DESIGN OF A CONTROLLER FOR CAR SUSPENSION SYSTEM VIA STATE SPACE APPROACH

By

SHARIFAH HANISAH BT SYED MOHD KHALID

Dessertation submitted in partial fulfilment of
the requirements for the
Bachelor of Engineering (Hons)
(Electrical Engineering)

AUGUST 2012

Universiti Teknologi PETRONAS

Bandar Seri Iskandar

31750 Tronoh

Perak Darul Ridzuan

CERTIFICATION OF APPROVAL

DESIGN OF A CONTROLLER FOR CAR SUSPENSION SYSTEM VIA STATE SPACE APPROACH

by

Sharifah Hanisah bt Syed Mohd Khalid

A project dissertation submitted to the
Department of Electrical & Electronic Engineering
Universiti Teknologi PETRONAS
in partial fulfilment of the requirement for the
Bachelor of Engineering (Hons)
(Electrical & Electronic Engineering)

Approved:

____

Assoc. Prof. Dr. Nordin Bin Saad
Project Supervisor

UNIVERSITI TEKNOLOGI PETRONAS

TRONOH, PERAK

AUGUST 2012

CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

____

[SHARIFAH HANISAH BT SYED MOHD KHALID 12440]

ABSTRACT

When it come to automobile performance, people will normally think about horse power torque and zero-to -60 acceleration. However with a good power generated by the piston engine it is still useless if the driver cannot manage to control the car. On the other hand, a good car performance will come together with a good suspension controlling. The propose of car suspension system is to minimize the friction between the tires and the road surface, to provide steering stability with good handling and having a comfort ride.

In this project , a brief information about car suspension system and how it work will be presented. In addition , state space approach have been chose for the method in the derivation of the mathematical modeling in designing controller to improve the performance of the suspension system.

Besides that, the controller design method using MATLAB software are also presented. The simulation result will help in analyzing the characteristic of the system and also be discussed to prove the controlling effects of the suspension system.

LIST OF TABLE

LIST OF FIGURE

LIST OF TABLE

CHAPTER 1
INTPRODUCTION

1.1 BACKGROUND

A car chassis comprises all the important system located under the car body ans suspension is one of it. Others main part of chassis are, the steering system, tires and the frame. Suspension system is basically connecting the wheels of the car to the car body. When a car hits a bump or hit a hole on the road, it will experience jolts and this suspension system will act as a body cushioned to give a stable and comfort ride.Besides that, this system are meant to support the vehicle weight, to maintains the body at relatively constant height , and to maintain the traction force between the tire and the road surface.

There are three type of car suspension system :

- Passive suspension system
- Semi- active suspension system
- Active suspension system

Passive show that the suspension elements cannot supply energy to the suspension system. This system includes the tradisional springs and shock absorbers. It has the ability to store energy using spring and dissipate the energy using the dampers. Semi- active suspension system can control the dissipation of energy. In this system, controlled damper or semi -active damper under closed loop control is needed to regulate damping force by dissipate the energy.

In this project, state space approach use for designing and modeling the controller for semi –active suspension system. The controller need to able to control the relative motion between car body and wheels. Others than, overall performance of a semi- active suspension system will be determine. Result from this, will be compare with the passive system using MATLAB simulink.

1.2 PROBLEM STATEMENT

The primary goal for a semi- active suspension design is to find a solution for good ride quality, handling performance and suspension travel. When a car hit a bumps or a hole, it experiance vertical accelaration that will lower down car handling an comfort. Thus to reduce the vertical acceleration of the car body is the main function of suspension. A good suspension should have the ability to behave differently on smooth and rough roads, in order to enhance passenger comfort the desired response should be soft in the other hand suspension should stiffen up to avoid hitting its limit when the road surface is to rough.

However, this system ia a nonliner and unstable system, in such it providing a challenge to the control engineer or reseacrhers. Thus many efforts, approach and method that have been done to develop the controller for this system, yet there is no result for the best system approach.

.

1.3 OBJECTIVES

- To give a good comfort ride and a good car handling experience
- To model quarter car system using state space
- To design a suitable controller and for suspension system
- To simulate the design and analyze the best parameter for the system

1.4 SCOPE OF STUDY

This project scope of study is about the construction of the mathematical modeling for semi- active and passive suspension system for a quarter car model. Then proceed with the designing the controller for both o the system using state space approach. MATLAB software will be used to simulate the result, and the result will be analyze and compare to find the best controller for the systems.

CHAPTER 2

LITERATURE REVIEW

2.1 Semi –Active suspension system

The diffrent between semi active and passive susupension system is the controlable damper. This system can provide real time dissipation of energy or can be said as rapid change in rat eof spring damping coeeficient. The controller can detemine the level of damping based on control strategy used by adjusting the damper into disired value.

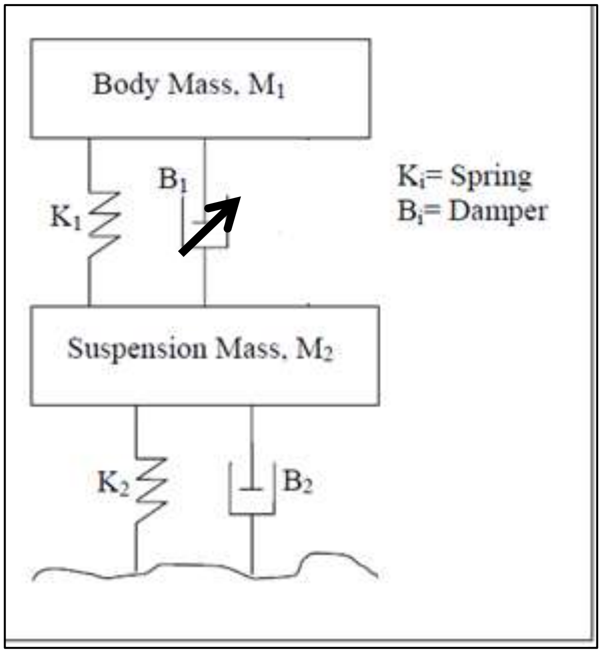


Figure 2.1: Semi-active car suspension 1

2.2 Passive Suspension System

For Passive suspension system its contains, spring and damper where spring absorb the energy and damper dissipate the energy. The spring located parallel with the damper placed at each corner of the vehicle, and the combination of spring-damper in this class is called a “strut”[5] . To support the strut some additional element or special geometrical arrangement are also used to upgrade the performance of passive system. For example, to increase the stiffness to roll motion, an additional roll spring (anti-roll bars)can be used. Trailing arms could be implemented between the sprung mass and the wheel hubs to reduce the dive and squat motion of the vehicle body during braking and acceleration.[5].

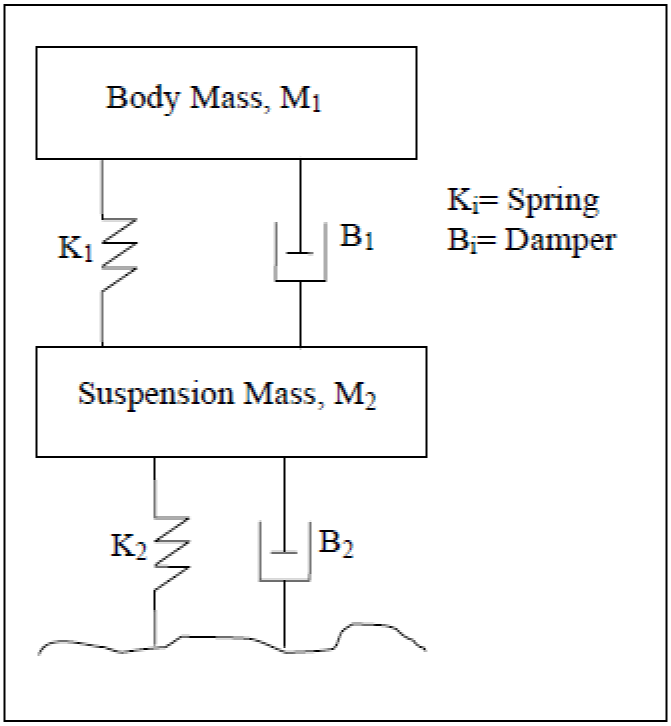


Figure 2.2: Passive car suspension

2.3 Controllability and Observability

The original theoretical concepts of controllability and observability was introduced by R. Kalma in 1960 are particularly important for practical implementation[6]. Controllability mean that a system is controllable where the system state can be change under control input. In the other hand, observability mean in order to see what going on inside the system under observation, the system must be observable[6].A linear controllable system may be define as a system which can be steered to any state from the zero to initial state. Kalman's canonical decomposition provides the basic theory and computational algorithm to remove unnecessary state from a realization, while preserving the input output[6].

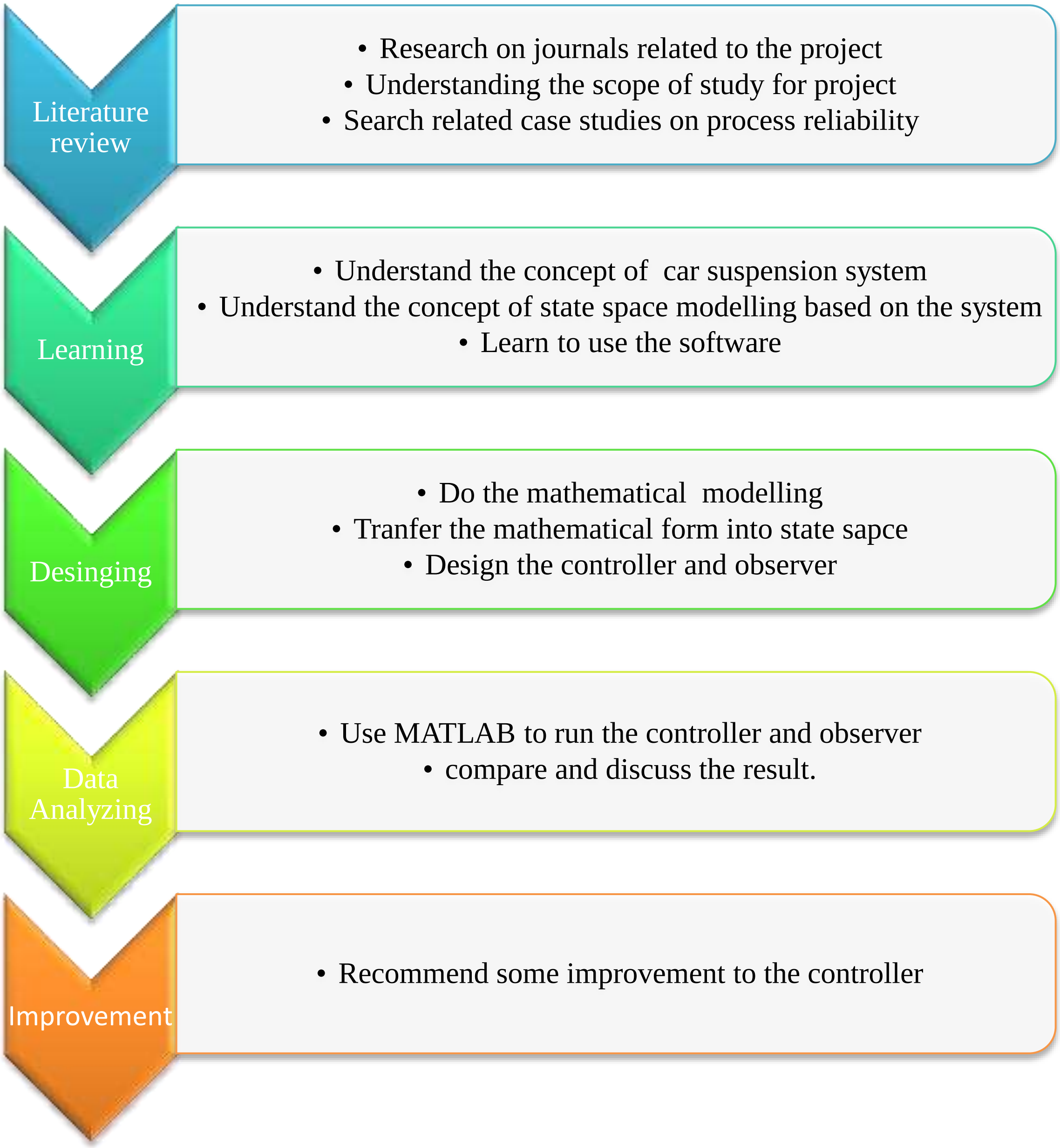
2.4 State space controller

In 1997, Carneige Mellon from University of Michigan had develop a procedure of designing state space controller in MATLAB for bus suspension system, [2]. The proposed method was applied to a suspension system, and simulation results show that substantial improvement in the performance was achieved compared with other local observer. An active suspension system gives better performance in term of comfort ride compared to passive suspension system was proved by study conducted by Yahya md. Sam[3][4]. An active system also proves to increases a tires to road contact in order to make the vehicle more stable.

CHAPTER 3

METHODOLOGY

3.1 PROJECT FLOW CHART



Literature review

- Research on journals related to the project
- Understanding the scope of study for project
- Search related case studies on process reliability

Learning

- Understand the concept of car suspension system
- Understand the concept of state space modelling based on the system
- Learn to use the software

Desinging

- Do the mathematical modelling
- Tranfer the mathematical form into state sapce
- Design the controller and observer

Data Analyzing

- Use MATLAB to run the controller and observer
- compare and discuss the result.

Improvement

- Recommend some improvement to the controller

Figure 3.1 : Project Flow

3.2 TOOLS AND EQUIPMENT

- MATLAB Software for data analysis and comparison
- Microsoft Word - report writing

W
MATLAB®
The Language of Technical Computing
Copyright 1984-2006, The MathWorks Inc.
Protected by U.S. patents. See www.mathworks.com/patents
The MathWorks

Figure 3.2 : Tools

3.3 GANTT CHART

	FYP 1														FYP 2														
Activity/Week	1	2	3	4	5	6	7	8	9	10	11	12	13	14	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Literature review	🟨	🟨	🟨	🟨	🟨	🟨																							
Previous research			🟨	🟨	🟨	🟨																							
Extended proposal defence						🟥																							
Porposal defence								🟥																					
Interim Draft Refort													🟥																
Interim Report														🟥															
Project Work																													
- tutorial ANSYS						🟨	🟨																						
-Mathematical Modeling								🟨	🟨	🟨	🟨	🟨	🟨	🟨															
- Derivation of state space equation														🟨	🟨	🟨	🟨												
- Analysis using MATLAB																🟨	🟨	🟨	🟨	🟨	🟨	🟨	🟨						
Submission of Dissertation																											🟥		
Submission of technical paper																											🟥		
Oral presentation																												🟥	
Project Submission																													🟥

Table 3.1 : Project Gantt chart

CHAPTER 4
RESULT AND DISCUSSION

4.1 Introduction

The main purpose of mathematical modeling is to obtain a state space representation of the quarter car model. This mathematical model allow to describe the relationship between the input and output, enable one to understand the behavior of the system.

A dynamic system can be describe by differential equation presentation via state space modelling. It use 1st order differential system. The state variables of a dynamic system are the variables making up the smallest set of variables that determine the state of the dynamic system. If at least n variables x1, x2… xn are needed to completely describe the behavior of a dynamic system (so that once the input is given for $t \leq t_0$and the initial state at $t = t_0$ is specified, the future state of the system is completely determined), then such n variables are a set of state variables. (Margaret Galvin, 2005) As for that, the state variables must not be physically measurable quantities. State variables must be choose in term of variables that do not represent physical quantities and those that are not measurable. It is an advantage of the state space methods where state variables can be freely choose. (Durbin and Koopman, 2004).

4.2 Mathematical Modeling

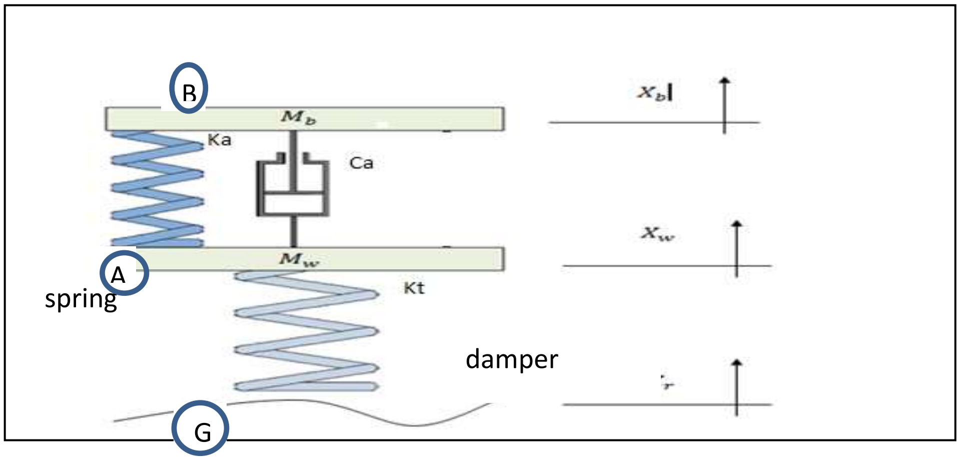


Figure 4.1: Passive suspension system

Refer to the figure 4.1, all the term use in the diagram can be define in the table below:

Term	Defination
Mw	mass of the wheel /unsprung mass (kg)
Mb	mass of the car body/sprung mass (kg)
Xr	road disturbance/road profile
Xw	wheel displacement (m)
Xb	car body displacement (m)
Ka	Ka = stiffness of car body spring (N/m)
Kt	stiffness of tire (N/m)
Ca	damper (Ns/m)
Xb –Xw	Suspension Travel
$\dot{X}_b$	Car velocity
$\ddot{X}_b$	Car body Acceleration
$Xw - Xr$	Wheel deflection
$\dot{X_w}$)	wheel vertical velocity

Table 4.1 : Term and defination for Passive suspension Modeling

From the figure 4.1, passive suspension system can be model by separating the body part and tire part. The two masses are connected together by a suspension that is made up of a spring of stiffness Ks and absorber having a damping factor Cs.Each part, will have it own free body diagram, where every force acting in the parts can be classified. For this passive suspension system, the modeling analysis start with the free body diagram at the body part (sprung mass.).

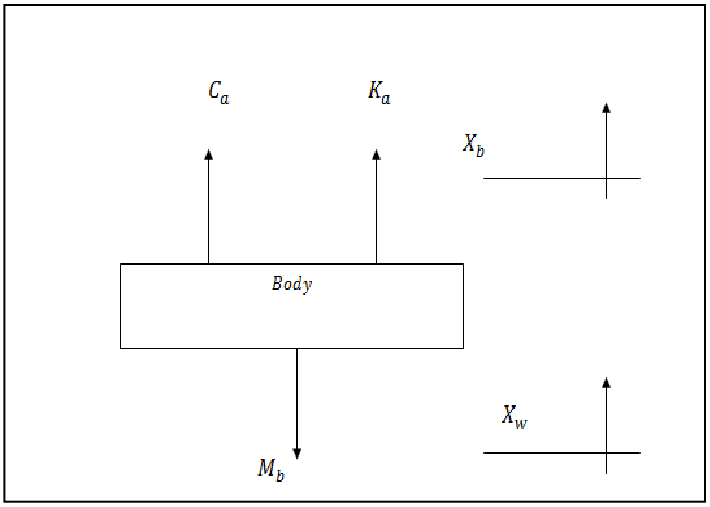


Figure4.2 : Free body diagram for Sprun g

$For\ M_b$

$F = Ma$

$$K_a(X_b - X_w) + C_a(\dot{X}_b - \dot{X}_w) + M_b\,\ddot{X}_b = 0 \quad \text{..................} \ (1)$$

$$K_a\,X_b - K_a\,X_w + C_a\dot{X}_b - C_a\dot{X}_w + M_b\,\ddot{X}_b = 0$$

$$M_b\,\ddot{X}_b = -K_a\,X_b + K_a\,X_w - C_a\dot{X}_b + C_a\dot{X}_w$$

Dividing equation (1) by M_b , we get,

$$\ddot{X}_b = \frac{-K_a\,X_b + K_a\,X_w - C_a\dot{X}_b + C_a\dot{X}_w}{M_b} \ldots\ldots\ldots\ldots\ldots\ldots..(2)$$

Now, it’s for Unsprung mass.

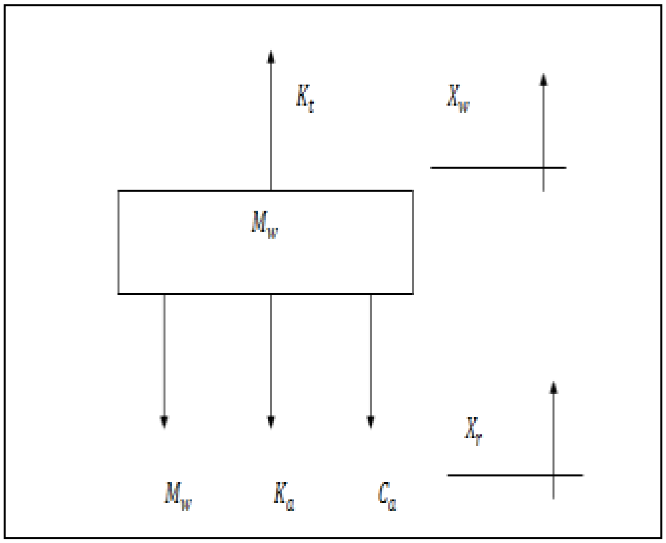


Figure 4.3: Free body diagram for Unsprung

$For\ M_w$

$F = Ma$

$$-K_t(X_r - X_w)+K_a(X_w - X_b) + C_a(\dot{X}_w - \dot{X}_b) + M_w\ddot{X}_w = 0$$

$$K_tX_w - K_tX_r+ K_aX_w - K_aX_b + C_a\dot{X}_w - \dot{C}_aX_b + M_w\ddot{X}_w = 0 \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$M_w\ddot{X}_w = -K_tX_w + K_tX_r - K_aX_w + K_aX_b - C_a\dot{X}_w + \dot{C}_aX_b$$

$$M_w\ddot{X}_w = -(K_t+K_a)X_w \ - C_a\dot{X}_w + K_aX_b + \dot{C}_aX_b + K_tX_r$$

Dividing equation (2) by M_w , we get,

$$\ddot{X_w} = \frac{-(K_t+K_a)X_w \ - C_a\dot{X_w} + K_aX_b + \dot{C_a}X_b + K_tX_r}{M_w} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(4)$$

The state space equation can be written as:

$X_1 = X_b$

$X_2 = \dot{X_b}$

$X_3 = X_w$

$X_4 = \dot{X_w}$

Thus, in state space the equation can be write as

$\dot{X_1} = \dot{X_b}$

$$\dot{X_2} = \ddot{X_b} = \frac{-K_a\, X_b + K_a\, X_w - C_a\dot{X_b} + C_a\dot{X_w}}{M_b}$$

$\dot{X_3} = \dot{X_w}$

$$\dot{X_4} = \ddot{X_w} = \frac{-(K_t+K_a)X_w \ - C_a\dot{X_w} + K_aX_b + C_a\,\dot{X_b} + K_tX_r}{M_w}$$

In State space Matric form,

$$\begin{bmatrix} \dot{X}_b \\ \ddot{X}_b \\ \dot{X}_w \\ \ddot{X}_w \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-K_a}{M_b} & \frac{-C_a}{M_b} & \frac{K_a}{M_b} & \frac{C_a}{M_b} \\ 0 & 0 & 0 & 1 \\ \frac{K_a}{M_w} & \frac{C_a}{M_w} & \frac{-(K_t+K_a)}{M_w} & -\frac{C_a}{M_w} \end{bmatrix} \begin{bmatrix} X_b \\ \dot{X}_b \\ X_w \\ \dot{X}_w \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\frac{K_t}{M_w} \end{bmatrix} X_r$$

$$Y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_b \\ \dot{X}_b \\ X_w \\ \dot{X}_w \end{bmatrix} + [0]\,[X_r]$$

This matric form can be write in ontrol canonical form.

$$\dot{X} = AX + BU$$

$$Y = CX + DU$$

$$\dot{X} = \begin{bmatrix} \dot{X}_b \\ \ddot{X}_b \\ \dot{X}_w \\ \ddot{X}_w \end{bmatrix} \qquad X = \begin{bmatrix} X_b \\ \dot{X}_b \\ X_w \\ \dot{X}_w \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-K_a}{M_b} & \frac{-C_a}{M_b} & \frac{K_a}{M_b} & \frac{C_a}{M_b} \\ 0 & 0 & 0 & 1 \\ \frac{K_a}{M_w} & \frac{C_a}{M_w} & \frac{-(K_t+K_a)}{M_w} & -\frac{C_a}{M_w} \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\frac{K_t}{M_w} \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

X_b and X_w are the vertical displacement of mechanical point B, (Car body), and A (wheel) respectively, relatively to the initial position , and Xr is the ground reaction(given by the height of the road profile). Ka and Kt are the spring constant of the car and tyre and care represent the constant of the car shock absorber.

Thus, in this sub chapter, the system for quarter car modelling have been develop . A definite between input and output for quarter car model have been obtain using state space, thus, complete one of the objective of this project. Equation obtained will be used to simulate the system responses in following chapter.

4.3 System Responses

Usually road surfaces vary from one to another, it sometime can be bumpy and sometimes there are hole in it. Thus when a car pass through the bumper or the holes the excitation input ‘Step Response’ which leads to vertical vibrations in car. In result of that, various shock absorbers were design to restore the steady state position of the car in the shortest possible time. Settling time is known as the time which the system takes to attain steady state equilibrium position is very important as the shorter the settling time, the more stable is the system. Figure 4.4 below shown the important parameters which will be used to analyse the result later.

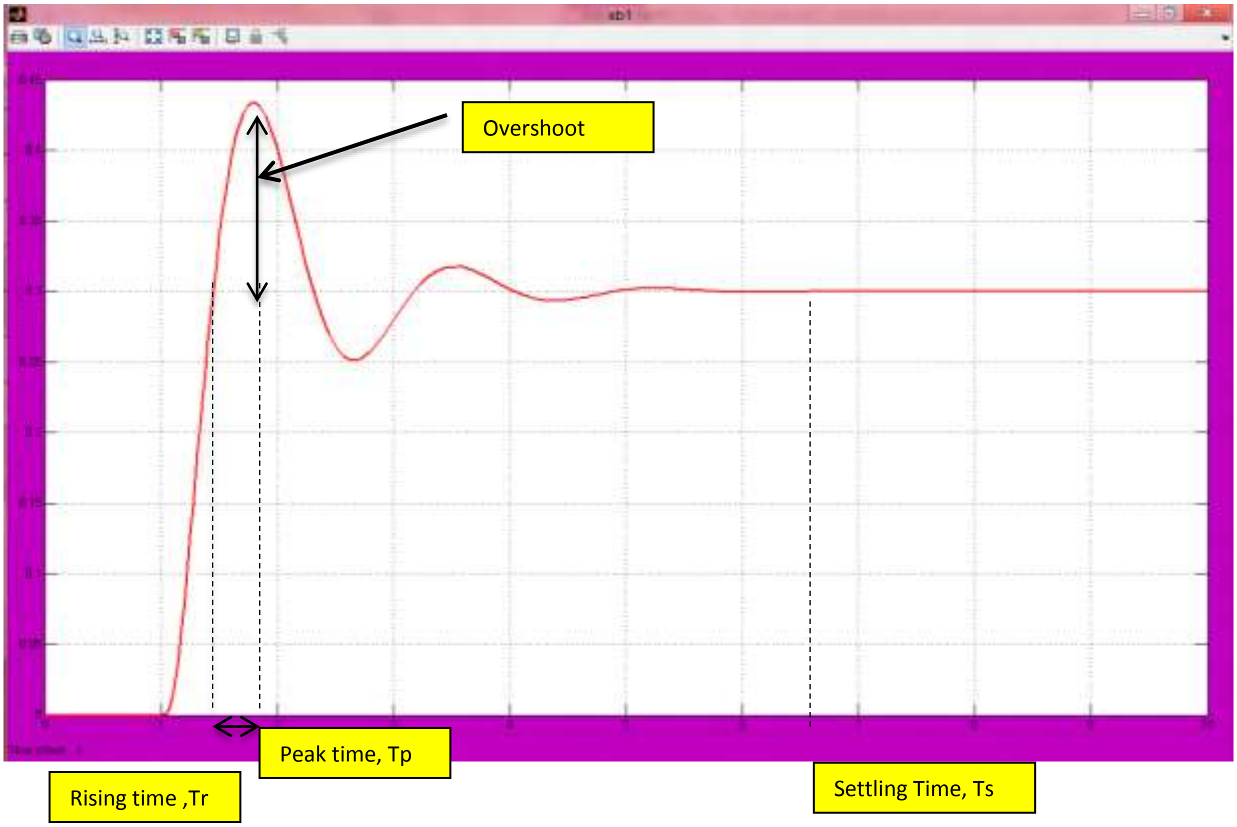


Figure 4.4 Syetem Step Response

Rising Time, Tr

Rising time is amount of time that the system takes to reach around 90% of its new set point.

Settling Time, Ts

Settling time is the time system takes to reach equilibrium to some level

Peak Time, Tp

Peak time is the time that requires the system to reach the maximum overshoots point

Over shoot,

This is the maximum distance beyond the final value that the response reaches. It is usually expressed as a percentage of different between the original value and the final steady state value.

Steady state error

This is measure of how far the final value reached in the step response is from the actual desired value.

Thus in the next sub topic, controller for the system will be design via state space using several values of gain that will be obtain. Comparison of step response result among all the gain values will be done, and one of the gain value will choose as the best controller as it give the best result in the step response system.

4.4 State Space Controller Design

4.41 Introduction

First a verification of important property is needed, in order to apply state-space controller design techniques ,which is controllability. It is necessary to have this controllability property in corresponds to the ability to place the closed-loop poles of the system anywhere in the complex s-plane.

For the system to be completely state controllable, the controllability matrix

$C = [B|AB| \ldots\ldots |A^{n-1}\, B|]$

must have rank n. The rank of a matrix is the number of independent rows . The number *n* corresponds to the number of state variables of the system. Adding additional terms to the controllability matrix with higher powers of the matrix A will not increase the rank of the controllability matrix since these additional terms will just be linear combinations of the earlier terms.

Since the model controllability matrix is 4x4, the rank of the matrix must be 4. The MATLAB command rank can give you the rank of this matrix.

However before proceed with the system controllability, let list down all the parameters.

Parameters	value
Mb	500kg
Mw	100kg
Ka	7500 N/m
Kt	170000 N/m
Ca	1250 Ns/m

Table 4.2 : Car suspension parameters

The reference Xr for the simulation refelects the road profile. For a road of 50m length , assuming the obstacle occur after 10m and the car speed is v=10m/s, the following define the corresponding reference input:

V= 10m/s

L= 2m

H = 0.3m

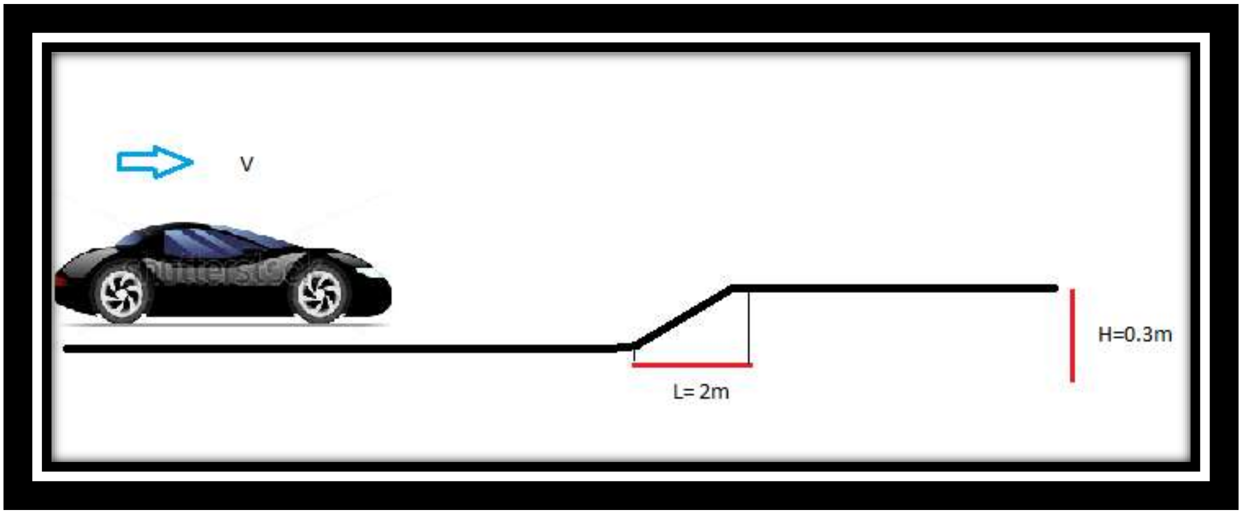


Figure 4.5: Situation to simulate

The MATLAB command rank can give you the rank of this matrix. Refer appendix for matlab coding for the matrix.

```
co = ctrb(A,B);

>> Controllability = rank(co)

Controllability =

    4
```

From the matlab, the system is completely state controllable since the controllability matrix has rank 4.

Below is the block diagram for quarter car suspension model. Let see how this suspension model run without controller.

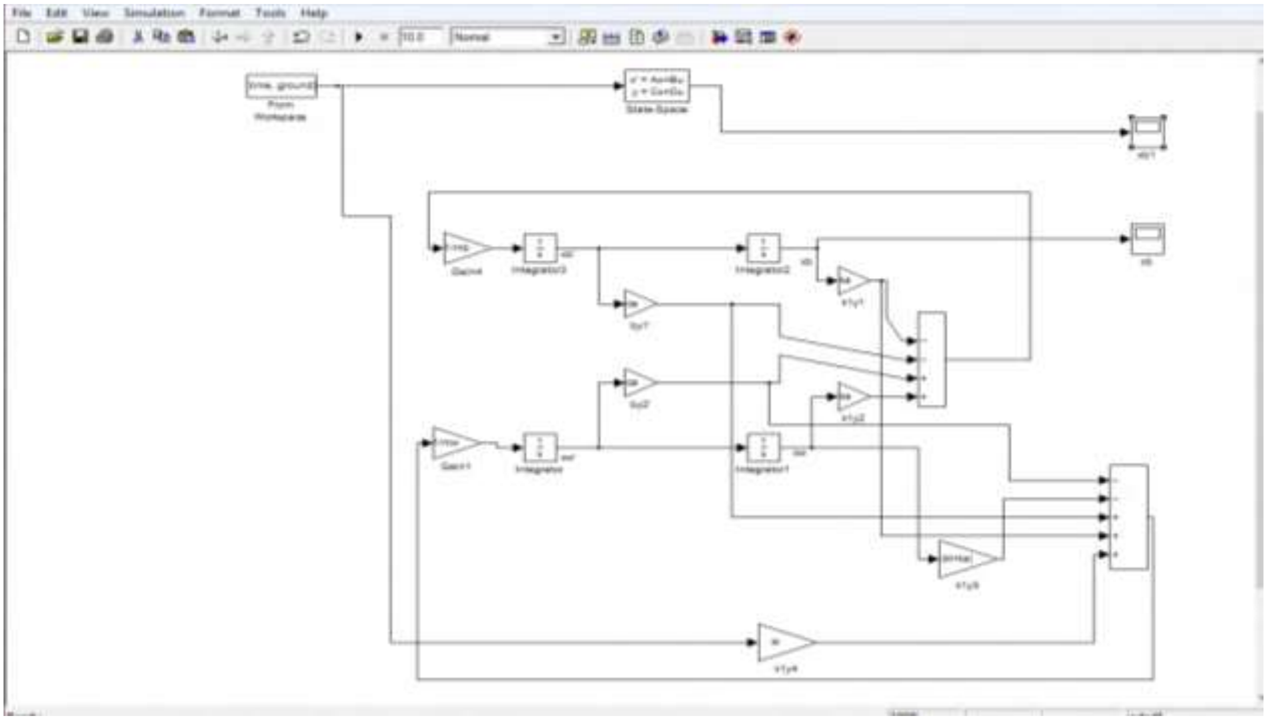


Figure 4.6 : Block diagram simulation model without controller

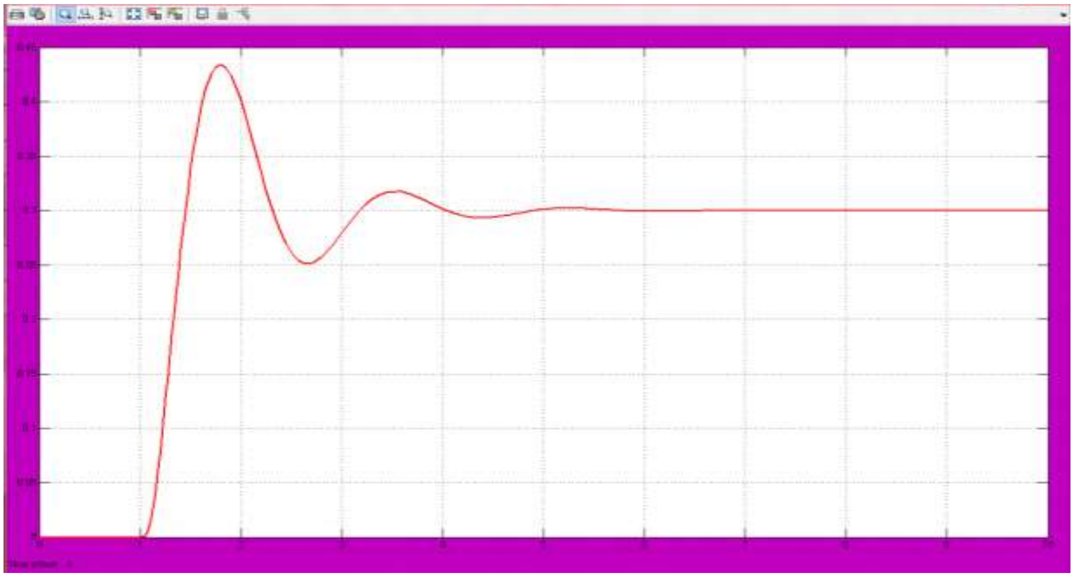


Figure 4.7 : result for state space mode

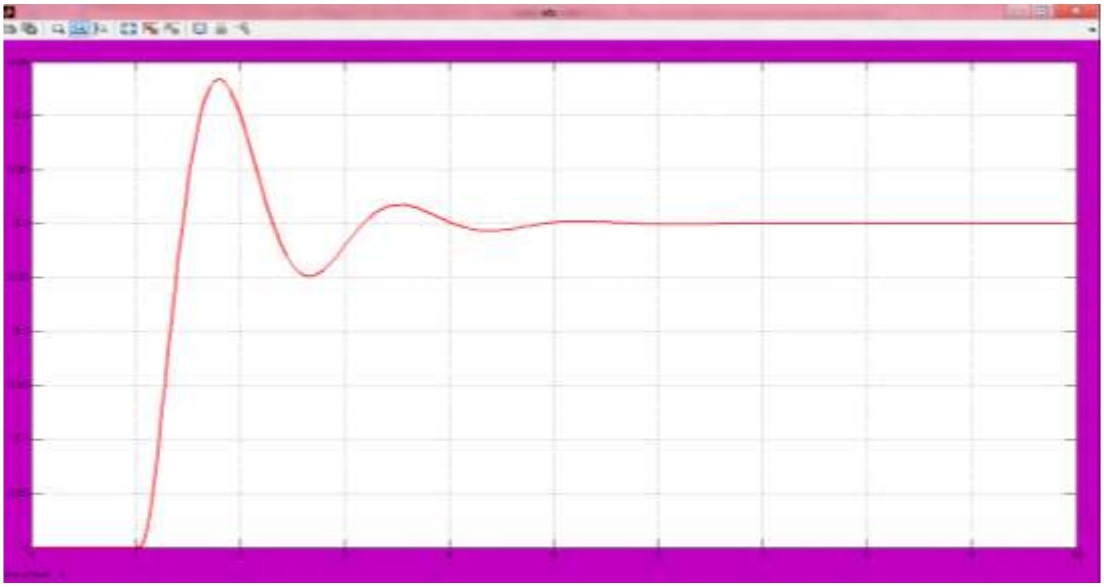


Figure 4.8 : result for block simulation of the model XB

For XB n XB1, the result is the same, this mean, my block simulation that I build is correct since the result is same with the state space block from matlab. From the graph we can calculate rise time, settling time and overshoot of the system. This calculation result later will be compare with the result after applying the controller.

Rise time, $T_r = 1.45s$

Peak = 0.433m $T_p = 1.801s$

Settling Time, $T_s = 6.38s$

Overshoot% $= \frac{C}{D} \times 100\%, where\ C = 0.433, D = 0.3$

$= 44.3\%$

4.42 Feedback Controller

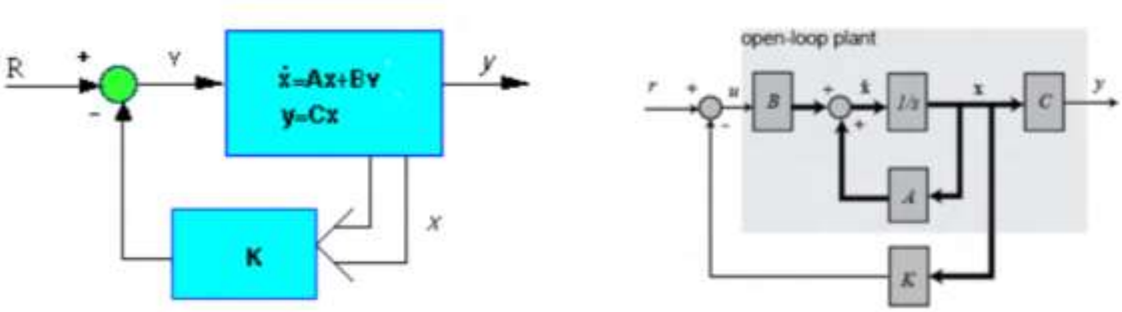


Figure 4.9 : Full Feedback Controller

In typical feedback control system the output y, is feedback to the summing junction. It is now the topology of design changes. Instead of feeding back y, it will then feedback all the state variable. If each state variable is feedback to the control u, through a gain K, there would be n gain that could be adjust to yield the required closed loop pole values. The feedback through the gain k is represent in figure by feedback vector K. The new state equation for closed-loop system can be write as

$$u = -Kx$$

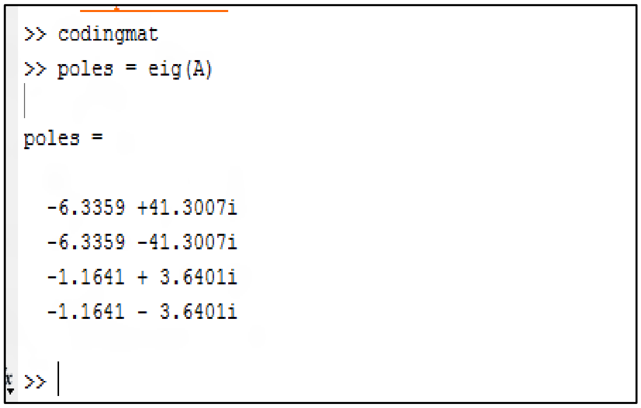
$$\dot{x} = (A - BK)x$$

$$y = (C - DK)x$$

The stability and time domain performance of the closed-loop feedback system are determined primarily by the location of the poles (eigenvalues) of the matrix (A-BK). Since the matrices A and B*K are both 4 by 4 matrices, there will be 3 poles for the system. By choosing an appropriate K matrix we can place these closed-loop poles anywhere we want. We can use the MATLAB function placeto find the control matrix, K, which will give the desired poles.

Before attempting this method, we have to decide where we want the closed-loop poles to be.

Using mat lab command , the eigen value of the system can be determine.



```
>> codingmat
>> poles = eig(A)

poles =

  -6.3359 +41.3007i
  -6.3359 -41.3007i
  -1.1641 + 3.6401i
  -1.1641 - 3.6401i

>>
```

Below is the simulation block diagram with feedback controller in matlab

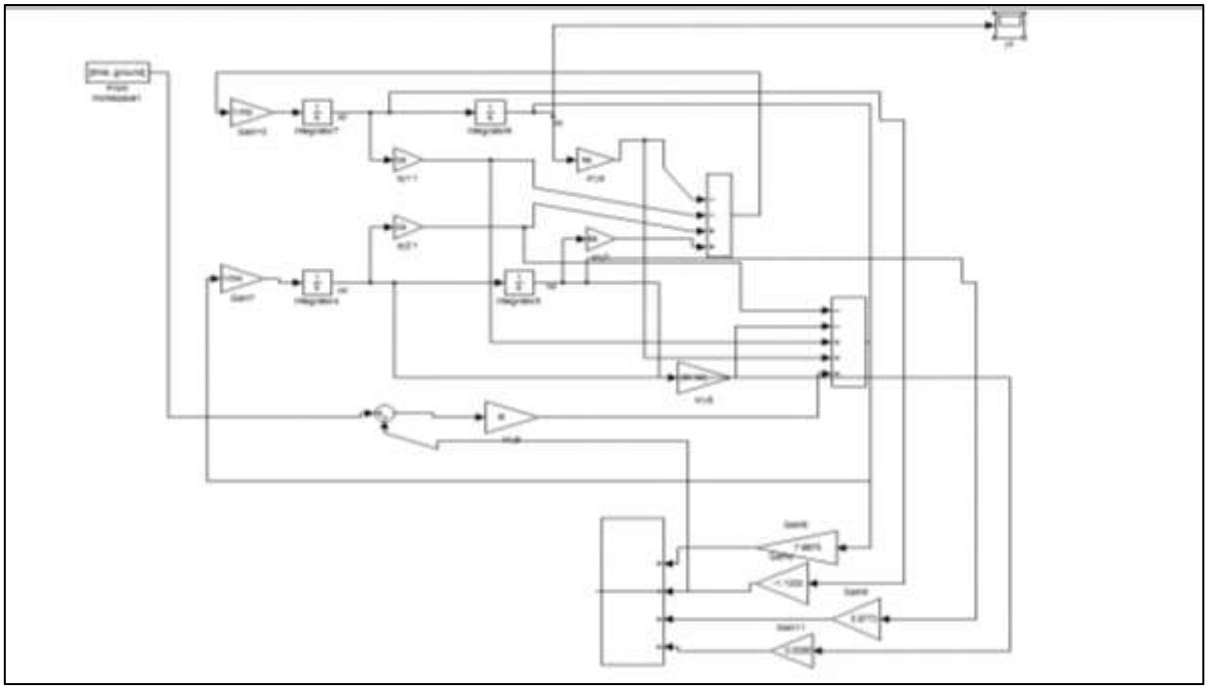


Figure 4.10 : Simulation block diagram with feedback controller

Thus to obtain gain K, we need to adjust these poles value to desired value which approaching the left of s-lane. The desired pole values that been chose are:

Value 1

p1 = -12.3 +82i

p2= -12.3 - 82i

p3 = -2.4 +7i

p4= -2.4 -7i

From this value, matrix gain can be obtain by this matlab command.

```
k = place(A,B, [p1 p2 p3 p4])



k =  7.8870   -1.1222    5.8773    0.0085
```

Then , putting all the feedback gain value1 in the system model simulation, we can see the result in figure 4.10:

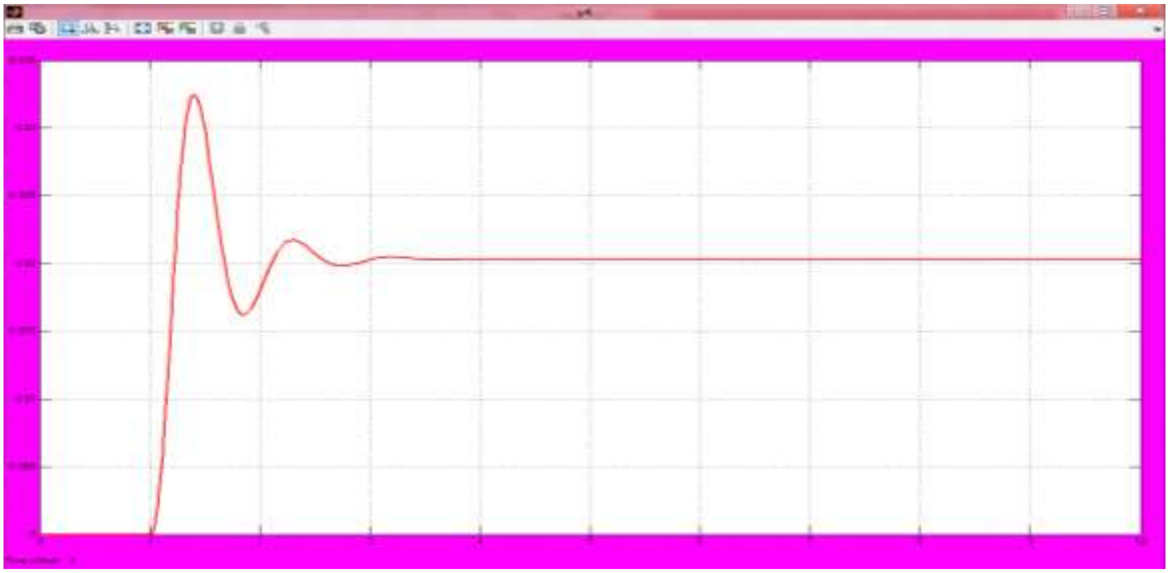


Figure 4.11: Result for k =   7.8870   -1.1222   5.8773   0.0085

<u>Value 2</u>

p1= -9+82i;

p2= -9 - 82i;

p3= -1.7 +7i;

p4= -1.7 - 7i;

From this value, matrix gain can be obtain by this matlab command

```
k = place(A,B, [p1 p2 p3 p4])

k =

    6.4211   -1.3677    6.4264    0.0038
```

Then , putting all the feedback gain value 2 in the system model simulation, we can see the result in figure 4.11.

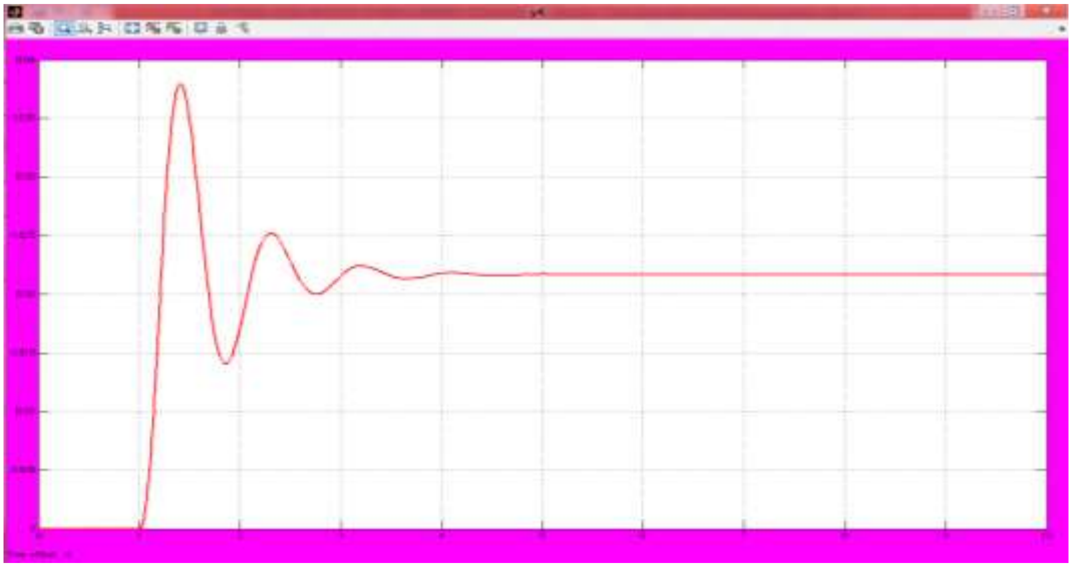


Figure 4.12: Result for k =   6.4211   -1.3677   6.4264   0.0038

The comparison of gain 1 and gain 2,

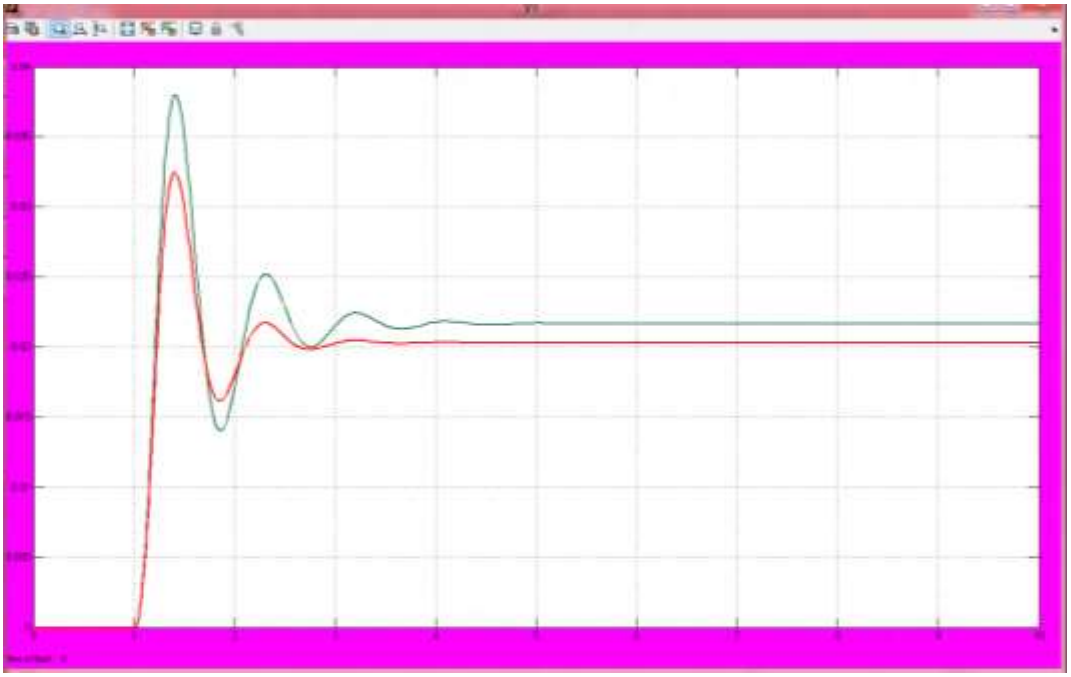


Figure 4.13: Comparison result for value K1 and K2

From figure 4.12 we can see that gain 1 give better result since . Controller with gain 2 have less overshoot and faster setteling time which is 3 second compare to the controller with gain 1 settleing time at 4.5 second.

4.42 Feedforward Controller

Feedback controllerwith reference input(feed forward controller)

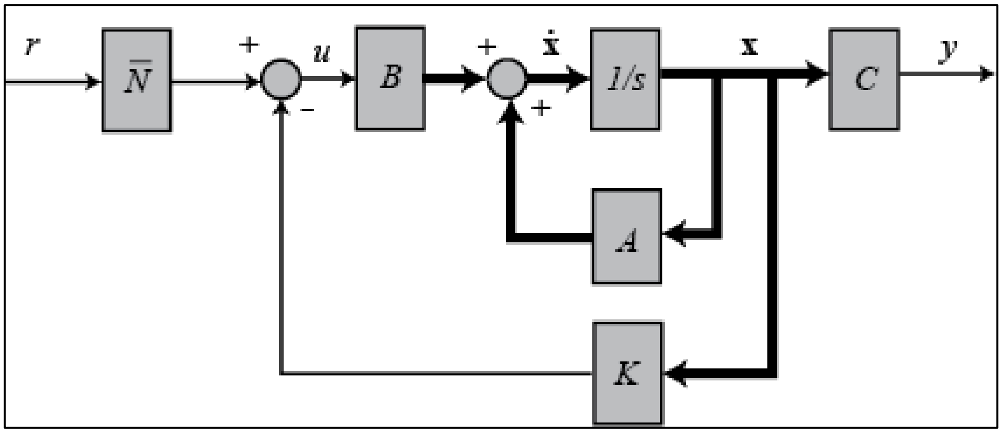


Figure 4.14: Full Feed forward Controller

This is a linear equation, and hence we have

This is a linear equation, and hence we have

$u(t) = -Ky(t) + r(t)$……………………. (1)

$\dot{x}(t) = Ax(t) + Bu(t)$…………………….(2)

$y(t) = Cx(t) - Du(t)$…………………….(3)

By substuting eqution 1 into eqution 2 and 3, we get:

$\dot{x}(t) = Ax(t) - BKy(t) + \mathrm{Br(t)}$

$y(t) = Cx(t) - DKy(t) + Dr(t)$

We can now rewrite the control law to be of the form

$u = -K(x - Nxr) + Nur = -Kx + Nr$

1where

$N = Nu + KNx.$

The scalar N represents a feedforward gain. The closed loop system is

$\dot{x} = (A - BK)x + BNr$

$y = (C - DK)x + DNr$

where r represents the reference input and y is the output for the closed loop plant. If N is chosen correctly, the steady state error for the closed loop system should be zero.

For this system model, I had try with different values of reference input Nr and gain K.

The simulation was run in in matlab with addition Nr block in the simulation, refer figure 4.14.

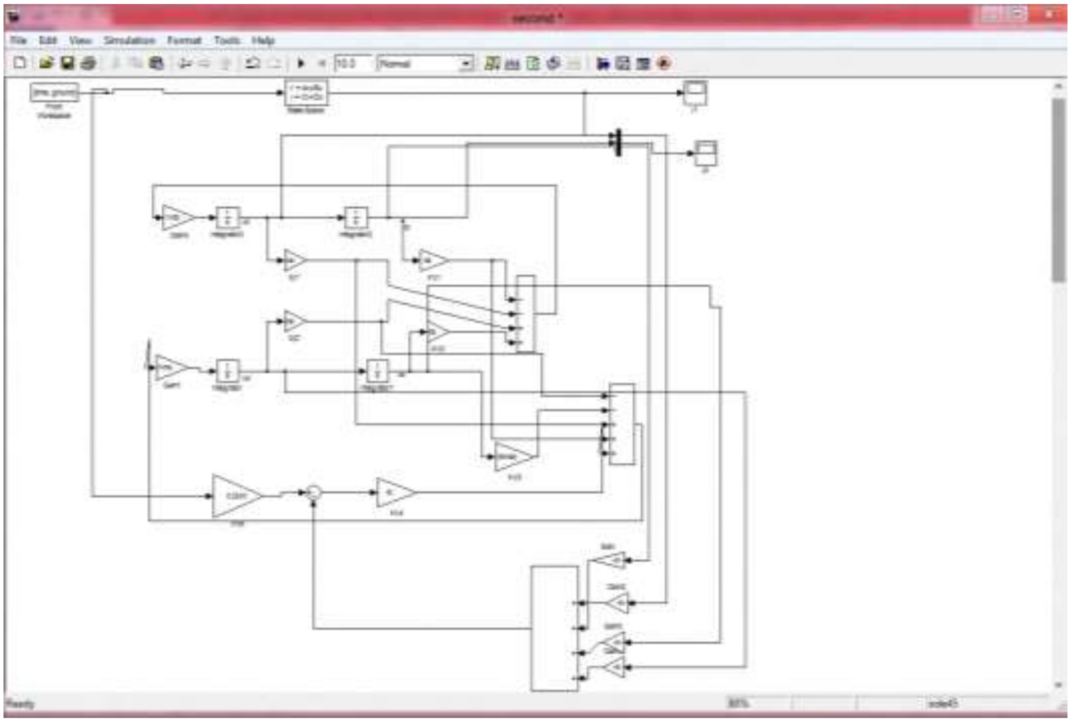


Figure 4.15 : Simulation block diagram with feed forward controller

Thus, from the result obtain we will determine the best value of Nr for the best controller.

For this feed forward controller, we are going to search the best value of Nr for the system. Thus, five different values of Nr will be test, and to get the value of Nr , different value of poles and will be choose. Value of Nr will be obtain by using matlab command.

Value 1

Using matlab command we can get the value of Nr.

```
>>k = place(A,B, [p1 p2 p3 p4])

k =

    7.8870   -1.1222    5.8773    0.0085

>> u= zeros (size(t));

>> x0= [0.1 0.1 0.1 0.1];

>> lsim(A-B*k,B,C,0,u,t,x0)

u= 0.001*ones(size(t));

>> u= 0.001*ones(size(t));

>> s = size(A,1);

>> Z = [zeros([1,s]) 1];

>> N = inv([A,B;C,D])*Z';

>> N = inv([A,B;C,D])*Z'

>> Nx = N(1:s);

>> Nu = N(1+s);

>> Nbar=Nu + k*Nx;

>> Nbar=Nu + k*Nx

Nbar =

   14.7643
```

For value 1, the value of reference input is 14.7643

Value 2

```
>> p1= -9+82i;

>> p2= -9 - 82i;

>> p3= -1.7 +7i;

>> p4= -1.7 - 7i;

> k = place(A,B, [p1 p2 p3 p4])

k =

   6.4211   -1.3677    6.4264    0.0038

>> x0= [0.1 0.1 0.1 0.1];

>> x0= [0.1 0.1 0.1 0.1];

>> u= 0.001*ones(size(t));

>> s = size(A,1);

>> Z = [zeros([1,s]) 1];

>> N = inv([A,B;C,D])*Z';

>> Nx = N(1:s);

>> Nu = N(1+s);

>> Nbar=Nu + k*Nx;

>> Nbar=Nu + k*Nx

Nbar =

  13.8475
```

For value 2 , the value of reference input is 13.847

Value 3

```
>> p1= -9+20i;

>> p2= -9 - 20i;

>> p3= -1.7 +3i;

>> p4= -1.7 - 3i;

>> k = place(A,B, [p1 p2 p3 p4])

k =

    0.0392    0.0314   -0.8149    0.0038

>> x0= [0.1 0.1 0.1 0.1];

>> u= 0.001*ones(size(t));

>> s = size(A,1);

>> Z = [zeros([1,s]) 1];

>> N = inv([A,B;C,D])*Z';

>> Nx = N(1:s);

>> Nu = N(1+s);

>> Nbar=Nu + k*Nx;

>> Nbar=Nu + k*Nx

Nbar =

    0.2243
```

For value 3, the value of reference input is 0.2243

Value 4

```
p1= -31.6795 + 82i;

>> p2= -31.6795 - 82i;

>> p3= -5.8205 + 7i;

>> p4= -5.8205 - 7i;

>> k = place(A,B, [p1 p2 p3 p4])

k =

   19.0096   -0.4876    5.1061    0.0353

>> x0= [0.1 0.1 0.1 0.1];

>> s = size(A,1);

>> Z = [zeros([1,s]) 1];

>> N = inv([A,B;C,D])*Z'

>> Nx = N(1:s);

>> Nu = N(1+s);

>> Nbar=Nu + k*Nx;

>> Nbar=Nu + k*Nx

Nbar =



   25.1156
```

For value 4 , the value of reference input is 25.115

Value 5

```
>>  p1= -31.6795 + 20i;

>> p2= -31.6795 - 20i;

>> p3= -5.8205 + 3i;

>> p4= -5.8205 - 3i;

>> k = place(A,B, [p1 p2 p3 p4])

k =

    2.0132    0.3186   -0.6531    0.0353



>> x0= [0.1 0.1 0.1 0.1];

>> s = size(A,1);

>> Z = [zeros([1,s]) 1];

>> N = inv([A,B;C,D])*Z

>> Nx = N(1:s);

>> Nbar=Nu + k*Nx;

>> Nbar=Nu + k*Nx

Nbar =

2.3601
```

For value 5 ,the value of reference input is 2.3601

After getting five different values of Nr, the values then were use in the simulation block. The response of each values of Nr is then display in the time scope. Below is the result and response of the system for every Nr value.

Simulation result

Result for Value 1, Nr =14.7643

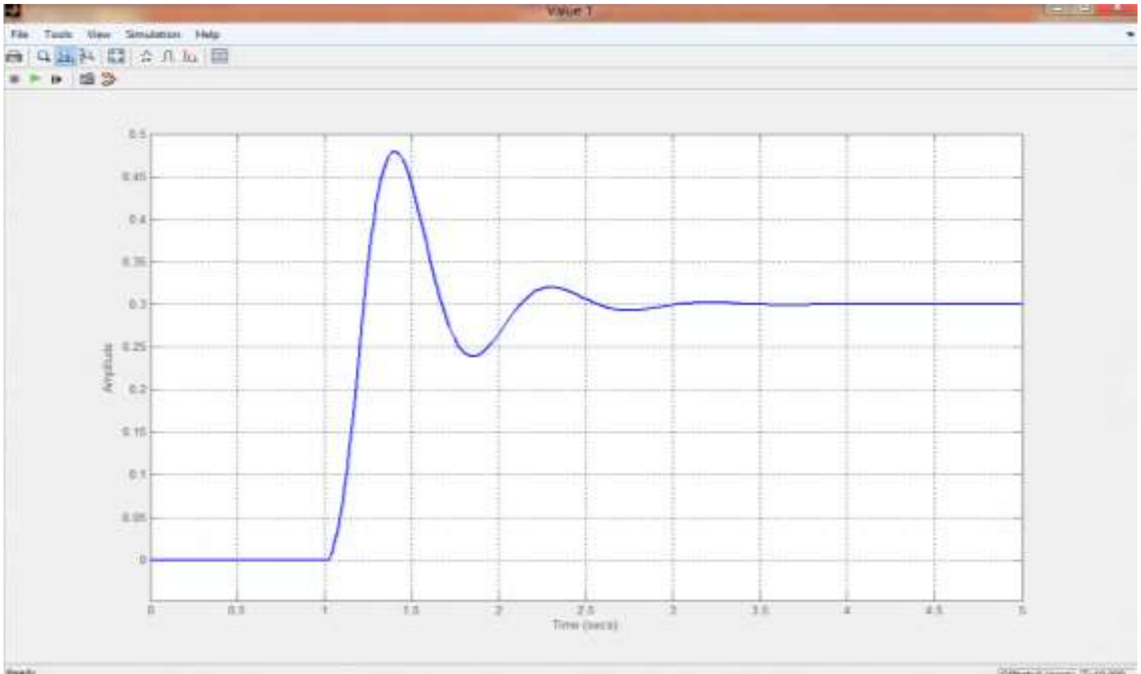


Figure 4.16: Result for Nr =14.7643

Rise time, $T_r = 1.25s$

Peak = 0.479m $T_p = 1.38s$

Settling Time, $T_s = 3.78s$

Overshoot% $= \frac{C}{D} \times 100\%, where\ C = 0.179, D = 0.3$

$= 59.6\%$

Result for Value 2 ,Nr =13.8475,

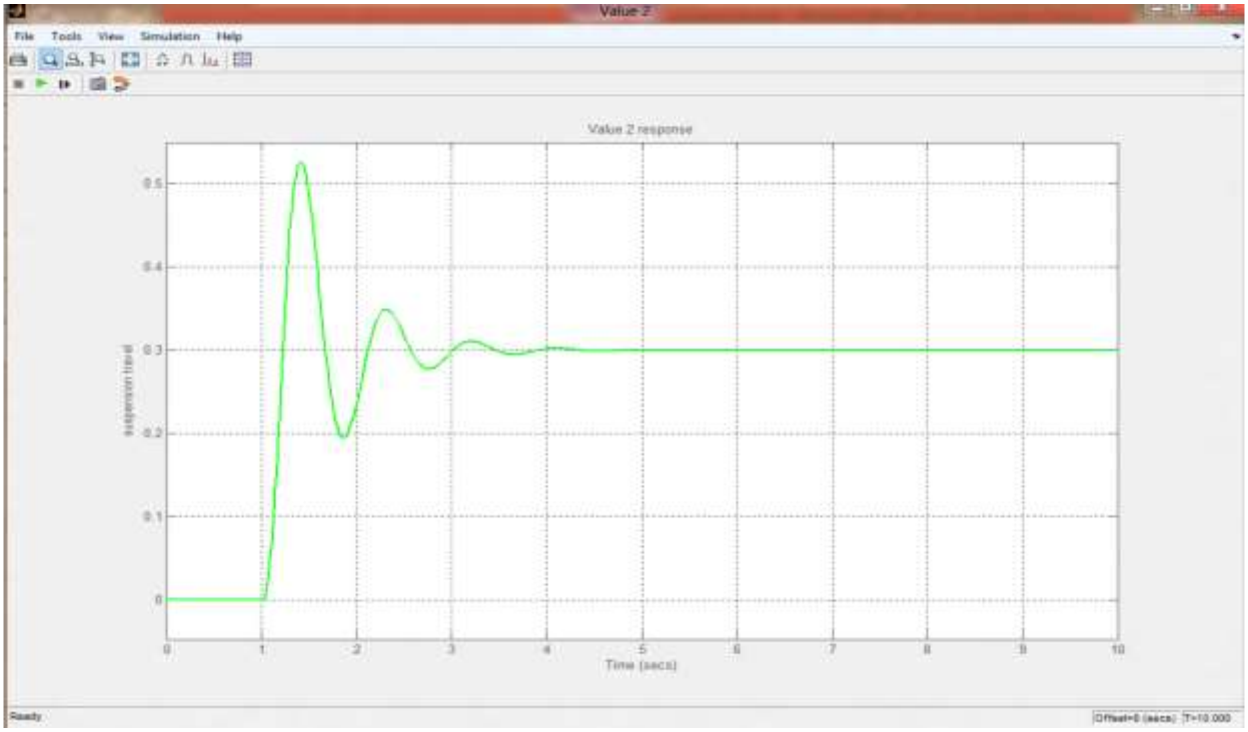


Figure 4.17: Result for Nr =13.8475

Rise time, $T_r = 1.23s$

Peak = 0.526m $T_p = 1.414s$

Settling Time, $T_s = 5.13s$

Overshoot% $= \frac{C}{D} \times 100\%, where\ C = 0.226, D = 0.3$

$= 75.3\%$

Result for Value 3, Nr =0.2243

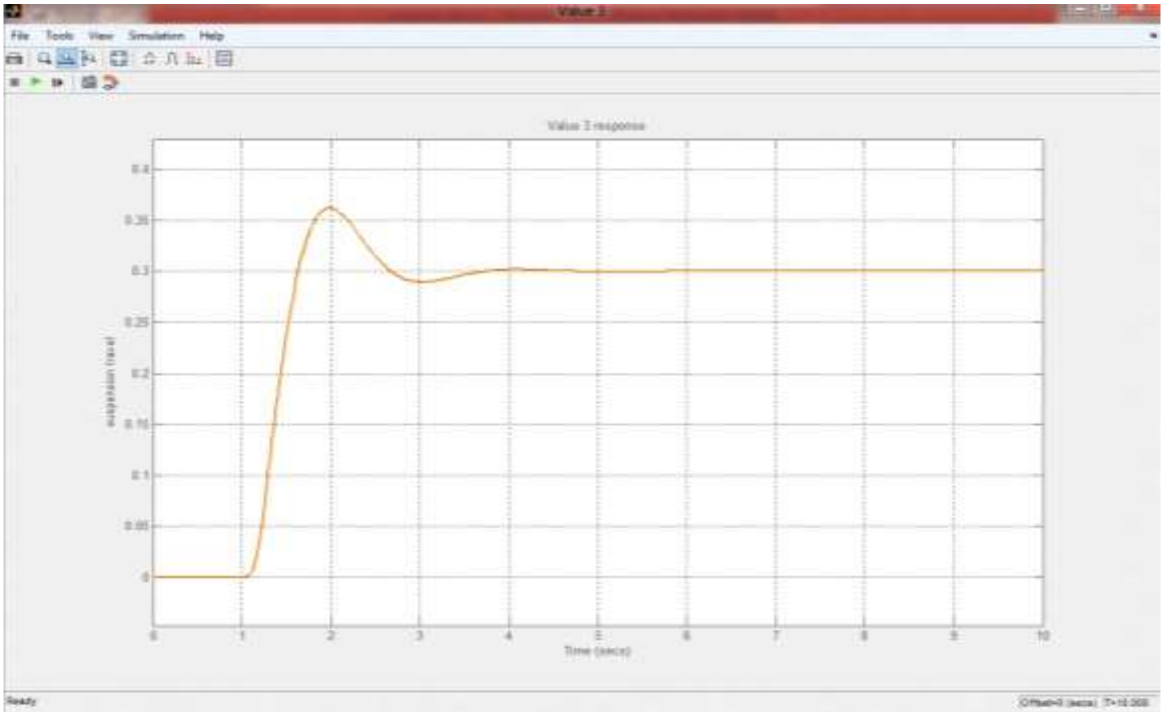


Figure 4.18: Result for Nr =0.2243

Rise time, $T_r = 1.66s$

Peak = 0.362m $T_p = 1.99s$

Settling Time, $T_s = 5.61s$

Overshoot% $= \frac{C}{D} \times 100\%, where\ C = 0.062, D = 0.3$

$= 20.67\%$

Result for Value 4, Nr =25.1156

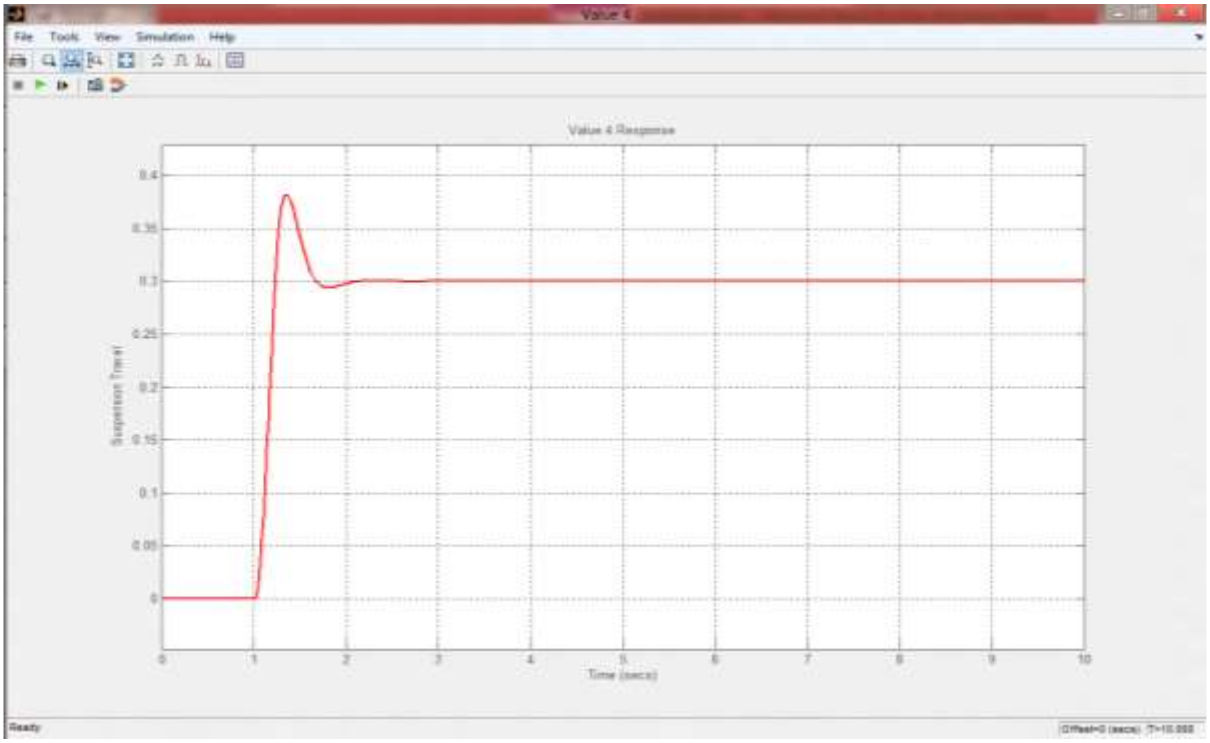


Figure 4.19: Result for Nr =25.1156

Rise time, $T_r = 1.26s$

Peak = 0.381m $T_p = 1.35s$

Settling Time, $T_s = 2.56s$

Overshoot% $= \frac{C}{D} \times 100\%, where\ C = 0.081, D = 0.3$

$= 27\%$

Result for Value 5, Nr =2.3601

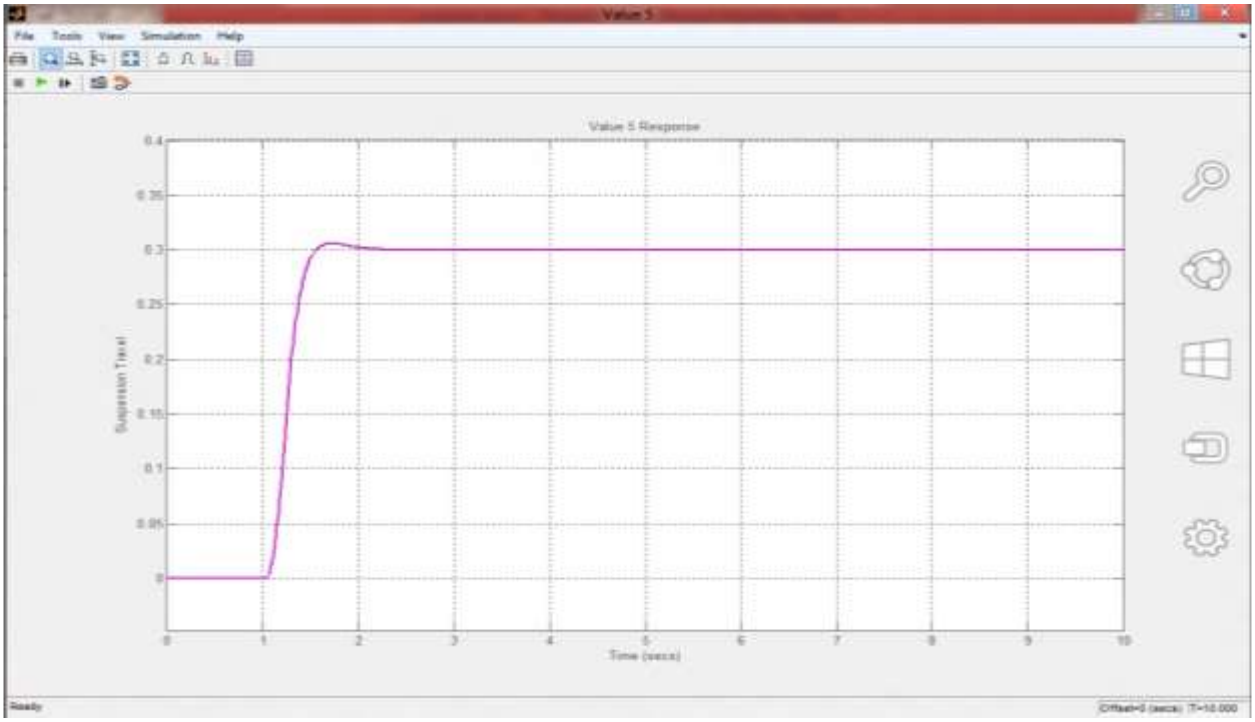


Figure 4.20: Result for Nr = 2.3601

Rise time, $T_r = 1.55s$

Peak = 0.3058m $T_p = 1.719s$

Settling Time, $T_s = 2.44s$

Overshoot% $= \frac{C}{D} \times 100\%, where\ C = 0.0058, D = 0.3$

$= 1.93\%$

Below in figure 4.20 , is the comparison between these three value of Nr. From the result we can define the best iput reference for the system

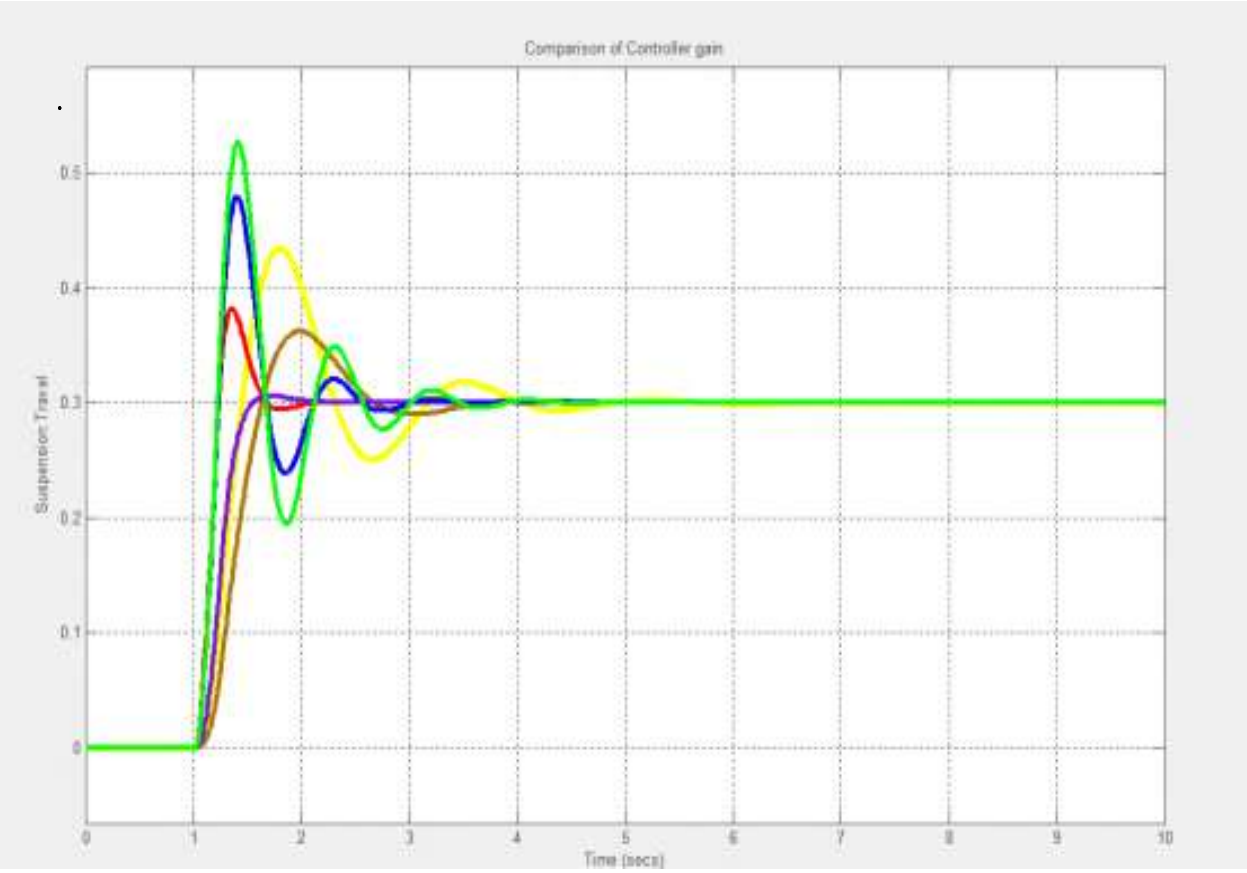


Figure 4.21: Comparison result for different value of Nr, feed forward gain

🟦 Value 1 , N=14.7643

🟩 Value 2 , N= 13.8475

🟧 Value 3, N= 0.2243

🟥 Value 4, N= 25.1156

🟪 Value 5 N=2.3601

🟨 Without controller

To simplified the result, below is the table showing the comparison of rise time, settling time and overshoot of every diffrent value of feedforward gain.

Values of gain	**Rise Time, T_r**	**Settleing Time**	**Overshoot**
Without controller gain	$1.45s$	$6.38s$	44.3%
Value 1	$1.25s$	$3.78s$	59.60%
Value 2	$1.23s$	$5.13s$	75.30%
Value 3	$1.66s$	$5.16s$	20.67%
Value 4	$1.26s$	$2.56s$	27.00%
Value 5	$1.55s$	$2.44s$	1.930%

Table 4.3 : Comparison of step response with diffrent value of feedforward gain

From the simulation result, value 5 can be choose as the best controller . Value 5, with the value Nr = 2.3601 give the most faster settling time and lowest overshoot range.

Using Value 5 gain it have reduced both the overshoot and the settling time, and had small effect on the rise time and the steady-state error. As state in sub topic 4.3, the shorter the settling time and small overshoot value, the stable teh system will be. Thus as for the result, value 5 feedforward gain via state space was choose as the best controller paremeter.

In the next subtopic, PID controller will be develop and the step response result will be compare with the state space controller result, and the best controller will be choose for the best step response result.

4.5 PID Controller Design

4.51 Introduction

PID are stands for propotional, intergeral and derivative. The controller takes a measured value from a system, process or other apparatus and compares it with a reference set point value. Error is defined as adiffrence between set point and measurement. PID is named after its three correcting terms, whose sum constitues the out put of the PID controller.

Propotional

To handle the immediate error, the error is multiplied by a constant P. When the zero is zero, apropotional controller's output is zero. However the P, controller cannot always gurantee the the set point will be reached if the set point oi not fixed in time.

Intergeral

To learn from the past, the error is intergerated and multiplied by constant. Without intergeral term , a PID controller cannot eliminate erro is the system requires a non- null input to produce desired set point.

Derivative

To anticcipitate the future , the first derivative over time is calculated and multiplied by another constant D.

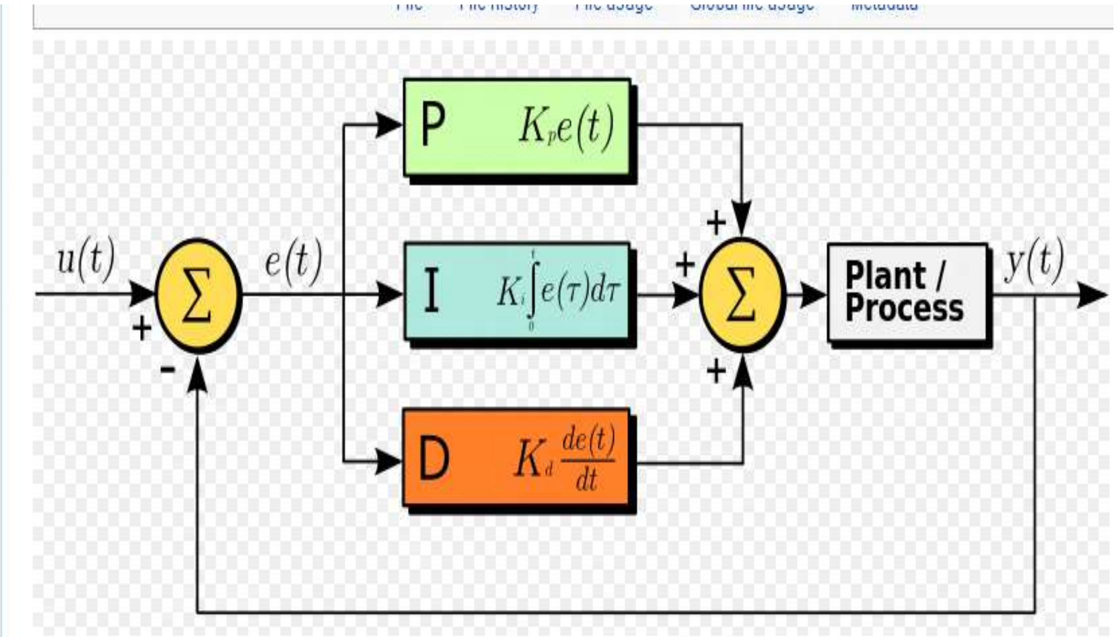


Figure 4.22: Block Diagram for PID controller

4.52 Tuning PID controller

Loop tuning is method to do an adjustment of control parameters to the optimum values for desired control response The optimum behavior on a system change varies depending on the application. Basically stability response is needed and the system must not oscillate for any combination of system conditions . One of the method to tune a PID controller is Zeigler –Nichols. This method suggest a set of value Kp, Ti, Td that will give a stable operation os the system. However, the resulting system may experiance a large maximum ovrshoot in the steo response. In that case series of fine tuning are needed until an acceptable result obtain.

Zeigler and Nichols proposed rules for determining the proportional gain Kp, integral Time Ti, and derivative time Td based on the transient response characteristics of a given system. In this method, we first set Ti = ∞ and Td = 0. Using the proportional control action only, increase Kp from 0 to a critical value Kcr at which the output first exhibits sustained oscillations. Thus the critical gain Kcr and the corresponding period Pcr are experimentally determined. Zeigler and Nichols suggested that we set the values of the parameter Kp, Ti,and Td according to the formula shown in Table .

Type of controller	**Kp**	**Ti**	**Td**
P	0.5 Kc	∞	0
PI	0.45 Kc	0.83 T	0
PID	0.6 Kc	0.6 T	0.125 T

Table 4.4 : Zeigler –Nichols tuning rules

4.53 Ziegler Nichols tuning method: closed loop

First, controller was place automaticly with lo gain, no reset or derivative. Then, slowly increase gain, making small changes in the set point, until oscillations start. Then adjust gain to make the oscillations continue with a constant amplitude. Ultimate Gain, Kc, and Period Ultimate Period, T. Need to be noted. The Ultimate Gain, Gu, is the gain at which the oscillations continue with a constant amplitude.

Simulink

Mat lab was used to run the tuning. Figure   show the block diagram of PID controller that will be used to obtain the best parameter for PID controller.

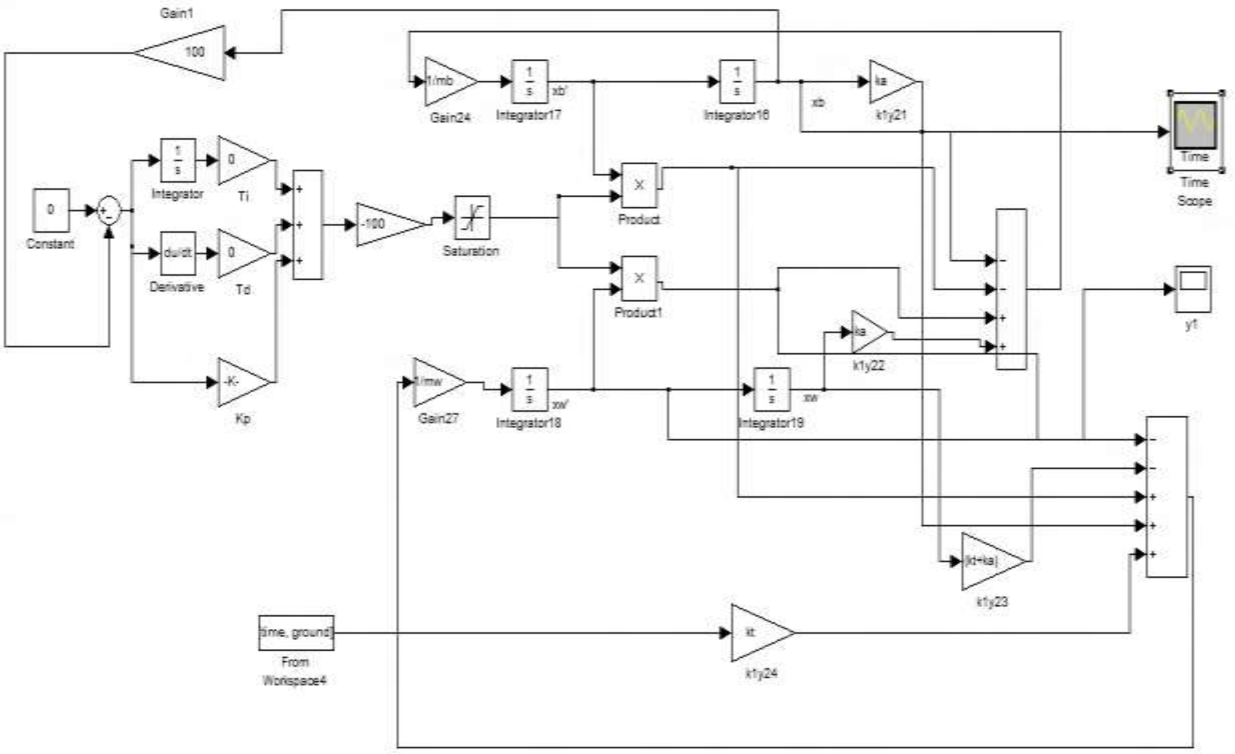


Figure 4.23:   Block diagarm for PID controller.

Several attempt were made to obtain the oscillations continue with a constant amplitude. Fisrt value of Kc choose wsa 0.007, and the response was in figure

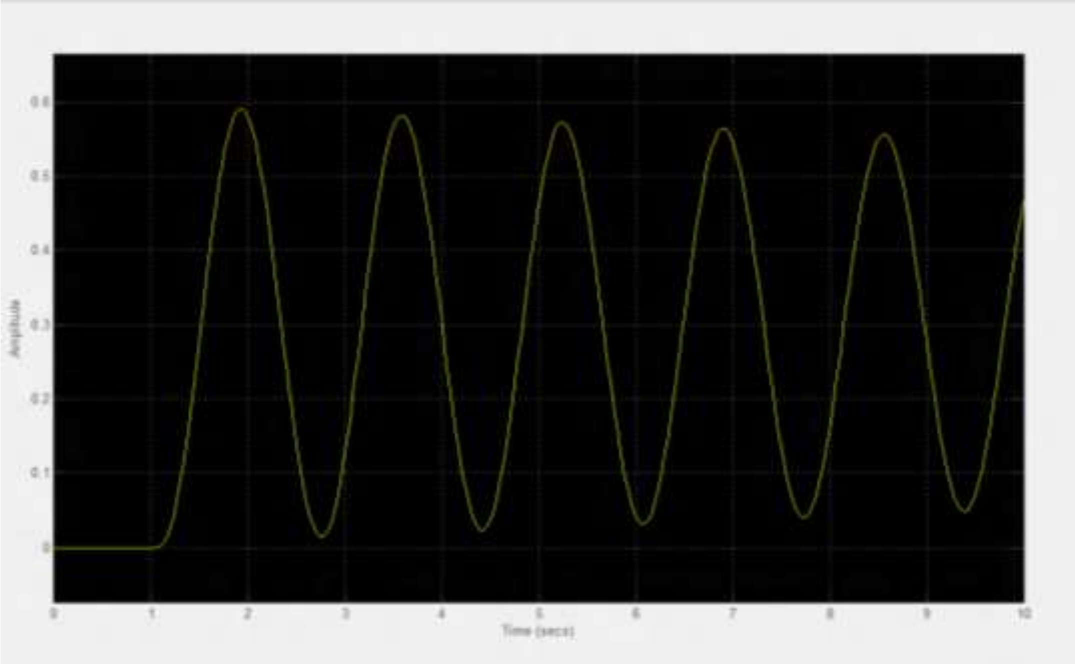


Figure 4.24 : Closed loop unit step response (kc= 0.007) for sustained oscillation

The amplitude obtain is not constant, thus next value was choose fine constant amplitude response. Then next value of Kc choosen was 0.00001. The response for Kc = 0.00001 shown in figure

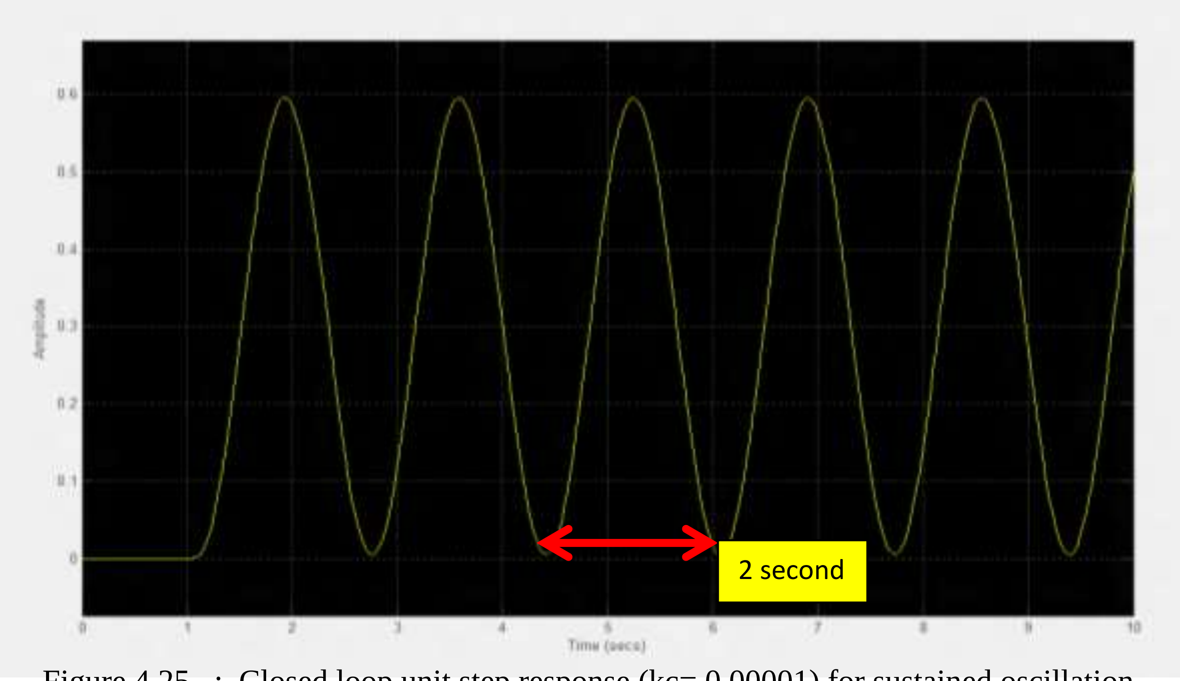


Figure 4.25 : Closed loop unit step response (kc= 0.00001) for sustained oscillation

Since Kc= 0.00001 give a constant amplitude, thus ultimate gain was choose to be 0.00001 and the ultimate period calculated is 2 second.

The critical gain (Kc) and critical time period (T), determined above are used to set the tuning rules for the quarter car model using the Zeigler-Nichols tuning rules. As discussed,the values of the P, PI and PID controller are obtained and are tabulated in Table 4.5 .

Type of controller	**Kp**	**Ti**	**Td**
P	0.000005	∞	0
PI	0.0000045	1.66	0
PID	0.000006	1.2	0.25

Table 4.5 :Zeigler-Nichols tuning values

After getting all the tuning value, the value will be use in the matlab simulink to obtain step rsponse for each P , PI and PID controller .

P controller,

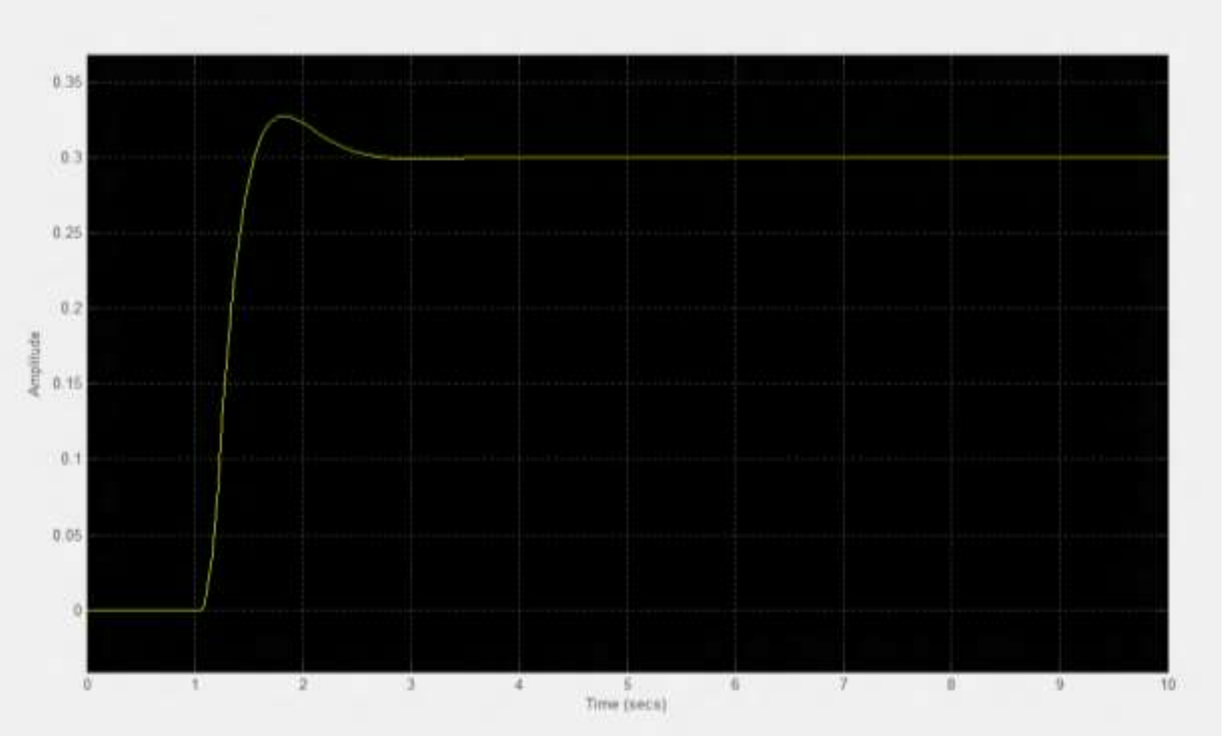


Figure 4.26 : step response result for P controller

Rise time, $T_r = 1.55s$

Peak = 0.327m $T_p = 1.875s$

Settling Time, $T_s = 2.82s$

Overshoot% $= \frac{C}{D} \times 100\%, where\ C = 0.027, D = 0.3$

$= 9\%$

PI controller

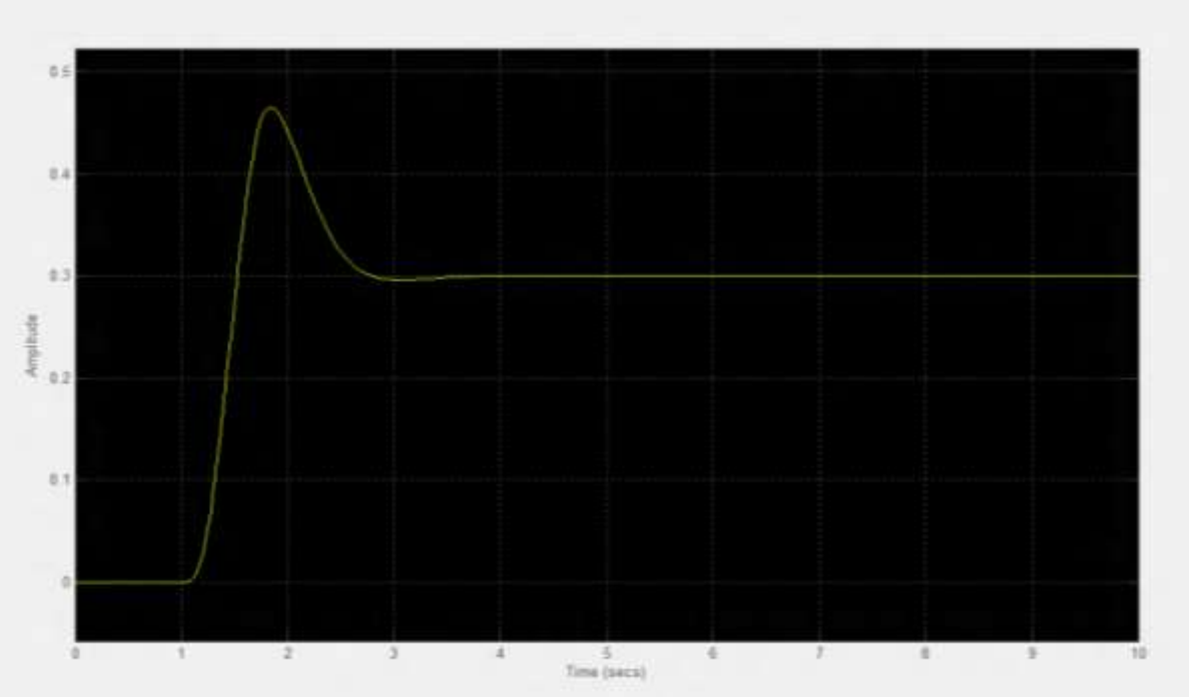


Figure 4.27 : step response result for PI controller

Rise time, $T_r = 1.53s$

Peak = 0.465m $T_p = 1.8475s$

Settling Time, $T_s = 4.05s$

Overshoot% $= \frac{C}{D} \times 100\%, where\ C = 0.165, D = 0.3$

$= 55\%$

PID controller

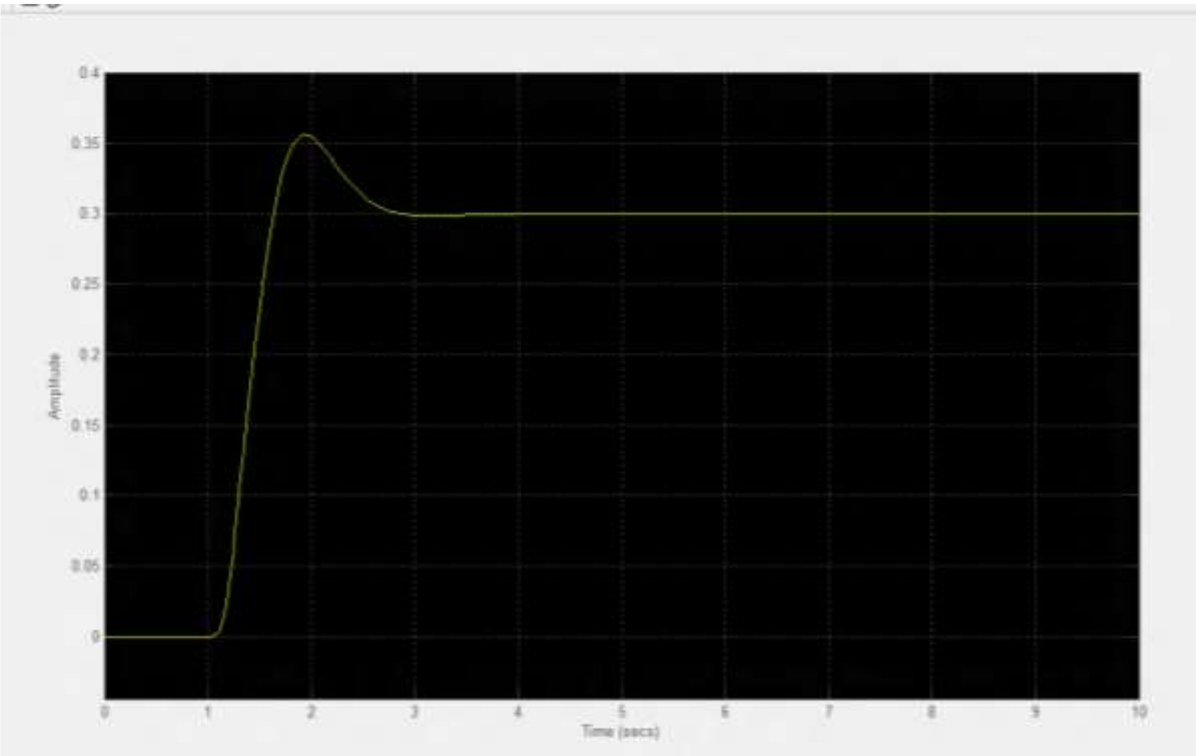


Figure 4.28 : step response result for PID controller

Rise time, $T_r = 1.6s$

Peak = 0.356m $T_p = 1.92s$

Settling Time, $T_s = 4.94s$

Overshoot% $= \frac{C}{D} \times 100\%, where\ C = 0.056, D = 0.3$

$= 18.6\%$

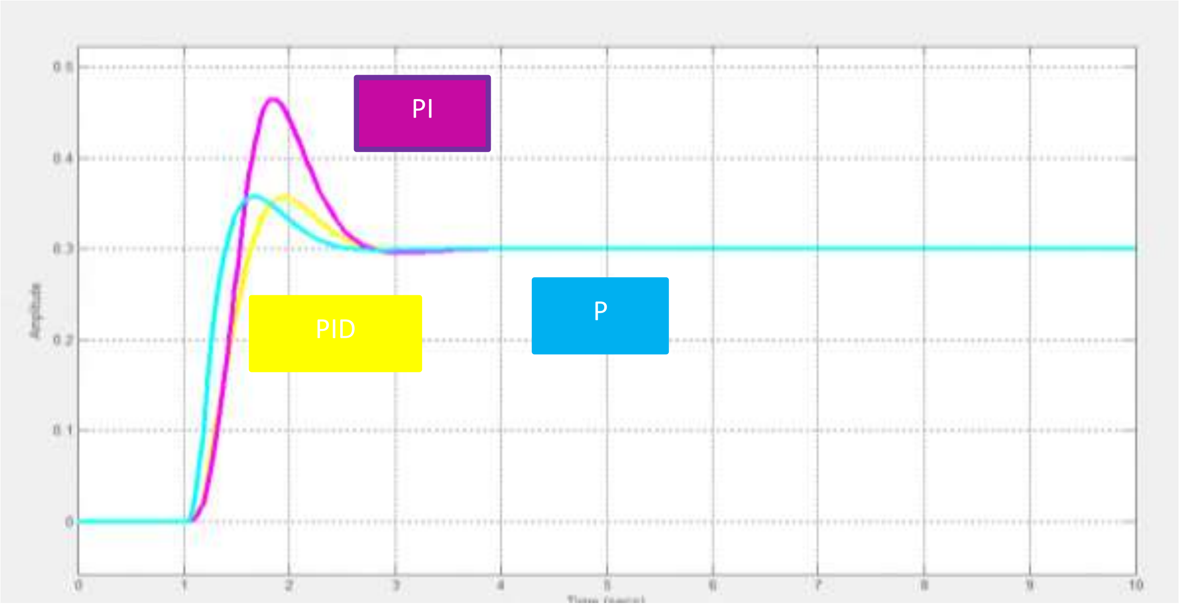


Figure 4.29 : Comparison step response between P, PI and PID controller

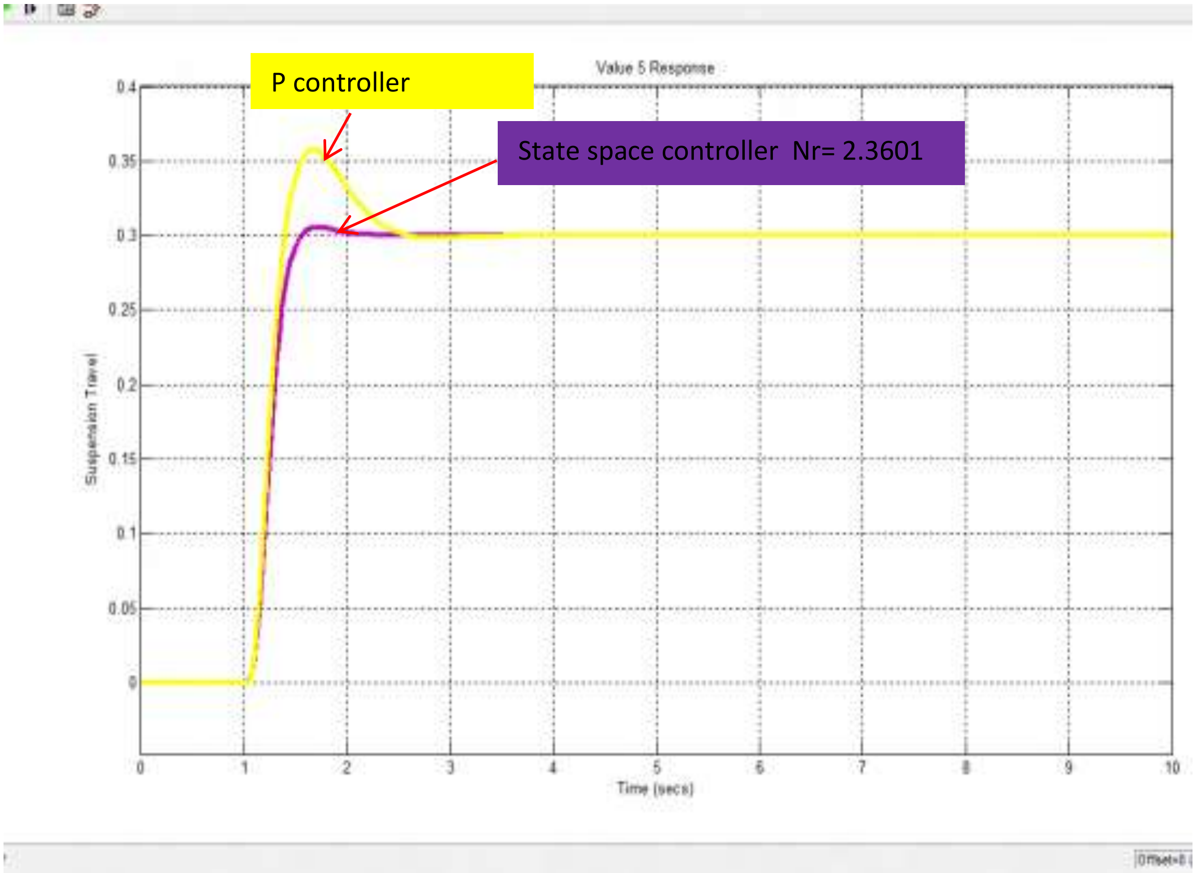
PID Controller	**Rise** **Time, T_r**	**Settleing Time**	**Overshoot**
P	$1.55s$	$2.82s$	9%
PI	$1.53s$	$4.05s$	55%
PID	$1.6s$	$4.94s$	18.6%

Table 4.6 show the comparision ofstep response parameter for P, PI, and PID controller.

From the simulation result, P controller can be choose as the best controller . P controller give the most faster settling time and lowest overshoot range.

P controller have reduced both the overshoot and the settling time, and had small effect on the rise time and the steady-state error. As state in sub topic 4.3, the shorter the settling time and small overshoot value, the stable teh system will be. Thus as for the result, P controller was choose as the best controller in PID controller design.

Next, the best controller from PID controller and best controller from state space will be compare. P controller and state space controller with value 5 gain are the best controller from each diffrent design. Here these two controller will be compare and the best controller will be choose based on the good step response parameters
Figure show the comparison betwen P controller and state space controller with
feedforward gain Nr= 2.3601.



P controller

State space controller Nr= 2.3601

Figure 4.30 : Comparison step response between PID controller and state space controller

Controller	Rise Time, T_r	Settleing Time	Overshoot
State space	1.55s	2.44s	59.60%
P controller	1.23s	5.13s	75.30%

Table 4.7 : Comparison step response between PID controller and state space controller

Thus from the comparison, the best controller from this project anylysis will be state space controller with Nr= 2.3061.

CHAPTER 5
CONCLUSION

5.1 Introduction

A quater car model suspension system controller was completely design , as for that this project is consider as satisfactory. The system modelling was obatain and implemented using MATLAB. A simulink block diagram by using MATLAB command is also develops where it allow the user to view performance of suspension system by comparing the different controller response, which is state space and PID controller.

For the controller design part, it is important to get desired rise time, peak time , setlling time, and overshoot of the system. As a result, it is necessary to find the best gain in state space controller to obtain the best system step response parameters.

5.2 Conclusion

Troughout this project, the desired objectives have been acieved. A dynamic model of a quater car suspension system have been formulate and derive. Along this semester running this project, some problem have occured especially during modeling the suspension and draw the correct block diagram. Yet, this problem were manage to solved successfully and new experiance were face while solving it.

The controller design were manage to simulate in the SIMULINK and give a good result as a controller. The mat lab SIMULINK allow to understand the simulation result easily.The comparison between two controller state space controller and PID controller, we compare and the best controller were choosed.

5.3 Suggestion for future work

This project use a quater model of car. Thus in the next future work, complete car model could be use. We can design a contorller for a full car model, yet it can achive a better control perfomance.

Next, we can use high level controller such as Fuzzy Logig controller and neutral Network, where we can compare all the possible controller for this system, and choose the best controller.

Since the result gain it all gain from simulation result, we can consider a hardware design for suspension system. However high budget is needed to build suspension plant, yet it still worth it because if the project was successful, the hardware can be pattern and commercialize since there is not much reseacrh on the hardware design.

REFERENCES

1. Muhammad Nuruddin bin Yahya 2007, *State Space Approach For Controlling 1-Dof Vehicle Active Suspension System* , thesis
2. Carneuge Mellon, Control Tutorial for MATLAB, The University of Michigan, 1997, http://www.engin.umich.edu/group/ctm/example/susp/ss1.html
3. Yahaya M. S and Mohd .R and Nasarudin .A *LQR Controller for Active car Suspension* .IEEE.2000
4. Yahaya M. S and Johari .H.S Osman, *Modelling and Control of the Active Suspension System Using Proportional Integral Sliding Mode Approach* , Asian Journal of cControl, Vol.7, No2, June 2005,pp. 91-98\
5. Fu-Cheng Wang, Design and Synthesis ofActive and Passive Vehicle Suspensions, 2001 Thesis.
6. R.Kalman, *Journal of Applied Mechanics* (1980),Volume: 47, Issue: 2, Pages: 415

7. Ahmad Faheem 2006 , *Thesis of Study of Dynamic Modelling and Stability of Passenger Cars,* RMIT University

8. Abu Bakar Adham Bin Hell Mee 2009, *Thesis of Modeling and Controller Design For An Active Car Suspesion System Using Half Car Model,* Universiti Teknologi Malaysia

9. Rosheila Binti Idrus 2008, *Thesis of Modeling and Control Of Active Suspension System For A Full Car Model,* Universiti Teknologi Malausia

Appendix

Mat lab command

Setting parametes

```
mw= 100;
>> mb = 500;
>> ka= 7500;
>> ca = 1250;
>> kt = 170000;
>> A =[0 1 0 0; -ka/mb -ca/mb ka/mb ca/mb; 0 0 0 1; ka/mw ca/mw -(ka+kt)/mw -ca/mw];
>> B = [0; 0; 0; kt/mw;];
>> C =[ 1 0 0 0];
>> D = 0;
>> v = 10;
>> l = 2;
>> h = 0.3;
>> time =[0;10/v;(10+l)/v;50/v];
>> ground = [0;0;h;h];

>> t_stop = 50/v;
```

Checking controllability of system

```
co = ctrb(A,B);

>> Controllability = rank(co)

Controllability =

    4
```