M     22
         (T1*T6).H     25
/

C2P(P,T)          DISCOUNTED FIXED COST
COEFFICIENT FOR INSTALLATION OF
PRODUCTION PLATFORM IN MILLIONS OF
DOLLARS
/
         P1.T1 70
         P1.T2 69
         P1.T3 68
         P1.T4 67
         P1.T5 66
         P1.T6 65
/
C2NP(N,P,T)       DISCOUNTED FIXED COST
COEFFICIENT FOR INSTALLATION OF WELL
PLATFORM IN MILLIONS OF DOLLARS
/
         N1.P1.T1 11
         N1.P1.T2 10
         N1.P1.T3 9
         N1.P1.T4 8
         N1.P1.T5 7
         N1.P1.T6 6
/
C2WNP(W,N,P,T)    DISCOUNTED FIXED COST
COEFFICIENT FOR DRILLING THE WELL IN
MILLIONS OF DOLLARS
/
         W1.N1.P1.(T1*T6) 6.69
         W2.N1.P1.(T1*T6) 6.34
         W3.N1.P1.(T1*T6) 5.76
         W4.N1.P1.(T1*T6) 5.83
/
C3P(P,T)          DISCOUNTED VARIABLE
COST OEFFICIENT FOR INSTALLATION OF
PRODUCTION PLATFORM IN MILLIONS OF
DOLLARS
/
         P1.T1 0.47
         P1.T2 0.46
         P1.T3 0.45
         P1.T4 0.44
         P1.T5 0.43
         P1.T6 0.42
/
C3NP(N,P,T)       DISCOUNTED VARIABLE
COST OEFFICIENT FOR INSTALLATION OF
WELL PLATFORM IN MILLIONS OF DOLLARS
/
         N1.P1.T1 0.19
         N1.P1.T2 0.18
```

```
         N1.P1.T3 0.17
         N1.P1.T4 0.16
         N1.P1.T5 0.15
         N1.P1.T6 0.14
/
C4(T)            DISCOUNTED COST
COEFFICIENT FOR MOVING RIGS IN
MILLIONS OF DOLLARS
/
         T1 50
         T2 49
         T3 48
         T4 47
         T5 46
         T6 45
/
V5(R,F,J,T)      INDEX OF PIECE J USED
TO CALCULATE PRESSURE IN RESERVOIR R
IN FIELD F AT TIME T IN BAR
(PRESPECIFIED FROM RESERVOIR
PERFORMANCE CURVE)
/
         R1.F1.J1.(T1*T6)     10
/


PROB(SP)
/
         L              0.3
         M              0.4
         H              0.3
/

PROBA(S)
/
         S1             0.09
         S2             0.12
         S3             0.09
         S4             0.12
         S5             0.16
         S6             0.12
         S7             0.09
         S8             0.12
         S9             0.09
/

TABLE

UID(W,N,P,T,S)     SCENARIOS FOR
UNCERTAINTY IN INITIAL
DELIVERABILITIES
                           S1      S2
S3      S4      S5      S6      S7
S8      S9
(W1*W4).N1.P1.(T1*T6)      4       4
4       5       5       5       6
6       6
;


TABLE

UIS(W,N,P,T,S)     SCENARIOS FOR
UNCERTAINTY IN SIZE
                           S1      S2
S3      S4      S5      S6      S7
S8      S9
(W1*W4).N1.P1.(T1*T6)      30      60
90      30      60      90      30
60      90
;


SCALAR
```

```
OMEGAU1            UPPER BOUND FOR OIL
FLOW RATE IN MILLION BARREL /5/
OMEGAU2            UPPER BOUND FOR OIL
FLOW RATE IN MILLION BARREL /5/
OMEGAU3            UPPER BOUND FOR OIL
FLOW RATE IN MILLION BARREL /5/
OMEGAU4            UPPER BOUND FOR
EXPANSION VARIABLE FOR WELL PLATFORM
/5/
OMEGAU5            UPPER BOUND FOR
EXPANSION VARIABLE FOR PRODUCTION
PLATFORM /5/
OMEGAU6            UPPER BOUND FOR
DRILLING RIG LOCATION /5/
OMEGAU7            UPPER BOUND FOR
FLOW PROFILE CONSTRAINT /5/
;

ALIAS

(T,THETA,TD,T1,TDASH)
;

FREE VARIABLES

Z                        TOTAL
PRODUCTION PROFIT FOR ALL SCENARIOS
AND PROBABILITIES IN DOLLARS
;

BINARY VARIABLES

Z1(W,N,P,T,S)        1 IF WELL W
CONNECTED TO WELL PLATFORM N TO
PRODUCTION PLATFORM P IS DRILLED IN
TIME T
Z2(N,P,T,S)          1 IF WELL
PLATFORM N CONNECTED TO PRODUCTION
PLATFORM P IS INSTALLED IN TIME T
Z3(P,T,S)            1 IF PRODUCTION
PLATFORM P IS INSTALLED IN TIME T
Z4(W,N,P,TD,S)       DUMMY PERIOD
VALUES AT PERIOD T
Z5(N,P,TD,S)         DUMMY PERIOD
VALUES AT PERIOD T
Z6(P,TD,S)           DUMMY PERIOD
VALUES AT PERIOD T
YD(R,F,J,T,S)        1 IF PIECE J USED
FOR LINEAR INTERPOLATION IN PERIOD T
ZD(N,P,D,T,S)        1 IF DRILLING RIG
K I LOCATED ON FACILITY IN PERIOD T
ZF(N,P,D,T,S)        1 IF THE K-TH RIG
IS LOCATED ON WELL PLATFORM N FIRST IN
PERIOD T
ZL(N,P,D,T,S)        1 IF THE K-TH RIG
IS LOCATED ON WELL PLATFORM N LAST IN
PERIOD T
SL(P,D,T,S)          1 WHEN ONLY THE
N-TH PLATFORM IS USED FOR DRILLING
USING THE K-TH RIG IN PERIOD T
ZT(N,P,D,T,S)        1 IF DRILLING RIG
D IS MOVED AFTER PERIOD T
;

POSITIVE VARIABLES

Z7(P,D,T,S)          NUMBER OF TIMES
DRILLING RIG D IS MOVED IN PERIOD T
X1(W,N,P,T,S)        AMOUNT OF GAS OR
OIL TO TRANSFER BETWEEN WELL W TO WELL
PLATFORM N TO PRODUCTION PLATFORM P IN
TIME T IN KG
X2(N,P,T,S)          AMOUNT OF GAS OR
OIL TO TRANSFER BETWEEN WELL PLATFORM
N TO PRODUCTION PLATFORM P IN TIME T
IN KG
```

```
X3(P,T,S)            AMOUNT OF GAS OR
OIL FLOWING AT PRODUCTION PLATFORM P
IN TIME T IN KG
X4(T,S)              SUM OF ALL GAS OR
OIL FLOWING AT PRODUCTION PLATFORM P
IN TIME T IN KG
V1(W,N,P,T,S)        PRESSURE FOR FLOW
BETWEEN WELL W TO WELL PLATFORM N TO
PRODUCTION PLATFORM P IN TIME T IN BAR
V2(N,P,T,S)          PRESSURE FOR FLOW
BETWEEN WELL PLATFORM N TO PRODUCTION
PLATFORM P IN TIME T IN BAR
V3(P,T,S)            PRESSURE FOR FLOW
AT PRODUCTION PLATFORM P IN TIME T IN
BAR
GF1(W,N,P,T,S)       GAS FLOW
(VOLUMETRIC) BETWEEN WELL W TO WELL
PLATFORM N TO PRODUCTION PLATFORM P IN
TIME T IN M3
GF2(N,P,T,S)         GAS FLOW
(VOLUMETRIC) BETWEEN WELL PLATFORM N
TO PRODUCTION PLATFORM P IN TIME T IN
M3
DELTA1(W,N,P,T,S)    PRESSURE DROP AT
CHOKE FOR FLOW BETWEEN WELL W TO WELL
PLATFORM N TO PRODUCTION PLATFORM P IN
TIME T IN BAR
DELTA2(N,P,T,S)      PRESSURE DROP AT
CHOKE FOR FLOW BETWEEN WELL PLATFORM N
TO PRODUCTION PLATFORM P IN TIME T IN
BAR
LF(W,N,P,T,S)        OIL FLOW FROM
WELL W TO WELL PLATFORM N TO
PRODUCTION PLATFORM P IN TIME T IN KG
GF(W,N,P,T,S)        GAS FLOW FROM
WELL W TO WELL PLATFORM N TO
PRODUCTION PLATFORM P IN TIME T IN M3
X5(W,N,P,THETA,S)    THE CUMULATIVE
AMOUNT OF OIL TO TRANSFER BETWEEN WELL
W TO WELL PLATFORM N TO PRODUCTION
PLATFORM P IN TIME THETA IN KG
X6(R,F,THETA,S)      THE CUMULATIVE
AMOUNT OF OIL FROM RESERVOIR R IN
FIELD F IN TIME THETA IN KG
V4(R,F,T,S)          PRESSURE IN
RESERVOIR R IN FIELD F AT TIME T IN
BAR
LAMBDA(R,F,J,T,S)    INTERPOLATION
VARIABLE AT PERIOD T
X7(N,P,THETA,S)      THE CUMULATIVE
AMOUNT OF OIL TO TRANSFER BETWEEN WELL
PLATFORM N TO PRODUCTION PLATFORM P IN
TIME THETA IN KG
X8(P,THETA,S)        THE CUMULATIVE
AMOUNT OF OIL TO FLOWING FROM
PRODUCTION PLATFORM P IN TIME THETA IN
KG
DV1(N,P,T,S)         DESIGN VARIABLE
FOR WELL PLATFORM
DV2(P,T,S)           DESIGN VARIABLE
FOR PRODUCTION PLATFORM
E1(N,P,T,S)          DESIGN EXPANSION
VARIABLE FOR WELL PLATFORM
E2(P,T,S)            DESIGN EXPANSION
VARIABLE FOR PRODUCTION PLATFORM
XD(W,N,P,T,S)        DELIVERABILITY OF
OIL FROM WELL
;

EQUATIONS

PROFIT               OBJECTIVE
FUNCTION IN MILLION DOLLAR
XTOTAL1(N,P,T,S)     SUM OF ALL GAS OR
OIL RELATED TO WELL PLATFORM N AND
PRODUCTION PLATFORM P IN TIME T IN KG
XTOTAL2(P,T,S)       SUM OF ALL GAS OR
OIL RELATED TO PRODUCTION PLATFORM P
IN TIME T IN KG
XTOTAL3(T,S)         SUM OF ALL GAS OR
OIL IN TIME T IN KG
VTOTAL1(W,N,P,T,S)   SUM OF ALL
PRESSURE RELATED TO WELL PLATFORM N1
AND PRODUCTION PLATFORM P1 IN TIME T1
IN BAR
VTOTAL2(N,P,T,S)     SUM OF ALL
PRESSURE RELATED TO PRODUCTION
PLATFORM P IN TIME T IN BAR
XFLOW1(W,N,P,T,S)    FLOW OF GAS OR
OIL FROM WELL W TO WELL PLATFORM N TO
PRODUCTION PLATFORM P IN TIME T IN KG
XFLOW2(W,N,P,T,S)    FLOW OF OIL FROM
WELL W TO WELL PLATFORM N TO
PRODUCTION PLATFORM P IN TIME T IN KG
XFLOW3(W,N,P,T,S)    FLOW OF GAS FROM
WELL W TO WELL PLATFORM N TO
PRODUCTION PLATFORM P IN TIME T IN KG
XCUM1(W,N,P,THETA,S) SUM OF THE AMOUNT
OF OIL FROM ALL PERIODS UP TO TIME
PERIOD THETA IN KG
XCUM2(R,F,THETA,S)   SUM OF THE AMOUNT
OF OIL FROM ALL PERIODS UP TO TIME
PERIOD THETA IN KG
PIECE1(R,F,T,S)      VALUE OF
INTERPOLATED VARIABLE FOR EACH
RESERVOIR R IN FIELD F IN TIME T
PIECE2(R,F,T,S)      CONSTRAINT FOR
THE INTERPOLATION VARIABLE AT PERIOD T
PIECE3(R,F,J,T,S)    CONSTRAINT FOR
GAMMA TO BE USED IN LINEAR
INTERPOLATION
PIECE4(R,F,T,S)      CONSTRAINT FOR
BINARY VARIABLE Y1=1 IF J USED FOR
LINEAR INTERPOLATION IN PERIOD T
INSTALL1(W,N,P,T,S)  Z=1 IF WELL W
CONNECTED TO WELL PLATFORM N TO
PRODUCTION PLATFORM P IS DRILLED IN
TIME T
INSTALL2(N,P,T,S)    Z=1 IF WELL
PLATFORM N CONNECTED TO PRODUCTION
PLATFORM P IS INSTALLED IN TIME T
INSTALL3(P,T,S)      Z=1 IF PRODUCTION
PLATFORM P IS INSTALLED IN TIME T
DUMMY1(W,N,P,TD,S)   TO REDUCE THE
NUMBER OF NODES ENUMERATED IN A BRANCH
AND BOUND TREE
DUMMY2(N,P,TD,S)     TO REDUCE THE
NUMBER OF NODES ENUMERATED IN A BRANCH
AND BOUND TREE
DUMMY3(P,TD,S)       TO REDUCE THE
NUMBER OF NODES ENUMERATED IN A BRANCH
AND BOUND TREE
XTOT1(W,N,P,THETA,S) THE FLOW OF OIL
FROM WELL W TO WELL PLATFORM N TO
PRODUCTION PLATFORM P IN PERIOD THETA
IN KG
XTOT2(N,P,THETA,S)   THE FLOW OF OIL
FROM WELL PLATFORM N TO PRODUCTION
PLATFORM P IN PERIOD THETA IN KG
XTOT3(P,THETA,S)     THE FLOW OF OIL
IN PRODUCTION PLATFORM P IN PERIOD
THETA IN KG
ZCON1(W,N,P,THETA,S) THE WELL PLATFORM
ASSOCIATED WITH A WELL MUST BE
INSTALLED BEFORE DRILLING THAT WELL
ZCON2(N,P,THETA,S)   THE PRODUCTION
PLATFORMS MUST BE INSTALLED BEFORE
ASSOCIATED WELL PLATFORMS
DES1(N,P,T,S)        THE FLOW FROM
WELL PLATFORM N TO PRODUCTION PLATFORM
P IN TIME T
```

```
DES2(N,P,T,S)        DESIGN VARIABLE
FOR WELL PLATFORM N TO PRODUCTION
PLATFORM P IN TIME T
DES3(N,P,T,S)        DESIGN EXPANSION
VARIABLE FOR WELL PLATFORM N TO
PRODUCTION PLATFORM P IN TIME T
DES4(P,T,S)          THE FLOW FROM
PRODUCTION PLATFORM P IN TIME T
DES5(P,T,S)          DESIGN VARIABLE
FOR PRODUCTION PLATFORM P IN TIME T
DES6(P,T,S)          DESIGN EXPANSION
VARIABLE FOR PRODUCTION PLATFORM P IN
TIME T
DRSC1(W,N,P,T,S)     IF ANY WELL
ASSOCIATED WITH WELL PLATFORM N IS
DRILLED IN PERIOD T _ SUMMATION OF Z(N
P K T) MUST BE 1 IN THAT TIME PERIOD
DRSC2(N,P,D,T,S)     THE K-TH RIG IS
LOCATED ON THE N-TH PLATFORM IN THAT
TIME PERIOD (SEE DRSC3)
DRSC3(P,D,T,S)       THE SLACK
VARIABLE TAKES A NON-ZERO VALUE IF ALL
WELLS ARE DRILLED FROM A SINGLE N-TH
PLATFORM IN THAT TIME PERIOD
DRSC4(N,P,D,T,S)     CONSTRAINT FOR
WELL DRILLED FROM WELL PLATFORM
DRSC5(N,P,D,T,S)     CONSTRAINT FOR
WELL DRILLED FROM WELL PLATFORM
DRSC6(N,P,D,T,S)     MOVEMENT OF RIG
CONSTRAINT
DRSC7(P,D,T,S)       MOVEMENT OF RIG
CONSTRAINT
FLOWPC(W,N,P,T,S)    FLOW PROFILE
CONSTRAINT
ENDO1(W,N,P,T,S)     ENDOGENEOUS
UNCERTAINTY IN SIZE AND INITIAL
DELIVERABILITY OF WELL
ENDO2(W,N,P,T,S)     OIL FLOW
CONSTRAINTS FOR ENDOGENEOUS
UNCERTAINTY
ENDO3(W,N,P,T,S)     CUMULATIVE OIL
CONSTRAINTS FOR ENDOGENEOUS
UNCERTAINTY
NAC1                 NON-
ANTICIPATIVITY CONSTRAINT
NAC2                 NON-
ANTICIPATIVITY CONSTRAINT
NAC3                 NON-
ANTICIPATIVITY CONSTRAINT
NAC4                 NON-
ANTICIPATIVITY CONSTRAINT
NAC5                 NON-
ANTICIPATIVITY CONSTRAINT
NAC6                 NON-
ANTICIPATIVITY CONSTRAINT
NAC7                 NON-
ANTICIPATIVITY CONSTRAINT
;

PROFIT..
Z =E=
SUM((S,SP,T),PROBA(S)*PROB(SP)*C1(T,SP
)*X4(T,S)) - SUM(S,
SUM(T,SUM(P,C2P(P,T)*Z3(P,T,S) +
C3P(P,T)*E2(P,T,S))) -
SUM(T,SUM(P,SUM(N$N1(N,P),(C2NP(N,P,T)
*Z2(N,P,T,S) +
C3NP(N,P,T)*E1(N,P,T,S))))) -
SUM(T,SUM(P,SUM(N,SUM(W$W1(W,N),C2WNP(
W,N,P,T)*Z1(W,N,P,T,S))))) -
(SUM(T,SUM(P,SUM(D,SUM(N$N1(N,P),ZT(N,
P,D,T,S)))))));

*1.MASS BALANCE

XTOTAL1(N,P,T,S)$N1(N,P)..
X2(N,P,T,S) =E=
SUM(W$W1(W,N),X1(W,N,P,T,S))$N1(N,P);
XTOTAL2(P,T,S)..
SUM(N$N1(N,P),X2(N,P,T,S)) =E=
X3(P,T,S);
XTOTAL3(T,S)..
SUM(P,X3(P,T,S)) =E= X4(T,S);

*2.PRESSURE BALANCE

VTOTAL1(W,N,P,T,S)$(N1(N,P)$W1(W,N))..
V1(W,N,P,T,S) - ALPHAN*X1(W,N,P,T,S) -
BETAN*GF1(W,N,P,T,S) -
DELTA1(W,N,P,T,S) =E=
V2(N,P,T,S)$(W1(W,N)$(N1(N,P)));
VTOTAL2(N,P,T,S)$N1(N,P)..
V2(N,P,T,S) - ALPHAP*X2(N,P,T,S) -
BETAP*GF2(N,P,T,S) - DELTA2(N,P,T,S)
=E= V3(P,T,S)$N1(N,P);

*3.FLOW CONSTRAINTS IN WELLS

XFLOW1(W,N,P,T,S)$(N1(N,P)$W1(W,N))..
X1(W,N,P,T,S) =E= LF(W,N,P,T,S) +
GF(W,N,P,T,S)$(W1(W,N)$(N1(N,P)));
XFLOW2(W,N,P,T,S)$(N1(N,P)$W1(W,N))..
LF(W,N,P,T,S) =L=
RO(W,N,P)*PMAX$(W1(W,N)$(N1(N,P)));
XFLOW3(W,N,P,T,S)$(N1(N,P)$W1(W,N))..
GF(W,N,P,T,S) =L=
LF(W,N,P,T,S)*GORMAX$(W1(W,N)$(N1(N,P)
));

*4.CUMULATIVE FLOW AMOUNT FROM WELLS
UP TO PERIOD T

XCUM1(W,N,P,THETA,S)$(N1(N,P)$W1(W,N))
..         X5(W,N,P,THETA,S) =E=
SUM(T$(ORD(T) LE (ORD(THETA)-1)),
X1(W,N,P,T,S)*DELTAT);
XCUM2(R,F,THETA,S)$R1(R,F)..
X6(R,F,THETA,S) =E=
SUM((W,N,P)$(W1(W,N)$(N1(N,P)$(W4(W,R,
F)))),X5(W,N,P,THETA,S));

*5.PIECEWISE LINEAR INTERPOLATION AT
WELL AND RESERVOIR LEVEL

PIECE1(R,F,T,S)..
SUM(J$J1(J,R),LAMBDA(R,F,J,T,S)*V5(R,F
,J,T)) =E= V4(R,F,T,S)$R1(R,F);
PIECE2(R,F,T,S)..
SUM(J$J1(J,R),LAMBDA(R,F,J,T,S)) =E=
1$R1(R,F);
PIECE3(R,F,J,T,S)..
LAMBDA(R,F,J,T,S) =L= YD(R,F,J,T,S) +
YD(R,F,J-1,T,S)$R1(R,F);
PIECE4(R,F,T,S)..
SUM(J,YD(R,F,J,T,S)) =E= 1$R1(R,F);

*6.LOGICAL CONSTRAINTS FOR
INSTALLATION AND FLOW FROM FACILITIES

INSTALL1(W,N,P,T,S)$(W1(W,N)$(N1(N,P))
)..        Z1(W,N,P,T,S) =L= 1;
INSTALL2(N,P,T,S)..
Z2(N,P,T,S) =L= 1$N1(N,P);
INSTALL3(P,T,S)..
Z3(P,T,S) =L= 1;
DUMMY1(W,N,P,TD,S)..
SUM(T,Z1(W,N,P,T,S)) + Z4(W,N,P,TD,S)
=E= 1$(N1(N,P)$W1(W,N));
DUMMY2(N,P,TD,S)..
SUM(T,Z2(N,P,T,S)) + Z5(N,P,TD,S) =E=
1;
```

```
DUMMY3(P,TD,S)..
SUM(T,Z3(P,T,S)) + Z6(P,TD,S) =E= 1;
XTOT1(W,N,P,T,S)$(N1(N,P)$W1(W,N))..
X1(W,N,P,T,S) =L=
OMEGAU1*Z1(W,N,P,T,S);
XTOT2(N,P,THETA,S)$N1(N,P)..
X2(N,P,THETA,S) =L=
OMEGAU2*SUM(T$((ORD(T)LE
ORD(THETA))),Z2(N,P,T,S));
XTOT3(P,THETA,S)..
X3(P,THETA,S) =L=
OMEGAU3*SUM(T$((ORD(T) LE
ORD(THETA))),Z3(P,T,S));
ZCON1(W,N,P,THETA,S)$(N1(N,P)$W1(W,N))
..        Z1(W,N,P,THETA,S) =L=
SUM(T$(ORD(T) LE
ORD(THETA)),Z2(N,P,T,S));
ZCON2(N,P,THETA,S)$N1(N,P)..
Z2(N,P,THETA,S) =L= SUM(T$(ORD(T) LE
ORD(THETA)),Z3(P,T,S));

*7.DESIGN OF FACILITY

DES1(N,P,T,S)$N1(N,P)..
X2(N,P,T,S) =L= DV1(N,P,T,S);
DES2(N,P,T,S)$N1(N,P)..
DV1(N,P,T,S) =E= DV1(N,P,T-1,S) +
E1(N,P,T,S);
DES3(N,P,T,S)$N1(N,P)..
E1(N,P,T,S) =L= OMEGAU4*Z2(N,P,T,S);
DES4(P,T,S)..
X3(P,T,S) =L= DV2(P,T,S);
DES5(P,T,S)..
DV2(P,T,S) =E= DV2(P,T-1,S) +
E2(P,T,S);
DES6(P,T,S)..
E2(P,T,S) =L= OMEGAU5*Z3(P,T,S);

*8.NUMBER OF WELLS DRILLED IN TIME
PERIOD T

DRSC1(W,N,P,T,S)$(N1(N,P)$W1(W,N))..
Z1(W,N,P,T,S) =L=
SUM(D,ZD(N,P,D,T,S));
DRSC2(N,P,D,T,S)$N1(N,P)..
ZF(N,P,D,T,S) + ZL(N,P,D,T,S) =L= 1 +
SL(P,D,T,S);
DRSC3(P,D,T,S)..
SUM(N$(N1(N,P)),ZD(N,P,D,T,S)) - 1 =L=
OMEGAU6*(1 - SL(P,D,T,S));
DRSC4(N,P,D,T,S)$N1(N,P)..
ZF(N,P,D,T,S) =L= ZD(N,P,D,T,S);
DRSC5(N,P,D,T,S)$N1(N,P)..
ZL(N,P,D,T,S) =L= ZD(N,P,D,T,S);
DRSC6(N,P,D,T,S)$N1(N,P)..
ZF(N,P,D,T,S) - ZL(N,P,D,T-1,S) =L=
ZT(N,P,D,T,S);
DRSC7(P,D,T,S)..
SUM(N$(N1(N,P)), ZD(N,P,D,T,S)) - 1
=L= Z7(P,D,T,S);

*9.DRILLING RIG SCHEDULING CONSTRAINTS

*10.FLOW PROFILE CONSTRAINTS

FLOWPC(W,N,P,T,S)$(N1(N,P)$W1(W,N))..
X1(W,N,P,T,S) =G= X1(W,N,P,T+1,S) -
OMEGAU7*(1 - SUM(TDASH$(ORD(TDASH) LE
ORD(T)),Z1(W,N,P,TDASH,S)));

*11.ENDOGENEOUS UNCERTAINTY IN SIZE
AND INITIAL DELIVERABILITY OF WELL

ENDO1(W,N,P,T,S)$(N1(N,P)$W1(W,N))..
(XD(W,N,P,T,S)/UID(W,N,P,T,S)) +
(X5(W,N,P,T,S)/UIS(W,N,P,T,S)) =E= 1;
ENDO2(W,N,P,T,S)$(N1(N,P)$W1(W,N))..
X1(W,N,P,T,S) =L= XD(W,N,P,T,S);
ENDO3(W,N,P,T,S)$(N1(N,P)$W1(W,N))..
X5(W,N,P,T,S) =L= UIS(W,N,P,T,S);

*12.NONANTICIPATIVITY CONSTRAINTS

NAC1(W,N,P,T,S)$((ORD(S) NE
CARD(S))$(N1(N,P)$W1(W,N)))..
Z1(W,N,P,T,S) =E= Z1(W,N,P,T,S+1);
NAC2(N,P,T,S)$((ORD(S) LT
9)$(N1(N,P)))..
Z2(N,P,T,S) =E= Z2(N,P,T,S+1);
NAC3(P,T,S)$((ORD(S) LT 9))..
Z3(P,T,S) =E= Z3(P,T,S+1);
NAC4(N,P,D,T,S)$((ORD(S) LT
9)$(N1(N,P)))..
ZD(N,P,D,T,S) =E= ZD(N,P,D,T,S+1);
NAC5(N,P,D,T,S)$((ORD(S) LT 9))..
ZT(N,P,D,T,S) =E= ZT(N,P,D,T,S+1);
NAC6(N,P,T,S)$((ORD(S) LT
9)$N1(N,P))..
E1(N,P,T,S) =E= E1(N,P,T,S+1);
NAC7(P,T,S)$((ORD(S) LT 9))..
E2(P,T,S) =E= E2(P,T,S+1);

*VARIABLE BOUNDS

X1.UP(W,N,P,T,S) = 1000;

MODEL OILPLAN /ALL/
;

OPTION LIMROW = 100;
OPTION LIMCOL = 100;
OPTION OPTCR = 0.0;

SOLVE OILPLAN USING MIP MAXIMIZING Z;
```

APPENDIX V: GAMS OUTPUT FILE ON STOCHASTIC MODEL

```
GAMS Rev 149  x86/MS Windows
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: OIL PRODUCTION PLANNING (STOCHASTIC
MODEL)
E x e c u t i o n


----    374 VARIABLE Z.L
=      145.580  TOTAL PRODUCTION
PROFIT FOR ALL SCENARIOS AND
PROBABILITIES IN

DOLLARS

----    374 VARIABLE Z1.L  1 IF WELL W
CONNECTED TO WELL PLATFORM N TO
PRODUCTIO
                           N PLATFORM
P IS DRILLED IN TIME T

INDEX 1 = W1  INDEX 2 = N1

               S1          S2
S3          S4          S5          S6

P1.T3       1.000       1.000
1.000       1.000       1.000
1.000

    +          S7          S8
S9

P1.T3       1.000       1.000
1.000

INDEX 1 = W2  INDEX 2 = N1

               S1          S2
S3          S4          S5          S6

P1.T1       1.000       1.000
1.000       1.000       1.000
1.000

    +          S7          S8
S9

P1.T1       1.000       1.000
1.000

INDEX 1 = W3  INDEX 2 = N1

               S1          S2
S3          S4          S5          S6

P1.T2       1.000       1.000
1.000       1.000       1.000
1.000

    +          S7          S8
S9

P1.T2       1.000       1.000
1.000

INDEX 1 = W4  INDEX 2 = N1

               S1          S2
S3          S4          S5          S6

P1.T5       1.000       1.000
1.000       1.000       1.000
1.000

    +          S7          S8
S9

P1.T5       1.000       1.000
1.000


----    374 VARIABLE Z2.L  1 IF WELL
PLATFORM N CONNECTED TO PRODUCTION
PLATFORM
                           P IS
INSTALLED IN TIME T

INDEX 1 = N1

               S1          S2
S3          S4          S5          S6

P1.T1       1.000       1.000
1.000       1.000       1.000
1.000

    +          S7          S8
S9

P1.T1       1.000       1.000
1.000


----    374 VARIABLE Z3.L  1 IF
PRODUCTION PLATFORM P IS INSTALLED IN
TIME T

               S1          S2
S3          S4          S5          S6

P1.T1       1.000       1.000
1.000       1.000       1.000
1.000

    +          S7          S8
S9

P1.T1       1.000       1.000
1.000


----    374 VARIABLE X1.L  AMOUNT OF
GAS OR OIL TO TRANSFER BETWEEN WELL W
TO WELL PLATFORM N TO PRODUCTION
PLATFORM P IN TIME T IN K
                           G

INDEX 1 = W1  INDEX 2 = N1

               S1          S2
S3          S4          S5          S6

P1.T3       4.000       4.000
4.000       5.000       5.000
5.000

    +          S7          S8
S9

P1.T3       5.000       5.000
5.000

INDEX 1 = W2  INDEX 2 = N1
```

```
              S1          S2
S3          S4          S5          S6

P1.T1       4.000       4.000
4.000       5.000       5.000
5.000

   +          S7          S8
S9

P1.T1       5.000       5.000
5.000

INDEX 1 = W3  INDEX 2 = N1

              S1          S2
S3          S4          S5          S6

P1.T2       4.000       4.000
4.000       5.000       5.000
5.000

   +          S7          S8
S9

P1.T2       5.000       5.000
5.000

INDEX 1 = W4  INDEX 2 = N1

              S1          S2
S3          S4          S5          S6

P1.T5       4.000       4.000
4.000       5.000       5.000
5.000

   +          S7          S8
S9

P1.T5       5.000       5.000
5.000


----    374 VARIABLE X2.L  AMOUNT OF
GAS OR OIL TO TRANSFER BETWEEN WELL
PLATFORM N TO PRODUCTION PLATFORM P IN
TIME T IN KG

INDEX 1 = N1

              S1          S2
S3          S4          S5          S6

P1.T1       4.000       4.000
4.000       5.000       5.000
5.000
P1.T2       4.000       4.000
4.000       5.000       5.000
5.000
P1.T3       4.000       4.000
4.000       5.000       5.000
5.000
P1.T5       4.000       4.000
4.000       5.000       5.000
5.000

   +          S7          S8
S9

P1.T1       5.000       5.000
5.000
P1.T2       5.000       5.000
5.000
P1.T3       5.000       5.000
5.000
P1.T5       5.000       5.000
5.000


----    374 VARIABLE X3.L  AMOUNT OF
GAS OR OIL FLOWING AT PRODUCTION
PLATFORM P IN TIME T IN KG

              S1          S2
S3          S4          S5          S6

P1.T1       4.000       4.000
4.000       5.000       5.000
5.000
P1.T2       4.000       4.000
4.000       5.000       5.000
5.000
P1.T3       4.000       4.000
4.000       5.000       5.000
5.000
P1.T5       4.000       4.000
4.000       5.000       5.000
5.000

   +          S7          S8
S9

P1.T1       5.000       5.000
5.000
P1.T2       5.000       5.000
5.000
P1.T3       5.000       5.000
5.000
P1.T5       5.000       5.000
5.000


----    374 VARIABLE X4.L  SUM OF ALL
GAS OR OIL FLOWING AT PRODUCTION
PLATFORM P IN TIME T IN KG

            S1          S2          S3
S4          S5          S6

T1       4.000       4.000       4.000
5.000       5.000       5.000
T2       4.000       4.000       4.000
5.000       5.000       5.000
T3       4.000       4.000       4.000
5.000       5.000       5.000
T5       4.000       4.000       4.000
5.000       5.000       5.000

 +          S7          S8          S9

T1       5.000       5.000       5.000
T2       5.000       5.000       5.000
T3       5.000       5.000       5.000
T5       5.000       5.000       5.000


----    374 VARIABLE Z7.L  NUMBER OF
TIMES DRILLING RIG D IS MOVED IN
PERIOD T

                      ( ALL
0.000 )


**** REPORT FILE SUMMARY

RESPLAN2 C:\Documents and
Settings\Kippi\My
Documents\gamsdir\projdir\UPSTREAM2.
         res
```

```
EXECUTION TIME       =        0.078
SECONDS      3 Mb  WIN226-149 Dec 19,
2007


USER: CS/IE 635, Spring 2007
G061206/0001AS-WIN
      Prof. Ferris
DC2937
      License for teaching and
research at degree granting
institutions
```

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APPENDIX VI: COMPUTER CODE FOR GAMS INPUT FILE ON BRANCH CONTRACT ALGORITHM MODEL

```
$TITLE: SCHEDULING REFINERY CRUDE OIL
OPERATIONS

$EOLCOM !

$ONEMPTY

*number of equations listed per block
option limrow = 0;

*number of variables listed per block
option limcol=0;

*do you want solver's solution output
printed?
option solprint=on;

*do you want solver's system output
printed?
option sysout=off;

*milp relative termination tolerance
option optcr=0;

*milp absolute termination tolerance
option optca=0;

*Decimals to display
option decimals=8;

*File to write results
FILE RESCRU  / cos1.res /;

PUT RESCRU;

PUT "--------------------------------
---------------------------------"
PUT / " SUBSYSTEM B.DOWNSTREAM CRUDE
OIL SCHEDULING AT REFINERY FRONT-END "
PUT / "------------------------------
-----------------------------------"
*------------------------------------
--------Choosing solvers

option lp=minos5;
option nlp=conopt;
option mip=osl;
option rmip=osl;
option minlp=DICOPT;

SETS

         A                SET OF TANK
INPUT SOURCES               /
                          U1, U2, U3,
ST1, ST2, ST3               /

         B                SET OF CRUDE
TANKS                      /
                          ST1  Storage
Tank 1
                          ST2  Storage
Tank 2
                          ST3  Storage
Tank 3
                          CT1
Charging Tank 1
                          CT2
Charging Tank 2
                          CT3
Charging Tank 3                   /

         H                SET OF TANK
OUTPUT DESTINATIONS         /
                          CT1, CT2,
CT3, D1, D2                   /

         D(H)             SET OF
DISTILLATION UNITS               /
                          D1 * D2
/

         G(B)             SET OF
CHARGING TANKS                   /
                          CT1 * CT3
/

         I                SET OF
COMPONENTS                       /
                          I1 * I2
/

         M                SOURCE UNIT
FOR SPLIT PIPELINE          /
                          MST2, MST3,
MCT2                        /

         K                DESTINATION
UNIT OF SPLIT PIPELINE      /
                          KCT3, KD2
/

         U(A)             SET OF CRUDE
SUPPLY STREAMS             /
                          U1 * U3
/

         Y(B)             SET OF
STORAGE TANKS                    /
                          ST1 * ST3
/

         L                TRANSFER
EVENT                          /
                          L0 * L2
/

         Q                SET OF SUB-
STRUCTURES                    /
                          Q1 * Q2
/

ALIAS

         (A , ADASH)
         (H , HDASH)
         (G , GDASH)
         (L , LDASH)

;


* ============================ for
MAPPING purposes
============================= *

SETS
```

```
         A1(A,B)          SET OF
INPUTS A TO TANK B
/

U1.(ST1,ST2),U2.ST2,U3.(ST2,ST3),

ST1.(CT1,CT2),ST2.(CT1,CT2,CT3),ST3.(C
T2,CT3) /

         A2(A,Y)          SET OF
INPUTS A TO STORAGE TANK Y
/

U1.(ST1,ST2),U2.ST2,U3.(ST2,ST3)
/


         D3(D,G)          SET OF
DISTILLATION UNITS D THAT CAN BE
CHARGED BY CHARGING TANK G /
                          D1.CT1,
D1.CT2, D2.CT2, D2.CT3
/


         G1(G,D)          SET OF
CHARGING TANKS G THAT CHARGE
DISTILLATION UNIT D            /
                          CT1.D1,
CT2.D1, CT2.D2, CT3.D2
/


         H1(B,H)          SET OF
OUTPUTS H FROM TANK B
/

CT1.D1,CT2.(D1,D2),CT3.D2
/

         H2(Y,H)          SET OF
OUTPUTS H FROM STORAGE TANK Y
/

/

         H3(G,H)          SET OF
OUTPUTS H FROM CHARGING TANK G
/

CT1.D1,CT2.(D1,D2),CT3.D2
/


         H4(Y,H)          SET OF
OUTPUTS H FROM STORAGE TANK Y
/

ST1.CT1,ST2.(CT1,CT2,CT3),ST3.CT3  /

         Y1(Y,U)          SET OF
STORAGE TANKS Y CONNECTED TO CRUDE
SUPPLY STREAMS U         /
                          ST1.U1,
ST2.U1, ST2.U2, ST2.U3, ST3.U3
/



* ==================== for
DECOMPOSITION purposes
==================== *

         B1(B)            SET OF TANKS
B BELONGING TO SUB-STRUCTURE Q1 /
                          ST1, ST2,
ST3, CT1, CT2                      /

         B2(B)            SET OF TANKS
B BELONGING TO SUB-STRUCTURE Q2 /
                          CT3
/


         D1(D)            SET OF
DISTILLATION UNITS D PRESENT IN SUB-
STRUCTURE Q1 /
                          D1
/

         D2(D)            SET OF
DISTILLATION UNITS D PRESENT IN SUB-
STRUCTURE Q2 /
                          D2
/


         U1(U)            SET OF CRUDE
SUPPLY STREAMS U PRESENT IN SUB-
STRUCTURE Q1 /
                          U1, U2, U3
/

         U2(U)            SET OF CRUDE
SUPPLY STREAMS U PRESENT IN SUB-
STRUCTURE Q2 /

/


         K1(M,K)          SET OF
DESTINATION UNIT K OF SPLIT PIPELINES
WITH SOURCE M /
                          MST2.KCT3,
MST3.KCT3, MCT2.KD2
/
;




*========================= BRANCH AND
CONTRACT ALGORITHM
========================

SETS

         CVI      / I1, I2
/
         CVA      / U1, U2, U3, ST1,
ST2, ST3                /
         CVB      / ST1, ST2, ST3,
CT1, CT2, CT3        /
         CVH      / CT1, CT2, CT3, D1,
D2                 /
         CVL      / L0 * L2
/

         SBLUEITOT(B,L)
         SREDITOT(B,L)
         SGREENITOT(B,L)
         SBLACKITOT(B,L)

ALIAS    (I,J)
         (A,X)
         (B,N)
         (H,C)
         (L,T);
```

```
         SBLUEITOT(B,L) = YES;
         SREDITOT(B,L) = NO;
         SGREENITOT(B,L) = NO;
         SBLACKITOT(B,L) = NO;



*======================== BRANCH AND
CONTRACT ALGORITHM
========================


POSITIVE VARIABLE

        ITOTINI(B,L)
        ITOTBEST(B,L)
        ITOTOLD(B,L)
        ITOTNEW(B,L)



        CONTADOR

        MAXDIS
        MAXPCDIS
        MAXABSDIS
        AUXABS
        AUXREL
        INFACTIBLE
        VARCONVER
        TIEMPOTOT
        LBEST
        ABSGAP
        RELGAP


FREE VARIABLE

        FREEITOTST1L0
        FREEITOTST1L1
        FREEITOTST1L2
        FREEITOTST2L0
        FREEITOTST2L1
        FREEITOTST2L2
        FREEITOTST3L0
        FREEITOTST3L1
        FREEITOTST3L2
        FREEITOTCT1L0
        FREEITOTCT1L1
        FREEITOTCT1L2
        FREEITOTCT2L0
        FREEITOTCT2L1
        FREEITOTCT2L2
        FREEITOTCT3L0
        FREEITOTCT3L1
        FREEITOTCT3L2

*=================================
MAIN MODEL
=================================


POSITIVE VARIABLE

         ITOT(B,L)        TOTAL
INVENTORY OF TANK B AT THE END OF
TRANSFER EVENT L
         INVI(I,B,L)      INVENTORY OF
COMPONENT I IN TANK B AT THE END OF
TRANSFER EVENT L

         TS1(A,B,L)       STARTING
TIME OF A TRANSFER FROM A TO B IN
TRANSFER EVENT L
         TE1(A,B,L)       ENDING TIME
OF A TRANSFER FROM A TO B IN TRANSFER
EVENT L
         TS2(B,H,L)       STARTING
TIME OF A TRANSFER FROM B TO H IN
TRANSFER EVENT L
         TE2(B,H,L)       ENDING TIME
OF A TRANSFER FROM B TO H IN TRANSFER
EVENT L


         TS1Q1(A,B,L)     STARTING
TIME OF A TRANSFER FROM A TO B IN
TRANSFER EVENT L
         TE1Q1(A,B,L)     ENDING TIME
OF A TRANSFER FROM A TO B IN TRANSFER
EVENT L
         TS2Q1(B,H,L)     STARTING
TIME OF A TRANSFER FROM B TO H IN
TRANSFER EVENT L
         TE2Q1(B,H,L)     ENDING TIME
OF A TRANSFER FROM B TO H IN TRANSFER
EVENT L


         TS1Q2(A,B,L)     STARTING
TIME OF A TRANSFER FROM A TO B IN
TRANSFER EVENT L
         TE1Q2(A,B,L)     ENDING TIME
OF A TRANSFER FROM A TO B IN TRANSFER
EVENT L
         TS2Q2(B,H,L)     STARTING
TIME OF A TRANSFER FROM B TO H IN
TRANSFER EVENT L
         TE2Q2(B,H,L)     ENDING TIME
OF A TRANSFER FROM B TO H IN TRANSFER
EVENT L


         TEO(U)           OVERALL
ENDING TIME OF CRUDE TRANSFER FROM
CRUDE SUPPLY STREAM U
         TSI(U)           INITIAL
STARTING TIME OF CRUDE TRANSFER FROM
CRUDE SUPPLY STREAM U


* ============================ Flow
variables
=============================== *

         FT1(A,B,L)       TOTAL FLOW
FROM A TO B IN TRANSFER EVENT L
         FT2(B,H,L)       TOTAL FLOW
FROM B TO H IN TRANSFER EVENT L


         FL1(I,A,B,L)     FLOW OF
COMPONENT I FROM A TO B IN TRANSFER
EVENT L
         FL2(I,B,H,L)     FLOW OF
COMPONENT I FROM B TO H IN TRANSFER
EVENT L


         FT1Q1(A,B,L)     TOTAL FLOW
FROM A TO B IN TRANSFER EVENT L
         FT2Q1(B,H,L)     TOTAL FLOW
FROM B TO H IN TRANSFER EVENT L
```

```
        FL1Q1(I,A,B,L)   FLOW OF
COMPONENT I FROM A TO B IN TRANSFER
EVENT L
        FL2Q1(I,B,H,L)   FLOW OF
COMPONENT I FROM B TO H IN TRANSFER
EVENT L


        FT1Q2(A,B,L)     TOTAL FLOW
FROM A TO B IN TRANSFER EVENT L
        FT2Q2(B,H,L)     TOTAL FLOW
FROM B TO H IN TRANSFER EVENT L


        FL1Q2(I,A,B,L)   FLOW OF
COMPONENT I FROM A TO B IN TRANSFER
EVENT L
        FL2Q2(I,B,H,L)   FLOW OF
COMPONENT I FROM B TO H IN TRANSFER
EVENT L



* ============== Variables to replace
bilinear terms to solve LBP
============== *

        INVVJ(I,B,H,L)
INVENTORY COMPONENT I FOR LBP
        INVVT(I,B,H,L)
INVENTORY COMPONENT I FOR LBP

        INVVJQ1(I,B,H,L)
INVENTORY COMPONENT I FOR LBP
        INVVTQ1(I,B,H,L)
INVENTORY COMPONENT I FOR LBP

        INVVJQ2(I,B,H,L)
INVENTORY COMPONENT I FOR LBP
        INVVTQ2(I,B,H,L)
INVENTORY COMPONENT I FOR LBP

        FLOWVJ(I,A,B,H,L)       FLOW
OF COMPONENT I FOR LBP
        FLOWVT(I,A,B,H,L)       FLOW
OF COMPONENT I FOR LBP

        FLOWVJQ(I,A,B,H,L,Q)    FLOW
OF COMPONENT I FOR LBP
        FLOWVTQ(I,A,B,H,L,Q)    FLOW
OF COMPONENT I FOR LBP

        FLOWVJQ1(I,A,B,H,L)     FLOW
OF COMPONENT I FOR LBP
        FLOWVTQ1(I,A,B,H,L)     FLOW
OF COMPONENT I FOR LBP

        FLOWVJQ2(I,A,B,H,L)     FLOW
OF COMPONENT I FOR LBP
        FLOWVTQ2(I,A,B,H,L)     FLOW
OF COMPONENT I FOR LBP



* ====================== Duplicate
variables declaration
============================ *

        FLTQ(M,K,L)      TOTAL FLOW
VARIABLE
        FLTQ1(M,K,L)     DUPLICATE
VARIABLE FOR SUB-STRUCTURE Q1
        FLTQ2(M,K,L)     DUPLICATE
VARIABLE FOR SUB-STRUCTURE Q2
        FLQ(I,M,K,L)     FLOW
VARIABLE
        FLQ1(I,M,K,L)    DUPLICATE
VARIABLE FOR SUB-STRUCTURE Q1
        FLQ2(I,M,K,L)    DUPLICATE
VARIABLE FOR SUB-STRUCTURE Q2

        TSQ(M,K,L)       START TIME
VARIABLE
        TSQ1(M,K,L)      DUPLICATE
VARIABLE FOR SUB-STRUCTURE Q1
        TSQ2(M,K,L)      DUPLICATE
VARIABLE FOR SUB-STRUCTURE Q2

        TEQ(M,K,L)       END TIME
VARIABLE
        TEQ1(M,K,L)      DUPLICATE
VARIABLE FOR SUB-STRUCTURE Q1
        TEQ2(M,K,L)      DUPLICATE
VARIABLE FOR SUB-STRUCTURE Q2

        WBQ(M,K,L)       FLOW
EXISTENCE VARIABLE
        WBQ1(M,K,L)      DUPLICATE
VARIABLE FOR SUB-STRUCTURE Q1
        WBQ2(M,K,L)      DUPLICATE
VARIABLE FOR SUB-STRUCTURE Q2

        ONEG1Q1(B,L)       NEGATIVE
SLACK VARIABLE
        OPOS1Q1(B,L)       POSITIVE
SLACK VARIABLE
        ONEG2Q1(I,B,L)     NEGATIVE
SLACK VARIABLE
        OPOS2Q1(I,B,L)     POSITIVE
SLACK VARIABLE
        ONEG3Q1(B,H,L)     NEGATIVE
SLACK VARIABLE
        OPOS3Q1(B,H,L)     POSITIVE
SLACK VARIABLE
        ONEG4Q1(I,B,L)     NEGATIVE
SLACK VARIABLE
        OPOS4Q1(I,B,L)     POSITIVE
SLACK VARIABLE
        ONEG5Q1(I,B,H,L)   NEGATIVE
SLACK VARIABLE
        OPOS5Q1(I,B,H,L)   POSITIVE
SLACK VARIABLE
        ONEG6Q1(I,B,H,L)   NEGATIVE
SLACK VARIABLE
        OPOS6Q1(I,B,H,L)   POSITIVE
SLACK VARIABLE
        ONEG7Q1(I,B,H,L)   NEGATIVE
SLACK VARIABLE
        OPOS7Q1(I,B,H,L)   POSITIVE
SLACK VARIABLE

        ONEG1Q2(B,L)       NEGATIVE
SLACK VARIABLE
        OPOS1Q2(B,L)       POSITIVE
SLACK VARIABLE
        ONEG2Q2(I,B,L)     NEGATIVE
SLACK VARIABLE
        OPOS2Q2(I,B,L)     POSITIVE
SLACK VARIABLE
        ONEG3Q2(B,H,L)     NEGATIVE
SLACK VARIABLE
        OPOS3Q2(B,H,L)     POSITIVE
SLACK VARIABLE
        ONEG4Q2(I,B,L)     NEGATIVE
SLACK VARIABLE
        OPOS4Q2(I,B,L)     POSITIVE
SLACK VARIABLE
        ONEG5Q2(I,B,H,L)   NEGATIVE
SLACK VARIABLE
```

```
        OPOS5Q2(I,B,H,L)   POSITIVE
SLACK VARIABLE
        ONEG6Q2(I,B,H,L)   NEGATIVE
SLACK VARIABLE
        OPOS6Q2(I,B,H,L)   POSITIVE
SLACK VARIABLE
        ONEG7Q2(I,B,H,L)   NEGATIVE
SLACK VARIABLE
        OPOS7Q2(I,B,H,L)   POSITIVE
SLACK VARIABLE
;


FREE VARIABLES

        ZZ                OBJECTIVE
FUNCTION OF COST
        ZZUB              OBJECTIVE
FUNCTION FOR UPPER BOUND
        ZZFIXBIN
        ZZQ1              OBJECTIVE
FUNCTION OF COST FOR STRUCTURE Q1
        ZZQ2              OBJECTIVE
FUNCTION OF COST FOR STRUCTURE Q2
        ZZU               OBJECTIVE
FUNCTION UPPER BOUND
*       ZZUB              OBJECTIVE
FUNCTION FOR UPPER BOUND
*       ZZOPTQ1           OBJECTIVE
FUNCTION (OPTIMAL) OF COST FOR
STRUCTURE Q1
*       ZZOPTQ2           OBJECTIVE
FUNCTION (OPTIMAL) OF COST FOR
STRUCTURE Q2

;



BINARY VARIABLES

        WB1(A,B,L)        EQUAL TO 1
IF THERE IS A FLOW FROM A TO B IN
TRANSFER EVENT L
        WB2(B,H,L)        EQUAL TO 1
IF THERE IS A FLOW FROM B TO H IN
TRANSFER EVENT L

        WB1Q1(A,B,L)      EQUAL TO 1
IF THERE IS A FLOW FROM A TO B IN
TRANSFER EVENT L
        WB2Q1(B,H,L)      EQUAL TO 1
IF THERE IS A FLOW FROM B TO H IN
TRANSFER EVENT L


        WB1Q2(A,B,L)      EQUAL TO 1
IF THERE IS A FLOW FROM A TO B IN
TRANSFER EVENT L
        WB2Q2(B,H,L)      EQUAL TO 1
IF THERE IS A FLOW FROM B TO H IN
TRANSFER EVENT L



;

*======================== BRANCH AND
CONTRACT ALGORITHM
========================

PARAMETERS

        ZERO;

ZERO = 0;



PARAMETER

        DELINIITOT(B,L)
        DIFABSITOT(B,L)
        DIFABSITOTUP(B,L)
        DIFABSITOTLO(B,L)

        DIFRELITOT(B,L)
        DIFRELITOTUP(B,L)
        DIFRELITOTLO(B,L)

        SOLOUNO
        INDICAREL

        FLAGLOITOT(B,L)
        FLAGUPITOT(B,L)

        OPTIMALL
        EPSILONABS
        EPSILONREL
        MINREDABS
        MINREDREL;

OPTIMALL = 0;
MINREDABS = 1E-5;
MINREDREL = 20;
EPSILONABS = 1E-5;
EPSILONREL = 0;




*=================================
MAIN MODEL
=================================

PARAMETERS


         CSEA            WAITING COST
FOR CRUDE SUPPLY STREAMS
/ 5  /
         CSET            CHANGEOVER
COST FOR CHARGED OIL SWITCH
/ 50 /
         CUNLOAD         UNLOADING
COST FOR CRUDE SUPPLY STREAMS
/ 10 /

         THOR            TIME HORIZON
FOR SCHEDULING
/ 12 /

         ND              NUMBER OF
DISTILLATION UNITS IN THE NETWORK
/ 2  /
         ND1             NUMBER OF
DISTILLATION UNITS IN SUB-STRUCTURE Q1
/ 1  /
```

```
         ND2              NUMBER OF
DISTILLATION UNITS IN SUB-STRUCTURE Q2
/ 1  /

         NE               NUMBER OF
TRANSFER EVENTS
/ 3  /

         TSO              SCALAR STEP
SIZE FOR UPDATING LAGRANGE MULTIPLIERS
/ 1  /

         ALPHA            SCALAR
BETWEEN 0 AND 2
/ 0.4 /

         ZZ1              OPTIMAL
OBJECTIVE FOR LQ1-R
/ 0  /
         ZZ2              OPTIMAL
OBJECTIVE FOR LQ2-R
/ 1  /
         ZZLB             SUM OF
OBJECTIVES ZZQ1 AND ZZQ2
/ 1  /
*        ZZUB             UPPER BOUND
VALUE FOR OBJECTIVE FUNCTION
/ 2  /


         CINV(B)          INVENTORY
MAINTENANCE COST FOR TANK B
/
                          ST1 * ST3
0.04
                          CT1 * CT3
0.08
/


         DM(G)            DEMAND OF
CRUDE-MIX TO BE CHARED FROM CHARGING
TANK G          /
                          CT1 * CT3
50
/

         LB1(I,B)         LOWER BOUND
ON FRACTION OF COMPONENT I INSIDE TANK
B           /
                          I1.ST1
0.01
                          I1.ST2
0.04
                          I1.ST3
0.07
                          I1.CT1
0.025
                          I1.CT2
0.045
                          I1.CT3
0.075
/

         UB1(I,B)         UPPER BOUND
ON FRACTION OF COMPONENT I INSIDE TANK
B           /
                          I1.ST1
0.03
                          I1.ST2
0.06
                          I1.ST3
0.09
                          I1.CT1
0.035
```

```
                          I1.CT2
0.065
                          I1.CT3
0.085
/

         FRACT(I,U)       FRACTION OF
COMPONENT I IN CRUDE SUPPLY STREAM U
/
                          I1.U1
0.01
                          I1.U2
0.085
                          I1.U3
0.06
/

         INITITOT(B)      INITIAL
TOTAL INVENTORY OF TANK B
/
                          ST1 * ST3
20
                          CT1
30
                          CT2
50
                          CT3
30
/

         INITINVI(I,B)    INITIAL
INVENTORY OF COMPONENT I IN TANK B
/
                          I1.ST1
0.02
                          I1.ST2
0.05
                          I1.ST3
0.08
                          I1.CT1
0.03
                          I1.CT2
0.05
                          I1.CT3
0.08
/

         LBI1(B)          LOWER BOUND
ON TOTAL INVENTORY IN A TANK B
/
                          ST1 * ST3
1
                          CT1 * CT3
1
/

         UBI1(B)          UPPER BOUND
ON TOTAL INVENTORY IN A TANK B
/
                          ST1 * ST3
100
                          CT1 * CT3
100
/

         TARR(U)          ARRIVAL TIME
OF CRUDE IN CRUDE SUPPLY STREAM U
/
                          U1
1
                          U2
5
                          U3
9
/
```

```
         VCRUDE(U)        TOTAL VOLUME
OF CRUDE OIL ARRIVING IN CRUDE SUPPLY
SYSTEM U    /
                          U1*U3
50
/


* ================== value for these
PARAMETERS are assigned below
=============== *

         LBF1(A,B)        LOWER BOUND
ON FLOWRATE FROM A TO B
         UBF1(A,B)        UPPER BOUND
ON FLOWRATE FROM A TO B

         LBF2(B,H)        LOWER BOUND
ON FLOWRATE FROM B TO H
         UBF2(B,H)        UPPER BOUND
ON FLOWRATE FROM B TO H

         LBFV1(A,B)       LOWER BOUND
ON FLOW VOLUME FROM A TO B
         UBFV1(A,B)       UPPER BOUND
ON FLOW VOLUME FROM A TO B
         LBFV2(B,H)       LOWER BOUND
ON FLOW VOLUME FROM B TO H
         UBFV2(B,H)       UPPER BOUND
ON FLOW VOLUME FROM B TO H


* =========================== Lagrange
multipliers
================================ *

         LMTS(M,K,L)      LAGRANGE
MULTIPLIER
         LMTE(M,K,L)      LAGRANGE
MULTIPLIER
         LMFLT(M,K,L)     LAGRANGE
MULTIPLIER
         LMFL(I,M,K,L)    LAGRANGE
MULTIPLIER
         LMWB(M,K,L)      LAGRANGE
MULTIPLIER


* ======================== Binary
variables assignation
============================= *

         WBUB1(A,B,L)     EQUAL TO
OPTIMAL VALUE OBTAINED FROM THE
SOLUTION OF CRUDE_LB
         WBUB2(B,H,L)     EQUAL TO
OPTIMAL VALUE OBTAINED FROM THE
SOLUTION OF CRUDE_LB


* ====================== Duplicate
variables assignation
============================ *

         DFLTQ(M,K,L)      TOTAL FLOW
VARIABLE
         DFLTQ1(M,K,L)     DUPLICATE
VARIABLE FOR SUB-STRUCTURE Q1
         DFLTQ2(M,K,L)     DUPLICATE
VARIABLE FOR SUB-STRUCTURE Q2

         DFLQ(I,M,K,L)     FLOW
VARIABLE
         DFLQ1(I,M,K,L)    DUPLICATE
VARIABLE FOR SUB-STRUCTURE Q1
         DFLQ2(I,M,K,L)    DUPLICATE
VARIABLE FOR SUB-STRUCTURE Q2

         DTSQ(M,K,L)       START TIME
VARIABLE
         DTSQ1(M,K,L)      DUPLICATE
VARIABLE FOR SUB-STRUCTURE Q1
         DTSQ2(M,K,L)      DUPLICATE
VARIABLE FOR SUB-STRUCTURE Q2

         DTEQ(M,K,L)       END TIME
VARIABLE
         DTEQ1(M,K,L)      DUPLICATE
VARIABLE FOR SUB-STRUCTURE Q1
         DTEQ2(M,K,L)      DUPLICATE
VARIABLE FOR SUB-STRUCTURE Q2

         DWBQ(M,K,L)       FLOW
EXISTENCE VARIABLE
         DWBQ1(M,K,L)      DUPLICATE
VARIABLE FOR SUB-STRUCTURE Q1
         DWBQ2(M,K,L)      DUPLICATE
VARIABLE FOR SUB-STRUCTURE Q2

;


* ========================= PARAMETER
value assignment
============================== *

         LBF1(A,B) = 1           ;
         UBF1(A,B) = 40          ;

         LBF2(B,H) = 1           ;
         UBF2(B,H) = 40          ;

         LBFV1(A,B) = 1          ;
         UBFV1(A,B) = 100        ;

         LBFV2(B,H) = 1          ;
         UBFV2(B,H) = 100        ;

         LMTS(M,K,L) = 0         ;
         LMTE(M,K,L) = 0         ;
         LMFLT(M,K,L) = 0        ;
         LMFL(I,M,K,L) = 0       ;
         LMWB(M,K,L) = 0         ;



*======================== BRANCH AND
CONTRACT ALGORITHM
========================

EQUATIONS

        COTAOF
        ITOTLIBREST1L0
        ITOTLIBREST1L1
        ITOTLIBREST1L2
        ITOTLIBREST2L0
        ITOTLIBREST2L1
        ITOTLIBREST2L2
        ITOTLIBREST3L0
        ITOTLIBREST3L1
        ITOTLIBREST3L2
        ITOTLIBRECT1L0
        ITOTLIBRECT1L1
        ITOTLIBRECT1L2
        ITOTLIBRECT2L0
        ITOTLIBRECT2L1
        ITOTLIBRECT2L2
        ITOTLIBRECT3L0
        ITOTLIBRECT3L1
```

```
        ITOTLIBRECT3L2;

COTAOF..        ZZ =L= ZZUB;

ITOTLIBREST1L0..        FREEITOTST1L0
=E= ITOT('ST1','L0');
ITOTLIBREST1L1..        FREEITOTST1L1
=E= ITOT('ST1','L1');
ITOTLIBREST1L2..        FREEITOTST1L2
=E= ITOT('ST1','L2');
ITOTLIBREST2L0..        FREEITOTST2L0
=E= ITOT('ST2','L0');
ITOTLIBREST2L1..        FREEITOTST2L1
=E= ITOT('ST2','L1');
ITOTLIBREST2L2..        FREEITOTST2L2
=E= ITOT('ST2','L2');
ITOTLIBREST3L0..        FREEITOTST3L0
=E= ITOT('ST3','L0');
ITOTLIBREST3L1..        FREEITOTST3L1
=E= ITOT('ST3','L1');
ITOTLIBREST3L2..        FREEITOTST3L2
=E= ITOT('ST3','L2');
ITOTLIBRECT1L0..        FREEITOTCT1L0
=E= ITOT('CT1','L0');
ITOTLIBRECT1L1..        FREEITOTCT1L1
=E= ITOT('CT1','L1');
ITOTLIBRECT1L2..        FREEITOTCT1L2
=E= ITOT('CT1','L2');
ITOTLIBRECT2L0..        FREEITOTCT2L0
=E= ITOT('CT2','L0');
ITOTLIBRECT2L1..        FREEITOTCT2L1
=E= ITOT('CT2','L1');
ITOTLIBRECT2L2..        FREEITOTCT2L2
=E= ITOT('CT2','L2');
ITOTLIBRECT3L0..        FREEITOTCT3L0
=E= ITOT('CT3','L0');
ITOTLIBRECT3L1..        FREEITOTCT3L1
=E= ITOT('CT3','L1');
ITOTLIBRECT3L2..        FREEITOTCT3L2
=E= ITOT('CT3','L2');



*=================================
MAIN MODEL
=================================

EQUATIONS

         OBJFUN      OBJECTIVE
FUNCTION
         OBJFIXBIN   OBJECTIVE
FUNCTION
         OBJQ1       OBJECTIVE
FUNCTION OF COST FOR STRUCTURE Q1
         OBJQ2       OBJECTIVE
FUNCTION OF COST FOR STRUCTURE Q2


         TC1         TANK CONSTRAINT 1
         TC2         TANK CONSTRAINT 2
         TC3         TANK CONSTRAINT 3
         TC4         TANK CONSTRAINT 4
         TC5         TANK CONSTRAINT 5
         TC6         TANK CONSTRAINT 6
         TC7         TANK CONSTRAINT 7
         TC8         TANK CONSTRAINT 8
         TC9         TANK CONSTRAINT 9
         TC10        TANK CONSTRAINT
10
         TC11        TANK CONSTRAINT
11
         TC12        TANK CONSTRAINT
12
         TC13        TANK CONSTRAINT
13
         TC14        TANK CONSTRAINT
14
         TC15        TANK CONSTRAINT
15
         TC16        TANK CONSTRAINT
16
         TC17        TANK CONSTRAINT
17
         TC18        TANK CONSTRAINT
18
         TC19        TANK CONSTRAINT
19
         TC20        TANK CONSTRAINT
20
         TC21        TANK CONSTRAINT
21
         TC22        TANK CONSTRAINT
22
         TC23        TANK CONSTRAINT
23
         TC24        TANK CONSTRAINT
24
         TC25        TANK CONSTRAINT
25
         TC26        TANK CONSTRAINT
26
         TC27        TANK CONSTRAINT
27
         TC28        TANK CONSTRAINT
28
         TC29        TANK CONSTRAINT
29
         TC30        TANK CONSTRAINT
30
         TC31        TANK CONSTRAINT
31

*$ONTEXT
         TC32        TANK CONSTRAINT
32
         TC33        TANK CONSTRAINT
33
         TC34        TANK CONSTRAINT
34
         TC35        TANK CONSTRAINT
35
         TC36        TANK CONSTRAINT
36
*$OFFTEXT


         DC1         DISTILLATION UNIT
CONSTRAINT 1
         DC2         DISTILLATION UNIT
CONSTRAINT 2
         DC3         DISTILLATION UNIT
CONSTRAINT 3
         DC4         DISTILLATION UNIT
CONSTRAINT 4
         DC5         DISTILLATION UNIT
CONSTRAINT 5


         CC1         CRUDE SUPPLY
STREAM CONSTRAINT 1
         CC2         CRUDE SUPPLY
STREAM CONSTRAINT 2
         CC3         CRUDE SUPPLY
STREAM CONSTRAINT 3
         CC4         CRUDE SUPPLY
STREAM CONSTRAINT 4

         VB1         VARIABLE BOUND 1
```

```
         VB2         VARIABLE BOUND 2
         VB3         VARIABLE BOUND 3
         VB4         VARIABLE BOUND 4
         VB5         VARIABLE BOUND 5
         VB6         VARIABLE BOUND 6
         VB7         VARIABLE BOUND 7
         VB8         VARIABLE BOUND 8
         VB9         VARIABLE BOUND 9
         VB10        VARIABLE BOUND 10
         VB11        VARIABLE BOUND 11
         VB12        VARIABLE BOUND 12
         VB13        VARIABLE BOUND 13
         VB14        VARIABLE BOUND 14
         VB15        VARIABLE BOUND 15


         LBP1        LOWER BOUNDING
PROBLEM 1
         LBP2        LOWER BOUNDING
PROBLEM 2
         LBP3        LOWER BOUNDING
PROBLEM 3
         LBP4        LOWER BOUNDING
PROBLEM 4
         LBP5        LOWER BOUNDING
PROBLEM 5
         LBP6        LOWER BOUNDING
PROBLEM 6
         LBP7        LOWER BOUNDING
PROBLEM 7
         LBP8        LOWER BOUNDING
PROBLEM 8
         LBP9        LOWER BOUNDING
PROBLEM 9
         LBP10       LOWER BOUNDING
PROBLEM 10
         LBP11       LOWER BOUNDING
PROBLEM 11
         LBP12       LOWER BOUNDING
PROBLEM 12
         LBP13       LOWER BOUNDING
PROBLEM 13
         LBP14       LOWER BOUNDING
PROBLEM 14
         LBP15       LOWER BOUNDING
PROBLEM 15
         LBP16       LOWER BOUNDING
PROBLEM 16
         LBP17       LOWER BOUNDING
PROBLEM 17

         LBP18       LOWER BOUNDING
PROBLEM 18
         LBP19       LOWER BOUNDING
PROBLEM 19
         LBP20       LOWER BOUNDING
PROBLEM 20
         LBP21       LOWER BOUNDING
PROBLEM 21
         LBP22       LOWER BOUNDING
PROBLEM 22


* ======================= LBP for sub-
structures Q1 and Q2
============================ *

         LBP10Q       LOWER BOUNDING
PROBLEM 10
         LBP11Q       LOWER BOUNDING
PROBLEM 11
         LBP12Q       LOWER BOUNDING
PROBLEM 12
         LBP13Q       LOWER BOUNDING
PROBLEM 13
         LBP14Q       LOWER BOUNDING
PROBLEM 14
         LBP15Q       LOWER BOUNDING
PROBLEM 15
         LBP16Q       LOWER BOUNDING
PROBLEM 16
         LBP17Q       LOWER BOUNDING
PROBLEM 17

         LBP1Q1       LOWER BOUNDING
PROBLEM 1
         LBP2Q1       LOWER BOUNDING
PROBLEM 2
         LBP3Q1       LOWER BOUNDING
PROBLEM 3
         LBP4Q1       LOWER BOUNDING
PROBLEM 4
         LBP5Q1       LOWER BOUNDING
PROBLEM 5
         LBP6Q1       LOWER BOUNDING
PROBLEM 6
         LBP7Q1       LOWER BOUNDING
PROBLEM 7
         LBP8Q1       LOWER BOUNDING
PROBLEM 8
         LBP9Q1       LOWER BOUNDING
PROBLEM 9
         LBP10Q1      LOWER BOUNDING
PROBLEM 10
         LBP11Q1      LOWER BOUNDING
PROBLEM 11
         LBP12Q1      LOWER BOUNDING
PROBLEM 12
         LBP13Q1      LOWER BOUNDING
PROBLEM 13
         LBP14Q1      LOWER BOUNDING
PROBLEM 14
         LBP15Q1      LOWER BOUNDING
PROBLEM 15
         LBP16Q1      LOWER BOUNDING
PROBLEM 16
         LBP17Q1      LOWER BOUNDING
PROBLEM 17

         LBP1Q2       LOWER BOUNDING
PROBLEM 1
         LBP2Q2       LOWER BOUNDING
PROBLEM 2
         LBP3Q2       LOWER BOUNDING
PROBLEM 3
         LBP4Q2       LOWER BOUNDING
PROBLEM 4
         LBP5Q2       LOWER BOUNDING
PROBLEM 5
         LBP6Q2       LOWER BOUNDING
PROBLEM 6
         LBP7Q2       LOWER BOUNDING
PROBLEM 7
         LBP8Q2       LOWER BOUNDING
PROBLEM 8
         LBP9Q2       LOWER BOUNDING
PROBLEM 9
         LBP10Q2      LOWER BOUNDING
PROBLEM 10
         LBP11Q2      LOWER BOUNDING
PROBLEM 11
         LBP12Q2      LOWER BOUNDING
PROBLEM 12
         LBP13Q2      LOWER BOUNDING
PROBLEM 13
         LBP14Q2      LOWER BOUNDING
PROBLEM 14
         LBP15Q2      LOWER BOUNDING
PROBLEM 15
         LBP16Q2      LOWER BOUNDING
PROBLEM 16
```

```
         LBP17Q2      LOWER BOUNDING PROBLEM 17

* ====================== Equations to update Lagrange multiplier ======================== *

         ULM1        UPDATING LAGRANGE MULTIPLIER 1
         ULM2        UPDATING LAGRANGE MULTIPLIER 2
         ULM3        UPDATING LAGRANGE MULTIPLIER 3
         ULM4        UPDATING LAGRANGE MULTIPLIER 4
         ULM5        UPDATING LAGRANGE MULTIPLIER 5
         ULM6        UPDATING LAGRANGE MULTIPLIER 6


* ============================ Equations to generate VLC ================================= *

         VLC1        GENERATING VALID LINEAR CUTS
         VLC2        GENERATING VALID LINEAR CUTS

* ============== Equations used to evaluate UB after setting binary variables ============ *

         TCUB1       DUPLICATE EQUATION FOR UB EVALUATION
         TCUB2       DUPLICATE EQUATION FOR UB EVALUATION
         TCUB3       DUPLICATE EQUATION FOR UB EVALUATION
         TCUB4       DUPLICATE EQUATION FOR UB EVALUATION
         TCUB5       DUPLICATE EQUATION FOR UB EVALUATION
         TCUB6       DUPLICATE EQUATION FOR UB EVALUATION
         TCUB8       DUPLICATE EQUATION FOR UB EVALUATION
         TCUB11      DUPLICATE EQUATION FOR UB EVALUATION
         TCUB14      DUPLICATE EQUATION FOR UB EVALUATION
         TCUB15      DUPLICATE EQUATION FOR UB EVALUATION
         TCUB16      DUPLICATE EQUATION FOR UB EVALUATION
         TCUB17      DUPLICATE EQUATION FOR UB EVALUATION
         TCUB18      DUPLICATE EQUATION FOR UB EVALUATION

         DCUB1       DUPLICATE EQUATION FOR UB EVALUATION
         DCUB2       DUPLICATE EQUATION FOR UB EVALUATION
         DCUB4       DUPLICATE EQUATION FOR UB EVALUATION
         DCUB5       DUPLICATE EQUATION FOR UB EVALUATION

         CCUB1       DUPLICATE EQUATION FOR UB EVALUATION
         CCUB2       DUPLICATE EQUATION FOR UB EVALUATION

* ========================== DUPLICATE EQUATIONS AND VARIABLES FOR LB1 ======== *

         TC1Q1        TANK CONSTRAINT 1
         TC2Q1        TANK CONSTRAINT 2
         TC3Q1        TANK CONSTRAINT 3
         TC4Q1        TANK CONSTRAINT 4
         TC5Q1        TANK CONSTRAINT 5
         TC6Q1        TANK CONSTRAINT 6
         TC7Q1        TANK CONSTRAINT 7
         TC8Q1        TANK CONSTRAINT 8
         TC9Q1        TANK CONSTRAINT 9
         TC10Q1       TANK CONSTRAINT 10
         TC11Q1       TANK CONSTRAINT 11
         TC12Q1       TANK CONSTRAINT 12
         TC13Q1       TANK CONSTRAINT 13
         TC14Q1       TANK CONSTRAINT 14
         TC15Q1       TANK CONSTRAINT 15
         TC16Q1       TANK CONSTRAINT 16
         TC17Q1       TANK CONSTRAINT 17
         TC18Q1       TANK CONSTRAINT 18
         TC19Q1       TANK CONSTRAINT 19
         TC20Q1       TANK CONSTRAINT 20
         TC21Q1       TANK CONSTRAINT 21
         TC22Q1       TANK CONSTRAINT 22
         TC23Q1       TANK CONSTRAINT 23
         TC24Q1       TANK CONSTRAINT 24
*        TC25Q1       TANK CONSTRAINT 25
         TC26Q1       TANK CONSTRAINT 26
         TC27Q1       TANK CONSTRAINT 27
         TC28Q1       TANK CONSTRAINT 28
         TC29Q1       TANK CONSTRAINT 29
         TC30Q1       TANK CONSTRAINT 30
         TC31Q1       TANK CONSTRAINT 31

*$ONTEXT
         TC32Q1       TANK CONSTRAINT 32
         TC33Q1       TANK CONSTRAINT 33
         TC34Q1       TANK CONSTRAINT 34
```

```
        TC35Q1        TANK CONSTRAINT
35
        TC36Q1        TANK CONSTRAINT
36
*$OFFTEXT

        DC1Q1         DISTILLATION
UNIT CONSTRAINT 1
        DC2Q1         DISTILLATION
UNIT CONSTRAINT 2
        DC3Q1         DISTILLATION
UNIT CONSTRAINT 3
        DC4Q1         DISTILLATION
UNIT CONSTRAINT 4
        DC5Q1         DISTILLATION
UNIT CONSTRAINT 5

        CC1Q1         CRUDE SUPPLY
STREAM CONSTRAINT 1
        CC2Q1         CRUDE SUPPLY
STREAM CONSTRAINT 2
        CC3Q1         CRUDE SUPPLY
STREAM CONSTRAINT 3
        CC4Q1         CRUDE SUPPLY
STREAM CONSTRAINT 4

        VB1Q1         VARIABLE BOUND
1
        VB2Q1         VARIABLE BOUND
2
        VB3Q1         VARIABLE BOUND
3
        VB4Q1         VARIABLE BOUND
4
        VB5Q1         VARIABLE BOUND
5
        VB6Q1         VARIABLE BOUND
6
        VB7Q1         VARIABLE BOUND
7
        VB8Q1         VARIABLE BOUND
8
        VB9Q1         VARIABLE BOUND
9
        VB10Q1        VARIABLE BOUND
10
        VB11Q1        VARIABLE BOUND
11
        VB12Q1        VARIABLE BOUND
12
        VB13Q1        VARIABLE BOUND
13
        VB14Q1        VARIABLE BOUND
14
        VB15Q1        VARIABLE BOUND
15

* ======================== DUPLICATE
EQUATIONS AND VARIABLES FOR LB
======== *
        TC1Q2         TANK CONSTRAINT
1
        TC2Q2         TANK CONSTRAINT
2
        TC3Q2         TANK CONSTRAINT
3
        TC4Q2         TANK CONSTRAINT
4
        TC5Q2         TANK CONSTRAINT
5
        TC6Q2         TANK CONSTRAINT
6
        TC7Q2         TANK CONSTRAINT
7
        TC8Q2         TANK CONSTRAINT
8
        TC9Q2         TANK CONSTRAINT
9
        TC10Q2        TANK CONSTRAINT
10
        TC11Q2        TANK CONSTRAINT
11
        TC12Q2        TANK CONSTRAINT
12
        TC13Q2        TANK CONSTRAINT
13
        TC14Q2        TANK CONSTRAINT
14
        TC15Q2        TANK CONSTRAINT
15
        TC16Q2        TANK CONSTRAINT
16
        TC17Q2        TANK CONSTRAINT
17
        TC18Q2        TANK CONSTRAINT
18
        TC19Q2        TANK CONSTRAINT
19
        TC20Q2        TANK CONSTRAINT
20
        TC21Q2        TANK CONSTRAINT
21
        TC22Q2        TANK CONSTRAINT
22
        TC23Q2        TANK CONSTRAINT
23
        TC24Q2        TANK CONSTRAINT
24
*       TC25Q2        TANK CONSTRAINT
25
        TC26Q2        TANK CONSTRAINT
26
        TC27Q2        TANK CONSTRAINT
27
        TC28Q2        TANK CONSTRAINT
28
        TC29Q2        TANK CONSTRAINT
29
        TC30Q2        TANK CONSTRAINT
30
        TC31Q2        TANK CONSTRAINT
31
*$ONTEXT
        TC32Q2        TANK CONSTRAINT
32
        TC33Q2        TANK CONSTRAINT
33
        TC34Q2        TANK CONSTRAINT
34
        TC35Q2        TANK CONSTRAINT
35
        TC36Q2        TANK CONSTRAINT
36
*$OFFTEXT

        DC1Q2         DISTILLATION
UNIT CONSTRAINT 1
        DC2Q2         DISTILLATION
UNIT CONSTRAINT 2
        DC3Q2         DISTILLATION
UNIT CONSTRAINT 3
        DC4Q2         DISTILLATION
UNIT CONSTRAINT 4
```

```
         DC5Q2         DISTILLATION
UNIT CONSTRAINT 5

         CC1Q2         CRUDE SUPPLY
STREAM CONSTRAINT 1
         CC2Q2         CRUDE SUPPLY
STREAM CONSTRAINT 2
         CC3Q2         CRUDE SUPPLY
STREAM CONSTRAINT 3
         CC4Q2         CRUDE SUPPLY
STREAM CONSTRAINT 4

         VB1Q2         VARIABLE BOUND
1
         VB2Q2         VARIABLE BOUND
2
         VB3Q2         VARIABLE BOUND
3
         VB4Q2         VARIABLE BOUND
4
         VB5Q2         VARIABLE BOUND
5
         VB6Q2         VARIABLE BOUND
6
         VB7Q2         VARIABLE BOUND
7
         VB8Q2         VARIABLE BOUND
8
         VB9Q2         VARIABLE BOUND
9
         VB10Q2        VARIABLE BOUND
10
         VB11Q2        VARIABLE BOUND
11
         VB12Q2        VARIABLE BOUND
12
         VB13Q2        VARIABLE BOUND
13
         VB14Q2        VARIABLE BOUND
14
         VB15Q2        VARIABLE BOUND
15

;

*                 --------------------
---------
*                |         EQUATION
|
*                |        DEFINITION
|
*                 --------------------
---------


* ======================= OBJECTIVE
FUNCTION ============================
*

 OBJFUN..
         ZZ =E= CSEA*SUM(U,TSI(U)-
TARR(U)) + CUNLOAD*SUM(U,TEO(U)-
TSI(U)) +

THOR*(SUM(B,CINV(B))*SUM(B,SUM(L,(ITOT
(B,L)))) + SUM(B,CINV(B))*
SUM(B,SUM(L,SUM(A$A1(A,B),FT1(A,B,L)))
) +

SUM(B,CINV(B))*SUM(B,SUM(L$(ORD(L) LT
CARD(L)),ITOT(B,L))) +
THOR*SUM(B,CINV(B))*SUM(B,INITITOT(B))
) / (2*NE + 1) +

CSET*(SUM(D,SUM(G$G1(G,D),SUM(L,WB2(G,
D,L)))) - ND)            ;

 OBJFIXBIN..
         ZZFIXBIN =E=
CSEA*SUM(U,TSI(U)-TARR(U)) +
CUNLOAD*SUM(U,TEO(U)-TSI(U)) +

THOR*(SUM(B,CINV(B))*SUM(B,SUM(L,(ITOT
(B,L)))) + SUM(B,CINV(B))*
SUM(B,SUM(L,SUM(A$A1(A,B),FT1(A,B,L)))
) +

SUM(B,CINV(B))*SUM(B,SUM(L$(ORD(L) LT
CARD(L)),ITOT(B,L))) +
THOR*SUM(B,CINV(B))*SUM(B,INITITOT(B))
) / (2*NE + 1) +

CSET*(SUM(D,SUM(G$G1(G,D),SUM(L,WBUB2(
G,D,L)))) - ND)            ;

 OBJQ1..
         ZZQ1 =E= CSEA*SUM(U1,TSI(U1)-
TARR(U1)) + CUNLOAD*SUM(U1,TEO(U1)-
TSI(U1)) +

THOR*(SUM(B1,CINV(B1))*SUM(B1,SUM(L,(I
TOT(B1,L)))) +
SUM(B1,CINV(B1))*SUM(B1,SUM(L,SUM(A,FT
1Q1(A,B1,L)))) +

SUM(B1,CINV(B1))*SUM(B1,SUM(L$(ORD(L)
LT CARD(L)),ITOT(B1,L))) +
THOR*SUM(B1,CINV(B1))*SUM(B2,INITITOT(
B2))) / (2*NE + 1) +

CSET*(SUM(D1,SUM(G$G1(G,D1),SUM(L,WB2Q
1(G,D1,L)))) - ND1) +
SUM(M,SUM(K$K1(M,K),SUM(L,LMFLT(M,K,L)
*FLTQ1(M,K,L)))) +

SUM(I,SUM(M,SUM(K$K1(M,K),SUM(L,LMFL(I
,M,K,L)*FLQ1(I,M,K,L))))) +
SUM(M,SUM(K$K1(M,K),SUM(L,LMTS(M,K,L)*
TSQ1(M,K,L)))) +

SUM(M,SUM(K$K1(M,K),SUM(L,LMTE(M,K,L)*
TEQ1(M,K,L)))) +
SUM(M,SUM(K$K1(M,K),SUM(L,LMWB(M,K,L)*
WBQ1(M,K,L))));

 OBJQ2..
         ZZQ2 =E= CSEA*SUM(U2,TSI(U2)-
TARR(U2)) + CUNLOAD*SUM(U2,TEO(U2)-
TSI(U2)) +

THOR*(SUM(B2,CINV(B2))*SUM(B2,SUM(L,(I
TOT(B2,L)))) +
SUM(B2,CINV(B2))*SUM(B2,SUM(L,SUM(A,FT
1(A,B2,L)))) +

SUM(B2,CINV(B2))*SUM(B2,SUM(L$(ORD(L)
LT CARD(L)),ITOT(B2,L))) +
THOR*SUM(B2,CINV(B2))*SUM(B2,INITITOT(
B2))) / (2*NE + 1) +

CSET*(SUM(D2,SUM(G$G1(G,D2),SUM(L,WB2(
G,D2,L)))) - ND2) -
SUM(M,SUM(K$K1(M,K),SUM(L,LMFLT(M,K,L)
*FLTQ2(M,K,L)))) -

SUM(I,SUM(M,SUM(K$K1(M,K),SUM(L,LMFL(I
,M,K,L)*FLQ2(I,M,K,L))))) -
SUM(M,SUM(K$K1(M,K),SUM(L,LMTS(M,K,L)*
TSQ2(M,K,L)))) -
```

```
SUM(M,SUM(K$K1(M,K),SUM(L,LMTE(M,K,L)*
TEQ2(M,K,L)))) -
SUM(M,SUM(K$K1(M,K),SUM(L,LMWB(M,K,L)*
WBQ2(M,K,L))));


* =========================== 1.TANK
CONSTRAINTS
============================== *


**(i)Constraints for Flow Transfers

 TC1(A,B,L)$A1(A,B)..
         FT1(A,B,L) =L=
         UBFV1(A,B)*WB1(A,B,L)
;

 TC2(B,H,L)$H1(B,H)..
         FT2(B,H,L) =L=
         UBFV2(B,H)*WB2(B,H,L)
;

 TCUB1(A,B,L)$A1(A,B)..
         FT1(A,B,L) =L=
         UBFV1(A,B)*WBUB1(A,B,L)
;

 TCUB2(B,H,L)$H1(B,H)..
         FT2(B,H,L) =L=
         UBFV2(B,H)*WBUB2(B,H,L)
;


**(ii)Duration Constraints

 TC3(A,B,L)$A1(A,B)..
         UBF1(A,B)*(TE1(A,B,L)-
TS1(A,B,L)) + UBF1(A,B)*THOR*(1-
WB1(A,B,L)) =G=
         FT1(A,B,L)
;

 TC4(B,H,L)$H1(B,H)..
         UBF2(B,H)*(TE2(B,H,L)-
TS2(B,H,L)) + UBF2(B,H)*THOR*(1-
WB2(B,H,L)) =G=
         FT2(B,H,L)
;

 TC5(A,Y,L)$A2(A,Y)..
         LBF1(A,Y)*(TE1(A,Y,L)-
TS1(A,Y,L)) - LBF1(A,Y)*THOR*(1-
WB1(A,Y,L)) =L=
         FT1(A,Y,L)
;

 TC6(Y,H,L)$H2(Y,H)..
         LBF2(Y,H)*(TE2(Y,H,L)-
TS2(Y,H,L)) - LBF2(Y,H)*THOR*(1-
WB2(Y,H,L)) =L=
         FT2(Y,H,L)
;

 TC7(G,H,L)$H3(G,H)..
         LBF2(G,H)*(TE2(G,H,L)-
TS2(G,H,L)) =L=
         FT2(G,H,L)
;

 TCUB3(A,B,L)$A1(A,B)..
         UBF1(A,B)*(TE1(A,B,L)-
TS1(A,B,L)) + UBF1(A,B)*THOR*(1-
WBUB1(A,B,L)) =G=
         FT1(A,B,L)
;

 TCUB4(B,H,L)$H1(B,H)..
         UBF2(B,H)*(TE2(B,H,L)-
TS2(B,H,L)) + UBF2(B,H)*THOR*(1-
WBUB2(B,H,L)) =G=
         FT2(B,H,L)
;

 TCUB5(A,Y,L)$A2(A,Y)..
         LBF1(A,Y)*(TE1(A,Y,L)-
TS1(A,Y,L)) - LBF1(A,Y)*THOR*(1-
WBUB1(A,Y,L)) =L=
         FT1(A,Y,L)
;

 TCUB6(Y,H,L)$H2(Y,H)..
         LBF2(Y,H)*(TE2(Y,H,L)-
TS2(Y,H,L)) - LBF2(Y,H)*THOR*(1-
WBUB2(Y,H,L)) =L=
         FT2(Y,H,L)
;


**(iii)Simple Sequencing Constraints

 TC8(A,B,L)$(ORD(L) LT
CARD(L)$A1(A,B))..
         TS1(A,B,L+1) =G=
         TE1(A,B,L) - THOR*(1-
WB1(A,B,L))
;

 TC9(A,B,L)$(ORD(L) LT
CARD(L)$A1(A,B))..
         TS1(A,B,L+1) =G=
         TS1(A,B,L)
;

 TC10(A,B,L)$(ORD(L) LT
CARD(L)$A1(A,B))..
         TE1(A,B,L+1) =G=
         TE1(A,B,L)
;

 TC11(B,H,L)$(ORD(L) LT
CARD(L)$H1(B,H))..
         TS2(B,H,L+1) =G=
         TE2(B,H,L) - THOR*(1-
WB2(B,H,L))
;

 TC12(B,H,L)$(ORD(L) LT
CARD(L)$H1(B,H))..
         TS2(B,H,L+1) =G=
         TS2(B,H,L)
;

 TC13(B,H,L)$(ORD(L) LT
CARD(L)$H1(B,H))..
         TE2(B,H,L+1) =G=
         TE2(B,H,L)
;


 TCUB8(A,B,L)$(ORD(L) LT
CARD(L)$A1(A,B))..
         TS1(A,B,L+1) =G=
         TE1(A,B,L) - THOR*(1-
WBUB1(A,B,L))
;
```

```
 TCUB11(B,H,L)$(ORD(L) LT
CARD(L)$H1(B,H))..
         TS2(B,H,L+1) =G=
         TE2(B,H,L) - THOR*(1-
WBUB2(B,H,L))
;


**(iv)Input and Output Restraints for
the Entire Horizon

 TC14(A,ADASH,B,L)$(ORD(L) LT
CARD(L)$(ORD(A) NE
ORD(ADASH)$A1(A,B)))..
         TS1(A,B,L+1) =G=
         TE1(ADASH,B,L) - THOR*(1-
WB1(ADASH,B,L))
;

 TC15(A,B,H,L)$(ORD(L) LT
CARD(L)$(A1(A,B)$H1(B,H)))..
         TS1(A,B,L+1) =G=
         TE2(B,H,L) - THOR*(1-
WB2(B,H,L))
;

 TC16(A,B,H,L)$(ORD(L) LT
CARD(L)$(A1(A,B)$H1(B,H)))..
         TS2(B,H,L+1) =G=
         TE1(A,B,L) - THOR*(1-
WB1(A,B,L))
;

 TC17(B,H,HDASH,L)$(ORD(L) LT
CARD(L)$(ORD(H) NE
ORD(HDASH)$H1(B,H)))..
         TS2(B,H,L+1) =G=
         TE2(B,HDASH,L) - THOR*(1-
WB2(B,HDASH,L))
;

 TC18(A,B,H,L)$(A1(A,B)$H1(B,H))..
         TE1(A,B,L) - THOR*(1-
WB1(A,B,L)) =L=
         TS2(B,H,L) + THOR*(1-
WB2(B,H,L))
;


 TCUB14(A,ADASH,B,L)$(ORD(L) LT
CARD(L)$(ORD(A) NE
ORD(ADASH)$A1(A,B)))..
         TS1(A,B,L+1) =G=
         TE1(ADASH,B,L) - THOR*(1-
WBUB1(ADASH,B,L))
;

 TCUB15(A,B,H,L)$(ORD(L) LT
CARD(L)$(A1(A,B)$H1(B,H)))..
         TS1(A,B,L+1) =G=
         TE2(B,H,L) - THOR*(1-
WBUB2(B,H,L))
;

 TCUB16(A,B,H,L)$(ORD(L) LT
CARD(L)$(A1(A,B)$H1(B,H)))..
         TS2(B,H,L+1) =G=
         TE1(A,B,L) - THOR*(1-
WBUB1(A,B,L))
;

 TCUB17(B,H,HDASH,L)$(ORD(L) LT
CARD(L)$(ORD(H) NE
ORD(HDASH)$H1(B,H)))..
         TS2(B,H,L+1) =G=
         TE2(B,HDASH,L) - THOR*(1-
WBUB2(B,HDASH,L))
;

 TCUB18(A,B,H,L)$(A1(A,B)$H1(B,H))..
         TE1(A,B,L) - THOR*(1-
WBUB1(A,B,L)) =L=
         TS2(B,H,L) + THOR*(1-
WBUB2(B,H,L))
;


**(v)Mass Balances

 TC19(B,L)..
         ITOT(B,L-1) +
SUM(A$A1(A,B),FT1(A,B,L)) =E=
         ITOT(B,L) +
SUM(H$H1(B,H),FT2(B,H,L))
;

 TC20(B)..
         ITOT(B,'L0') =E=
         INITITOT(B)
;

 TC21(I,B,L)..
         INVI(I,B,L-1) +
SUM(A$A1(A,B),FL1(I,A,B,L)) =E=
         INVI(I,B,L)
+SUM(H$H1(B,H),FL2(I,B,H,L))
;

 TC22(I,B)..
         INVI(I,B,'L0') =E=
         INITINVI(I,B)
;

 TC23(A,B,L)$A1(A,B)..
         FT1(A,B,L) =E=
         SUM(I,FL1(I,A,B,L))
;

 TC24(B,H,L)$H1(B,H)..
         FT2(B,H,L) =E=
         SUM(I,FL2(I,B,H,L))
;



**(vi)Component Balances (Replaced by
LBP)

 TC25(I,B,H,L)$H1(B,H)..
         FL2(I,B,H,L)*(ITOT(B,L-
1)+SUM(A$A1(A,B),FT1(A,B,L))) =E=
         FT2(B,H,L)*(INVI(I,B,L-
1)+SUM(A$A1(A,B),FL1(I,A,B,L)));


 LBP1(I,B,H,L)$H1(B,H)..
         INVVJ(I,B,H,L) +
SUM(A$A1(A,B),FLOWVJ(I,A,B,H,L)) =E=
         INVVT(I,B,H,L) +
SUM(A$A1(A,B),FLOWVT(I,A,B,H,L))
;


 LBP2(I,B,H,L)$H1(B,H)..
         INVVJ(I,B,H,L) =G=
         LBI1(B)*FL2(I,B,H,L) +
LBFV2(B,H)*ITOT(B,L-1) -
LBI1(B)*LBFV2(B,H)      ;

 LBP3(I,B,H,L)$H1(B,H)..
         INVVJ(I,B,H,L) =G=
```

```
         UBI1(B)*FL2(I,B,H,L) +
UBFV2(B,H)*ITOT(B,L-1) -
UBI1(B)*UBFV2(B,H)      ;

 LBP4(I,B,H,L)$H1(B,H)..
         INVVJ(I,B,H,L) =L=
         LBI1(B)*FL2(I,B,H,L) +
UBFV2(B,H)*ITOT(B,L-1) -
LBI1(B)*UBFV2(B,H)      ;

 LBP5(I,B,H,L)$H1(B,H)..
         INVVJ(I,B,H,L) =L=
         UBI1(B)*FL2(I,B,H,L) +
LBFV2(B,H)*ITOT(B,L-1) -
LBI1(B)*LBFV2(B,H)      ;


 LBP6(I,B,H,L)$H1(B,H)..
         INVVT(I,B,H,L) =G=
         LBI1(B)*FT2(B,H,L) +
LBFV2(B,H)*INVI(I,B,L-1) -
LBI1(B)*LBFV2(B,H)      ;

 LBP7(I,B,H,L)$H1(B,H)..
         INVVT(I,B,H,L) =G=
         UBI1(B)*FT2(B,H,L) +
UBFV2(B,H)*INVI(I,B,L-1) -
UBI1(B)*UBFV2(B,H)      ;

 LBP8(I,B,H,L)$H1(B,H)..
         INVVT(I,B,H,L) =L=
         LBI1(B)*FT2(B,H,L) +
UBFV2(B,H)*INVI(I,B,L-1) -
LBI1(B)*UBFV2(B,H)      ;

 LBP9(I,B,H,L)$H1(B,H)..
         INVVT(I,B,H,L) =L=
         UBI1(B)*FT2(B,H,L) +
LBFV2(B,H)*INVI(I,B,L-1) -
UBI1(B)*LBFV2(B,H)      ;


 LBP10(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
         FLOWVJ(I,A,B,H,L) =G=
         LBFV1(A,B)*FL2(I,B,H,L) +
LBFV2(B,H)*FT1(A,B,L) -
LBFV1(A,B)*LBFV2(B,H) ;

 LBP11(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
         FLOWVJ(I,A,B,H,L) =G=
         UBFV1(A,B)*FL2(I,B,H,L) +
UBFV2(B,H)*FT1(A,B,L) -
UBFV1(A,B)*UBFV2(B,H) ;

 LBP12(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
         FLOWVJ(I,A,B,H,L) =L=
         LBFV1(A,B)*FL2(I,B,H,L) +
UBFV2(B,H)*FT1(A,B,L) -
LBFV1(A,B)*UBFV2(B,H) ;

 LBP13(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
         FLOWVJ(I,A,B,H,L) =L=
         UBFV1(A,B)*FL2(I,B,H,L) +
LBFV2(B,H)*FT1(A,B,L) -
UBFV1(A,B)*LBFV2(B,H) ;


 LBP14(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
         FLOWVT(I,A,B,H,L) =G=
         LBFV1(A,B)*FT2(B,H,L) +
LBFV2(B,H)*FL1(I,A,B,L) -
LBFV1(A,B)*LBFV2(B,H) ;

 LBP15(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
         FLOWVT(I,A,B,H,L) =G=
         UBFV1(A,B)*FT2(B,H,L) +
UBFV2(B,H)*FL1(I,A,B,L) -
UBFV1(A,B)*UBFV2(B,H) ;

 LBP16(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
         FLOWVT(I,A,B,H,L) =L=
         LBFV1(A,B)*FT2(B,H,L) +
UBFV2(B,H)*FL1(I,A,B,L) -
LBFV1(A,B)*UBFV2(B,H) ;

 LBP17(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
         FLOWVT(I,A,B,H,L) =L=
         UBFV1(A,B)*FT2(B,H,L) +
LBFV2(B,H)*FL1(I,A,B,L) -
UBFV1(A,B)*LBFV2(B,H) ;



* ================ for generating
cutting planes using Lagrangean
decomposition ============ *


LBP10Q(I,A,B,H,L,Q)$(A1(A,B)$H1(B,H)).
.
         FLOWVJQ(I,A,B,H,L,Q) =G=
         LBFV1(A,B)*FL2(I,B,H,L) +
LBFV2(B,H)*FT1(A,B,L) -
LBFV1(A,B)*LBFV2(B,H) ;

*
LBP10Q(I,A,B,H,L,Q)$(A1(A,B)$H1(B,H)).
.
*FLOWVJQ(I,M,N,H,L,Q) =G=
*LBFV1(A,B)*FL2(I,B,H,L) +
LBFV2(B,H)*FT1(A,B,L) -
LBFV1(A,B)*LBFV2(B,H);



LBP11Q(I,A,B,H,L,Q)$(A1(A,B)$H1(B,H)).
.
         FLOWVJQ(I,A,B,H,L,Q) =G=
         UBFV1(A,B)*FL2(I,B,H,L) +
UBFV2(B,H)*FT1(A,B,L) -
UBFV1(A,B)*UBFV2(B,H) ;


LBP12Q(I,A,B,H,L,Q)$(A1(A,B)$H1(B,H)).
.
         FLOWVJQ(I,A,B,H,L,Q) =L=
         LBFV1(A,B)*FL2(I,B,H,L) +
UBFV2(B,H)*FT1(A,B,L) -
LBFV1(A,B)*UBFV2(B,H) ;


LBP13Q(I,A,B,H,L,Q)$(A1(A,B)$H1(B,H)).
.
         FLOWVJQ(I,A,B,H,L,Q) =L=
         UBFV1(A,B)*FL2(I,B,H,L) +
LBFV2(B,H)*FT1(A,B,L) -
UBFV1(A,B)*LBFV2(B,H) ;



LBP14Q(I,A,B,H,L,Q)$(A1(A,B)$H1(B,H)).
.
         FLOWVTQ(I,A,B,H,L,Q) =G=
         LBFV1(A,B)*FT2(B,H,L) +
LBFV2(B,H)*FL1(I,A,B,L) -
LBFV1(A,B)*LBFV2(B,H) ;


LBP15Q(I,A,B,H,L,Q)$(A1(A,B)$H1(B,H)).
.
         FLOWVTQ(I,A,B,H,L,Q) =G=
```

```
        UBFV1(A,B)*FT2(B,H,L) +
UBFV2(B,H)*FL1(I,A,B,L) -
UBFV1(A,B)*UBFV2(B,H) ;


LBP16Q(I,A,B,H,L,Q)$(A1(A,B)$H1(B,H)).
.
        FLOWVTQ(I,A,B,H,L,Q) =L=
        LBFV1(A,B)*FT2(B,H,L) +
UBFV2(B,H)*FL1(I,A,B,L) -
LBFV1(A,B)*UBFV2(B,H) ;


LBP17Q(I,A,B,H,L,Q)$(A1(A,B)$H1(B,H)).
.
        FLOWVTQ(I,A,B,H,L,Q) =L=
        UBFV1(A,B)*FT2(B,H,L) +
LBFV2(B,H)*FL1(I,A,B,L) -
UBFV1(A,B)*LBFV2(B,H) ;



* =========================== sub-
structure Q1 (w/o Q index)
============================ *

 LBP1Q1(I,B,H,L)$H1(B,H)..
        INVVJQ1(I,B,H,L) +
SUM(A$A1(A,B),FLOWVJQ1(I,A,B,H,L)) =E=
        INVVTQ1(I,B,H,L) +
SUM(A$A1(A,B),FLOWVTQ1(I,A,B,H,L))
;


 LBP2Q1(I,B,H,L)$H1(B,H)..
        INVVJQ1(I,B,H,L) =G=
        LBI1(B)*FL2Q1(I,B,H,L) +
LBFV2(B,H)*ITOT(B,L-1) -
LBI1(B)*LBFV2(B,H)      ;

 LBP3Q1(I,B,H,L)$H1(B,H)..
        INVVJQ1(I,B,H,L) =G=
        UBI1(B)*FL2Q1(I,B,H,L) +
UBFV2(B,H)*ITOT(B,L-1) -
UBI1(B)*UBFV2(B,H)      ;

 LBP4Q1(I,B,H,L)$H1(B,H)..
        INVVJQ1(I,B,H,L) =L=
        LBI1(B)*FL2Q1(I,B,H,L) +
UBFV2(B,H)*ITOT(B,L-1) -
LBI1(B)*UBFV2(B,H)      ;

 LBP5Q1(I,B,H,L)$H1(B,H)..
        INVVJQ1(I,B,H,L) =L=
        UBI1(B)*FL2Q1(I,B,H,L) +
LBFV2(B,H)*ITOT(B,L-1) -
LBI1(B)*LBFV2(B,H)      ;

 LBP6Q1(I,B,H,L)$H1(B,H)..
        INVVTQ1(I,B,H,L) =G=
        LBI1(B)*FT2Q1(B,H,L) +
LBFV2(B,H)*INVI(I,B,L-1) -
LBI1(B)*LBFV2(B,H)      ;

 LBP7Q1(I,B,H,L)$H1(B,H)..
        INVVTQ1(I,B,H,L) =G=
        UBI1(B)*FT2Q1(B,H,L) +
UBFV2(B,H)*INVI(I,B,L-1) -
UBI1(B)*UBFV2(B,H)      ;

 LBP8Q1(I,B,H,L)$H1(B,H)..
        INVVTQ1(I,B,H,L) =L=
        LBI1(B)*FT2Q1(B,H,L) +
UBFV2(B,H)*INVI(I,B,L-1) -
LBI1(B)*UBFV2(B,H)      ;

 LBP9Q1(I,B,H,L)$H1(B,H)..
        INVVTQ1(I,B,H,L) =L=
        UBI1(B)*FT2Q1(B,H,L) +
LBFV2(B,H)*INVI(I,B,L-1) -
UBI1(B)*LBFV2(B,H)      ;


LBP10Q1(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
        FLOWVJQ1(I,A,B,H,L) =G=
        LBFV1(A,B)*FL2Q1(I,B,H,L) +
LBFV2(B,H)*FT1Q1(A,B,L) -
LBFV1(A,B)*LBFV2(B,H) ;


LBP11Q1(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
        FLOWVJQ1(I,A,B,H,L) =G=
        UBFV1(A,B)*FL2Q1(I,B,H,L) +
UBFV2(B,H)*FT1Q1(A,B,L) -
UBFV1(A,B)*UBFV2(B,H) ;


LBP12Q1(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
        FLOWVJQ1(I,A,B,H,L) =L=
        LBFV1(A,B)*FL2Q1(I,B,H,L) +
UBFV2(B,H)*FT1Q1(A,B,L) -
LBFV1(A,B)*UBFV2(B,H) ;


LBP13Q1(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
        FLOWVJQ1(I,A,B,H,L) =L=
        UBFV1(A,B)*FL2Q1(I,B,H,L) +
LBFV2(B,H)*FT1Q1(A,B,L) -
UBFV1(A,B)*LBFV2(B,H) ;



LBP14Q1(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
        FLOWVTQ1(I,A,B,H,L) =G=
        LBFV1(A,B)*FT2Q1(B,H,L) +
LBFV2(B,H)*FL1Q1(I,A,B,L) -
LBFV1(A,B)*LBFV2(B,H) ;


LBP15Q1(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
        FLOWVTQ1(I,A,B,H,L) =G=
        UBFV1(A,B)*FT2Q1(B,H,L) +
UBFV2(B,H)*FL1Q1(I,A,B,L) -
UBFV1(A,B)*UBFV2(B,H) ;


LBP16Q1(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
        FLOWVTQ1(I,A,B,H,L) =L=
        LBFV1(A,B)*FT2Q1(B,H,L) +
UBFV2(B,H)*FL1Q1(I,A,B,L) -
LBFV1(A,B)*UBFV2(B,H) ;


LBP17Q1(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
        FLOWVTQ1(I,A,B,H,L) =L=
        UBFV1(A,B)*FT2Q1(B,H,L) +
LBFV2(B,H)*FL1Q1(I,A,B,L) -
UBFV1(A,B)*LBFV2(B,H) ;



* =========================== sub-
structure Q2 (w/o Q index)
============================ *

 LBP1Q2(I,B,H,L)$H1(B,H)..
        INVVJQ2(I,B,H,L) +
SUM(A$A1(A,B),FLOWVJQ2(I,A,B,H,L)) =E=
        INVVTQ2(I,B,H,L) +
SUM(A$A1(A,B),FLOWVTQ2(I,A,B,H,L))
;
```

```
 LBP2Q2(I,B,H,L)$H1(B,H)..
         INVVJQ2(I,B,H,L) =G=
         LBI1(B)*FL2Q2(I,B,H,L) +
LBFV2(B,H)*ITOT(B,L-1) -
LBI1(B)*LBFV2(B,H)      ;

 LBP3Q2(I,B,H,L)$H1(B,H)..
         INVVJQ2(I,B,H,L) =G=
         UBI1(B)*FL2Q2(I,B,H,L) +
UBFV2(B,H)*ITOT(B,L-1) -
UBI1(B)*UBFV2(B,H)      ;

 LBP4Q2(I,B,H,L)$H1(B,H)..
         INVVJQ2(I,B,H,L) =L=
         LBI1(B)*FL2Q2(I,B,H,L) +
UBFV2(B,H)*ITOT(B,L-1) -
LBI1(B)*UBFV2(B,H)      ;

 LBP5Q2(I,B,H,L)$H1(B,H)..
         INVVJQ2(I,B,H,L) =L=
         UBI1(B)*FL2Q2(I,B,H,L) +
LBFV2(B,H)*ITOT(B,L-1) -
LBI1(B)*LBFV2(B,H)      ;

 LBP6Q2(I,B,H,L)$H1(B,H)..
         INVVTQ2(I,B,H,L) =G=
         LBI1(B)*FT2Q2(B,H,L) +
LBFV2(B,H)*INVI(I,B,L-1) -
LBI1(B)*LBFV2(B,H)      ;

 LBP7Q2(I,B,H,L)$H1(B,H)..
         INVVTQ2(I,B,H,L) =G=
         UBI1(B)*FT2Q2(B,H,L) +
UBFV2(B,H)*INVI(I,B,L-1) -
UBI1(B)*UBFV2(B,H)      ;

 LBP8Q2(I,B,H,L)$H1(B,H)..
         INVVTQ2(I,B,H,L) =L=
         LBI1(B)*FT2Q2(B,H,L) +
UBFV2(B,H)*INVI(I,B,L-1) -
LBI1(B)*UBFV2(B,H)      ;

 LBP9Q2(I,B,H,L)$H1(B,H)..
         INVVTQ2(I,B,H,L) =L=
         UBI1(B)*FT2Q2(B,H,L) +
LBFV2(B,H)*INVI(I,B,L-1) -
UBI1(B)*LBFV2(B,H)      ;


LBP10Q2(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
         FLOWVJQ2(I,A,B,H,L) =G=
         LBFV1(A,B)*FL2Q2(I,B,H,L) +
LBFV2(B,H)*FT1Q2(A,B,L) -
LBFV1(A,B)*LBFV2(B,H) ;


LBP11Q2(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
         FLOWVJQ2(I,A,B,H,L) =G=
         UBFV1(A,B)*FL2Q2(I,B,H,L) +
UBFV2(B,H)*FT1Q2(A,B,L) -
UBFV1(A,B)*UBFV2(B,H) ;


LBP12Q2(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
         FLOWVJQ2(I,A,B,H,L) =L=
         LBFV1(A,B)*FL2Q2(I,B,H,L) +
UBFV2(B,H)*FT1Q2(A,B,L) -
LBFV1(A,B)*UBFV2(B,H) ;


LBP13Q2(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
         FLOWVJQ2(I,A,B,H,L) =L=
         UBFV1(A,B)*FL2Q2(I,B,H,L) +
LBFV2(B,H)*FT1Q2(A,B,L) -
UBFV1(A,B)*LBFV2(B,H) ;
```

```
LBP14Q2(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
         FLOWVTQ2(I,A,B,H,L) =G=
         LBFV1(A,B)*FT2Q2(B,H,L) +
LBFV2(B,H)*FL1Q2(I,A,B,L) -
LBFV1(A,B)*LBFV2(B,H) ;


LBP15Q2(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
         FLOWVTQ2(I,A,B,H,L) =G=
         UBFV1(A,B)*FT2Q2(B,H,L) +
UBFV2(B,H)*FL1Q2(I,A,B,L) -
UBFV1(A,B)*UBFV2(B,H) ;


LBP16Q2(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
         FLOWVTQ2(I,A,B,H,L) =L=
         LBFV1(A,B)*FT2Q2(B,H,L) +
UBFV2(B,H)*FL1Q2(I,A,B,L) -
LBFV1(A,B)*UBFV2(B,H) ;


LBP17Q2(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
         FLOWVTQ2(I,A,B,H,L) =L=
         UBFV1(A,B)*FT2Q2(B,H,L) +
LBFV2(B,H)*FL1Q2(I,A,B,L) -
UBFV1(A,B)*LBFV2(B,H) ;



 LBP18(M,K,L)$K1(M,K)..
         FLTQ1(M,K,L) =E=
         FLTQ2(M,K,L)
;

 LBP19(I,M,K,L)$K1(M,K)..
         FLQ1(I,M,K,L) =E=
         FLQ1(I,M,K,L)
;

 LBP20(M,K,L)$K1(M,K)..
         TSQ1(M,K,L) =E=
         TSQ2(M,K,L)
;

 LBP21(M,K,L)$K1(M,K)..
         TEQ1(M,K,L) =E=
         TEQ2(M,K,L)
;

 LBP22(M,K,L)$K1(M,K)..
         WBQ1(M,K,L) =E=
         WBQ2(M,K,L)
;


**(vii)Inventory Bounds

 TC26(B,L)$(ORD(L) GT 1)..
         ITOT(B,L-
1)+SUM(A$A1(A,B),FT1(A,B,L)) =L=
         UBI1(B)
;


**(viii)Bounds on Components Fractions
inside a Tank

 TC27(I,B,L)..
         LB1(I,B)*ITOT(B,L) =L=
         INVI(I,B,L)
;
```

```
 TC28(I,B,L)..
         INVI(I,B,L) =L=
         UB1(I,B)*ITOT(B,L)
;

 TC29(I,B,H,L)$H1(B,H)..
         LB1(I,B)*FT2(B,H,L) =L=
         FL2(I,B,H,L)
;

 TC30(I,B,H,L)$H1(B,H)..
         FL2(I,B,H,L) =L=
         UB1(I,B)*FT2(B,H,L)
;

**(ix)Crude-mix Demand Constraints

 TC31(G)..
SUM(D$D3(D,G),SUM(L,FT2(G,D,L))) =E=
         DM(G)
;

**(x)Bound Strengthening Cuts

 TC32(I,B,H,L)$H1(B,H)..
         ITOT(B,L-1)*FL2(I,B,H,L) +
SUM(A$A1(A,B),FT1(A,B,L)*FL2(I,B,H,L))
=E=
         INVI(I,B,L-
1)*FT2(B,H,L)+SUM(A$A1(A,B),FL1(I,A,B,
L)*FT2(B,H,L))         ;

 TC33(B,H,L)$H1(B,H)..
         SUM(I,ITOT(B,L-
1)*FL2(I,B,H,L)) =E=
         ITOT(B,L-1)*FT2(B,H,L)
;

 TC34(A,B,H,L)$(A1(A,B)$H1(B,H))..
SUM(I,FT1(A,B,L)*FL2(I,B,H,L)) =E=
         FT1(A,B,L)*FT2(B,H,L)
;

 TC35(B,H,L)$H1(B,H)..
         SUM(I,INVI(I,B,L-
1)*FT2(B,H,L)) =E=
         ITOT(B,L-1)*FT2(B,H,L)
;

 TC36(A,B,H,L)$(A1(A,B)$H1(B,H))..
SUM(I,FL1(I,A,B,L)*FT2(B,H,L)) =E=
         FT1(A,B,L)*FT2(B,H,L)
;

* ===========================
2.DISTILLATION UNIT CONSTRAINTS
=========================== *

**(i)Allocation Constraints

 DC1(D,L)..
         SUM(G$G1(G,D),WB2(G,D,L)) =L=
         1
;

 DC2(G,L)..
         SUM(D$D3(D,G),WB2(G,D,L)) =L=
         1
;

 DCUB1(D,L)..
         SUM(G$G1(G,D),WBUB2(G,D,L))
=L=
         1
;

 DCUB2(G,L)..
         SUM(D$D3(D,G),WBUB2(G,D,L))
=L=
         1
;

**(ii)Continuous Operation Constraints

 DC3(D)..
SUM(L,SUM(G$G1(G,D),TE2(G,D,L)-
TS2(G,D,L))) =E=
         THOR
;

 DC4(G,GDASH,D,L)$(ORD(L) LT
CARD(L)$(ORD(G) NE
ORD(GDASH)$G1(G,D)))..
         TS2(G,D,L+1) =G=
         TE2(GDASH,D,L) - THOR*(1-
WB2(GDASH,D,L))
;

 DC5(G,GDASH,D,L)$(ORD(L) LT
CARD(L)$(ORD(G) NE
ORD(GDASH)$G1(G,D)))..
         TS2(G,D,L+1) =L=
         TE2(GDASH,D,L) + THOR*(1-
WB2(GDASH,D,L))
;

 DCUB4(G,GDASH,D,L)$(ORD(L) LT
CARD(L)$(ORD(G) NE
ORD(GDASH)$G1(G,D)))..
         TS2(G,D,L+1) =G=
         TE2(GDASH,D,L) - THOR*(1-
WBUB2(GDASH,D,L))
;

 DCUB5(G,GDASH,D,L)$(ORD(L) LT
CARD(L)$(ORD(G) NE
ORD(GDASH)$G1(G,D)))..
         TS2(G,D,L+1) =L=
         TE2(GDASH,D,L) + THOR*(1-
WBUB2(GDASH,D,L))
;

*3.CRUDE SUPPLY STREAM CONSTRAINTS

**(i)Timing Constraints

 CC1(U,Y,L)$Y1(Y,U)..
         TSI(U) =L=
         TS1(U,Y,L) + THOR*(1-
WB1(U,Y,L))
;

 CC2(U,Y,L)$Y1(Y,U)..
         TEO(U) =G=
         TE1(U,Y,L) - THOR*(1-
WB1(U,Y,L))
;
```

```
 CCUB1(U,Y,L)$Y1(Y,U)..
         TSI(U) =L=
         TS1(U,Y,L) + THOR*(1-
WBUB1(U,Y,L))
;

 CCUB2(U,Y,L)$Y1(Y,U)..
         TEO(U) =G=
         TE1(U,Y,L) - THOR*(1-
WBUB1(U,Y,L))
;


**(ii)Overall Mass Balances

 CC3(U)..

SUM(L,SUM(Y$Y1(Y,U),FT1(U,Y,L))) =E=
         VCRUDE(U)
;


**(iii)Component Balances

 CC4(I,U,Y,L)$Y1(Y,U)..
         FL1(I,U,Y,L) =E=
         FRACT(I,U)*FT1(U,Y,L)
;



*4.VARIABLE BOUNDS

 VB1(I,B,L)..
         INVI(I,B,L) =L=
         UBI1(B)
;

 VB2(B,L)..
         LBI1(B) =L=
         ITOT(B,L)
;

 VB3(B,L)..
         ITOT(B,L) =L=
         UBI1(B)
;

 VB4(I,A,B,L)$A1(A,B)..
         FL1(I,A,B,L) =L=
         UBFV1(A,B)
;

 VB5(I,B,H,L)$H1(B,H)..
         FL2(I,B,H,L) =L=
         UBFV2(B,H)
;

 VB6(A,B,L)$A1(A,B)..
         FT1(A,B,L) =L=
         UBFV1(A,B)
;

 VB7(B,H,L)$H1(B,H)..
         FT2(B,H,L) =L=
         UBFV2(B,H)
;

 VB8(A,B,L)$A1(A,B)..
         TS1(A,B,L) =L=
         THOR
;

 VB9(A,B,L)$A1(A,B)..
         TE1(A,B,L) =L=
         THOR
;

 VB10(B,H,L)$H1(B,H)..
         TS2(B,H,L) =L=
         THOR
;

 VB11(B,H,L)$H1(B,H)..
         TE2(B,H,L) =L=
         THOR
;

 VB12(U)..
         TARR(U) =L=
         TSI(U)
;

 VB13(U)..
         TSI(U) =L=
         THOR
;

 VB14(U)..
         TARR(U) =L=
         TEO(U)
;

 VB15(U)..
         TEO(U) =L=
         THOR
;



* ==============================
GENERATE VALID LINEAR CUTS
=============================== *

 VLC1..
         ZZ1 =L=
         CSEA*SUM(U1,TSI(U1)-TARR(U1))
+ CUNLOAD*SUM(U1,TEO(U1)-TSI(U1)) +
THOR*(SUM(B1,CINV(B1))*SUM(B1,SUM(L,(I
TOT(B1,L)))) +

SUM(B1,CINV(B1))*SUM(B1,SUM(L,SUM(A,FT
1(A,B1,L)))) +
SUM(B1,CINV(B1))*SUM(B1,SUM(L$(ORD(L)
LT CARD(L)),ITOT(B1,L))) +

THOR*SUM(B1,CINV(B1))*SUM(B2,INITITOT(
B2))) / (2*NE + 1) +
CSET*(SUM(D1,SUM(G$G1(G,D1),SUM(L,WB2(
G,D1,L)))) - ND1) +

SUM(M,SUM(K$K1(M,K),SUM(L,LMFLT(M,K,L)
*FLTQ(M,K,L)))) +
SUM(I,SUM(M,SUM(K$K1(M,K),SUM(L,LMFL(I
,M,K,L)*FLQ(I,M,K,L))))) +

SUM(M,SUM(K$K1(M,K),SUM(L,LMTS(M,K,L)*
TSQ(M,K,L)))) +
SUM(M,SUM(K$K1(M,K),SUM(L,LMTE(M,K,L)*
TEQ(M,K,L)))) +

SUM(M,SUM(K$K1(M,K),SUM(L,LMWB(M,K,L)*
WBQ(M,K,L))))
;


 VLC2..
         ZZ2 =L=
         CSEA*SUM(U2,TSI(U2)-TARR(U2))
+ CUNLOAD*SUM(U2,TEO(U2)-TSI(U2)) +
```

```
THOR*(SUM(B2,CINV(B2))*SUM(B2,SUM(L,(I
TOT(B2,L)))) +

SUM(B2,CINV(B2))*SUM(B2,SUM(L,SUM(A,FT
1(A,B2,L)))) +
SUM(B2,CINV(B2))*SUM(B2,SUM(L$(ORD(L)
LT CARD(L)),ITOT(B2,L))) +

THOR*SUM(B2,CINV(B2))*SUM(B2,INITITOT(
B2))) / (2*NE + 1) +
CSET*(SUM(D2,SUM(G$G1(G,D2),SUM(L,WB2(
G,D2,L)))) - ND2) -

SUM(M,SUM(K$K1(M,K),SUM(L,LMFLT(M,K,L)
*FLTQ(M,K,L)))) -
SUM(I,SUM(M,SUM(K$K1(M,K),SUM(L,LMFL(I
,M,K,L)*FLQ(I,M,K,L))))) -

SUM(M,SUM(K$K1(M,K),SUM(L,LMTS(M,K,L)*
TSQ(M,K,L)))) -
SUM(M,SUM(K$K1(M,K),SUM(L,LMTE(M,K,L)*
TEQ(M,K,L)))) -

SUM(M,SUM(K$K1(M,K),SUM(L,LMWB(M,K,L)*
WBQ(M,K,L))))
;



* =================== FOR DUPLICATING
EQUATIONS AND VARIABLES IN Q1
============= *

 TC1Q1(A,B,L)$A1(A,B)..
         FT1Q1(A,B,L) =L=
         UBFV1(A,B)*WB1Q1(A,B,L)
;

 TC2Q1(B,H,L)$H1(B,H)..
         FT2Q1(B,H,L) =L=
         UBFV2(B,H)*WB2Q1(B,H,L)
;

 TC3Q1(A,B,L)$A1(A,B)..
         UBF1(A,B)*(TE1Q1(A,B,L)-
TS1Q1(A,B,L)) + UBF1(A,B)*THOR*(1-
WB1Q1(A,B,L)) =G=
         FT1Q1(A,B,L)
;

 TC4Q1(B,H,L)$H1(B,H)..
         UBF2(B,H)*(TE2Q1(B,H,L)-
TS2Q1(B,H,L)) + UBF2(B,H)*THOR*(1-
WB2Q1(B,H,L)) =G=
         FT2Q1(B,H,L)
;

 TC5Q1(A,Y,L)$A2(A,Y)..
         LBF1(A,Y)*(TE1Q1(A,Y,L)-
TS1Q1(A,Y,L)) - LBF1(A,Y)*THOR*(1-
WB1Q1(A,Y,L)) =L=
         FT1Q1(A,Y,L)
;

 TC6Q1(Y,H,L)$H2(Y,H)..
         LBF2(Y,H)*(TE2Q1(Y,H,L)-
TS2Q1(Y,H,L)) - LBF2(Y,H)*THOR*(1-
WB2Q1(Y,H,L)) =L=
         FT2Q1(Y,H,L)
;

 TC7Q1(G,H,L)$H3(G,H)..
         LBF2(G,H)*(TE2Q1(G,H,L)-
TS2Q1(G,H,L)) =L=
         FT2Q1(G,H,L)
;

 TC8Q1(A,B,L)$(ORD(L) LT
CARD(L)$A1(A,B))..
         TS1Q1(A,B,L+1) =G=
         TE1Q1(A,B,L) - THOR*(1-
WB1Q1(A,B,L))
;

 TC9Q1(A,B,L)$(ORD(L) LT
CARD(L)$A1(A,B))..
         TS1Q1(A,B,L+1) =G=
         TS1Q1(A,B,L)
;

 TC10Q1(A,B,L)$(ORD(L) LT
CARD(L)$A1(A,B))..
         TE1Q1(A,B,L+1) =G=
         TE1Q1(A,B,L)
;

 TC11Q1(B,H,L)$(ORD(L) LT
CARD(L)$H1(B,H))..
         TS2Q1(B,H,L+1) =G=
         TE2Q1(B,H,L) - THOR*(1-
WB2Q1(B,H,L))
;

 TC12Q1(B,H,L)$(ORD(L) LT
CARD(L)$H1(B,H))..
         TS2Q1(B,H,L+1) =G=
         TS2Q1(B,H,L)
;

 TC13Q1(B,H,L)$(ORD(L) LT
CARD(L)$H1(B,H))..
         TE2Q1(B,H,L+1) =G=
         TE2Q1(B,H,L)
;

 TC14Q1(A,ADASH,B,L)$(ORD(L) LT
CARD(L)$(ORD(A) NE
ORD(ADASH)$A1(A,B)))..
         TS1Q1(A,B,L+1) =G=
         TE1Q1(ADASH,B,L) - THOR*(1-
WB1Q1(ADASH,B,L))
;

 TC15Q1(A,B,H,L)$(ORD(L) LT
CARD(L)$(A1(A,B)$H1(B,H)))..
         TS1Q1(A,B,L+1) =G=
         TE2Q1(B,H,L) - THOR*(1-
WB2Q1(B,H,L))
;

 TC16Q1(A,B,H,L)$(ORD(L) LT
CARD(L)$(A1(A,B)$H1(B,H)))..
         TS2Q1(B,H,L+1) =G=
         TE1Q1(A,B,L) - THOR*(1-
WB1Q1(A,B,L))
;

 TC17Q1(B,H,HDASH,L)$(ORD(L) LT
CARD(L)$(ORD(H) NE
ORD(HDASH)$H1(B,H)))..
         TS2Q1(B,H,L+1) =G=
         TE2Q1(B,HDASH,L) - THOR*(1-
WB2Q1(B,HDASH,L))
;

 TC18Q1(A,B,H,L)$(A1(A,B)$H1(B,H))..
         TE1Q1(A,B,L) - THOR*(1-
WB1Q1(A,B,L)) =L=
         TS2Q1(B,H,L) + THOR*(1-
WB2Q1(B,H,L))
;

 TC19Q1(B,L)..
```

```
         ITOT(B,L-1) +
SUM(A$A1(A,B),FT1Q1(A,B,L)) =E=
         ITOT(B,L) +
SUM(H$H1(B,H),FT2Q1(B,H,L))
;
*- ONEG1Q1(B,L) + OPOS1Q1(B,L)
;

 TC20Q1(B)..
         ITOT(B,'L0') =E=
         INITITOT(B)
;

 TC21Q1(I,B,L)..
         INVI(I,B,L-1) +
SUM(A$A1(A,B),FL1Q1(I,A,B,L)) =E=
         INVI(I,B,L)
+SUM(H$H1(B,H),FL2Q1(I,B,H,L)) -
ONEG2Q1(I,B,L) + OPOS2Q1(I,B,L)
;

 TC22Q1(I,B)..
         INVI(I,B,'L0') =E=
         INITINVI(I,B)
;

 TC23Q1(A,B,L)$A1(A,B)..
         FT1Q1(A,B,L) =E=
         SUM(I,FL1Q1(I,A,B,L))
;

 TC24Q1(B,H,L)$H1(B,H)..
         FT2Q1(B,H,L) =E=
         SUM(I,FL2Q1(I,B,H,L))
;

 TC26Q1(B,L)$(ORD(L) GT 1)..
         ITOT(B,L-
1)+SUM(A$A1(A,B),FT1Q1(A,B,L)) =L=
         UBI1(B)
;

 TC27Q1(I,B,L)..
         LB1(I,B)*ITOT(B,L) =L=
         INVI(I,B,L)
* - ONEG1Q1(I,B,L) + OPOS1Q1(I,B,L)
;
 ;
 TC28Q1(I,B,L)..
         INVI(I,B,L) =L=
         UB1(I,B)*ITOT(B,L)
;

 TC29Q1(I,B,H,L)$H1(B,H)..
         LB1(I,B)*FT2Q1(B,H,L) =L=
         FL2Q1(I,B,H,L)
;

 TC30Q1(I,B,H,L)$H1(B,H)..
         FL2Q1(I,B,H,L) =L=
         UB1(I,B)*FT2Q1(B,H,L)
;

 TC31Q1(G)..

SUM(D$D3(D,G),SUM(L,FT2Q1(G,D,L))) =E=
         DM(G)
;

 TC32Q1(I,B,H,L)$H1(B,H)..
         ITOT(B,L-1)*FL2Q1(I,B,H,L) +
SUM(A$A1(A,B),FT1Q1(A,B,L)*FL2Q1(I,B,H
,L)) =E=
         INVI(I,B,L-
1)*FT2Q1(B,H,L)+SUM(A$A1(A,B),FL1Q1(I,
A,B,L)*FT2Q1(B,H,L))         ;

 TC33Q1(B,H,L)$H1(B,H)..
         SUM(I,ITOT(B,L-
1)*FL2Q1(I,B,H,L)) =E=
         ITOT(B,L-1)*FT2Q1(B,H,L)
;

 TC34Q1(A,B,H,L)$(A1(A,B)$H1(B,H))..

SUM(I,FT1Q1(A,B,L)*FL2Q1(I,B,H,L)) =E=
         FT1Q1(A,B,L)*FT2Q1(B,H,L)
;

 TC35Q1(B,H,L)$H1(B,H)..
         SUM(I,INVI(I,B,L-
1)*FT2Q1(B,H,L)) =E=
         ITOT(B,L-1)*FT2Q1(B,H,L)
;

 TC36Q1(A,B,H,L)$(A1(A,B)$H1(B,H))..

SUM(I,FL1Q1(I,A,B,L)*FT2Q1(B,H,L)) =E=
         FT1Q1(A,B,L)*FT2Q1(B,H,L)
;

 DC1Q1(D,L)..
         SUM(G$G1(G,D),WB2Q1(G,D,L))
=L=
         1
;

 DC2Q1(G,L)..
         SUM(D$D3(D,G),WB2Q1(G,D,L))
=L=
         1        ;

 DC3Q1(D)..

SUM(L,SUM(G$G1(G,D),TE2Q1(G,D,L)-
TS2Q1(G,D,L))) =E=
         THOR
;

 DC4Q1(G,GDASH,D,L)$(ORD(L) LT
CARD(L)$(ORD(G) NE
ORD(GDASH)$G1(G,D)))..
         TS2Q1(G,D,L+1) =G=
         TE2Q1(GDASH,D,L) - THOR*(1-
WB2Q1(GDASH,D,L))
;

 DC5Q1(G,GDASH,D,L)$(ORD(L) LT
CARD(L)$(ORD(G) NE
ORD(GDASH)$G1(G,D)))..
         TS2Q1(G,D,L+1) =L=
         TE2Q1(GDASH,D,L) + THOR*(1-
WB2Q1(GDASH,D,L))
;

 CC1Q1(U,Y,L)$Y1(Y,U)..
         TSI(U) =L=
         TS1Q1(U,Y,L) + THOR*(1-
WB1Q1(U,Y,L))
;

 CC2Q1(U,Y,L)$Y1(Y,U)..
         TEO(U) =G=
         TE1Q1(U,Y,L) - THOR*(1-
WB1Q1(U,Y,L))
;

 CC3Q1(U)..

SUM(L,SUM(Y$Y1(Y,U),FT1Q1(U,Y,L))) =E=
         VCRUDE(U)
;
```

```
 CC4Q1(I,U,Y,L)$Y1(Y,U)..
         FL1Q1(I,U,Y,L) =E=
         FRACT(I,U)*FT1Q1(U,Y,L)
;

 VB1Q1(I,B,L)..
         INVI(I,B,L) =L=
         UBI1(B)
;

 VB2Q1(B,L)..
         LBI1(B) =L=
         ITOT(B,L)
;

 VB3Q1(B,L)..
         ITOT(B,L) =L=
         UBI1(B)
;

 VB4Q1(I,A,B,L)$A1(A,B)..
         FL1Q1(I,A,B,L) =L=
         UBFV1(A,B)
;

 VB5Q1(I,B,H,L)$H1(B,H)..
         FL2Q1(I,B,H,L) =L=
         UBFV2(B,H)
;

 VB6Q1(A,B,L)$A1(A,B)..
         FT1Q1(A,B,L) =L=
         UBFV1(A,B)
;

 VB7Q1(B,H,L)$H1(B,H)..
         FT2Q1(B,H,L) =L=
         UBFV2(B,H)
;

 VB8Q1(A,B,L)$A1(A,B)..
         TS1Q1(A,B,L) =L=
         THOR
;

 VB9Q1(A,B,L)$A1(A,B)..
         TE1Q1(A,B,L) =L=
         THOR
;

 VB10Q1(B,H,L)$H1(B,H)..
         TS2Q1(B,H,L) =L=
         THOR
;

 VB11Q1(B,H,L)$H1(B,H)..
         TE2Q1(B,H,L) =L=
         THOR
;

 VB12Q1(U)..
         TARR(U) =L=
         TSI(U)
;

 VB13Q1(U)..
         TSI(U) =L=
         THOR
;

 VB14Q1(U)..
         TARR(U) =L=
         TEO(U)
;

 VB15Q1(U)..
         TEO(U) =L=
         THOR
;

* =================== FOR DUPLICATING
EQUATIONS AND VARIABLES IN Q2
============= *

 TC1Q2(A,B,L)$A1(A,B)..
         FT1Q2(A,B,L) =L=
         UBFV1(A,B)*WB1Q2(A,B,L)
;

 TC2Q2(B,H,L)$H1(B,H)..
         FT2Q2(B,H,L) =L=
         UBFV2(B,H)*WB2Q2(B,H,L)
;

 TC3Q2(A,B,L)$A1(A,B)..
         UBF1(A,B)*(TE1Q2(A,B,L)-
TS1Q2(A,B,L)) + UBF1(A,B)*THOR*(1-
WB1Q2(A,B,L)) =G=
         FT1Q2(A,B,L)
;

 TC4Q2(B,H,L)$H1(B,H)..
         UBF2(B,H)*(TE2Q2(B,H,L)-
TS2Q2(B,H,L)) + UBF2(B,H)*THOR*(1-
WB2Q2(B,H,L)) =G=
         FT2Q2(B,H,L)
;

 TC5Q2(A,Y,L)$A2(A,Y)..
         LBF1(A,Y)*(TE1Q2(A,Y,L)-
TS1Q2(A,Y,L)) - LBF1(A,Y)*THOR*(1-
WB1Q2(A,Y,L)) =L=
         FT1Q2(A,Y,L)
;

 TC6Q2(Y,H,L)$H2(Y,H)..
         LBF2(Y,H)*(TE2Q2(Y,H,L)-
TS2Q2(Y,H,L)) - LBF2(Y,H)*THOR*(1-
WB2Q2(Y,H,L)) =L=
         FT2Q2(Y,H,L)
;

 TC7Q2(G,H,L)$H3(G,H)..
         LBF2(G,H)*(TE2Q2(G,H,L)-
TS2Q2(G,H,L)) =L=
         FT2Q2(G,H,L)
;
 TC8Q2(A,B,L)$(ORD(L) LT
CARD(L)$A1(A,B))..
         TS1Q2(A,B,L+1) =G=
         TE1Q2(A,B,L) - THOR*(1-
WB1Q2(A,B,L))
;

 TC9Q2(A,B,L)$(ORD(L) LT
CARD(L)$A1(A,B))..
         TS1Q2(A,B,L+1) =G=
         TS1Q2(A,B,L)
;

 TC10Q2(A,B,L)$(ORD(L) LT
CARD(L)$A1(A,B))..
         TE1Q2(A,B,L+1) =G=
         TE1Q2(A,B,L)
;

 TC11Q2(B,H,L)$(ORD(L) LT
CARD(L)$H1(B,H))..
         TS2Q2(B,H,L+1) =G=
```

```
         TE2Q2(B,H,L) - THOR*(1-
WB2Q2(B,H,L))
;

 TC12Q2(B,H,L)$(ORD(L) LT
CARD(L)$H1(B,H))..
         TS2Q2(B,H,L+1) =G=
         TS2Q2(B,H,L)
;

 TC13Q2(B,H,L)$(ORD(L) LT
CARD(L)$H1(B,H))..
         TE2Q2(B,H,L+1) =G=
         TE2Q2(B,H,L)
;

 TC14Q2(A,ADASH,B,L)$(ORD(L) LT
CARD(L)$(ORD(A) NE
ORD(ADASH)$A1(A,B)))..
         TS1Q2(A,B,L+1) =G=
         TE1Q2(ADASH,B,L) - THOR*(1-
WB1Q2(ADASH,B,L))
;

 TC15Q2(A,B,H,L)$(ORD(L) LT
CARD(L)$(A1(A,B)$H1(B,H)))..
         TS1Q2(A,B,L+1) =G=
         TE2Q2(B,H,L) - THOR*(1-
WB2Q2(B,H,L))
;

 TC16Q2(A,B,H,L)$(ORD(L) LT
CARD(L)$(A1(A,B)$H1(B,H)))..
         TS2Q2(B,H,L+1) =G=
         TE1Q2(A,B,L) - THOR*(1-
WB1Q2(A,B,L))
;

 TC17Q2(B,H,HDASH,L)$(ORD(L) LT
CARD(L)$(ORD(H) NE
ORD(HDASH)$H1(B,H)))..
         TS2Q2(B,H,L+1) =G=
         TE2Q2(B,HDASH,L) - THOR*(1-
WB2Q2(B,HDASH,L))
;

 TC18Q2(A,B,H,L)$(A1(A,B)$H1(B,H))..
         TE1Q2(A,B,L) - THOR*(1-
WB1Q2(A,B,L)) =L=
         TS2Q2(B,H,L) + THOR*(1-
WB2Q2(B,H,L))
;

 TC19Q2(B,L)..
         ITOT(B,L-1) +
SUM(A$A1(A,B),FT1Q2(A,B,L)) =E=
         ITOT(B,L) +
SUM(H$H1(B,H),FT2Q2(B,H,L))
;

 TC20Q2(B)..
         ITOT(B,'L0') =E=
         INITITOT(B)
;

 TC21Q2(I,B,L)..
         INVI(I,B,L-1) +
SUM(A$A1(A,B),FL1Q2(I,A,B,L)) =E=
         INVI(I,B,L)
+SUM(H$H1(B,H),FL2Q2(I,B,H,L))
;

 TC22Q2(I,B)..
         INVI(I,B,'L0') =E=
         INITINVI(I,B)
;

 TC23Q2(A,B,L)$A1(A,B)..
         FT1Q2(A,B,L) =E=
         SUM(I,FL1Q2(I,A,B,L))
;

 TC24Q2(B,H,L)$H1(B,H)..
         FT2Q2(B,H,L) =E=
         SUM(I,FL2Q2(I,B,H,L))
;

 TC26Q2(B,L)$(ORD(L) GT 1)..
         ITOT(B,L-
1)+SUM(A$A1(A,B),FT1Q2(A,B,L)) =L=
         UBI1(B)
;

 TC27Q2(I,B,L)..
         LB1(I,B)*ITOT(B,L) =L=
         INVI(I,B,L)
;

 TC28Q2(I,B,L)..
         INVI(I,B,L) =L=
         UB1(I,B)*ITOT(B,L)
;

 TC29Q2(I,B,H,L)$H1(B,H)..
         LB1(I,B)*FT2Q2(B,H,L) =L=
         FL2Q2(I,B,H,L)
;

 TC30Q2(I,B,H,L)$H1(B,H)..
         FL2Q2(I,B,H,L) =L=
         UB1(I,B)*FT2Q2(B,H,L)
;

 TC31Q2(G)..

SUM(D$D3(D,G),SUM(L,FT2Q2(G,D,L))) =E=
         DM(G)
;

 TC32Q2(I,B,H,L)$H1(B,H)..
         ITOT(B,L-1)*FL2Q2(I,B,H,L) +
SUM(A$A1(A,B),FT1Q2(A,B,L)*FL2Q2(I,B,H
,L)) =E=
         INVI(I,B,L-
1)*FT2Q2(B,H,L)+SUM(A$A1(A,B),FL1Q2(I,
A,B,L)*FT2Q2(B,H,L))         ;

 TC33Q2(B,H,L)$H1(B,H)..
         SUM(I,ITOT(B,L-
1)*FL2Q2(I,B,H,L)) =E=
         ITOT(B,L-1)*FT2Q2(B,H,L)
;

 TC34Q2(A,B,H,L)$(A1(A,B)$H1(B,H))..

SUM(I,FT1Q2(A,B,L)*FL2Q2(I,B,H,L)) =E=
         FT1Q2(A,B,L)*FT2Q2(B,H,L)
;

 TC35Q2(B,H,L)$H1(B,H)..
         SUM(I,INVI(I,B,L-
1)*FT2Q2(B,H,L)) =E=
         ITOT(B,L-1)*FT2Q2(B,H,L)
;

 TC36Q2(A,B,H,L)$(A1(A,B)$H1(B,H))..

SUM(I,FL1Q2(I,A,B,L)*FT2Q2(B,H,L)) =E=
         FT1Q2(A,B,L)*FT2Q2(B,H,L)
;

 DC1Q2(D,L)..
```

```
        SUM(G$G1(G,D),WB2Q2(G,D,L))
=L=
        1
;

 DC2Q2(G,L)..
        SUM(D$D3(D,G),WB2Q2(G,D,L))
=L=
        1        ;

 DC3Q2(D)..

SUM(L,SUM(G$G1(G,D),TE2Q2(G,D,L)-
TS2Q2(G,D,L))) =E=
        THOR
;

 DC4Q2(G,GDASH,D,L)$(ORD(L) LT
CARD(L)$(ORD(G) NE
ORD(GDASH)$G1(G,D)))..
        TS2Q2(G,D,L+1) =G=
        TE2Q2(GDASH,D,L) - THOR*(1-
WB2Q2(GDASH,D,L))
;

 DC5Q2(G,GDASH,D,L)$(ORD(L) LT
CARD(L)$(ORD(G) NE
ORD(GDASH)$G1(G,D)))..
        TS2Q2(G,D,L+1) =L=
        TE2Q2(GDASH,D,L) + THOR*(1-
WB2Q2(GDASH,D,L))
;

 CC1Q2(U,Y,L)$Y1(Y,U)..
        TSI(U) =L=
        TS1Q2(U,Y,L) + THOR*(1-
WB1Q2(U,Y,L))
;

 CC2Q2(U,Y,L)$Y1(Y,U)..
        TEO(U) =G=
        TE1Q2(U,Y,L) - THOR*(1-
WB1Q2(U,Y,L))
;

 CC3Q2(U)..

SUM(L,SUM(Y$Y1(Y,U),FT1Q2(U,Y,L))) =E=
        VCRUDE(U)
;

 CC4Q2(I,U,Y,L)$Y1(Y,U)..
        FL1Q2(I,U,Y,L) =E=
        FRACT(I,U)*FT1Q2(U,Y,L)
;

 VB1Q2(I,B,L)..
        INVI(I,B,L) =L=
        UBI1(B)
;

 VB2Q2(B,L)..
        LBI1(B) =L=
        ITOT(B,L)
;

 VB3Q2(B,L)..
        ITOT(B,L) =L=
        UBI1(B)
;

 VB4Q2(I,A,B,L)$A1(A,B)..
        FL1Q2(I,A,B,L) =L=
        UBFV1(A,B)
;

 VB5Q2(I,B,H,L)$H1(B,H)..
        FL2Q2(I,B,H,L) =L=
        UBFV2(B,H)
;

 VB6Q2(A,B,L)$A1(A,B)..
        FT1Q2(A,B,L) =L=
        UBFV1(A,B)
;

 VB7Q2(B,H,L)$H1(B,H)..
        FT2Q2(B,H,L) =L=
        UBFV2(B,H)
;

 VB8Q2(A,B,L)$A1(A,B)..
        TS1Q2(A,B,L) =L=
        THOR
;

 VB9Q2(A,B,L)$A1(A,B)..
        TE1Q2(A,B,L) =L=
        THOR
;

 VB10Q2(B,H,L)$H1(B,H)..
        TS2Q2(B,H,L) =L=
        THOR
;

 VB11Q2(B,H,L)$H1(B,H)..
        TE2Q2(B,H,L) =L=
        THOR
;

 VB12Q2(U)..
        TARR(U) =L=
        TSI(U)
;

 VB13Q2(U)..
        TSI(U) =L=
        THOR
;

 VB14Q2(U)..
        TARR(U) =L=
        TEO(U)
;

 VB15Q2(U)..
        TEO(U) =L=
        THOR
;


*======================== BRANCH AND
CONTRACT ALGORITHM
========================

MODEL ACOTAVARS
/
OBJFUN
TC1
TC2
*TC3
TC4
TC5
TC6
TC7
TC8
TC9
TC10
TC11
TC12
TC13
```

```
TC14
TC15
TC16
TC17
TC18
*TC19
TC20
*TC21
TC22
*TC23
*TC24

TC26
*TC27
TC28
TC29
TC30
TC31
*DC1
*DC2
DC3
DC4
DC5
CC1
CC2
CC3
CC4
VB1
VB2
VB3
VB4
VB5
VB6
VB7
VB8
VB9
VB10
VB11
VB12
VB13
VB14
VB15
LBP1
LBP2
LBP3
*LBP4
*LBP5
*LBP6
LBP7
*LBP8
LBP9
LBP10
LBP11
LBP12
*LBP13
*LBP14
LBP15
LBP16
LBP17
COTAOF
ITOTLIBREST1L0
ITOTLIBREST1L1
ITOTLIBREST1L2
ITOTLIBREST2L0
ITOTLIBREST2L1
ITOTLIBREST2L2
ITOTLIBREST3L0
ITOTLIBREST3L1
ITOTLIBREST3L2
ITOTLIBRECT1L0
ITOTLIBRECT1L1
ITOTLIBRECT1L2
ITOTLIBRECT2L0
ITOTLIBRECT2L1
ITOTLIBRECT2L2
ITOTLIBRECT3L0
ITOTLIBRECT3L1
ITOTLIBRECT3L2
/

MODEL MR00

/
OBJFUN
TC1
TC2
*TC3
TC4
TC5
TC6
TC7
TC8
TC9
TC10
TC11
TC12
TC13
TC14
TC15
TC16
TC17
TC18
*TC19
TC20
*TC21
TC22
*TC23
*TC24

TC26
*TC27
TC28
TC29
TC30
TC31

*DC1
*DC2
DC3
DC4
DC5
CC1
CC2
CC3
CC4
VB1
VB2
VB3
VB4
VB5
VB6
VB7
VB8
VB9
VB10
VB11
VB12
VB13
VB14
VB15
LBP1
LBP2
LBP3
*LBP4
*LBP5
*LBP6
LBP7
*LBP8
LBP9
LBP10
LBP11
LBP12
```

```
*LBP13
*LBP14
LBP15
LBP16
LBP17
/

MODEL MF00
/
TCUB1
TCUB2
*TCUB3
TCUB4
TCUB5
TCUB6
TC7
TCUB8
TC9
TC10
TCUB11
TC12
TC13
TCUB14
TCUB15
TCUB16
TCUB17
TCUB18
*TC19
TC20
*TC21
TC22
*TC23
*TC24

TC26
*TC27
TC28
TC29
TC30
TC31
*DCUB1
*DCUB2
DC3
DCUB4
DCUB5
CCUB1
CCUB2
CC3
CC4
VB1
VB2
VB3
VB4
VB5
VB6
VB7
VB8
VB9
VB10
VB11
VB12
VB13
VB14
VB15
OBJFIXBIN
/

MODEL MINLPOUB

/
TC1
TC2
TC3
TC4
TC5
TC6
TC7
TC8
TC9
TC10
TC11
TC12
TC13
TC14
TC15
TC16
TC17
TC18
TC19
TC20
TC21
TC22
TC23
TC24
TC25
TC26
TC27
TC28
TC29
TC30
TC31
DC1
DC2
DC3
DC4
DC5
CC1
CC2
CC3
CC4
VB1
VB2
VB3
VB4
VB5
VB6
VB7
VB8
VB9
VB10
VB11
VB12
VB13
VB14
VB15
OBJFUN
/

;

MODEL LBMIP                    /
TC1
TC2
*TC3
TC4
TC5
TC6
TC7
TC8
TC9
TC10
TC11
TC12
TC13
TC14
TC15
TC16
TC17
TC18
*TC19
TC20
*TC21
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```
TC22
*TC23
*TC24

TC26
*TC27
TC28
TC29
TC30
TC31
*DC1
*DC2
DC3
DC4
DC5
CC1
CC2
CC3
CC4
VB1
VB2
VB3
VB4
VB5
VB6
VB7
VB8
VB9
VB10
VB11
VB12
VB13
VB14
VB15
LBP1
LBP2
LBP3
*LBP4
*LBP5
*LBP6
LBP7
*LBP8
LBP9
LBP10
LBP11
LBP12

*LBP13

*LBP14
LBP15
LBP16
LBP17
OBJFUN
/

;

MODEL CRUDE_LB1
/
TC1Q1
TC2Q1
TC3Q1
TC4Q1
TC5Q1
TC6Q1
TC7Q1
TC8Q1
TC9Q1
TC10Q1
TC11Q1
TC12Q1
TC13Q1
TC14Q1
TC15Q1
TC16Q1
TC17Q1
TC18Q1
TC19Q1
TC20Q1
*TC21Q1
TC22Q1
TC23Q1
TC24Q1

TC26Q1
*TC27Q1
TC28Q1
TC29Q1
TC30Q1
TC31Q1
DC1Q1
DC2Q1
DC3Q1
DC4Q1
DC5Q1
CC1Q1
CC2Q1
CC3Q1
CC4Q1
VB1Q1
VB2Q1
VB3Q1
VB4Q1
VB5Q1
VB6Q1
VB7Q1
VB8Q1
VB9Q1
VB10Q1
VB11Q1
VB12Q1
VB13Q1
VB14Q1
VB15Q1


LBP1Q1
LBP2Q1
LBP3Q1
*LBP4Q1
*LBP5Q1


*LBP6Q1
LBP7Q1
LBP8Q1
LBP9Q1
LBP10Q1
LBP11Q1
LBP12Q1

*LBP13Q1

LBP14Q1
LBP15Q1
LBP16Q1
LBP17Q1
LBP18
LBP19
LBP20
LBP21
LBP22
*OBJF
OBJQ1
*OBJQ2
VLC1
VLC2                                /

;

MODEL CRUDE_LB2                     /
```

```
TC1Q2
TC2Q2
TC3Q2
TC4Q2
TC5Q2
TC6Q2
TC7Q2
TC8Q2
TC9Q2
TC10Q2
TC11Q2
TC12Q2
TC13Q2
TC14Q2
TC15Q2
TC16Q2
TC17Q2
TC18Q2
*TC19Q2
TC20Q2
*TC21Q2
TC22Q2
TC23Q2
TC24Q2

TC26Q2
*TC27Q2
TC28Q2
TC29Q2
TC30Q2
TC31Q2
DC1Q2
DC2Q2
DC3Q2
DC4Q2
DC5Q2
CC1Q2
CC2Q2
CC3Q2
CC4Q2
VB1Q2
VB2Q2
VB3Q2
VB4Q2
VB5Q2
VB6Q2
VB7Q2
VB8Q2
VB9Q2
VB10Q2
VB11Q2
VB12Q2
VB13Q2
VB14Q2
VB15Q2

LBP1Q2
LBP2Q2
LBP3Q2
*LBP4Q2
*LBP5Q2
*LBP6Q2
LBP7Q2
LBP8Q2
LBP9Q2
LBP10Q2
LBP11Q2
LBP12Q2

*LBP13Q2

LBP14Q2
LBP15Q2
LBP16Q2
LBP17Q2
LBP18
LBP19
LBP20
LBP21
LBP22
*OBJF
*OBJQ1
OBJQ2
VLC1
VLC2                                  /

;


MODEL CRUDE_UB                        /
TCUB1
TCUB2
TCUB3
TCUB4
TCUB5
TCUB6
TC7
TCUB8
TC9
TC10
TCUB11
TC12
TC13
TCUB14
TCUB15
TCUB16
TCUB17
TCUB18
TC19
TC20
TC21
TC22
TC23
TC24

TC26
TC27
TC28
TC29
TC30
TC31
DCUB1
DCUB2
DC3
DCUB4
DCUB5
CCUB1
CCUB2
CC3
CC4
VB1
VB2
VB3
VB4
VB5
VB6
VB7
VB8
VB9
VB10
VB11
VB12
VB13
VB14
VB15
OBJFIXBIN
/

;
```

```
ZZUB.FX = INF;

*================== BOUNDS FOR BRANCH
AND CONTRACT ALGORITHM
===================

ITOT.LO(B,L) = 0;        ITOT.UP(B,L)
= 100;

LOOP( (B,L),
 ITOTINI.LO(B,L) = ITOT.LO(B,L);
 ITOTINI.UP(B,L) = ITOT.UP(B,L);
 DELINIITOT(B,L) = ITOTINI.UP(B,L) -
ITOTINI.LO(B,L);
);


FILE RESCRUBND / CBOUND.RES /;

PUT RESCRUBND;

PUT //

LOOP( (B,L),
 PUT / "ITOT.LO('
"B.TL:3,",",L.TL:3,"')=
",ITOT.LO(B,L):18:12";"
 "      ITOT.UP('
"B.TL:3,",",L.TL:3,"')=
",ITOT.UP(B,L):18:12";";
);

*-------------------------------------
---------------------------
*------------------------------
SoluTion to model MIP


SOLVE MR00 minimizing ZZ using mip;

*-------------------------------------
---------------------------
*---------------------------------
creating file with results

PUT RESCRU;

PUT ///   "---------------------------
-------------------";
PUT /     "NEXT THE SOLUTION OF THE
RELAXED PROBLEM (MIP)"
PUT /     "---------------------------
-------------------";

PUT //
LOOP( (B,L),
 PUT /"ITOT.l('
"B.TL:3,",",L.TL:3,"')=
",ITOT.l(B,L):18:12";";
);

 PUT  / "ZZ.l=" ZZ.l:20:12 ";";

*---------------

 PUT /// "LP solution time : "
MR00.resusd:10:5;
 PUT / "Model Status: "
 PUT$(MR00.modelstat eq 1) "Optimal.";
 PUT$(MR00.modelstat eq 2) "Locally
optimal.";
 PUT$(MR00.modelstat eq 3)
"Unbounded.";
 PUT$(MR00.modelstat eq 4)
"Infeasible.";
 PUT$(MR00.modelstat eq 5) "Locally
infeasible.";
 PUT$(MR00.modelstat eq 6)
"Intermediate infeasible.";
 PUT$(MR00.modelstat eq 7)
"Intermediate nonoptimal.";
 PUT$(MR00.modelstat eq 8) "Integer
solution.";
 PUT$(MR00.modelstat eq 9)
"Intermediate non-integer.";
 PUT$(MR00.modelstat eq 10) "Integer
infeasible.";
 PUT$(MR00.modelstat eq 12) "Error
unknown.";
 PUT$(MR00.modelstat eq 13) "Error no
solution.";

*--------------------------
*NEXT USED LEVELS ARE REPORTED


PUT //
LOOP( (B,L),
 PUT / "ITOT.LO('
"B.TL:3,",",L.TL:3,"')=
",ITOT.LO(B,L):18:12";"
 "      ITOT.UP('
"B.TL:3,",",L.TL:3,"')=
",ITOT.UP(B,L):18:12";";
);

*-------------------------------------
--------------------------
*-------------------------------------
--------------------------
*MULTIPLICATION


PUT //
LOOP( (B,L),
 PUT / "ITOT.M('
"B.TL:3,",",L.TL:3,"')=
",ITOT.M(B,L):18:12";";
);

IF(MR00.MODELSTAT EQ 1,
  LOOP((B,L),
    ITOTBEST.L(B,L) = ITOT.L(B,L);
  );
);

*=================== TO FIX BINARY
VARIABLES TO PARAMETER VALUES
==================

PUT RESCRU;

PUT //
PUT / "------------------------------
------"
PUT / "BINARY VARIABLES OBTAINED FROM
MIP-LB"
PUT / "------------------------------
------"

PUT //

LOOP( (A,B)$A1(A,B),
 LOOP( L,
  WBUB1(A,B,L) = WB1.L(A,B,L)
  PUT "WB1('
"A.TL:3,",",B.TL:3,",",L.TL:3," ')=
",WBUB1(A,B,L):18:12;
  PUT ";   "
 );
 PUT /
```

```
);

PUT /

LOOP( (B,H)$H1(B,H),
 LOOP( L,
  WBUB2(B,H,L) = WB2.L(B,H,L)
  PUT "WB2('
"B.TL:3,",",H.TL:3,",",L.TL:3," ')=
",WBUB2(B,H,L):18:12;
  PUT ";   "
 );
 PUT /
);

LOOP( (A,B,L)$A1(A,B),
 WB1.FX(A,B,L) = WB1.L(A,B,L);
);

LOOP( (B,H,L)$H1(B,H),
 WB2.FX(B,H,L) = WB2.L(B,H,L);
);

*------------------------------
Solution to NLP non convex model (UB)

SOLVE MF00 MINIMIZING ZZFIXBIN USING
NLP;

*---------------------------------
creating file with results

PUT RESCRU;

PUT ///   "---------------------------
-----------------------------------";
PUT / "NEXT THE SOLUTION OF THE
NONCONVEX PROBLEM  (UB)"
PUT / "-------------------------------
-------------------------------";

PUT //
PUT$(MF00.modelstat eq 2 and
ZZFIXBIN.l lt ZZUB.l)
    "**********This solution improves
current upper bound value to ";
ZZUB.fx$(MF00.modelstat eq 2 and
ZZFIXBIN.l lt ZZUB.l)=ZZFIXBIN.l;
PUT // "             ZZUB =
",ZZUB.l:20:12;

PUT //
LOOP( (B,L),
 PUT /"ITOT.l('
"B.TL:3,",",L.TL:3,"')=
",ITOT.l(B,L):18:12";";
);

PUT  / "ZZFIXBIN.l=" ZZFIXBIN.l:20:12
";";

PUT /// "Nonconvex NLP solution time :
" MF00.resusd:10:5;
PUT / "Model Status: "
PUT$(MF00.modelstat eq 1) "Optimal.";
PUT$(MF00.modelstat eq 2) "Locally
optimal.";
PUT$(MF00.modelstat eq 3)
"Unbounded.";
PUT$(MF00.modelstat eq 4)
"Infeasible.";
PUT$(MF00.modelstat eq 5) "Locally
infeasible.";
PUT$(MF00.modelstat eq 6)
"Intermediate infeasible.";
PUT$(MF00.modelstat eq 7)
"Intermediate nonoptimal.";
PUT$(MF00.modelstat eq 8) "Integer
solution.";
PUT$(MF00.modelstat eq 9)
"Intermediate non-integer.";
PUT$(MF00.modelstat eq 10) "Integer
infeasible.";
PUT$(MF00.modelstat eq 12) "Error
unknown.";
PUT$(MF00.modelstat eq 13) "Error no
solution.";

*NEXT USED LEVELS ARE REPORTED


PUT //
LOOP( (B,L),
 PUT / "ITOT.LO('
"B.TL:3,",",L.TL:3,"')=
",ITOT.LO(B,L):18:12";"
 "      ITOT.UP('
"B.TL:3,",",L.TL:3,"')=
",ITOT.UP(B,L):18:12";";
);

 PUT //   "---------------------------
-----------------------------------";
 PUT ///


IF(MF00.MODELSTAT EQ 2,
  LOOP((B,L),
    ITOTBEST.L(B,L) = ITOT.L(B,L);
  );
);

IF(MF00.MODELSTAT EQ 10,
  LOOP((B,L),
    ITOTBEST.L(B,L) = ITOT.L(B,L);
  );
);


*DEFINE VARIABLES AND PARAMETERS FOR
FEASIBILITY BASED TESTS


*----------------SE ESPECIFICA ARCHIVO
PARA RESULTADOS (SPECIFIC FILES FOR
RESULTS)

PUT RESCRU;

*--------ESPECIFICA EL VALOR DE LA
MEJOR COTA SUPERIOR DISPONIBLE
(SPECIFIC THE VALUE OF THE BEST LEVEL
SUPERIOR AVAILABLE)

LBEST.L=-INF;


*------------------------NUMERO MAXIMO
DE SUBPROBLEMAS A RESOLVER (NUMBER OF
MAXIMUM SUBPROBLEMS TO SOLVE)
SET ITERATIONS /1*500/;

*-------------------------------
INICIALIZA LAS MAXIMAS DISTANCIAS
(INITIALIZES PRINCIPLE DISTANCES)
MAXDIS.L=100000.0;
MAXPCDIS.L=100000.0;

*--------------------------INICIALIZA
LAS REDUCCIONES ALCANZADAS
(INITIALIZES THE REACHED REDUCTIONS)
```

```
ABSGAP.L=100.0;
RELGAP.L=100.0;

AUXABS.L=100.0;
AUXREL.L=100.0;

*---------------INDICA QUE EL PROBLEMA
ES FACTIBLE Y TIENE OPTIMO (INDICATES
PROBLEM IS FEASIBLE AND HAS OPTIMUM)
INFACTIBLE.L=0;
ACOTAVARS.MODELSTAT=1;

*------------------------------
INICIALIZA TIEMPO TOTAL DE COMPUTO
(INITIALIZES TOTAL TIME OF
CALCULATIONS)
TIEMPOTOT.L=0.00000000000;

*--------INICIALIZA CONTADOR DEL
NUMERO DE SUBPROBLEMAS RESUELTOS
(INITIALIZES ACCOUNTANT FOR NUMBER OF
RESOLUTE SUBPROBS)
CONTADOR.L=0;


FLAGLOITOT(B,L)=1;
FLAGUPITOT(B,L)=1;

LOOP(ITERATIONS $(CARD(SBLUEITOT) NE 0
AND ABSGAP.L GT EPSILONABS AND
RELGAP.L GT EPSILONABS),

 PUT ///
 PUT /   "----------------------------
----------------------------------";
 PUT /   "REDUCTION. IT INITIATES
GREATER ITERATION NUMBERS:
"ORD(ITERATIONS):5:0
 PUT /   "----------------------------
----------------------------------";


 ITOT.FX(B,L)$( (ITOT.UP(B,L)-
ITOT.LO(B,L)) LT
EPSILONABS)=ITOTBEST.L(B,L);
 SGREENITOT(B,L)$(ITOTBEST.L(B,L)-
ITOT.LO(B,L) LT EPSILONABS)=YES;
 SBLACKITOT(B,L)$(ITOT.UP(B,L)-
ITOTBEST.L(B,L) LT EPSILONABS)=YES;
 ITOT.FX(B,L)$(SGREENITOT(B,L) AND
SBLACKITOT(B,L))=ITOTBEST.L(B,L);

 LOOP( (B,L),

FLAGLOITOT(B,L)$(SGREENITOT(B,L))=0;

FLAGUPITOT(B,L)$(SBLACKITOT(B,L))=0;

   IF(FLAGLOITOT(B,L) EQ 0 AND
FLAGUPITOT(B,L) EQ 0,
   SBLUEITOT(B,L)=NO;
   SREDITOT(B,L)=YES;
     IF(SGREENITOT(B,L) AND
SBLACKITOT(B,L),
      SREDITOT(B,L)=NO;
     );
   );
 );


AUXREL.L=100.0;

*--------------------------

 PUT /
```

```
 PUT /"NUMBER OF ELEMENTS IN SBLUE:
"CARD(SBLUEITOT):5:0 /;
 LOOP( (B,L) $(SBLUEITOT(B,L)),
   PUT / B.TL:10,L.TL:10
 );

 PUT /
 PUT /"NUMBER OF ELEMENTS IN SRED:
"CARD(SREDITOT):5:0 /;
 LOOP( (B,L) $(SREDITOT(B,L)),
   PUT / B.TL:10,L.TL:10
 );

 PUT /
 PUT /"NUMBER OF ELEMENTS IN SGREEN:
"CARD(SGREENITOT):5:0 /;
 LOOP( (B,L) $(SGREENITOT(B,L) ),
   PUT / B.TL:10,L.TL:10
 );

 PUT /
 PUT /"NUMBER OF ELEMENTS IN SBLACK:
"CARD(SBLACKITOT):5:0;
 LOOP( (B,L) $(SBLACKITOT(B,L) ),
   PUT / B.TL:10,L.TL:10
 );

 PUT /;

*--------------ALMACENA COTAS ANTES DE
REDUCIR (STORES LEVELS BEFORE
REDUCING)
 LOOP( (B,L),
   ITOTOLD.LO(B,L)=ITOT.LO(B,L);
ITOTOLD.UP(B,L)=ITOT.UP(B,L);
 );

*------------CALCULA DISTANCIA DESDE
LAS COTAS HACIA LA MEJOR SOLUCION
(CALCULATE DISTANCE FROM LEVELS
TOWARDS BEST SOLUTION)

 LOOP( (B,L),
   DIFABSITOT(B,L)=ITOT.UP(B,L)-
ITOT.LO(B,L);
   DIFABSITOTUP(B,L)=ITOT.UP(B,L)-
ITOTBEST.L(B,L);
   DIFABSITOTLO(B,L)=ITOTBEST.L(B,L)-
ITOT.LO(B,L);

   DIFRELITOT(B,L)=ZERO;
   DIFRELITOTUP(B,L)=ZERO;
   DIFRELITOTLO(B,L)=ZERO;
   DIFRELITOT(B,L)$(DELINIITOT(B,L) NE
0)=100*DIFABSITOT(B,L)/DELINIITOT(B,L)
;
   DIFRELITOTUP(B,L)$(DELINIITOT(B,L)
NE
0)=100*DIFABSITOTUP(B,L)/DELINIITOT(B,
L);
   DIFRELITOTLO(B,L)$(DELINIITOT(B,L)
NE
0)=100*DIFABSITOTLO(B,L)/DELINIITOT(B,
L);
 );

 MAXABSDIS.L=0.0;
 LOOP(SBLUEITOT,

MAXABSDIS.L=MAX[MAXABSDIS.L,DIFABSITOT
LO(SBLUEITOT),DIFABSITOTUP(SBLUEITOT)]
 );

 MAXPCDIS.L=0.0;
 LOOP(SBLUEITOT,
```

```
   MAXPCDIS.L$(FLAGLOITOT(SBLUEITOT)
EQ
1)=MAX[MAXPCDIS.L,DIFRELITOTLO(SBLUEIT
OT)];
   MAXPCDIS.L$(FLAGUPITOT(SBLUEITOT)
EQ
1)=MAX[MAXPCDIS.L,DIFRELITOTUP(SBLUEIT
OT)];
 );

*----

PUT //
LOOP( (N,T),
PUT /"ITOT.LO('"N.TL:3,",",T.TL:3,"')=
",ITOT.LO(N,T):18:12";"
 "    ITOT.UP('"N.TL:3,",",T.TL:3,"')=
",ITOT.UP(N,T):18:12";";
);

PUT //
LOOP(SBLUEITOT(B,L),
PUT
/"DIFABSITOTLO('"B.TL:3,",",L.TL:3,"')
= ",DIFABSITOTLO(SBLUEITOT):18:12";"
 "
DIFABSITOTUP('"B.TL:3,",",L.TL:3,"')=
",DIFABSITOTUP(SBLUEITOT):18:12";" /
 "
DIFABSITOT('"B.TL:3,",",L.TL:3,"')=
",DIFABSITOT(SBLUEITOT):18:12";";
);
PUT /   "DISTANCE ABSOLUTE
PRINCIPLE:"MAXABSDIS.L:18:12;

PUT //
LOOP(SBLUEITOT(B,L),
PUT
/"DIFRELITOTLO('"B.TL:3,",",L.TL:3,"')
= ",DIFRELITOTLO(SBLUEITOT):18:12";"
 "
DIFRELITOTUP('"B.TL:3,",",L.TL:3,"')=
",DIFRELITOTUP(SBLUEITOT):18:12";" /
 "
DIFRELITOT('"B.TL:3,",",L.TL:3,"')=
",DIFRELITOT(SBLUEITOT):18:12";";
);
PUT /   "DISTANCE PERCENTAGE
PRINCIPLE:"MAXPCDIS.L:18:12;
*---------------

SOLOUNO=0;
LOOP( (N,T) $(SBLUEITOT(N,T) AND
MAXPCDIS.L GT EPSILONREL AND SOLOUNO
EQ 0 AND
              (MAXPCDIS.L EQ
DIFRELITOTLO(N,T) OR MAXPCDIS.L EQ
DIFRELITOTUP(N,T)) ),
  CONTADOR.L=CONTADOR.L+1;
  PUT //   "SUBPROBLEM
NUMBER:"CONTADOR.L:5:0;
  PUT / "LEVELS OF ITOT ARE
UPDATED("N.TL:3,",",T.TL:3,")"
        " FROM:" ITOT.LO(N,T):18:12"--
---"ITOT.UP(N,T):18:12;


*============ STORING TANKS
======================================


*---------------ST1-------------------
----
IF(SOLOUNO EQ 0 AND
SBLUEITOT('ST1','L0') AND MAXPCDIS.L
EQ DIFRELITOT('ST1','L0'),
  SOLVE ACOTAVARS MINIMIZING
FREEITOTST1L0 USING RMIP;
  AUXABS.L=FREEITOTST1L0.L-
ITOT.LO(N,T);

ITOT.LO(N,T)=MAX[ITOT.LO(N,T),FREEITOT
ST1L0.L];

  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) NE
0)=100*AUXABS.L/(ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T));
  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) EQ 0)=0.00000000000;
  FLAGLOITOT(N,T)$(AUXREL.L LT
MINREDREL)=0;
  SOLOUNO=1;
);

IF(SOLOUNO EQ 0 AND
SBLUEITOT('ST1','L0') AND MAXPCDIS.L
EQ DIFRELITOTUP('ST1','L0'),
  SOLVE ACOTAVARS MAXIMIZING
FREEITOTST1L0 USING RMIP;
  AUXABS.L=ITOT.UP(N,T)-
FREEITOTST1L0.L;

ITOT.UP(N,T)=MIN[ITOT.UP(N,T),FREEITOT
ST1L0.L];

  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) NE
0)=100*AUXABS.L/(ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T));
  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) EQ 0)=0.00000000000;
  FLAGUPITOT(N,T)$(AUXREL.L LT
MINREDREL)=0;
  SOLOUNO=1;
);
*---------------

IF(SOLOUNO EQ 0 AND
SBLUEITOT('ST1','L1') AND MAXPCDIS.L
EQ DIFRELITOT('ST1','L1'),
  SOLVE ACOTAVARS MINIMIZING
FREEITOTST1L1 USING RMIP;
  AUXABS.L=FREEITOTST1L1.L-
ITOT.LO(N,T);

ITOT.LO(N,T)=MAX[ITOT.LO(N,T),FREEITOT
ST1L1.L];

  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) NE
0)=100*AUXABS.L/(ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T));
  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) EQ 0)=0.00000000000;
  FLAGLOITOT(N,T)$(AUXREL.L LT
MINREDREL)=0;
  SOLOUNO=1;
);

IF(SOLOUNO EQ 0 AND
SBLUEITOT('ST1','L1') AND MAXPCDIS.L
EQ DIFRELITOTUP('ST1','L1'),
  SOLVE ACOTAVARS MAXIMIZING
FREEITOTST1L1 USING RMIP;
  AUXABS.L=ITOT.UP(N,T)-
FREEITOTST1L1.L;
```

```
ITOT.UP(N,T)=MIN[ITOT.UP(N,T),FREEITOT
ST1L1.L];

  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) NE
0)=100*AUXABS.L/(ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T));
  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) EQ 0)=0.00000000000;
  FLAGUPITOT(N,T)$(AUXREL.L LT
MINREDREL)=0;
  SOLOUNO=1;
);
*---------------

IF(SOLOUNO EQ 0 AND
SBLUEITOT('ST1','L2') AND MAXPCDIS.L
EQ DIFRELITOT('ST1','L2'),
  SOLVE ACOTAVARS MINIMIZING
FREEITOTST1L2 USING RMIP;
  AUXABS.L=FREEITOTST1L2.L-
ITOT.LO(N,T);

ITOT.LO(N,T)=MAX[ITOT.LO(N,T),FREEITOT
ST1L2.L];

  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) NE
0)=100*AUXABS.L/(ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T));
  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) EQ 0)=0.00000000000;
  FLAGLOITOT(N,T)$(AUXREL.L LT
MINREDREL)=0;
  SOLOUNO=1;
);

IF(SOLOUNO EQ 0 AND
SBLUEITOT('ST1','L2') AND MAXPCDIS.L
EQ DIFRELITOTUP('ST1','L2'),
  SOLVE ACOTAVARS MAXIMIZING
FREEITOTST1L2 USING RMIP;
  AUXABS.L=ITOT.UP(N,T)-
FREEITOTST1L2.L;

ITOT.UP(N,T)=MIN[ITOT.UP(N,T),FREEITOT
ST1L2.L];

  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) NE
0)=100*AUXABS.L/(ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T));
  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) EQ 0)=0.00000000000;
  FLAGUPITOT(N,T)$(AUXREL.L LT
MINREDREL)=0;
  SOLOUNO=1;
);
*---------------

*---------------ST2-------------------
----

IF(SOLOUNO EQ 0 AND
SBLUEITOT('ST2','L0') AND MAXPCDIS.L
EQ DIFRELITOT('ST2','L0'),
  SOLVE ACOTAVARS MINIMIZING
FREEITOTST2L0 USING RMIP;
  AUXABS.L=FREEITOTST2L0.L-
ITOT.LO(N,T);

ITOT.LO(N,T)=MAX[ITOT.LO(N,T),FREEITOT
ST2L0.L];

  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) NE
0)=100*AUXABS.L/(ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T));
  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) EQ 0)=0.00000000000;
  FLAGLOITOT(N,T)$(AUXREL.L LT
MINREDREL)=0;
  SOLOUNO=1;
);

IF(SOLOUNO EQ 0 AND
SBLUEITOT('ST2','L0') AND MAXPCDIS.L
EQ DIFRELITOTUP('ST2','L0'),
  SOLVE ACOTAVARS MAXIMIZING
FREEITOTST2L0 USING RMIP;
  AUXABS.L=ITOT.UP(N,T)-
FREEITOTST2L0.L;

ITOT.UP(N,T)=MIN[ITOT.UP(N,T),FREEITOT
ST2L0.L];

  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) NE
0)=100*AUXABS.L/(ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T));
  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) EQ 0)=0.00000000000;
  FLAGUPITOT(N,T)$(AUXREL.L LT
MINREDREL)=0;
  SOLOUNO=1;
);
*---------------

IF(SOLOUNO EQ 0 AND
SBLUEITOT('ST2','L1') AND MAXPCDIS.L
EQ DIFRELITOT('ST2','L1'),
  SOLVE ACOTAVARS MINIMIZING
FREEITOTST2L1 USING RMIP;
  AUXABS.L=FREEITOTST2L1.L-
ITOT.LO(N,T);

ITOT.LO(N,T)=MAX[ITOT.LO(N,T),FREEITOT
ST2L1.L];

  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) NE
0)=100*AUXABS.L/(ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T));
  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) EQ 0)=0.00000000000;
  FLAGLOITOT(N,T)$(AUXREL.L LT
MINREDREL)=0;
  SOLOUNO=1;
);

IF(SOLOUNO EQ 0 AND
SBLUEITOT('ST2','L1') AND MAXPCDIS.L
EQ DIFRELITOTUP('ST2','L1'),
  SOLVE ACOTAVARS MAXIMIZING
FREEITOTST2L1 USING RMIP;
  AUXABS.L=ITOT.UP(N,T)-
FREEITOTST2L1.L;

ITOT.UP(N,T)=MIN[ITOT.UP(N,T),FREEITOT
ST2L1.L];

  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) NE
0)=100*AUXABS.L/(ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T));
  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) EQ 0)=0.00000000000;
  FLAGUPITOT(N,T)$(AUXREL.L LT
MINREDREL)=0;
  SOLOUNO=1;
```

```
);
*---------------

IF(SOLOUNO EQ 0 AND
SBLUEITOT('ST2','L2') AND MAXPCDIS.L
EQ DIFRELITOT('ST2','L2'),
  SOLVE ACOTAVARS MINIMIZING
FREEITOTST2L2 USING RMIP;
  AUXABS.L=FREEITOTST2L2.L-
ITOT.LO(N,T);

ITOT.LO(N,T)=MAX[ITOT.LO(N,T),FREEITOT
ST2L2.L];

  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) NE
0)=100*AUXABS.L/(ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T));
  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) EQ 0)=0.00000000000;
  FLAGLOITOT(N,T)$(AUXREL.L LT
MINREDREL)=0;
  SOLOUNO=1;
);

IF(SOLOUNO EQ 0 AND
SBLUEITOT('ST2','L2') AND MAXPCDIS.L
EQ DIFRELITOTUP('ST2','L2'),
  SOLVE ACOTAVARS MAXIMIZING
FREEITOTST2L2 USING RMIP;
  AUXABS.L=ITOT.UP(N,T)-
FREEITOTST2L2.L;

ITOT.UP(N,T)=MIN[ITOT.UP(N,T),FREEITOT
ST2L2.L];

  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) NE
0)=100*AUXABS.L/(ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T));
  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) EQ 0)=0.00000000000;
  FLAGUPITOT(N,T)$(AUXREL.L LT
MINREDREL)=0;
  SOLOUNO=1;
);
*---------------

*---------------ST3-------------------
----

IF(SOLOUNO EQ 0 AND
SBLUEITOT('ST3','L0') AND MAXPCDIS.L
EQ DIFRELITOT('ST3','L0'),
  SOLVE ACOTAVARS MINIMIZING
FREEITOTST3L0 USING RMIP;
  AUXABS.L=FREEITOTST3L0.L-
ITOT.LO(N,T);

ITOT.LO(N,T)=MAX[ITOT.LO(N,T),FREEITOT
ST3L0.L];

  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) NE
0)=100*AUXABS.L/(ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T));
  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) EQ 0)=0.00000000000;
  FLAGLOITOT(N,T)$(AUXREL.L LT
MINREDREL)=0;
  SOLOUNO=1;
);

IF(SOLOUNO EQ 0 AND
SBLUEITOT('ST3','L0') AND MAXPCDIS.L
EQ DIFRELITOTUP('ST3','L0'),
  SOLVE ACOTAVARS MAXIMIZING
FREEITOTST3L0 USING RMIP;
  AUXABS.L=ITOT.UP(N,T)-
FREEITOTST3L0.L;

ITOT.UP(N,T)=MIN[ITOT.UP(N,T),FREEITOT
ST3L0.L];

  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) NE
0)=100*AUXABS.L/(ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T));
  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) EQ 0)=0.00000000000;
  FLAGUPITOT(N,T)$(AUXREL.L LT
MINREDREL)=0;
  SOLOUNO=1;
);
*---------------

IF(SOLOUNO EQ 0 AND
SBLUEITOT('ST3','L1') AND MAXPCDIS.L
EQ DIFRELITOT('ST3','L1'),
  SOLVE ACOTAVARS MINIMIZING
FREEITOTST3L1 USING RMIP;
  AUXABS.L=FREEITOTST3L1.L-
ITOT.LO(N,T);

ITOT.LO(N,T)=MAX[ITOT.LO(N,T),FREEITOT
ST3L1.L];

  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) NE
0)=100*AUXABS.L/(ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T));
  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) EQ 0)=0.00000000000;
  FLAGLOITOT(N,T)$(AUXREL.L LT
MINREDREL)=0;
  SOLOUNO=1;
);

IF(SOLOUNO EQ 0 AND
SBLUEITOT('ST3','L1') AND MAXPCDIS.L
EQ DIFRELITOTUP('ST3','L1'),
  SOLVE ACOTAVARS MAXIMIZING
FREEITOTST3L1 USING RMIP;
  AUXABS.L=ITOT.UP(N,T)-
FREEITOTST3L1.L;

ITOT.UP(N,T)=MIN[ITOT.UP(N,T),FREEITOT
ST3L1.L];

  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) NE
0)=100*AUXABS.L/(ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T));
  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) EQ 0)=0.00000000000;
  FLAGUPITOT(N,T)$(AUXREL.L LT
MINREDREL)=0;
  SOLOUNO=1;
);
*---------------

IF(SOLOUNO EQ 0 AND
SBLUEITOT('ST3','L2') AND MAXPCDIS.L
EQ DIFRELITOT('ST3','L2'),
  SOLVE ACOTAVARS MINIMIZING
FREEITOTST3L2 USING RMIP;
  AUXABS.L=FREEITOTST3L2.L-
ITOT.LO(N,T);

ITOT.LO(N,T)=MAX[ITOT.LO(N,T),FREEITOT
ST3L2.L];
```

```
  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) NE
0)=100*AUXABS.L/(ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T));
  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) EQ 0)=0.00000000000;
  FLAGLOITOT(N,T)$(AUXREL.L LT
MINREDREL)=0;
  SOLOUNO=1;
);

IF(SOLOUNO EQ 0 AND
SBLUEITOT('ST3','L2') AND MAXPCDIS.L
EQ DIFRELITOTUP('ST3','L2'),
  SOLVE ACOTAVARS MAXIMIZING
FREEITOTST3L2 USING RMIP;
  AUXABS.L=ITOT.UP(N,T)-
FREEITOTST3L2.L;

ITOT.UP(N,T)=MIN[ITOT.UP(N,T),FREEITOT
ST3L2.L];

  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) NE
0)=100*AUXABS.L/(ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T));
  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) EQ 0)=0.00000000000;
  FLAGUPITOT(N,T)$(AUXREL.L LT
MINREDREL)=0;
  SOLOUNO=1;
);
*---------------

*============ CHARGING TANKS
=====================================

*---------------CT1-------------------
----

IF(SOLOUNO EQ 0 AND
SBLUEITOT('CT1','L0') AND MAXPCDIS.L
EQ DIFRELITOT('CT1','L0'),
  SOLVE ACOTAVARS MINIMIZING
FREEITOTCT1L0 USING RMIP;
  AUXABS.L=FREEITOTCT1L0.L-
ITOT.LO(N,T);

ITOT.LO(N,T)=MAX[ITOT.LO(N,T),FREEITOT
CT1L0.L];

  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) NE
0)=100*AUXABS.L/(ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T));
  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) EQ 0)=0.00000000000;
  FLAGLOITOT(N,T)$(AUXREL.L LT
MINREDREL)=0;
  SOLOUNO=1;
);

IF(SOLOUNO EQ 0 AND
SBLUEITOT('CT1','L0') AND MAXPCDIS.L
EQ DIFRELITOTUP('CT1','L0'),
  SOLVE ACOTAVARS MAXIMIZING
FREEITOTCT1L0 USING RMIP;
  AUXABS.L=ITOT.UP(N,T)-
FREEITOTCT1L0.L;

ITOT.UP(N,T)=MIN[ITOT.UP(N,T),FREEITOT
CT1L0.L];

  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) NE
0)=100*AUXABS.L/(ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T));
  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) EQ 0)=0.00000000000;
  FLAGUPITOT(N,T)$(AUXREL.L LT
MINREDREL)=0;
  SOLOUNO=1;
);
*---------------

IF(SOLOUNO EQ 0 AND
SBLUEITOT('CT1','L1') AND MAXPCDIS.L
EQ DIFRELITOT('CT1','L1'),
  SOLVE ACOTAVARS MINIMIZING
FREEITOTCT1L1 USING RMIP;
  AUXABS.L=FREEITOTCT1L1.L-
ITOT.LO(N,T);

ITOT.LO(N,T)=MAX[ITOT.LO(N,T),FREEITOT
CT1L1.L];

  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) NE
0)=100*AUXABS.L/(ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T));
  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) EQ 0)=0.00000000000;
  FLAGLOITOT(N,T)$(AUXREL.L LT
MINREDREL)=0;
  SOLOUNO=1;
);

IF(SOLOUNO EQ 0 AND
SBLUEITOT('CT1','L1') AND MAXPCDIS.L
EQ DIFRELITOTUP('CT1','L1'),
  SOLVE ACOTAVARS MAXIMIZING
FREEITOTCT1L1 USING RMIP;
  AUXABS.L=ITOT.UP(N,T)-
FREEITOTCT1L1.L;

ITOT.UP(N,T)=MIN[ITOT.UP(N,T),FREEITOT
CT1L1.L];

  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) NE
0)=100*AUXABS.L/(ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T));
  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) EQ 0)=0.00000000000;
  FLAGUPITOT(N,T)$(AUXREL.L LT
MINREDREL)=0;
  SOLOUNO=1;
);
*---------------
*---------------

IF(SOLOUNO EQ 0 AND
SBLUEITOT('CT1','L2') AND MAXPCDIS.L
EQ DIFRELITOT('CT1','L2'),
  SOLVE ACOTAVARS MINIMIZING
FREEITOTCT1L2 USING RMIP;
  AUXABS.L=FREEITOTCT1L2.L-
ITOT.LO(N,T);

ITOT.LO(N,T)=MAX[ITOT.LO(N,T),FREEITOT
CT1L2.L];

  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) NE
0)=100*AUXABS.L/(ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T));
  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) EQ 0)=0.00000000000;
  FLAGLOITOT(N,T)$(AUXREL.L LT
MINREDREL)=0;
  SOLOUNO=1;
```

```
);

IF(SOLOUNO EQ 0 AND
SBLUEITOT('CT1','L2') AND MAXPCDIS.L
EQ DIFRELITOTUP('CT1','L2'),
  SOLVE ACOTAVARS MAXIMIZING
FREEITOTCT1L2 USING RMIP;
  AUXABS.L=ITOT.UP(N,T)-
FREEITOTCT1L2.L;

ITOT.UP(N,T)=MIN[ITOT.UP(N,T),FREEITOT
CT1L2.L];

  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) NE
0)=100*AUXABS.L/(ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T));
  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) EQ 0)=0.00000000000;
  FLAGUPITOT(N,T)$(AUXREL.L LT
MINREDREL)=0;
  SOLOUNO=1;
);
*---------------

*---------------CT2-------------------
----

IF(SOLOUNO EQ 0 AND
SBLUEITOT('CT2','L0') AND MAXPCDIS.L
EQ DIFRELITOT('CT2','L0'),
  SOLVE ACOTAVARS MINIMIZING
FREEITOTCT2L0 USING RMIP;
  AUXABS.L=FREEITOTCT2L0.L-
ITOT.LO(N,T);

ITOT.LO(N,T)=MAX[ITOT.LO(N,T),FREEITOT
CT2L0.L];

  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) NE
0)=100*AUXABS.L/(ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T));
  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) EQ 0)=0.00000000000;
  FLAGLOITOT(N,T)$(AUXREL.L LT
MINREDREL)=0;
  SOLOUNO=1;
);

IF(SOLOUNO EQ 0 AND
SBLUEITOT('CT2','L0') AND MAXPCDIS.L
EQ DIFRELITOTUP('CT2','L0'),
  SOLVE ACOTAVARS MAXIMIZING
FREEITOTCT2L0 USING RMIP;
  AUXABS.L=ITOT.UP(N,T)-
FREEITOTCT2L0.L;

ITOT.UP(N,T)=MIN[ITOT.UP(N,T),FREEITOT
CT2L0.L];

  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) NE
0)=100*AUXABS.L/(ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T));
  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) EQ 0)=0.00000000000;
  FLAGUPITOT(N,T)$(AUXREL.L LT
MINREDREL)=0;
  SOLOUNO=1;
);
*---------------

*---------------
IF(SOLOUNO EQ 0 AND
SBLUEITOT('CT2','L1') AND MAXPCDIS.L
EQ DIFRELITOT('CT2','L1'),
  SOLVE ACOTAVARS MINIMIZING
FREEITOTCT2L1 USING RMIP;
  AUXABS.L=FREEITOTCT2L1.L-
ITOT.LO(N,T);

ITOT.LO(N,T)=MAX[ITOT.LO(N,T),FREEITOT
CT2L1.L];

  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) NE
0)=100*AUXABS.L/(ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T));
  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) EQ 0)=0.00000000000;
  FLAGLOITOT(N,T)$(AUXREL.L LT
MINREDREL)=0;
  SOLOUNO=1;
);

IF(SOLOUNO EQ 0 AND
SBLUEITOT('CT2','L1') AND MAXPCDIS.L
EQ DIFRELITOTUP('CT2','L1'),
  SOLVE ACOTAVARS MAXIMIZING
FREEITOTCT2L1 USING RMIP;
  AUXABS.L=ITOT.UP(N,T)-
FREEITOTCT2L1.L;

ITOT.UP(N,T)=MIN[ITOT.UP(N,T),FREEITOT
CT2L1.L];

  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) NE
0)=100*AUXABS.L/(ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T));
  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) EQ 0)=0.00000000000;
  FLAGUPITOT(N,T)$(AUXREL.L LT
MINREDREL)=0;
  SOLOUNO=1;
);
*---------------
*---------------

IF(SOLOUNO EQ 0 AND
SBLUEITOT('CT2','L2') AND MAXPCDIS.L
EQ DIFRELITOT('CT2','L2'),
  SOLVE ACOTAVARS MINIMIZING
FREEITOTCT2L2 USING RMIP;
  AUXABS.L=FREEITOTCT2L2.L-
ITOT.LO(N,T);

ITOT.LO(N,T)=MAX[ITOT.LO(N,T),FREEITOT
CT2L2.L];

  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) NE
0)=100*AUXABS.L/(ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T));
  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) EQ 0)=0.00000000000;
  FLAGLOITOT(N,T)$(AUXREL.L LT
MINREDREL)=0;
  SOLOUNO=1;
);

IF(SOLOUNO EQ 0 AND
SBLUEITOT('CT2','L2') AND MAXPCDIS.L
EQ DIFRELITOTUP('CT2','L2'),
  SOLVE ACOTAVARS MAXIMIZING
FREEITOTCT2L2 USING RMIP;
  AUXABS.L=ITOT.UP(N,T)-
FREEITOTCT2L2.L;
```

```
ITOT.UP(N,T)=MIN[ITOT.UP(N,T),FREEITOT
CT2L2.L];

  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) NE
0)=100*AUXABS.L/(ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T));
  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) EQ 0)=0.00000000000;
  FLAGUPITOT(N,T)$(AUXREL.L LT
MINREDREL)=0;
  SOLOUNO=1;
);
*---------------



*---------------CT3-------------------
----

IF(SOLOUNO EQ 0 AND
SBLUEITOT('CT3','L0') AND MAXPCDIS.L
EQ DIFRELITOT('CT3','L0'),
  SOLVE ACOTAVARS MINIMIZING
FREEITOTCT3L0 USING RMIP;
  AUXABS.L=FREEITOTCT3L0.L-
ITOT.LO(N,T);

ITOT.LO(N,T)=MAX[ITOT.LO(N,T),FREEITOT
CT3L0.L];

  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) NE
0)=100*AUXABS.L/(ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T));
  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) EQ 0)=0.00000000000;
  FLAGLOITOT(N,T)$(AUXREL.L LT
MINREDREL)=0;
  SOLOUNO=1;
);

IF(SOLOUNO EQ 0 AND
SBLUEITOT('CT3','L0') AND MAXPCDIS.L
EQ DIFRELITOTUP('CT3','L0'),
  SOLVE ACOTAVARS MAXIMIZING
FREEITOTCT3L0 USING RMIP;
  AUXABS.L=ITOT.UP(N,T)-
FREEITOTCT3L0.L;

ITOT.UP(N,T)=MIN[ITOT.UP(N,T),FREEITOT
CT3L0.L];

  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) NE
0)=100*AUXABS.L/(ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T));
  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) EQ 0)=0.00000000000;
  FLAGUPITOT(N,T)$(AUXREL.L LT
MINREDREL)=0;
  SOLOUNO=1;
);
*---------------

*---------------

IF(SOLOUNO EQ 0 AND
SBLUEITOT('CT3','L1') AND MAXPCDIS.L
EQ DIFRELITOT('CT3','L1'),
  SOLVE ACOTAVARS MINIMIZING
FREEITOTCT3L1 USING RMIP;
  AUXABS.L=FREEITOTCT3L1.L-
ITOT.LO(N,T);

ITOT.LO(N,T)=MAX[ITOT.LO(N,T),FREEITOT
CT3L1.L];

  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) NE
0)=100*AUXABS.L/(ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T));
  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) EQ 0)=0.00000000000;
  FLAGLOITOT(N,T)$(AUXREL.L LT
MINREDREL)=0;
  SOLOUNO=1;
);

IF(SOLOUNO EQ 0 AND
SBLUEITOT('CT3','L1') AND MAXPCDIS.L
EQ DIFRELITOTUP('CT3','L1'),
  SOLVE ACOTAVARS MAXIMIZING
FREEITOTCT3L1 USING RMIP;
  AUXABS.L=ITOT.UP(N,T)-
FREEITOTCT3L1.L;

ITOT.UP(N,T)=MIN[ITOT.UP(N,T),FREEITOT
CT3L1.L];

  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) NE
0)=100*AUXABS.L/(ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T));
  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) EQ 0)=0.00000000000;
  FLAGUPITOT(N,T)$(AUXREL.L LT
MINREDREL)=0;
  SOLOUNO=1;
);
*---------------
*---------------

IF(SOLOUNO EQ 0 AND
SBLUEITOT('CT3','L2') AND MAXPCDIS.L
EQ DIFRELITOT('CT3','L2'),
  SOLVE ACOTAVARS MINIMIZING
FREEITOTCT3L2 USING RMIP;
  AUXABS.L=FREEITOTCT3L2.L-
ITOT.LO(N,T);

ITOT.LO(N,T)=MAX[ITOT.LO(N,T),FREEITOT
CT3L2.L];

  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) NE
0)=100*AUXABS.L/(ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T));
  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) EQ 0)=0.00000000000;
  FLAGLOITOT(N,T)$(AUXREL.L LT
MINREDREL)=0;
  SOLOUNO=1;
);

IF(SOLOUNO EQ 0 AND
SBLUEITOT('CT3','L2') AND MAXPCDIS.L
EQ DIFRELITOTUP('CT3','L2'),
  SOLVE ACOTAVARS MAXIMIZING
FREEITOTCT3L2 USING RMIP;
  AUXABS.L=ITOT.UP(N,T)-
FREEITOTCT3L2.L;

ITOT.UP(N,T)=MIN[ITOT.UP(N,T),FREEITOT
CT3L2.L];

  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) NE
0)=100*AUXABS.L/(ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T));
```

```
  AUXREL.L$( (ITOTOLD.UP(N,T)-
ITOTOLD.LO(N,T)) EQ 0)=0.00000000000;
  FLAGUPITOT(N,T)$(AUXREL.L LT
MINREDREL)=0;
  SOLOUNO=1;
);
*---------------

  PUT / "
HASTA:"
        ITOT.LO(N,T):18:12"-----
"ITOT.UP(N,T):18:12;
  PUT // "                  RED ABS:"
AUXABS.L:18:12;
  PUT / "               -----RED %:"
AUXREL.L:18:12;
  PUT / "             MODEL STATUS: "
  PUT$(ACOTAVARS.MODELSTAT EQ 1) "
OPTIMAL";

TIEMPOTOT.L=TIEMPOTOT.L+ACOTAVARS.RESU
SD;
  PUT / "       SUBPROBLEM SOL. TIME:
" ACOTAVARS.RESUSD:10:5;
  PUT / "      ACCUMULATED SOL. TIME:
" TIEMPOTOT.L:10:5;

  IF( ACOTAVARS.MODELSTAT NE 1,
  PUT // "STOP. SUBPROBLEM SOLUCION IS
NOT OPTIMAL.";
  PUT /  "ACOTAVARS MODEL STATUS:
"ACOTAVARS.MODELSTAT;
  ABORT "STOP. SUBPROBLEM SOLUCION IS
NOT OPTIMAL.";  );

  PUT /   "
MULTIPLICADOR=" COTAOF.M:18:12;
  LBEST.L$(COTAOF.M NE
0)=MAX[LBEST.L,ZZUB.L-(ITOT.UP(N,T)-
ITOT.LO(N,T))/ABS(COTAOF.M)];
  PUT /   "
LBEST.L=" LBEST.L:18:12;
  ABSGAP.L=ZZUB.L-LBEST.L;
  RELGAP.L=ABS[100*ABSGAP.L/ZZUB.L];
  PUT /   "     ABSOLUTE GAP (UB-LB) =
" ABSGAP.L:18:12;
  PUT /   "   REL GAP 100*(UB-LB)/UB =
" RELGAP.L:18:12;



PUT //
PUT "INICIA REDUCCION POR
FACTIBILIDAD"

PUT //
PUT "TERMINA REDUCCION POR
FACTIBILIDAD"

);

);

PUT ///
PUT /   "-----------------------------
---------------------------------";
PUT /   "            RESULTADOS
FINALES DE LA REDUCCION"
PUT /   "-----------------------------
---------------------------------";

*------------CALCULA DISTANCIA DESDE
LAS COTAS HACIA LA MEJOR SOLUCION

LOOP( (B,L),
```

```
DIFABSITOT(B,L)=ITOT.UP(B,L)-
ITOT.LO(B,L);
DIFABSITOTUP(B,L)=ITOT.UP(B,L)-
ITOTBEST.L(B,L);
DIFABSITOTLO(B,L)=ITOTBEST.L(B,L)-
ITOT.LO(B,L);

DIFRELITOT(B,L)=ZERO;
DIFRELITOTUP(B,L)=ZERO;
DIFRELITOTLO(B,L)=ZERO;
DIFRELITOT(B,L)$(DELINIITOT(B,L) NE
0)=100*DIFABSITOT(B,L)/DELINIITOT(B,L)
;
DIFRELITOTUP(B,L)$(DELINIITOT(B,L) NE
0)=100*DIFABSITOTUP(B,L)/DELINIITOT(B,
L);
DIFRELITOTLO(B,L)$(DELINIITOT(B,L) NE
0)=100*DIFABSITOTLO(B,L)/DELINIITOT(B,
L);
);

MAXABSDIS.L=0.0;
LOOP( SBLUEITOT(B,L),
MAXABSDIS.L=MAX[MAXABSDIS.L,DIFABSITOT
LO(SBLUEITOT),DIFABSITOTUP(SBLUEITOT)]
);

MAXPCDIS.L=0.0;
LOOP( SBLUEITOT(B,L),
MAXPCDIS.L=MAX[MAXPCDIS.L,DIFRELITOTLO
(SBLUEITOT)];
MAXPCDIS.L=MAX[MAXPCDIS.L,DIFRELITOTUP
(SBLUEITOT)];
);

*------------SE IMPRIMEN RESULTADOS

PUT //
LOOP( SBLUEITOT(B,L),
PUT /"ITOT.LO('"B.TL:3,",",L.TL:3,"')=
",ITOT.LO(SBLUEITOT):18:12";"
 "    ITOT.UP('"B.TL:3,",",L.TL:3,"')=
",ITOT.UP(SBLUEITOT):18:12";";
);

PUT //
LOOP( SBLUEITOT(B,L),
PUT
/"DAITOTLO('"B.TL:3,",",L.TL:3,"')=
",DIFABSITOTLO(SBLUEITOT):18:12";"
 "
DAITOTUP('"B.TL:3,",",L.TL:3,"')=
",DIFABSITOTUP(SBLUEITOT):18:12";" /
 "
DAITOT('"B.TL:3,",",L.TL:3,"')=
",DIFABSITOT(SBLUEITOT):18:12";";
);
PUT /   "DISTANCIA MAXIMA
ABSOLUTA:"MAXABSDIS.L:18:12;

PUT //
LOOP( SBLUEITOT(B,L),
PUT
/"DRITOTLO('"B.TL:3,",",L.TL:3,"')=
",DIFRELITOTLO(SBLUEITOT):18:12";"
 "
DRITOTUP('"B.TL:3,",",L.TL:3,"')=
",DIFRELITOTUP(SBLUEITOT):18:12";" /
 "
DRITOT('"B.TL:3,",",L.TL:3,"')=
",DIFRELITOT(SBLUEITOT):18:12";";
);
PUT /   "DISTANCIA MAXIMA
PORCENTUAL:"MAXPCDIS.L:18:12;
*---------------
```

```
PUT /   "-----------------------------
--------------------------------";
*------------------------------
SoluTion to model MIP


SOLVE MR00 minimizing ZZ using mip;
*---------------------------------
creating file with results

PUT RESCRU;

PUT ///   "---------------------------
-------------------";
PUT /     "NEXT THE SOLUTION OF THE
RELAXED PROBLEM (MIP)"
PUT /     "---------------------------
-------------------";

PUT //
LOOP( (B,L),
 PUT /"ITOT.l('
"B.TL:3,",",L.TL:3,"')=
",ITOT.l(B,L):18:12";";
);

 PUT  / "ZZ.l=" ZZ.l:20:12 ";";


 PUT /// "LP solution time : "
MR00.resusd:10:5;
 PUT / "Model Status: "
 PUT$(MR00.modelstat eq 1) "Optimal.";
 PUT$(MR00.modelstat eq 2) "Locally
optimal.";
 PUT$(MR00.modelstat eq 3)
"Unbounded.";
 PUT$(MR00.modelstat eq 4)
"Infeasible.";
 PUT$(MR00.modelstat eq 5) "Locally
infeasible.";
 PUT$(MR00.modelstat eq 6)
"Intermediate infeasible.";
 PUT$(MR00.modelstat eq 7)
"Intermediate nonoptimal.";
 PUT$(MR00.modelstat eq 8) "Integer
solution.";
 PUT$(MR00.modelstat eq 9)
"Intermediate non-integer.";
 PUT$(MR00.modelstat eq 10) "Integer
infeasible.";
 PUT$(MR00.modelstat eq 12) "Error
unknown.";
 PUT$(MR00.modelstat eq 13) "Error no
solution.";

*NEXT USED LEVELS ARE REPORTED


PUT //
LOOP( (B,L),
 PUT / "ITOT.LO('
"B.TL:3,",",L.TL:3,"')=
",ITOT.LO(B,L):18:12";"
 "      ITOT.UP('
"B.TL:3,",",L.TL:3,"')=
",ITOT.UP(B,L):18:12";";
);

*MULTIPLICATION


PUT //
LOOP( (B,L),
 PUT / "ITOT.M('
"B.TL:3,",",L.TL:3,"')=
",ITOT.M(B,L):18:12";";
);

PUT ///
PUT "MODELSTAT FOR LB:"MR00.MODELSTAT;
PUT / "MODELSTAT FOR
UB:"MF00.MODELSTAT;
```

APPENDIX VII: GAMS OUTPUT FILE ON BRANCH AND CONTRACT ALGORITHM MODEL

```
GAMS Rev 149  x86/MS Windows
04/27/07 19:28:59 Page 7
: SCHEDULING REFINERY CRUDE OIL
OPERATIONS
Model Statistics    SOLVE ACOTAVARS
Using RMIP From line 3295


LOOPS
ITERATIONS   1
                                *
ST1
                                *
L1


MODEL STATISTICS

BLOCKS OF EQUATIONS          77
SINGLE EQUATIONS        2,015
BLOCKS OF VARIABLES          38
SINGLE VARIABLES          836  1
projected
NON ZERO ELEMENTS         5,277
DISCRETE VARIABLES        100


GENERATION TIME      =        0.047
SECONDS      3 Mb  WIN226-149 Dec 19,
2007

**** SOLVE from line 3295 ABORTED,
EXECERROR = 1

**** REPORT FILE SUMMARY

RESCRU C:\Documents and
Settings\Kippi\My
Documents\gamsdir\projdir\cos1.res
RESCRUBND C:\Documents and
Settings\Kippi\My
Documents\gamsdir\projdir\CBOUND.RE
          S


EXECUTION TIME       =        0.093
SECONDS      3 Mb  WIN226-149 Dec 19,
2007


USER: CS/IE 635, Spring 2007
G061206/0001AS-WIN
      Prof. Ferris
DC2937
      License for teaching and
research at degree granting
institutions


**** FILE SUMMARY

Input      C:\Documents and
Settings\Kippi\My
Documents\gamsdir\projdir\COS_ITOT
           (FOR DISSERTATION
APPENDIX).gms
Output     C:\Documents and
Settings\Kippi\My
Documents\gamsdir\projdir\COS_ITOT
           (FOR DISSERTATION
APPENDIX).lst
```

APPENDIX VIII: COMPUTER CODE FOR GAMS INPUT FILE ON CRUDE OIL SCHEDULING WITH SLACK VARIABLES MODEL

```
$TITLE: SCHEDULING REFINERY CRUDE OIL OPERATIONS

$EOLCOM !

$ONEMPTY

*number of equations listed per block
option limrow = 0;

*number of variables listed per block
option limcol=0;

*do you want solver's solution output printed?
option solprint=on;

*do you want solver's system output printed?
option sysout=off;

*milp relative termination tolerance
option optcr=0;

*milp absolute termination tolerance
option optca=0;

*Decimals to display
option decimals=8;

*File to write results
FILE RESCRU  / cosslavar.res /;

PUT RESCRU;

PUT "------------------------------------------------------------------"
PUT / " SUBSYSTEM B.DOWNSTREAM CRUDE OIL SCHEDULING AT REFINERY FRONT-END "
PUT / "------------------------------------------------------------------"
*--------------------------------------------Choosing solvers

option lp=minos5;
option nlp=conopt;
option nlp=BARON;
option mip=CPLEX;
option rmip=osl;
option minlp=DICOPT;

SETS

         A                    SET OF TANK INPUT SOURCES     /
                              U1, U2, U3, ST1, ST2, ST3     /

         B                    SET OF CRUDE TANKS     /
                              ST1  Storage Tank 1
                              ST2  Storage Tank 2
                              ST3  Storage Tank 3
                              CT1  Charging Tank 1
                              CT2  Charging Tank 2
                              CT3  Charging Tank 3     /

         H                    SET OF TANK OUTPUT DESTINATIONS     /
                              CT1, CT2, CT3, D1, D2     /

         D(H)                 SET OF DISTILLATION UNITS     /
                              D1 * D2     /

         G(B)                 SET OF CHARGING TANKS     /
                              CT1 * CT3     /

         I                    SET OF COMPONENTS     /
                              I1 * I2     /

         M                    SOURCE UNIT FOR SPLIT PIPELINE     /
                              MST2, MST3, MCT2     /

         K                    DESTINATION UNIT OF SPLIT PIPELINE     /
                              KCT3, KD2     /

         U(A)                 SET OF CRUDE SUPPLY STREAMS     /
                              U1 * U3     /

         Y(B)                 SET OF STORAGE TANKS     /
                              ST1 * ST3     /

         L                    TRANSFER EVENT     /
                              L0 * L2     /

         Q                    SET OF SUB-STRUCTURES     /
                              Q1 * Q2     /

         S                    SET OF SLACK VARIABLES     /
                              Q1 * Q2     /

ALIAS

         (A , ADASH)
         (H , HDASH)
         (G , GDASH)
         (L , LDASH)

;
```

```
* =============================== for
MAPPING purposes
================================= *

SETS


         A1(A,B)          SET OF
INPUTS A TO TANK B
/

U1.(ST1,ST2),U2.ST2,U3.(ST2,ST3),

ST1.(CT1,CT2),ST2.(CT1,CT2,CT3),ST3.(C
T2,CT3)  /

         A2(A,Y)          SET OF
INPUTS A TO STORAGE TANK Y
/

U1.(ST1,ST2),U2.ST2,U3.(ST2,ST3)
/

         D3(D,G)          SET OF
DISTILLATION UNITS D THAT CAN BE
CHARGED BY CHARGING TANK G /
                          D1.CT1,
D1.CT2, D2.CT2, D2.CT3
/

         G1(G,D)          SET OF
CHARGING TANKS G THAT CHARGE
DISTILLATION UNIT D            /
                          CT1.D1,
CT2.D1, CT2.D2, CT3.D2
/

         H1(B,H)          SET OF
OUTPUTS H FROM TANK B
/

ST1.(CT1,CT2),ST2.(CT1,CT2,CT3),ST3.(C
T2,CT3),

CT1.D1,CT2.(D1,D2),CT3.D2
/

         H2(Y,H)          SET OF
OUTPUTS H FROM STORAGE TANK Y
/

ST1.(CT1,CT2),ST2.(CT1,CT2,CT3),ST3.(C
T2,CT3)  /

         H3(G,H)          SET OF
OUTPUTS H FROM CHARGING TANK G
/

CT1.D1,CT2.(D1,D2),CT3.D2
/

         H4(Y,H)          SET OF
OUTPUTS H FROM STORAGE TANK Y
/

ST1.CT1,ST2.(CT1,CT2,CT3),ST3.CT3  /

         Y1(Y,U)          SET OF
STORAGE TANKS Y CONNECTED TO CRUDE
SUPPLY STREAMS U         /
                          ST1.U1,
ST2.U1, ST2.U2, ST2.U3, ST3.U3
/
```

```
* ======================== for
DECOMPOSITION purposes
=============================== *

         B1(B)            SET OF TANKS
B BELONGING TO SUB-STRUCTURE Q1 /
                          ST1, ST2,
ST3, CT1, CT2                      /

         B2(B)            SET OF TANKS
B BELONGING TO SUB-STRUCTURE Q2 /
                          CT3
/

         D1(D)            SET OF
DISTILLATION UNITS D PRESENT IN SUB-
STRUCTURE Q1 /
                          D1
/

         D2(D)            SET OF
DISTILLATION UNITS D PRESENT IN SUB-
STRUCTURE Q2 /
                          D2
/

         U1(U)            SET OF CRUDE
SUPPLY STREAMS U PRESENT IN SUB-
STRUCTURE Q1 /
                          U1, U2, U3
/

         U2(U)            SET OF CRUDE
SUPPLY STREAMS U PRESENT IN SUB-
STRUCTURE Q2 /

/

         K1(M,K)          SET OF
DESTINATION UNIT K OF SPLIT PIPELINES
WITH SOURCE M /
                          MST2.KCT3,
MST3.KCT3, MCT2.KD2
/
;


*======================== SLACK
VARIABLES ========================

FREE VARIABLE

        SLAVARTC3FT1(A,B,L)
        SLAVARTC19ITOT(B,L)
        SLAVARTC21INVI(I,B,L)
        SLAVARTC23FT1(A,B,L)
        SLAVARTC24FT2(B,H,L)
        SLAVARTC27INVI(I,B,L)
        SLAVARDC1WB2(D,L)
        SLAVARDC2WB2(G,L)
        SLAVARLBP4INVVJ(I,B,H,L)
        SLAVARLBP5INVVJ(I,B,H,L)
        SLAVARLBP6INVVT(I,B,H,L)
        SLAVARLBP8INVVT(I,B,H,L)
        SLAVARLBP13FLOWVJ(I,A,B,H,L)
        SLAVARLBP14FLOWVT(I,A,B,H,L)
        SLAVARDCUB1WBUB2(D,L)
        SLAVARDCUB2WBUB2(G,L)
```

```
        SLAVARTC3FT1Q1(A,B,L)
        SLAVARTC19ITOTQ1(B,L)
        SLAVARTC21INVIQ1(I,B,L)
        SLAVARTC23FT1Q1(A,B,L)
        SLAVARTC24FT2Q1(B,H,L)
        SLAVARTC27INVIQ1(I,B,L)
        SLAVARDC1WB2Q1(D,L)
        SLAVARDC2WB2Q1(G,L)
        SLAVARLBP4INVVJQ1(I,B,H,L)
        SLAVARLBP5INVVJQ1(I,B,H,L)
        SLAVARLBP6INVVTQ1(I,B,H,L)
        SLAVARLBP8INVVTQ1(I,B,H,L)
        SLAVARLBP13FLOWVJQ1(I,A,B,H,L)
        SLAVARLBP14FLOWVTQ1(I,A,B,H,L)
        SLAVARDCUB1WBUB2Q1(D,L)
        SLAVARDCUB2WBUB2Q1(G,L)

        SLAVARTC3FT1Q2(A,B,L)
        SLAVARTC19ITOTQ2(B,L)
        SLAVARTC21INVIQ2(I,B,L)
        SLAVARTC23FT1Q2(A,B,L)
        SLAVARTC24FT2Q2(B,H,L)
        SLAVARTC27INVIQ2(I,B,L)
        SLAVARDC1WB2Q2(D,L)
        SLAVARDC2WB2Q2(G,L)
        SLAVARLBP4INVVJQ2(I,B,H,L)
        SLAVARLBP5INVVJQ2(I,B,H,L)
        SLAVARLBP6INVVTQ2(I,B,H,L)
        SLAVARLBP8INVVTQ2(I,B,H,L)
        SLAVARLBP13FLOWVJQ2(I,A,B,H,L)
        SLAVARLBP14FLOWVTQ2(I,A,B,H,L)
        SLAVARDCUB1WBUB2Q2(D,L)
         SLAVARDCUB2WBUB2Q2(G,L)
;



*=================================
MAIN MODEL
=================================


POSITIVE VARIABLE

         ITOT(B,L)        TOTAL
INVENTORY OF TANK B AT THE END OF
TRANSFER EVENT L
         INVI(I,B,L)      INVENTORY OF
COMPONENT I IN TANK B AT THE END OF
TRANSFER EVENT L

         TS1(A,B,L)       STARTING
TIME OF A TRANSFER FROM A TO B IN
TRANSFER EVENT L
         TE1(A,B,L)       ENDING TIME
OF A TRANSFER FROM A TO B IN TRANSFER
EVENT L
         TS2(B,H,L)       STARTING
TIME OF A TRANSFER FROM B TO H IN
TRANSFER EVENT L
         TE2(B,H,L)       ENDING TIME
OF A TRANSFER FROM B TO H IN TRANSFER
EVENT L



         TS1Q1(A,B,L)     STARTING
TIME OF A TRANSFER FROM A TO B IN
TRANSFER EVENT L
         TE1Q1(A,B,L)     ENDING TIME
OF A TRANSFER FROM A TO B IN TRANSFER
EVENT L
         TS2Q1(B,H,L)     STARTING
TIME OF A TRANSFER FROM B TO H IN
TRANSFER EVENT L
         TE2Q1(B,H,L)     ENDING TIME
OF A TRANSFER FROM B TO H IN TRANSFER
EVENT L



         TS1Q2(A,B,L)     STARTING
TIME OF A TRANSFER FROM A TO B IN
TRANSFER EVENT L
         TE1Q2(A,B,L)     ENDING TIME
OF A TRANSFER FROM A TO B IN TRANSFER
EVENT L
         TS2Q2(B,H,L)     STARTING
TIME OF A TRANSFER FROM B TO H IN
TRANSFER EVENT L
         TE2Q2(B,H,L)     ENDING TIME
OF A TRANSFER FROM B TO H IN TRANSFER
EVENT L


         TEO(U)           OVERALL
ENDING TIME OF CRUDE TRANSFER FROM
CRUDE SUPPLY STREAM U
         TSI(U)           INITIAL
STARTING TIME OF CRUDE TRANSFER FROM
CRUDE SUPPLY STREAM U


* =================================
Flow variables
======================================
==== *

         FT1(A,B,L)       TOTAL FLOW
FROM A TO B IN TRANSFER EVENT L
         FT2(B,H,L)       TOTAL FLOW
FROM B TO H IN TRANSFER EVENT L


         FL1(I,A,B,L)     FLOW OF
COMPONENT I FROM A TO B IN TRANSFER
EVENT L
         FL2(I,B,H,L)     FLOW OF
COMPONENT I FROM B TO H IN TRANSFER
EVENT L


         FT1Q1(A,B,L)     TOTAL FLOW
FROM A TO B IN TRANSFER EVENT L
         FT2Q1(B,H,L)     TOTAL FLOW
FROM B TO H IN TRANSFER EVENT L


         FL1Q1(I,A,B,L)   FLOW OF
COMPONENT I FROM A TO B IN TRANSFER
EVENT L
         FL2Q1(I,B,H,L)   FLOW OF
COMPONENT I FROM B TO H IN TRANSFER
EVENT L



         FT1Q2(A,B,L)     TOTAL FLOW
FROM A TO B IN TRANSFER EVENT L
         FT2Q2(B,H,L)     TOTAL FLOW
FROM B TO H IN TRANSFER EVENT L


         FL1Q2(I,A,B,L)   FLOW OF
COMPONENT I FROM A TO B IN TRANSFER
EVENT L
```

```
        FL2Q2(I,B,H,L)   FLOW OF
COMPONENT I FROM B TO H IN TRANSFER
EVENT L



* ================== Variables to
replace bilinear terms to solve LBP
====================== *

        INVVJ(I,B,H,L)
INVENTORY COMPONENT I FOR LBP
        INVVT(I,B,H,L)
INVENTORY COMPONENT I FOR LBP

        INVVJQ1(I,B,H,L)
INVENTORY COMPONENT I FOR LBP
        INVVTQ1(I,B,H,L)
INVENTORY COMPONENT I FOR LBP

        INVVJQ2(I,B,H,L)
INVENTORY COMPONENT I FOR LBP
        INVVTQ2(I,B,H,L)
INVENTORY COMPONENT I FOR LBP

        FLOWVJ(I,A,B,H,L)       FLOW
OF COMPONENT I FOR LBP
        FLOWVT(I,A,B,H,L)       FLOW
OF COMPONENT I FOR LBP

        FLOWVJQ(I,A,B,H,L,Q)    FLOW
OF COMPONENT I FOR LBP
        FLOWVTQ(I,A,B,H,L,Q)    FLOW
OF COMPONENT I FOR LBP

        FLOWVJQ1(I,A,B,H,L)     FLOW
OF COMPONENT I FOR LBP
        FLOWVTQ1(I,A,B,H,L)     FLOW
OF COMPONENT I FOR LBP

        FLOWVJQ2(I,A,B,H,L)     FLOW
OF COMPONENT I FOR LBP
        FLOWVTQ2(I,A,B,H,L)     FLOW
OF COMPONENT I FOR LBP



* ====================== Duplicate
variables declaration
============================ *

        FLTQ(M,K,L)      TOTAL FLOW
VARIABLE
        FLTQ1(M,K,L)     DUPLICATE
VARIABLE FOR SUB-STRUCTURE Q1
        FLTQ2(M,K,L)     DUPLICATE
VARIABLE FOR SUB-STRUCTURE Q2

        FLQ(I,M,K,L)     FLOW
VARIABLE
        FLQ1(I,M,K,L)    DUPLICATE
VARIABLE FOR SUB-STRUCTURE Q1
        FLQ2(I,M,K,L)    DUPLICATE
VARIABLE FOR SUB-STRUCTURE Q2

        TSQ(M,K,L)       START TIME
VARIABLE
        TSQ1(M,K,L)      DUPLICATE
VARIABLE FOR SUB-STRUCTURE Q1
        TSQ2(M,K,L)      DUPLICATE
VARIABLE FOR SUB-STRUCTURE Q2

        TEQ(M,K,L)       END TIME
VARIABLE
        TEQ1(M,K,L)      DUPLICATE
VARIABLE FOR SUB-STRUCTURE Q1
        TEQ2(M,K,L)      DUPLICATE
VARIABLE FOR SUB-STRUCTURE Q2

        WBQ(M,K,L)       FLOW
EXISTENCE VARIABLE
        WBQ1(M,K,L)      DUPLICATE
VARIABLE FOR SUB-STRUCTURE Q1
        WBQ2(M,K,L)      DUPLICATE
VARIABLE FOR SUB-STRUCTURE Q2

;



FREE VARIABLES

        ZZ               OBJECTIVE
FUNCTION OF COST
        ZZUB             OBJECTIVE
FUNCTION FOR UPPER BOUND
        ZZFIXBIN
        ZZQ1             OBJECTIVE
FUNCTION OF COST FOR STRUCTURE Q1
        ZZQ2             OBJECTIVE
FUNCTION OF COST FOR STRUCTURE Q2
        ZZU              OBJECTIVE
FUNCTION UPPER BOUND
;



BINARY VARIABLES

        WB1(A,B,L)       EQUAL TO 1
IF THERE IS A FLOW FROM A TO B IN
TRANSFER EVENT L
        WB2(B,H,L)       EQUAL TO 1
IF THERE IS A FLOW FROM B TO H IN
TRANSFER EVENT L

        WB1Q1(A,B,L)     EQUAL TO 1
IF THERE IS A FLOW FROM A TO B IN
TRANSFER EVENT L
        WB2Q1(B,H,L)     EQUAL TO 1
IF THERE IS A FLOW FROM B TO H IN
TRANSFER EVENT L


        WB1Q2(A,B,L)     EQUAL TO 1
IF THERE IS A FLOW FROM A TO B IN
TRANSFER EVENT L
        WB2Q2(B,H,L)     EQUAL TO 1
IF THERE IS A FLOW FROM B TO H IN
TRANSFER EVENT L



;






*=================================
MAIN MODEL
================================

PARAMETERS


        CSEA             WAITING COST
FOR CRUDE SUPPLY STREAMS
/ 5  /
```

```
         CSET             CHANGEOVER
COST FOR CHARGED OIL SWITCH
/ 50 /
         CUNLOAD          UNLOADING
COST FOR CRUDE SUPPLY STREAMS
/ 10 /

         THOR             TIME HORIZON
FOR SCHEDULING
/ 12 /

         ND               NUMBER OF
DISTILLATION UNITS IN THE NETWORK
/ 2  /
         ND1              NUMBER OF
DISTILLATION UNITS IN SUB-STRUCTURE Q1
/ 1  /
         ND2              NUMBER OF
DISTILLATION UNITS IN SUB-STRUCTURE Q2
/ 1  /

         NE               NUMBER OF
TRANSFER EVENTS
/ 3  /

         TSO              SCALAR STEP
SIZE FOR UPDATING LAGRANGE MULTIPLIERS
/ 1  /

         ALPHA            SCALAR
BETWEEN 0 AND 2
/ 0.4 /

         ZZ1              OPTIMAL
OBJECTIVE FOR LQ1-R
/ 0  /
         ZZ2              OPTIMAL
OBJECTIVE FOR LQ2-R
/ 1  /
         ZZLB             SUM OF
OBJECTIVES ZZQ1 AND ZZQ2
/ 1  /


         CINV(B)          INVENTORY
MAINTENANCE COST FOR TANK B
/
                          ST1 * ST3
0.04
                          CT1 * CT3
0.08
/


         DM(G)            DEMAND OF
CRUDE-MIX TO BE CHARED FROM CHARGING
TANK G          /
                          CT1 * CT3
50
/

         LB1(I,B)         LOWER BOUND
ON FRACTION OF COMPONENT I INSIDE TANK
B           /
                          I1.ST1
0.01
                          I1.ST2
0.04
                          I1.ST3
0.07
                          I1.CT1
0.025
                          I1.CT2
0.045
                          I1.CT3
0.075
/

         UB1(I,B)         UPPER BOUND
ON FRACTION OF COMPONENT I INSIDE TANK
B           /
                          I1.ST1
0.03
                          I1.ST2
0.06
                          I1.ST3
0.09
                          I1.CT1
0.035
                          I1.CT2
0.065
                          I1.CT3
0.085
/

         FRACT(I,U)       FRACTION OF
COMPONENT I IN CRUDE SUPPLY STREAM U
/
                          I1.U1
0.01
                          I1.U2
0.085
                          I1.U3
0.06
/

         INITITOT(B)      INITIAL
TOTAL INVENTORY OF TANK B
/
                          ST1 * ST3
20
                          CT1
30
                          CT2
50
                          CT3
30
/

         INITINVI(I,B)    INITIAL
INVENTORY OF COMPONENT I IN TANK B
/
                          I1.ST1
0.02
                          I1.ST2
0.05
                          I1.ST3
0.08
                          I1.CT1
0.03
                          I1.CT2
0.05
                          I1.CT3
0.08
/

         LBI1(B)          LOWER BOUND
ON TOTAL INVENTORY IN A TANK B
/
                          ST1 * ST3
1
                          CT1 * CT3
1
/

         UBI1(B)          UPPER BOUND
ON TOTAL INVENTORY IN A TANK B
/
                          ST1 * ST3
100
```

```
                          CT1 * CT3
100
/

         TARR(U)          ARRIVAL TIME
OF CRUDE IN CRUDE SUPPLY STREAM U
/
                          U1
1
                          U2
5
                          U3
9
/

         VCRUDE(U)        TOTAL VOLUME
OF CRUDE OIL ARRIVING IN CRUDE SUPPLY
SYSTEM U    /
                          U1*U3
50
/


* =================== value for these
PARAMETERS are assigned below
=============== *

         LBF1(A,B)        LOWER BOUND
ON FLOWRATE FROM A TO B
         UBF1(A,B)        UPPER BOUND
ON FLOWRATE FROM A TO B

         LBF2(B,H)        LOWER BOUND
ON FLOWRATE FROM B TO H
         UBF2(B,H)        UPPER BOUND
ON FLOWRATE FROM B TO H

         LBFV1(A,B)       LOWER BOUND
ON FLOW VOLUME FROM A TO B
         UBFV1(A,B)       UPPER BOUND
ON FLOW VOLUME FROM A TO B
         LBFV2(B,H)       LOWER BOUND
ON FLOW VOLUME FROM B TO H
         UBFV2(B,H)       UPPER BOUND
ON FLOW VOLUME FROM B TO H


* =========================== Lagrange
multipliers
================================ *

         LMTS(M,K,L)      LAGRANGE
MULTIPLIER
         LMTE(M,K,L)      LAGRANGE
MULTIPLIER
         LMFLT(M,K,L)     LAGRANGE
MULTIPLIER
         LMFL(I,M,K,L)    LAGRANGE
MULTIPLIER
         LMWB(M,K,L)      LAGRANGE
MULTIPLIER


* ======================== Binary
variables assignation
============================= *

         WBUB1(A,B,L)     EQUAL TO
OPTIMAL VALUE OBTAINED FROM THE
SOLUTION OF CRUDE_LB
         WBUB2(B,H,L)     EQUAL TO
OPTIMAL VALUE OBTAINED FROM THE
SOLUTION OF CRUDE_LB

* ====================== Duplicate
variables assignation
============================ *

         DFLTQ(M,K,L)      TOTAL FLOW
VARIABLE
         DFLTQ1(M,K,L)     DUPLICATE
VARIABLE FOR SUB-STRUCTURE Q1
         DFLTQ2(M,K,L)     DUPLICATE
VARIABLE FOR SUB-STRUCTURE Q2

         DFLQ(I,M,K,L)     FLOW
VARIABLE
         DFLQ1(I,M,K,L)    DUPLICATE
VARIABLE FOR SUB-STRUCTURE Q1
         DFLQ2(I,M,K,L)    DUPLICATE
VARIABLE FOR SUB-STRUCTURE Q2

         DTSQ(M,K,L)       START TIME
VARIABLE
         DTSQ1(M,K,L)      DUPLICATE
VARIABLE FOR SUB-STRUCTURE Q1
         DTSQ2(M,K,L)      DUPLICATE
VARIABLE FOR SUB-STRUCTURE Q2

         DTEQ(M,K,L)       END TIME
VARIABLE
         DTEQ1(M,K,L)      DUPLICATE
VARIABLE FOR SUB-STRUCTURE Q1
         DTEQ2(M,K,L)      DUPLICATE
VARIABLE FOR SUB-STRUCTURE Q2

         DWBQ(M,K,L)       FLOW
EXISTENCE VARIABLE
         DWBQ1(M,K,L)      DUPLICATE
VARIABLE FOR SUB-STRUCTURE Q1
         DWBQ2(M,K,L)      DUPLICATE
VARIABLE FOR SUB-STRUCTURE Q2

;


* ========================= PARAMETER
value assignment
============================== *

         LBF1(A,B) = 1           ;
         UBF1(A,B) = 40          ;

         LBF2(B,H) = 1           ;
         UBF2(B,H) = 40          ;

         LBFV1(A,B) = 1          ;
         UBFV1(A,B) = 100        ;

         LBFV2(B,H) = 1          ;
         UBFV2(B,H) = 100        ;

         LMTS(M,K,L) = 0         ;
         LMTE(M,K,L) = 0         ;
         LMFLT(M,K,L) = 0        ;
         LMFL(I,M,K,L) = 0       ;
         LMWB(M,K,L) = 0         ;




*================================
MAIN MODEL
================================

EQUATIONS
```

```
         OBJFUN      OBJECTIVE
FUNCTION
         OBJFIXBIN   OBJECTIVE
FUNCTION
         OBJQ1       OBJECTIVE
FUNCTION OF COST FOR STRUCTURE Q1
         OBJQ2       OBJECTIVE
FUNCTION OF COST FOR STRUCTURE Q2

         TC1         TANK CONSTRAINT 1
         TC2         TANK CONSTRAINT 2
         TC3         TANK CONSTRAINT 3
         TC4         TANK CONSTRAINT 4
         TC5         TANK CONSTRAINT 5
         TC6         TANK CONSTRAINT 6
         TC7         TANK CONSTRAINT 7
         TC8         TANK CONSTRAINT 8
         TC9         TANK CONSTRAINT 9
         TC10        TANK CONSTRAINT
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         TC11        TANK CONSTRAINT
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         TC12        TANK CONSTRAINT
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         TC13        TANK CONSTRAINT
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         TC14        TANK CONSTRAINT
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         TC15        TANK CONSTRAINT
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         TC16        TANK CONSTRAINT
16
         TC17        TANK CONSTRAINT
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         TC18        TANK CONSTRAINT
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         TC19        TANK CONSTRAINT
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         TC20        TANK CONSTRAINT
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         TC21        TANK CONSTRAINT
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         TC22        TANK CONSTRAINT
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         TC23        TANK CONSTRAINT
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         TC24        TANK CONSTRAINT
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         TC25        TANK CONSTRAINT
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         TC26        TANK CONSTRAINT
26
         TC27        TANK CONSTRAINT
27
         TC28        TANK CONSTRAINT
28
         TC29        TANK CONSTRAINT
29
         TC30        TANK CONSTRAINT
30
         TC31        TANK CONSTRAINT
31

*$ONTEXT
         TC32        TANK CONSTRAINT
32
         TC33        TANK CONSTRAINT
33
         TC34        TANK CONSTRAINT
34
         TC35        TANK CONSTRAINT
35
         TC36        TANK CONSTRAINT
36
*$OFFTEXT
```

```
         DC1         DISTILLATION UNIT
CONSTRAINT 1
         DC2         DISTILLATION UNIT
CONSTRAINT 2
         DC3         DISTILLATION UNIT
CONSTRAINT 3
         DC4         DISTILLATION UNIT
CONSTRAINT 4
         DC5         DISTILLATION UNIT
CONSTRAINT 5

         CC1         CRUDE SUPPLY
STREAM CONSTRAINT 1
         CC2         CRUDE SUPPLY
STREAM CONSTRAINT 2
         CC3         CRUDE SUPPLY
STREAM CONSTRAINT 3
         CC4         CRUDE SUPPLY
STREAM CONSTRAINT 4

         VB1         VARIABLE BOUND 1
         VB2         VARIABLE BOUND 2
         VB3         VARIABLE BOUND 3
         VB4         VARIABLE BOUND 4
         VB5         VARIABLE BOUND 5
         VB6         VARIABLE BOUND 6
         VB7         VARIABLE BOUND 7
         VB8         VARIABLE BOUND 8
         VB9         VARIABLE BOUND 9
         VB10        VARIABLE BOUND 10
         VB11        VARIABLE BOUND 11
         VB12        VARIABLE BOUND 12
         VB13        VARIABLE BOUND 13
         VB14        VARIABLE BOUND 14
         VB15        VARIABLE BOUND 15

         LBP1        LOWER BOUNDING
PROBLEM 1
         LBP2        LOWER BOUNDING
PROBLEM 2
         LBP3        LOWER BOUNDING
PROBLEM 3
         LBP4        LOWER BOUNDING
PROBLEM 4
         LBP5        LOWER BOUNDING
PROBLEM 5
         LBP6        LOWER BOUNDING
PROBLEM 6
         LBP7        LOWER BOUNDING
PROBLEM 7
         LBP8        LOWER BOUNDING
PROBLEM 8
         LBP9        LOWER BOUNDING
PROBLEM 9
         LBP10       LOWER BOUNDING
PROBLEM 10
         LBP11       LOWER BOUNDING
PROBLEM 11
         LBP12       LOWER BOUNDING
PROBLEM 12
         LBP13       LOWER BOUNDING
PROBLEM 13
         LBP14       LOWER BOUNDING
PROBLEM 14
         LBP15       LOWER BOUNDING
PROBLEM 15
         LBP16       LOWER BOUNDING
PROBLEM 16
         LBP17       LOWER BOUNDING
PROBLEM 17
```

```
        LBP18       LOWER BOUNDING
PROBLEM 18
        LBP19       LOWER BOUNDING
PROBLEM 19
        LBP20       LOWER BOUNDING
PROBLEM 20
        LBP21       LOWER BOUNDING
PROBLEM 21
        LBP22       LOWER BOUNDING
PROBLEM 22


* ====================== LBP for sub-
structures Q1 and Q2
============================ *

        LBP10Q       LOWER BOUNDING
PROBLEM 10
        LBP11Q       LOWER BOUNDING
PROBLEM 11
        LBP12Q       LOWER BOUNDING
PROBLEM 12
        LBP13Q       LOWER BOUNDING
PROBLEM 13
        LBP14Q       LOWER BOUNDING
PROBLEM 14
        LBP15Q       LOWER BOUNDING
PROBLEM 15
        LBP16Q       LOWER BOUNDING
PROBLEM 16
        LBP17Q       LOWER BOUNDING
PROBLEM 17

        LBP1Q1       LOWER BOUNDING
PROBLEM 1
        LBP2Q1       LOWER BOUNDING
PROBLEM 2
        LBP3Q1       LOWER BOUNDING
PROBLEM 3
        LBP4Q1       LOWER BOUNDING
PROBLEM 4
        LBP5Q1       LOWER BOUNDING
PROBLEM 5
        LBP6Q1       LOWER BOUNDING
PROBLEM 6
        LBP7Q1       LOWER BOUNDING
PROBLEM 7
        LBP8Q1       LOWER BOUNDING
PROBLEM 8
        LBP9Q1       LOWER BOUNDING
PROBLEM 9
        LBP10Q1      LOWER BOUNDING
PROBLEM 10
        LBP11Q1      LOWER BOUNDING
PROBLEM 11
        LBP12Q1      LOWER BOUNDING
PROBLEM 12
        LBP13Q1      LOWER BOUNDING
PROBLEM 13
        LBP14Q1      LOWER BOUNDING
PROBLEM 14
        LBP15Q1      LOWER BOUNDING
PROBLEM 15
        LBP16Q1      LOWER BOUNDING
PROBLEM 16
        LBP17Q1      LOWER BOUNDING
PROBLEM 17

        LBP1Q2       LOWER BOUNDING
PROBLEM 1
        LBP2Q2       LOWER BOUNDING
PROBLEM 2
        LBP3Q2       LOWER BOUNDING
PROBLEM 3
        LBP4Q2       LOWER BOUNDING
PROBLEM 4
        LBP5Q2       LOWER BOUNDING
PROBLEM 5
        LBP6Q2       LOWER BOUNDING
PROBLEM 6
        LBP7Q2       LOWER BOUNDING
PROBLEM 7
        LBP8Q2       LOWER BOUNDING
PROBLEM 8
        LBP9Q2       LOWER BOUNDING
PROBLEM 9
        LBP10Q2      LOWER BOUNDING
PROBLEM 10
        LBP11Q2      LOWER BOUNDING
PROBLEM 11
        LBP12Q2      LOWER BOUNDING
PROBLEM 12
        LBP13Q2      LOWER BOUNDING
PROBLEM 13
        LBP14Q2      LOWER BOUNDING
PROBLEM 14
        LBP15Q2      LOWER BOUNDING
PROBLEM 15
        LBP16Q2      LOWER BOUNDING
PROBLEM 16
        LBP17Q2      LOWER BOUNDING
PROBLEM 17


* ====================== Equations to
update Lagrange multiplier
======================== *

        ULM1        UPDATING LAGRANGE
MULTIPLIER 1
        ULM2        UPDATING LAGRANGE
MULTIPLIER 2
        ULM3        UPDATING LAGRANGE
MULTIPLIER 3
        ULM4        UPDATING LAGRANGE
MULTIPLIER 4
        ULM5        UPDATING LAGRANGE
MULTIPLIER 5
        ULM6        UPDATING LAGRANGE
MULTIPLIER 6


* ============================
Equations to generate VLC
================================= *

        VLC1        GENERATING VALID
LINEAR CUTS
        VLC2        GENERATING VALID
LINEAR CUTS


* ============== Equations used to
evaluate UB after setting binary
variables ============ *

        TCUB1       DUPLICATE
EQUATION FOR UB EVALUATION
        TCUB2       DUPLICATE
EQUATION FOR UB EVALUATION
        TCUB3       DUPLICATE
EQUATION FOR UB EVALUATION
        TCUB4       DUPLICATE
EQUATION FOR UB EVALUATION
        TCUB5       DUPLICATE
EQUATION FOR UB EVALUATION
        TCUB6       DUPLICATE
EQUATION FOR UB EVALUATION
        TCUB8       DUPLICATE
EQUATION FOR UB EVALUATION
        TCUB11      DUPLICATE
EQUATION FOR UB EVALUATION
```

```
        TCUB14      DUPLICATE
EQUATION FOR UB EVALUATION
        TCUB15      DUPLICATE
EQUATION FOR UB EVALUATION
        TCUB16      DUPLICATE
EQUATION FOR UB EVALUATION
        TCUB17      DUPLICATE
EQUATION FOR UB EVALUATION
        TCUB18      DUPLICATE
EQUATION FOR UB EVALUATION

        DCUB1       DUPLICATE
EQUATION FOR UB EVALUATION
        DCUB2       DUPLICATE
EQUATION FOR UB EVALUATION
        DCUB4       DUPLICATE
EQUATION FOR UB EVALUATION
        DCUB5       DUPLICATE
EQUATION FOR UB EVALUATION

        CCUB1       DUPLICATE
EQUATION FOR UB EVALUATION
        CCUB2       DUPLICATE
EQUATION FOR UB EVALUATION

* ========================= DUPLICATE
EQUATIONS AND VARIABLES FOR LB1
======== *
        TC1Q1         TANK CONSTRAINT
1
        TC2Q1         TANK CONSTRAINT
2
        TC3Q1         TANK CONSTRAINT
3
        TC4Q1         TANK CONSTRAINT
4
        TC5Q1         TANK CONSTRAINT
5
        TC6Q1         TANK CONSTRAINT
6
        TC7Q1         TANK CONSTRAINT
7
        TC8Q1         TANK CONSTRAINT
8
        TC9Q1         TANK CONSTRAINT
9
        TC10Q1        TANK CONSTRAINT
10
        TC11Q1        TANK CONSTRAINT
11
        TC12Q1        TANK CONSTRAINT
12
        TC13Q1        TANK CONSTRAINT
13
        TC14Q1        TANK CONSTRAINT
14
        TC15Q1        TANK CONSTRAINT
15
        TC16Q1        TANK CONSTRAINT
16
        TC17Q1        TANK CONSTRAINT
17
        TC18Q1        TANK CONSTRAINT
18
        TC19Q1        TANK CONSTRAINT
19
        TC20Q1        TANK CONSTRAINT
20
        TC21Q1        TANK CONSTRAINT
21
        TC22Q1        TANK CONSTRAINT
22
        TC23Q1        TANK CONSTRAINT
23
        TC24Q1        TANK CONSTRAINT
24
*       TC25Q1        TANK CONSTRAINT
25
        TC26Q1        TANK CONSTRAINT
26
        TC27Q1        TANK CONSTRAINT
27
        TC28Q1        TANK CONSTRAINT
28
        TC29Q1        TANK CONSTRAINT
29
        TC30Q1        TANK CONSTRAINT
30
        TC31Q1        TANK CONSTRAINT
31

*$ONTEXT
        TC32Q1        TANK CONSTRAINT
32
        TC33Q1        TANK CONSTRAINT
33
        TC34Q1        TANK CONSTRAINT
34
        TC35Q1        TANK CONSTRAINT
35
        TC36Q1        TANK CONSTRAINT
36
*$OFFTEXT

        DC1Q1         DISTILLATION
UNIT CONSTRAINT 1
        DC2Q1         DISTILLATION
UNIT CONSTRAINT 2
        DC3Q1         DISTILLATION
UNIT CONSTRAINT 3
        DC4Q1         DISTILLATION
UNIT CONSTRAINT 4
        DC5Q1         DISTILLATION
UNIT CONSTRAINT 5

        CC1Q1         CRUDE SUPPLY
STREAM CONSTRAINT 1
        CC2Q1         CRUDE SUPPLY
STREAM CONSTRAINT 2
        CC3Q1         CRUDE SUPPLY
STREAM CONSTRAINT 3
        CC4Q1         CRUDE SUPPLY
STREAM CONSTRAINT 4

        VB1Q1         VARIABLE BOUND
1
        VB2Q1         VARIABLE BOUND
2
        VB3Q1         VARIABLE BOUND
3
        VB4Q1         VARIABLE BOUND
4
        VB5Q1         VARIABLE BOUND
5
        VB6Q1         VARIABLE BOUND
6
        VB7Q1         VARIABLE BOUND
7
        VB8Q1         VARIABLE BOUND
8
        VB9Q1         VARIABLE BOUND
9
        VB10Q1        VARIABLE BOUND
10
        VB11Q1        VARIABLE BOUND
11
```

```
         VB12Q1        VARIABLE BOUND 12
         VB13Q1        VARIABLE BOUND 13
         VB14Q1        VARIABLE BOUND 14
         VB15Q1        VARIABLE BOUND 15

* ========================= DUPLICATE EQUATIONS AND VARIABLES FOR LB ======== *

         TC1Q2         TANK CONSTRAINT 1
         TC2Q2         TANK CONSTRAINT 2
         TC3Q2         TANK CONSTRAINT 3
         TC4Q2         TANK CONSTRAINT 4
         TC5Q2         TANK CONSTRAINT 5
         TC6Q2         TANK CONSTRAINT 6
         TC7Q2         TANK CONSTRAINT 7
         TC8Q2         TANK CONSTRAINT 8
         TC9Q2         TANK CONSTRAINT 9
         TC10Q2        TANK CONSTRAINT 10
         TC11Q2        TANK CONSTRAINT 11
         TC12Q2        TANK CONSTRAINT 12
         TC13Q2        TANK CONSTRAINT 13
         TC14Q2        TANK CONSTRAINT 14
         TC15Q2        TANK CONSTRAINT 15
         TC16Q2        TANK CONSTRAINT 16
         TC17Q2        TANK CONSTRAINT 17
         TC18Q2        TANK CONSTRAINT 18
         TC19Q2        TANK CONSTRAINT 19
         TC20Q2        TANK CONSTRAINT 20
         TC21Q2        TANK CONSTRAINT 21
         TC22Q2        TANK CONSTRAINT 22
         TC23Q2        TANK CONSTRAINT 23
         TC24Q2        TANK CONSTRAINT 24
*        TC25Q2        TANK CONSTRAINT 25
         TC26Q2        TANK CONSTRAINT 26
         TC27Q2        TANK CONSTRAINT 27
         TC28Q2        TANK CONSTRAINT 28
         TC29Q2        TANK CONSTRAINT 29
         TC30Q2        TANK CONSTRAINT 30
         TC31Q2        TANK CONSTRAINT 31
*$ONTEXT
         TC32Q2        TANK CONSTRAINT 32
         TC33Q2        TANK CONSTRAINT 33
         TC34Q2        TANK CONSTRAINT 34
         TC35Q2        TANK CONSTRAINT 35
         TC36Q2        TANK CONSTRAINT 36
*$OFFTEXT

         DC1Q2         DISTILLATION UNIT CONSTRAINT 1
         DC2Q2         DISTILLATION UNIT CONSTRAINT 2
         DC3Q2         DISTILLATION UNIT CONSTRAINT 3
         DC4Q2         DISTILLATION UNIT CONSTRAINT 4
         DC5Q2         DISTILLATION UNIT CONSTRAINT 5

         CC1Q2         CRUDE SUPPLY STREAM CONSTRAINT 1
         CC2Q2         CRUDE SUPPLY STREAM CONSTRAINT 2
         CC3Q2         CRUDE SUPPLY STREAM CONSTRAINT 3
         CC4Q2         CRUDE SUPPLY STREAM CONSTRAINT 4

         VB1Q2         VARIABLE BOUND 1
         VB2Q2         VARIABLE BOUND 2
         VB3Q2         VARIABLE BOUND 3
         VB4Q2         VARIABLE BOUND 4
         VB5Q2         VARIABLE BOUND 5
         VB6Q2         VARIABLE BOUND 6
         VB7Q2         VARIABLE BOUND 7
         VB8Q2         VARIABLE BOUND 8
         VB9Q2         VARIABLE BOUND 9
         VB10Q2        VARIABLE BOUND 10
         VB11Q2        VARIABLE BOUND 11
         VB12Q2        VARIABLE BOUND 12
         VB13Q2        VARIABLE BOUND 13
         VB14Q2        VARIABLE BOUND 14
         VB15Q2        VARIABLE BOUND 15

;

*                  -----------------------------
*                 |         EQUATION            |
*                 |        DEFINITION           |
```

```
*                 -----------------------------

* ============================
OBJECTIVE FUNCTION
======================================= *

 OBJFUN..
         ZZ =E= CSEA*SUM(U,TSI(U)-TARR(U)) + CUNLOAD*SUM(U,TEO(U)-TSI(U)) +

THOR*(SUM(B,CINV(B))*SUM(B,SUM(L,(ITOT(B,L)))) + SUM(B,CINV(B))*
SUM(B,SUM(L,SUM(A$A1(A,B),FT1(A,B,L)))) +

SUM(B,CINV(B))*SUM(B,SUM(L$(ORD(L) LT CARD(L)),ITOT(B,L))) +
THOR*SUM(B,CINV(B))*SUM(B,INITITOT(B))) / (2*NE + 1) +

CSET*(SUM(D,SUM(G$G1(G,D),SUM(L,WB2(G,D,L)))) - ND)
$ontext
*                + 10*SUM((A,B,L), SLAVARTC3FT1(A,B,L) +
SLAVARTC23FT1(A,B,L))
                 + 10*SUM((B,L), SLAVARTC19ITOT(B,L))
                 + 10*SUM ( (I,B,L), SLAVARTC21INVI(I,B,L) +
SLAVARTC27INVI(I,B,L) )
*                 + 10*SUM ( (B,H,L), SLAVARTC24FT2(B,H,L) )
                 + 10*SUM ( (D,L), SLAVARDC1WB2(D,L) )
                 + 10*SUM ( (G,L), SLAVARDC2WB2(G,L) )
*                 + 10*SUM ( (I,B,H,L), SLAVARLBP4INVVJ(I,B,H,L) )
*                 + 10*SUM ( (I,B,H,L), SLAVARLBP5INVVJ(I,B,H,L) )
*                 + 10*SUM ( (I,B,H,L), SLAVARLBP8INVVT(I,B,H,L) )
*                 + 10*SUM ( (I,A,B,H,L),
SLAVARLBP13FLOWVJ(I,A,B,H,L) +
SLAVARLBP14FLOWVT(I,A,B,H,L) )
*                 + 10*SUM( S, SLACK(S))
$offtext
          ;



 OBJFIXBIN..
         ZZFIXBIN =E=
CSEA*SUM(U,TSI(U)-TARR(U)) +
CUNLOAD*SUM(U,TEO(U)-TSI(U)) +

THOR*(SUM(B,CINV(B))*SUM(B,SUM(L,(ITOT(B,L)))) + SUM(B,CINV(B))*
SUM(B,SUM(L,SUM(A$A1(A,B),FT1(A,B,L)))) +

SUM(B,CINV(B))*SUM(B,SUM(L$(ORD(L) LT CARD(L)),ITOT(B,L))) +
THOR*SUM(B,CINV(B))*SUM(B,INITITOT(B))) / (2*NE + 1) +

CSET*(SUM(D,SUM(G$G1(G,D),SUM(L,WBUB2(G,D,L)))) - ND)            ;
```

```
 OBJQ1..
         ZZQ1 =E= CSEA*SUM(U1,TSI(U1)-TARR(U1)) + CUNLOAD*SUM(U1,TEO(U1)-TSI(U1)) +

THOR*(SUM(B1,CINV(B1))*SUM(B1,SUM(L,(ITOT(B1,L)))) +
SUM(B1,CINV(B1))*SUM(B1,SUM(L,SUM(A,FT1Q1(A,B1,L)))) +

SUM(B1,CINV(B1))*SUM(B1,SUM(L$(ORD(L) LT CARD(L)),ITOT(B1,L))) +
THOR*SUM(B1,CINV(B1))*SUM(B2,INITITOT(B2))) / (2*NE + 1) +

CSET*(SUM(D1,SUM(G$G1(G,D1),SUM(L,WB2Q1(G,D1,L)))) - ND1) +
SUM(M,SUM(K$K1(M,K),SUM(L,LMFLT(M,K,L)*FLTQ1(M,K,L)))) +

SUM(I,SUM(M,SUM(K$K1(M,K),SUM(L,LMFL(I,M,K,L)*FLQ1(I,M,K,L))))) +
SUM(M,SUM(K$K1(M,K),SUM(L,LMTS(M,K,L)*TSQ1(M,K,L)))) +

SUM(M,SUM(K$K1(M,K),SUM(L,LMTE(M,K,L)*TEQ1(M,K,L)))) +
SUM(M,SUM(K$K1(M,K),SUM(L,LMWB(M,K,L)*WBQ1(M,K,L))));

 OBJQ2..
         ZZQ2 =E= CSEA*SUM(U2,TSI(U2)-TARR(U2)) + CUNLOAD*SUM(U2,TEO(U2)-TSI(U2)) +

THOR*(SUM(B2,CINV(B2))*SUM(B2,SUM(L,(ITOT(B2,L)))) +
SUM(B2,CINV(B2))*SUM(B2,SUM(L,SUM(A,FT1(A,B2,L)))) +

SUM(B2,CINV(B2))*SUM(B2,SUM(L$(ORD(L) LT CARD(L)),ITOT(B2,L))) +
THOR*SUM(B2,CINV(B2))*SUM(B2,INITITOT(B2))) / (2*NE + 1) +

CSET*(SUM(D2,SUM(G$G1(G,D2),SUM(L,WB2(G,D2,L)))) - ND2) -
SUM(M,SUM(K$K1(M,K),SUM(L,LMFLT(M,K,L)*FLTQ2(M,K,L)))) -

SUM(I,SUM(M,SUM(K$K1(M,K),SUM(L,LMFL(I,M,K,L)*FLQ2(I,M,K,L))))) -
SUM(M,SUM(K$K1(M,K),SUM(L,LMTS(M,K,L)*TSQ2(M,K,L)))) -

SUM(M,SUM(K$K1(M,K),SUM(L,LMTE(M,K,L)*TEQ2(M,K,L)))) -
SUM(M,SUM(K$K1(M,K),SUM(L,LMWB(M,K,L)*WBQ2(M,K,L))));



* ============================ 1.TANK CONSTRAINTS
======================================= *



**(i)Constraints for Flow Transfers

 TC1(A,B,L)$A1(A,B)..
         FT1(A,B,L) =L=
         UBFV1(A,B)*WB1(A,B,L)
;
```

```
 TC2(B,H,L)$H1(B,H)..
         FT2(B,H,L) =L=
         UBFV2(B,H)*WB2(B,H,L)
;

 TCUB1(A,B,L)$A1(A,B)..
         FT1(A,B,L) =L=
         UBFV1(A,B)*WBUB1(A,B,L)
;

 TCUB2(B,H,L)$H1(B,H)..
         FT2(B,H,L) =L=
         UBFV2(B,H)*WBUB2(B,H,L)
;


**(ii)Duration Constraints

 TC3(A,B,L)$A1(A,B)..
         UBF1(A,B)*(TE1(A,B,L)-
TS1(A,B,L)) + UBF1(A,B)*THOR*(1-
WB1(A,B,L)) =G=
         FT1(A,B,L) +
SLAVARTC3FT1(A,B,L)
;


 TC4(B,H,L)$H1(B,H)..
         UBF2(B,H)*(TE2(B,H,L)-
TS2(B,H,L)) + UBF2(B,H)*THOR*(1-
WB2(B,H,L)) =G=
         FT2(B,H,L)
;

 TC5(A,Y,L)$A2(A,Y)..
         LBF1(A,Y)*(TE1(A,Y,L)-
TS1(A,Y,L)) - LBF1(A,Y)*THOR*(1-
WB1(A,Y,L)) =L=
         FT1(A,Y,L)
;

 TC6(Y,H,L)$H2(Y,H)..
         LBF2(Y,H)*(TE2(Y,H,L)-
TS2(Y,H,L)) - LBF2(Y,H)*THOR*(1-
WB2(Y,H,L)) =L=
         FT2(Y,H,L)
;

 TC7(G,H,L)$H3(G,H)..
         LBF2(G,H)*(TE2(G,H,L)-
TS2(G,H,L)) =L=
         FT2(G,H,L)
;

 TCUB3(A,B,L)$A1(A,B)..
         UBF1(A,B)*(TE1(A,B,L)-
TS1(A,B,L)) + UBF1(A,B)*THOR*(1-
WBUB1(A,B,L)) =G=
         FT1(A,B,L)
;

 TCUB4(B,H,L)$H1(B,H)..
         UBF2(B,H)*(TE2(B,H,L)-
TS2(B,H,L)) + UBF2(B,H)*THOR*(1-
WBUB2(B,H,L)) =G=
         FT2(B,H,L)
;

 TCUB5(A,Y,L)$A2(A,Y)..
         LBF1(A,Y)*(TE1(A,Y,L)-
TS1(A,Y,L)) - LBF1(A,Y)*THOR*(1-
WBUB1(A,Y,L)) =L=
         FT1(A,Y,L)
;

 TCUB6(Y,H,L)$H2(Y,H)..
         LBF2(Y,H)*(TE2(Y,H,L)-
TS2(Y,H,L)) - LBF2(Y,H)*THOR*(1-
WBUB2(Y,H,L)) =L=
         FT2(Y,H,L)
;


**(iii)Simple Sequencing Constraints

 TC8(A,B,L)$(ORD(L) LT
CARD(L)$A1(A,B))..
         TS1(A,B,L+1) =G=
         TE1(A,B,L) - THOR*(1-
WB1(A,B,L))
;

 TC9(A,B,L)$(ORD(L) LT
CARD(L)$A1(A,B))..
         TS1(A,B,L+1) =G=
         TS1(A,B,L)
;

 TC10(A,B,L)$(ORD(L) LT
CARD(L)$A1(A,B))..
         TE1(A,B,L+1) =G=
         TE1(A,B,L)
;

 TC11(B,H,L)$(ORD(L) LT
CARD(L)$H1(B,H))..
         TS2(B,H,L+1) =G=
         TE2(B,H,L) - THOR*(1-
WB2(B,H,L))
;

 TC12(B,H,L)$(ORD(L) LT
CARD(L)$H1(B,H))..
         TS2(B,H,L+1) =G=
         TS2(B,H,L)
;

 TC13(B,H,L)$(ORD(L) LT
CARD(L)$H1(B,H))..
         TE2(B,H,L+1) =G=
         TE2(B,H,L)
;


 TCUB8(A,B,L)$(ORD(L) LT
CARD(L)$A1(A,B))..
         TS1(A,B,L+1) =G=
         TE1(A,B,L) - THOR*(1-
WBUB1(A,B,L))
;

 TCUB11(B,H,L)$(ORD(L) LT
CARD(L)$H1(B,H))..
         TS2(B,H,L+1) =G=
         TE2(B,H,L) - THOR*(1-
WBUB2(B,H,L))
;


**(iv)Input and Output Restraints for
the Entire Horizon

 TC14(A,ADASH,B,L)$(ORD(L) LT
CARD(L)$(ORD(A) NE
ORD(ADASH)$A1(A,B)))..
         TS1(A,B,L+1) =G=
         TE1(ADASH,B,L) - THOR*(1-
WB1(ADASH,B,L))
;
```

```
 TC15(A,B,H,L)$(ORD(L) LT
CARD(L)$(A1(A,B)$H1(B,H)))..
         TS1(A,B,L+1) =G=
         TE2(B,H,L) - THOR*(1-
WB2(B,H,L))
;

 TC16(A,B,H,L)$(ORD(L) LT
CARD(L)$(A1(A,B)$H1(B,H)))..
         TS2(B,H,L+1) =G=
         TE1(A,B,L) - THOR*(1-
WB1(A,B,L))
;

 TC17(B,H,HDASH,L)$(ORD(L) LT
CARD(L)$(ORD(H) NE
ORD(HDASH)$H1(B,H)))..
         TS2(B,H,L+1) =G=
         TE2(B,HDASH,L) - THOR*(1-
WB2(B,HDASH,L))
;

 TC18(A,B,H,L)$(A1(A,B)$H1(B,H))..
         TE1(A,B,L) - THOR*(1-
WB1(A,B,L)) =L=
         TS2(B,H,L) + THOR*(1-
WB2(B,H,L))
;


 TCUB14(A,ADASH,B,L)$(ORD(L) LT
CARD(L)$(ORD(A) NE
ORD(ADASH)$A1(A,B)))..
         TS1(A,B,L+1) =G=
         TE1(ADASH,B,L) - THOR*(1-
WBUB1(ADASH,B,L))
;

 TCUB15(A,B,H,L)$(ORD(L) LT
CARD(L)$(A1(A,B)$H1(B,H)))..
         TS1(A,B,L+1) =G=
         TE2(B,H,L) - THOR*(1-
WBUB2(B,H,L))
;

 TCUB16(A,B,H,L)$(ORD(L) LT
CARD(L)$(A1(A,B)$H1(B,H)))..
         TS2(B,H,L+1) =G=
         TE1(A,B,L) - THOR*(1-
WBUB1(A,B,L))
;

 TCUB17(B,H,HDASH,L)$(ORD(L) LT
CARD(L)$(ORD(H) NE
ORD(HDASH)$H1(B,H)))..
         TS2(B,H,L+1) =G=
         TE2(B,HDASH,L) - THOR*(1-
WBUB2(B,HDASH,L))
;

 TCUB18(A,B,H,L)$(A1(A,B)$H1(B,H))..
         TE1(A,B,L) - THOR*(1-
WBUB1(A,B,L)) =L=
         TS2(B,H,L) + THOR*(1-
WBUB2(B,H,L))
;


**(v)Mass Balances

 TC19(B,L)..
         ITOT(B,L-1) +
SUM(A$A1(A,B),FT1(A,B,L)) =E=
         ITOT(B,L) +
SUM(H$H1(B,H),FT2(B,H,L)) +
SLAVARTC19ITOT(B,L)             ;
```

```
 TC20(B)..
         ITOT(B,'L0') =E=
         INITITOT(B)
;

 TC21(I,B,L)..
         INVI(I,B,L-1) +
SUM(A$A1(A,B),FL1(I,A,B,L)) =E=
         INVI(I,B,L)
+SUM(H$H1(B,H),FL2(I,B,H,L)) +
SLAVARTC21INVI(I,B,L)        ;

 TC22(I,B)..
         INVI(I,B,'L0') =E=
         INITINVI(I,B)
;

 TC23(A,B,L)$A1(A,B)..
         FT1(A,B,L) =E=
         SUM(I,FL1(I,A,B,L)) +
SLAVARTC23FT1(A,B,L)
;

 TC24(B,H,L)$H1(B,H)..
         FT2(B,H,L) =E=
         SUM(I,FL2(I,B,H,L)) +
SLAVARTC24FT2(B,H,L)
;


**(vi)Component Balances (Replaced by
LBP)

 TC25(I,B,H,L)$H1(B,H)..
         FL2(I,B,H,L)*(ITOT(B,L-
1)+SUM(A$A1(A,B),FT1(A,B,L))) =E=
         FT2(B,H,L)*(INVI(I,B,L-
1)+SUM(A$A1(A,B),FL1(I,A,B,L)));


 LBP1(I,B,H,L)$H1(B,H)..
         INVVJ(I,B,H,L) +
SUM(A$A1(A,B),FLOWVJ(I,A,B,H,L)) =E=
         INVVT(I,B,H,L) +
SUM(A$A1(A,B),FLOWVT(I,A,B,H,L))
;


 LBP2(I,B,H,L)$H1(B,H)..
         INVVJ(I,B,H,L) =G=
         LBI1(B)*FL2(I,B,H,L) +
LBFV2(B,H)*ITOT(B,L-1) -
LBI1(B)*LBFV2(B,H)      ;

 LBP3(I,B,H,L)$H1(B,H)..
         INVVJ(I,B,H,L) =G=
         UBI1(B)*FL2(I,B,H,L) +
UBFV2(B,H)*ITOT(B,L-1) -
UBI1(B)*UBFV2(B,H)      ;

 LBP4(I,B,H,L)$H1(B,H)..
         INVVJ(I,B,H,L) =L=
         LBI1(B)*FL2(I,B,H,L) +
UBFV2(B,H)*ITOT(B,L-1) -
LBI1(B)*UBFV2(B,H) +
SLAVARLBP4INVVJ(I,B,H,L) ;

 LBP5(I,B,H,L)$H1(B,H)..
         INVVJ(I,B,H,L) =L=
         UBI1(B)*FL2(I,B,H,L) +
LBFV2(B,H)*ITOT(B,L-1) -
LBI1(B)*LBFV2(B,H) +
SLAVARLBP5INVVJ(I,B,H,L) ;
```

```
 LBP6(I,B,H,L)$H1(B,H)..
         INVVT(I,B,H,L) =G=
         LBI1(B)*FT2(B,H,L) +
LBFV2(B,H)*INVI(I,B,L-1) -
LBI1(B)*LBFV2(B,H) +
SLAVARLBP6INVVT(I,B,H,L)     ;

 LBP7(I,B,H,L)$H1(B,H)..
         INVVT(I,B,H,L) =G=
         UBI1(B)*FT2(B,H,L) +
UBFV2(B,H)*INVI(I,B,L-1) -
UBI1(B)*UBFV2(B,H)      ;

 LBP8(I,B,H,L)$H1(B,H)..
         INVVT(H,B,H,L) =L=
         LBI1(B)*FT2(B,H,L) +
UBFV2(B,H)*INVI(I,B,L-1) -
LBI1(B)*UBFV2(B,H) +
SLAVARLBP8INVVT(I,B,H,L) ;

 LBP9(I,B,H,L)$H1(B,H)..
         INVVT(I,B,H,L) =L=
         UBI1(B)*FT2(B,H,L) +
LBFV2(B,H)*INVI(I,B,L-1) -
UBI1(B)*LBFV2(B,H)      ;


 LBP10(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
         FLOWVJ(I,A,B,H,L) =G=
         LBFV1(A,B)*FL2(I,B,H,L) +
LBFV2(B,H)*FT1(A,B,L) -
LBFV1(A,B)*LBFV2(B,H) ;

 LBP11(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
         FLOWVJ(I,A,B,H,L) =G=
         UBFV1(A,B)*FL2(I,B,H,L) +
UBFV2(B,H)*FT1(A,B,L) -
UBFV1(A,B)*UBFV2(B,H) ;

 LBP12(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
         FLOWVJ(I,A,B,H,L) =L=
         LBFV1(A,B)*FL2(I,B,H,L) +
UBFV2(B,H)*FT1(A,B,L) -
LBFV1(A,B)*UBFV2(B,H) ;

 LBP13(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
         FLOWVJ(I,A,B,H,L) =L=
         UBFV1(A,B)*FL2(I,B,H,L) +
LBFV2(B,H)*FT1(A,B,L) -
UBFV1(A,B)*LBFV2(B,H) +
SLAVARLBP13FLOWVJ(I,A,B,H,L) ;


 LBP14(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
         FLOWVT(I,A,B,H,L) =G=
         LBFV1(A,B)*FT2(B,H,L) +
LBFV2(B,H)*FL1(I,A,B,L) -
LBFV1(A,B)*LBFV2(B,H) +
SLAVARLBP14FLOWVT(I,A,B,H,L)  ;

 LBP15(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
         FLOWVT(I,A,B,H,L) =G=
         UBFV1(A,B)*FT2(B,H,L) +
UBFV2(B,H)*FL1(I,A,B,L) -
UBFV1(A,B)*UBFV2(B,H) ;

 LBP16(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
         FLOWVT(I,A,B,H,L) =L=
         LBFV1(A,B)*FT2(B,H,L) +
UBFV2(B,H)*FL1(I,A,B,L) -
LBFV1(A,B)*UBFV2(B,H) ;

 LBP17(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
         FLOWVT(I,A,B,H,L) =L=
         UBFV1(A,B)*FT2(B,H,L) +
LBFV2(B,H)*FL1(I,A,B,L) -
UBFV1(A,B)*LBFV2(B,H) ;



* ================ for generating
cutting planes using Lagrangean
decomposition ============ *


LBP10Q(I,A,B,H,L,Q)$(A1(A,B)$H1(B,H)).
.
         FLOWVJQ(I,A,B,H,L,Q) =G=
         LBFV1(A,B)*FL2(I,B,H,L) +
LBFV2(B,H)*FT1(A,B,L) -
LBFV1(A,B)*LBFV2(B,H) ;

*
LBP10Q(I,A,B,H,L,Q)$(A1(A,B)$H1(B,H)).
.
*FLOWVJQ(I,M,N,H,L,Q) =G=
*LBFV1(A,B)*FL2(I,B,H,L) +
LBFV2(B,H)*FT1(A,B,L) -
LBFV1(A,B)*LBFV2(B,H);



LBP11Q(I,A,B,H,L,Q)$(A1(A,B)$H1(B,H)).
.
         FLOWVJQ(I,A,B,H,L,Q) =G=
         UBFV1(A,B)*FL2(I,B,H,L) +
UBFV2(B,H)*FT1(A,B,L) -
UBFV1(A,B)*UBFV2(B,H) ;


LBP12Q(I,A,B,H,L,Q)$(A1(A,B)$H1(B,H)).
.
         FLOWVJQ(I,A,B,H,L,Q) =L=
         LBFV1(A,B)*FL2(I,B,H,L) +
UBFV2(B,H)*FT1(A,B,L) -
LBFV1(A,B)*UBFV2(B,H) ;


LBP13Q(I,A,B,H,L,Q)$(A1(A,B)$H1(B,H)).
.
         FLOWVJQ(I,A,B,H,L,Q) =L=
         UBFV1(A,B)*FL2(I,B,H,L) +
LBFV2(B,H)*FT1(A,B,L) -
UBFV1(A,B)*LBFV2(B,H) ;


LBP14Q(I,A,B,H,L,Q)$(A1(A,B)$H1(B,H)).
.
         FLOWVTQ(I,A,B,H,L,Q) =G=
         LBFV1(A,B)*FT2(B,H,L) +
LBFV2(B,H)*FL1(I,A,B,L) -
LBFV1(A,B)*LBFV2(B,H) ;

LBP15Q(I,A,B,H,L,Q)$(A1(A,B)$H1(B,H)).
.
         FLOWVTQ(I,A,B,H,L,Q) =G=
         UBFV1(A,B)*FT2(B,H,L) +
UBFV2(B,H)*FL1(I,A,B,L) -
UBFV1(A,B)*UBFV2(B,H) ;


LBP16Q(I,A,B,H,L,Q)$(A1(A,B)$H1(B,H)).
.
         FLOWVTQ(I,A,B,H,L,Q) =L=
         LBFV1(A,B)*FT2(B,H,L) +
UBFV2(B,H)*FL1(I,A,B,L) -
LBFV1(A,B)*UBFV2(B,H) ;
```

```
LBP17Q(I,A,B,H,L,Q)$(A1(A,B)$H1(B,H)).
.
         FLOWVTQ(I,A,B,H,L,Q) =L=
         UBFV1(A,B)*FT2(B,H,L) +
LBFV2(B,H)*FL1(I,A,B,L) -
UBFV1(A,B)*LBFV2(B,H) ;



* ========================== sub-
structure Q1 (w/o Q index)
============================ *

 LBP1Q1(I,B,H,L)$H1(B,H)..
         INVVJQ1(I,B,H,L) +
SUM(A$A1(A,B),FLOWVJQ1(I,A,B,H,L)) =E=
         INVVTQ1(I,B,H,L) +
SUM(A$A1(A,B),FLOWVTQ1(I,A,B,H,L))
;


 LBP2Q1(I,B,H,L)$H1(B,H)..
         INVVJQ1(I,B,H,L) =G=
         LBI1(B)*FL2Q1(I,B,H,L) +
LBFV2(B,H)*ITOT(B,L-1) -
LBI1(B)*LBFV2(B,H)      ;

 LBP3Q1(I,B,H,L)$H1(B,H)..
         INVVJQ1(I,B,H,L) =G=
         UBI1(B)*FL2Q1(I,B,H,L) +
UBFV2(B,H)*ITOT(B,L-1) -
UBI1(B)*UBFV2(B,H)      ;

 LBP4Q1(I,B,H,L)$H1(B,H)..
         INVVJQ1(I,B,H,L) =L=
         LBI1(B)*FL2Q1(I,B,H,L) +
UBFV2(B,H)*ITOT(B,L-1) -
LBI1(B)*UBFV2(B,H)+
SLAVARLBP4INVVJQ1(I,B,H,L)      ;

 LBP5Q1(I,B,H,L)$H1(B,H)..
         INVVJQ1(I,B,H,L) =L=
         UBI1(B)*FL2Q1(I,B,H,L) +
LBFV2(B,H)*ITOT(B,L-1) -
LBI1(B)*LBFV2(B,H) +
SLAVARLBP5INVVJQ1(I,B,H,L)       ;

 LBP6Q1(I,B,H,L)$H1(B,H)..
         INVVTQ1(I,B,H,L) =G=
         LBI1(B)*FT2Q1(B,H,L) +
LBFV2(B,H)*INVI(I,B,L-1) -
LBI1(B)*LBFV2(B,H) +
SLAVARLBP6INVVTQ1(I,B,H,L)     ;

 LBP7Q1(I,B,H,L)$H1(B,H)..
         INVVTQ1(I,B,H,L) =G=
         UBI1(B)*FT2Q1(B,H,L) +
UBFV2(B,H)*INVI(I,B,L-1) -
UBI1(B)*UBFV2(B,H)      ;

 LBP8Q1(I,B,H,L)$H1(B,H)..
         INVVTQ1(I,B,H,L) =L=
         LBI1(B)*FT2Q1(B,H,L) +
UBFV2(B,H)*INVI(I,B,L-1) -
LBI1(B)*UBFV2(B,H)  +
SLAVARLBP8INVVTQ1(I,B,H,L)     ;

 LBP9Q1(I,B,H,L)$H1(B,H)..
         INVVTQ1(I,B,H,L) =L=
         UBI1(B)*FT2Q1(B,H,L) +
LBFV2(B,H)*INVI(I,B,L-1) -
UBI1(B)*LBFV2(B,H)      ;


LBP10Q1(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
         FLOWVJQ1(I,A,B,H,L) =G=
         LBFV1(A,B)*FL2Q1(I,B,H,L) +
LBFV2(B,H)*FT1Q1(A,B,L) -
LBFV1(A,B)*LBFV2(B,H) ;


LBP11Q1(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
         FLOWVJQ1(I,A,B,H,L) =G=
         UBFV1(A,B)*FL2Q1(I,B,H,L) +
UBFV2(B,H)*FT1Q1(A,B,L) -
UBFV1(A,B)*UBFV2(B,H) ;


LBP12Q1(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
         FLOWVJQ1(I,A,B,H,L) =L=
         LBFV1(A,B)*FL2Q1(I,B,H,L) +
UBFV2(B,H)*FT1Q1(A,B,L) -
LBFV1(A,B)*UBFV2(B,H) ;


LBP13Q1(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
         FLOWVJQ1(I,A,B,H,L) =L=
         UBFV1(A,B)*FL2Q1(I,B,H,L) +
LBFV2(B,H)*FT1Q1(A,B,L) -
UBFV1(A,B)*LBFV2(B,H) +
SLAVARLBP13FLOWVJQ1(I,A,B,H,L) ;



LBP14Q1(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
         FLOWVTQ1(I,A,B,H,L) =G=
         LBFV1(A,B)*FT2Q1(B,H,L) +
LBFV2(B,H)*FL1Q1(I,A,B,L) -
LBFV1(A,B)*LBFV2(B,H) +
SLAVARLBP14FLOWVTQ1(I,A,B,H,L);


LBP15Q1(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
         FLOWVTQ1(I,A,B,H,L) =G=
         UBFV1(A,B)*FT2Q1(B,H,L) +
UBFV2(B,H)*FL1Q1(I,A,B,L) -
UBFV1(A,B)*UBFV2(B,H) ;


LBP16Q1(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
         FLOWVTQ1(I,A,B,H,L) =L=
         LBFV1(A,B)*FT2Q1(B,H,L) +
UBFV2(B,H)*FL1Q1(I,A,B,L) -
LBFV1(A,B)*UBFV2(B,H) ;


LBP17Q1(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
         FLOWVTQ1(I,A,B,H,L) =L=
         UBFV1(A,B)*FT2Q1(B,H,L) +
LBFV2(B,H)*FL1Q1(I,A,B,L) -
UBFV1(A,B)*LBFV2(B,H) ;



* ========================== sub-
structure Q2 (w/o Q index)
============================ *

 LBP1Q2(I,B,H,L)$H1(B,H)..
         INVVJQ2(I,B,H,L) +
SUM(A$A1(A,B),FLOWVJQ2(I,A,B,H,L)) =E=
         INVVTQ2(I,B,H,L) +
SUM(A$A1(A,B),FLOWVTQ2(I,A,B,H,L))
;


 LBP2Q2(I,B,H,L)$H1(B,H)..
         INVVJQ2(I,B,H,L) =G=
         LBI1(B)*FL2Q2(I,B,H,L) +
LBFV2(B,H)*ITOT(B,L-1) -
LBI1(B)*LBFV2(B,H)      ;
```

```
 LBP3Q2(I,B,H,L)$H1(B,H)..
         INVVJQ2(I,B,H,L) =G=
         UBI1(B)*FL2Q2(I,B,H,L) +
UBFV2(B,H)*ITOT(B,L-1) -
UBI1(B)*UBFV2(B,H)      ;

 LBP4Q2(I,B,H,L)$H1(B,H)..
         INVVJQ2(I,B,H,L) =L=
         LBI1(B)*FL2Q2(I,B,H,L) +
UBFV2(B,H)*ITOT(B,L-1) -
LBI1(B)*UBFV2(B,H) +
SLAVARLBP4INVVJQ2(I,B,H,L)     ;

 LBP5Q2(I,B,H,L)$H1(B,H)..
         INVVJQ2(I,B,H,L) =L=
         UBI1(B)*FL2Q2(I,B,H,L) +
LBFV2(B,H)*ITOT(B,L-1) -
LBI1(B)*LBFV2(B,H)  +
SLAVARLBP5INVVJQ2(I,B,H,L)      ;

 LBP6Q2(I,B,H,L)$H1(B,H)..
         INVVTQ2(I,B,H,L) =G=
         LBI1(B)*FT2Q2(B,H,L) +
LBFV2(B,H)*INVI(I,B,L-1) -
LBI1(B)*LBFV2(B,H) +
SLAVARLBP6INVVTQ2(I,B,H,L)     ;

 LBP7Q2(I,B,H,L)$H1(B,H)..
         INVVTQ2(I,B,H,L) =G=
         UBI1(B)*FT2Q2(B,H,L) +
UBFV2(B,H)*INVI(I,B,L-1) -
UBI1(B)*UBFV2(B,H)      ;

 LBP8Q2(I,B,H,L)$H1(B,H)..
         INVVTQ2(I,B,H,L) =L=
         LBI1(B)*FT2Q2(B,H,L) +
UBFV2(B,H)*INVI(I,B,L-1) -
LBI1(B)*UBFV2(B,H)  +
SLAVARLBP8INVVTQ2(I,B,H,L)     ;

 LBP9Q2(I,B,H,L)$H1(B,H)..
         INVVTQ2(I,B,H,L) =L=
         UBI1(B)*FT2Q2(B,H,L) +
LBFV2(B,H)*INVI(I,B,L-1) -
UBI1(B)*LBFV2(B,H)      ;


LBP10Q2(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
         FLOWVJQ2(I,A,B,H,L) =G=
         LBFV1(A,B)*FL2Q2(I,B,H,L) +
LBFV2(B,H)*FT1Q2(A,B,L) -
LBFV1(A,B)*LBFV2(B,H) ;


LBP11Q2(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
         FLOWVJQ2(I,A,B,H,L) =G=
         UBFV1(A,B)*FL2Q2(I,B,H,L) +
UBFV2(B,H)*FT1Q2(A,B,L) -
UBFV1(A,B)*UBFV2(B,H) ;


LBP12Q2(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
         FLOWVJQ2(I,A,B,H,L) =L=
         LBFV1(A,B)*FL2Q2(I,B,H,L) +
UBFV2(B,H)*FT1Q2(A,B,L) -
LBFV1(A,B)*UBFV2(B,H) ;


LBP13Q2(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
         FLOWVJQ2(I,A,B,H,L) =L=
         UBFV1(A,B)*FL2Q2(I,B,H,L) +
LBFV2(B,H)*FT1Q2(A,B,L) -
UBFV1(A,B)*LBFV2(B,H)+
SLAVARLBP13FLOWVJQ2(I,A,B,H,L) ;

LBP14Q2(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
         FLOWVTQ2(I,A,B,H,L) =G=
         LBFV1(A,B)*FT2Q2(B,H,L) +
LBFV2(B,H)*FL1Q2(I,A,B,L) -
LBFV1(A,B)*LBFV2(B,H) +
SLAVARLBP14FLOWVTQ2(I,A,B,H,L);


LBP15Q2(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
         FLOWVTQ2(I,A,B,H,L) =G=
         UBFV1(A,B)*FT2Q2(B,H,L) +
UBFV2(B,H)*FL1Q2(I,A,B,L) -
UBFV1(A,B)*UBFV2(B,H) ;


LBP16Q2(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
         FLOWVTQ2(I,A,B,H,L) =L=
         LBFV1(A,B)*FT2Q2(B,H,L) +
UBFV2(B,H)*FL1Q2(I,A,B,L) -
LBFV1(A,B)*UBFV2(B,H) ;


LBP17Q2(I,A,B,H,L)$(A1(A,B)$H1(B,H))..
         FLOWVTQ2(I,A,B,H,L) =L=
         UBFV1(A,B)*FT2Q2(B,H,L) +
LBFV2(B,H)*FL1Q2(I,A,B,L) -
UBFV1(A,B)*LBFV2(B,H) ;



 LBP18(M,K,L)$K1(M,K)..
         FLTQ1(M,K,L) =E=
         FLTQ2(M,K,L)
;

 LBP19(I,M,K,L)$K1(M,K)..
         FLQ1(I,M,K,L) =E=
         FLQ1(I,M,K,L)
;

 LBP20(M,K,L)$K1(M,K)..
         TSQ1(M,K,L) =E=
         TSQ2(M,K,L)
;

 LBP21(M,K,L)$K1(M,K)..
         TEQ1(M,K,L) =E=
         TEQ2(M,K,L)
;

 LBP22(M,K,L)$K1(M,K)..
         WBQ1(M,K,L) =E=
         WBQ2(M,K,L)
;


**(vii)Inventory Bounds

 TC26(B,L)$(ORD(L) GT 1)..
         ITOT(B,L-
1)+SUM(A$A1(A,B),FT1(A,B,L)) =L=
         UBI1(B)
;


**(viii)Bounds on Components Fractions
inside a Tank

 TC27(I,B,L)..
         LB1(I,B)*ITOT(B,L) =L=
         INVI(I,B,L) +
SLAVARTC27INVI(I,B,L)
;
```

```
 TC28(I,B,L)..
         INVI(I,B,L) =L=
         UB1(I,B)*ITOT(B,L)
;

 TC29(I,B,H,L)$H1(B,H)..
         LB1(I,B)*FT2(B,H,L) =L=
         FL2(I,B,H,L)
;

 TC30(I,B,H,L)$H1(B,H)..
         FL2(I,B,H,L) =L=
         UB1(I,B)*FT2(B,H,L)
;


**(ix)Crude-mix Demand Constraints

 TC31(G)..

SUM(D$D3(D,G),SUM(L,FT2(G,D,L))) =E=
         DM(G)
;


**(x)Bound Strengthening Cuts

 TC32(I,B,H,L)$H1(B,H)..
         ITOT(B,L-1)*FL2(I,B,H,L) +
SUM(A$A1(A,B),FT1(A,B,L)*FL2(I,B,H,L))
=E=
         INVI(I,B,L-
1)*FT2(B,H,L)+SUM(A$A1(A,B),FL1(I,A,B,
L)*FT2(B,H,L))         ;

 TC33(B,H,L)$H1(B,H)..
         SUM(I,ITOT(B,L-
1)*FL2(I,B,H,L)) =E=
         ITOT(B,L-1)*FT2(B,H,L)
;

 TC34(A,B,H,L)$(A1(A,B)$H1(B,H))..

SUM(I,FT1(A,B,L)*FL2(I,B,H,L)) =E=
         FT1(A,B,L)*FT2(B,H,L)
;

 TC35(B,H,L)$H1(B,H)..
         SUM(I,INVI(I,B,L-
1)*FT2(B,H,L)) =E=
         ITOT(B,L-1)*FT2(B,H,L)
;

 TC36(A,B,H,L)$(A1(A,B)$H1(B,H))..

SUM(I,FL1(I,A,B,L)*FT2(B,H,L)) =E=
         FT1(A,B,L)*FT2(B,H,L)
;



* ===========================
2.DISTILLATION UNIT CONSTRAINTS
=========================== *


**(i)Allocation Constraints

 DC1(D,L)..
         SUM(G$G1(G,D),WB2(G,D,L)) =L=
         1 + SLAVARDC1WB2(D,L)
;

 DC2(G,L)..
         SUM(D$D3(D,G),WB2(G,D,L)) =L=
         1 + SLAVARDC2WB2(G,L)
;


 DCUB1(D,L)..
         SUM(G$G1(G,D),WBUB2(G,D,L))
=L=
         1 + SLAVARDCUB1WBUB2(D,L)
;

 DCUB2(G,L)..
         SUM(D$D3(D,G),WBUB2(G,D,L))
=L=
         1 + SLAVARDCUB2WBUB2(G,L)
;


**(ii)Continuous Operation Constraints

 DC3(D)..

SUM(L,SUM(G$G1(G,D),TE2(G,D,L)-
TS2(G,D,L))) =E=
         THOR
;

 DC4(G,GDASH,D,L)$(ORD(L) LT
CARD(L)$(ORD(G) NE
ORD(GDASH)$G1(G,D)))..
         TS2(G,D,L+1) =G=
         TE2(GDASH,D,L) - THOR*(1-
WB2(GDASH,D,L))
;

 DC5(G,GDASH,D,L)$(ORD(L) LT
CARD(L)$(ORD(G) NE
ORD(GDASH)$G1(G,D)))..
         TS2(G,D,L+1) =L=
         TE2(GDASH,D,L) + THOR*(1-
WB2(GDASH,D,L))
;


 DCUB4(G,GDASH,D,L)$(ORD(L) LT
CARD(L)$(ORD(G) NE
ORD(GDASH)$G1(G,D)))..
         TS2(G,D,L+1) =G=
         TE2(GDASH,D,L) - THOR*(1-
WBUB2(GDASH,D,L))
;

 DCUB5(G,GDASH,D,L)$(ORD(L) LT
CARD(L)$(ORD(G) NE
ORD(GDASH)$G1(G,D)))..
         TS2(G,D,L+1) =L=
         TE2(GDASH,D,L) + THOR*(1-
WBUB2(GDASH,D,L))
;


*3.CRUDE SUPPLY STREAM CONSTRAINTS

**(i)Timing Constraints

 CC1(U,Y,L)$Y1(Y,U)..
         TSI(U) =L=
         TS1(U,Y,L) + THOR*(1-
WB1(U,Y,L))
;

 CC2(U,Y,L)$Y1(Y,U)..
         TEO(U) =G=
         TE1(U,Y,L) - THOR*(1-
WB1(U,Y,L))
;
```

```
 CCUB1(U,Y,L)$Y1(Y,U)..
         TSI(U) =L=
         TS1(U,Y,L) + THOR*(1-
WBUB1(U,Y,L))
;

 CCUB2(U,Y,L)$Y1(Y,U)..
         TEO(U) =G=
         TE1(U,Y,L) - THOR*(1-
WBUB1(U,Y,L))
;


**(ii)Overall Mass Balances

 CC3(U)..

SUM(L,SUM(Y$Y1(Y,U),FT1(U,Y,L))) =E=
         VCRUDE(U)
;


**(iii)Component Balances

 CC4(I,U,Y,L)$Y1(Y,U)..
         FL1(I,U,Y,L) =E=
         FRACT(I,U)*FT1(U,Y,L)
;



*4.VARIABLE BOUNDS

 VB1(I,B,L)..
         INVI(I,B,L) =L=
         UBI1(B)
;

 VB2(B,L)..
         LBI1(B) =L=
         ITOT(B,L)
;

 VB3(B,L)..
         ITOT(B,L) =L=
         UBI1(B)
;

 VB4(I,A,B,L)$A1(A,B)..
         FL1(I,A,B,L) =L=
         UBFV1(A,B)
;

 VB5(I,B,H,L)$H1(B,H)..
         FL2(I,B,H,L) =L=
         UBFV2(B,H)
;

 VB6(A,B,L)$A1(A,B)..
         FT1(A,B,L) =L=
         UBFV1(A,B)
;

 VB7(B,H,L)$H1(B,H)..
         FT2(B,H,L) =L=
         UBFV2(B,H)
;

 VB8(A,B,L)$A1(A,B)..
         TS1(A,B,L) =L=
         THOR
;

 VB9(A,B,L)$A1(A,B)..
         TE1(A,B,L) =L=
         THOR
;

 VB10(B,H,L)$H1(B,H)..
         TS2(B,H,L) =L=
         THOR
;

 VB11(B,H,L)$H1(B,H)..
         TE2(B,H,L) =L=
         THOR
;

 VB12(U)..
         TARR(U) =L=
         TSI(U)
;

 VB13(U)..
         TSI(U) =L=
         THOR
;

 VB14(U)..
         TARR(U) =L=
         TEO(U)
;

 VB15(U)..
         TEO(U) =L=
         THOR
;



* =============================
GENERATE VALID LINEAR CUTS
=============================== *

 VLC1..
         ZZQ1.L =L=
         CSEA*SUM(U1,TSI(U1)-TARR(U1))
+ CUNLOAD*SUM(U1,TEO(U1)-TSI(U1)) +
THOR*(SUM(B1,CINV(B1))*SUM(B1,SUM(L,(I
TOT(B1,L)))) +

SUM(B1,CINV(B1))*SUM(B1,SUM(L,SUM(A,FT
1(A,B1,L)))) +
SUM(B1,CINV(B1))*SUM(B1,SUM(L$(ORD(L)
LT CARD(L)),ITOT(B1,L))) +

THOR*SUM(B1,CINV(B1))*SUM(B2,INITITOT(
B2))) / (2*NE + 1) +
CSET*(SUM(D1,SUM(G$G1(G,D1),SUM(L,WB2(
G,D1,L)))) - ND1) +

SUM(M,SUM(K$K1(M,K),SUM(L,LMFLT(M,K,L)
*FLTQ(M,K,L)))) +
SUM(I,SUM(M,SUM(K$K1(M,K),SUM(L,LMFL(I
,M,K,L)*FLQ(I,M,K,L))))) +

SUM(M,SUM(K$K1(M,K),SUM(L,LMTS(M,K,L)*
TSQ(M,K,L)))) +
SUM(M,SUM(K$K1(M,K),SUM(L,LMTE(M,K,L)*
TEQ(M,K,L)))) +

SUM(M,SUM(K$K1(M,K),SUM(L,LMWB(M,K,L)*
WBQ(M,K,L))))
;


 VLC2..
         ZZQ2.L =L=
         CSEA*SUM(U2,TSI(U2)-TARR(U2))
+ CUNLOAD*SUM(U2,TEO(U2)-TSI(U2)) +
```

```
THOR*(SUM(B2,CINV(B2))*SUM(B2,SUM(L,(I
TOT(B2,L)))) +

SUM(B2,CINV(B2))*SUM(B2,SUM(L,SUM(A,FT
1(A,B2,L)))) +
SUM(B2,CINV(B2))*SUM(B2,SUM(L$(ORD(L)
LT CARD(L)),ITOT(B2,L))) +

THOR*SUM(B2,CINV(B2))*SUM(B2,INITITOT(
B2))) / (2*NE + 1) +
CSET*(SUM(D2,SUM(G$G1(G,D2),SUM(L,WB2(
G,D2,L)))) - ND2) -

SUM(M,SUM(K$K1(M,K),SUM(L,LMFLT(M,K,L)
*FLTQ(M,K,L)))) -
SUM(I,SUM(M,SUM(K$K1(M,K),SUM(L,LMFL(I
,M,K,L)*FLQ(I,M,K,L))))) -

SUM(M,SUM(K$K1(M,K),SUM(L,LMTS(M,K,L)*
TSQ(M,K,L)))) -
SUM(M,SUM(K$K1(M,K),SUM(L,LMTE(M,K,L)*
TEQ(M,K,L)))) -

SUM(M,SUM(K$K1(M,K),SUM(L,LMWB(M,K,L)*
WBQ(M,K,L))))
;



* ================== FOR DUPLICATING
EQUATIONS AND VARIABLES IN Q1
============= *

 TC1Q1(A,B,L)$A1(A,B)..
         FT1Q1(A,B,L) =L=
         UBFV1(A,B)*WB1Q1(A,B,L)
;

 TC2Q1(B,H,L)$H1(B,H)..
         FT2Q1(B,H,L) =L=
         UBFV2(B,H)*WB2Q1(B,H,L)
;

 TC3Q1(A,B,L)$A1(A,B)..
         UBF1(A,B)*(TE1Q1(A,B,L)-
TS1Q1(A,B,L)) + UBF1(A,B)*THOR*(1-
WB1Q1(A,B,L)) =G=
         FT1Q1(A,B,L) +
SLAVARTC3FT1Q1(A,B,L)
;

 TC4Q1(B,H,L)$H1(B,H)..
         UBF2(B,H)*(TE2Q1(B,H,L)-
TS2Q1(B,H,L)) + UBF2(B,H)*THOR*(1-
WB2Q1(B,H,L)) =G=
         FT2Q1(B,H,L)
;

 TC5Q1(A,Y,L)$A2(A,Y)..
         LBF1(A,Y)*(TE1Q1(A,Y,L)-
TS1Q1(A,Y,L)) - LBF1(A,Y)*THOR*(1-
WB1Q1(A,Y,L)) =L=
         FT1Q1(A,Y,L)
;

 TC6Q1(Y,H,L)$H2(Y,H)..
         LBF2(Y,H)*(TE2Q1(Y,H,L)-
TS2Q1(Y,H,L)) - LBF2(Y,H)*THOR*(1-
WB2Q1(Y,H,L)) =L=
         FT2Q1(Y,H,L)
;

 TC7Q1(G,H,L)$H3(G,H)..
         LBF2(G,H)*(TE2Q1(G,H,L)-
TS2Q1(G,H,L)) =L=
         FT2Q1(G,H,L)
;
 TC8Q1(A,B,L)$(ORD(L) LT
CARD(L)$A1(A,B))..
         TS1Q1(A,B,L+1) =G=
         TE1Q1(A,B,L) - THOR*(1-
WB1Q1(A,B,L))
;

 TC9Q1(A,B,L)$(ORD(L) LT
CARD(L)$A1(A,B))..
         TS1Q1(A,B,L+1) =G=
         TS1Q1(A,B,L)
;

 TC10Q1(A,B,L)$(ORD(L) LT
CARD(L)$A1(A,B))..
         TE1Q1(A,B,L+1) =G=
         TE1Q1(A,B,L)
;

 TC11Q1(B,H,L)$(ORD(L) LT
CARD(L)$H1(B,H))..
         TS2Q1(B,H,L+1) =G=
         TE2Q1(B,H,L) - THOR*(1-
WB2Q1(B,H,L))
;

 TC12Q1(B,H,L)$(ORD(L) LT
CARD(L)$H1(B,H))..
         TS2Q1(B,H,L+1) =G=
         TS2Q1(B,H,L)
;

 TC13Q1(B,H,L)$(ORD(L) LT
CARD(L)$H1(B,H))..
         TE2Q1(B,H,L+1) =G=
         TE2Q1(B,H,L)
;

 TC14Q1(A,ADASH,B,L)$(ORD(L) LT
CARD(L)$(ORD(A) NE
ORD(ADASH)$A1(A,B)))..
         TS1Q1(A,B,L+1) =G=
         TE1Q1(ADASH,B,L) - THOR*(1-
WB1Q1(ADASH,B,L))
;

 TC15Q1(A,B,H,L)$(ORD(L) LT
CARD(L)$(A1(A,B)$H1(B,H)))..
         TS1Q1(A,B,L+1) =G=
         TE2Q1(B,H,L) - THOR*(1-
WB2Q1(B,H,L))
;

 TC16Q1(A,B,H,L)$(ORD(L) LT
CARD(L)$(A1(A,B)$H1(B,H)))..
         TS2Q1(B,H,L+1) =G=
         TE1Q1(A,B,L) - THOR*(1-
WB1Q1(A,B,L))
;

 TC17Q1(B,H,HDASH,L)$(ORD(L) LT
CARD(L)$(ORD(H) NE
ORD(HDASH)$H1(B,H)))..
         TS2Q1(B,H,L+1) =G=
         TE2Q1(B,HDASH,L) - THOR*(1-
WB2Q1(B,HDASH,L))
;

 TC18Q1(A,B,H,L)$(A1(A,B)$H1(B,H))..
         TE1Q1(A,B,L) - THOR*(1-
WB1Q1(A,B,L)) =L=
         TS2Q1(B,H,L) + THOR*(1-
WB2Q1(B,H,L))
;
```

```
 TC19Q1(B,L)..
         ITOT(B,L-1) +
SUM(A$A1(A,B),FT1Q1(A,B,L)) =E=
         ITOT(B,L) +
SUM(H$H1(B,H),FT2Q1(B,H,L))  +
SLAVARTC19ITOTQ1(B,L)
;
*- ONEG1Q1(B,L) + OPOS1Q1(B,L)
;

 TC20Q1(B)..
         ITOT(B,'L0') =E=
         INITITOT(B)
;

 TC21Q1(I,B,L)..
         INVI(I,B,L-1) +
SUM(A$A1(A,B),FL1Q1(I,A,B,L)) =E=
         INVI(I,B,L)
+SUM(H$H1(B,H),FL2Q1(I,B,H,L)) +
SLAVARTC21INVIQ1(I,B,L)
;

 TC22Q1(I,B)..
         INVI(I,B,'L0') =E=
         INITINVI(I,B)
;

 TC23Q1(A,B,L)$A1(A,B)..
         FT1Q1(A,B,L) =E=
         SUM(I,FL1Q1(I,A,B,L)) +
SLAVARTC23FT1Q1(A,B,L)
;

 TC24Q1(B,H,L)$H1(B,H)..
         FT2Q1(B,H,L) =E=
         SUM(I,FL2Q1(I,B,H,L)) +
SLAVARTC24FT2Q1(B,H,L)
;

 TC26Q1(B,L)$(ORD(L) GT 1)..
         ITOT(B,L-
1)+SUM(A$A1(A,B),FT1Q1(A,B,L)) =L=
         UBI1(B)
;

 TC27Q1(I,B,L)..
         LB1(I,B)*ITOT(B,L) =L=
         INVI(I,B,L) +
SLAVARTC27INVIQ1(I,B,L)
;

* - ONEG1Q1(I,B,L) + OPOS1Q1(I,B,L)
;
 ;
 TC28Q1(I,B,L)..
         INVI(I,B,L) =L=
         UB1(I,B)*ITOT(B,L)
;

 TC29Q1(I,B,H,L)$H1(B,H)..
         LB1(I,B)*FT2Q1(B,H,L) =L=
         FL2Q1(I,B,H,L)
;

 TC30Q1(I,B,H,L)$H1(B,H)..
         FL2Q1(I,B,H,L) =L=
         UB1(I,B)*FT2Q1(B,H,L)
;

 TC31Q1(G)..

SUM(D$D3(D,G),SUM(L,FT2Q1(G,D,L))) =E=
         DM(G)
;
```

```
 TC32Q1(I,B,H,L)$H1(B,H)..
         ITOT(B,L-1)*FL2Q1(I,B,H,L) +
SUM(A$A1(A,B),FT1Q1(A,B,L)*FL2Q1(I,B,H
,L)) =E=
         INVI(I,B,L-
1)*FT2Q1(B,H,L)+SUM(A$A1(A,B),FL1Q1(I,
A,B,L)*FT2Q1(B,H,L))         ;

 TC33Q1(B,H,L)$H1(B,H)..
         SUM(I,ITOT(B,L-
1)*FL2Q1(I,B,H,L)) =E=
         ITOT(B,L-1)*FT2Q1(B,H,L)
;

 TC34Q1(A,B,H,L)$(A1(A,B)$H1(B,H))..

SUM(I,FT1Q1(A,B,L)*FL2Q1(I,B,H,L)) =E=
         FT1Q1(A,B,L)*FT2Q1(B,H,L)
;

 TC35Q1(B,H,L)$H1(B,H)..
         SUM(I,INVI(I,B,L-
1)*FT2Q1(B,H,L)) =E=
         ITOT(B,L-1)*FT2Q1(B,H,L)
;

 TC36Q1(A,B,H,L)$(A1(A,B)$H1(B,H))..

SUM(I,FL1Q1(I,A,B,L)*FT2Q1(B,H,L)) =E=
         FT1Q1(A,B,L)*FT2Q1(B,H,L)
;

 DC1Q1(D,L)..
         SUM(G$G1(G,D),WB2Q1(G,D,L))
=L=
         1 + SLAVARDC1WB2Q1(D,L)
;

 DC2Q1(G,L)..
         SUM(D$D3(D,G),WB2Q1(G,D,L))
=L=
         1   + SLAVARDC2WB2Q2(G,L)
;

 DC3Q1(D)..

SUM(L,SUM(G$G1(G,D),TE2Q1(G,D,L)-
TS2Q1(G,D,L))) =E=
         THOR
;

 DC4Q1(G,GDASH,D,L)$(ORD(L) LT
CARD(L)$(ORD(G) NE
ORD(GDASH)$G1(G,D)))..
         TS2Q1(G,D,L+1) =G=
         TE2Q1(GDASH,D,L) - THOR*(1-
WB2Q1(GDASH,D,L))
;

 DC5Q1(G,GDASH,D,L)$(ORD(L) LT
CARD(L)$(ORD(G) NE
ORD(GDASH)$G1(G,D)))..
         TS2Q1(G,D,L+1) =L=
         TE2Q1(GDASH,D,L) + THOR*(1-
WB2Q1(GDASH,D,L))
;

 CC1Q1(U,Y,L)$Y1(Y,U)..
         TSI(U) =L=
         TS1Q1(U,Y,L) + THOR*(1-
WB1Q1(U,Y,L))
;

 CC2Q1(U,Y,L)$Y1(Y,U)..
         TEO(U) =G=
```

```
         TE1Q1(U,Y,L) - THOR*(1-
WB1Q1(U,Y,L))
;

 CC3Q1(U)..

SUM(L,SUM(Y$Y1(Y,U),FT1Q1(U,Y,L))) =E=
         VCRUDE(U)
;

 CC4Q1(I,U,Y,L)$Y1(Y,U)..
         FL1Q1(I,U,Y,L) =E=
         FRACT(I,U)*FT1Q1(U,Y,L)
;

 VB1Q1(I,B,L)..
         INVI(I,B,L) =L=
         UBI1(B)
;

 VB2Q1(B,L)..
         LBI1(B) =L=
         ITOT(B,L)
;

 VB3Q1(B,L)..
         ITOT(B,L) =L=
         UBI1(B)
;

 VB4Q1(I,A,B,L)$A1(A,B)..
         FL1Q1(I,A,B,L) =L=
         UBFV1(A,B)
;

 VB5Q1(I,B,H,L)$H1(B,H)..
         FL2Q1(I,B,H,L) =L=
         UBFV2(B,H)
;

 VB6Q1(A,B,L)$A1(A,B)..
         FT1Q1(A,B,L) =L=
         UBFV1(A,B)
;

 VB7Q1(B,H,L)$H1(B,H)..
         FT2Q1(B,H,L) =L=
         UBFV2(B,H)
;

 VB8Q1(A,B,L)$A1(A,B)..
         TS1Q1(A,B,L) =L=
         THOR
;

 VB9Q1(A,B,L)$A1(A,B)..
         TE1Q1(A,B,L) =L=
         THOR
;

 VB10Q1(B,H,L)$H1(B,H)..
         TS2Q1(B,H,L) =L=
         THOR
;

 VB11Q1(B,H,L)$H1(B,H)..
         TE2Q1(B,H,L) =L=
         THOR
;

 VB12Q1(U)..
         TARR(U) =L=
         TSI(U)
;

 VB13Q1(U)..
         TSI(U) =L=
         THOR
;

 VB14Q1(U)..
         TARR(U) =L=
         TEO(U)
;

 VB15Q1(U)..
         TEO(U) =L=
         THOR
;

* =================== FOR DUPLICATING
EQUATIONS AND VARIABLES IN Q2
============= *

 TC1Q2(A,B,L)$A1(A,B)..
         FT1Q2(A,B,L) =L=
         UBFV1(A,B)*WB1Q2(A,B,L)
;

 TC2Q2(B,H,L)$H1(B,H)..
         FT2Q2(B,H,L) =L=
         UBFV2(B,H)*WB2Q2(B,H,L)
;

 TC3Q2(A,B,L)$A1(A,B)..
         UBF1(A,B)*(TE1Q2(A,B,L)-
TS1Q2(A,B,L)) + UBF1(A,B)*THOR*(1-
WB1Q2(A,B,L)) =G=
         FT1Q2(A,B,L) +
SLAVARTC3FT1Q2(A,B,L)
;

 TC4Q2(B,H,L)$H1(B,H)..
         UBF2(B,H)*(TE2Q2(B,H,L)-
TS2Q2(B,H,L)) + UBF2(B,H)*THOR*(1-
WB2Q2(B,H,L)) =G=
         FT2Q2(B,H,L)
;

 TC5Q2(A,Y,L)$A2(A,Y)..
         LBF1(A,Y)*(TE1Q2(A,Y,L)-
TS1Q2(A,Y,L)) - LBF1(A,Y)*THOR*(1-
WB1Q2(A,Y,L)) =L=
         FT1Q2(A,Y,L)
;

 TC6Q2(Y,H,L)$H2(Y,H)..
         LBF2(Y,H)*(TE2Q2(Y,H,L)-
TS2Q2(Y,H,L)) - LBF2(Y,H)*THOR*(1-
WB2Q2(Y,H,L)) =L=
         FT2Q2(Y,H,L)
;

 TC7Q2(G,H,L)$H3(G,H)..
         LBF2(G,H)*(TE2Q2(G,H,L)-
TS2Q2(G,H,L)) =L=
         FT2Q2(G,H,L)
;
 TC8Q2(A,B,L)$(ORD(L) LT
CARD(L)$A1(A,B))..
         TS1Q2(A,B,L+1) =G=
         TE1Q2(A,B,L) - THOR*(1-
WB1Q2(A,B,L))
;

 TC9Q2(A,B,L)$(ORD(L) LT
CARD(L)$A1(A,B))..
         TS1Q2(A,B,L+1) =G=
         TS1Q2(A,B,L)
;
```

```
 TC10Q2(A,B,L)$(ORD(L) LT
CARD(L)$A1(A,B))..
         TE1Q2(A,B,L+1) =G=
         TE1Q2(A,B,L)
;

 TC11Q2(B,H,L)$(ORD(L) LT
CARD(L)$H1(B,H))..
         TS2Q2(B,H,L+1) =G=
         TE2Q2(B,H,L) - THOR*(1-
WB2Q2(B,H,L))
;

 TC12Q2(B,H,L)$(ORD(L) LT
CARD(L)$H1(B,H))..
         TS2Q2(B,H,L+1) =G=
         TS2Q2(B,H,L)
;

 TC13Q2(B,H,L)$(ORD(L) LT
CARD(L)$H1(B,H))..
         TE2Q2(B,H,L+1) =G=
         TE2Q2(B,H,L)
;

 TC14Q2(A,ADASH,B,L)$(ORD(L) LT
CARD(L)$(ORD(A) NE
ORD(ADASH)$A1(A,B)))..
         TS1Q2(A,B,L+1) =G=
         TE1Q2(ADASH,B,L) - THOR*(1-
WB1Q2(ADASH,B,L))
;

 TC15Q2(A,B,H,L)$(ORD(L) LT
CARD(L)$(A1(A,B)$H1(B,H)))..
         TS1Q2(A,B,L+1) =G=
         TE2Q2(B,H,L) - THOR*(1-
WB2Q2(B,H,L))
;

 TC16Q2(A,B,H,L)$(ORD(L) LT
CARD(L)$(A1(A,B)$H1(B,H)))..
         TS2Q2(B,H,L+1) =G=
         TE1Q2(A,B,L) - THOR*(1-
WB1Q2(A,B,L))
;

 TC17Q2(B,H,HDASH,L)$(ORD(L) LT
CARD(L)$(ORD(H) NE
ORD(HDASH)$H1(B,H)))..
         TS2Q2(B,H,L+1) =G=
         TE2Q2(B,HDASH,L) - THOR*(1-
WB2Q2(B,HDASH,L))
;

 TC18Q2(A,B,H,L)$(A1(A,B)$H1(B,H))..
         TE1Q2(A,B,L) - THOR*(1-
WB1Q2(A,B,L)) =L=
         TS2Q2(B,H,L) + THOR*(1-
WB2Q2(B,H,L))
;

 TC19Q2(B,L)..
         ITOT(B,L-1) +
SUM(A$A1(A,B),FT1Q2(A,B,L)) =E=
         ITOT(B,L) +
SUM(H$H1(B,H),FT2Q2(B,H,L)) +
SLAVARTC19ITOTQ2(B,L)
;

 TC20Q2(B)..
         ITOT(B,'L0') =E=
         INITITOT(B)
;

 TC21Q2(I,B,L)..
         INVI(I,B,L-1) +
SUM(A$A1(A,B),FL1Q2(I,A,B,L)) =E=
         INVI(I,B,L)
+SUM(H$H1(B,H),FL2Q2(I,B,H,L)) +
SLAVARTC21INVIQ2(I,B,L)
;

 TC22Q2(I,B)..
         INVI(I,B,'L0') =E=
         INITINVI(I,B)
;

 TC23Q2(A,B,L)$A1(A,B)..
         FT1Q2(A,B,L) =E=
         SUM(I,FL1Q2(I,A,B,L)) +
SLAVARTC23FT1Q2(A,B,L)
;

 TC24Q2(B,H,L)$H1(B,H)..
         FT2Q2(B,H,L) =E=
         SUM(I,FL2Q2(I,B,H,L)) +
SLAVARTC24FT2Q2(B,H,L)
;

 TC26Q2(B,L)$(ORD(L) GT 1)..
         ITOT(B,L-
1)+SUM(A$A1(A,B),FT1Q2(A,B,L)) =L=
         UBI1(B)
;

 TC27Q2(I,B,L)..
         LB1(I,B)*ITOT(B,L) =L=
         INVI(I,B,L) +
SLAVARTC27INVIQ2(I,B,L)
;

;

 TC28Q2(I,B,L)..
         INVI(I,B,L) =L=
         UB1(I,B)*ITOT(B,L)
;

 TC29Q2(I,B,H,L)$H1(B,H)..
         LB1(I,B)*FT2Q2(B,H,L) =L=
         FL2Q2(I,B,H,L)
;

 TC30Q2(I,B,H,L)$H1(B,H)..
         FL2Q2(I,B,H,L) =L=
         UB1(I,B)*FT2Q2(B,H,L)
;

 TC31Q2(G)..

SUM(D$D3(D,G),SUM(L,FT2Q2(G,D,L))) =E=
         DM(G)
;

 TC32Q2(I,B,H,L)$H1(B,H)..
         ITOT(B,L-1)*FL2Q2(I,B,H,L) +
SUM(A$A1(A,B),FT1Q2(A,B,L)*FL2Q2(I,B,H
,L)) =E=
         INVI(I,B,L-
1)*FT2Q2(B,H,L)+SUM(A$A1(A,B),FL1Q2(I,
A,B,L)*FT2Q2(B,H,L))         ;

 TC33Q2(B,H,L)$H1(B,H)..
         SUM(I,ITOT(B,L-
1)*FL2Q2(I,B,H,L)) =E=
         ITOT(B,L-1)*FT2Q2(B,H,L)
;

 TC34Q2(A,B,H,L)$(A1(A,B)$H1(B,H))..

SUM(I,FT1Q2(A,B,L)*FL2Q2(I,B,H,L)) =E=
```

```
        FT1Q2(A,B,L)*FT2Q2(B,H,L)
;

 TC35Q2(B,H,L)$H1(B,H)..
        SUM(I,INVI(I,B,L-
1)*FT2Q2(B,H,L)) =E=
        ITOT(B,L-1)*FT2Q2(B,H,L)
;

 TC36Q2(A,B,H,L)$(A1(A,B)$H1(B,H))..

SUM(I,FL1Q2(I,A,B,L)*FT2Q2(B,H,L)) =E=
        FT1Q2(A,B,L)*FT2Q2(B,H,L)
;

 DC1Q2(D,L)..
        SUM(G$G1(G,D),WB2Q2(G,D,L))
=L=
        1 +  SLAVARDC1WB2Q2(D,L)
;

 DC2Q2(G,L)..
        SUM(D$D3(D,G),WB2Q2(G,D,L))
=L=
        1  + SLAVARDC2WB2Q2(G,L)
;

 DC3Q2(D)..

SUM(L,SUM(G$G1(G,D),TE2Q2(G,D,L)-
TS2Q2(G,D,L))) =E=
        THOR
;

 DC4Q2(G,GDASH,D,L)$(ORD(L) LT
CARD(L)$(ORD(G) NE
ORD(GDASH)$G1(G,D)))..
        TS2Q2(G,D,L+1) =G=
        TE2Q2(GDASH,D,L) - THOR*(1-
WB2Q2(GDASH,D,L))
;

 DC5Q2(G,GDASH,D,L)$(ORD(L) LT
CARD(L)$(ORD(G) NE
ORD(GDASH)$G1(G,D)))..
        TS2Q2(G,D,L+1) =L=
        TE2Q2(GDASH,D,L) + THOR*(1-
WB2Q2(GDASH,D,L))
;

 CC1Q2(U,Y,L)$Y1(Y,U)..
        TSI(U) =L=
        TS1Q2(U,Y,L) + THOR*(1-
WB1Q2(U,Y,L))
;

 CC2Q2(U,Y,L)$Y1(Y,U)..
        TEO(U) =G=
        TE1Q2(U,Y,L) - THOR*(1-
WB1Q2(U,Y,L))
;

 CC3Q2(U)..

SUM(L,SUM(Y$Y1(Y,U),FT1Q2(U,Y,L))) =E=
        VCRUDE(U)
;

 CC4Q2(I,U,Y,L)$Y1(Y,U)..
        FL1Q2(I,U,Y,L) =E=
        FRACT(I,U)*FT1Q2(U,Y,L)
;

 VB1Q2(I,B,L)..
        INVI(I,B,L) =L=
        UBI1(B)
;

 VB2Q2(B,L)..
        LBI1(B) =L=
        ITOT(B,L)
;

 VB3Q2(B,L)..
        ITOT(B,L) =L=
        UBI1(B)
;

 VB4Q2(I,A,B,L)$A1(A,B)..
        FL1Q2(I,A,B,L) =L=
        UBFV1(A,B)
;

 VB5Q2(I,B,H,L)$H1(B,H)..
        FL2Q2(I,B,H,L) =L=
        UBFV2(B,H)
;

 VB6Q2(A,B,L)$A1(A,B)..
        FT1Q2(A,B,L) =L=
        UBFV1(A,B)
;

 VB7Q2(B,H,L)$H1(B,H)..
        FT2Q2(B,H,L) =L=
        UBFV2(B,H)
;

 VB8Q2(A,B,L)$A1(A,B)..
        TS1Q2(A,B,L) =L=
        THOR
;

 VB9Q2(A,B,L)$A1(A,B)..
        TE1Q2(A,B,L) =L=
        THOR
;

 VB10Q2(B,H,L)$H1(B,H)..
        TS2Q2(B,H,L) =L=
        THOR
;

 VB11Q2(B,H,L)$H1(B,H)..
        TE2Q2(B,H,L) =L=
        THOR
;

 VB12Q2(U)..
        TARR(U) =L=
        TSI(U)
;

 VB13Q2(U)..
        TSI(U) =L=
        THOR
;

 VB14Q2(U)..
        TARR(U) =L=
        TEO(U)
;

 VB15Q2(U)..
        TEO(U) =L=
        THOR
;


*======================== LAGRANGEAN
ALGORITHM ========================
```

```
$ontext

1. Decompose into Q1 and Q2
2. Solve Q1 to obtain objective value
of ZZQ1
3. Use value of ZZQ1 to generate
linear cut
4. repeat for Q2
5. solve MR00 with the two linear cuts
6. Solve CRUDE_UB

$offtext


MODEL CRUDE_LB1
* to generate linear cut for Q1
/
TC1Q1
TC2Q1
TC3Q1
TC4Q1
TC5Q1
TC6Q1
TC7Q1
TC8Q1
TC9Q1
TC10Q1
TC11Q1
TC12Q1
TC13Q1
TC14Q1
TC15Q1
TC16Q1
TC17Q1
TC18Q1
TC19Q1
TC20Q1
TC21Q1
TC22Q1
TC23Q1
TC24Q1

TC26Q1
TC27Q1
TC28Q1
TC29Q1
TC30Q1
TC31Q1
DC1Q1
DC2Q1
DC3Q1
DC4Q1
DC5Q1
CC1Q1
CC2Q1
CC3Q1
CC4Q1
VB1Q1
VB2Q1
VB3Q1
VB4Q1
VB5Q1
VB6Q1
VB7Q1
VB8Q1
VB9Q1
VB10Q1
VB11Q1
VB12Q1
VB13Q1
VB14Q1
VB15Q1


LBP1Q1
LBP2Q1
LBP3Q1
LBP4Q1
LBP5Q1


LBP6Q1
LBP7Q1
LBP8Q1
LBP9Q1
LBP10Q1
LBP11Q1
LBP12Q1

LBP13Q1

LBP14Q1
LBP15Q1
LBP16Q1
LBP17Q1
LBP18
LBP19
LBP20
LBP21
LBP22
*OBJF
OBJQ1
*OBJQ2
/

;

MODEL CRUDE_LB2
* to generate linear cut for Q2
/
TC1Q2
TC2Q2
TC3Q2
TC4Q2
TC5Q2
TC6Q2
TC7Q2
TC8Q2
TC9Q2
TC10Q2
TC11Q2
TC12Q2
TC13Q2
TC14Q2
TC15Q2
TC16Q2
TC17Q2
TC18Q2
TC19Q2
TC20Q2
TC21Q2
TC22Q2
TC23Q2
TC24Q2

TC26Q2
TC27Q2
TC28Q2
TC29Q2
TC30Q2
TC31Q2
DC1Q2
DC2Q2
DC3Q2
DC4Q2
DC5Q2
CC1Q2
CC2Q2
CC3Q2
CC4Q2
VB1Q2
```

```
VB2Q2
VB3Q2
VB4Q2
VB5Q2
VB6Q2
VB7Q2
VB8Q2
VB9Q2
VB10Q2
VB11Q2
VB12Q2
VB13Q2
VB14Q2
VB15Q2

LBP1Q2
LBP2Q2
LBP3Q2
LBP4Q2
LBP5Q2
LBP6Q2
LBP7Q2
LBP8Q2
LBP9Q2
LBP10Q2
LBP11Q2
LBP12Q2

LBP13Q2

LBP14Q2
LBP15Q2
LBP16Q2
LBP17Q2
LBP18
LBP19
LBP20
LBP21
LBP22
*OBJF
*OBJQ1
OBJQ2
*VLC1
*VLC2
/
;


MODEL MR00
*MIP with convex envelopes
/
OBJFUN
TC1
TC2
TC3
TC4
TC5
TC6
TC7
TC8
TC9
TC10
TC11
TC12
TC13
TC14
TC15
TC16
TC17
TC18
TC19
TC20
TC21
TC22
TC23
TC24
TC26
TC27
TC28
TC29
TC30
TC31
DC1
DC2
DC3
DC4
DC5
CC1
CC2
CC3
CC4
VB1
VB2
VB3
VB4
VB5
VB6
VB7
VB8
VB9
VB10
VB11
VB12
VB13
VB14
VB15
LBP1
LBP2
LBP3
LBP4
LBP5
LBP6
LBP7
LBP8
LBP9
LBP10
LBP11
LBP12
LBP13
LBP14
LBP15
LBP16
LBP17
VLC1
VLC2
/
;

MODEL MF00
*NLP to obtain upper bound to original
MINLP
/
TCUB1
TCUB2
TCUB3
TCUB4
TCUB5
TCUB6
TC7
TCUB8
TC9
TC10
TCUB11
TC12
TC13
TCUB14
TCUB15
TCUB16
TCUB17
TCUB18
TC19
```

```
TC20
TC21
TC22
TC23
TC24
TC25
TC26
TC27
TC28
TC29
TC30
TC31
DCUB1
DCUB2
DC3
DCUB4
DCUB5
CCUB1
CCUB2
CC3
CC4
VB1
VB2
VB3
VB4
VB5
VB6
VB7
VB8
VB9
VB10
VB11
VB12
VB13
VB14
VB15
OBJFIXBIN
/
;

ZZUB.FX = INF;

*to generate linear cuts for Q1
SOLVE CRUDE_LB1 USING MIP MINIMIZING
ZZQ1       ;

*to generate linear cuts for Q2
SOLVE CRUDE_LB2 USING MIP MINIMIZING
ZZQ2       ;

*-------------------------------------
---------------------------
*------------------------------
SoluTion to model MIP


SOLVE MR00 minimizing ZZ using mip;


*==================== TO FIX BINARY
VARIABLES TO PARAMETER VALUES
=====================

PUT RESCRU;

PUT //
PUT / "-------------------------------
------"
PUT / "BINARY VARIABLES OBTAINED FROM
MIP-LB"
PUT / "-------------------------------
------"

PUT //

LOOP( (A,B)$A1(A,B),
 LOOP( L,
  WBUB1(A,B,L) = WB1.L(A,B,L)
  PUT "WB1('
"A.TL:3,",",B.TL:3,",",L.TL:3," ')=
",WBUB1(A,B,L):18:12;
  PUT ";   "
 );
 PUT /
);

PUT /

LOOP( (B,H)$H1(B,H),
 LOOP( L,
  WBUB2(B,H,L) = WB2.L(B,H,L)
  PUT "WB2('
"B.TL:3,",",H.TL:3,",",L.TL:3," ')=
",WBUB2(B,H,L):18:12;
  PUT ";   "
 );
 PUT /
);

*------------------------------
Solution to NLP non convex model (UB)

SOLVE MF00 MINIMIZING ZZFIXBIN USING
NLP;
```

APPENDIX IX: GAMS OUTPUT FILE ON CRUDE OIL SCHEDULING WITH SLACK VARIABLES MODEL

```
GAMS Rev 149 x86/MS Windows
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: SCHEDULING REFINERY CRUDE OIL
OPERATIONS
Solution Report SOLVE CRUDE_LB1
Using MIP From line 2455


      S O L V E      S U M M A R Y

     MODEL                    CRUDE_LB1
OBJECTIVE  ZZQ1
     TYPE                           MIP
DIRECTION  MINIMIZE
     SOLVER  CPLEX                 FROM
LINE  2455

****  SOLVER  STATUS          1  NORMAL
COMPLETION
**** MODEL STATUS      1 OPTIMAL
****          OBJECTIVE          VALUE
538.6000

 RESOURCE USAGE, LIMIT           0.125
1000.000
 ITERATION  COUNT,  LIMIT          207
10000

ILOG CPLEX     Dec 24, 2007 WIN.CP.CP
22.6 035.037.041.vis For Cplex 11.0
Cplex 11.0.0, GAMS Link 34
Cplex licensed for 1 use of lp, qp,
mip and barrier, with 2 parallel
threads.

Proven optimal solution.

MIP  Solution:               538.600000
(34 iterations, 0 nodes)
Final  Solve:                538.600000
(173 iterations)

Best possible:         538.600000
Absolute gap:            0.000000
Relative gap:            0.000000
```

```
GAMS Rev 149 x86/MS Windows
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: SCHEDULING REFINERY CRUDE OIL
OPERATIONS
Solution Report SOLVE CRUDE_LB2
Using MIP From line 2458


      S O L V E      S U M M A R Y

     MODEL                    CRUDE_LB2
OBJECTIVE  ZZQ2
     TYPE                           MIP
DIRECTION  MINIMIZE
     SOLVER  CPLEX                 FROM
LINE  2458

****  SOLVER  STATUS          1  NORMAL
COMPLETION
**** MODEL STATUS      1 OPTIMAL
****          OBJECTIVE          VALUE
8.0114

 RESOURCE USAGE, LIMIT           0.125
1000.000
 ITERATION  COUNT,  LIMIT          173
10000

ILOG CPLEX     Dec 24, 2007 WIN.CP.CP
22.6 035.037.041.vis For Cplex 11.0
Cplex 11.0.0, GAMS Link 34
Cplex licensed for 1 use of lp, qp,
mip and barrier, with 2 parallel
threads.

Proven optimal solution.

MIP  Solution:                 8.011429
(31 iterations, 0 nodes)
Final  Solve:                  8.011429
(142 iterations)

Best possible:           8.011429
Absolute gap:            0.000000
Relative gap:            0.000000
```

```
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: SCHEDULING REFINERY CRUDE OIL
OPERATIONS
Solution Report SOLVE MR00 Using
MIP From line 2464


     S O L V E      S U M M A R Y

     MODEL                       MR00
OBJECTIVE  ZZ
     TYPE                        MIP
DIRECTION  MINIMIZE
     SOLVER  CPLEX               FROM
LINE  2464

**** SOLVER STATUS         1 NORMAL
COMPLETION
**** MODEL STATUS      1 OPTIMAL
****        OBJECTIVE          VALUE
1980.4400

 RESOURCE USAGE, LIMIT          0.156
1000.000
 ITERATION COUNT, LIMIT           214
10000

ILOG CPLEX     Dec 24, 2007 WIN.CP.CP
22.6 035.037.041.vis For Cplex 11.0
Cplex 11.0.0, GAMS Link 34
Cplex licensed for 1 use of lp, qp,
mip and barrier, with 2 parallel
threads.

Proven optimal solution.

MIP Solution:              1980.440000
(34 iterations, 0 nodes)
Final Solve:               1980.440000
(180 iterations)

Best possible:        1980.440000
Absolute gap:            0.000000
Relative gap:            0.000000
```

```
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: SCHEDULING REFINERY CRUDE OIL
OPERATIONS
Solution Report SOLVE MF00 Using
NLP From line 2500


     S O L V E      S U M M A R Y

     MODEL                       MF00
OBJECTIVE  ZZFIXBIN
     TYPE                        NLP
DIRECTION  MINIMIZE
     SOLVER  BARON               FROM
LINE  2500

**** SOLVER STATUS         1 NORMAL
COMPLETION
**** MODEL STATUS      1 OPTIMAL
****        OBJECTIVE          VALUE
2148.1050

 RESOURCE USAGE, LIMIT          1.840
1000.000
 ITERATION COUNT, LIMIT             0
10000
 EVALUATION ERRORS                  0
0

GAMS/BARON     Dec 24, 2007 WIN.BA.NA
22.6 011.000.000.vis P3PC

Branch And Reduce Optimization
Navigator
Nikolaos Sahinidis and Mohit
Tawarmalani
The Optimization Firm, LLC.

  Total time elapsed :     000:00:02,
in seconds:      2.09
   on parsing       :      000:00:01,
in seconds:         1.17
   on preprocessing:       000:00:01,
in seconds:         0.92
   on navigating    :      000:00:00,
in seconds:         0.00
   on relaxed       :      000:00:00,
in seconds:         0.00
   on local         :      000:00:00,
in seconds:         0.00
   on tightening    :      000:00:00,
in seconds:         0.00
   on marginals     :      000:00:00,
in seconds:         0.00
   on probing       :      000:00:00,
in seconds:         0.00

   Total no. of BaR iterations:      -
1
   Best solution found at node:      -
1
   Max. no. of nodes in memory:
0


Solution        = 2148.10499999  best
solution found during preprocessing
Best possible = 2148.105
Absolute gap   = 9.99989424599335E-9
optca = 1E-9
Relative gap  = 0  optcr = 0
```