

**DEVELOPMENT OF CONTROLLER AND OBSERVER FOR
CONTINUOUS STIRRED TANK REACTOR VIA STATE SPACE
APPROACH**

By

MOHAMAD FAEZ BIN AHMAD ADNAN

DISSERTATION

Submitted to the Department of Electrical & Electronic Engineering
in Partial Fulfillment of the Requirements
for the Degree
Bachelor of Engineering (Hons)
(Electrical & Electronic Engineering)

Universiti Teknologi PETRONAS

Bandar Seri Iskandar

31750 Tronoh

Perak Darul Ridzuan

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CERTIFICATION OF APPROVAL

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A project dissertation submitted to the
Department of Electrical & Electronic Engineering
Universiti Teknologi PETRONAS
in partial fulfilment of the requirement for the
Bachelor of Engineering (Hons)
(Electrical & Electronic Engineering)

Approved:

Assoc. Prof Dr. Nordin Saad
Project Supervisor

Dr. Radzuan Razali
Project Co-Supervisor

UNIVERSITI TEKNOLOGI PETRONAS
TRONOH, PERAK

May 2012

CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

Mohamad Faez Bin Ahmad Adnan

ABSTRACT

This paper describes the designing, simulation and analysis of controller and observer for a continuous stirred tank reactor via state space approach. Many industries uses the conventional control system approach, as opposed to the modern control approach commonly used in aerospace industries. Conventional controls possess several drawbacks, for example PID controllers are not adaptive and not robust. Thus, qualities such as robustness, optimality and adaptivity could have been overlooked. This project is looking at modern control approach for plant control which is expected to be better in terms of the system's controllability and stability. The entire project involves understanding the process control and state space, grasping the concept of system identification as well as mastering the function of MATLAB and Simulink for controller and observer design and simulation. Extensive utilization of MATLAB and Simulink were involved in several experiments and simulations. Results from the project indicate the practicality of modern control in plant process control system. This project successfully achieved the theoretical implementation of modern control engineering in plant process control systems, paving way for a possible design of a new controller and observer strategy that are robust, optimal and adaptive via modern control approach.

ACKNOWLEDGEMENTS

Throughout the period of accomplishing this project, I had received ideas, supports and assistance from few individuals. My appreciation to my project supervisor Assoc. Prof. Dr. Nordin Saad, for taking me under his wing and providing me all the necessary assets and resources, not only to accomplish this project, but to enrich my character and knowledge further. My utmost appreciation and gratitude is also extended to my co-supervisor, Dr. Razduan Razali for the dedication of the time and effort, relentlessly teaching and guiding me despite his many other obligation. Many thanks to my family back home for their sacrifices and heading me towards the stars. Special thanks to all the members of the Electrical and Electronics Engineering Department, for establishing continuous support and backing me up. My appreciation is also extended to my friends and everyone who encouraged and supported me throughout the successful completion of this project.

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LIST OF ABBREVIATIONS

α_1	Discharge Coefficient from Tank 1
α_2	Discharge Coefficient from Tank 2
α_3	Discharge Coefficient between the Two Tanks
A1	Area of cross section of Tank 1
A2	Area of cross section of Tank 2
CSTR	Continuous Stirred Tank Reactor
H1	Operating point in Tank 1
H2	Operating point in Tank 2
LTI	Linear Time Invariant
PID	Proportional-Integral-Derivative
Q_{i1}	Rate of Flow of Fluid into Tank 1
Q_{i2}	Rate of Flow of Fluid into Tank 2
Q_{o1}	Rate of Flow of Fluid from Tank 1
Q_{o2}	Rate of Flow of Fluid from Tank 2
Q_{o3}	Rate of Flow of Fluid from Tank 1 and Tank 2 or From Tank 2 to Tank 1
UTP	Universiti Teknologi PETRONAS

CHAPTER 1

INTRODUCTION

This chapter introduces and explains the project topic, “Development of Controller for Continuous Stirred Tank Reactor via State-Space Approach”. A background study on this topic is highlighted followed by problem statement, objectives and finally the scope of study.

1.1 Background of Study

In chemical and petrochemical industries, chemical reactors are vessels designed to contain the chemical reactions. The design and operation of chemical reactor is crucial in determining the whole success of industrial operation. Continuous stirred tank reactor (CSTR) is one of the models used to estimate the most important variables of different chemical reactor. In continuous stirred tank reactor, one or more fluid reagents are introduced into a tank reactor equipped with an impeller while the reactor effluent is removed whereby the impeller stirs the reagents to ensure proper mix [4]. Continuous stirred tank reactor belongs to a class of nonlinear systems where both steady-state and dynamics behavior are nonlinear [2]. The process nonlinearities may cause difficulties when controlling using conventional controllers with fix parameters. Since the chemical reaction has a sufficiently high heat of reaction, the continuous stirred tank reactor exhibits significant nonlinearities, extreme parametric sensitivity and nonlinear oscillations for some operating conditions [7]. A small amount of uncertainty may cause a large variation from the desired performance behavior, even often caused an unstable plant. One possible method to cope with this problem is by implementing the modern control approach.

The project aims to apply the concepts in modern control to a continuous stirred tank reactor system control. This will involves understanding of the system dynamics, the equations associated to it, analysis of its controllability and observability, which leads to the controller and observer design. Presently, different control strategies such as PID type controllers, fuzzy logic based method and genetic algorithms based

performance improvement method have all been investigated for concentration control of continuous stirred tank reactor [13].

From the system engineering point of view, since continuous stirred tank reactor belongs to the class of the nonlinear systems with continuous distributed parameters [8]; it is significant to understand a mathematical relationship or the governing dynamics between the input and the output of the system before formulating a controller. The underlying principle and knowledge of the system should be investigated to comprehend the occurrence of nonlinearity in the system dynamics.

1.2 Problem Statement

Today's chemical and petrochemical industries focus only on stability in plant control systems. Other qualities such as robustness, optimality and adaptivity are often overlooked. However, growing demands for consistency in these qualities have urged the development of system that not only stable, but also robust, optimal and adaptive.

A part from that, the modeling of plant control systems are presently carried out only using first-order state space equation, providing only limited capability in modern plant control. A study on modeling using second-order state space equations is implemented to enable more flexibility in plant control, whereby more state can be controlled.

Otherwise, the conventional control approaches are widely used for controlling a plant process. However, the conventional control possesses several drawbacks, for example PID controllers are not adaptive and not robust. Thus, the project is looking at modern control approach for plant control which is expected to be better in terms of the system's controllability and stability.

1.2.1 Significant and Feasibility of the Project

The notion of the project is to design and developed a controller for continuous stirred tank reactor by using modern control theory. The aim of the project is to improve the smoothness and efficiency of the plant process by applied the concept of state space approach into the controller in order to have better plant's controllability and stability. Opposing to the classical control theory like PID controllers with required on-site tuning

due to design approximations, designing the controller via modern control theory is carried out in the state space and can deal with multi-input and multi output (MIMO) system. This approach design could overcome the limitations of classical control in more sophisticated design problems.

The project can be complete within the given of two semesters. The project scope and time frame is referred to the project key milestone and Gantt chart in Table 2. The project will be conducted starting with the collection of related such as books, journals and technical paper specifically in CSTR process control and modern control theory. Research will be done from time to time to get better understanding on the subject. In the end of the project, new controller will be successfully design and through analysis of its performance will be measured.

1.3 Objectives

The main objective of the project is to design and analyze a controller for continuous stirred tank reactor (CSTR) via state space approach. In doing so, the following objectives below are set:

- To study the control system of continuous stirred tank reactor (CSTR) as in second-order state space representation.
- To design a controller and observer for continuous stirred tank reactor (CSTR) and apply the concept of modern control engineering in the analysis.

1.4 Scope of Study

The scope of the project is on the continuous stirred tank reactor (CSTR) solely. Therefore, the project requires details study of CSTR principle. Besides, the following should also be achieved:

- Modeling a plant process control system on MATLAB and Simulink.
- Designing and implementing observer and controller strategies for plant process.

CHAPTER 2

LITERATURE REVIEW

2.1. Continuous Stirred Tank Reactor (CSTR)

The continuous stirred tank reactor (CSTR) also known as vat or bacmix reactor is common ideal type in chemical engineering. A CSTR often refers to a model used to estimate the key unit operation variables when using a continuous agitated-tank reactor to reach a specified output [18].

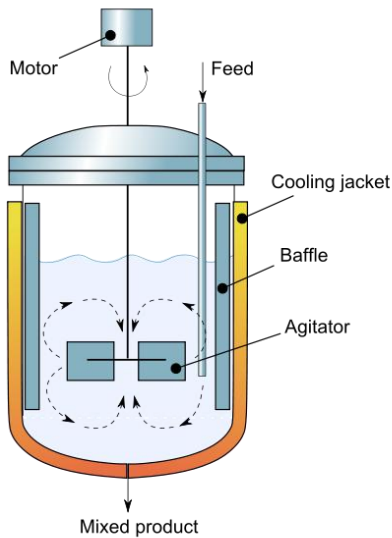


Figure 1: Cross-Sectional Diagram of Continuous Stirred Tank Reactor [3]

In a CSTR, reactants and products are continuously added and withdrawn. In practice, mechanical or hydraulic agitation is required to achieve uniform composition and temperature, a strong choice influenced by process consideration [6]. The CSTR is the idealized opposite of the well-stirred batch and tubular plug-flow reactors. Because the compositions of mixtures leaving a CSTR are those within the reactor, the reaction driving forces, usually the reactant concentrations are necessarily low. Thus, except for reaction orders zero and negative, a CSTR required the largest volume of the reactor types to obtain desired conversion. However, the long driving force makes possible better control of rapid exothermic and endothermic reactions [19]. When high conversions of reactants are needed, several CSTRs in series can be used. Equally good results can be obtained by dividing a single vessel into compartments while minimizing

back-mixing and short-circuiting. The larger number of CSTR stages, the closer the performance approaches that of a tubular plug-flow reactor [25].

In a CSTR, one or more fluid reagents are introduced into a tank reactor equipped with impeller while the reactor effluent is removed [19]. The impeller stirs the reagents to ensure proper mixing. Simply dividing the volume of the tank by average volumetric flow rate through the tank gives the residence time, or the average amount of time a discrete quantity of reagent spends inside the tank. Using chemical kinetics, the reaction's expected percent completion can be calculated. Some important aspects of the CSTR are:

- At steady state, the flow rate in must equal the mass flow rate out, otherwise the tank will overflow or go empty (transient state). While reactor is in transient state, the model equation must be derived from the differential mass energy balance [5]
- The reaction proceeds at the reaction rate associated with the final (output) concentration [5].
- Often, it is economically beneficial to operate several CSTRs in series. This allows for example, the first CSTR to operate at a higher reagent concentration cause a higher reaction rate. In these cases, the sizes of the reactors may be varied in order to minimize the total capital investment required to implement the process.

2.2. Process Controller

In every plant and system in process industries a reliable process control is needed. A control system is crucial as the variables need to be maintained at their desired reading even when disturbances occur and also to respond to change in the targeted value [4]. The desired values are based on the details analysis of the plant operation and objectives. These are the main objectives of the process control; safety, environment protection, equipment protection, product quality, profit optimization, monitoring, diagnosis and smooth plant operation [3].

2.3. Conventional Controller

As control system is needed, PID controller was developed many decades ago and being used as industrial controller until today. Proportional-Integral-Derivative Controller or known as PID Controller is the most common form of feedback and became the standard tool when process control was developed in 1940s. PID controllers have been utilized for control of diverse dynamical systems ranged from industrial process to aircraft and ship dynamics [1].

It is so popular that more than 95% of the control loops are of PID type in the process control nowadays, and most are actually PI controller. PID Controller remains the most often algorithm used today because of its simplicity, robustness, performance characteristics and successful practical applications [7].

It is as well, an important element of a Distributed Control System (DCS). Basically, all PID controllers made today are based on microprocessors. Therefore, the controller is given opportunities to provide additional features such as automatic tuning, gain scheduling, and continuous adaptation [21]. The controller operates by calculating the inaccurate value as the difference between a Measured Variable (MV) and a desired Set Point (SP). The controller will then try to minimize the error by adjusting the process control inputs [1].

Although PID controller is the most popular controller for the majority of control systems, the classic tuning methods involved in the controller suffers with a few systematic design problems [1]. It is difficult to adjust the PID parameters and once the parameters are adjusted, they remain unchanged during the control systems operation.

Linear fixed-gain PID controllers are often acceptable for controlling a minor physical process; however the requirements for high-performance control with changes in operating conditions or environmental parameters are usually beyond the capabilities of simple PID controllers [13]. Nevertheless the most difficult part of PID controllers is how to alter the three parameters with the change of operating conditions and

environmental parameters. It takes longer time to tune and get the best tuning of PID parameters.

2.4. Modern Control Theory

In control to the frequency domain analysis of the classical control, modern control theory utilizes the time-domain state space representation, a mathematical model of a physical system as a set input, output and state variable related by first-order differential equations [8]. To abstract from the number of inputs, outputs and states, the variables are expressed as vectors and the differential and algebraic equations are written in matrix form (the latter only being possible when the dynamics system is linear) [11].

2.4.1. State Space Representation

The state space representation (also known as the “time-domain approach”) modeling method appear novel to practitioners who are accustomed to think in terms of frequency response function or transfer function but it is not a new way of looking at dynamics systems [11].

State space modeling enables noise and vibration engineer to have access to and put to use a wealth of knowledge and analysis technique from the linear system discipline, including designing estimators and controller for single-input-single output systems.

2.4.2. State Space Advantage

There are many advantages to modeling system in state space. The most important advantage is the matrix formulation. Computer can easily manipulate matrices. Having the A, B, C and D matrices, one can calculate stability, controllability, observability and many other useful attributes of a system [8].

The second most important aspect of state space modeling is that it allows us to model the internal dynamics of the system, as well as the overall input/output relationship as in transfer function [15]. As stated earlier, state space modeling makes

the vast, existing, linear system knowledge such as estimation and optimal control theory available to the user.

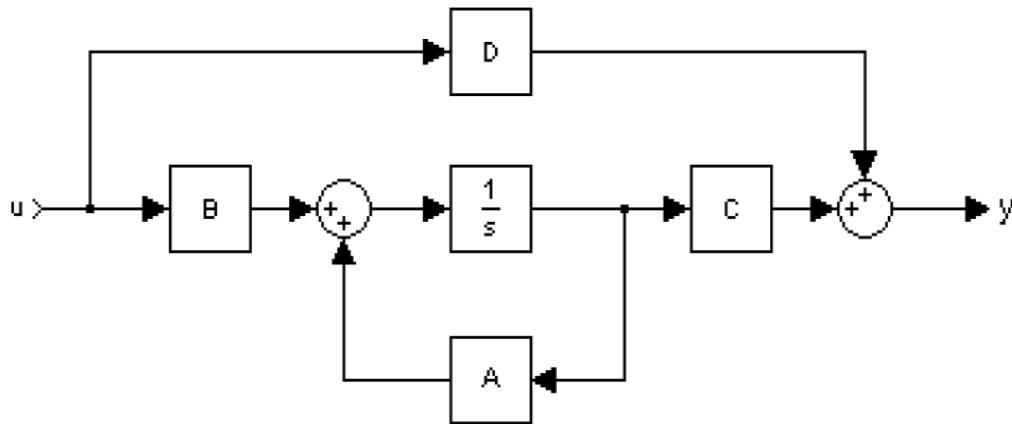


Figure 2: Block Diagram Representation of the State Equations

2.5. Process Control

Process control is a statistics and engineering discipline that deals with architectures, mechanisms, and algorithm for controlling the output of a specific process [21].

For instance, increasing in a vessel is a process that has specific, desired outcome to reach and maintain a defined pressure, kept constant over time [14]. Here, the pressure is the controlled variable. At the same time, it is the input variable since it is measured by a pressure sensor and used to decide whether to increase or decrease the opening of valves connected to the vessel. The desired pressure is set point. The state of the valve (e.g. the setting of the valve allowing more gas to flow through it) is called the manipulated variable since it is subject to control actions

Process control system can be characterized as one or more of the following forms:

- Batch: Some application require that specific quantities of raw materials be combined in specific ways for particular durations to produce an intermediate or end result [13]. An example is the production of adhesives and glue, which normally require the mixing of raw materials in a heated vessel for a period to

form a quantity of product. Others important examples are generally used to produce a relatively low to intermediate quantity of product per year [6].

- Continuous: Often, a physical system is represented through variables that are smooth and uninterrupted in time. The control of the water temperature in a heating jacket is an example of continuous process control [12]. Some important continuous processes are the production of fuels, chemicals and plastics. Continuous processes in manufacturing are used to produce very large quantities of product per year.
- Discrete: Mostly found in many manufacturing, motion and packaging applications. Robotics assembly can be characterized as discrete process control. Most discrete manufacturing involves the production of discrete piece of product such as metal stamping [2].

2.6. Terminology and Definition

In controlling a process in plant system there exist two types of variables.

2.6.1. Input Variable

The variable shows the effect of surroundings on the process. It normally refers to those factors that influence the process. Input variable typically include flow rates of stream entering or leaving a process [5]. Compositions or temperatures of streams entering a process are typical input variables. Input variables are often manipulated by process controller in order to achieve desired performance [6]. There are effects of the surrounding that are controllable and some are not. These are broken down into two types of inputs which are:

- Manipulated inputs: variable in the surroundings can be control by an operator or the control system in place.
- Disturbances: inputs that cannot be controlled by an operator or control system. There exist both measurable and immeasurable disturbances [24].

2.6.2. Output Variable

An output variable is often a state variable that is measured, particularly for control process [5]. These are the variables that are process outputs that affect the surroundings. An example of this would be the amount of CO² gas that comes out of a combustion reaction [20]. These variables may or may not be measured.

As considering a controls problem, two major control structures can be looked on.

- Single Input-Single Output (SISO) - for one control output variable there exist one manipulate input variable that is used to affect the process.
- Multiple Input-Multiple Output (MIMO) - there are several control output variable that are affected by several manipulated input variables used in a given process [16].

2.7. Controllability and Observability

Controllability and observability represent two major concepts of modern control system theory. These originally theoretical concepts, introduced by R. Kalma in 1960, are particularly important for practical implementation [15]. Controllability mean in order to be able to do whatever we want with the given dynamic system under control input, the system must be controllable and observability mean in order to see what is going on inside the system under observation, the system must be observable [11]. Even though the concepts of controllability and observability are almost abstractly defined, now intuitively understand their meanings. The remaining problem is to produce some mathematical checkup tests and to define controllability and observability more rigorously. The observability of linear discrete systems is very naturally introduced using only elementary linear algebra [17].

Controllability is an important property of a system to be controlled. A linear controllable system may be defined as a system which can be steered to any state from the zero initial state [21]. The controllability property plays an important role in many control problems, such as stabilization of an unstable system by feedback or optimal control. Since the output is a linear combination of the input and states, one or more

poles can be canceled by the zeros induced by this linear combination [16]. When that happens, the cancelled modes are said to be unobservable. Of course, since started with a transfer function, any pole-zero cancellations should be dealt with at that point, so that the state space realization will always be controllable and observable. If a mode is uncontrollable, the input cannot affect it; if it is unobservable, it has no effect on the output [11]. Therefore, there is usually no reason to include unobservable or uncontrollable modes in a state-space model.

Kalman's canonical decomposition provides the basic theory and computational algorithm to remove unnecessary states from a realization, while preserving the input-output map [17]. Its reliance on the controllability and observability matrices makes the approach somewhat susceptible to numerical problems, as these matrices are often poorly conditioned. A more serious drawback, however, is that this reduction is based on structural properties of the system (linear independence) but without explicitly considering the quantitative aspects of the problem [16]. In practical applications, especially when numerical computations are involved, one is rarely faced with perfectly dependent or perfectly orthogonal vectors. Moreover, a commonly encountered problem is that of a model reduction where modes that have independent but small contributions should be eliminated [15]. With such an objective in mind the previous algorithm is inadequate.

2.8. Mathematical Modeling of Two isothermals Continuous Stirred Tank Reactor (CSTR) in series with interacting tank

2.8.1. First Principle method for parameter determination

The state space model of CSTR system is based on the Mass Balance Equation for the volume of fluid present inside the two CSTR tanks. At any given instant the rate of change of volume of fluid present inside the tank can be expressed in terms of the rate at which fluid flows into the CSTR through the outlet valves. The CSTR in series with interaction system dynamics can be visualized as shown in the **Figure 3**.

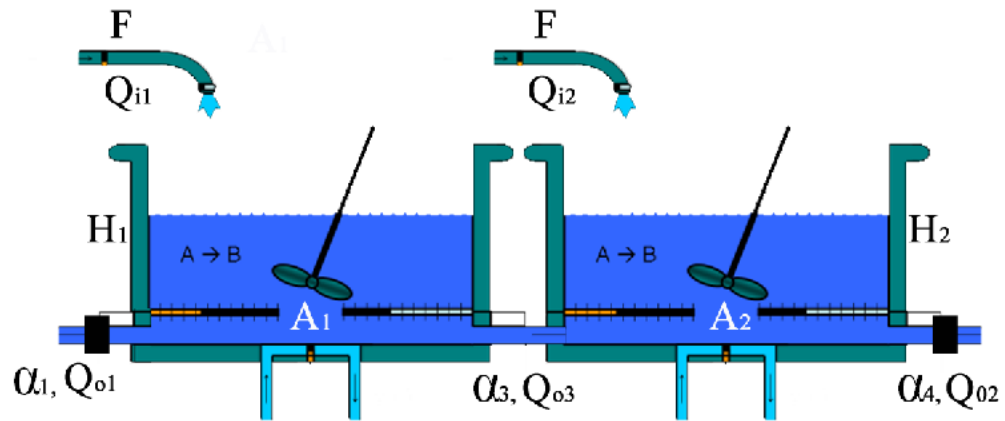


Figure 3: Two CSTR in Series with Interacting Tank

Where,

α_1 = discharge coefficient from Tank 1

α_2 = discharge coefficient from Tank 2

α_3 = discharge coefficient between the two tanks i.e. coupling coefficient between the two tank. (α_3 will be positive for $H_1 > H_2$ and negative for $H_1 < H_2$)

Q_{i1} = rate of flow of fluid into Tank 1 (manifested in the form of control input – U1)

Q_{i2} = rate of flow of fluid into Tank 2 (manifested in the form of control input –U2)

Q_{o1} = rate of flow of fluid from Tank 1

Q_{o2} = rate of flow of fluid from Tank 2

Q_{o3} = rate of flow of fluid from Tank 1 and Tank 2 or from Tank 2 to Tank 1 depending upon whether $H_1 > H_2$ or $H_1 < H_2$, respectively.

H1 = operating point in Tank 1 (chosen to be equal to the desired set point)

H2 = operating point in Tank 2 (chosen to be equal to the desired set point)

A1 = area of cross section of Tank 1

A2 = area of cross section of Tank 2

2.8.2. The Mass Balance Equation

At any given time, the height of fluid in either of the two CSTR tank is related to the fluid inlet rate, fluid outlet rate and the tank interactions.

$$A1 \frac{dH1}{dt} = Qi1 - Qo1 - Qo3 \quad (3.1)$$

$$A2 \frac{dH2}{dt} = Qi2 - Qo2 - Qo3 \quad (3.2)$$

Here,

$$Qo1 = \alpha1\sqrt{H1} \quad (3.3)$$

$$Qo2 = \alpha2\sqrt{H2} \quad (3.4)$$

$$Qo3 = \alpha3\sqrt{H1 - H2} \quad (3.5)$$

Substituting equation (3.3), (3.4) and (3.5) in equations (3.1) and (3.2), results as

$$A1 \frac{dH1}{dt} = Qi1 - \alpha1\sqrt{H1} - \alpha3\sqrt{H1 - H2} \quad (3.6)$$

$$A2 \frac{dH2}{dt} = Qi2 - \alpha2\sqrt{H2} - \alpha3\sqrt{H1 - H2} \quad (3.7)$$

The equation (3.6) and (3.7) represent a non-linear relationship between the fluid level (H1 and H2 in the two tanks, respectively) and the discharge coefficients. Since the operating point is known and does not change quite often then is convenient to linearize the system obtained by the first principle around the desired operating point. This makes the process significantly simpler and the model works well in a region around the chosen operating point. The stretch of operating band in which the linearized

system gives a response similar to the actual non-linear system determines the sensitivity of the linearized system.

2.8.3. Linearization and State Space Formation

Considering an incremental of q_{i1} and q_{i2} in the two control inputs respectively, which subsequently cause an incremental change in height in two tanks – h_1 and h_2 , respectively. Hence, the equation (3.6) and (3.7) can be re-written as,

$$A1 \frac{d(H1+h1)}{dt} = (Qi1 + qi1) - \alpha1\sqrt{(H1 + h1)} - \alpha3\sqrt{(H1 + h1) - (H2 + h2)} \quad (3.8)$$

$$A2 \frac{d(H2+h2)}{dt} = (Qi2 + qi2) - \alpha2\sqrt{(H2 + h2)} - \alpha3\sqrt{(H1 + h1) - (H2 + h2)} \quad (3.9)$$

Now, subtracting equation (3.6) from (3.8) and (3.7) from (3.9) results,

$$A1 \frac{dh1}{dt} = qi1 - \alpha1(\sqrt{H1 + h1}) - \sqrt{H1} - \alpha3\sqrt{H1 - H2 + h1 - h2} - \sqrt{H1 - H2} \quad (3.10)$$

$$A2 \frac{dh2}{dt} = qi2 - \alpha2(\sqrt{H2 + h2}) - \sqrt{H2} - \alpha3\sqrt{H1 - H2 + h1 - h2} - \sqrt{H1 - H2} \quad (3.11)$$

Knowing that as per Binomial expansion,

$$(1 + x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \dots \quad (3.12)$$

For $x \ll 1$, that can be approximated as

$$(1 + x)^n = 1 + nx \quad (3.13)$$

Now, putting the approximations given by equation (3.13) in equation (3.10) and (3.11) and rearrange results as,

$$A1 \frac{dh1}{dt} = q1 - \alpha1 \frac{h1}{2\sqrt{H1}} - \alpha3 \frac{(h1-h2)}{2\sqrt{H1-H2}} \quad (3.14)$$

$$A2 \frac{dh2}{dt} = q2 - \alpha2 \frac{h2}{2\sqrt{H2}} - \alpha3 \frac{(h1-h2)}{2\sqrt{H1-H2}} \quad (3.15)$$

Where $q_{i1} = q1$ and $q_{i2} = q2$

The continuous time state space equations of multiple input-multiple output (MIMO) system can be represented as,

$$\begin{bmatrix} \frac{dh1}{dt} \\ \frac{dh2}{dt} \end{bmatrix} = \begin{bmatrix} \frac{-1}{2A1} \left(\frac{\alpha1}{\sqrt{H1}} + \frac{\alpha3}{\sqrt{H1-H2}} \right) & \frac{\alpha3}{2A1\sqrt{H1-H2}} \\ \frac{\alpha3}{2A1\sqrt{H1-H2}} & \frac{-1}{2A2} \left(\frac{\alpha2}{\sqrt{H1}} + \frac{\alpha3}{\sqrt{H1-H2}} \right) \end{bmatrix} \begin{bmatrix} h1 \\ h2 \end{bmatrix} + \begin{bmatrix} \frac{1}{A1} & 0 \\ 0 & \frac{1}{A2} \end{bmatrix} \begin{bmatrix} q1 \\ q2 \end{bmatrix} \quad (3.16)$$

$$\begin{bmatrix} y1 \\ y2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} h1 \\ h2 \end{bmatrix} \quad (3.17)$$

Where,

$q1$ = input to Tank 1 (rate at which fluid is pumped into Tank 1 in cm^3/sec)

$q2$ = input to Tank 2 (rate at which fluid is pumped into Tank 2 in cm^3/sec)

$y1$ = output for Tank 1 (incremental change in height – $h1$ of fluid from the operating point height $H2$, in Tank 1 – cm)

$y2$ = output for Tank 2 (incremental change in height – $h2$ of fluid from the operating point height $H2$, in Tank 2 – cm)

CHAPTER 3 METHODOLOGY

This chapter discusses the research methodology, procedure identification as well as the tools utilized throughout the course of completing the project.

3.1. Research Methodology

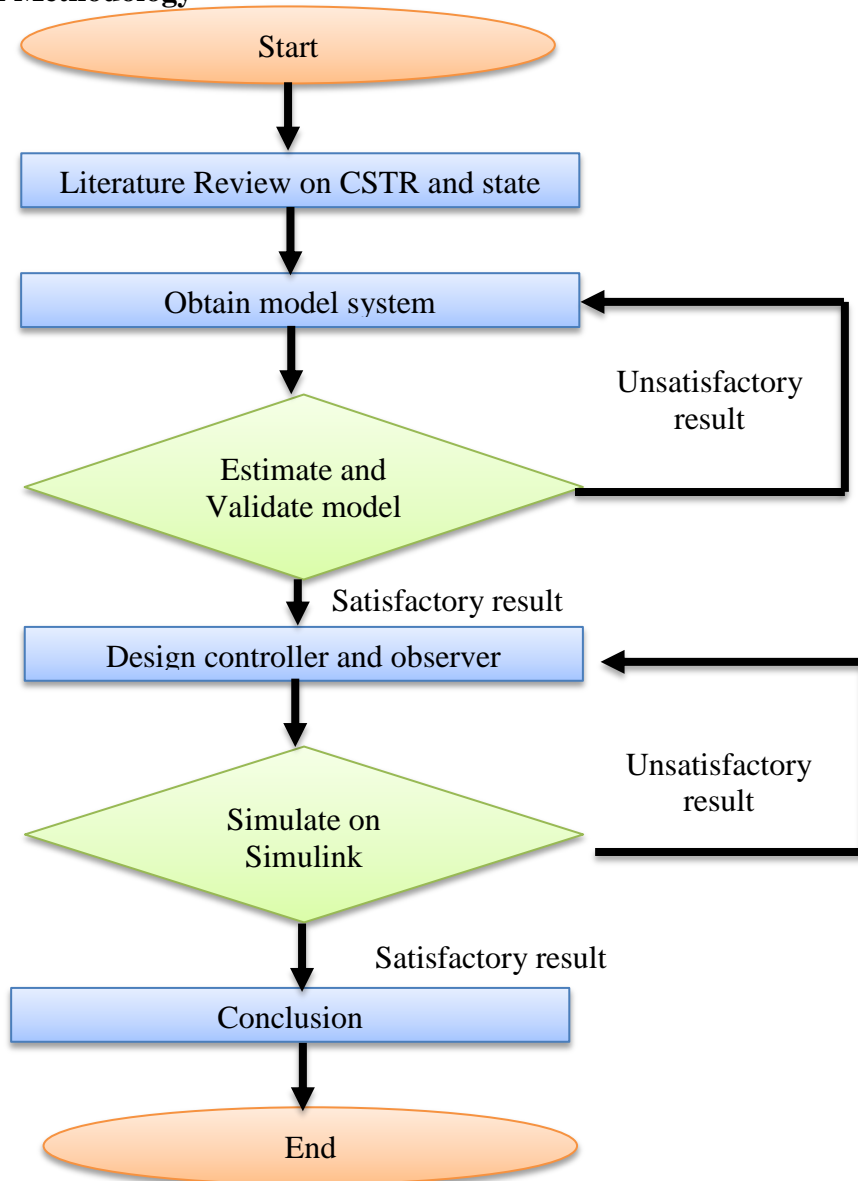


Figure 4: Project Flow Diagram

3.2. Project Activities

The project is generally divided into four main phase:

- Phase 1: Literature Review

Through study on the continuous stirred tank reactor (CSTR), the mathematical modeling of the system, model in state space and as well as the coding required for simulating in MATLAB. Other required studies which are related to the project were done too.

- Phase 2: Estimate and validate model

The model of the continuous stirred tank reactor is modeled using the MATLAB with the help of Simulink. The performance of the model was analyzed.

- Phase 3: Design controller and observer

The controller and observer poles and gains are determined based on the model parameters.

- Phase 4: Simulate controller and observer in Simulink

The designed controller and observers are then simulated on different system types on Simulink to observe their effects. Results found to be documented and compiled in the report.

- Phase 5: Analysis and Conclusion

Through analysis is conducted to test the behavior and performance of the controller and observer designed. The analysis included testing on the performance of controller with existing of disturbances and internal behavior effects.

3.3. Gantt Chart

Table 1: Gantt Chart for Final Year Project 1(FYP 1)

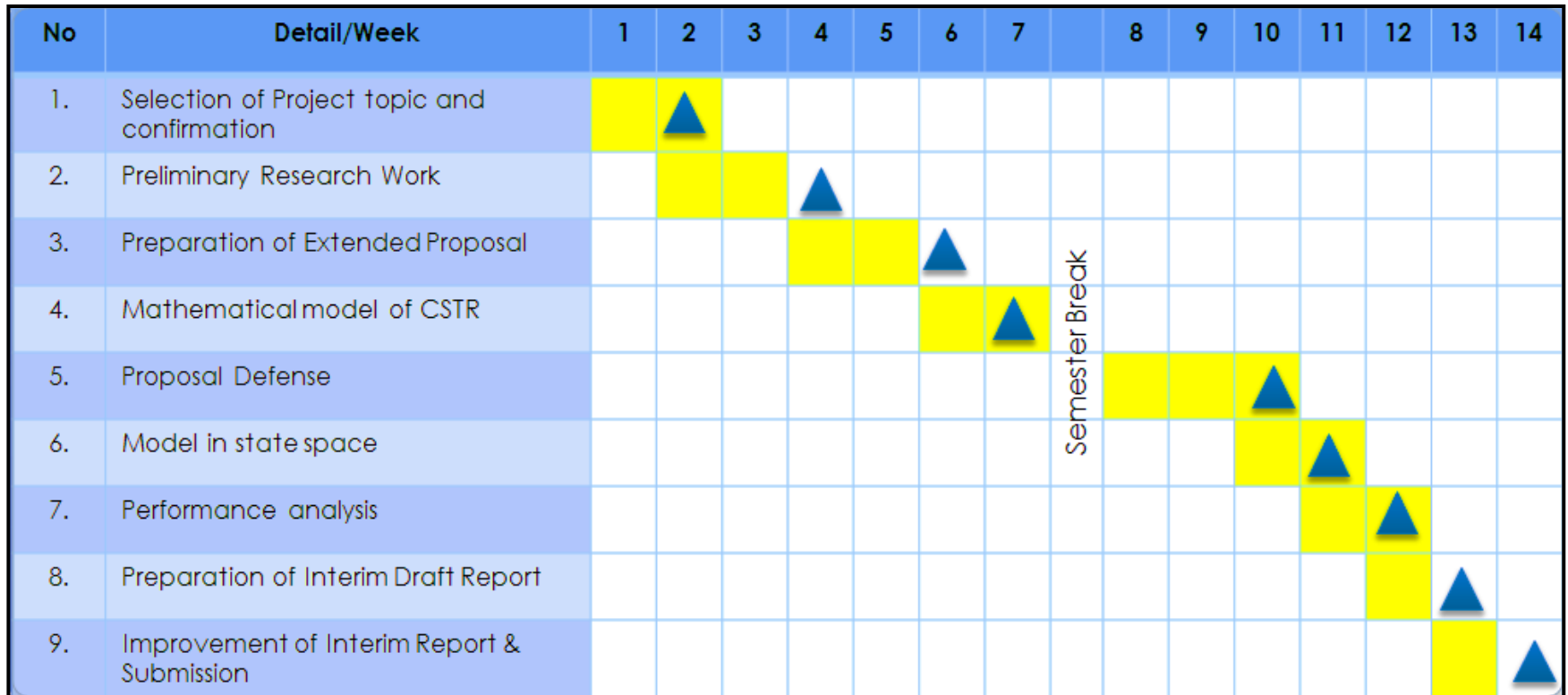
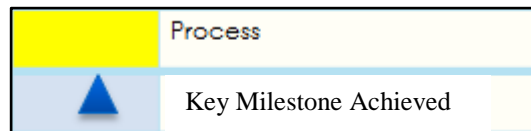
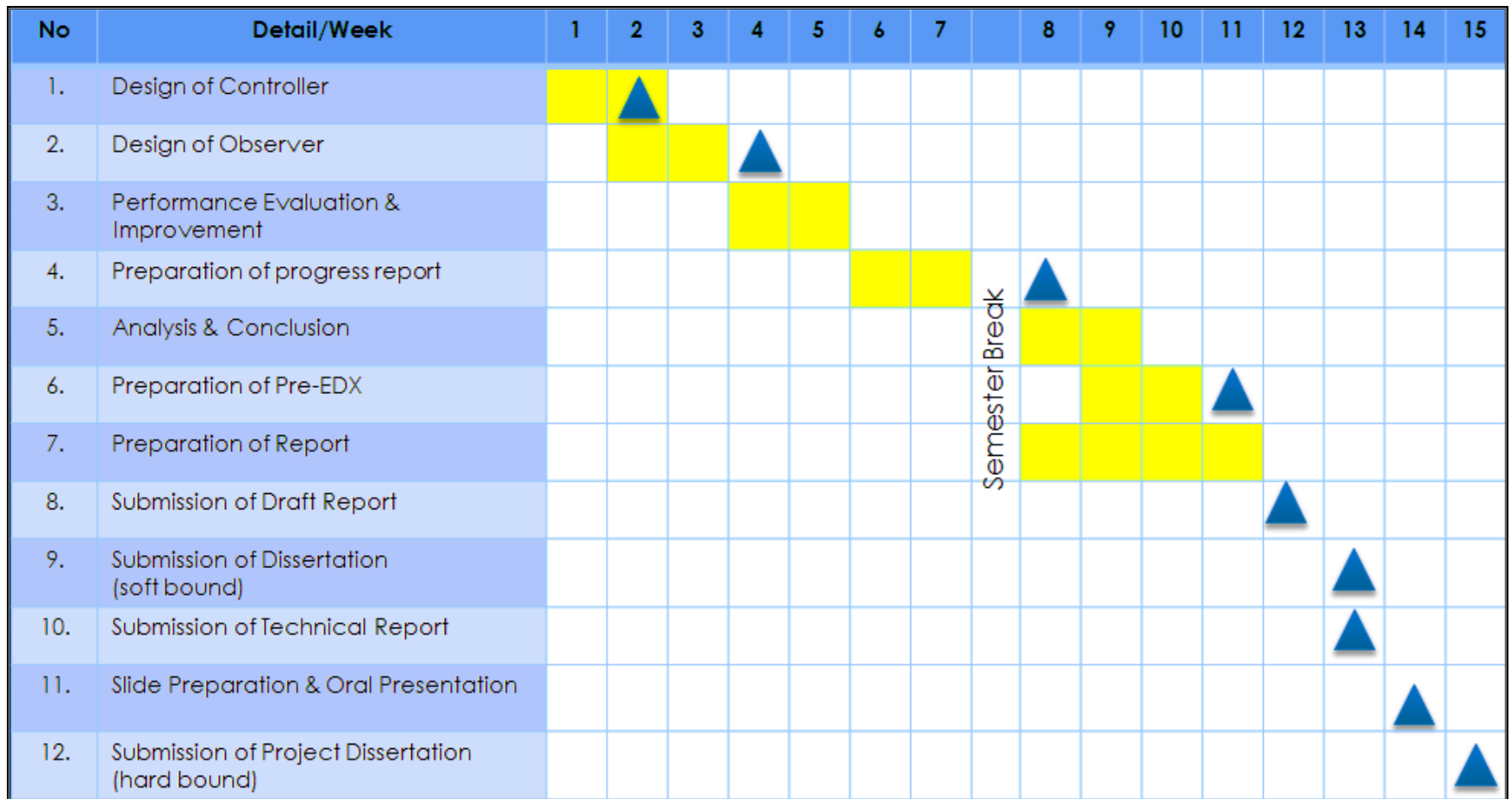


Table 2: Gantt Chart for Final Year Project 2 (FYP 2)



3.4. Tools and Equipment

3.4.1. *MATLAB*

MATLAB (short for MATrix LABoratory) is a special-purpose computer program optimized to perform engineering and scientific calculation [10]. It started life as a program designed to perform matrix mathematics, but over the years it grown into a flexible computing system capable of solving essentially any technical problem.

MATLAB program implements the MATLAB programming language, and provides an extensive library of predefined functions that make technical programming task easier and more efficient. In this project, MATLAB is used to perform the calculation in determining of controllability and observability of the system design as well as obtaining the transfer function.

3.4.2. *Simulink*

Simulink is a software package for modeling, simulating and analyzing dynamic systems [10]. It supports linear and nonlinear systems, modeled in continuous time, sampled time or a hybrid of the two. Systems can also be mutilates such as have different parts that are sampled or updated at different rates. For modeling, Simulink provides a graphical user interface (GUI) for building models as block diagram, using click-and-drag mouse operation [12]. The model of continuous stirred tank reactor (CSTR) and its controllers are designed using Simulink then the performance of the system is evaluated.

CHAPTER 4

RESULT AND DISCUSSION

In this chapter, the results of every stage and phase of the project are discussed.

4.1. Part 1: State Space Model of Two Continuous Stirred Tank Reactor (CSTR) in Series with Interacting Tank

From the system analyzed the following state space was model as:

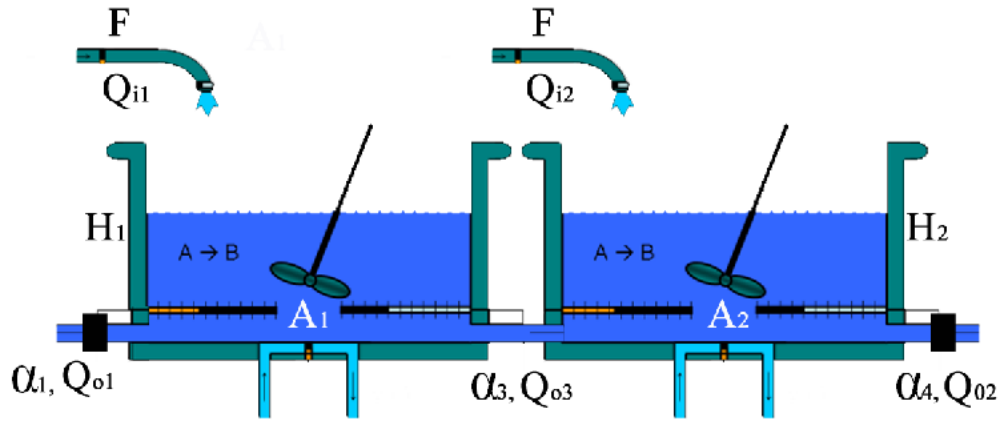


Figure 5: CSTR in series with interacting tank

$$\begin{bmatrix} \frac{dh_1}{dt} \\ \frac{dh_2}{dt} \end{bmatrix} = \begin{bmatrix} \frac{-1}{2A_1} \left(\frac{\alpha_1}{\sqrt{H_1}} + \frac{\alpha_3}{\sqrt{H_1-H_2}} \right) & \frac{\alpha_3}{2A_1\sqrt{H_1-H_2}} \\ \frac{\alpha_3}{2A_1\sqrt{H_1-H_2}} & \frac{-1}{2A_2} \left(\frac{\alpha_2}{\sqrt{H_1}} + \frac{\alpha_3}{\sqrt{H_1-H_2}} \right) \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{A_1} & 0 \\ 0 & \frac{1}{A_2} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \quad (4.1)$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} \quad (4.2)$$

Where,

$$A = \begin{bmatrix} \frac{-1}{2A_1} \left(\frac{\alpha_1}{\sqrt{H_1}} + \frac{\alpha_3}{\sqrt{H_1-H_2}} \right) & \frac{\alpha_3}{2A_1\sqrt{H_1-H_2}} \\ \frac{\alpha_3}{2A_1\sqrt{H_1-H_2}} & \frac{-1}{2A_2} \left(\frac{\alpha_2}{\sqrt{H_1}} + \frac{\alpha_3}{\sqrt{H_1-H_2}} \right) \end{bmatrix} \quad (4.3)$$

$$B = \begin{bmatrix} \frac{1}{A1} & 0 \\ 0 & \frac{1}{A2} \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad D=0 \quad \text{and} \quad \frac{dh1}{dx} = \dot{x}1, \quad \frac{dh2}{dx} = \dot{x}2 \quad (4.4)$$

The parameters of CSTR in series with interacting tank are given in **Table 3**.

Table 3: Parameters of CSTR system

Name	Expression	Value
Cross Sectional Area of CSTR system reservoir	A1 and A2	32cm ²
Discharge coefficients	$\alpha 1$	4.0993
	$\alpha 2$	4.3702
	$\alpha 3$	25.1952
Operating points	H1	15cm
	H2	12cm

The operating point can be chosen to be any value from 0 to 30 cm as per the tank capacity. The only limitation in choosing the operating points is that the difference in H1 and H2 cannot be more than 5cm. This limitation arises because of limitation in pumping rate of the motor pumps and the coupling between the two motor tanks. The maximum pumping rate (r) delivered by each of two motors at 5 volts come out to be 36 cm³/sec [26]. Hence the maximum difference in height H1 and H2 can be maintained if one of the pumps is working at its full capacity (36 cm³/sec) while the other pump is off (0 cm³/sec). Therefore, H1 and H2 are chosen as 15 cm and 12 cm for working out the state space matrices.

The state space function is derived based on the state space equations (4.1) and (4.2) by substituting the parameter given in Table 3 as such:

$$\begin{bmatrix} \frac{dh1}{dt} \\ \frac{dh2}{dt} \end{bmatrix} = \begin{bmatrix} -0.23155847 & 0.22728837 \\ 0.22728837 & -0.2449193 \end{bmatrix} \begin{bmatrix} h1 \\ h2 \end{bmatrix} + \begin{bmatrix} 0.03125 & 0 \\ 0 & 0.03125 \end{bmatrix} \begin{bmatrix} q1 \\ q2 \end{bmatrix} \quad (4.5)$$

$$\begin{bmatrix} y1 \\ y2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} h1 \\ h2 \end{bmatrix} \quad (4.6)$$

By using the parameters derived from the previous equation, the transfer function of CSTR system is formulated using transfer function matrix whereby

$$G(s) = C(sI - A)^{-1} B + D \quad (4.7)$$

Where;

$$A = \begin{bmatrix} -0.23155847 & 0.22728837 \\ 0.22728837 & -0.2449193 \end{bmatrix} \quad (4.8)$$

$$B = \begin{bmatrix} 0.03125 & 0 \\ 0 & 0.03125 \end{bmatrix} \quad (4.9)$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (4.10)$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (4.11)$$

Since the design system consists of multiple inputs and multiple outputs (MIMO), the second order transfer function of CSTR system is being developed.

$$\frac{Y_1(s)}{Q_1(s)} = \frac{0.0313s+0.0077}{s^2+0.4765s+0.0051} \quad (4.12)$$

4.2. Continuous Stirred Tank Reactor (CSTR) Model in Simulink

The CSTR system was modeled using Simulink as shown in **Figure 6**. The output is connected to a scope to be observed. A step input is applied to the system and corresponding input and output is observed simultaneously at the scope shown in **Figure 7**.

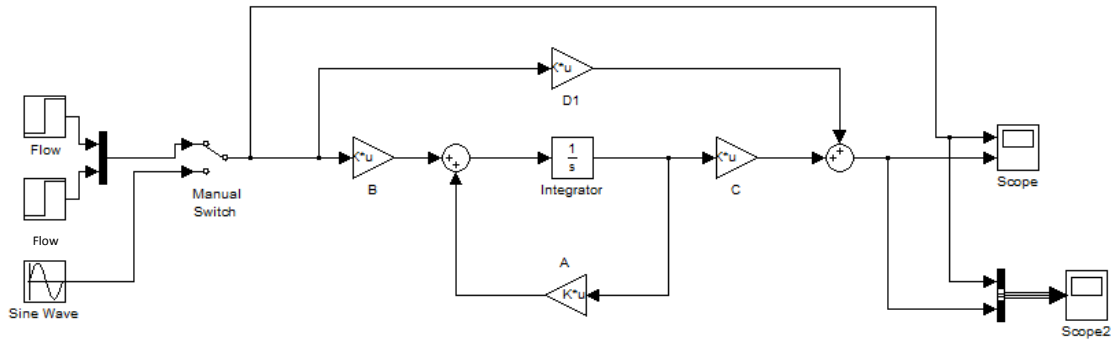


Figure 6: CSTR Model in Simulink

Following figures shown the input and output response of the CSTR system model for CSTR in series with interacting tank and CSTR in series with non-interacting tank. Step input of step 0.5 volts (step change in rate = $4.7688\text{cm}^3/\text{sec}$) is applied to motor pump 1 [26]. The output (liquid level) in the both CSTR is allowed to settle at the new steady state.

While maintaining the input to motor pump 1 unchanged, step input of 0.5 volts (step change in rate = $4.9013\text{cm}^3/\text{sec}$) is applied to motor pump 2 [26] and the output in both CSTR is allowed to settle to the new steady state value. The design of CSTR system consists of multiple inputs and multiple outputs (MIMO), the graph is plot represented the transfer function.

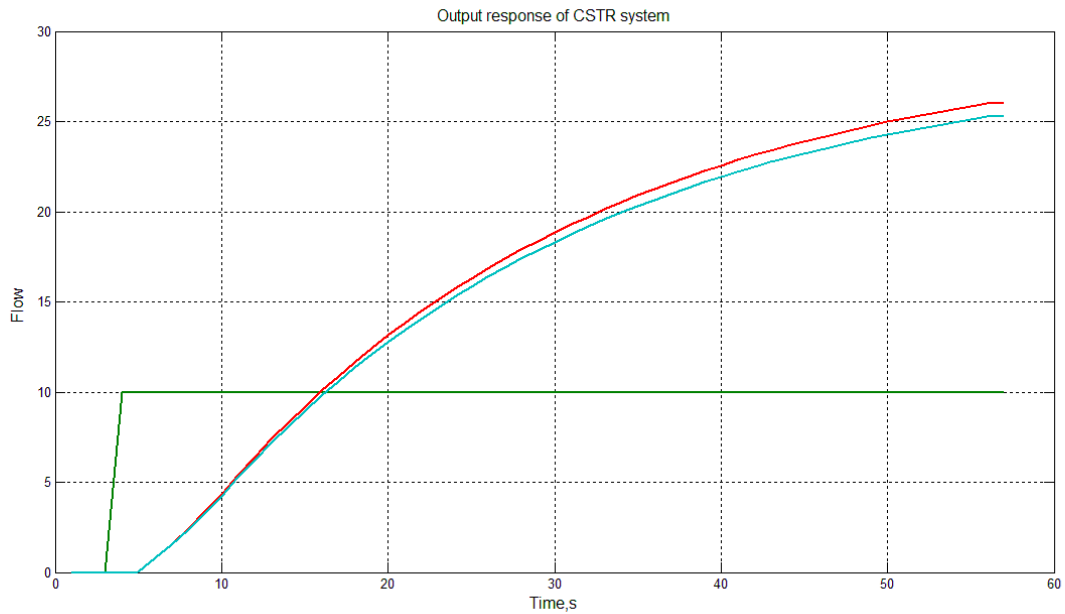


Figure 7: Output Response of the CSTR system when step input is applied

CSTR system yields product by polymerization of reactant by consuming catalyst in the presence of solvent. A coolant flows through the jacket in function to remove the heat of polymerization and to keep the temperature constant. The load is the supply rate of reactant which is equal to the output rate of product of CSTR, since the reactor volume is also kept constant. The quality of the molecular weight of the product is determined by concentration of the reactant and inhibitor [23].

Thus, the controlled outputs are the output flow rate and the concentration of reactant and inhibitor while the control inputs are supply rates of reactant, catalyst and inhibitor. The operation of CSTR is disturbed by external factors such as change in the feed flow rate and temperature. However, in this system, the feed flow rate is being manipulated and control action is performed to alleviate the impact of changing disturbance and keep the output flow rate and concentration at desired set point.

Based on result in **Figure 7**, the output response is linearly increased with respect to time. The graph keeps increasing to infinity with no overshoot and no settling time indicated the plant is in unstable condition. Some correction must be made to allow the output response to reach back to the desired points. An important problem is then to control the plant so that the quality of the product remains unchanged during the large load changes.

4.3. Controllability and Observability of CSTR Models

In the world of control engineering, there are slew of systems available that need be controlled. The task of a control engineer is to design controller and compensator units interact with these pre-existing system. However, some of systems simply cannot be controlled. The concept of controllability refers to the ability of a controller to arbitrarily alter functionality of the system plant.

The concepts of controllability and observability play important rule in design of control system in state space. In fact, the conditions of controllability and observability may govern the existence of a complete solution to control system design problem [4]. The solution to this problem may not exist if the system considered is not controllable. Since CSTR is belongs to nonlinear system, the plant models must be in controllable condition so that designing of controller can be precede. Thus, before proceeding to the modeling and simulation, the controllability and observability of the system are determined.

To check the controllability of the system, some calculation has been made as;

$$A = \begin{bmatrix} -0.23155847 & 0.22728837 \\ 0.22728837 & -0.2449193 \end{bmatrix} \quad (4.13)$$

$$B = \begin{bmatrix} 0.03125 & 0 \\ 0 & 0.03125 \end{bmatrix} \quad (4.14)$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (4.15)$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (4.16)$$

$$Mc = [B \quad AB] \neq 0$$

$$Mc = \begin{bmatrix} 0.03125 & -0.00723619 \\ 0 & 0.00710275 \end{bmatrix} \quad (4.17)$$

$$Mc^{-1} = \frac{\begin{bmatrix} 0.03125 & -0.00723619 \\ 0 & 0.00710275 \end{bmatrix}}{0.0002219} \quad (4.18)$$

$$Q_I = \frac{1}{0.0002219} [0 \quad 0.00710275] \quad (4.19)$$
$$Q_I = 32.008787 \neq 0$$

Therefore, the system is controllable.

Next, the observability of the plant system is calculated;

$$\begin{aligned} [C^T \quad A^T \quad C^T] &\neq 0 \\ [C^T \quad A^T \quad C^T] &= \begin{bmatrix} 0 & -0.231558 & 0.227288 \\ 1 & 0.227288 & -0.244919 \end{bmatrix} \\ &= -0.227288 \neq 0 \end{aligned} \tag{4.20}$$

Thus, the system is observable.

The system is found to be controllable and observable as proven using calculation and as well as computing in MATLAB. This is because in both calculations the rank is not equivalent to zero. Since the system is controllable and observable, the state feedback control can be designed. Previously, the CSTR model shown to has a nonlinear system and in the unstable condition, thus it is necessary to reduce the transient period of unstable condition in order to improve plant productivity performance including man power saving and efficient energy by developing a versatile controller.

4.4. Controller for Continuous Stirred Tank Reactor (CSTR)

4.4.1. Full State Feedback Controller Design

The concept of feed-backing all the state variable back to the input of the system through suitable feedback matrix in the control strategy is known as the full-state variable feedback control technique [26]. Using this approach, the desired location of the close-loop eigenvalues of the system will be specified. Thus, the aim is to design a feedback controller that will move some or all the open-loop pole location as specified.

4.4.1.1. Determine controller feedback gain, K

A gain, K for full-state feedback controller is calculated using Ackermann's formula for pole-placement technique as shown;

$$\begin{aligned} M_c &= [B \quad AB] \\ &= \begin{bmatrix} 0.03125 & -0.00723619 \\ 0 & 0.00710275 \end{bmatrix} \end{aligned} \quad (4.21)$$

$$\begin{aligned} M_c^{-1} &= \frac{\begin{bmatrix} 0.03125 & -0.00723619 \\ 0 & 0.00710275 \end{bmatrix}}{0.00022196093} \\ &= \begin{bmatrix} 140.790544 & 0 \\ -32.601188 & 32.000001 \end{bmatrix} \end{aligned} \quad (4.22)$$

$$Q_c = [-32.601188 \quad 32.000001] \quad (4.23)$$

The desired roots are chosen at -1.7 and -0.1, thus the desired characteristic polynomial is written as;

$$\alpha_c(s) = (s + 1.7)(s + 0.1) \quad (4.24)$$

$$= s^2 + 1.8s + 0.17 \quad (4.25)$$

$$\alpha_c(A) = A^2 + 1.8A + 0.17I \quad (4.26)$$

$$\begin{aligned} &= \begin{bmatrix} 0.105279 & -0.108298 \\ -0.108298 & 0.111645 \end{bmatrix} + \begin{bmatrix} -0.416804 & 0.409118 \\ 0.409118 & -0.440854 \end{bmatrix} + \begin{bmatrix} 0.17 & 0 \\ 0 & 0.17 \end{bmatrix} \\ &= \begin{bmatrix} 0.168475 & 0.30082 \\ 0.30082 & -0.159209 \end{bmatrix} \end{aligned}$$

Therefore, the controller gain, K is;

$$\begin{aligned}
 K &= [-32.601188 \quad 32.000001] \begin{bmatrix} 0.168475 & 0.30082 \\ 0.30082 & -0.159209 \end{bmatrix} \\
 &= [4.133755 \quad -14.903378]
 \end{aligned}
 \tag{4.27}$$

The equation of controller is derived as

$$\begin{aligned}
 u &= -kx \\
 u &= -[4.133755 \quad -14.903378] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\
 u &= -4.133755x_1 + 14.903378x_2
 \end{aligned}
 \tag{4.28}$$

4.1.1.2. Designing the Full State Feedback Controller

Most of the control system are based on the principle of feedback whereby the signal to be controlled is compared to a desired reference signal and the discrepancy used to compute corrective control action [27]. A feedback control system is valuable because it allowed the adjustment of the transient response been made. Besides, the sensitivity of the system and the effect of disturbance can be reduced significantly. The block diagram in shown **Figure 8** is constructed to simulate the plant system with full-state feedback controller.

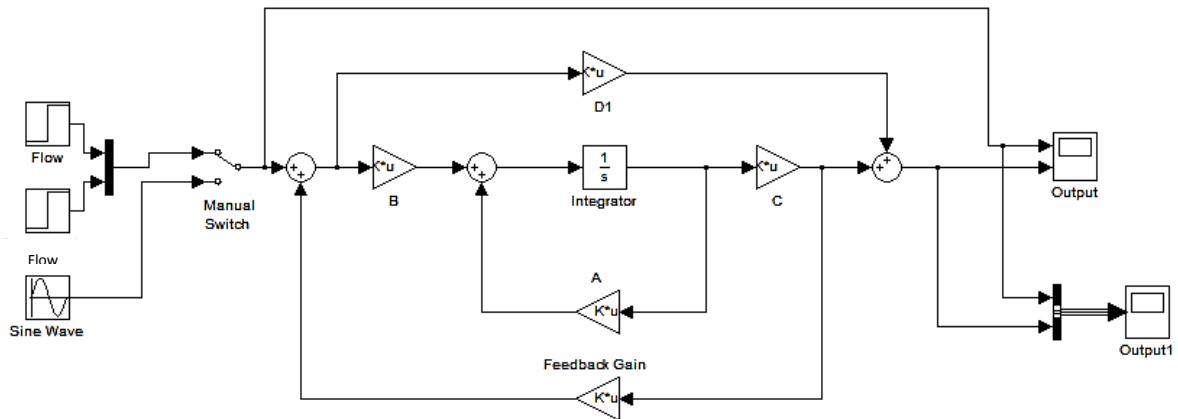


Figure 8: Feedback Controller

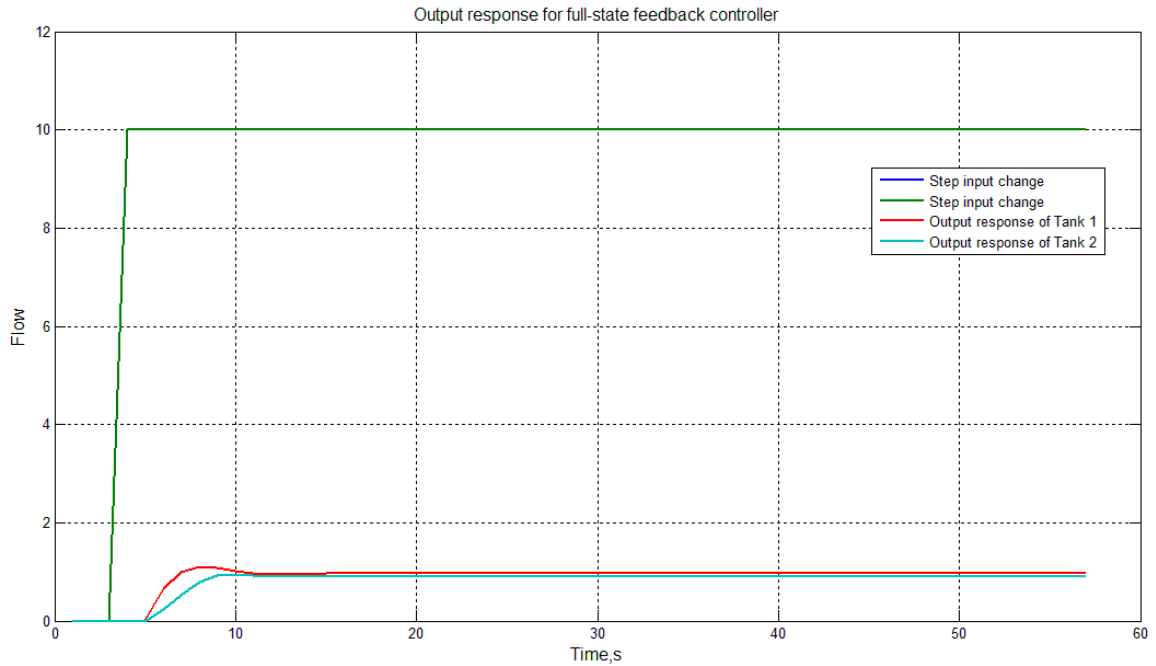


Figure 9: Output Response for Full State Feedback Controller

Figure 9 describes the result of the signal when the full-state feedback controller is introduced to the CSTR plant control system. The introduction of feedback controller gain to the system improves plant stability (the graph shown plant becoming stable after a period of time compared to the previous one; **Figure 7**). However, the controller is unsuccessfully in bringing the system output value reach up to the desired set point; 10 % and produced large steady state error.

Table 4: Full State Feedback Controller Performance

Parameters	Output 1	Output 2
Offset	9.0	9.0
Rise time, s	-	-
Settling time, s	10.4	10.4
Overshoot, %	-	-

4.4.2. Full State Feedback Feedforward Controller Design

The objective of introducing feedforward controller is to enhance the controllability of the designed controller from the previous design (using feedback controller only). In order to design the controllers using pole placement method, the controller poles are chosen to be further from zero than the plant poles so that the response will be more faster.

4.4.2.1. Determine feedforward controller gain, N

Following method is used to find the controller forward gain, N.

$$\begin{bmatrix} N_x \\ N_u \end{bmatrix} = \begin{bmatrix} -0.231558 & 0.227288 & 0.03125 \\ 0.227288 & -0.244919 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad (4.29)$$

$$\begin{aligned} M_c^{-1} &= \frac{\begin{bmatrix} -0.231558 & 0.227288 & 0 \\ 0.227288 & -0.244919 & 1 \\ 0.03125 & 0 & 0 \end{bmatrix}}{0.007102575} \\ &= \begin{bmatrix} -32.601979 & 32.000788 & 0 \\ 32.000788 & -34.483128 & 14.794008 \\ 4.399813 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 \\ 2.7214965 \\ 0 \end{bmatrix} \end{aligned}$$

$$N = N_u + N_x \quad (4.30)$$

$$= 0 + [-14.133755 \quad 4.133755] \begin{bmatrix} 0 \\ 2.7214965 \end{bmatrix} = 11.256761$$

4.1.1.2. Designing feedback-feedforward controller

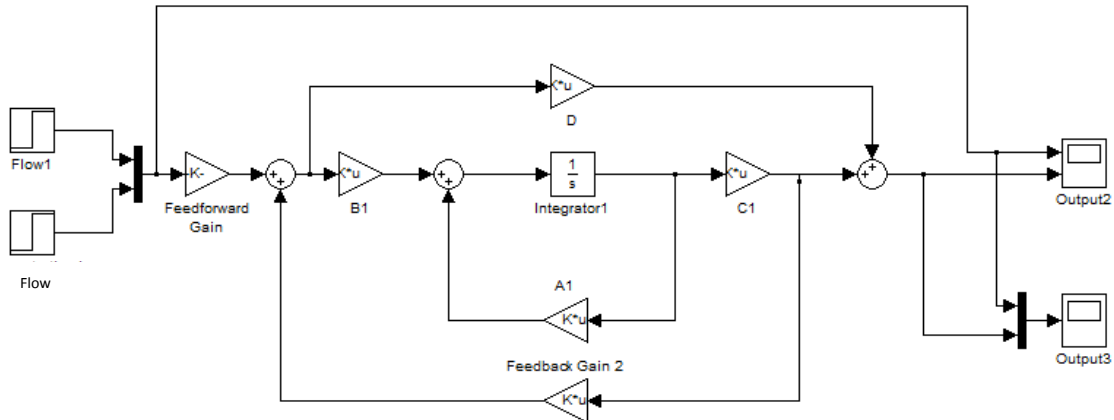


Figure 10: Feedback-Feedforward Controller

Previously, the full-state feedback controller results the correction of the transient response of the system from unstable to become stable condition. However, the results shown that the output response is not reached back to the desired output. Feedforward gain is introduced to cater with this problem so that the output response meets the expected result.

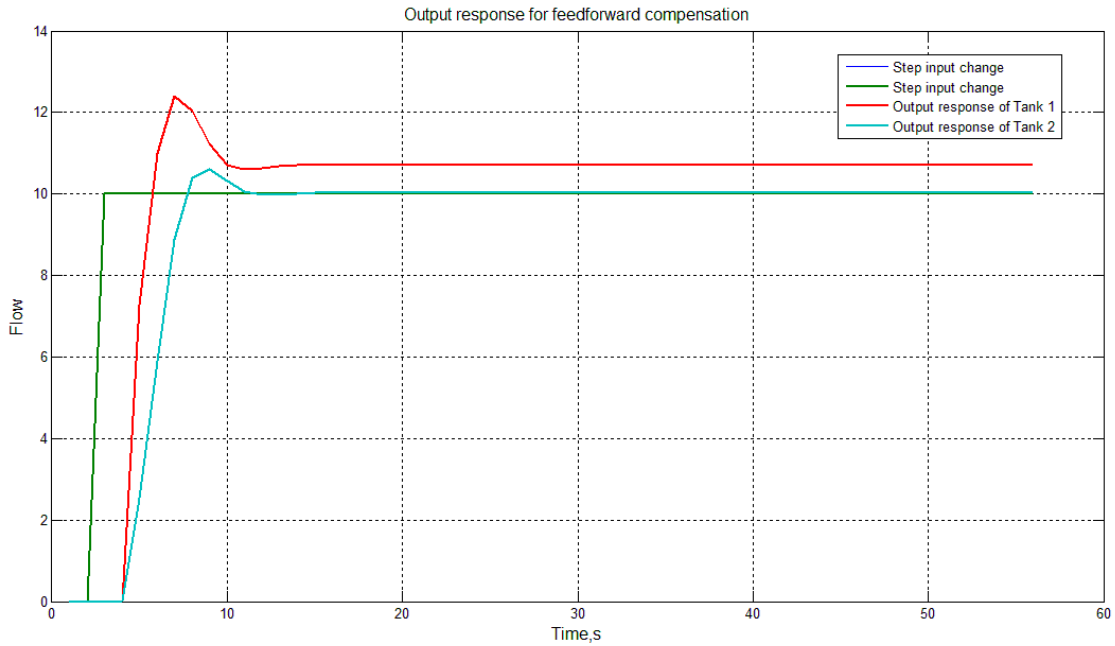


Figure 11: Output Response for Feedforward Compensation

Figure 11 shows the output of Scope 3 when the feedforward gain is implemented into the block diagram. The controller is virtually successful in bringing the system output back to the desired set point with the presence of slightly offset and overshoot.

Table 5: Full State Feedback-Feedforward Controller Performance

Parameters	Output 1	Output 2
Offset	1.5	0.0
Rise time, s	15.2	17.5
Settling time, s	14.7	12.4
Overshoot, %	12.7	10.0

4.4.3. Tracking Controller Design

Next, the modeling and simulation of tracking controller is performed to track the varying reference signal contains an integrating element. Previous designed controller shown one of the output unable to reach back to desired set point, therefore introducing of integral into the design system can helps this to eliminate the steady state error. **Figure 12** is shown the block diagram of tracking controller and **Figure 13** describes the results of the output response.

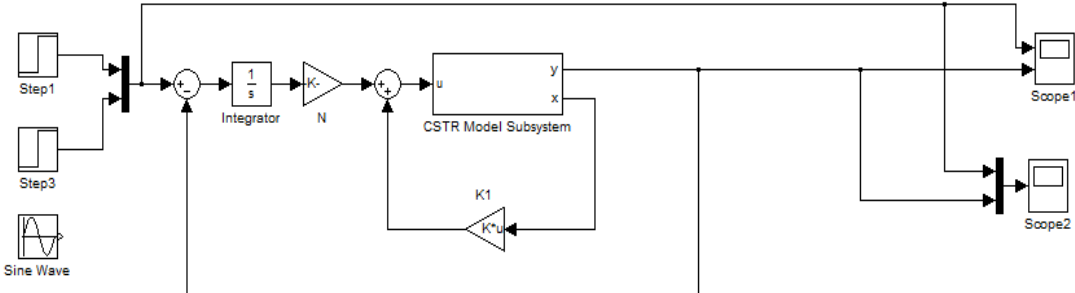


Figure 12: CSTR System with Tracking Controller

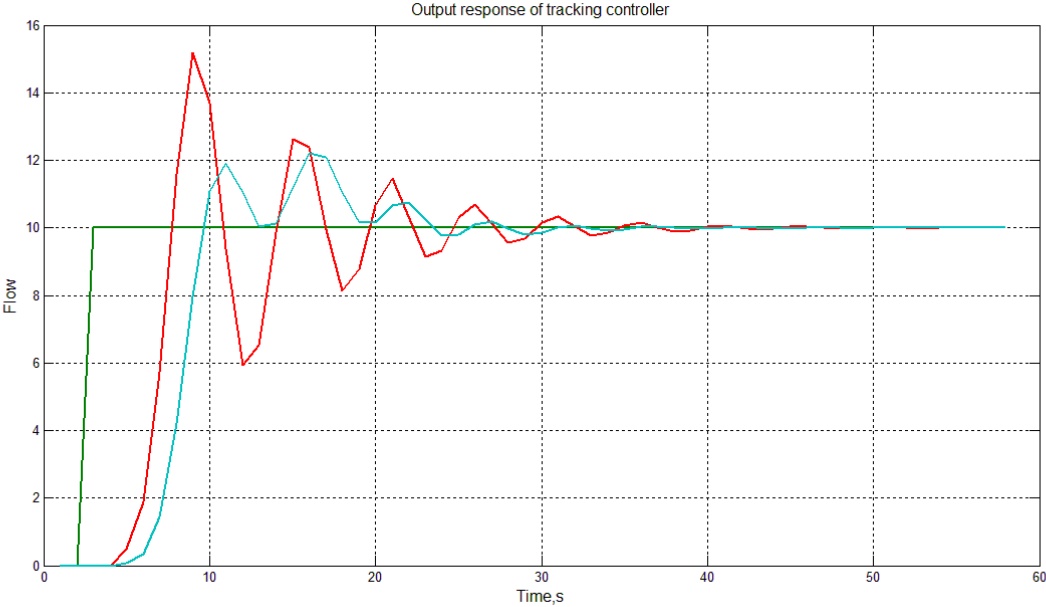


Figure 13: Output Response of Tracking Controller

Table 6: Tracking Controller Performance

Parameters	Output 1	Output 2
Offset	0.0	0.0
Rise Time, s	7.6	9.8
Settling time, s	40.2	40.1
Overshoot, %	54.0	18.0

Referring to the result from **Figure 13**, both output response are successfully bringing back to the set point and reaching the steady state condition. As the controlled output are the output flow rate and the concentration of reactant that required be maintained at the desired value while manipulating the supply rate of reactant and catalyst, the tracking controller must be able to alter with any disturbance in the system.

Even though, controlling using tracking controller results slow rise time and settling time compared to the CSTR system with full state feedback-feedforward controller, it give a better result in term of steady state error. Since, the main controlled objective is to maintain the both outputs at the desired set point, CSTR system with tracking controller provide the better result.

4.5. Observer Design for Continuous Stirred Tank Reactor (CSTR)

4.5.1. Full Order State Observer

A state observer is described as a system that models a real system in order to provide an estimate of its internal state with the given measurements of the input and output of the real system [9].

4.5.1.1. Determine full order state observer gain, L

$$[\lambda I - (A - LC)] = 0 \quad (4.31)$$

$$\begin{aligned} A - LC &= \begin{bmatrix} -0.23155847 & 0.22728837 \\ 0.22728837 & -0.2449193 \end{bmatrix} - \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} -0.23155847 & 0.22728837 \\ 0.22728837 & -0.2449193 \end{bmatrix} - \begin{bmatrix} 0 & l_1 \\ 0 & l_2 \end{bmatrix} \\ &= \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} -0.23155847 & 0.22728837 \\ 0.22728837 & -0.2449193 \end{bmatrix} - \begin{bmatrix} 0 & l_1 \\ 0 & l_2 \end{bmatrix} = 0 \\ &= \begin{bmatrix} \lambda - 0.23155847 & -(0.22728837 + l_1) \\ -0.22728837 & \lambda + 0.2449193 + l_2 \end{bmatrix} = 0 \\ &= \lambda^2 + (0.013361 + l_2)\lambda + (-0.108373 + 0.227288l_1 - 0.231558l_2) \quad (4.32) \end{aligned}$$

Desired poles by assuming greater than plant poles; -3.0 and -0.5

$$\begin{aligned} (\lambda + 3.0)(\lambda + 0.5) &= 0 \\ \lambda^2 + 3.5\lambda + 1.5 &= 0 \quad (4.33) \end{aligned}$$

Compare with calculated poles results;

$$\begin{aligned} 0.013361 + l_2 &= 3.5 \\ l_2 &= 3.486639 \quad (4.34) \end{aligned}$$

$$\begin{aligned} -0.108373 + 0.227288l_1 - 0.231558(3.486639) &= 0 \\ l_1 &= 4.028951 \quad (4.35) \end{aligned}$$

$$\text{Therefore, full-state observer gain is } \begin{bmatrix} 4.028951 \\ 3.486639 \end{bmatrix} \quad (4.36)$$

4.5.1.2. Designing full order state observer

The results shown in previous graphs are already meet the expectation however it is not a valid assumption to declare the design controllers are good enough to be implemented. To compensate for this, a full-state observer is designed to estimate those states that not measured. The block diagram in shown **Figure 14** describes the design of the full state observer using Simulink. Step inputs were applied and the respond was observed in the scope of **Figure 15**.

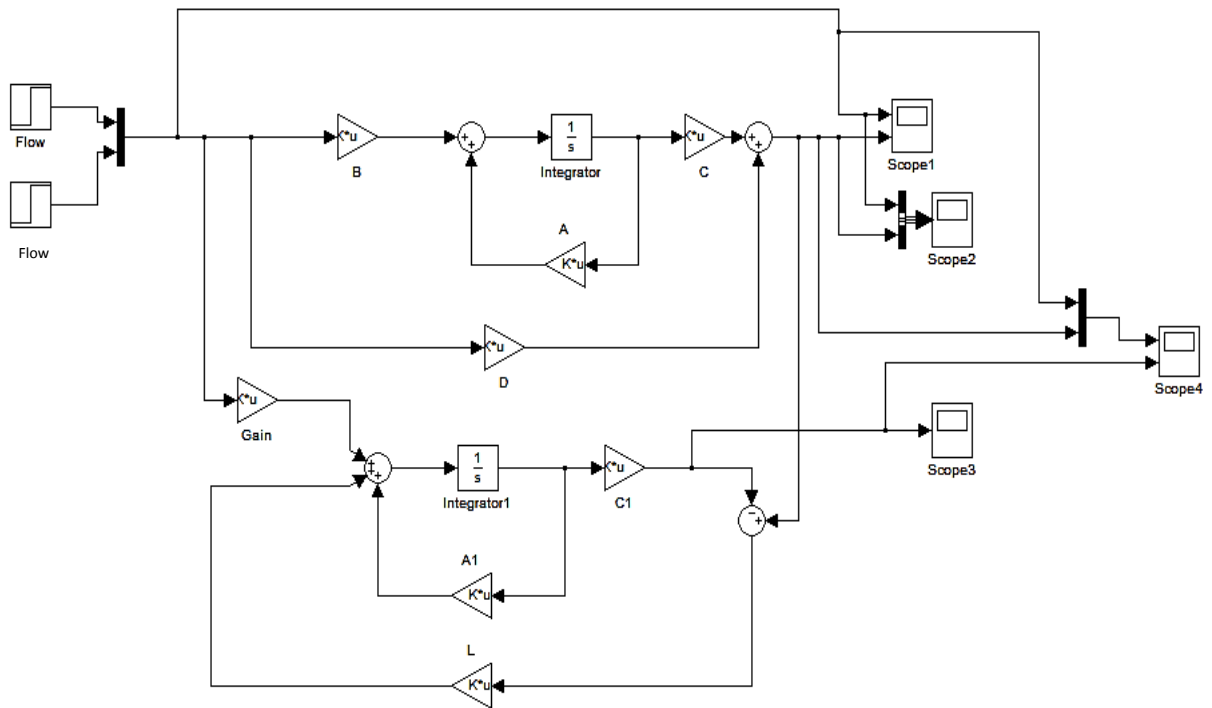


Figure 14: Full-State Observer

The results shown in **Figure 15** indicated that the observers give almost the same graph pattern with slightly different between the state variable and the output. This is probably due to the disturbance or noise of the system because when the desired poles are further onto the left plane, the bandwidth of the system increases whereby it becomes more sensitive to noise and disturbances.

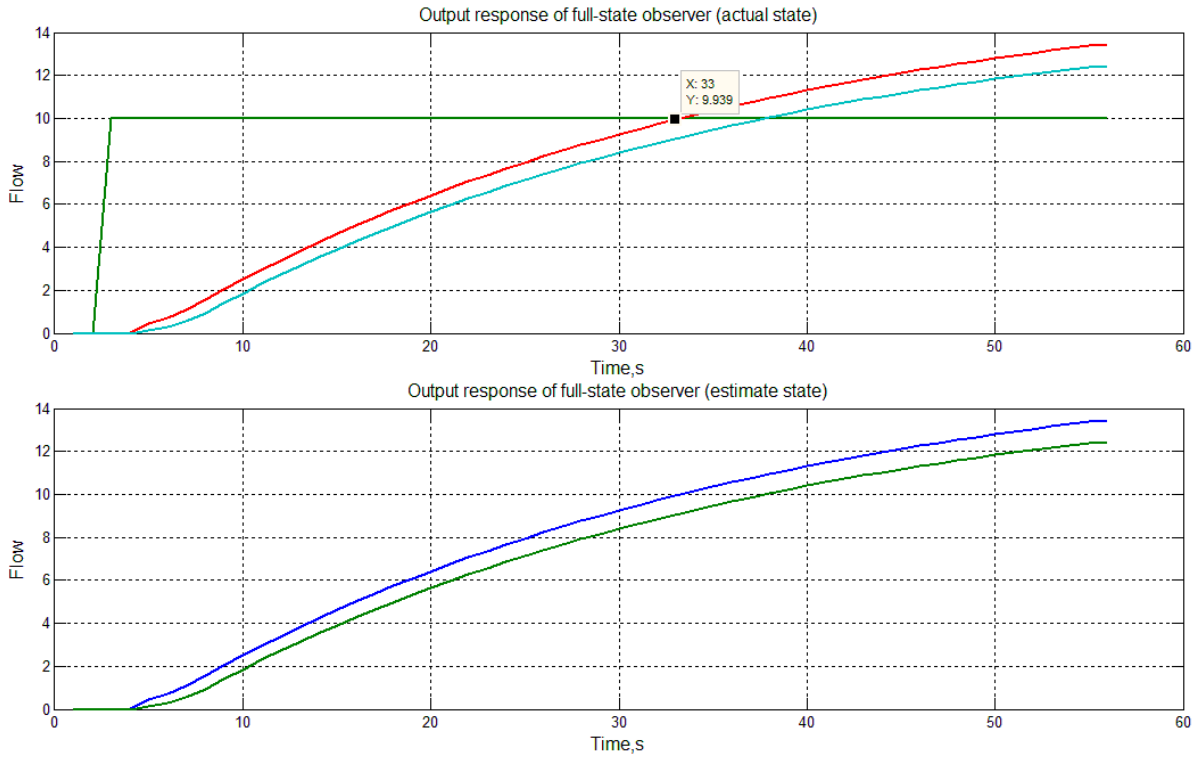


Figure 15: Output Response of Full-State Observer

4.5.2. *Reduced Order Observer*

The reduced order observer is required to preserve the steady-state gain matrix of the observed system and to minimize the transient estimation error [27]. The problem of designing a reduced order observer for a system subjected to unknown disturbance and deterministic input. The estimation error is independent of the deterministic error if and only if a full-state observer is used [23]. However, the observer structure property is not preserved in the standard reduced order estimation [27]. The designing of a reduced order observer helped to eliminate the steady-state error of the system, subjected to colored noises and deterministic input was solved.

4.5.2.1. *Determine reduced order observer gain, L*

Following approach is used to find the reduced order observer gain, L

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u \quad (4.37)$$

$$y_2 = [0 \quad 1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (4.38)$$

$$\dot{x}_1 = A_{11}x_1 + A_{12}x_2 + B_1u \quad (4.39)$$

$$\dot{x}_2 = A_{21}x_1 + A_{22}x_2 + B_2u$$

$$y_2 = x_2 \quad (4.40)$$

The state x_2 is measurable and the design is to estimate the state, x_1 .

For measured portion, the idea is that the observer can only observe the unmeasured state x_1 , hence, the derivative of x_2 is replaced with new state equation, \hat{w} as shown,

$$\dot{x}_2 = A_{21}x_1 + A_{22}x_2 + B_2u \quad (4.41)$$

$$\hat{w} = A_{22}x_2 \quad (4.42)$$

$$w = \dot{x}_2 - A_{21}x_1 - A_{22}x_2 - B_2u \quad (4.43)$$

$$w = y - A_{21}x_1 - A_{22}x_2 - B_2u \quad (4.44)$$

For unmeasured portion, unknown state is replaced by estimate and a corrective term is added to be multiplied by observer gain, L.

$$\dot{x}_1 = A_{11}x_1 + A_{12}x_2 + B_1u$$

$$\dot{\hat{x}}_1 = A_{11}x_1 + A_{12}x_2 + B_1u + L[\dot{y} - A_{21}\hat{x}_1 - B_2u - A_{22}x_2] \quad (4.45)$$

Rearrange,

$$(\dot{\hat{x}}_1 - L\dot{y}) = A_{11}x_1 + A_{12}x_2 + B_1u - LA_{21}\hat{x}_1 - LB_2u - LA_{22}x_2$$

$$(\dot{\hat{x}}_1 - L\dot{y}) = (A_{11}x_1 - LA_{21})\hat{x}_1 + (A_{12}x_2 - LA_{22})x_2 + B_1u - LB_2u \quad (4.46)$$

Substitute the value of matrix A into the equation

$$(\dot{\hat{x}}_1 - L\dot{y}) = (-0.231558 - 0.227288L)\hat{x}_1 + (0.227288 + 0.244919L)x_2 + 0.03125u$$

The eigenvalue for the observer is placed at -2.5, so that

$$-0.231558 - 0.227288L = -2.5$$

$$L = 9.980474112 \quad (4.47)$$

4.5.2.2. Designing reduced order observer

Later, the observer was only studied using reduced order observer which observe the unmeasured state. The reduced order observer was developed in Simulink as in **Figure 16**. Step inputs were applied and the respond was observed in the scope.

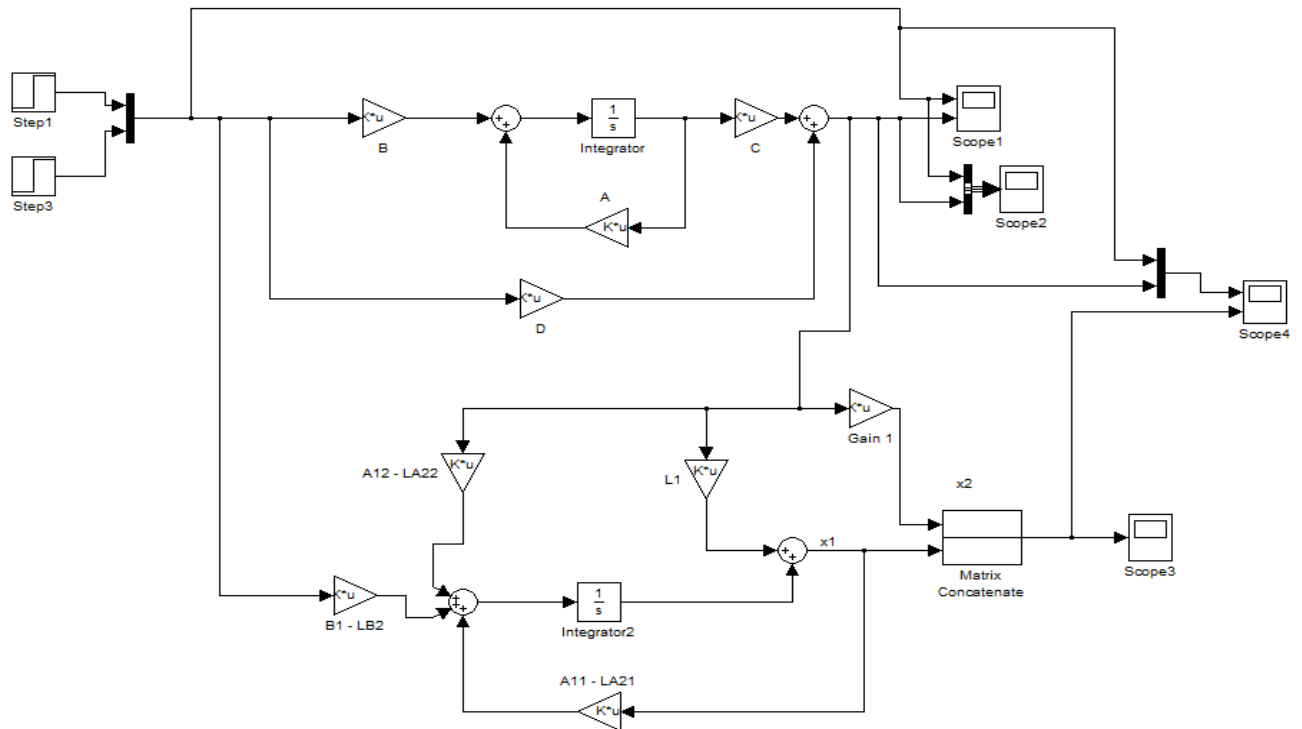


Figure 16: Reduced Order Observer

The reduced order observer can estimate the immeasurable states, and direct feedback path can be used to obtain the measured state value. The results shown in **Figure 17** indicated that the observer give almost the same graph pattern that prove the working observer. From the graph, the reduced order observer had shown it able to estimate the immeasurable states.

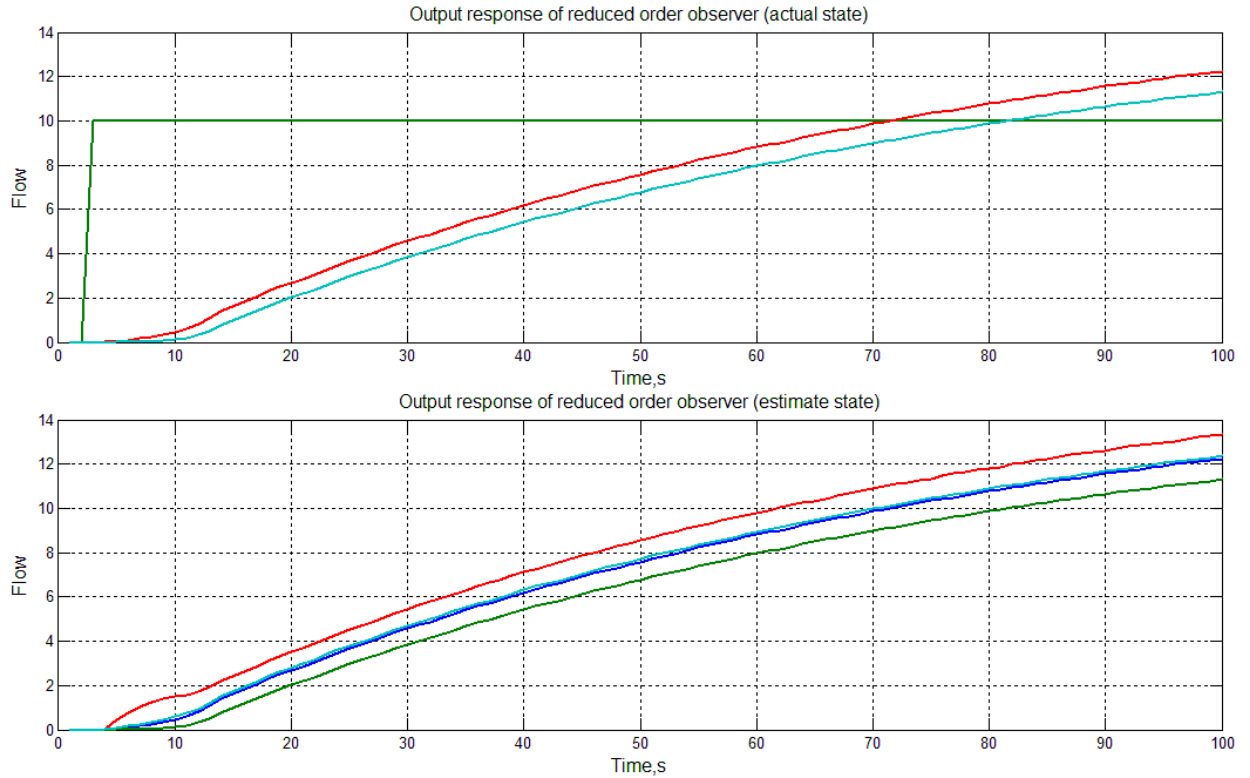


Figure 17: Output Response of Reduced Order Observer

4.6. Comparison of Full State Observer and Reduced Order Observer

Next, both types of observers are tested to monitor the output response when CSTR system is integrated with full state feedback feedforward controller. The objective of this part is to evaluate on the performance of each observer when the control action be implemented into CSTR system. In order to classify as good observer, output response from the observer must results closely enough to the output response obtained by the controller.

Figure 18 and 19 showed the block diagram for full state feedback-feedforward with full state observer and with reduced order observer.

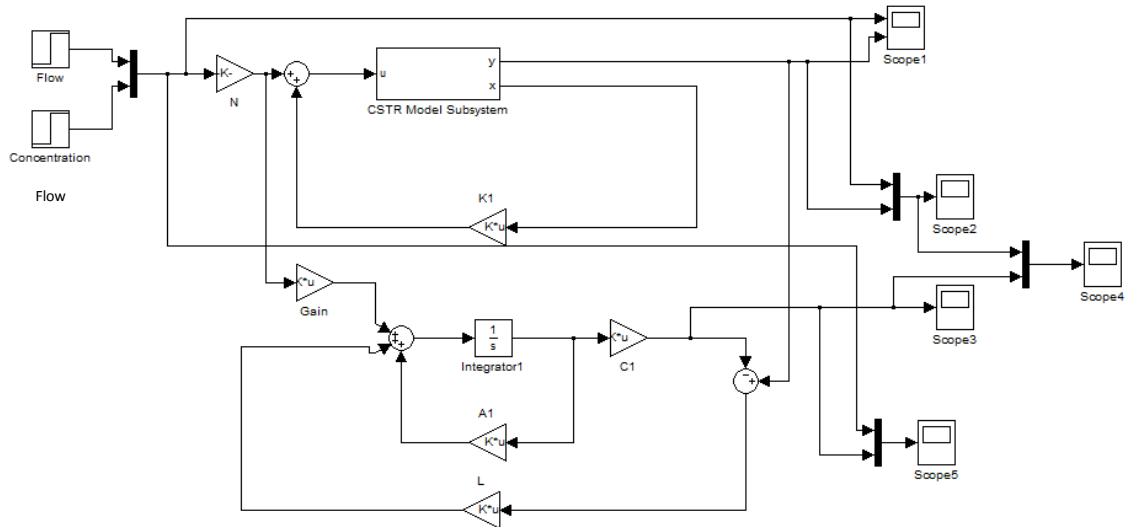


Figure 18: Full-State Feedback-Feedforward Controller with Full State Observer

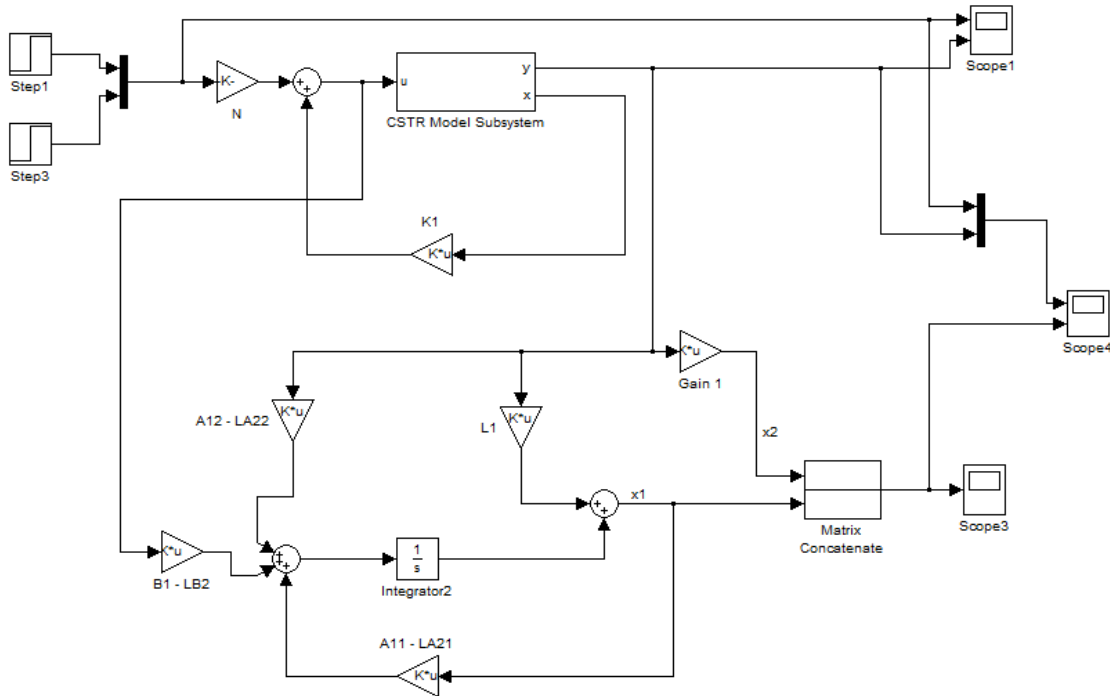


Figure 19: Full-State Feedback-Feedforward Controller with Reduced Order Observer

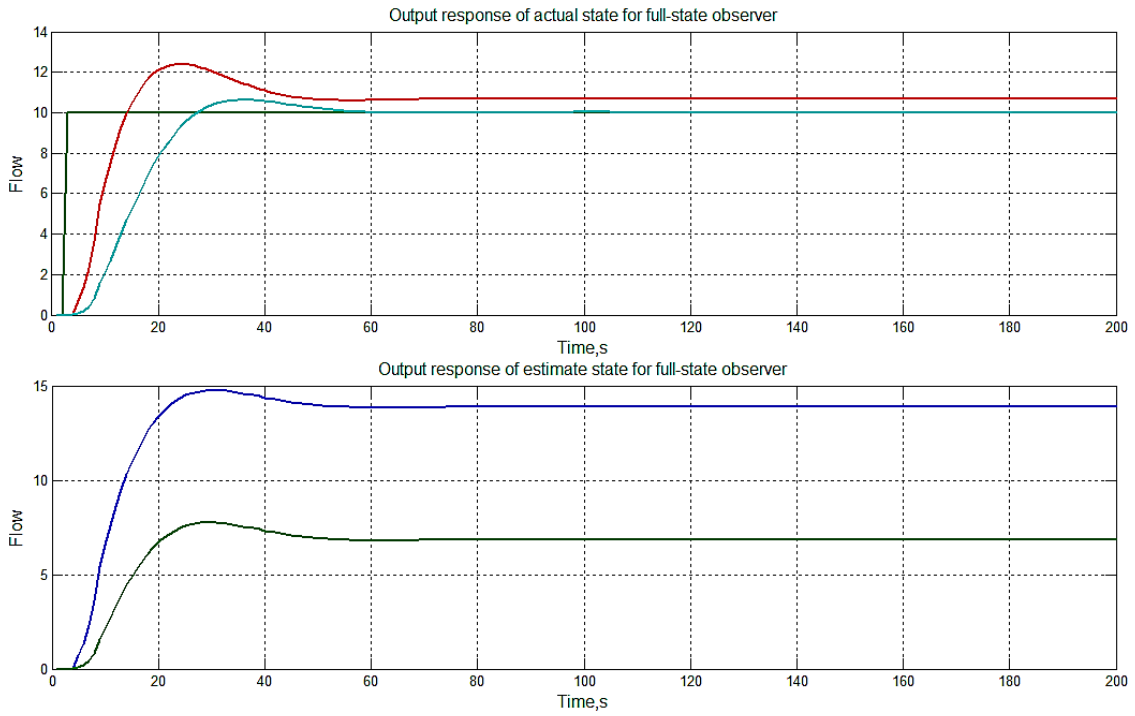


Figure 20: Output Response of Full-State Observer

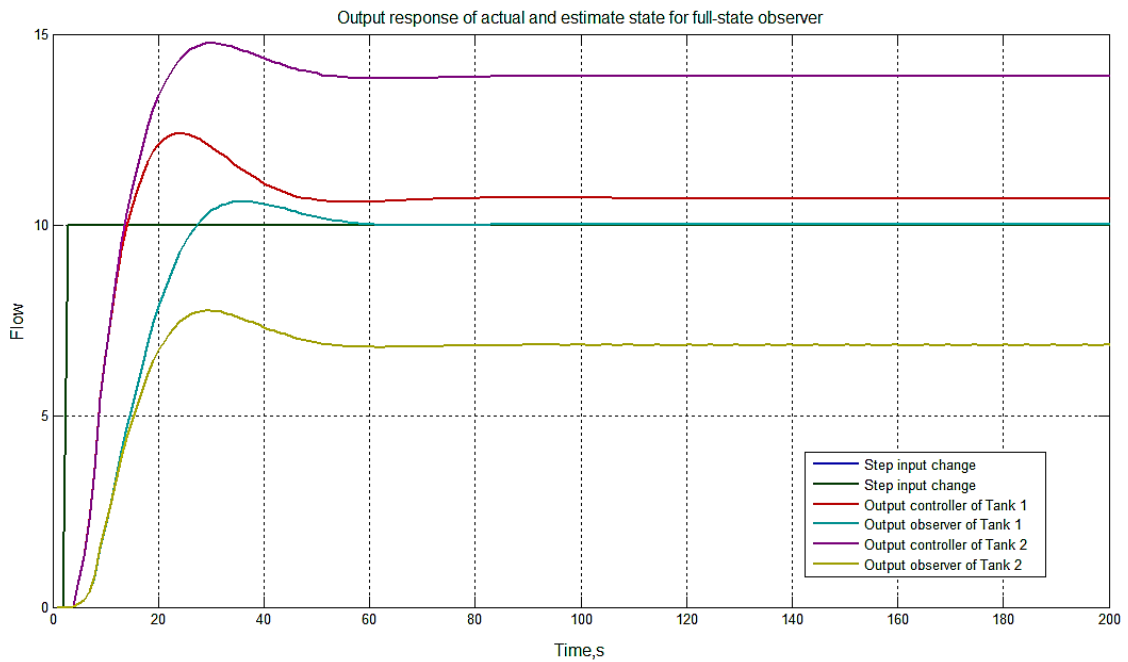


Figure 21 : Output Response of Estimated State for Full-State Observer

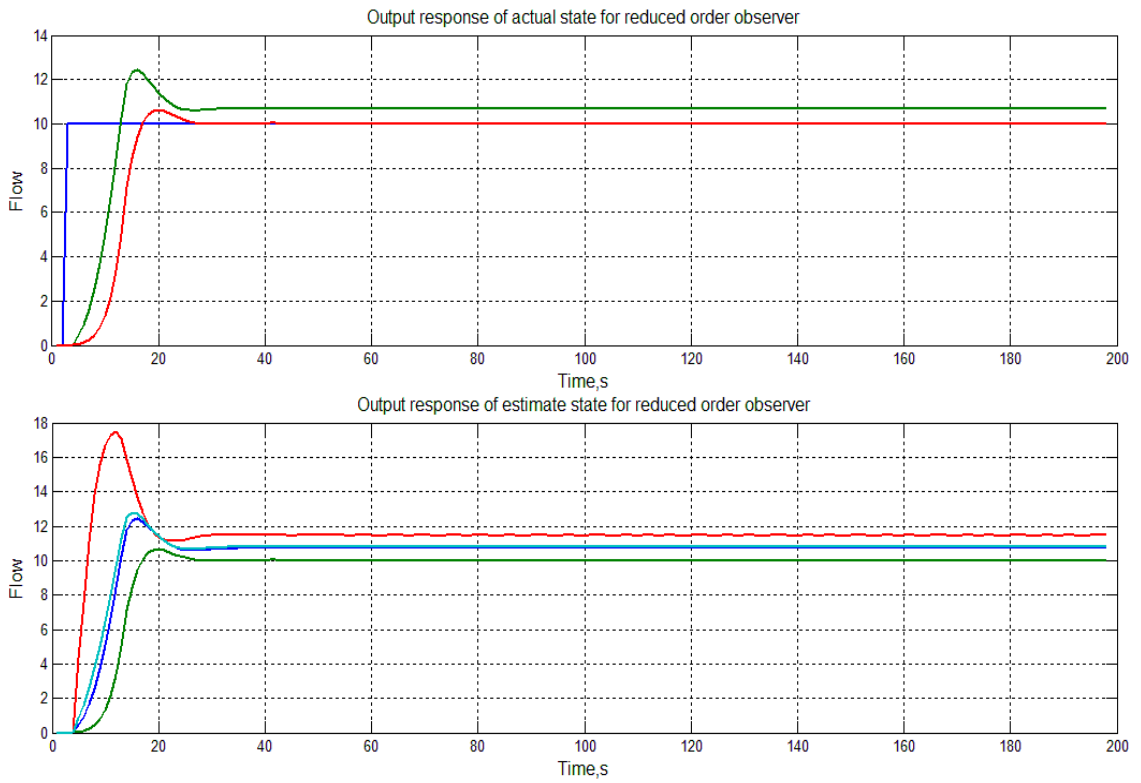


Figure 23: Output Response of Reduced Order Observer

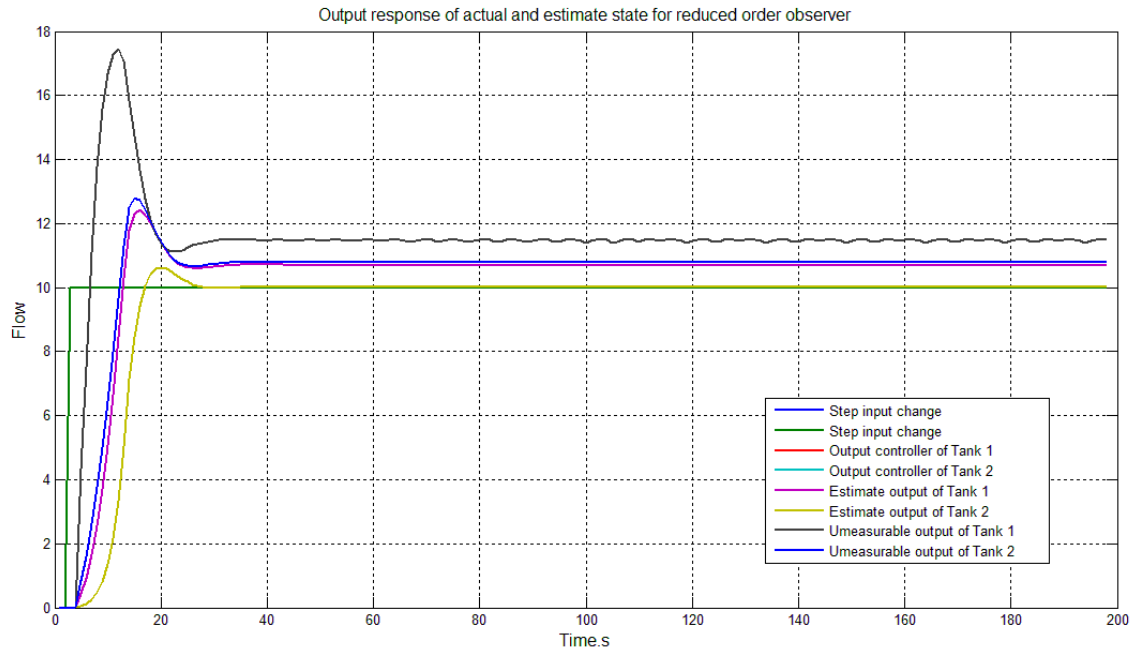


Figure 22: Output Response of Actual and Estimated State of Reduced Order Observer

4.7. Controller Performance Evaluation

4.7.1. Controlling CSTR with Existence of Disturbance

A similar system of CSTR is constructed, but with the exception that an observer and controller are included in the system. The block diagram in shown **Figure 24** and **26** are constructed to demonstrate the closed loop system with disturbance.

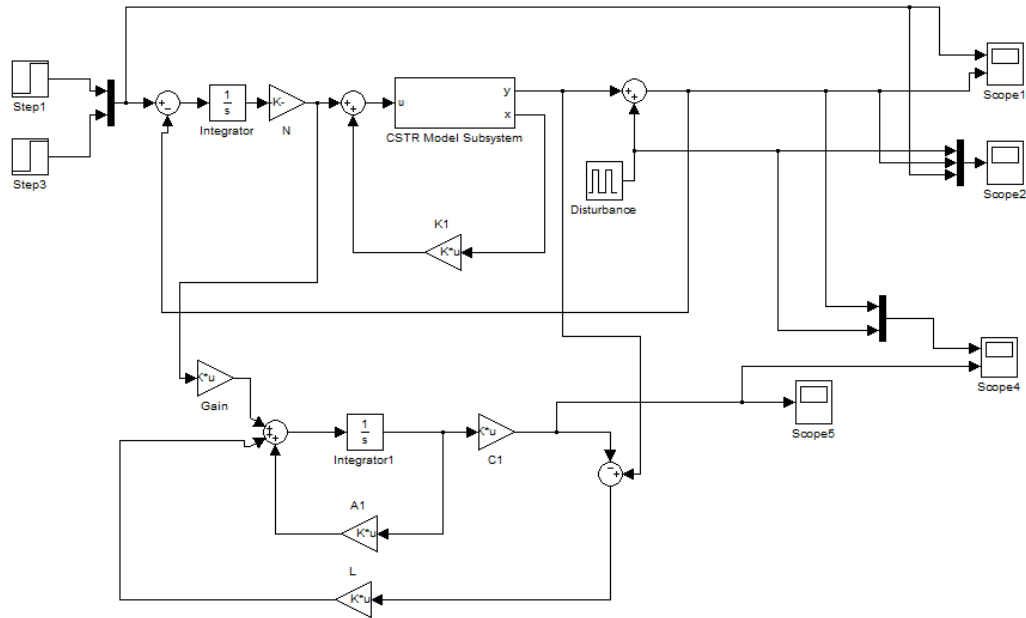


Figure 24: CSTR system with Tracking Controller and Full State Observer

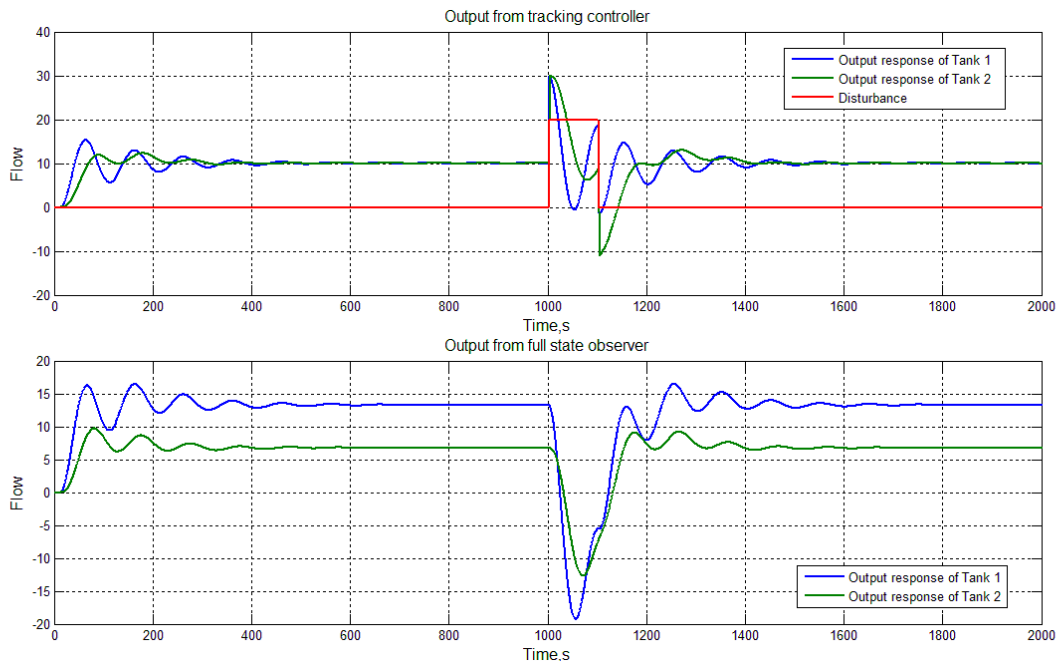


Figure 25: Output Response When Disturbance Is Applied To the System

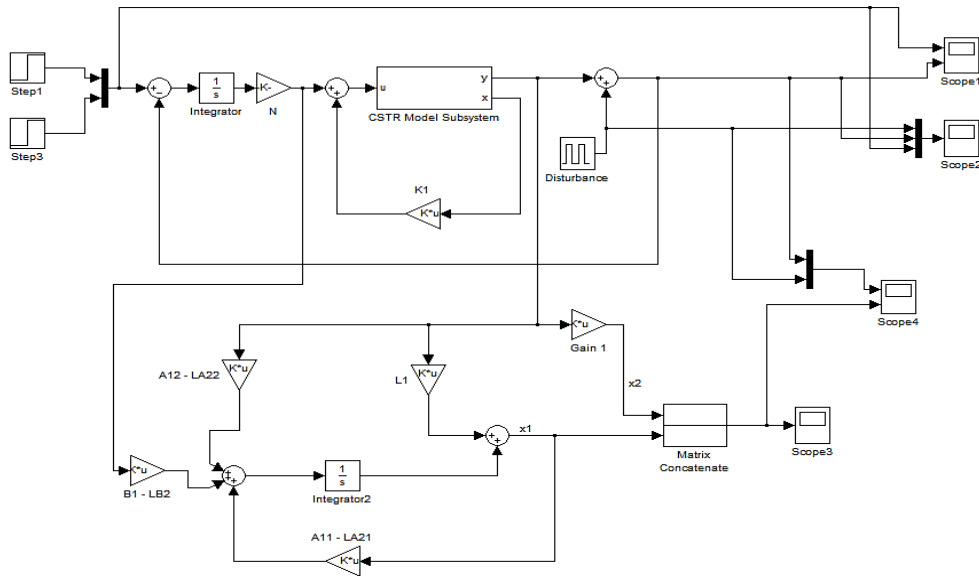


Figure 26: CSTR System with Tracking Controller and Reduced Order Observer

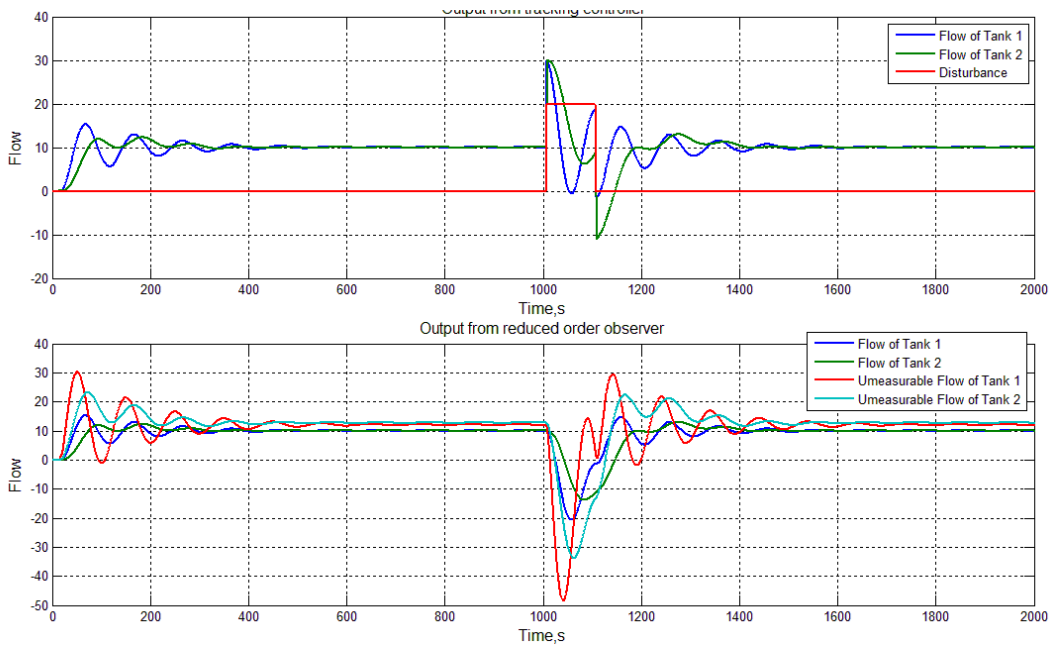


Figure 27: Output Response When Disturbance Is Applied To the System

The output of Scope is shown in **Figure 25** (when the full state observer is used in the system) and **Figure 27**(when the reduced order observer is used instead). The output shows the effects of the controller and observer on the system output during the presence of the disturbance. When the disturbance is added to the system from 1000th to 1500th seconds, the controller and observer successfully in bringing the system output value back to 10 which is the set point value, which the presence of slight offset.

4.7.2. Comparison with PID Controller

The objective of designing PID controller is to make performances comparison with state space controller. **Figure 26** showed the block diagram of CSTR with PID controller whereby **Figure 27** showed the output response of the system. Since the PID controller able to control single-input single-output system (SISO), only single step input is used and derivation of CSTR system transfer function been placed as the plant model.

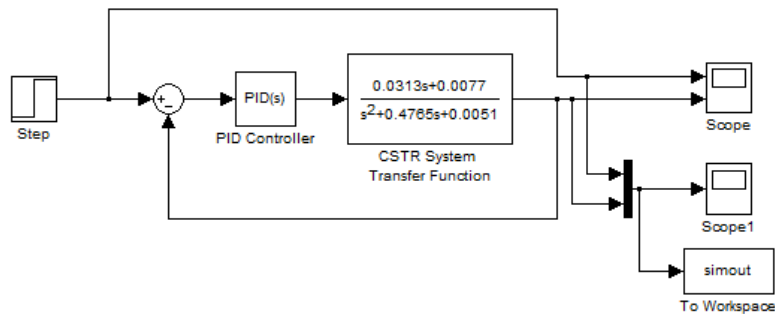


Figure 26: CSTR system with PID Controller

In the system, the product flow rate is being controlled to keep it maintain at desired set point while manipulating the flow rate of reactant and catalyst into the tank. Controlling using PID controller provides limitation of process control where are only single output can be control, it means the concentration of reactant need to be neglected.

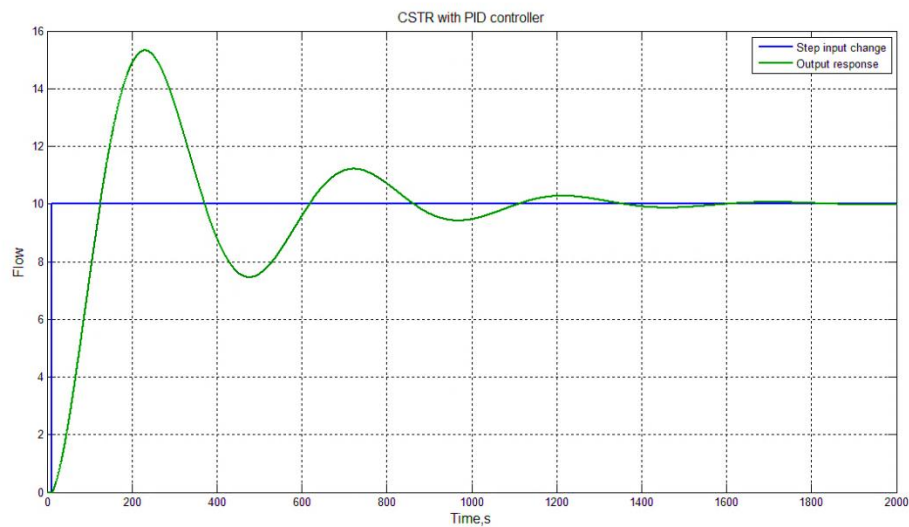


Figure 27: Output Response of PID Controller

Figure 28 shown the performance comparison of tracking controller with PID controller

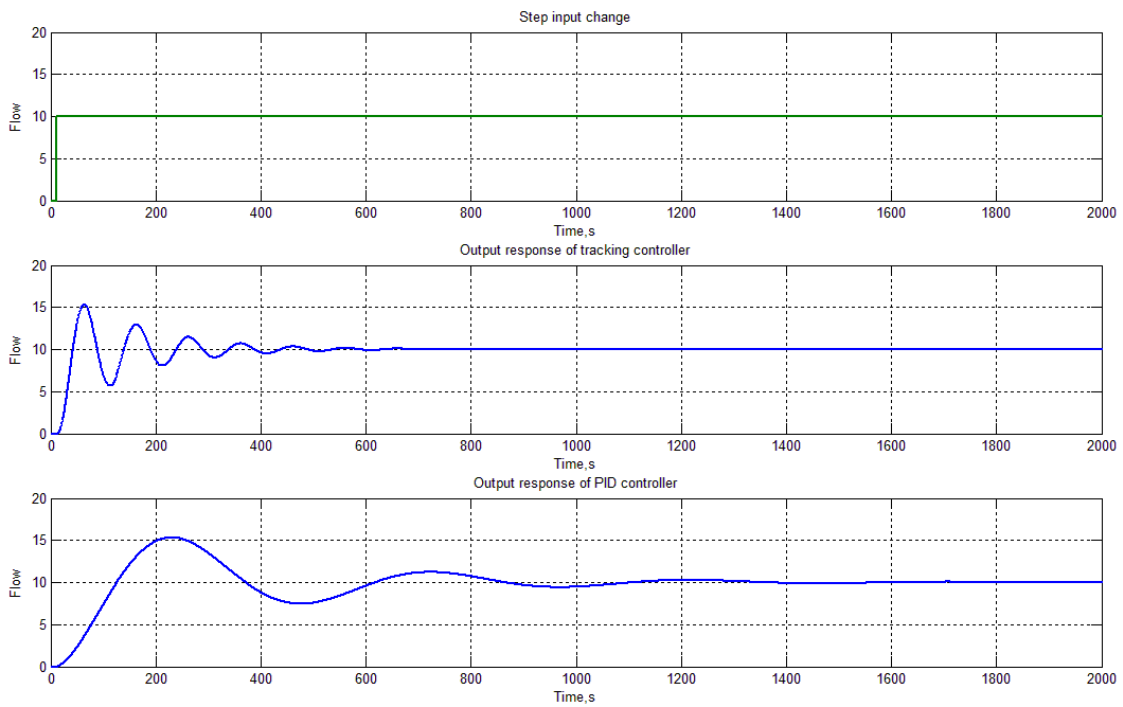


Figure 28: Output Response of CSTR with Tracking Controller and PID Controller

Table 7: Comparison of Tracking Controller And PID Controller Performance

Parameters	Tracking Controller	PID Controller
Offset	0	0
Rise Time, s	68	175
Settling time, s	650	1600
Overshoot, %	50	60

The result from the graph indicated that controlling CSTR via tracking controller results in faster rise time and settling time compared to PID controller where it is required long time in maintaining the stability and yield higher response time. The PID controller does not ensure the stability of the process and incapable suppressing the influence of external disturbances. In comparison, a good controller required faster rise time, settling time as well as smaller percentage overshoot; therefore tracking controller meets all requirements to be classified as a good controller with better controllability and stability.

CHAPTER 5

CONCLUSION AND RECOMMENDATIONS

This chapter concludes the entire project and proposes several recommendations which could improve the outcome of the project.

5.1. Conclusion

It is significant to precede the project as it proven that the conventional controller has several disadvantages and can be replaced with new control strategy via modern control theory. In this project, continuous stirred tank reactor (CSTR) system was successfully modeled on Simulink from through study on the mathematical modeling. Details study on the process in particular modeling and analysis of the system enhance the understanding on way to improve the control strategies of CSTR process. Controller and observer were also effectively designed using pole placement method and implemented in second order system, producing promising results that indicate the practicality of modern control in plant process control system. The success of this project signifies that an alternative to the current implementation of plant process control system can be made possible with the design of new controller and observer strategies that are robust, optimal and adaptive via modern control approach.

This project has achieved its objective that is to design the controller and observer by using the state space approach and performed analysis in second order system. This has been proven in the simulation results obtained using MATLAB and Simulink. In conclusion, with the accomplishment of theoretically implementing the concepts of modern control engineering in plant process control systems, this project is a **success**.

5.2. Suggested Future Work

Despite the overall objectives of the project achieved, there are several recommendations which could be considered in order to improve the project outcome as such:

- Implementation of plant modeling to obtain the parameters
 - Currently, the parameters of CSTR system are based on mathematical modeling and reference from standard parameters of CSTR. However, using the real plant modeling promise more complex design but with better output.
- Simulate using the transfer function of actual disturbances
 - The disturbance in the CSTR system was simulated using only a pulse generator, not an actual disturbance. Using the transfer function of actual disturbances will result in more realistic simulation.
- Implementation of controller and observer on actual plant
 - Currently, the real plant implementation of designed unable to be proceed due to unavailability of the CSTR plant. The promising results from the simulation only to one direction; implement the controller and observer on the actual plant increase its effectiveness as an alternative control strategy.
- A through study on disturbance and noise in the system
 - The bandwidth of the system increases as desired poles are further onto the left hand side of the plane. This caused the system to be more sensitive to noise and disturbance that can alter the output of the system. This problem can be encountered by having a thorough study on it.

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APPENDICES

APPENDIX A

```
>> % Determine transfer function of CSTR in series with interacting tank
```

```
>> A=[-2.7586 0.2273;0.2273 -0.2449];
```

```
>> B=[0.0313 0;0 0.0313];
```

```
>> C=[1 0;0 1];
```

```
>> D=[0 0;0 0];
```

```
>> [num,den]=ss2tf(A,B,C,D,1)
```

```
num =
```

```
0 0.0313 0.0077
0 0 0.0071
```

```
den =
```

```
1.0000 3.0035 0.6239
```

```
>> [num,den]=ss2tf(A,B,C,D,2)
```

```
num =
```

```
0 0 0.0071
0 0.0313 0.0863
```

```
den =
```

```
1.0000 3.0035 0.6239
```

```
>> % Determine transfer function of CSTR in series with non-interacting tank
```

```
>> A=[-0.0165 0;0 -0.0176];
```

```
>> B=[0.0313 0;0 0.0313];
```

```
>> C=[1 0;0 1];
```

```
>> D=[0 0;0 0];
```

```
>> [num,den]=ss2tf(A,B,C,D,1)
```

```
num =
```

```
0 0.0313 0.0006  
0 0 0
```

```
den =
```

```
1.0000 0.0341 0.0003
```

```
>> [num,den]=ss2tf(A,B,C,D,2)
```

```
num =
```

```
0 0 0  
0 0.0313 0.0005
```

```
den =
```

```
1.0000 0.0341 0.000
```

APPENDIX B

% Computing gains using robust pole placement for CSTR model

```
>> A=[-2.7586 0.2273;0.2273 -0.2449];
```

```
>> B=[0.0313 0;0 0.0313];
```

```
>> C=[1 0;0 1];
```

```
>> D=[0 0;0 0];
```


```
>> J=[-10 -5];
```

```
>> K=place(A,B,J)
```


K =

```
231.3546    7.2620
 7.2620    151.9201
```



APPENDIX C POSTER PRESENTATION



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ElectrEx
Engineering Excellence



STUDENT SOCIETY

Development of Controller for Continuous Stirred Tank Reactor (CSTR) via State Space Approach

1 INTRODUCTION

- The work describes the designing, simulation and analysis of controller for a continuous stirred tank reactor via state space approach.
- The project is looking at modern control approach for plant control which is expected to be better in terms of the system's controllability and stability.
- Results from the project indicate the practicality of modern control in plant process control system.

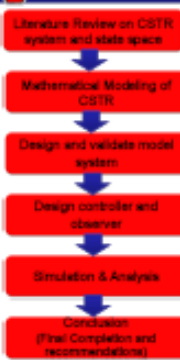
2 PROBLEM STATEMENT

- Today's industries are focusing only on stability in plant control system. Other qualities such as robustness, optimality and adaptively are overlooked.
- The modeling of plant control systems using classical control approach is mostly carried out only using first-order state space equation, providing only limited capability in modern plant control.
- Conventional control possesses several drawbacks. Therefore, modern control is used as alternative for control strategies.

3 OBJECTIVES

- To study the control system of continuous stirred tank reactor (CSTR) in second order state space representation.
- To design a controller and observer for CSTR and apply the concept of modern control engineering in the analysis.

4 METHODOLOGY



5 MATHEMATICAL MODELING

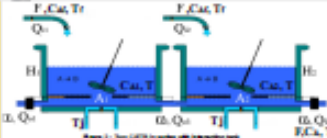


Figure 1: Two CSTR in series with interacting tank

Parameter	Value
V_1, V_2 (m ³)	1
K_1, K_2 (min ⁻¹)	1000
K_3, K_4 (min ⁻¹)	10000
ρ_1, ρ_2 (kg/m ³)	800
C_{A1}, C_{A2} (mol/m ³)	800
T_1, T_2 (K)	400
T_1, T_2 (K)	400

Table 1: Reactor parameter values

$$\frac{dx}{dt} = \begin{bmatrix} \frac{1}{V} (F_1 C_{A1} - F_2 C_{A2}) - K_1 e^{-T_1/T} C_{A1} \\ \frac{1}{V} (F_2 C_{A2} - F_3 C_{A3}) - K_2 e^{-T_2/T} C_{A2} \end{bmatrix} + \begin{bmatrix} C_0 - C_{A1} \\ T_0 - T_1 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{V \rho} (F_3 - F_2) \end{bmatrix} \begin{bmatrix} T_2 - T_1 \end{bmatrix}$$

The simplified transfer function of the process given as

$$Gp1(s) = \frac{2.7s + 2.742}{s^2 + 1.33s + 0.4966}$$

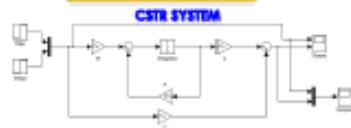


Figure 2: CSTR model's block

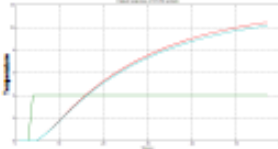


Figure 3: Step response for one stage

6 RESULT & DISCUSSION

FULL STATE FEEDBACK PREDFORWARD CONTROLLER

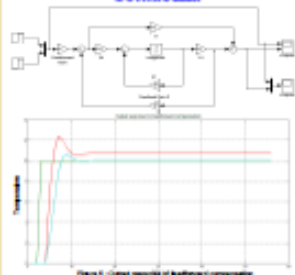


Figure 4: Step response of full state controller

FULL STATE OBSERVER

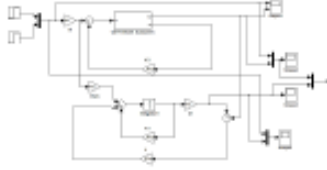


Figure 5: Step response of full state observer

REDUCED ORDER OBSERVER




Figure 6: Step response of reduced order observer

TRACKING CONTROLLER

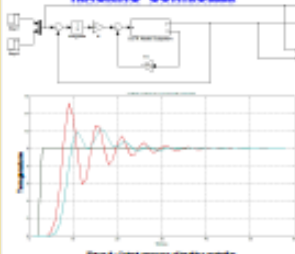


Figure 7: Step response of tracking controller

- The control action is formed to alleviate the impact of the changing disturbance and keep temperature and volume at the desired set point. The only manipulated variable is the coolant temperature.
- The control objectives are to ensure the stability of the process, suppressing the influence of external disturbances and optimizing the economic performance of a plant.

7 CONCLUSION

The success of the project signifies that an alternative to the current implementation of plant process control system can be made possible with the design of a new controller and observer strategy that are robust, optimal and adaptive via modern control approach.

Name : Mohamad Faez Bin Ahmad Adnan
ID Number : 11385
Supervisor : Assoc. Professor Dr. Nordin Bin Saad
Dr. Radzuan B. Razali

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APPENDIX D
TECHNICAL PAPER