

# **DEVELOPING A MATHEMATICAL MODEL FOR HYDRAULIC FRACTURING IN MICROSOFT EXCEL**

by

**NGUYEN THE THAO**

**Dissertation submitted in partial fulfilment of**

**the requirements for the**

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**Universiti Teknologi PETRONAS**

**Bandar Seri Iskandar**

**31750 Tronoh**

**Perak Darul Ridzuan**

**CERTIFICATION OF APPROVAL**

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**Approved by,**

---

**(Name of Main Supervisor)**

**UNIVERSITI TEKNOLOGI PETRONAS**

**TRONOH, PERAK**

**December 2012**

## **CERTIFICATION OF ORIGINALITY**

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

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**NGUYEN THE THAO**

## **ABSTRACT**

Many computer software programs have been developed to assist petroleum engineers and scientists in designing hydraulic fractures. These programs use analytical, numerical or empirical methods (or a combination of two or all three methods) to model fracture propagation in the reservoir. The user – which is usually experienced petroleum professional – provides the necessary input parameters to the model. These input parameters include reservoir characteristics, fluid and proppant characteristics. Usually among the major outputs of these models are the fracture geometry and conductivity.

The goal of this study is to provide a Mathematical Model of Hydraulic Fracturing Design that allows the engineer to solve the inverse problem in a fast and efficient manner. Using the model developed in this study the engineer identifies the desired fracture geometry that she/he would like to achieve and enters this value along with the reservoir characteristics into the intelligent software tool. The software tool would then solve the inverse problem and provides the engineer with fluid, proppant that would produce the desired fracture in that particular reservoir and well.

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## **ABBREVIATIONS**

FYP: Final Year Project

ME: Microsoft Excel

HFD: Hydraulic Fracture Design

PR: Production Ratio

# CHAPTER I

## INTRODUCTION

### 1.1 Background of Study

Hydraulic fracturing is a well stimulation process used to maximize the extraction of underground resources; including oil, natural gas, geothermal energy, and even water. The oil and gas industry uses hydraulic fracturing to enhance subsurface fracture systems to allow oil or natural gas to move more freely from the rock pores to production wells that bring the oil or gas to the surface.

The process of hydraulic fracturing begins with building the necessary site infrastructure including well construction. Production wells may be drilled in the vertical direction only or paired with horizontal or directional sections. Vertical well sections may be drilled hundreds to thousands of feet below the land surface and lateral sections may extend 1000 to 6000 feet away from the well.

Fluids, commonly made up of water and chemical additives, are pumped into a geologic formation at high pressure during hydraulic fracturing. When the pressure exceeds the rock strength, the fluids open or enlarge fractures that can extend several hundred feet away from the well. After the fractures are created, a propping agent is pumped into the fractures to keep them from closing when the pumping pressure is released. After fracturing is completed, the internal pressure of the geologic formation cause the injected fracturing fluids to rise to the surface where it may be stored in tanks or pits prior to disposal or recycling. Recovered fracturing fluids are referred to as flow-back. Disposal options for flow-back include discharge into surface water or underground injection.

## 1.2 Problem Statement

For the first issue, in the market a lot of software has developed to run hydraulic fracture design. These software were developed by several famous oil and gas companies, such as Halliburton, Schlumberger, Baker Hugh, BJ... but those softwares are not free, user have to pay for their license before installing and using them. And in other hand, those software need to run in a high performance system because of performing of high resolution and high contrast graphic.

With Microsoft Excel (ME), it is not only cheap for license but also friendly with user so ME will be a better candidate for user because almost everybody in the world who is using computer now knows how to use and control basic functions of Microsoft Office, especially ME. Therefore, developing mathematical model in Excel will very useful for student and engineering who is busy and does not have much time to learn using Hydraulic fracturing design. When this project is done, there is a free program to be applied for hydraulic fracturing design.

This is a second issue for this project; during studying and simulation of hydraulic fracturing to calculate for hydraulic fracturing, it will take more time for manual calculation and sometimes, the result of manual calculation is not corrected because of some user mistakes.

### **1.3 Objective**

The objective of this project is to develop a mathematical model for hydraulic fracturing designation by using Microsoft Excel (ME).

### **1.4 Scope of Study**

The scope of this project is studying how to design a mathematical model of hydraulic fracturing in Microsoft Excel environment. Author has to develop macro functions in Microsoft Excel using the formulas of Hydraulic Fracturing calculation and author creates necessary user form base in macro functions.

In the beginning, author needs to study and research calculations of hydraulic fracturing, what parameters and formulas need to use in Hydraulic Fracture Design (HFD). Author also need to understand the procedures of HFD to have an over view of software program. In the next step, author needs to learn how to create macro functions and user form in Microsoft Excel. In the final step, author will design a mathematical model using formulas, procedures of hydraulic fracturing program and create interface of software base on macro functions and user form.

## CHAPTER II

### LITERATURE REVIEW

#### 2.1 Summary of Previous Work In The Area of Hydraulic Fracture Design

*Ralph and Veatch (1986)* present the general concepts of hydraulic fracture treatments economics and introduce the net present value as a valuable tool for the optimal design of hydraulic fracture treatment. An optimal hydraulic fracture treatment design maximizes the net present value of the revenue after the treatment, considering the post-fracture production performance and the treatment costs.

*Poulsen and Soliman (1986)* used fluid volume and proppant concentration as treatment design variables, with a two dimensional fracture propagation model, accounting for proppant transport and sedimentation. No formal optimization procedure was used (trial and error), minimizing the difference between calculated and desired fracture length and conductivity.

*Balen et al. (1988)* used as design variables the fracturing fluid, injected fluid volume and proppant concentration, pumping rate, and proppant types. Their work used a two-dimensional fracture propagation model for predicting fracture geometry and an economic model. The optimization procedure was based on a sensitivity analysis of the design variables with respect to net present value.

*Hareland et al. (1993)* used fluid injection rate and fracturing fluid as design variables and a pseudo three-dimensional fracture propagation model coupled with a post-fracture production and economic models. The optimization procedure was similar to that used by *Balen et al. (1988)*.

*Rueda et al. (1994)* considered as treatment design variables the injected fluid volume, fracturing fluid type, proppant type, and pumping rate. Their work used a two-

dimensional fracture propagation model, accounting for fracture closure behavior, and a post-fracture production model coupled with an economic model. The optimization was posed as a mixed integer linear programming (MILP) problem and solved accordingly.

*Mohaghegh et al. (1999)* used as design variables the fluid volume injected, proppant concentration, and fluid injection rate. Their work used a surrogate model of a three-dimensional fracturing simulator accounting for fracture propagation and closure behavior, and proppant transport and sedimentation. The optimization procedure was a Genetic Algorithm.

The analysis of previous work shows limitations such as the absence of a global optimization procedure (*Poulsen and Soliman, 1986; Balen et al., 1988; Hareland et al., 1993; Rueda et al., 1994*), direct coupling of the hydraulic fracture models and optimization procedure (*Poulsen and Soliman, 1986; Balen et al., 1988; Hareland et al., 1993*), no error estimation (all the previous work), limited number of design variables (*Poulsen and Soliman, 1986; Hareland et al., 1993*), not account for fracture closure and proppant transport and sedimentation (*Poulsen and Soliman, 1986; Balen et al., 1988; Hareland et al., 1993*).

## **2.2 Fracture Design**

Engineering computations always precede a fracturing treatment. These consist of calculation of fluid volume and viscosity, injection rate, weight of proppant, volumes of different phases of the job (prepad, pad, slurry, and displacement), surface and bottomhole injection pressure, hydraulic horsepower required at the surface, and the mechanical equipment needed for this. A very important part of fracture design is determination of the fluid volume required to create a fracture with a given length. A hydraulic fracture is usually identified by three dimensions: length, width, and height (Fig. 5). Fracture length itself has two components: created and propped. Created length is the distance between the wellbore and farthest point into the formation. Propped length is the distance between the wellbore and farthest point where proppant has



travelled inside the fracture. Fracture width is the separation between the two faces of the fracture. Its value is largest at the wellbore and tapers toward the tip of the fracture. Fracture height is the distance between the top and bottom of the fracture. The relationships between these parameters and fracture design are as follows.

- a. Created fracture length influences total injected volume and fracture width. The longer the required fracture length, the larger the volume of fluid needed to create this length. Longer length also results in a wider fracture.
- b. Propped fracture length influences slurry volume, proppant weight, and production increase. Creating a longer propped fracture length requires injecting a larger amount/weight of sand. But this also results in a higher production increase.
- c. Fracture width depends on formation mechanical properties, fluid viscosity, and fracture dimensions. Higher-viscosity fluids create a wider fracture. The longer the fracture, the wider it will be at the wellbore.
- d. Fracture height is the big unknown and is usually assumed to be constant and related to formation thickness. The relationship between fracture width and other fracture dimensions has been a subject of debate among fracturing experts for many years. The two basic concepts are those of *Perkins and Kern (1961)*, who relate fracture width to its height, and *Khristianovic and Zheltov (1955)*, who express fracture width as a function of its length. Depending on the choice of basic equations, elaborate computations are needed to calculate various fracturing parameters as functions of injected fluid and formation properties. These are usually done using computer simulations, which are commercially available for this purpose.

### **2.3 Effecting factors for Hydraulic Fracturing**

Four factors control improvements in productivity (i.e., the productivity index) provided by hydraulic fracturing.

- a. Propped fracture area (sq ft). This is the area of the fracture adjacent to the porous interval that has been propped (length times height). All the fracture area adjacent to

the porous interval that is created may not be propped, and only the fracture area adjacent to the productive porosity that is propped is considered an effective area.

- b. Conductivity of the propped fracture (md-ft). This is a measurement of how well the propped fracture conducts the produced fluids. In addition to the effect of closure stress on the permeability of the proppant, factors such as embedment, proppant distribution, and resultant fracture width must be considered to determine the conductivity of the fracture at reservoir producing conditions.
- c. Reservoir permeability. This value is used to determine the fracture conductivity required to use the proposed fracture penetration effectively.
- d. Drainage radius. This value is used, as is reservoir permeability, to determine the length of fracture needed. A long fracture is needed if the well spacing is large and the reservoir permeability is low.

## **2.4 Requirements**

### **2.4.1 Hydraulic fluid requirement**

The usual hydraulic fluid requirement is for an oil phase material with viscosity between 50 and 150 centipoises or higher, depending on the individual job. It has been found that Napalm soap and similar soaps provide the desired characteristics. Napalm can be added to gasoline, kerosene or other refined petroleum cuts to produce gels having any desired viscosity up to considerably over 300 centipoises.

**Hydraulic Base Fluids:** The ideal fluid should be an oily one rather than a water base fluid, to avoid decreasing the permeability of the formation to oil or gas. This requirement is met by the Napalm gel being used. Water base fluids, however, would be advantageous in treating water wells, or water injection wells used in either the oil industry or industries where water is used in solution mining of salts, and in the Frasch process of mining sulphur. There is also some reason to believe that a water base fluid could be used successfully in oil or gas wells. In other words, it is entirely possible that the benefits derived from hydraulically fracturing the formation may be great enough to

overcome the decrease in permeability caused by water wetting of the oil or gas sand. It is known, for example, that limestone reservoirs can be acidized with an acid solution that is as high as 80 or 90 percent water, with a resulting increase in well productivity. Future work with this process may indicate that it is more economical to use a water base for the hydraulic fracturing fluids than the more expensive gasoline and crude oil base fluids, particularly in formations not appreciably contaminated with argillaceous materials.

**Sand Carrying Capacity of the Fluid:** It is often desirable that the hydraulic fluid should carry in suspension a sufficient amount of strong granular material such as sand to be used as a propping agent to keep the fracture from closing off after release of pressure. The high viscosity of the Napalm gels makes them well suited to transport such material.

#### **2.4.2 Pump requirement**

Field experience has shown that the successful Hydraulic fracturing treatment requires a definite fracturing of the formation, as indicated by a decrease in injection pressure; this decrease in pressure is clearly shown on several of the charts presented later.

Starting with well depth, fluid viscosity, formation thickness and permeability, and bottom hole pressure, it is possible to compute with fair accuracy, by an empirical method, the pump rate necessary to fracture a formation and to extend the fracture after it is made. It is also possible to determine the necessary fluid viscosity to fracture and to extend the fracture in the formation with a given pump rate

## 2.5 Calculation procedures

1. Set the required production ratio (PR). This number must be possible to compare with current production rate.
2. Select need data of formation and well should be using during calculation. Before selecting data, author has to do manual calculation of HFD.
3. Select the treatment fluid, proppant agent. Determine the maximum proppant concentration to be pumped. In this step, author need to understand properties of fluids and proppants before selecting.
4. Using the formation data and well properties, to determine fracture fluid coefficient, fracturing efficiency, then calculate fluid volume.
5. Using fracturing fluid and proppant calculated above author calculates for pumping rate of mixed fluids then calculating pressure required to inject into well.
6. Using calculated require pressure above, author calculates horsepower.
7. Using pressure of pump, type of sand, radius of formation author calculates production ratio and compares with required PR at the beginning. If PR does not reach required range, the calculation need to repeat from step 2.

## 2.6 Fracture Design Calculation

The objective of any type of fracture design calculation is to plan the most economical treatment that will result the desired increase in productivity. This type of calculation involves consideration of the following variables:

1. Fracture fluid coefficient,  $C$ .
2. Injection rate,  $q$ .
3. Total injection volume,  $V$ .
4. Area of the fracture,  $A$ .
5. Weight of propping agent,  $S$ .
6. Surface injection pressure  $P_s$  (pump pressure,  $P_p$ ).
7. Horsepower required,  $H_h$
8. Productivity ratio of the well, PR.

Variables can easily be seen that values for some of these parameters must be known or assumed before it is possible to determine the others. This suggests two possible methods of solution.

- a) The first approach is to assume an injection rate, a size of treatment (total volume of fracture fluid), and a fracture gradient based on previous fracture jobs in the area. From these assumptions it is possible to obtain the surface pressure, horsepower requirements, maximum quantity of propping agent necessary, and productivity. Due to assumptions made, we will have a design which will be economically feasible.
- b) In the second method all of these parameters can be determined from the desired productivity ratio and assumed fracture gradient. In this method, calculations are made for several assumed productivity ratios to provide a cost comparison.

In this project, author will apply the second approach for developing mathematical model.

## **2.7 Fracture Fluid and Proppant properties**

### **2.7.1 Fracture Fluid**

To design the optimum fracture treatment, it is necessary to pick the best fluid for the reservoir conditions.

To select the correct fluid, the design engineer must consider the following fracture fluid requirements, at a minimum.

The fracture fluid should be:

- a. Compatible with the formation material
- b. Compatible with the formation fluid
- c. Viscous enough to generate a pressure drop in the fracture to produce enough fracture width to allow propping agents to enter and more into the fracture

- d. Capable of suspending propping agents and transporting them deep into the fracture
- e. Capable of maintaining viscosity during the pumping portion of the treatment, but able to break to a lower viscosity fluid so it can either imbibe into the formation or flowback into the wellbore
- f. Easy to mix in the field
- g. Easy to control from a chemistry view point
- h. Cost effective

The first decision the design engineer must make is to evaluate the base fluid required for the formation. One can choose from water based, foam based or oil based. Most treatments are pumped with water based fluids. In low pressure reservoirs, foam based fluids may be used to aid in fracture fluid cleanup. In some oil reservoirs, oil based fluid might be used. However, the petroleum literature is full of examples where water-based fluids are used successfully to stimulate oil reservoirs. In fact, one of the best ways to get more oil out of the ground is to design a water flood. As such, one should not worry about pumping water in an oil reservoir as long as the proper fluid compatibility tests are run.

By using the fracture fluid coefficient, we can determine properties of fracture fluid.

$$\frac{1}{C} = \frac{1}{C_v} + \frac{1}{C_c} + \frac{1}{C_w} \text{-----(1)}$$

$$C_v = 0.0469 \left( \frac{k\Delta p\phi}{\mu} \right)^{\frac{1}{2}} ; \text{unit} \left( \frac{\text{ft}}{\sqrt{\text{min}}} \right) \text{-----(2)}$$

Where:

k = effective formation permeability to fracturing fluid, in darcies

φ = formation porosity, a fraction

μ = fracturing fluid viscosity, in centipoises

Δp = differential pressure across the face of the fracture, in psi

Δp = G<sub>f</sub> \* D – p<sub>ws</sub> = fracture gradient \* Depth – normal reservoir pressure

$$C_c = 0.0374 \Delta p \left( \frac{k c_f \phi}{\mu} \right)^{\frac{1}{2}} \text{-----}(3)$$

Where:

$\Delta p$  = differential pressure across the face of the fracture, in psi

$k$  = effective formation permeability to fracturing fluid, in darcies

$\phi$  = formation porosity, a fraction

$\mu$  = fracturing fluid viscosity, in centipoises

$c_f$  = isothermal compressibility of the reservoir fluid, in  $\text{psi}^{-1}$

$$C_w = \frac{1.0164m}{A_f} \text{-----}(4)$$

Where:

$$m = 2.254 \frac{k A_f^2 c \Delta p}{\mu} \text{ the slope of the fluid - loss curve in } \frac{\text{cm}^3}{\sqrt{\text{min}}}$$

$A_f$  = area of the test media

$\Delta p$  = differential pressure across the face of the fracture, in psi

$k$  = effective formation permeability to fracturing fluid, in darcies

$\phi$  = formation porosity, a fraction

$\mu$  = fracturing fluid viscosity, in centipoises

$c$  = proportionality constant

### 2.7.2 Proppant agent

Once the reservoir permeability is known and the optimum fracture length has been computed, then the desired conductivity of the fracture can be determined. If we assume the optimum dimensionless fracture conductivity ( $F_{cd}$ ) is 1.6, then in a low-permeability reservoir of 0.2 md with a design fracture half-length of 500 ft, then a fracture conductivity of 160 md-ft would be optimal for gas production. However, this low value of fracture conductivity may not allow for an efficient cleanup of a partially broken

fracture fluid. In tight formations, such as this one, a higher value of dimensionless conductivity is required (*Soliman and Hunt, 1985*) to be certain the fracture fluid can clean up. Other considerations include the degradation of conductivity with time and limitations imposed by flow profiles.

Many authors (*Richardson, 2000; Barree, 2003; Vincent et al., 1999*) have stated the primary variables in fracture conductivity are:

1. Multiphase flow effects that reduce hydrocarbon permeability
2. High closure stress that causes
  1. Proppant crushing or
  2. Proppant embedment
3. Partially broken fracture fluids that plug the fracture because of high yield points
4. Non-Darcy flow in the fracture that causes extra pressure drop
5. Gel damage that reduces fracture permeability
6. Filter cake deposition that reduces fracture width
7. Fines migration from the reservoir or from crushed proppant grains.

*Richardson (2000)* proposed that a dimensionless conductivity ( $F_{cd}$ ) of 30 should be used to compensate for these effects. In higher-permeability formations, this may prove to be cost prohibitive or impossible, even when using methods such as tip screen out to increase fracture width. *Hernandez et al. (2004)* presented a method to account for non-Darcy flow and calculate required conductivity. This model has a result of usually recommending shorter and wider fractures as drawdown increases, but can prevent overdesigning to achieve a high dimensionless conductivity that may not be necessary.

Although materials selection and fracture profile are designed around a desired conductivity, when specifying the conductivity, the probable materials need to be kept in mind. This is similar to the question about whether the chicken or the egg came first. Some assumptions can be made about the ability of the gel to clean up, sanding potential



of the reservoir, and fluid flow that should aid an experienced designer in specifying conductivity.

## 2.8 Necessary Equations for Calculating

### 2.8.1 Fracture Treating Pressure

#### Vertical Fracture

$$P_t = \frac{2\mu}{1-\mu} P_{ob} \text{-----(5)}$$

#### Horizontal Fracture

$$P_t = P_{ob} \text{-----(6)}$$

#### Angle Fracture

$$P_t = \frac{P_{ob}}{2} \left[ 1 + \left( \frac{2\mu}{1-\mu} \right) + \left( 1 - \frac{2\mu}{1-\mu} \right) \cos 2\phi \right] \text{-----(7)}$$

$P_t$ : Fracture Treating Pressure; *psi*

$P_{ob}$ : Overburden pressure; *psi*

$\mu$ : Poisson's Ratio

$\phi$ : Angle of fracture from horizontal; *degree*

### 2.8.2 Fracture Area

$$A_t = \frac{q_i W}{\pi C^2} \left[ e^{x^2} \operatorname{erfc}(x) + \frac{2x}{\sqrt{\pi}} - 1 \right] \text{-----(8)}$$

$$x = 2C \frac{\sqrt{\pi t}}{W} \text{-----(9)}$$

$q_i$  = Injection rate;  $\frac{\text{ft}^3}{\text{min}}$

W: Width of fracture; ft

t: Injection time; min

C: Fracture Fluid Coefficient;  $\frac{\text{ft}}{\sqrt{\text{min}}}$

erfc(x): Designation for error function of (x)

### 2.8.3 Fracture Fluid Coefficients

**Viscosity controlled fluids:**

$$C_v = 0.0469 \left( \frac{k \cdot \Delta P \cdot \phi}{\mu_f} \right)^{\frac{1}{2}} ; \frac{\text{ft}}{\sqrt{\text{min}}} \text{-----}$$

**Reservoir Controlled Fluids:**

$$C_c = 0.0374 \Delta P \left( \frac{k \cdot c_f \cdot \phi}{\mu_r} \right)^{\frac{1}{2}} ; \frac{\text{ft}}{\sqrt{\text{min}}} \text{-----}$$

**Wall Building Fluids:**

$$C_{w\text{act}} = \left( \frac{0.0164m}{A_f} \right) \left( \frac{\Delta P_{\text{act}}}{\Delta P} \right)^{\frac{1}{2}} ; \frac{\text{ft}}{\sqrt{\text{min}}} \text{-----}$$

$$m = \left( \frac{2.254 k \cdot A_f \cdot C \cdot \Delta P}{\mu} \right)^{\frac{1}{2}} \text{----- (10)}$$

*k*: Effective formation permeability;

*φ*: Formation porosity; Fraction

*μ<sub>f</sub>*: Fracture fluid viscosity; cp

*ΔP*: differential pressure across the face of fracture; psi *c<sub>f</sub>*: Isothermal Compressibility of

### 2.8.4 Fracture Efficiency

$$\text{Eff} = \frac{1}{x^2} \left[ e^{x^2} \text{erfc}(x) + \frac{2x}{\sqrt{\pi}} - 1 \right] \text{----- (11)}$$

$$x = \frac{2C \sqrt{\pi t}}{w} \text{-----}$$

Where:

*C: Coefficient of fluid*

*w: Fracture width; ft*

*t: Time of injection; min*

### 2.8.5 Injection time

Using trial and error method:

$$A = 2 * r_f * \text{formation thickness} \text{----- (12)}$$

$$\text{Eff1} = \frac{1}{x^2} \left[ e^{x^2} \text{erfc}(x) + \frac{2x}{\sqrt{\pi}} - 1 \right] \text{-----}$$

$$x = \frac{2C \sqrt{\pi t}}{w} \text{-----}$$

$$\text{Eff2} = \frac{WA}{q_i t} \text{----- (13)}$$

If Eff1 = Eff2; → t

Where:

*t: Time of injection; min*

*C: Coefficient of fluid*

*w: Fracture width; ft*

*r<sub>f</sub>: Fracture Radius; ft*

## 2.8.6 Type of Flows

### Flow in Pipe

$$v = 17.16 \frac{q}{d^2} \text{-----} (14)$$

$$N_{Re} = 928 \frac{d \cdot v \cdot \rho}{\mu} \text{-----} (15)$$

$$f = \frac{64}{N_{Re}}; N_{Re} < 2000 \text{-----} (16)$$

$$\frac{1}{\sqrt{f}} = -2 \log \left[ \frac{2.6}{3.7} - \frac{5.02}{N_{Re}} \log \left( \frac{2.6}{3.7} + \frac{13}{N_{Re}} \right) \right]; N_{Re} > 2000 \text{-----} (17)$$

$$\Delta P_f = \frac{(f \cdot L \cdot \rho \cdot v^2)}{25.80d} \text{-----} (18)$$

Where:

$v$ : Velocity of fluid;  $\frac{ft}{s}$

$q$ : Flowrate of fluid;  $\frac{ft^3}{s}$

$\rho$ : Density of fluid;  $\frac{lbm}{ft^3}$

$\mu$ : Viscosity of fluid;  $cp$

$L$ : Length of pipe;  $ft$

$\varepsilon$ : roughness number

$d$ : Pipe diameter;  $in$

### Annular flow

$$\alpha = \frac{d_c}{d_t} \text{-----} (19)$$

$$d_e = d_c * \frac{1}{2} [C_L^{0.25} + (1 + \alpha^2)^{0.5}] \text{-----} (20)$$

$$C_L = (1 - \alpha^2) \left[ (1 + \alpha^2) - \left( \frac{1 - \alpha^2}{\ln \frac{1}{\alpha}} \right) \right] \text{-----} (21)$$

$$v = 17.16 \frac{q}{d_e^2} \text{-----} (22)$$

$$N_{Re} = 928 \frac{d_e * v * \rho}{\mu} \text{-----} (23)$$

$$f = \frac{64}{N_{Re}}; N_{Re} < 2000 \text{-----}$$

$$\frac{1}{\sqrt{f}} = -2 \log \left[ \frac{\frac{2\varepsilon}{d_e}}{3.7} - \frac{5.02}{N_{Re}} \log \left( \frac{\frac{2\varepsilon}{d_e}}{3.7} + \frac{13}{N_{Re}} \right) \right]; N_{Re} > 2000 \text{-----} (24)$$

$$\Delta P_f = \frac{(f * L * \rho * v^2)}{25.80 d_e} \text{-----} (25)$$

*v*: Velocity of fluid;  $\frac{ft}{s}$

*q*: Flowrate of fluid;  $\frac{ft}{s}$

$\rho$ : Density of fluid;  $\frac{lbm}{ft^3}$

$\mu$ : Viscosity of fluid; *cp*

*L*: Length of pipe; *ft*

$\varepsilon$ : roughness number

$d_e$ : annular diameter; *in*

$d_c$ : Casing ID; *in*

$d_t$ : Tubing OD; *in*

### 2.8.7 Fracturing fluid volume

$$V_f = qt \text{-----} (26)$$

Where:

*q* = assumed flow rate

*t* = injection time

### 2.8.8 Proppant agent volume

$$\text{amount of proppant} = S * A \text{-----} (27)$$

$$V = (\text{volume per unit Area}) * \text{Area} \text{-----} (28)$$

$$S = V * (1 - \phi_{\text{sand}}) * \rho_{\text{sand}} \text{-----} (29)$$

$$A = \frac{q * t * \text{Eff}}{W} \text{-----} (30)$$

where:

q = flowrate

t = injection time

W = fracture width

### 2.8.9 Pump pressure

$$P_p = P_t + \Delta P_f + \Delta P_p - \Delta P_s \text{-----} (31)$$

$$\Delta P_s = 0.052 * \rho_{\text{fluid}} * D \text{-----} (32)$$

Where:

$P_p$  = pump pressure

$P_t$  = bottom hole treating pressure

$\Delta P_f$  = friction pressure drop in pipe

$\Delta P_p$  = pressure drop through the perforations

$\Delta P_s$  = hydrostatic pressure

D: Well depth; ft

$\rho_{\text{fluid}}$ : Density of fluid; lbm/ft<sup>3</sup>

### 2.8.10 Pump Horsepower

$$H_h = 0.0245 P_p q_t \text{-----} (33)$$

Where:

$P_p$  = Pump pressure

$q_t$  = fluid flowrate

### 2.8.11 Productivity Ratio

$$PR = \left[ \frac{k_f \cdot W}{k \cdot h} \right] \left[ \frac{\left( \frac{k \cdot h}{k_f \cdot w} + 1 \right) \ln \left( \frac{r_e}{r_w} \right)}{\left( \frac{k_f \cdot W}{k \cdot h} + 1 \right) \ln \left( \frac{r_e}{r_f} \right) + \ln \left( \frac{r_f}{r_w} \right)} \right] \text{-----} (33)$$

Where:

$k_f$ : Proppant Permeability; md

$k$ : Formation permeability; md

$w$ : Fracture width; ft

$r_e$ : Reservoir radius; ft

$r_w$ : Wellbore radius; ft

$r_f$ : Fracture radius; ft

## CHAPTER III

### METHODOLOGY

#### 3.1 Project Works

##### 3.1.1 Running Diagram of Mathematical Model

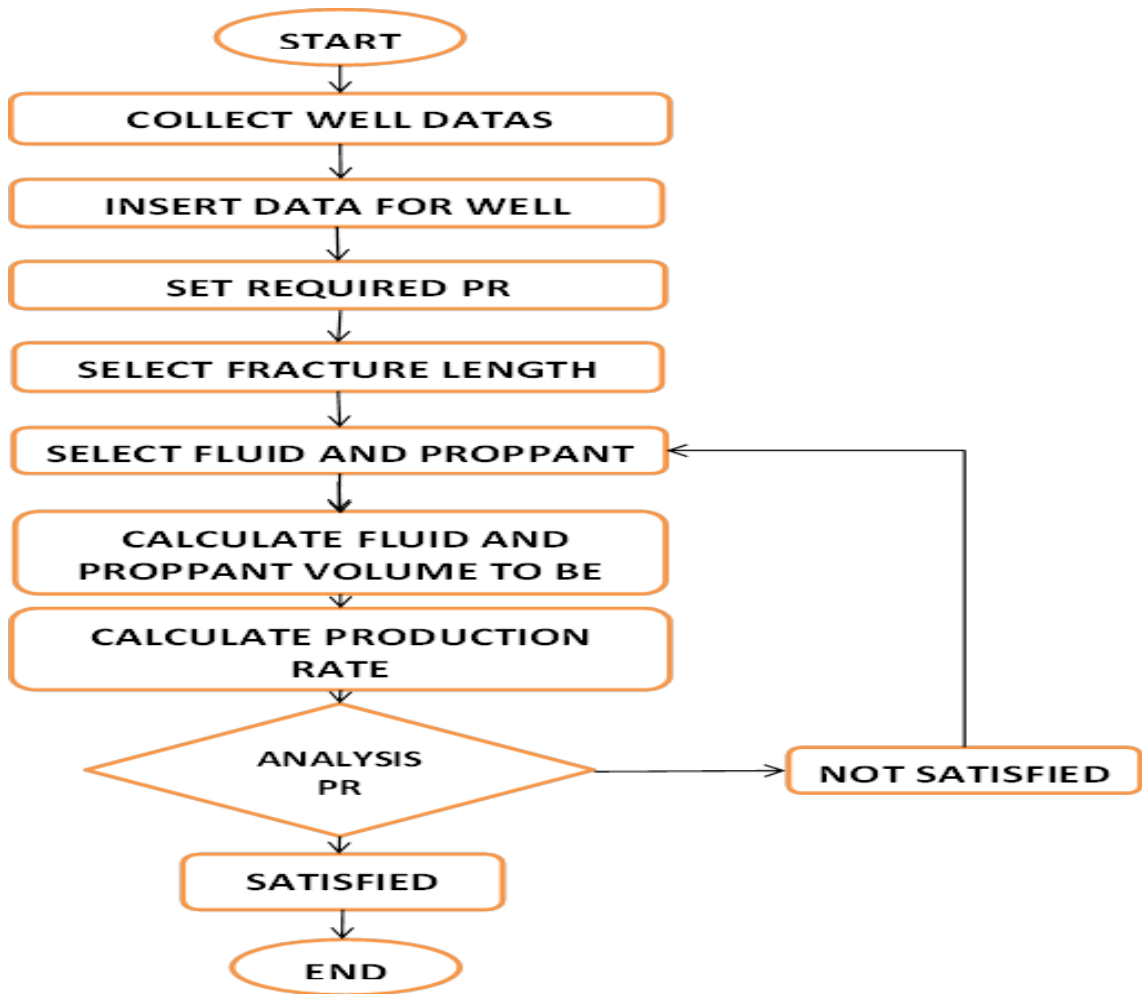


Figure 1: Mathematical Model Flow Chart



### 3.1.2 Manual Calculation

a. Proppant agent volume

Find  $C = \frac{k_f W}{k}$  -----

From the productivity ratio find  $r_f/r_e$

Calculate for  $r_f$

Calculate area of fracture  $A = 2 * r_f * \text{formation thickness}$  -----

Calculate volume of proppant per unit area  $V$

Calculate *weight of proppant*  $S = V * (1 - \phi_{\text{proppant}}) * \rho_{\text{proppant}}$  -----

Calculate amount of proppant  $= S * A$  -----

b. Fracture Fluid volume

Assume flow rate of fracture fluid

Calculate the fracture fluid volume by using trial and error method.

Find fracturing fluid coefficient  $C_w = \frac{1.0164m}{A_f}$  -----

Determine pressure differential across the fracture face:

$\Delta p = \text{Fracture treating pressure} - \text{bottom hole pressure}$  -----

$\text{fracturing pressure} = \text{Depth} * 0.7$  (vertical fracture) -----

$\text{fracturing pressure} = \text{Depth} * 1$  (horizontal pressure) -----

Find  $C_{w,act} = C_w * \left(\frac{\Delta P_{act}}{\Delta P}\right)^{\frac{1}{2}}$  -----

Find (x) in the equation:  $A_t = \frac{q_i W}{4\pi C^2} \left[ e^{x^2} \text{erfc}(x) + \frac{2x}{\sqrt{\pi}} - 1 \right]$  -----

Find fracturing efficiency  $Eff = \frac{W A_t}{q_i t}$  -----

Find (t) by using trial and error method

Find volume of fracture fluid  $V = qt$  -----

c. Pump pressure

Find proppant concentration  $x = \frac{\text{maximum amount of proppant}}{\text{volume of fracture fluid}}$  -----

Flow rate  $q_t$  (including proppant) in sp.gr of proppant

$q_t = q_i + \frac{q_i * x}{\gamma_T * 8.34}$  -----

*Specific gravity at temperature T*

$$\gamma_T = \gamma_{60}[1 - \beta(T - 60)] \text{-----}$$

$$\text{Density } \rho_T = \frac{8.34 \gamma_T + x}{1 + 0.0456x} \text{-----}$$

$$\text{Hydrostatic pressure } \Delta P_s = 0.052 \rho_T D \text{-----}$$

$$\text{Velocity in the casing } v = 17.16 * \frac{q_t}{D^2} \text{-----}$$

Reynolds number  $N_R$

Find friction factor using NR

$$\text{Frictional pressure drop } \Delta P_f = \frac{\text{friction factor} * \text{depth} * \rho_T * v^2}{25.80 * D} \text{-----}$$

$$\text{Pump pressure } P_p = \text{fracture pressure} + \Delta P_f - \Delta P_s \text{-----}$$

d. Pump horsepower

$$H_h = 0.0245 * P_s * q_t \text{-----}$$

e. Result of manual calculation (Appendix)

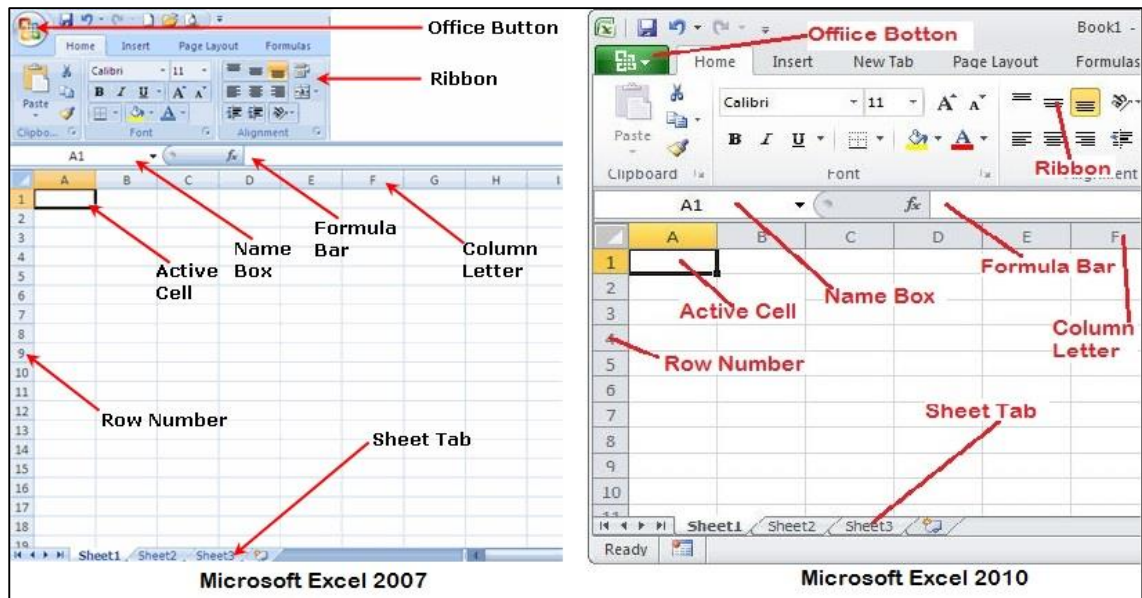
*Table 1: Manual calculation*

Sand Volume	Fluid Volume	Pump Pressure	Horsepower	PR
12000	29000	3500	2600	3.3

### 3.1.3 Mathematical Model in MS Excel.

This project has been working on Microsoft Excel; all the necessary formulas will be inserted in Excel file as the macro function and user form. User need to add data for specific well.

In this project, a mathematical model of Hydraulic fracturing is developed in Microsoft Office 2007 in the computer laboratory in Building 15 and Microsoft Office 2010 in



personal laptop. In the general, the function of both versions are quite same, they have a bit difference in the interface and option menu.

*Figure 2: Microsoft Excel 2007 vs Microsoft Excel 2010*

In the first, author need to activate Developer Tool.

Figure 3: Developer Tab

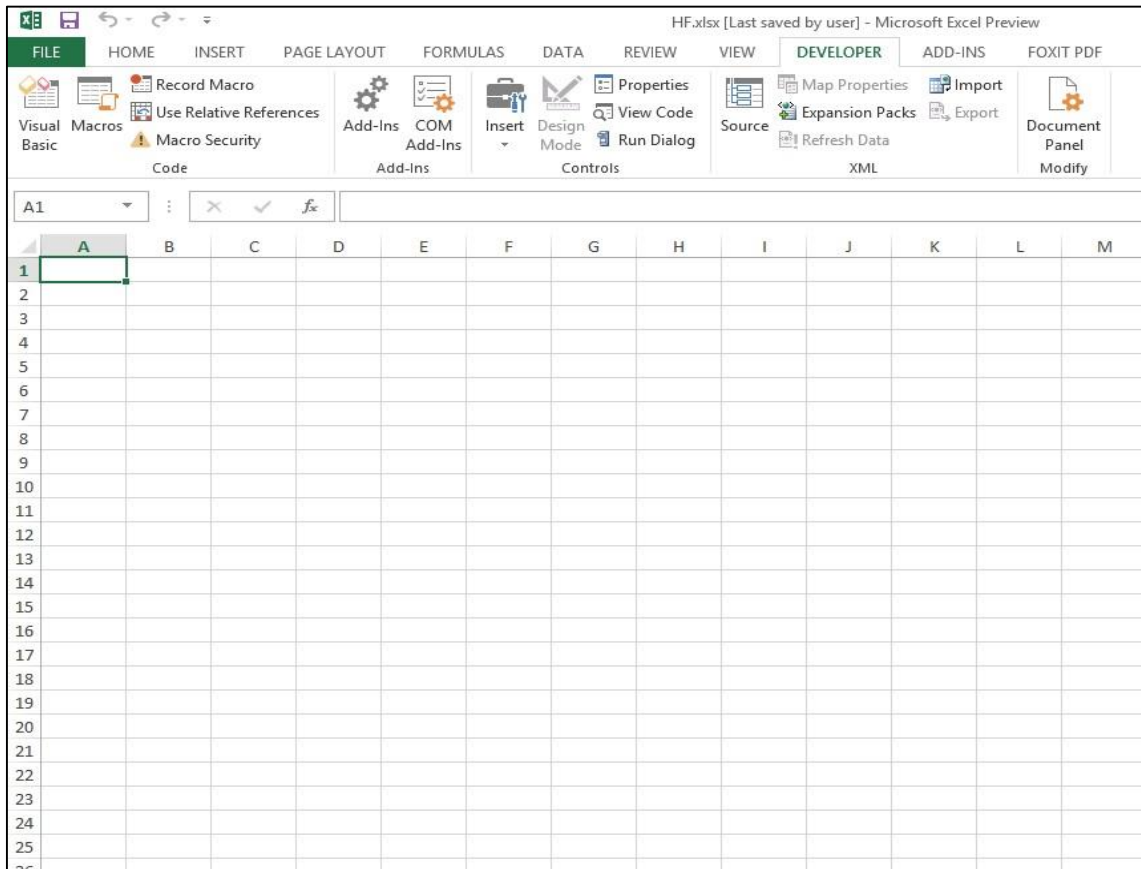
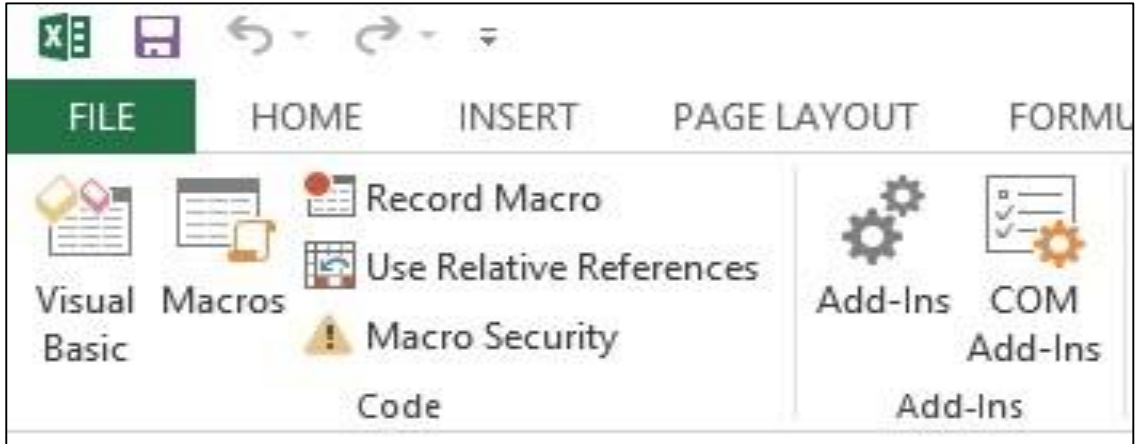


Figure 4: Developer Menu Tools

In the developer tab, there are showed Visual Basic and Macros

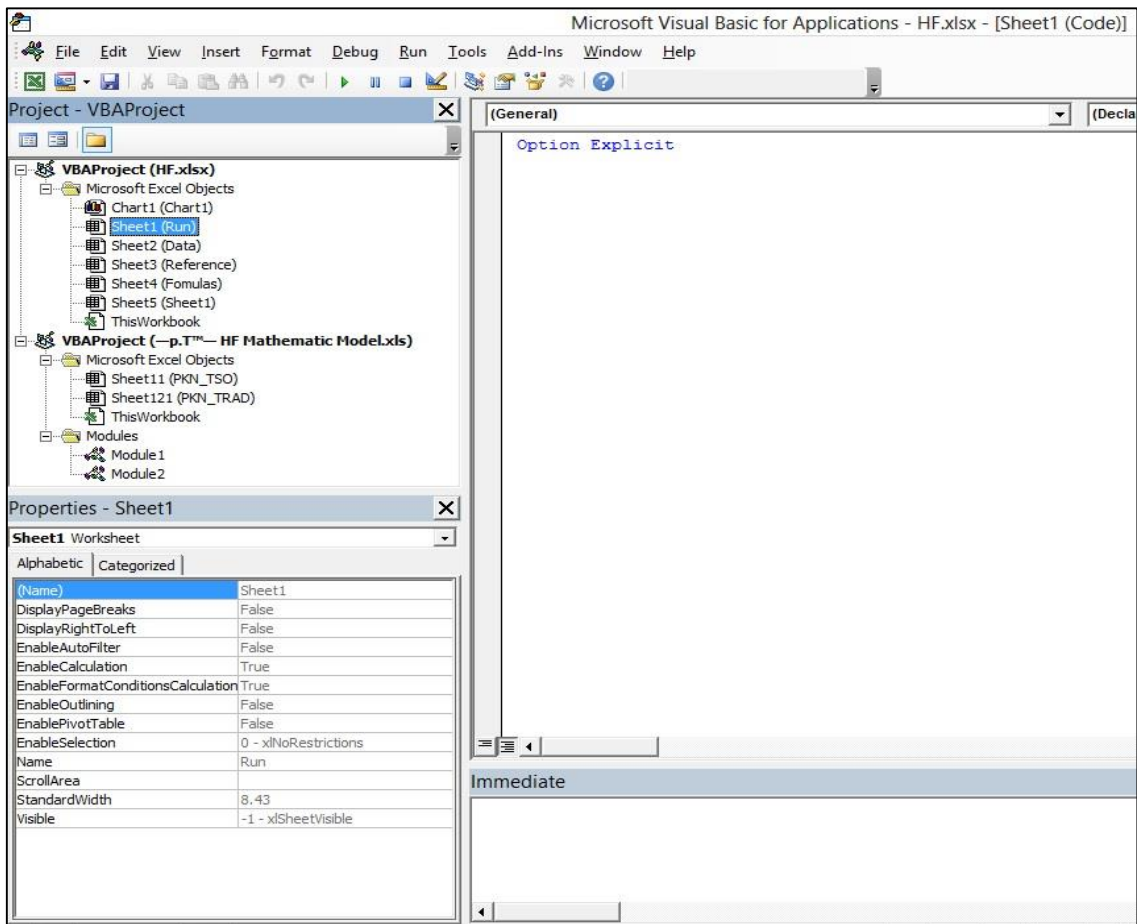


Figure 5: Visual Basic Window

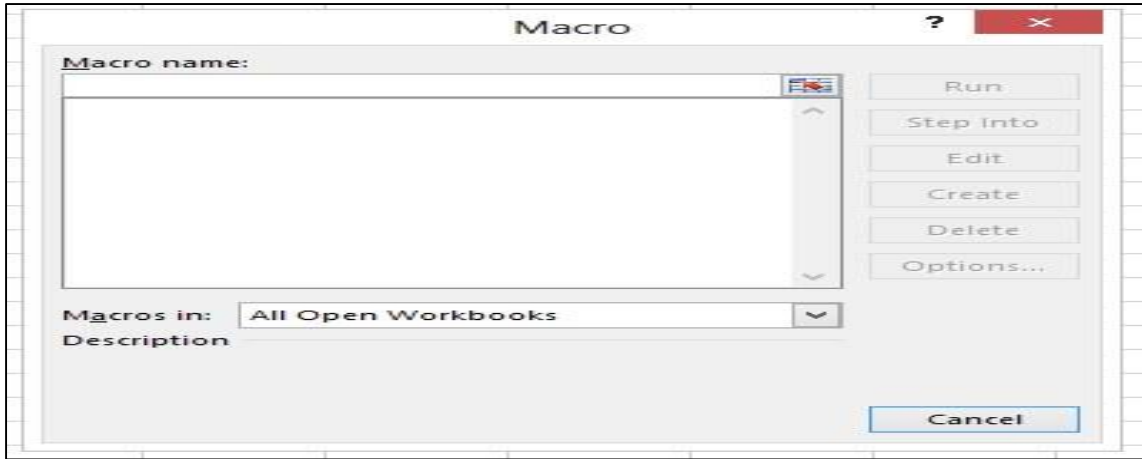


Figure 6: Macros Windows

In the visual basic window, author has written code to create macro function. An in macro window, all functions have been created will show there.

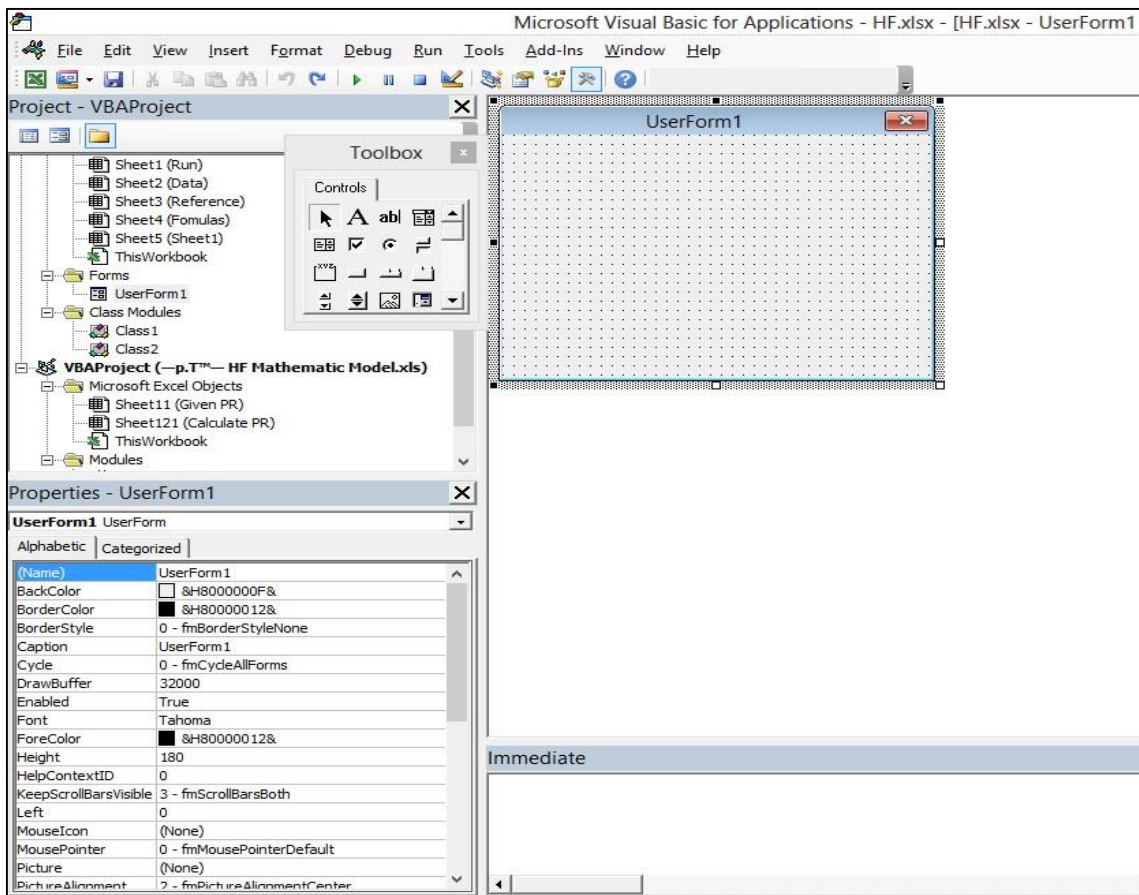
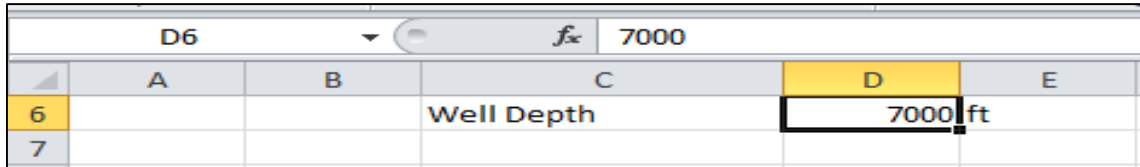


Figure 7: Userform in Visual Basic for Application Window

In userform, author assigned function in the buttons for calculation function.

### 3.2 Steps Of Designing Model

Step 1: Enter required data in table sheet and fix it.

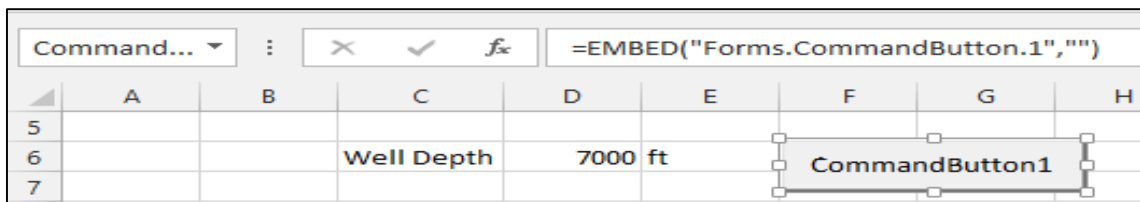


	A	B	C	D	E
6			Well Depth	7000	
7					

Figure 8: Insert data into table

In the table, value of well depth is in “D6”. This value cannot be changed to other position. It was fixed in “D6”.

Step 2: Create button needed.

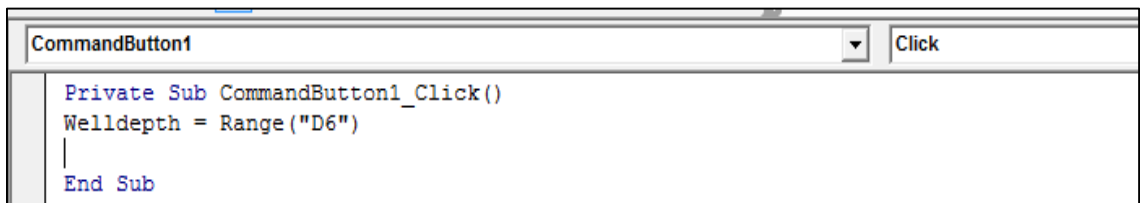


	A	B	C	D	E	F	G	H
5								
6			Well Depth	7000 ft		CommandButton1		
7								

Figure 9: Create button

CommandButton1 is click button.

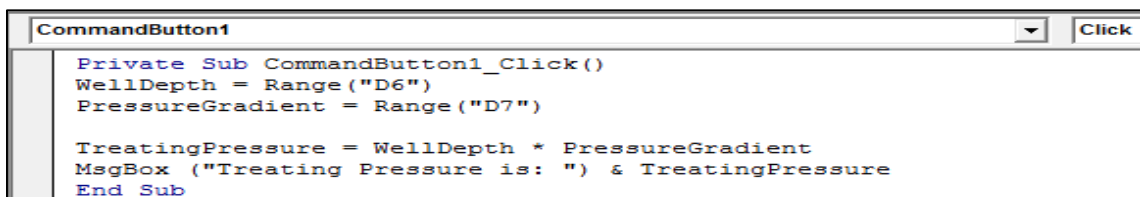
Step 3: Assign value of well depth in button. Value of welldepth will be in “D6”



```
Private Sub CommandButton1_Click()  
Welldepth = Range("D6")  
End Sub
```

Figure 10: Assign data

Step 4: Insert formulas should be run when click button.



```
Private Sub CommandButton1_Click()  
WellDepth = Range("D6")  
PressureGradient = Range("D7")  
TreatingPressure = WellDepth * PressureGradient  
MsgBox ("Treating Pressure is: ") & TreatingPressure  
End Sub
```

Figure 11: Insert formula into button's function

Step 5: Design for cases:

The screenshot shows the VBA Project window for 'VBAProject (Treating Pr)' with 'Module1' selected. The code in the 'General' tab is as follows:

```
Public Function Fracture(Vertical)
    If Vertical.Value Then
        Range("D7") = 0.7
    Else
        Range("D7") = 1
    End If
End Function
```

Below the code, two Excel spreadsheet screenshots illustrate the conditional logic. The top spreadsheet shows the 'Vertical Fracture' checkbox checked, resulting in a 'Treating Pressure' of 0.7. The bottom spreadsheet shows the checkbox unchecked, resulting in a 'Treating Pressure' of 1.

	A	B	C	D	E	F	G
6			Well Depth	7000 ft		Treating Pressure	
7			Pressure Gradient	0.7		Treating Pressure	
8			Treating Pressure	4900 psi		<input checked="" type="checkbox"/>	Vertical Fracture

	A	B	C	D	E	F	G
6			Well Depth	7000 ft		Treating Pressure	
7			Pressure Gradient	1		Treating Pressure	
8			Treating Pressure	7000 psi		<input type="checkbox"/>	Vertical Fracture

Figure 12: Select Cases

In this case, author design for vertical. If the check box is checked “D7” will be 0.7 or unchecked, “D7” will be 1.

The formula that author inserted is about to calculate “treating pressure” and result of this equation will be show in a message box. Value of “treating pressure” will be assigned in “D8”, name and unit are assigned in “C8” and “E8”

The screenshot shows the VBA Project window for 'VBAProject (Treating Pr)' with 'Module1' selected. The code in the 'CommandButton1' tab is as follows:

```
Private Sub CommandButton1_Click()
    WellDepth = Range("D6")
    Call Fracture(Vertical)
    PressureGradient = Range("D7")
    TreatingPressure = WellDepth * PressureGradient
    MsgBox ("Treating Pressure is: ") & TreatingPressure
    Range("D8") = TreatingPressure
    Range("C8") = "Treating Pressure"
    Range("E8") = "psi"
End Sub
```



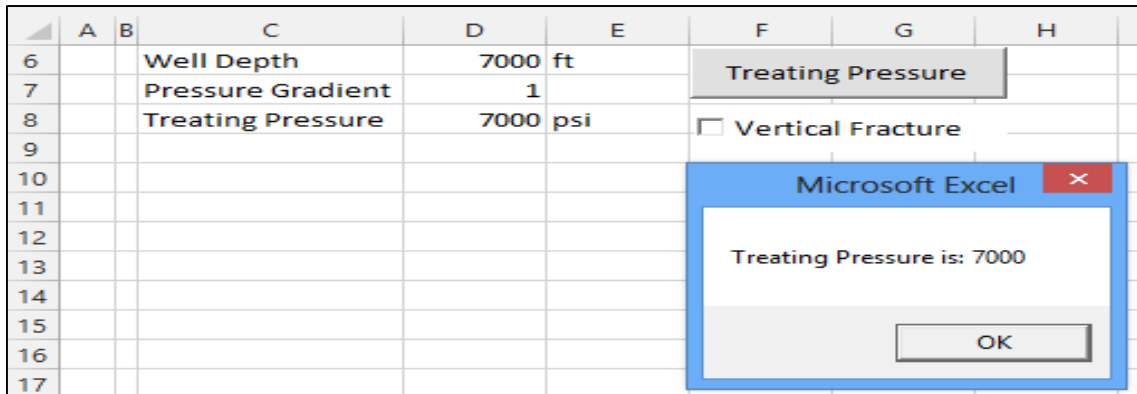


Figure 13: Check results

Step 5: Design an interface.

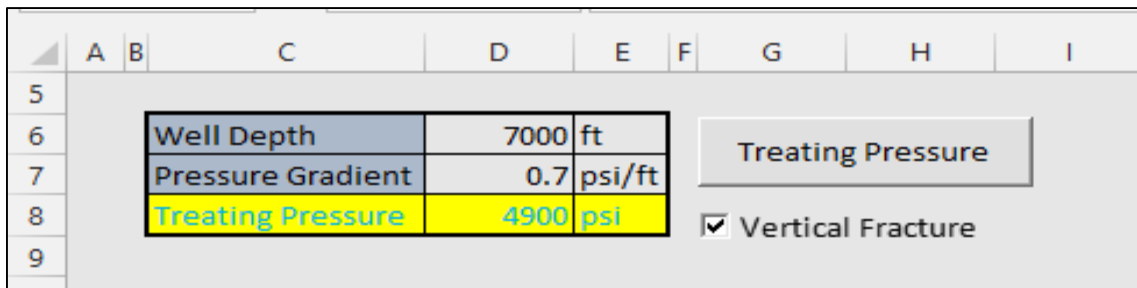


Figure 14: Design interface

Author arranged and made color for table and buttons.

Step 6: Save file in extension of “xlsm” for the best file of macro.

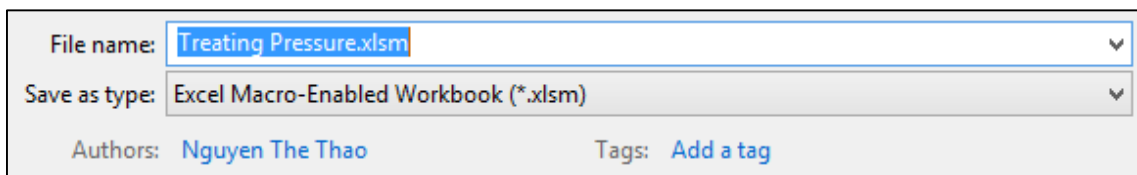


Figure 15: Save file

### 3.3 Plan of action and schedule of FYP II

No.	Detail/ Week	1	2	3	4	5	6	7		8	9	10	11	12	13	14	
1	Project Work Continue	Process	Process	Process					Mid-Semester Break								
3	Project Work Continue				Process	Process	Process	Process									
4	Submission of Progress Report										Milestone						
5	Seminar (compulsory)											Process	Process	Process			
5	Project work continue										Process	Process	Process	Process			
6	Poster Exhibition												Milestone				
7	Submission of Dissertation (soft bound)													Milestone			
8	Oral Presentation															Milestone	
9	Submission of Project Dissertation (Hard Bound)																Milestone

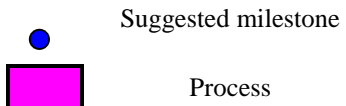


Figure 16: Schedule of FYP II

## CHAPTER IV

### RESULTS AND DISCUSSION

#### 4.1 Results

Author is still doing in coding for macro functions in Microsoft Excel. Author had to read and did the samples for coding in Excel. The coding is quite complicated with author who is doing Petroleum Engineer.

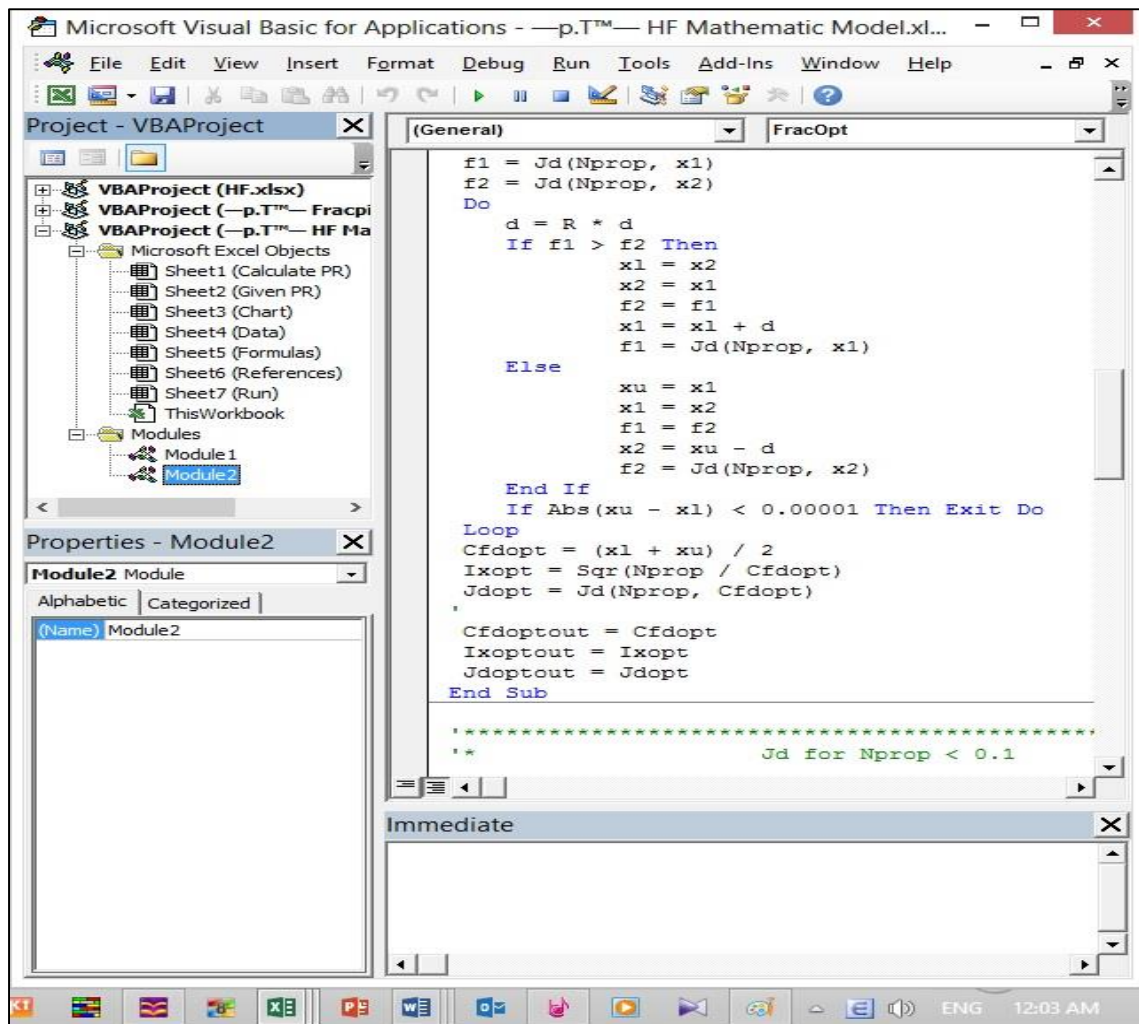


Figure 17: Coding Window I

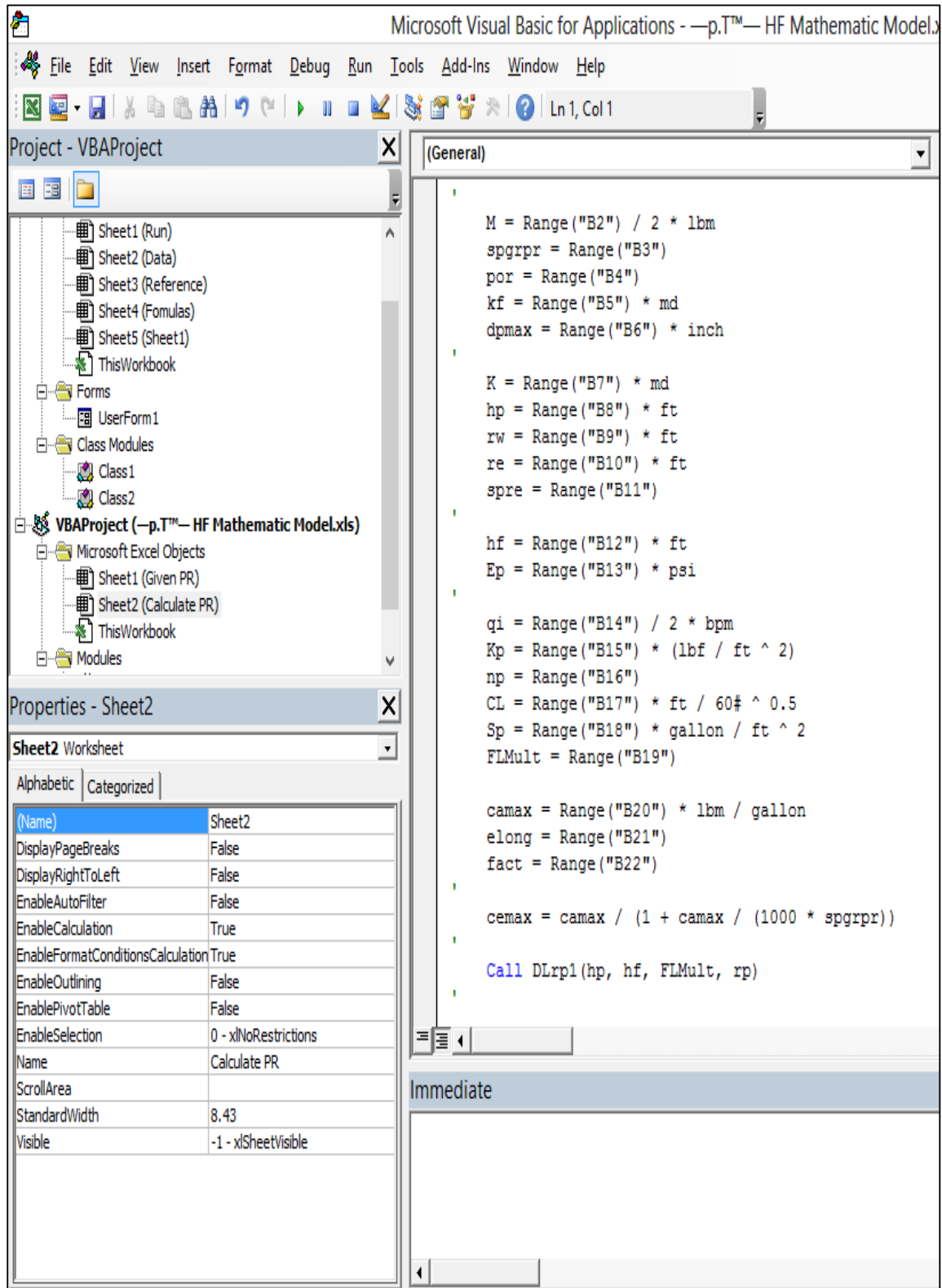


Figure 18: Code Window II

In this window, author assign the name boxes with value of data for reservoir and well.

User can enter the input data manually for specific reservoir and well. In this table some data are assumed due to limited data collection.

Table 2: Input data for reservoir and well

<b>Well</b>		
Well depth	7000.0000	ft
Well Spacing	40.0000	acres
Well Diameter	0.4000	ft
Casing Size ID	4.8920	in
Tubing Size ID	2.0000	in
Tubing Size OD	2.3750	in

<b>Fluid</b>		
<input checked="" type="checkbox"/> High Viscosity		
Fracture Fluid Name	Fluid 1	
Fluid Gravity	0.8760	API
Viscosity	500.0000	cp
Coefficient of Thermal	0.0005	%/F
<input type="checkbox"/> Low Viscosity		
Fracture Fluid Name	Fluid 2	
Fluid Gravity	0.8760	API
Viscosity	4.0000	cp
Compressibility	0.000010	
Coefficient of Thermal	0.000500	%/F
<input type="checkbox"/> Filter Cake		
Fracture Fluid Name	Fluid 3	
Fluid Gravity	0.8760	API
	35.0000	API
Slope of Fluid-loss Curve	1.2000	cu cm/Sqrt(min)
Fluid loss Measure at	125.0000	F
Fluid loss Measure at	1000.0000	psi
Viscosity of Crude	7.0000	cp
Area of filter paper	22.8000	cm <sup>2</sup>
Coefficient of Thermal	0.0005	%/F

<b>Proppant</b>		
Proppant Name	Sand	10-20 mesh
Gravity of Sand	2.6300	lb
Sand Density	164.1120	lb/cuft
Sand Porosity	0.3500	%
Sand Permeability	20000.0000	md
<b>Reservoir</b>		
Average Permeability	1.0000	mD
Formation Thickness	50.0000	ft
Static bottom Hole Pressure	2500.0000	psig
Average Well Temperature	125.0000	F
Isothermal compressibility	0.0040	psi-1
Reservoir Fluid Viscosity	4.0000	cp
Coefficient of thermal of fluid	0.0005	
Compressibility of Res Fluid	0.000010	psi-1
Reservoir Radius	660.0000	ft
Formation porosity	0.1350	%
<b>PR</b>		
Required PR	3.0000	times

Fracture		
Injection Rate	30.0000	bb/min
<b>Injection Time</b>	28.0000	min
Fracture Treatment Pressure	4900.0000	psi
Fracture width	0.00833	ft
Fracture Gradient	0.70000	psi/ft
Injection Type	<input checked="" type="checkbox"/> Casing	
	<input type="checkbox"/> Tubing	
	<input type="checkbox"/> Annular	
Fracture Type	<input type="checkbox"/> Angle	
	<input checked="" type="checkbox"/> Vertical Fracture	
	<input type="checkbox"/> Horizontal	

By linking the many function in one button so when user want to calculate, just click one click and wait. The process of calculate was separated to 4 button. So user can calculate each parameter individually.

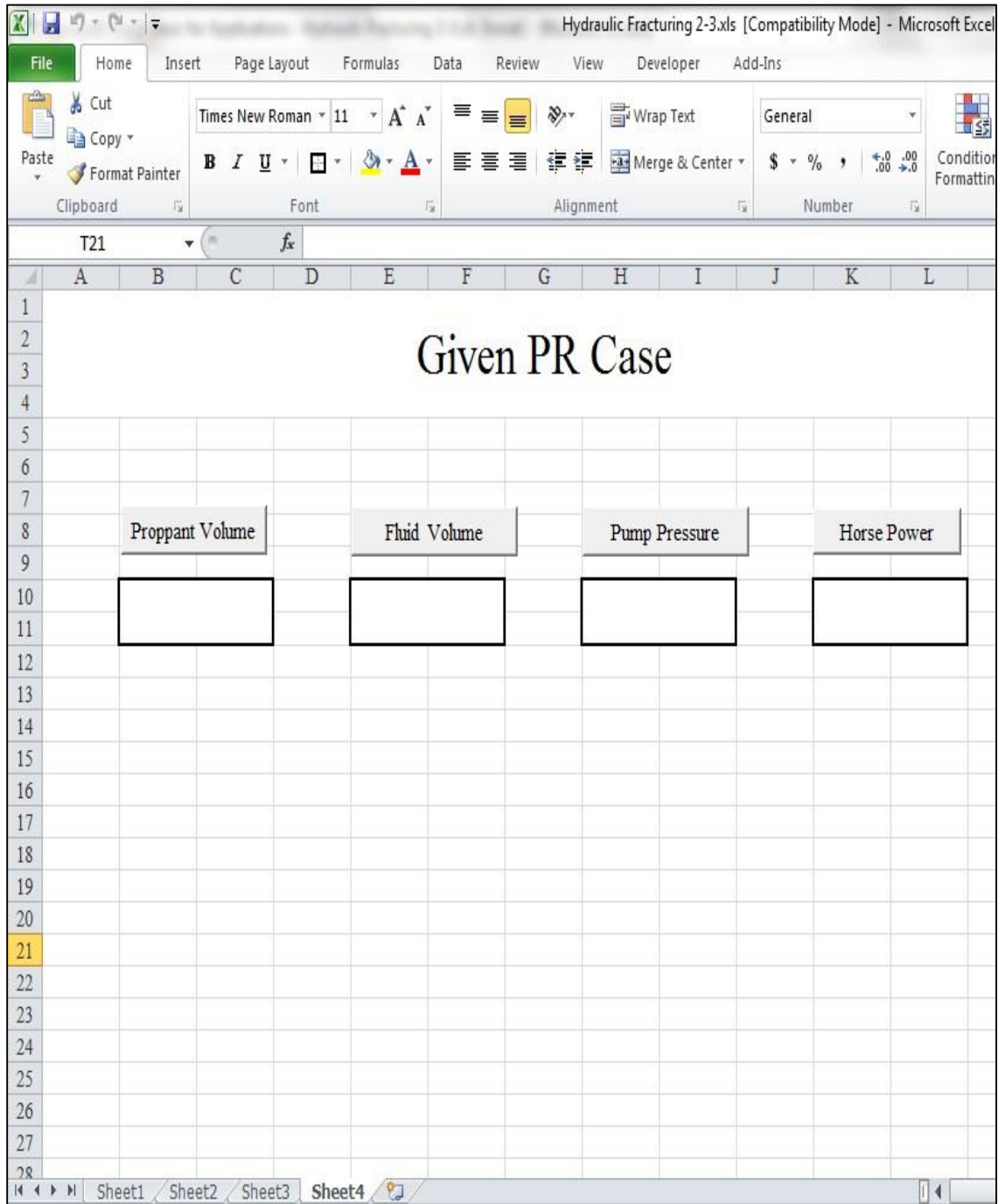
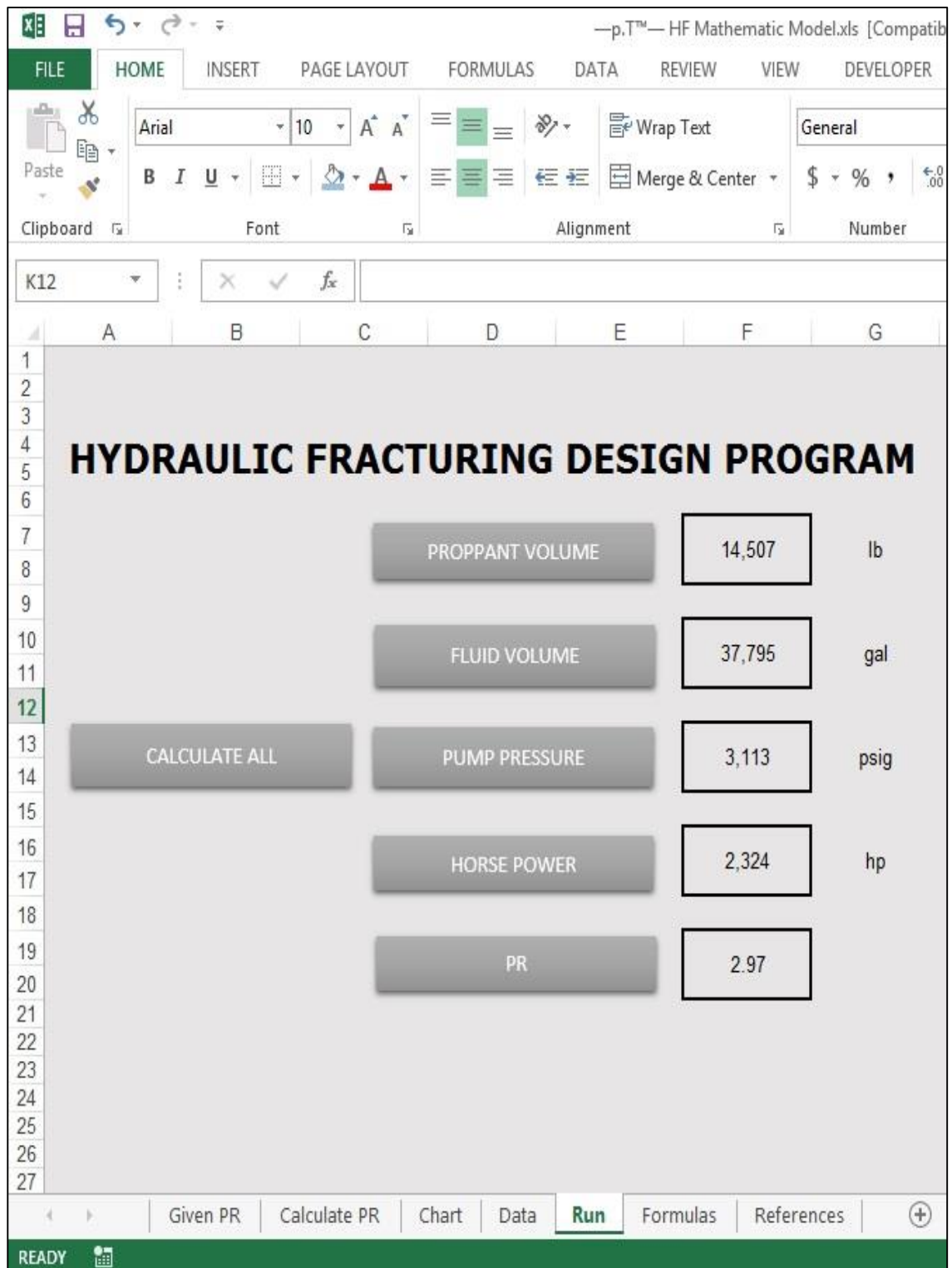


Figure 19: Interface of program in Microsoft Excel Version 1

In update version, author will design one more button to calculate all 4 parameters instead user has to click one by one.





*Figure 20: Interface of program in Microsoft Excel Version 2*

In this version, author arranged a bit for buttons and home button function.

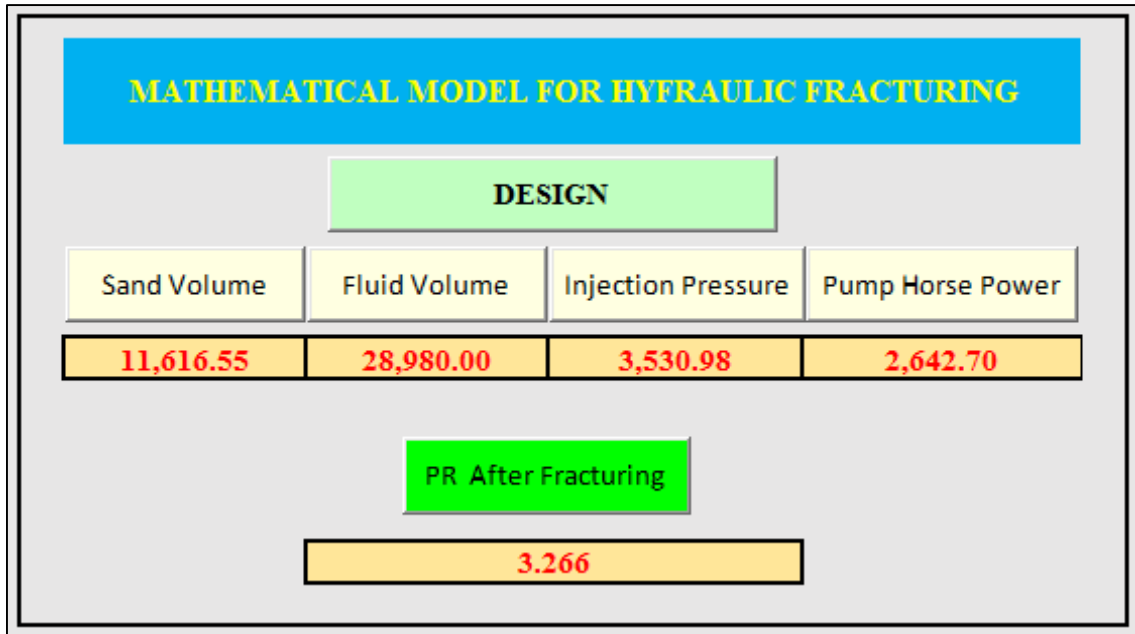


Figure 21: Interface of program in Microsoft Excel Version 3

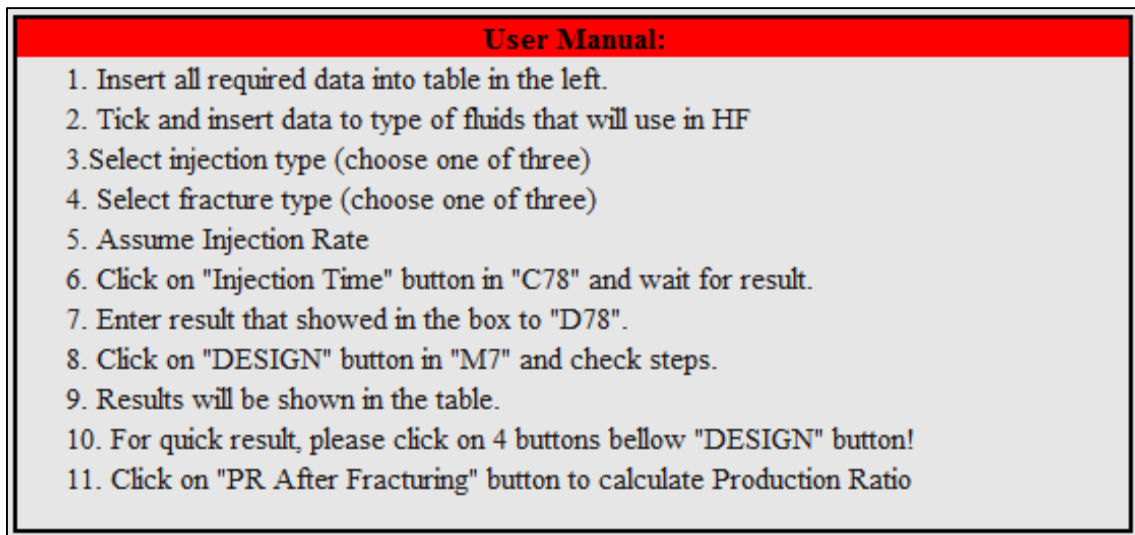


Figure 22: User Manual

## 4.2 Discussion

**Designing challenges:** author is studying in Petroleum Engineering field and not expert in coding. When author started doing this project, author starts from zero about coding in Excel but coding made author so excited.

**Data challenges:** due to the limited data for well and reservoir so author does not have much chance to test in real well and reservoir. Author had to create his own data for testing.

**Interface challenges:** the interface of program is still a problem with author because his program is basic and simple. Author will update and upgrade his program to be professional in function and interface.

**Result correction:** This model is still facing a problem. The problem is result is not corrected as manual calculation. Author will double check and update the corrected one.

## **CHAPTER V**

### **CONCLUSION AND RECOMMENDATION**

#### **5.1 Conclusion**

Author is satisfied with the project because the project is now on the right way and valuable knowledge was gained on hydraulic fracture especially in understanding the concept of mathematical model design.

In further, author will develop this model in other program language and author will add more functions to make this model more friendly and easy to use.

#### **5.2 Recommendation**

For UTP management, there is plenty of room to improve the current system in conducting Final Year Project in order to produce the quality research and product. This FYP course is carried out only in fourteen weeks times. There is limited time to conduct full research and prototype product when the students have to deal with the other important subject of the final year also.

## REFERENCE

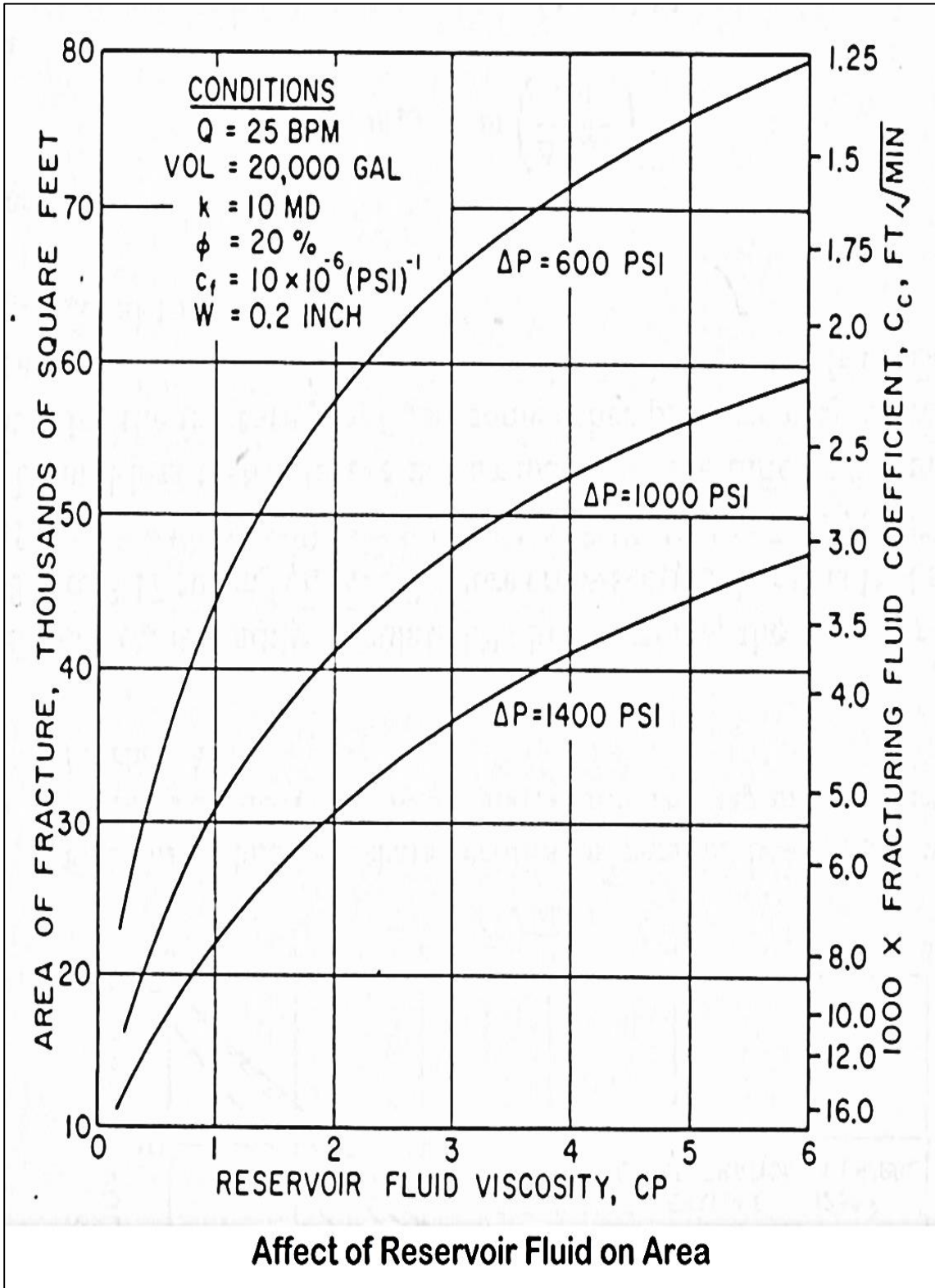
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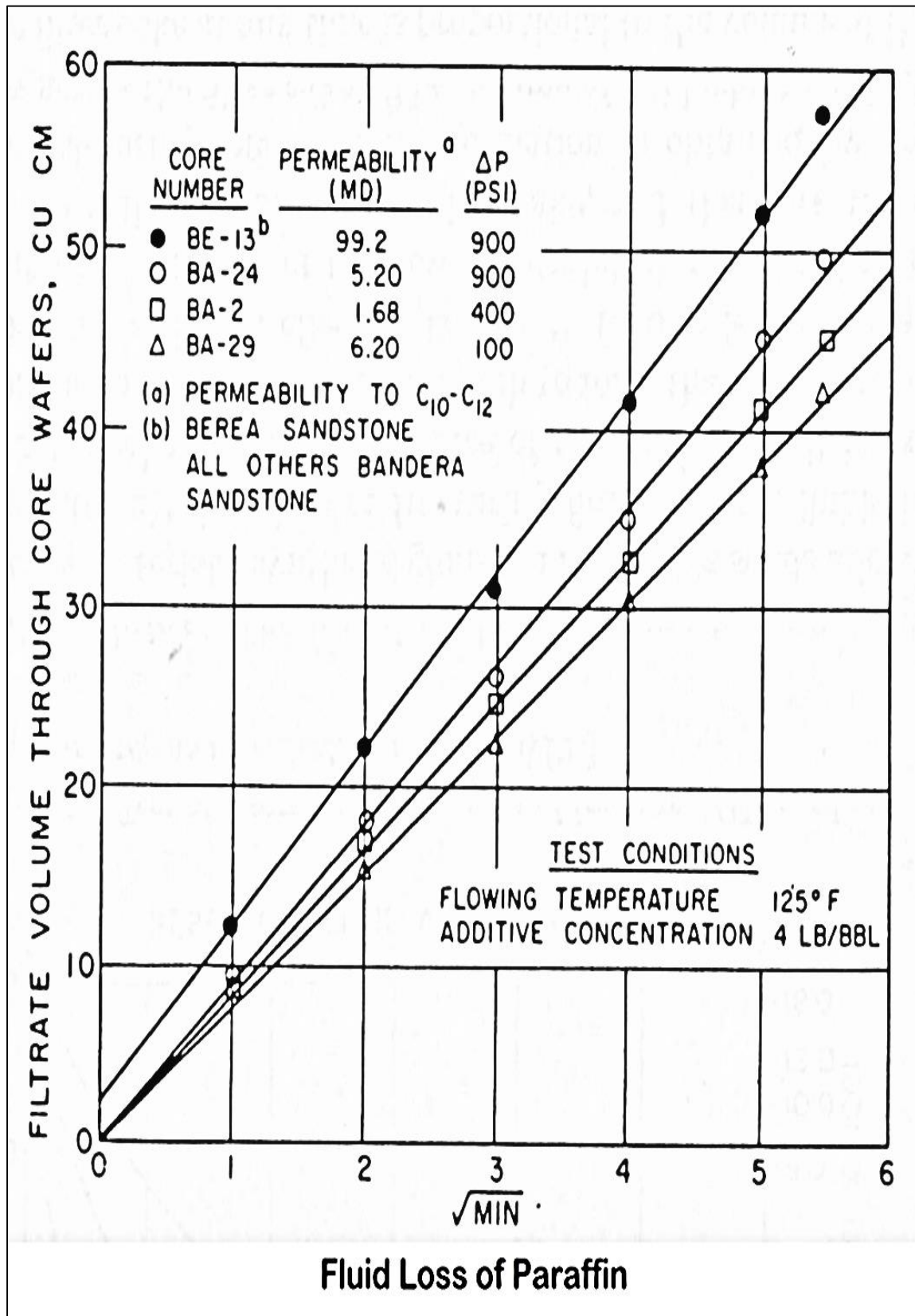
# APPENDICES

## Appendix 1

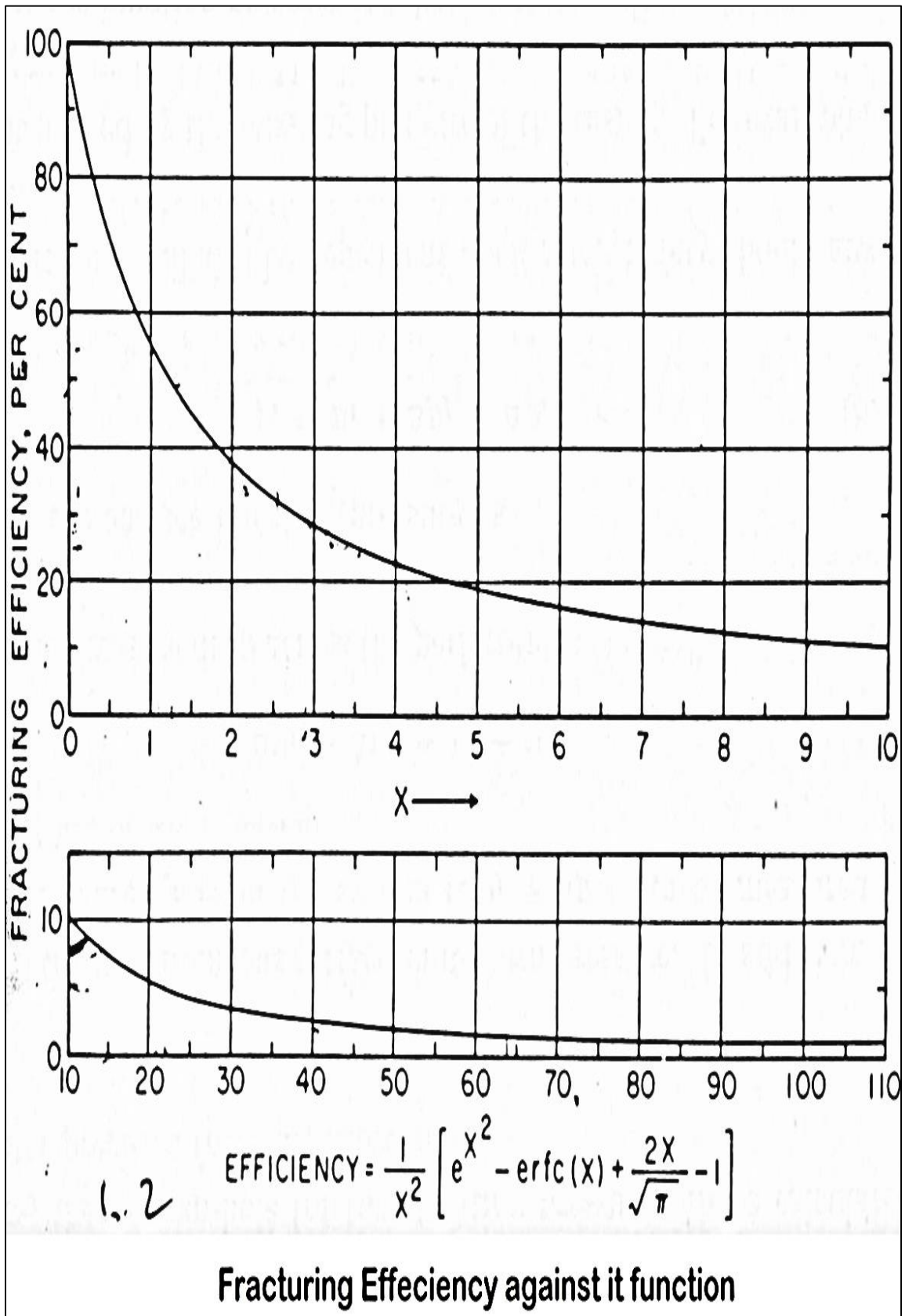




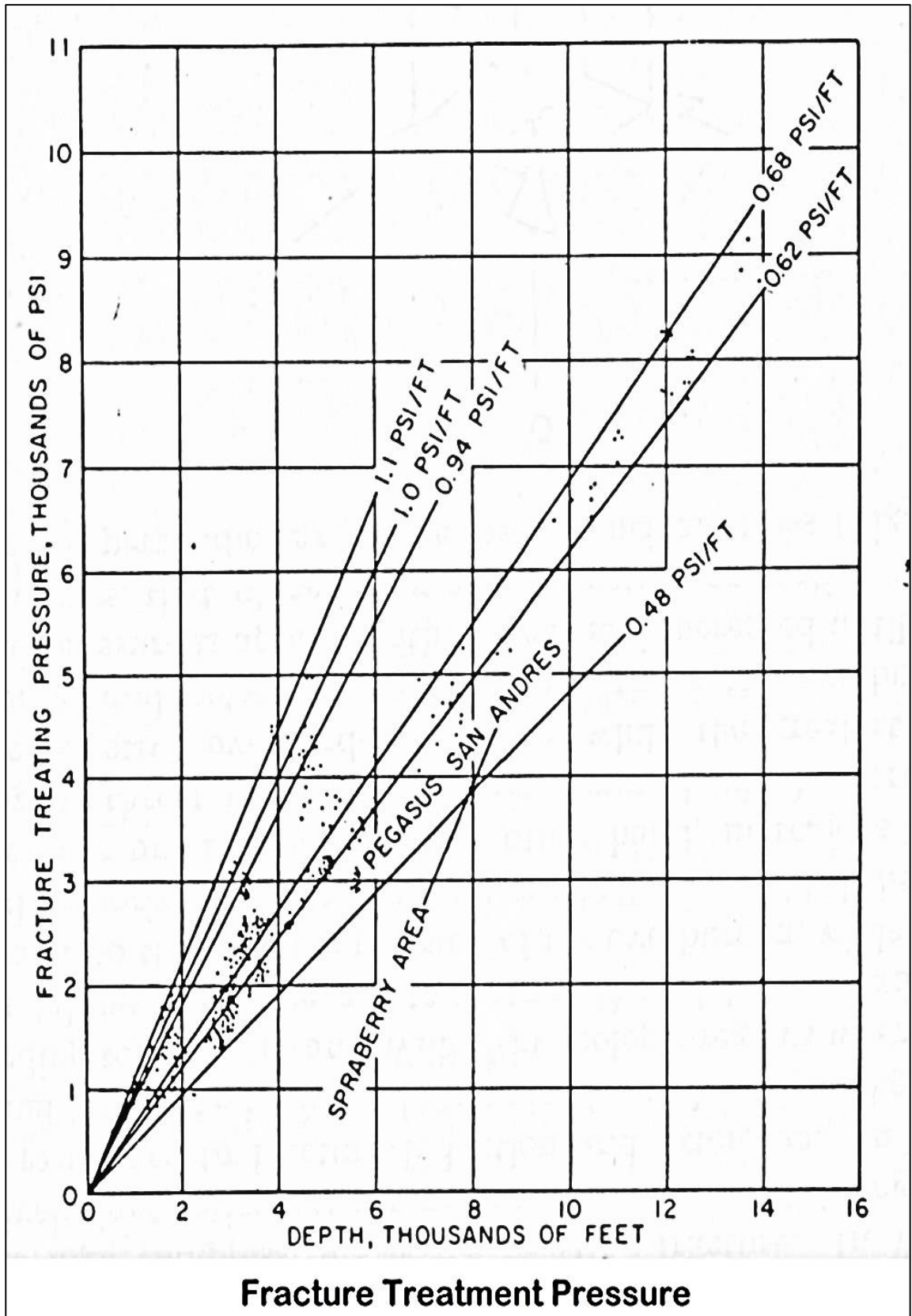
Appendix II



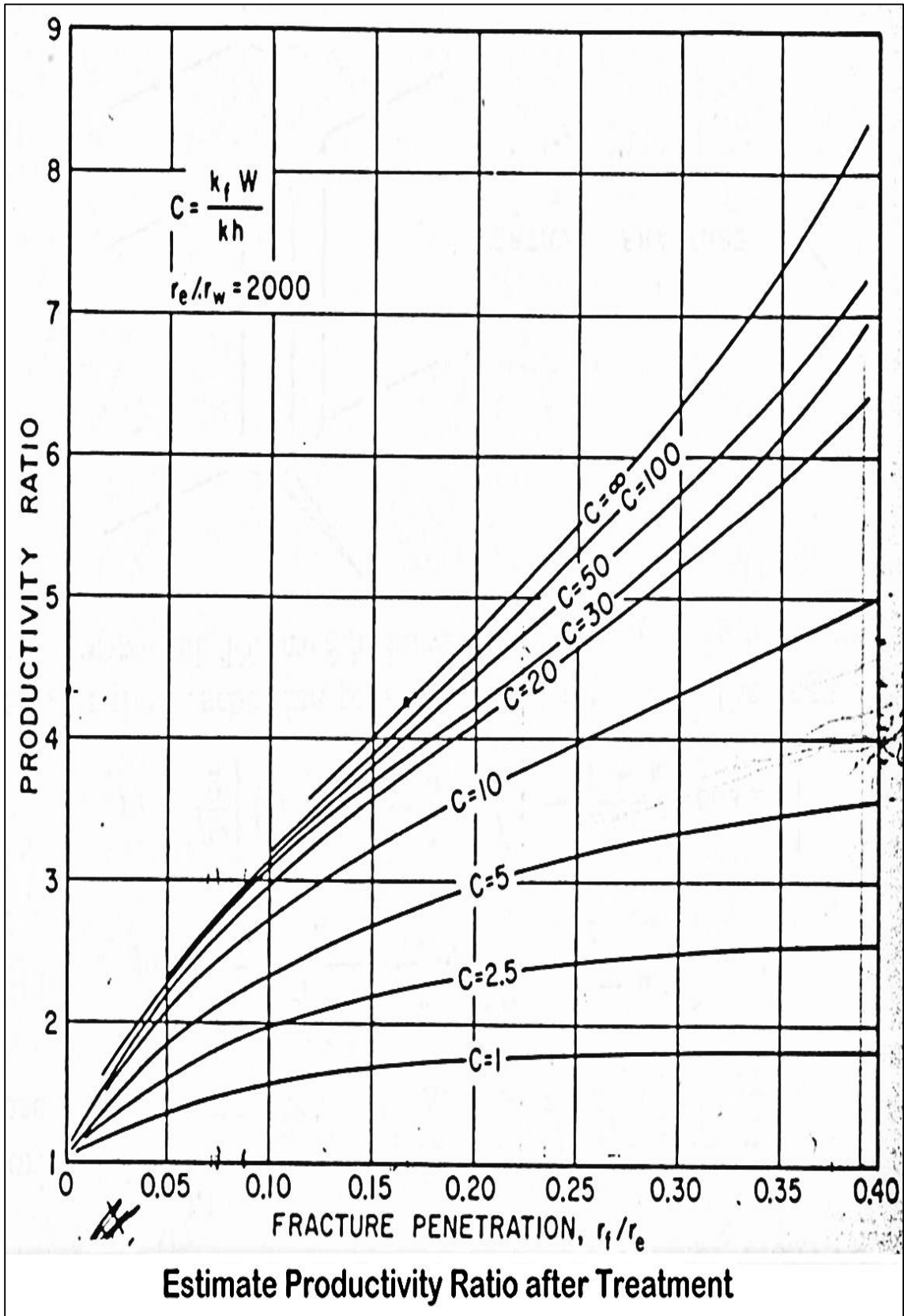
Appendix III



Appendix IV

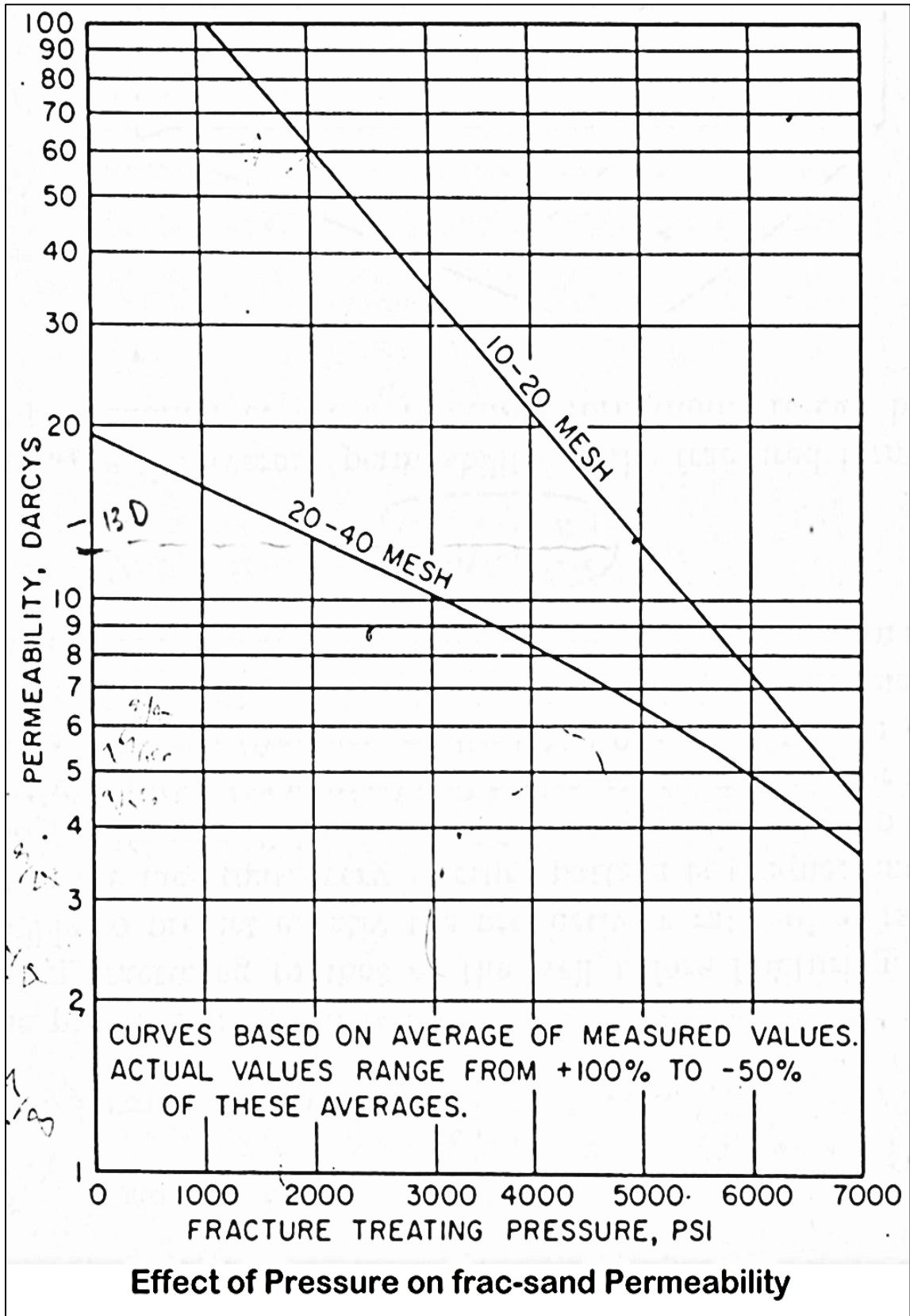


Appendix V





Appendix VI



# Appendix VII

7.1. COMPLEMENTARY ERROR FUNCTION OF $x^a$										
$x$	0	1	2	3	4	5	6	7	8	9
0.0	1.0000	0.9887	0.9774	0.9662	0.9549	0.9436	0.9324	0.9211	0.9099	0.8987
0.1	0.8875	0.8764	0.8652	0.8541	0.8431	0.8320	0.8210	0.8100	0.7991	0.7882
0.2	0.7773	0.7665	0.7557	0.7450	0.7343	0.7237	0.7131	0.7026	0.6921	0.6817
0.3	0.6714	0.6611	0.6509	0.6407	0.6306	0.6206	0.6107	0.6008	0.5910	0.5813
0.4	0.5716	0.5620	0.5525	0.5431	0.5338	0.5245	0.5153	0.5063	0.4973	0.4883
0.5	0.4795	0.4708	0.4621	0.4535	0.4451	0.4367	0.4284	0.4202	0.4121	0.4041
0.6	0.3961	0.3883	0.3806	0.3730	0.3654	0.3580	0.3506	0.3434	0.3362	0.3292
0.7	0.3222	0.3153	0.3086	0.3019	0.2953	0.2888	0.2825	0.2762	0.2700	0.2639
0.8	0.2579	0.2520	0.2462	0.2405	0.2349	0.2293	0.2239	0.2186	0.2133	0.2082
0.9	0.2031	0.1981	0.1932	0.1884	0.1837	0.1791	0.1746	0.1701	0.1658	0.1615
1.0	0.1573	0.1532	0.1492	0.1452	0.1414	0.1376	0.1339	0.1302	0.1267	0.1232
1.1	0.1198	0.1165	0.1132	0.1100	0.1069	0.1039	0.1009	0.0980	0.0952	0.0924
1.2	0.0897	0.0870	0.0845	0.0819	0.0795	0.0771	0.0748	0.0725	0.0703	0.0681
1.3	0.0660	0.0639	0.0619	0.0600	0.0581	0.0562	0.0544	0.0527	0.0510	0.0493
1.4	0.0477	0.0461	0.0446	0.0431	0.0417	0.0403	0.0389	0.0376	0.0363	0.0351
1.5	0.0339	0.0327	0.0316	0.0305	0.0294	0.0284	0.0274	0.0264	0.0255	0.0245
1.6	0.0237	0.0228	0.0220	0.0212	0.0204	0.0196	0.0189	0.0182	0.0175	0.0168
1.7	0.0162	0.0156	0.0150	0.0144	0.0139	0.0133	0.0128	0.0123	0.0118	0.0114
1.8	0.0109	0.0105	0.0101	0.0097	0.0093	0.0089	0.0085	0.0082	0.0078	0.0075
1.9	0.0072	0.0069	0.0066	0.0063	0.0061	0.0058	0.0056	0.0053	0.0051	0.0049
2.0	0.00468	0.00448	0.00428	0.00409	0.00391	0.00374	0.00358	0.00342	0.00327	0.00312
2.1	0.00298	0.00285	0.00272	0.00259	0.00247	0.00236	0.00225	0.00215	0.00205	0.00195
2.2	0.00186	0.00178	0.00169	0.00161	0.00154	0.00146	0.00139	0.00133	0.00126	0.00120
2.3	0.00114	0.00109	0.00103	0.00098	0.00094	0.00089	0.00085	0.00080	0.00076	0.00072
2.4	0.00069	0.00065	0.00062	0.00059	0.00056	0.00053	0.00050	0.00048	0.00045	0.00043
2.5	0.00041	0.00039	0.00037	0.00035	0.00033	0.00031	0.00029	0.00028	0.00026	0.00025
2.6	0.00024	0.00022	0.00021	0.00020	0.00019	0.00018	0.00017	0.00016	0.00015	0.00014
2.7	0.00013	0.00013	0.00012	0.00011	0.00011	0.00010	0.00009	0.00009	0.00008	0.00008
2.8	0.000075	0.000071	0.000067	0.000063	0.000059	0.000056	0.000052	0.000049	0.000046	0.000044
2.9	0.000041	0.000039	0.000036	0.000034	0.000032	0.000030	0.000028	0.000027	0.000025	0.000023

Appendix VIII

