UNIVERSITI TEKNOLOGI PETRONAS

Development of controller and observer for 2D Crane System via State-space approach

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Final Project Report.

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CERTIFICATION OF APPROVAL

Development of controller and observer for 2D Crane System via State-space approach

BY

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CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

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AHLAM MOHAMED ELSAFI ABD ELHAMID

ABSTRACT

This report actually presents researches and studies and progress that are being achieved for the chosen topic which is **Development of controller and observer for 2D Crane Systems via State-space approach**. This report contains an introduction and background studies about cranes and how does this topic it related to my studies as a final year & control system student all being represented in scope of studies section, in fact the main objectives of this research are getting the dynamic equation for the 2D crane systems and applies state-space approach to develop a controller and an observer for them. The dynamic equations for the systems are being obtained by using Euler-Langrange formulation for obtaining the state-space representation of the systems. Furthermore, control & observer canonical forms have been designed and then simulated using Matlab Simulink for testing the stability of the system before designing the controller and the observer for the system

ACKNOWLEDGEMENTS

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NOMENCLATURE

Nomenclature 1 : Symbols of the variables and its' indications .

CHAPTER 1

INTRODUCTION

1.0 Background study :

2D and 3D Cranes are very substantial structures being used for moving heavy loads that's over human capability, Cranes are widely used in factories and other facilities such as shipboards, ports and construction sites, factories, refineries and coke industries. Position control and swing suppression of payloads are needed for crane control. The behavior of the payload shows similar characteristics of an inverted pendulum as it uses flexible cables for suspending the payload, and the dynamics of a crane system are similar to those of an inverted pendulum.

In fact cranes are being operated manually by operators –human efforts - by using the joystick which makes it hard to depend on the system because the performance basically depends on human condition at that time whether he was tired or fully concentrated .

It is usually expected -that the payload can be brought to its destination and be still at the end of its journey. Since the payload exhibits a pendulum-like swinging motion, it is difficult to position the payload precisely. Cranes that are able to position the load in three dimensions are difficult to control due to their nonlinear behavior. In order to achieve fast and precise payload positioning, researchers have developed various control methodologies. In practical applications, However, it is necessary not only to have a fast position regulation toward the desired position in an economic sense, but also to suppress the swing motion of the payload since a severe payload swing can cause a payload damage or an accident, and also hinder an accurate placement of the payload. Accordingly, much research has been done on the overhead crane systems.
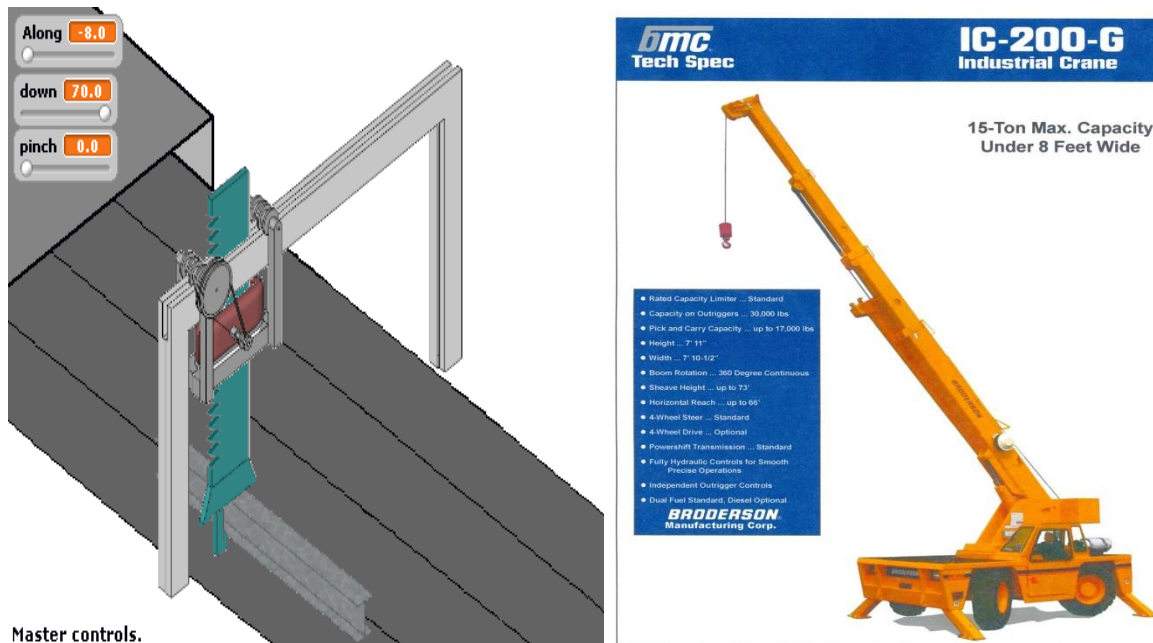
Figures 1 shows 2D&3D Cranes .

1.1. Problem statement

The problem in 3D and 2D cranes is to maintain a desired residual sway of the payload. At slow motion, the payload sway angle could be ignored but at fast motion the sway angle couldn't be controlled specially when the payload it heavy, Also it becomes hard to control it manually since it depends on the operator experience which brings us to do a closed loop control system in order to control the sway angle as much as possible otherwise it will damage the payload itself as well as the surroundings and human.

1.2.Project objective

As it is mentioned earlier in the problem statement that we are trying to reduce the sway angle as much as possible in order not to be any damage for the surroundings, so the objectives of the project will be:

a) Developing the mathematical model –state space – for the 2D crane.
b) Studying the dynamic behavior of the 2D crane .
c) To develop controller and observer for the cranes via state-space approach.
d) To Evaluate the developed controller's performances.

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1.3. Scope of study

    a) Operator –in –the- loop approach

This approach means that the input of the model will be the accelerations of the links of the 2D crane resulting from the control of an operator using pedals and joystick

    b) Controller and observer via state-space approach

This is an application of the studies which have been studied in MODERN CONTROL ENGINEERING by getting the control canonical, observer canonical forms for the system and designing controller by using The 'pole placement' method thus for getting the controller's gains. Moreover, observer will be designed for the system based on. Full-state observer approach.

CHAPTER 2

LITERATURE REVIEW

Cranes are consider to be very old structures first invented by ancient Greeks which were built from wood , nowadays there are many types of cranes such as 2D, 3D and spider cranes . Movement of 3D cranes system consists of three dimensional X,Y and Z , and it is different from the other types of cranes such as the spider crane in terms of safety and the structure . 3D and 2D cranes have a better structure and a very high safety integrity comparing to the other type of cranes.

Modern cranes usually use internal combustions engines or electric motors for providing a great lifting capability than was previously used. In designing the cranes two considerations must be taken, the crane must be able to lift a load of a demanded range of weight at the same time it must remain stable while moving the load by maintaining a small sway angle in order not to damage to surrounding.

Methods that have been investigated for controlling flexible structures in general and cranes in particular, can be roughly divided into feedback and feed-forward approaches. Feedback control methods use measurements and estimates of the system states to reduce vibration, while feed-forward techniques alter the actuator commands so that the system oscillations are reduced. The performance of feedback methods can often be improved by additionally using a feed forward Controller and properly designed feed-forward compensators can dramatically reduce the complexity of the required feedback controller for a given level of performance.

There are many method have been applied to achieve this goal such as input shaping technique has been applied in order to achieve it, the new shaping method was implemented on an overhead crane at the Robotics Laboratory of the University Carlos III of Madrid. Experimental results show that the method greatly reduces residual oscillations.

CHAPTER 3

METHODOLOGY

3.0Introduction

Methods that will be applied for carrying out the project are:

a) Literature review :

Many technical papers have been read for having enough information about 2D crane , also researches have been made about controller , observers and state-space .

b) Matlab software :

This software will be used for calculating the complex mathematical equations after getting **the mathematical model** for this dynamic system. Furthermore, it will be used for simulating the system and testing its stability after designing the controller and observer for the system.

3.1.FLOW CHART

Start

↓

Literature Research

↓

-Background study about 2D Crane.

-dynamic modeling of the 2D Crane

- studying state-space and how to design controller and observer

↓

Getting the mathematical model for the dynamic and getting the control & observer canonical forms for the system.

↓

Simulation &modeling on Matlab simulation.

↓

Designing a controller and observer via state-space

↓

Evaluating the stability of the system after using the controller and observer by using Matlab simulation.

↓

Ok

END

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Item	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Selection of the project topic														
Preliminary research work						✿								
Extended proposal submission														
Finalizing the experiments														
Proposal defence														
Conducting experiments and Gathering data														
Submission of Interim Draft report													✿	
Submission of Interim report														✿

Figure3: Gantt chart for FYP 1

Figure 4 : GANTT chart FYP 2 .

Item	1	2	3	4	5	6	7	8	9	10	11	12	13	14
MATLAB work for simulation 2D	■	■	■	■										
Designing a controller and observer via state-space					■	■								
Data analysis							■	■						
Submission progress report								✦						
Comparing the result with PID controller								■	■	■				
Pre EDX											✦			
Submission of Draft report												✦		
Submission of the dissertation													✦	
Submission of the technical paper													✦	
Oral presentation														✦
Submission of the project dissertation (hard bound)														✦

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Chapter 4

Result & Discussion:

-Now we are coming to apply step 3 in this research since step 1 and 2 have been applied already as shown in the flow chart above.

-For 2D crane system the result being obtained is basically the state-space or the matrix form for the system
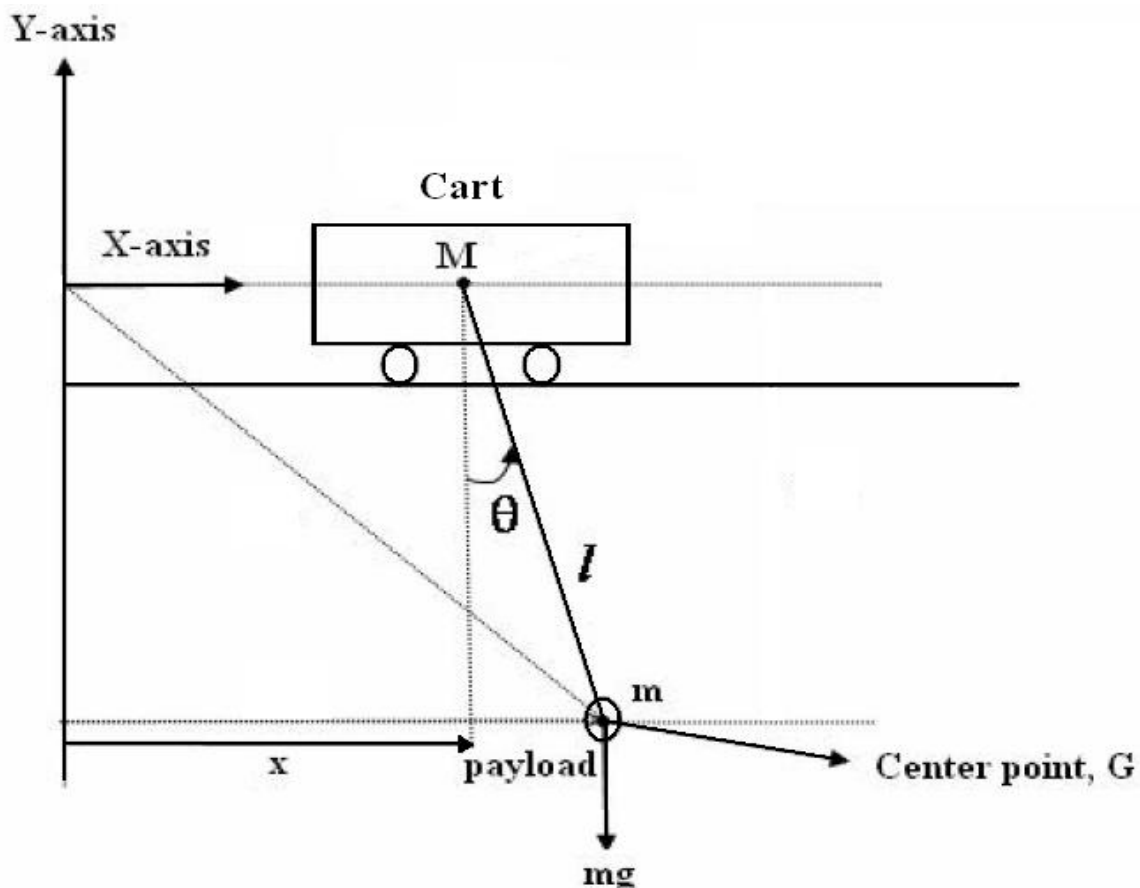
4.0 Dynamic Equation Derivation :

Figure 2; Sketch of the 2D crane system.

Symbol	Indication
l	*The length of the hoisting robe*
m	*The mass payload (kg)*
M	*The mass of the trolley (kg)*
θ	*The sway angle*
L	*Lagranian*
P	*Potential energy*
T	*Total kinetic energy of the system*
r	*Distance*
v	*Velocity*
x	*The horizontal position of the trolley (m)*
g	*The center of gravitation force.*

NOMENCLATURE 1 : Symbols of the variables and its indication .

4.0.1 Analyzing the system :

$\vec{r}_m$ = {x+l sin θ,-cos θ}   *for the payload*

$\vec{r}_{ro}$ ={x,0}   *for the trolley*

$\checkmark$ *kinetic energy of any system* = $\frac{1}{2} mv^2$

$$v = \frac{dr}{dt} = \dot{r} \ \& \ v^2 = \dot{r}^2$$

Total kinetic energy of the system = kinetic energy of the trolley + kinetic energy of the payload

$\therefore T = \frac{1}{2} m\dot{r}_m^2 + \frac{1}{2} M\dot{r}_O^2$

- $\frac{1}{2} M\dot{r}_O^2 = \frac{1}{2} M\dot{x}^2$

- $\frac{1}{2} m\dot{r}_m^2 = \frac{1}{2}m \ \left\| \vec{r}_m \right\|^2 = \frac{1}{2} m \ ((\dot{r}_{mx})^2 + (\dot{r}_{my})^2)$

$r_{mx} = x + l \sin θ \quad \& \quad r_{my} = -l \cos θ$

$\dot{r}_{mx} = \dot{x} + \dot{l} \sin θ + \dot{θ} l \cos θ \quad \& \quad \dot{r}_{my} = -\dot{l} \cos θ + (-l) \dot{θ} (-\sin θ)$

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$$\therefore (\dot{r}_{mx})^2 + (\dot{r}_{my})^2 = \dot{x}^2 + \dot{l}^2 \sin\theta^2 + \dot{\theta}^2 l^2 \cos\theta^2 + 2\dot{x}\dot{l}\sin\theta + 2\dot{l}\sin\theta\, l\dot{\theta}\cos\theta + 2\dot{x}l\dot{\theta}\cos\theta +$$

$$\dot{l}^2 \cos\theta^2 + \dot{\theta}^2 l^2 \sin\theta^2 - 2\dot{l}\sin\theta\, l\dot{\theta}\cos\theta$$

$$\therefore (\dot{r}_{mx})^2 + (\dot{r}_{my})^2 = \dot{x}^2 + \dot{\theta}^2 l^2 + \dot{l}^2 + 2\dot{x}l\dot{\theta}\cos\theta + \dot{l}^2 + 2\dot{x}\dot{l}\sin\theta$$

$$\therefore T = \frac{1}{2}m(\dot{x}^2 + \dot{\theta}^2 l^2 + \dot{l}^2 + 2\dot{x}l\dot{\theta}\cos\theta + \dot{l}^2 + 2\dot{x}\dot{l}\sin\theta) + \tfrac{1}{2}M\dot{x}^2$$

$$P = mgy_m = -m\,g\,l\,\cos\theta$$

- **By Using lagrangian approach**

$$L = T\text{-}P$$

$$\therefore L = \frac{1}{2}m(\dot{x}^2 + \dot{\theta}^2 l^2 + \dot{l}^2 + 2\dot{x}l\dot{\theta}\cos\theta + \dot{l}^2 + 2\dot{x}\dot{l}\sin\theta) + \tfrac{1}{2}M\dot{x}^2 + m\,g\,l\,\cos\theta \quad (1)$$

- **By using lagranian equations we could construct the lagrangian L=T-P**

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = F_j \qquad\qquad j=1,2$$

Since we are dealing with 2 dimensional crane system , the variables changes with the time are 2 ,represented in x (the distance travels by the crane) and θ (sway angle . so "j" in lagrangian equation will be 1 & 2 .

Firstly every "q" in lagrangian equation will be substituted by" x" to obtain the equation of motion for the crane.

So the new equation will be :

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = F_x$$

From equation 1 :

$$L = \frac{1}{2}m(\dot{x}^2 + \dot{\theta}^2 l^2 + \dot{l}^2 + 2\dot{x}l\dot{\theta}\cos\theta + \dot{l}^2 + 2\dot{x}\dot{l}\sin\theta) + \tfrac{1}{2}M\dot{x}^2 + m\,g\,l\,\cos\theta$$

$$\therefore \frac{\partial L}{\partial \dot{x}} = m\dot{x} + m\,l\,\dot{\theta}\cos\theta + m\dot{l}\sin\theta + M\dot{x}$$

$$\therefore \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = m\ddot{x} - m\,l\,\dot{\theta}^2\sin\theta + m\,\ddot{\theta}l\cos\theta + m\,\dot{l}\,\dot{\theta}\cos\theta + m\dot{l}\,\dot{\theta}\cos\theta + m\ddot{l}\sin\theta + M\ddot{x}$$

Also From equation 1

$$L = \frac{1}{2}m(\dot{x}^2 + \dot{\theta}^2 l^2 + \dot{l}^2 + 2\dot{x}l\dot{\theta}\cos\theta + \dot{l}^2 + 2\dot{x}\dot{l}\sin\theta) + \tfrac{1}{2}M\dot{x}^2 + m\,g\,l\,\cos\theta$$

Since the equation is not a function of x , we can say that :

$$\frac{\partial L}{\partial x} = 0$$

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$$\therefore F_x = (m+M)\ddot{x} + m\,l(-\dot{\theta}^2\sin\theta + \ddot{\theta}\cos\theta) + 2m\,\dot{l}\,\dot{\theta}\cos\theta + m\ddot{l}\sin\theta \quad (2)$$

Secondly, every "q" in lagranian equation will be substituted by"θ " to obtain the equation of sway angle for the crane.

<div align="center">

So the new equation will be

</div>

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = F_\theta$$

<div align="center">

Since the force F will be acted in the direction of x only so $F_\theta=0$

</div>

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = F_\theta=0$$

From equation 1 we have

$$L=\frac{1}{2}m(\dot{x}^2+\dot{\theta}^2 l^2+\dot{l}^2+2\dot{x}l\dot{\theta}\cos\theta+\dot{l}^2+2\dot{x}\dot{l}\sin\theta)+\tfrac{1}{2}M\dot{x}^2+m\,g\,l\cos\theta$$

$$\therefore \frac{\partial L}{\partial \theta}=\frac{1}{2}m(-2\dot{x}\,l\dot{\theta}\,\sin\theta+2\dot{x}\,\dot{l}\,\cos\theta)-m\,g\,l\sin\theta$$

And

$$\frac{\partial L}{\partial \dot{\theta}}=\frac{1}{2}m(2\dot{\theta}\,l^2+2\dot{x}l\cos\theta)$$

Therefore,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right)=\frac{1}{2}m(4l\dot{l}\dot{\theta}+2\ddot{\theta}l^2-2\dot{x}l\dot{\theta}\sin\theta+2\ddot{x}l\cos\theta+2\dot{x}\dot{l}\cos\theta)$$

$$\therefore \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right)-\frac{\partial L}{\partial \theta}=\frac{1}{2}m(4l\dot{l}\dot{\theta}+2\ddot{\theta}l^2-2\dot{x}l\dot{\theta}\sin\theta+2\ddot{x}l\cos\theta+2\dot{x}\dot{l}\cos\theta)-$$
$$\frac{1}{2}m(-2\dot{x}\,l\dot{\theta}\sin\theta+2\dot{x}\,\dot{l}\,\cos\theta)+m\,g\,l\sin\theta=0$$

$$=2ml\dot{l}\dot{\theta}+m\ddot{\theta}l^2-m\,\dot{x}l\dot{\theta}\sin\theta+m\ddot{x}l\cos\theta+m\dot{x}\dot{l}\cos\theta+m\dot{x}\,l\dot{\theta}\sin\theta-m\dot{x}\,\dot{l}\,\cos\theta+m\,g\,l\sin\theta=0$$

Divide all the variables by" ml" so we will have

$$2\dot{l}\dot{\theta}+\ddot{\theta}l-\dot{x}\dot{\theta}\sin\theta+\ddot{x}\cos\theta+\dot{x}\dot{l}\cos\theta+\dot{x}\dot{\theta}\sin\theta-\dot{x}\,\dot{l}\,\cos\theta+g\,\sin\theta=0$$
$$\therefore=2\dot{l}\dot{\theta}+\ddot{\theta}l+\ddot{x}\cos\theta+g\,\sin\theta=0 \qquad (3)$$

-Now we have 2 non linear equations in terms of x and θ

✓ $2\dot{l}\dot{\theta}+\ddot{\theta}l+\ddot{x}\cos\theta+g\,\sin\theta=0$ *(4)*

✓ $F_x=(m+M)\ddot{x}+m\,l(-\dot{\theta}^2\sin\theta+\ddot{\theta}\cos\theta)+2m\,\dot{l}\,\dot{\theta}\cos\theta+m\ddot{l}\sin\theta$ *(5)*

- Since those 2 equations are not linear, we have to linearlized them in order to get the state-space representation for the system.
- Assuming that the length of the robe is going to change in a very small amount ≈ 0 with respect to time. So we have $\dot{l} \approx \ddot{l} \approx 0$
- Also the sway angle will be very small ≈ 0 ≈ $\dot{\theta}$ & θ ≈ sin θ & cos θ = 1
- By substituting these values of in the non linear equations 4&5 we will get linear equations .

- ***After substituting these values The new equations will be***
  - $x : f_x = (M+m)\ddot{x} + ml\ddot{\theta}$ *(6)*
  - $\theta : 0 = l\ddot{\theta} + \ddot{x} + g\theta$ *(7)*

✓ ***The state-space general form representation is***
$$\dot{x} = Ax + Bu \quad \& \quad y = Cx$$

Where

$$x = \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} \qquad\qquad \dot{x} = \begin{bmatrix} \dot{x} \\ \dot{\theta} \\ \ddot{x} \\ \ddot{\theta} \end{bmatrix}$$

From equation 7 we have

- $\theta : 0 = l\ddot{\theta} + \ddot{x} + g\theta$ *so* $\ddot{\theta} = -(\ddot{x} + g\theta)/l$ *&* $\ddot{x} = -(l\ddot{\theta} + g\theta)$ *(7.1)*
- *by substituting these values into equation (6)*

∴ $fx = (M+m) * (-l\ddot{\theta} - g\theta) + ml\ddot{\theta}$ *and* $fx = -Ml\ddot{\theta} - mg\theta - Mg\theta - ml\ddot{\theta} + ml\ddot{\theta}$

$$\ddot{\theta} = (-g\theta(m+M) - fx)/lM \quad (8)$$

By Substituting 8 into 7.1
$$\ddot{x} = -(l\left(\frac{g\theta((-M-m)-f_x)}{lM}\right)) - g\theta$$

$$\ddot{x} = \frac{g\theta m}{M} + \frac{g\theta M}{M} - g\theta + \frac{fx}{M}$$

$$\ddot{x} = \frac{g\theta m}{M} + \frac{fx}{M} \quad (9)$$

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For obtaining the state-space representation we have

$\dot{x} = \dot{x}$

$\dot{\theta} = \dot{\theta}$

$\ddot{x} = \dfrac{g\theta m}{M} + \dfrac{fx}{M}$

$\ddot{\theta} = (-g\theta(m+M) - fx)/lM$

These equations can be arranged as the matrix form below:

$$\begin{bmatrix} \dot{x} \\ \dot{\theta} \\ \ddot{x} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{gm}{M} & 0 & 0 \\ 0 & -\frac{g(m+M)}{lM} & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{M} \\ \frac{-1}{Ml} \end{bmatrix} f_x$$

- **After getting the matrix form of the system and the 2 output equations we will get the transfer function for both outputs :**

From equations 6 and 7 we have:

- $x: f_x = (M+m)\ddot{x} + ml\ddot{\theta}$ *(6)*
- $\theta: 0 = l\ddot{\theta} + \ddot{x} + g\theta$ *(7)*

4.1 State–space representation:

Form equations 6 we can get the transfer function for the 1st output:

$x: f_x = (M+m)x(s)S^2 + ml\theta(s)S^2$

From equation 7 we will get the other transfer function for the 2nd output:

$\theta: 0 = l\theta(s)S^2 + x(s)S^2 + g\theta$

By solving those 2 equations we will get

1) $\dfrac{ls^2 + g}{mls^4 + g(M+m)s^2} = \dfrac{x(s)}{f_x(s)}$ *(10)*

2) $\dfrac{1}{-mls^2 - g(M+m)} = \dfrac{\theta(s)}{f_x(s)}$ *(11)*

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Equations 10 & 11 represent the transfer functions equations for the 2 outputs distance travelled by the trolley and sway angle respectively.

By choosing some values of l , g , m and M as .305meter ,9.8 meter/s^2 ,.8kg and 1 kg respectively and substituting these values into the output equations we will get .

- $\dfrac{.305s^2+9.8}{.244s^4+17.64s^2} = \dfrac{x(s)}{f_x(s)}$

- $\dfrac{1}{-.244s^2-17.64} = \dfrac{\theta(s)}{f_x(s)}$

4.1.1 Simulating the system:

After getting the transfer functions for both outputs, the next step is to simulate the system stability using Simulink software on Matlab in order to check the stability of the system before designing the controller.

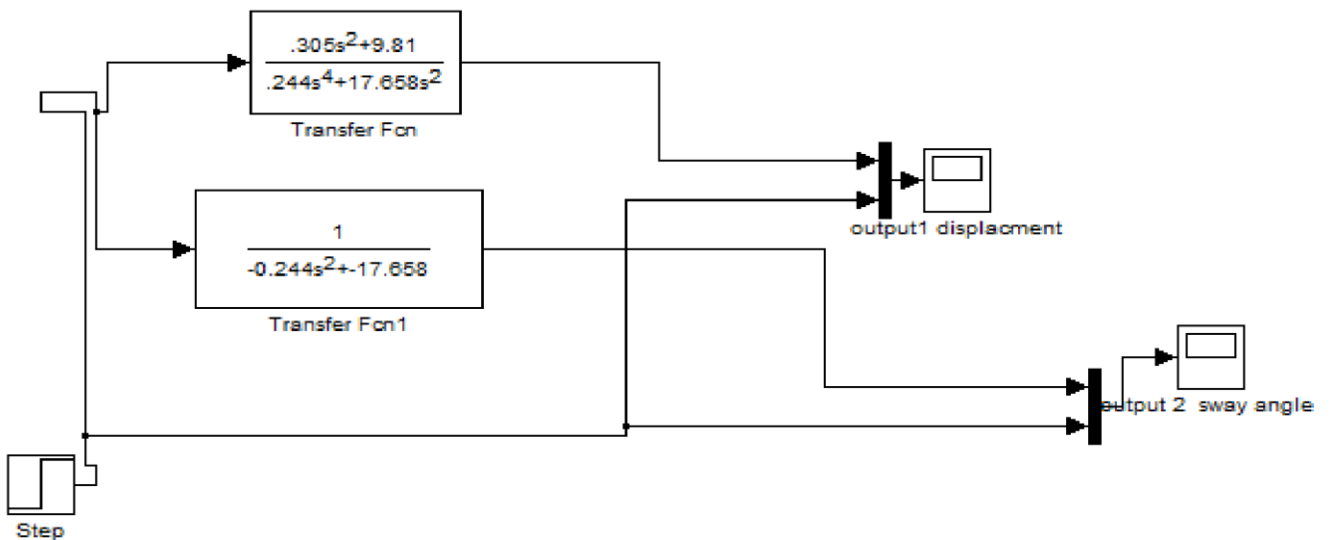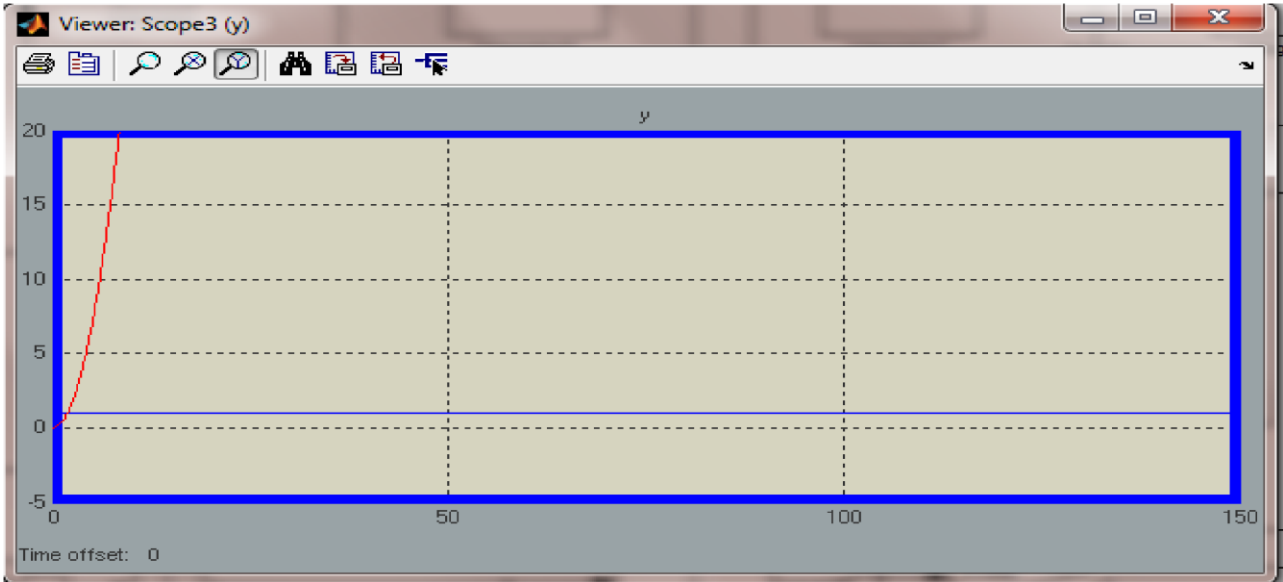Applying a step function as an input so we could check the system stability .

Figure 5 : shows the model on simulink " state space model " .

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After applying the step function as a input we will get these graphs as an output for the system. Graphs 6 and 7 show the behavior of distance travelled by trolley and sway angle respectively.

Figure 6: graph for the behavior of output 1 "distance travelled or displacement "
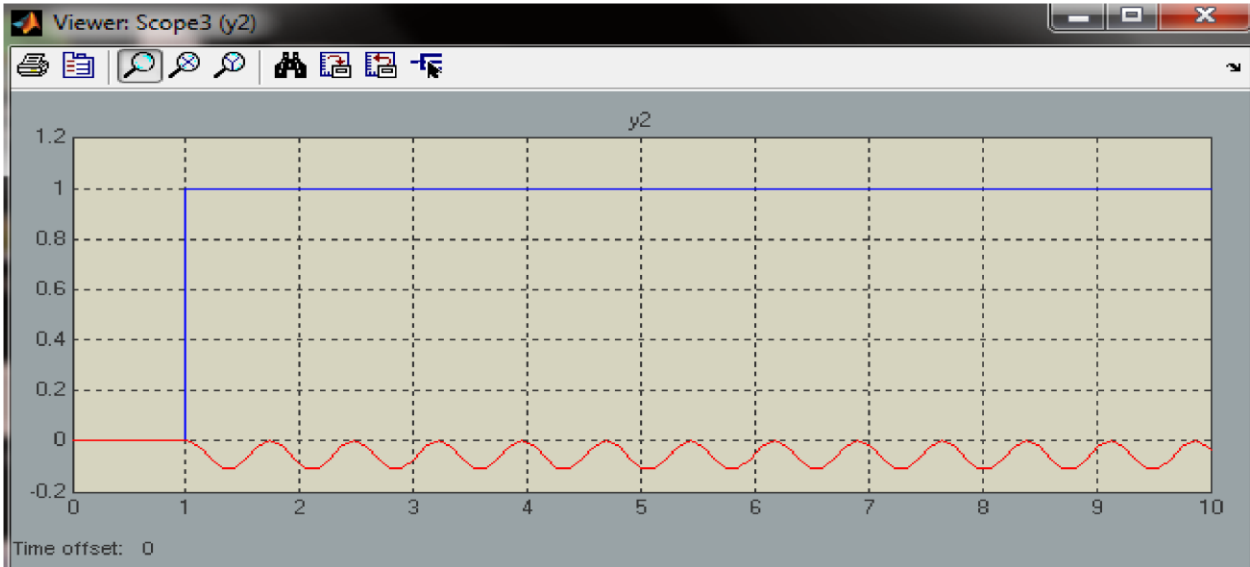
Figure 7A : graph for the behavior of output 2 "sway angle " , STEP function input .

applying a sine wave function as a input In order to know the system output 2 " sway angle" behavior .
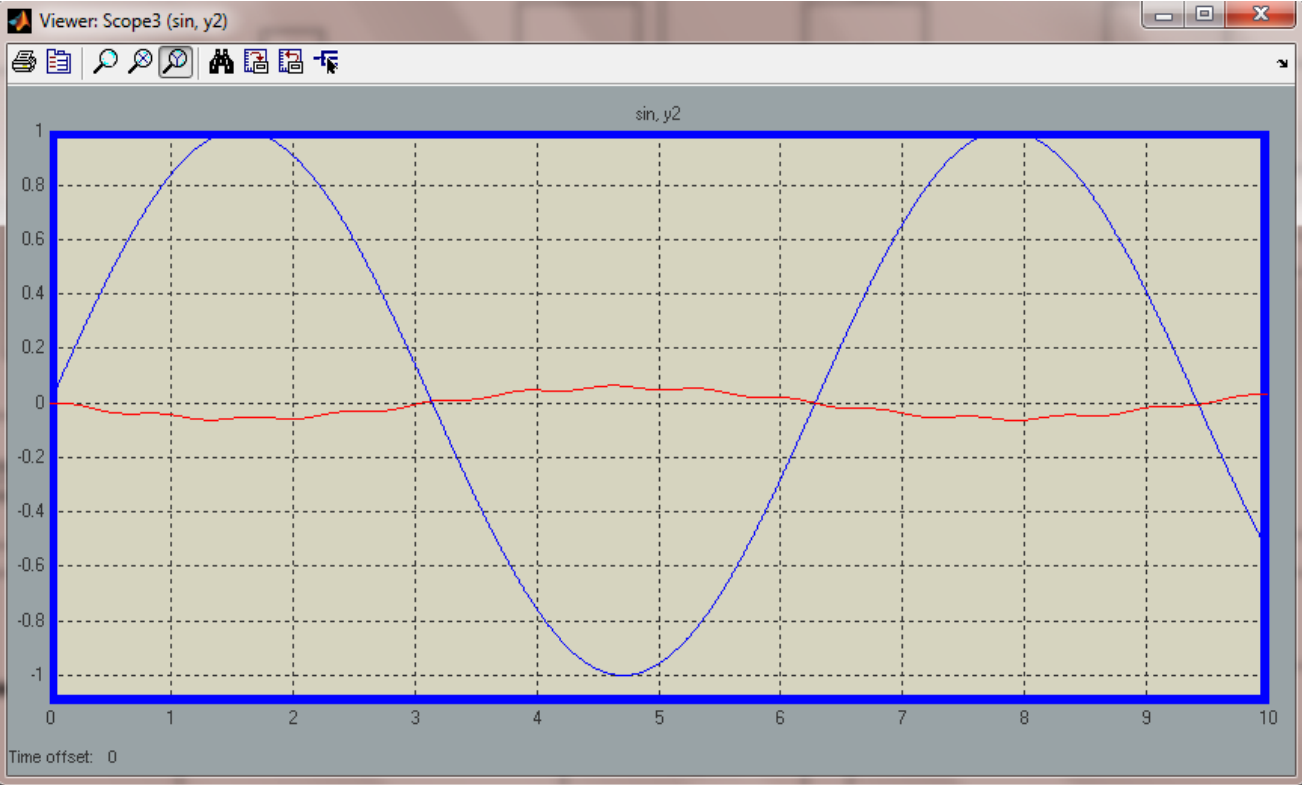

Figure 7B: *graph for the behavior of output 2 "sway angle " sine wave input .*

➢ **Next step is a confirmation step; to make sure that the signal graph produced is right.**

By getting the control canonical form for the system and the observer canonical form for it , the same graphs for the 2 outputs is expected as a result and this method is being applied in order to check the accuracy of the produced graph which will help later on when we design the controller and the observer .

4.2 Control canonical form & observer canonical forms.

1) *Control and observer canonical forms for the 1st output "distance travelled "*

- *Output 1 we have* $\dfrac{x(s)}{f_x(s)} = \dfrac{.305s^2 + 9.8}{.244s^4 + 17.64s^2}$

By writing a certain commands on Matlab we will get the control canonical form for the displacement "refer to the appendix 1 "

The matrixes which have been obtained from the Matlab is actually represented the control canonical form for 1st output "distance travelled by the trolley " which also could be written represented as :

Control canonical form:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -72.21 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$Y = \begin{bmatrix} 40.16 & 0 & 1.25 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + [0]\, u$$

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By writing certain commands on Matlab we could get the observer canonical form for the distance travelled by the trolley. "refer to appendix 2 "

Observer canonical form for output 1 "distance travelled by the trolley ":

$$
s\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -72.21 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 40.16 \\ 0 \\ 1.25 \\ 0 \end{bmatrix} u
$$

$$
Y = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}
$$

2) Control and observer canonical form for the 2nd output "sway angle "

For system 2 " sway angle " we have the transfer function :

$$
\frac{\theta(s)}{f_x(s)} = \frac{1}{-.244s^2 - 17.64}
$$

Using Matlab commands to get the control canonical form for the second output which is the sway angle "refer to appendix 3 " .

That could be represented as:

Control canonical form for output 2 " sway angle " :

$$
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -72.21 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u
$$

$$
Y = \begin{bmatrix} -4.098 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0]u
$$

Similarly in order to get the observer canonical form for the sway angle using Matlab commands "refer to appendix 4 "

Observer canonical form for output 2 " sway angle " :

$$s\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & -72.21 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -4.098 \\ 0 \end{bmatrix} u$$

$$Y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0]u$$

Table 1 shows the control and observer canonical forms for both outputs 1&2 as:

<table>
<tr>
<td>

Control canonical form for displacement :

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -72.21 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$Y = \begin{bmatrix} 40.16 & 0 & 1.25 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + [0]u$$

</td>
<td>

Observer canonical form for displacement

$$s\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -72.21 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 40.1639 \\ 0 \\ 1.25 \\ 0 \end{bmatrix} u$$

$$Y = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + [0]u$$

</td>
</tr>
<tr>
<td>

Control canonical form for sway angle :

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -72.21 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$Y = \begin{bmatrix} -4.098 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0]u$$

</td>
<td>

Observer canonical form for sway angle :

$$s\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & -72.21 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -4.098 \\ 0 \end{bmatrix} u$$

$$Y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0]u$$

</td>
</tr>
</table>

Table 1 : control and observer canonical form for outputs 1&2.

From these matrixes we can get the feed forward gain and the feedback gain for both outputs as shown in the table 2 bellow:

Control canonical form for 1ˢᵗ output displacement" x":	Observer canonical form for output 1 displacement x :

$-\alpha_0=0$
$-\alpha_1=0$
$-\alpha_2=-72.21$
$-\alpha_3=0$
                                      &
$\beta_0=40.16$
$\beta_1=0$
$\beta_2=1.25$
$\beta_3=0$
	$-\alpha_0=0$
	$-\alpha_1=0$
	$-\alpha_2=-72.21$
	$-\alpha_3=0$
	
	                                         &
	$\beta_0=40.16$
	$\beta_1=0$
	$\beta_2=1.25$
	$\beta_3=0$

Control canonical form for output 2 sway angle :	Observer canonical form for output 2 sway angle :

$-\alpha_0=-72.21$
$-\alpha_1=0$
                                        &
$\beta_0=-4.098$
$\beta_1=0$
	$-\alpha_0=-72.21$
	$-\alpha_1=0$
	                                        &
	$\beta_0=-4.098$
	$\beta_1=0$

Table 2 : feedback and feed-forward gains for both outputs .

By simulating the systems on Matlab using simulink , graph 8 shows the control canonical model
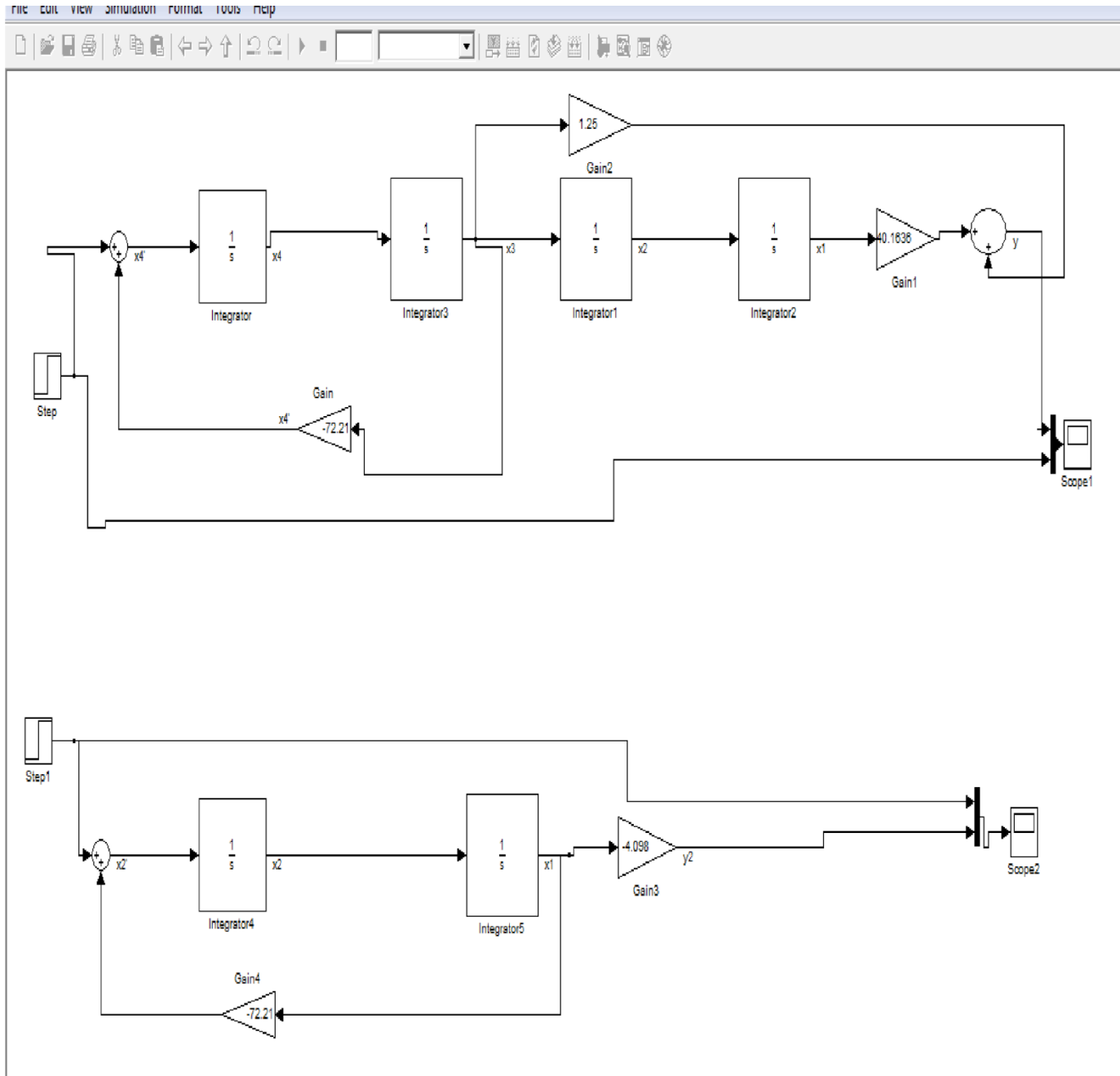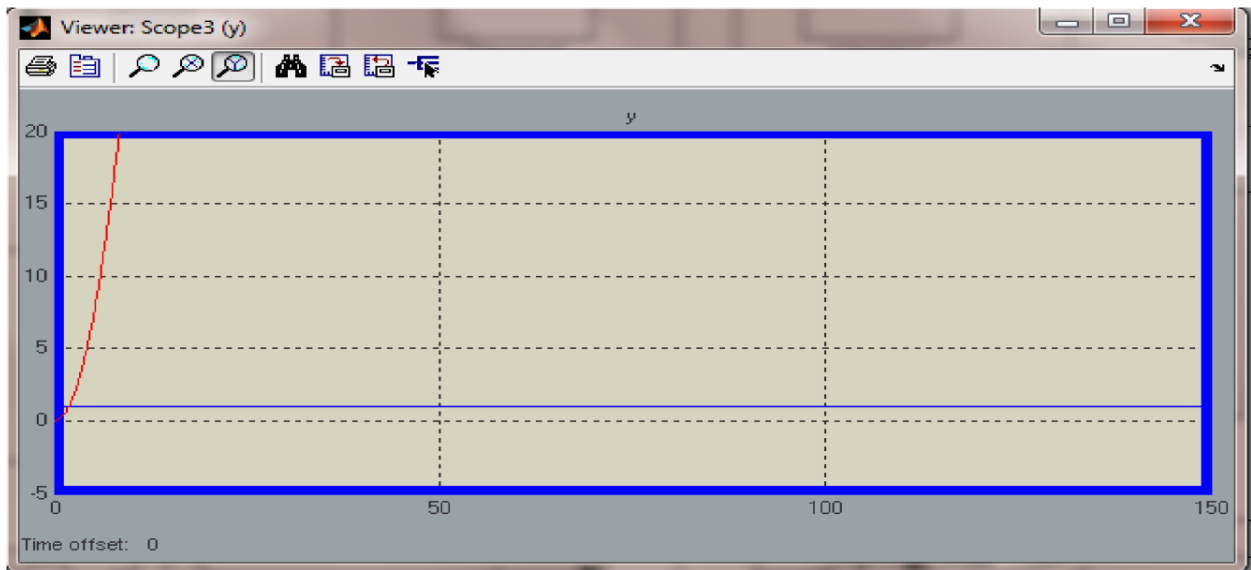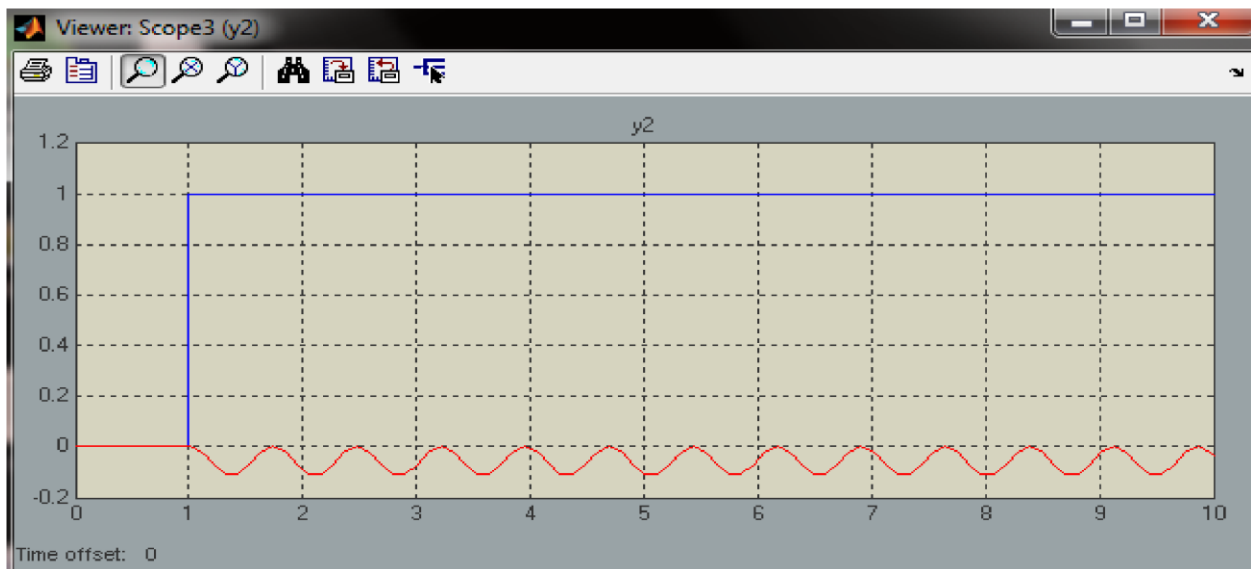for output 1 & 2 :

Figure 8: control canonical model for output 1 & 2 .

Figures 9 & 10 show the behavior of the distance travelled and sway angle respectively when we apply a step function as an input to the system:
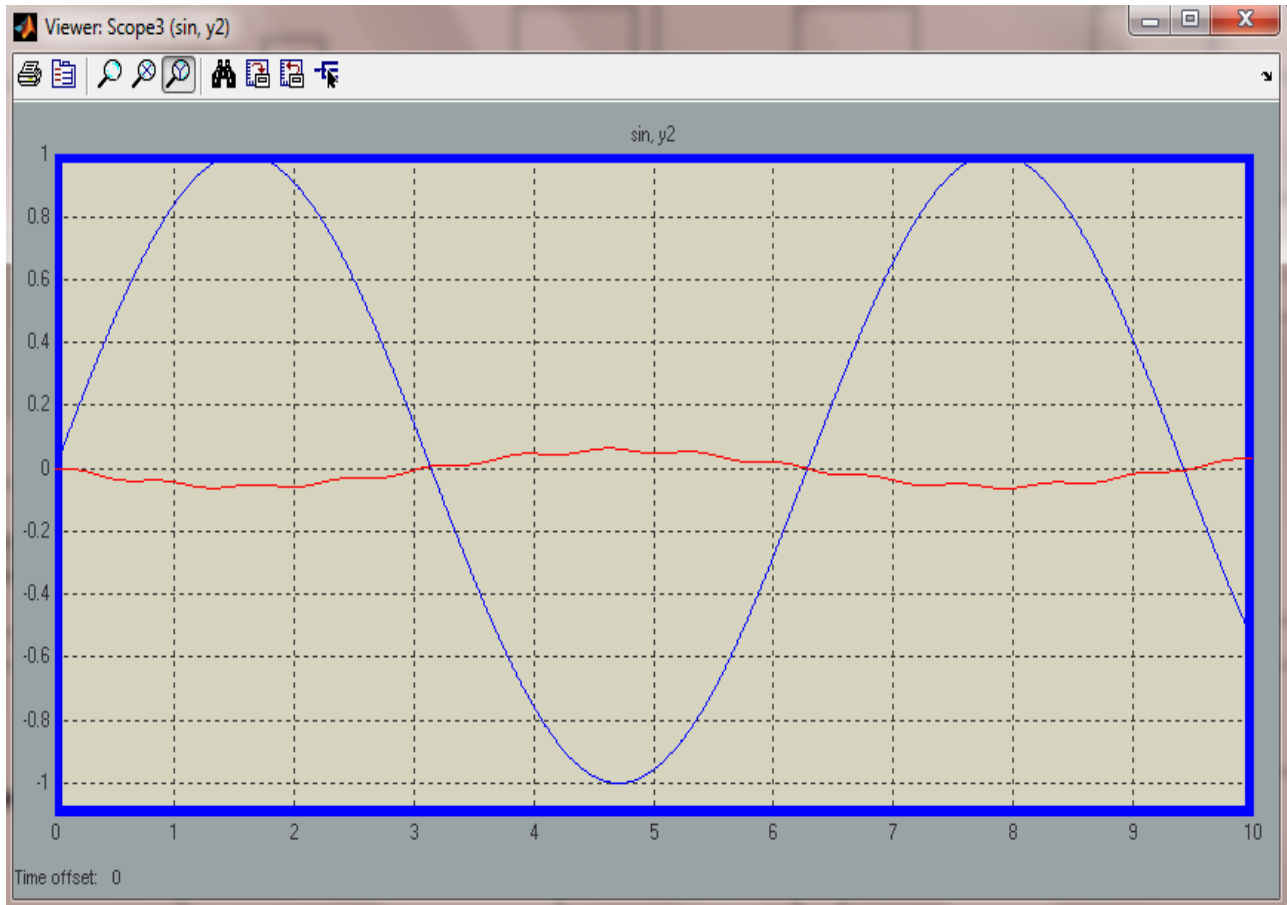

figure 9 above: graph for the behavior of distance travelled "control canonical form ".

Figure 10 A : graph for the behavior of sway angle " control canonical form " .

- *As shown in the figures above both system are not stable.*

Figure 10 B : graph for the behavior of sway angle " control canonical form " , sin wave input .

Similarly we simulate the system using observer canonical form, figure 11 show the observer canonical model for the 2 outputs.
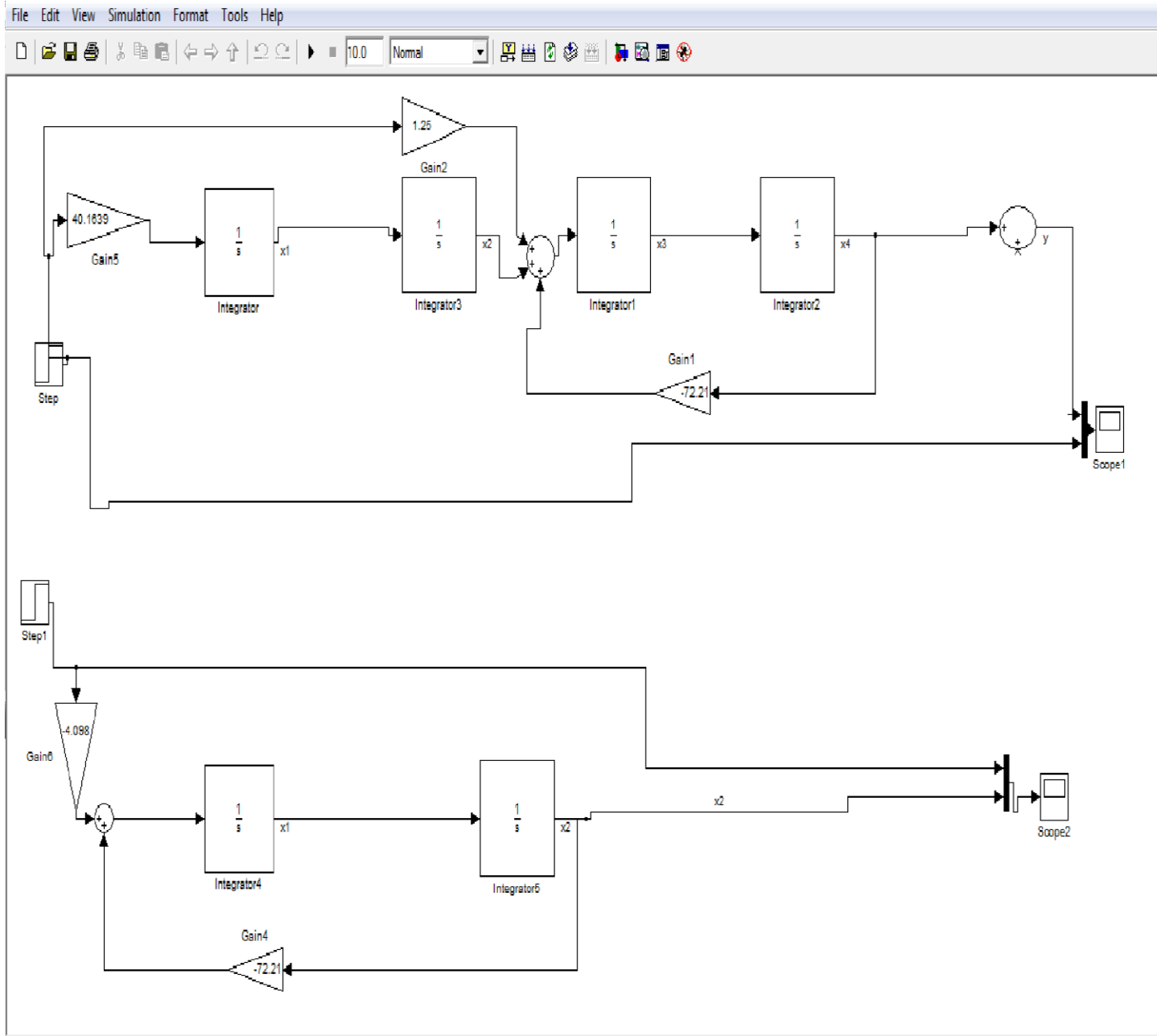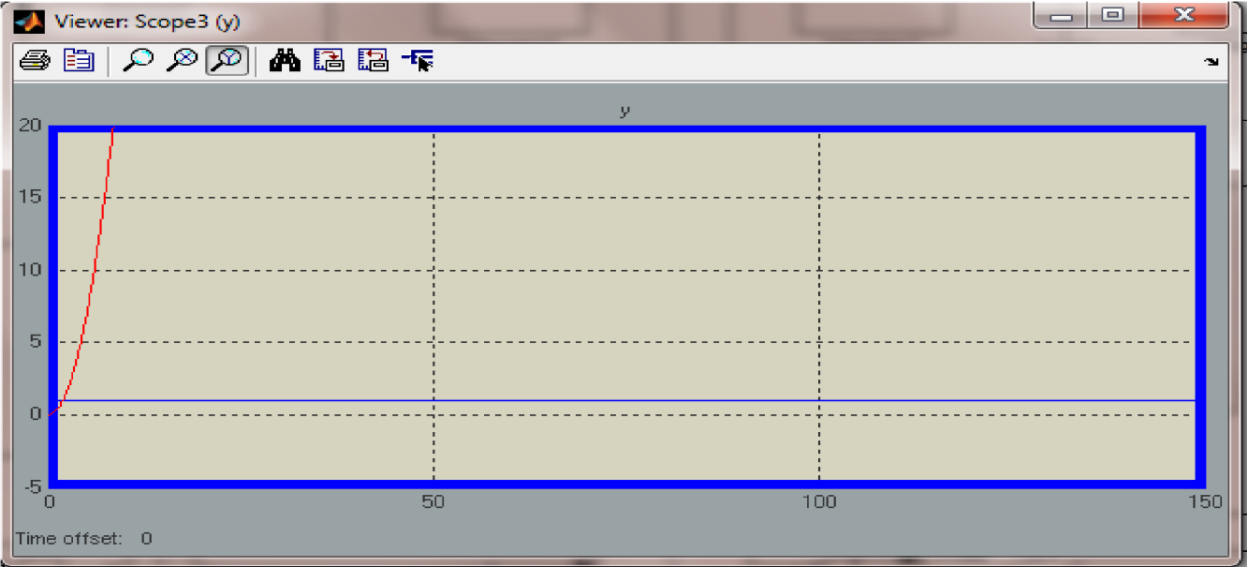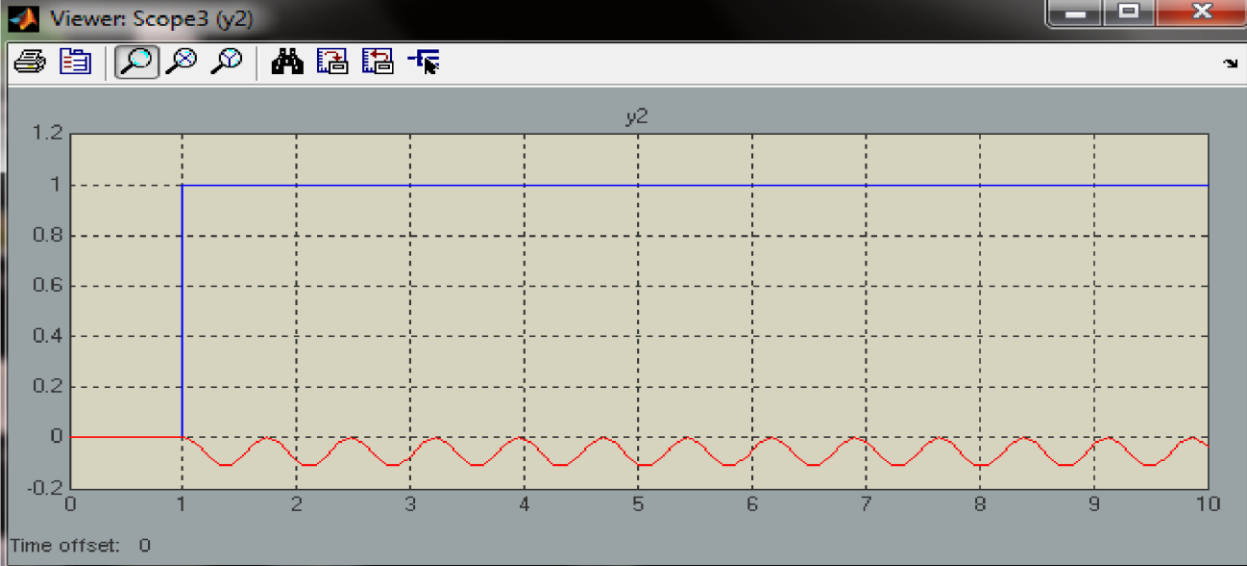

Figure 11 : observer canonical model for the distance travelled " scope 1" and sway angle "scope 2 ":

Also graphs 12 , 13 show the behavior of the 2 outputs distance travelled by the trolley and sway angle respectively in observer canonical form .

Figure 12 : graph for the behavior of distance travelled by the trolley " observer canonical form".

Figure 13 : graph for the behavior of the sway angle " observer canonical form".

Discussion:

- As shown in graphs 6, 7, 9, 10 , 12 and 13 clearly we can observe that graphs 6,9 and 12 are the same which shows that the behavior of output 1 " distance travelled by the trolley " is actually the same in state-space , control canonical and observer canonical forms , similarly graphs 7 , 10 and 13 are the same which also clearly pointed that the behavior of the output 2 " sway angle " is the same in state-space , control and observer canonical form .
- These graphs indicates that we are walking on the right track because it verified that the accuracy of the signal graph produced is high , further more it shows that the system is unstable which simply lead us to design a controller and observer for making the system stable as much as possible .

4.3 Designing a controller and observer for the 2 outputs system.

After simulating the systems through 3 methods simulating state-space , control and observer canonical models and making sure that the system gives the same output graph that obviously shows that the system is unstable , designing a controller and observer becomes a very important step in this project so that system become stable .

Checking the controllability and observability of the system is a very essential step In order to make sure that the system is controllable and observable in the first place

The next commands have been written in Matlab in order to know if the output 1 "displacement is controllable and observable or not:

>> a = [0   -72.2131   0   0 ; 1   0   0   0 ; 0   1   0   0 ; 0   0 1   0];

>> b= [1 ; 0 ; 0 ; 0];

>> c=[0   1.2500   0 40.1639];

>> d= [0];

>> M=[b a*b (a^2)*b (a^3)*b];

>> rank(M)

ans =   4

>> N=[c' a'*c' (a')^2* c' (a')^3* c'];

>> rank(N)

ans = 4

- ***This system is 4 by 4 system so the rank should be >= 4***
- ***rank (m)=4 and rank(n)= 4 show that the system is controllable and observable and we could design both controller and observer***
- ***getting rank m and n through those 2 equations respectively***

1) *M=[b a*b (a^2)*b (a^3)*b] : controllability test" 4 by 4 system "*

2) *N=[c' a'*c' (a')^2* c' (a')^3* c'] : observability test " 4 by 4 system "*

 similarly checking if the output 2 " sway angle " is controllable and observable or not :

A =[0   -72.2951 ; 1   0]

B= [1; 0];

C =   [0 -4.0984];

D =   [0];

>> M=[B A*B];

>> rank (M)

ans = 2

>> N=[C' A'*C'];

>> rank(N)

ans =   2

- ***This system is 2 by 2 system so the rank should be >= 2***
- ***rank (m)=2 and rank(n)= 2 show that the system is controllable and observable and we could design both controller and observer***
- ***getting rank m and n through those 2 equations respectively***

3) *M=[b a*b] : controllability test" 2 by 2 system "*

4) *N=[c' a'*c'] : observability test " 2 by 2 system .*

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4.3.1 Designing controller for the system, output 1 :

After making sure that the system is controllable we can write certain commands on Matlab in order to get the controller gains for output 1; displacement or distance travelled by the trolley.

Refer to appendix 5

>> poles=eig(a)

poles =

 0

 0

 0 + 8.4978i

 0 - 8.4978i

- ***The values shown above represent the real pole for the system***
- ***By choosing different values from the real poles we can get a better controller design so we will enter the next new values for the poles and get the controller gain.***

>> p1=-4;

>> p2=-4+8i;

>> p3=-4-8i;

>> p4=-3;

>> k=place(a,b,[p1 p2 p3 p4])

k = 15.0000 75.7869 656.0000 960.0000

We have controller gains for the controller being designed for the distance travelled by the trolley ad

$$K= 15\ \ 75\ \ \ 656\ \ \ 960$$

4.3.2 Designing controller for the system , output 2 :

Similarly we will enter the same commands in order to get the controller gain for output 2 ; sway angle " refer to appendix 6

A = [0   -72.2951 ; 1.0000   0]

B =[1; 0]

C=[0   -4.0984];

D = 0;

>> poles=eig(a)

poles =

    0 + 8.5027i

    0 - 8.5027i

- *Real poles for the system are 0 + 8.5027i , 0 - 8.5027i and we will enter new values of the poles in order to get a better design for the controller*

>>P1=-2;

> P2=-1.5;

>> K=place(A,B,[P1 P2])

K =

  3.5000 -69.2951

Table 3 shows the controller gains for output1 and output 2 for the system

Controller gains for Output 1 , displacement	*Controller gains for Output 2 sway angle*
K= **15 75 656 960**	**k = 3.5 -69.29**

By simulating the systems on Matlab using Simulink , graph 14 , 15 shows the system with the controller gains using the control canonical model for output 1 & 2 respectively *:*
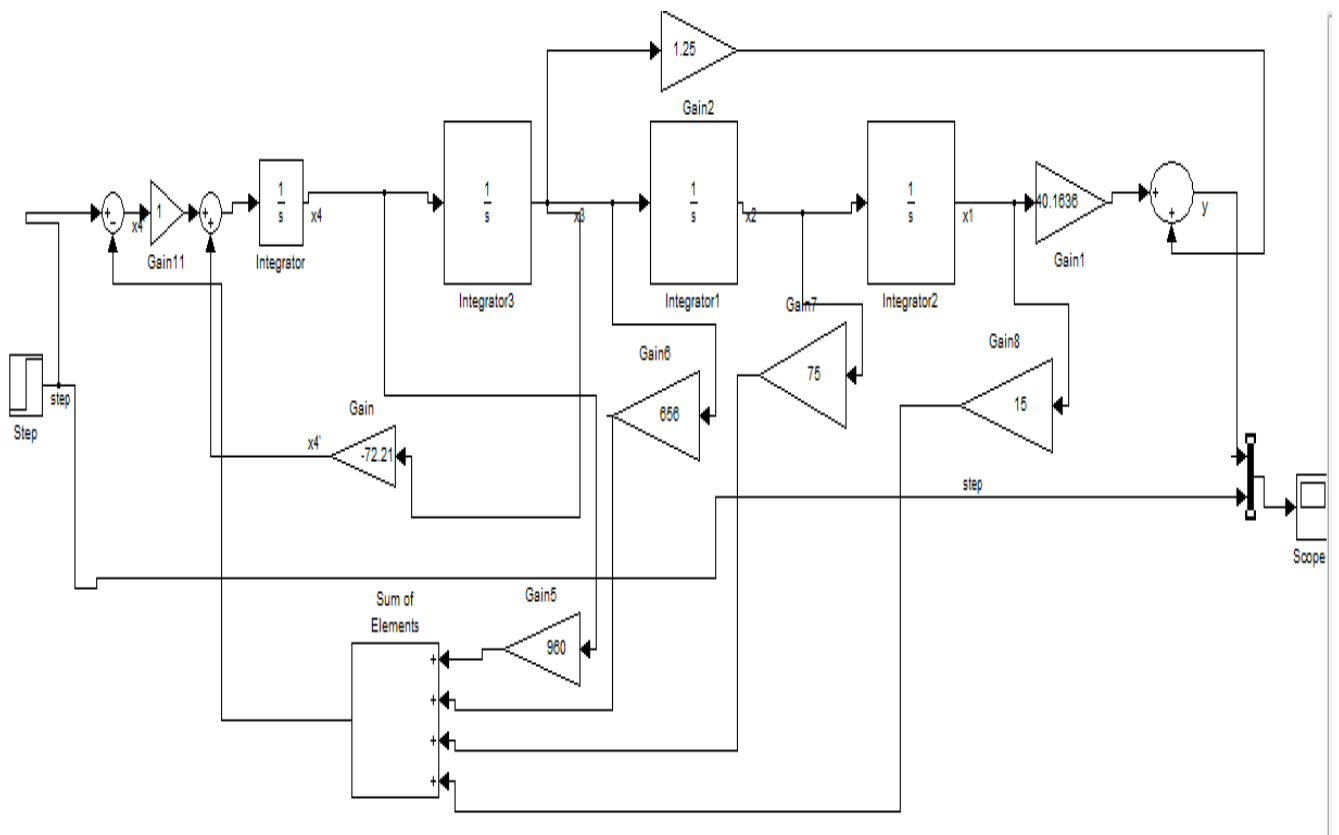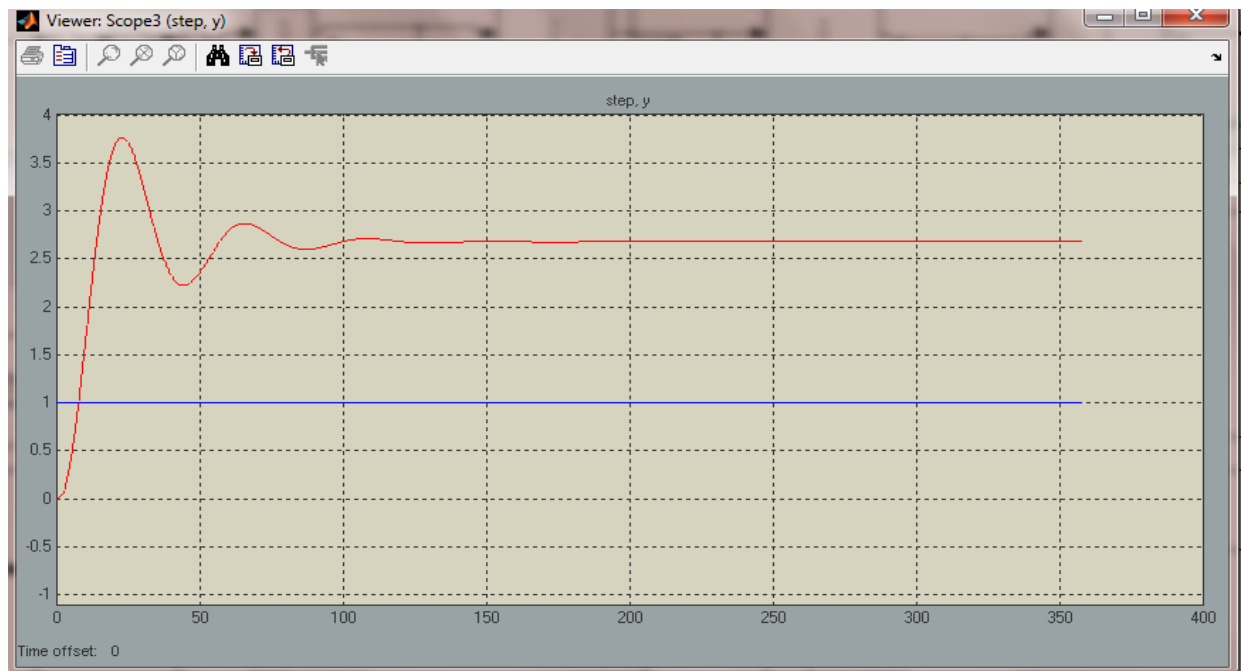

Graph 14 : control canonical form + the controller gains for the 1ˢᵗ output , displacement
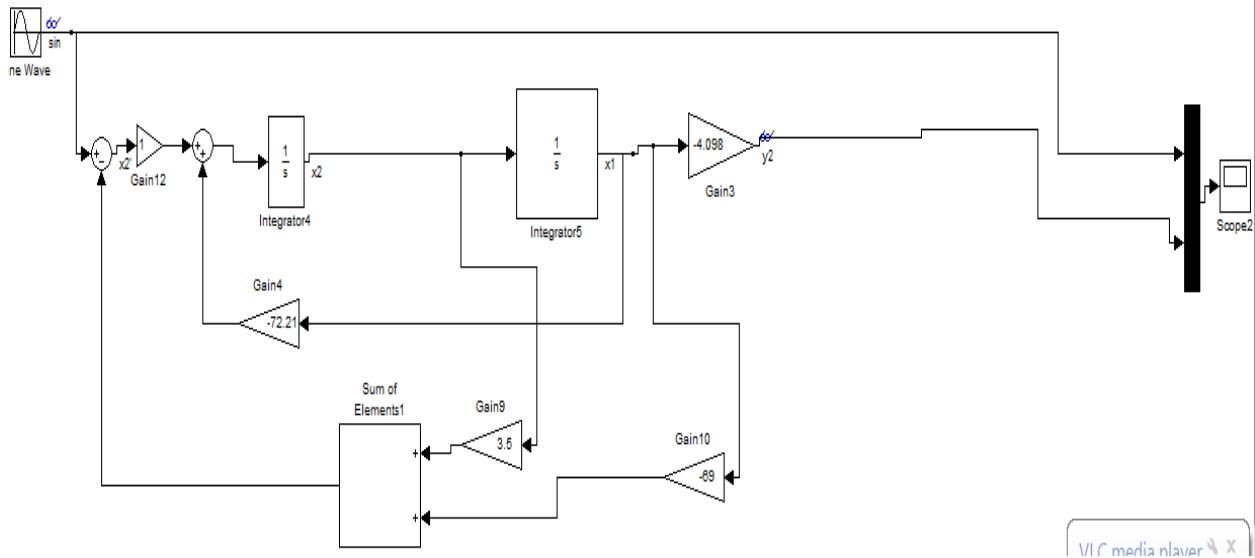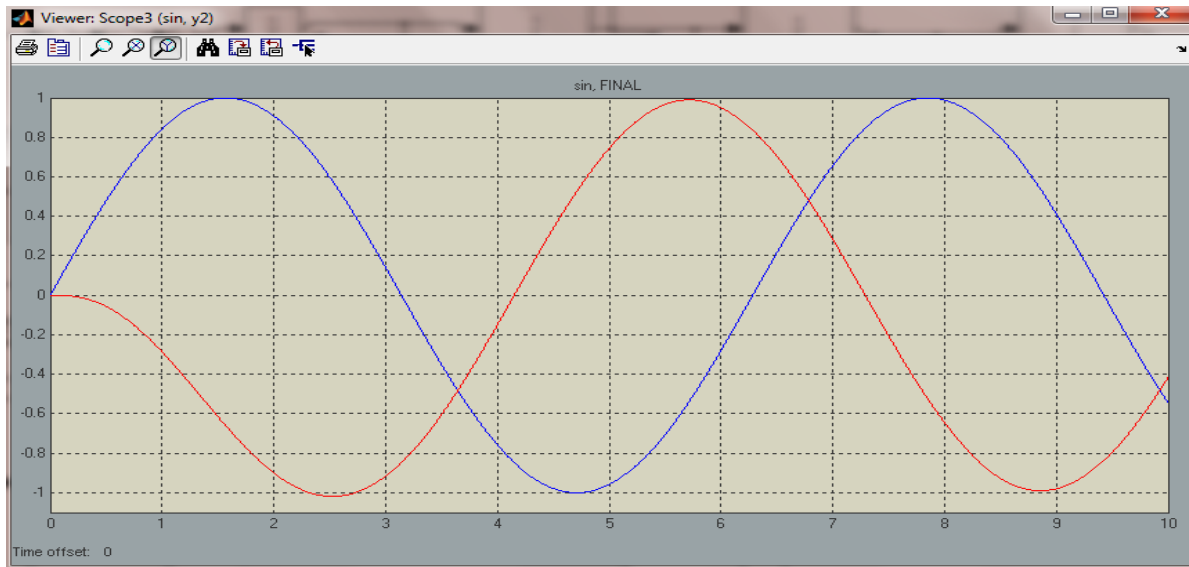
Graph 15 : control canonical form + the controller gains for the 2nd output , sway angle.

Graph 16, 17 shows the output behavior for the displacement and the sway angle respectively after designing the controller to the system .

Graph 16 : The output behavior of the displacement after designing the controller .

Graph 17 : The output behavior of the sway angle after designing the controller "sin wave onput" .

Discussion:

- As it is shown in graph 16 and 17 , the 2 systems have become more stable than before after designing the controller .
- Graph 16 shows that the distance travelled by the trolley graph become more stable than the original graph
- Graph 17 indicates that the sway angle graph become more stable since the swaying amplitude became same like the sin wave applied.

4.4 Designing an observer for the systems.

After simulating the systems with the controller design it becomes important to design an observer for both outputs to make the system more stable and to have a better result and a better graph.

4.4.1 Designing the observer for the system ; displacement .

Refer to appendix 7 to get the observer design using Matlab commands for output 1 , displacement or distance travelled by the trolley .

Form Matlab we got this result :

num =

 1.0e+005 * 0   0.0008   0.0388   0.6231   5.2424

den =

 1.0e+005 *

  0.0000   0.0010   0.0403   0.6296   5.2520

Which shows the observer design for the system that could be written as :

> ❖   **observer Design for displacement** $= \dfrac{80s^3 + 3880s^2 + 62310\,s + 524240}{100s^3 + 4030\,s^2 + 62960s + 52520}$

4.4.2 Designing the observer for the system; sway angle:

Similarly we will enter the same commands in order to get the observer equation for output 2 ; sway angle " refer to appendix 8"
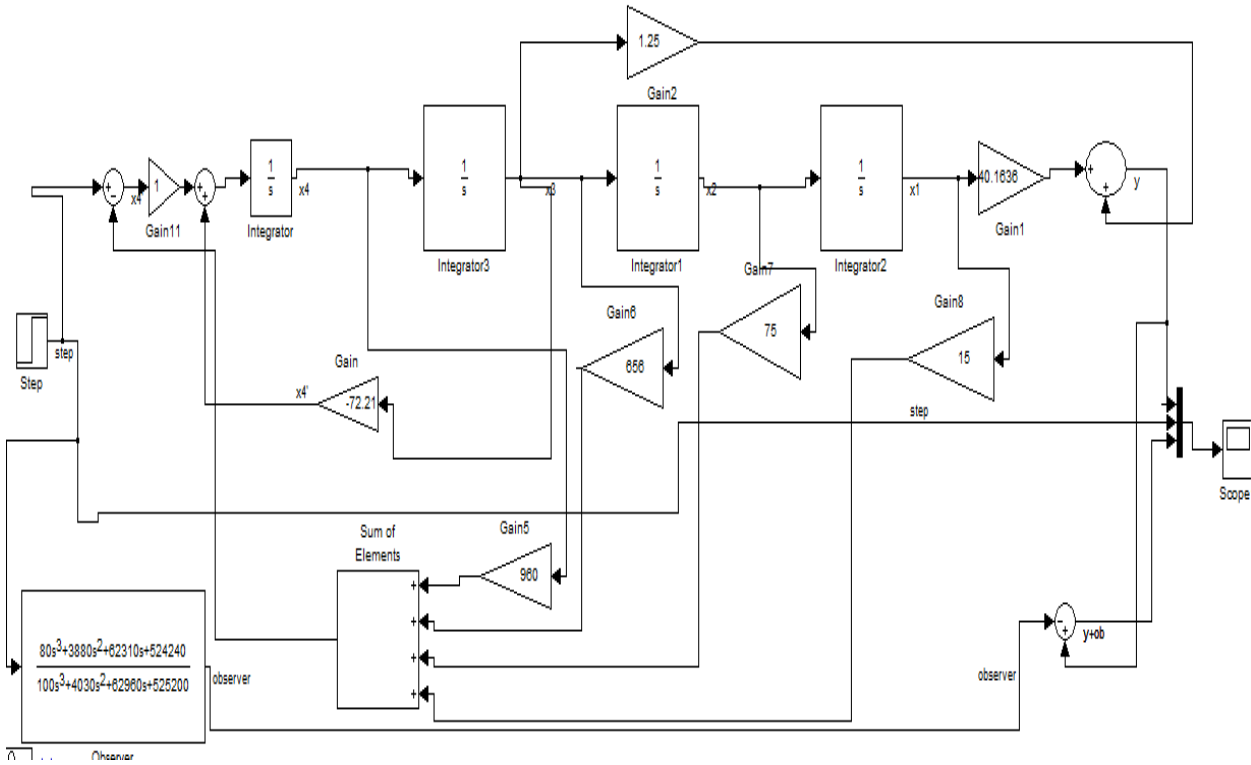
> ❖ **Observer Design for sway angle** $= \dfrac{-4}{s^2 + 44s + 499}$

Table 4 shows the observer design for output1 and output 2 for the system as a transfer functions:
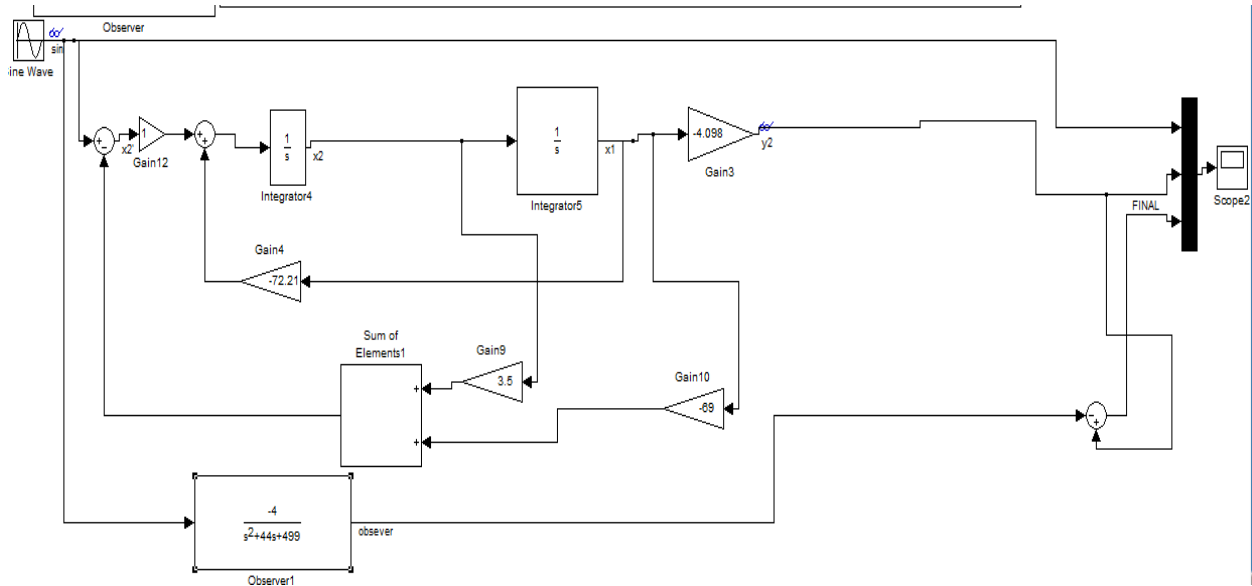
Observer design for Output 1 ; displacement .	*Observer design for Output 2 ; sway angle .*
$$\frac{80s^3 + 3880s^2 + 62310\ s + 524240}{100s^3 + 4030\ s^2 + 62960s + 52520}$$	$$\frac{-4}{s^2 + 44s + 499}$$

By simulating the system on Matlab in order to get the output behavior of the system .

Graphs 18, 19 show the controller model together with the observer design for the displacement and sway angle respectively.
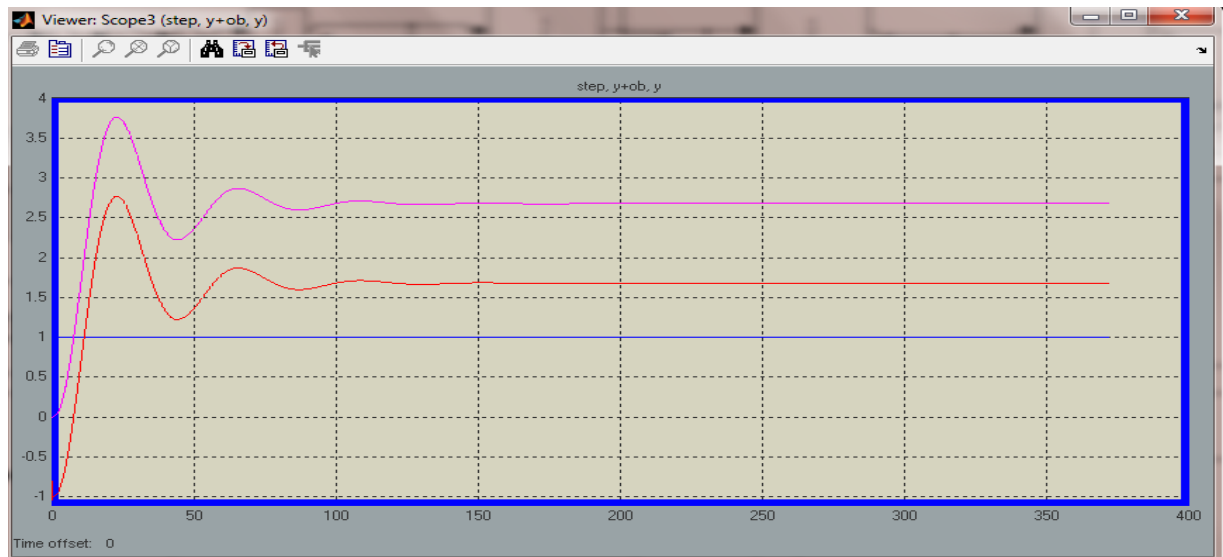

Graph 18 : controller model + observer designing for the 1ˢᵗ output , displacement .

Graph 19 : Controller model together with observer design for the sway angle .
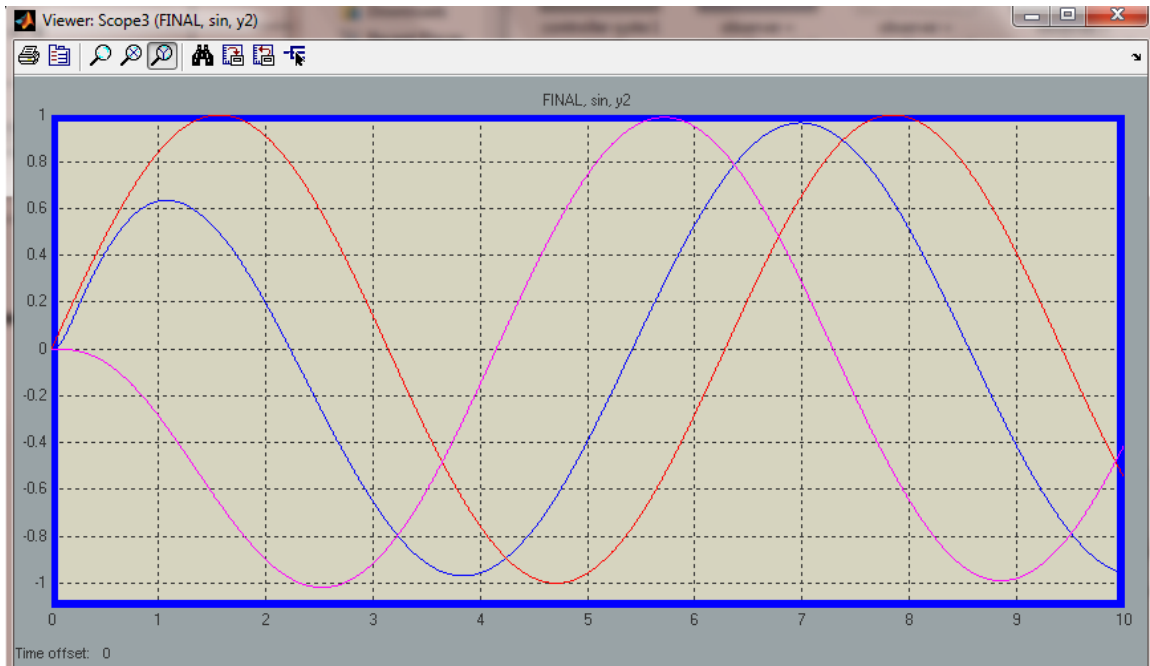
Graphs 20, 21 show the output behavior for the displacement and the sway angle respectively after combining the controller design with observer design to the system .

Graph 20 : The output behavior For the displacement after combining the controller with the observer .

- *The pink color indicates the graph for the controller only*
- *The red color indicates the output behavior for the system after combining the controller and the observer.*

Graph 21 above shows the sway angle behavior after combining the controller design together with the observer design

The purple color :   : shows the sway angle behavior (controller only)

The red color : the step input

the blue color shows the sway angle behavior (with the controller design and the observer design

Discussion:

Graphs 20 and 21 show the output behavior of the 2 systems - displacement and sway angle respectively - after combining the design of the controller and the observer together which clearly give us a better result and better graph because the graphs seems more stable and very near to the step signal in the 1st output or the sin wave in the 2nd output .

CHAPTER 5

Conclusion & Recommendations

As a conclusion for this report: Developing 2D crane system performances basically comes by reducing the its' sway angle as much as possible and improving the displacements' "distance travelled by the trolley "graph to make it more stable and steady like the input signal given to it , and there are several methods have been applied for achieving this issue such as input shaping method . In this research the main target is trying to develop a controller and observer via state-space approach for making the system stable as much as possible by reducing the sway angle.

Background studies, literature review and picking the proper method to be worked with besides getting the dynamic model of 2D crane system by using Euler-Langrange formulation achieved the main target of FYP 1 research.

The rest of the procedures is being done as a FYP2 work represented in simulating the system by using state-space , control and observer canonical forms in order to prove that system is unstable before going through next procedures which are designing a controller and observer and again simulating the system to check the stability .

As a conclusion I can say that the system is obviously unstable and the signal graphs produced is very accurate because the graphs are the same for 3 methods checking, state-space , control canonical and observer canonical form .

Doing these steps followed by designing the controller and observer , the objectives of this project have been achieved successfully; since the graphs have been produced after designing the controller and the observer show that the 2 outputs system behavior have became more stable and gives a better result using it than not using the observer and the controller at all , further for final year students this project is very helpful for understanding modern control system and automotive in such a way that the studies of those subjects have been applied for solving problems in the real life .

As a recommendation for the next year student research , it is recommend that they combine the 2 outputs together , as for my research I dealt with the two outputs for the system which they are the distance travelled by the trolley and the sway angle individually ; so it is recommend to combine them both as a new method of work .

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[6] Hongjun Chen, Bingtuan Gao, and Xiaohua Zhang, Dynamical Modeling and Nonlinear Control of a 3D Crane, International Conference on Control and Automation Budapest, Hungary, June 27-29, 2005

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APPENDICES

APPENDIX 1 : How to get The control canonical form for the displacement using Matlab commands .

>> num=[0 0 .305 0 9.8];

>> den=[.244 0 17.62 0 0];

>> [a,b,c,d]=tf2ss(num,den)

We will get this data

a =

0	-72.2131	0	0
1.0000	0	0	0
0	1.0000	0	0
0	0	1.0000	0

b =

1

0

0

0

c = 0   1.2500   0   40.1639

 d=0

APPENDIX 2 : How to get The observer canonical for the displacement using Matlab commands.

>> osys=ss(a',c',b',d)

a =	x1	x2	x3	x4
x1	0	0	0	0
x2	1	0	0	0
x3	0	1	0	-72.21
x4	0	0	1	0

APPENDIX 3 : How to get The control canonical form for the sway angle using Matlab commands .

 >> num=[1];

>> num=[0 0 1];

>> den=[-.244 0 -17.64];

>> [a,b,c,d]=tf2ss(num,den)

a =

0	-72.2951
1.0000	0

b = 1

   0

c =   0   -4.0984

d =   0

APPENDIX 4 : *How to get The observer canonical form for the sway angle using Matlab commands .*

osys=ss(e',g',f',h)

a =

```
      x1    x2

 x1    0  -72.3

 x2    1    0
```

b =

```
       u1

 x1  -4.089

 x2     0
```

c =

```
     x1  x2

 y1   0   1
```

d =

```
      u1

 y1   0
```

APPENDIX 5 : *How to get the controller gains using Matlab commands for output 1 , displacement or distance travelled by the trolley .*

>>a ;

>>b ;

>>c = ;

>> d ;

 >> *poles=eig(a)*

poles =

   0

   0

   0 + 8.4978i

   0 - 8.4978i

>> p1=-4;

>> p2=-4+8i;

>> p3=-4-8i;

>> p4=-3;

>> k=place(a,b,[p1 p2 p3 p4])

k =

   15.0000 75.7869 656.0000 960.0000

APPENDIX 6 : How to get the controller gains using Matlab commands for output 2 ; sway angle .

a = [0   -72.2951 ; 1.0000   0]

b =[1; 0]

c =[0   -4.0984];

d = 0;

>> poles=eig(a)

poles =

     0 + 8.5027i

     0 - 8.5027i

>>P1=-2;

> P2=-1.5;

>> K=place(A,B,[P1 P2])

>> K =

   3.5000 -69.2951

APPENDIX 7 : How to get the observer design using Matlab commands for output 1 , displacement or distance travelled by the trolley

>> a;

>> b;

>> c;

>> d;

>> M=[b a*b (a^2)*b (a^3)*b];

>> rank(M);

>>rank(M)

ans = 4

>> J=[-4 0 0 0 ;0 -4+8i 0 0 ; 0 0 -4-8i 0 ; 0 0 0 -3];

>> poly(J);

>> Phi=polyvalm(poly(J),a);

>> K=[0 0 0 1]*(inv(M))*Phi;

>> K

K =

   15.0000   75.7869   656.0000   960.0000

>> N=[c' a'*c' (a')^2* c' (a')^3* c'];

>> J0=[-20.1 0 0 0 ;0 -20.2 0 0 ; 0 0 -22.1 0 ; 0 0 0 -22.2];

>> poly(J0);

>> Ph=polyvalm(poly(J0),a);

>> Ke=Ph*(inv(N'))*[0;;0 ;0; 1];

>> Ke=Ph*(inv(N'))*[0;0 ;0; 1]

Ke =

 -214.4755

 -631.6944

   71.6516

   21.7662

>> aa =a -Ke*c -b*K;

>> bb=Ke;

>> cc=c;

>> dd=d;

>> [num,den]=ss2tf(aa,bb,cc,dd)

num = 1.0e+005 * 0   .0008   0.0388   0.623   5.244

den = 1.0e+005 0   .0010   0.0403   0.6296   5.252

APPENDIX 8 : How to get the observer design using Matlab commands for output 2 ; sway angle .

>> A = [0 -72.295 ; 1.0000   0]

>> B= [1 ; 0]

>> C = [0   -4.0984];

>> D=   0

>> M=[B A*B];

>> rank (M)

>>ans=2

>> J=[-2 0;0 -1.5];

>> poly(J);

>> Phi=polyvalm(poly(J),A);

>> K=[0 1]*(inv(M))*Phi

K =

   3.5000 -69.2951

>> poly(J0);

>> Ph=polyvalm(poly(J0),A);

>>

>> Ke=Ph*(inv(N'))*[0;1];

>> Ke=Ph*(inv(N'))*[0;1]

Ke =

 -84.8400

 -10.0040

>> AA =A -Ke*C -B*K

>> BB=B;

>> CC=C;

>> DD=D;

>> [NUM,DEN]=ss2tf(AA,BB,CC,DD)

NUM =

     0   0 -4.0984

DEN =

  1.0000 44.5000 494.2049

4