Identification and Control of Recycle Process

by

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Dissertation submitted in partial fulfilment of
the requirement for the
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CERTIFICATION OF APPROVAL

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A project dissertation submitted to the
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Approved by,

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(Dr Lemma Dendena Tuffa)

UNIVERSITI TEKNOLOGI PETRONAS

TRONOH, PERAK

December 2012

CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources of persons.

(RAJA NORAKMAL BIN RAJA BAKHTIAR AFFENDY)

ABSTRACT

The recycle stream is important to increase and optimize the conversion of the feed and also the selectivity of the desired product. Even though recycle brings a lot of advantages, in the process dynamics and control point of view, it causes a lot of problem to the whole dynamics of the process. In this project, three control strategies are used to control the recycle stream in two CSTR connected in series. The control strategies are recycle compensator, Ziegler-Nichols tuning method and Skogestad tuning method. The performances of these control strategies and their combinations are compared based on certain aspects such as response time and offset value to find the best control strategy or combination of control strategies. To gain a more precise way to compare the performance of the control strategies, each one of them are analysed by using the integral error analysis which are the Integral of the absolute value of the error, Integral of the squared error, Integral of time-weighed absolute error.

ACKNOWLEDGEMENT

I wish to express my sincere and deepest gratitude to my final year project supervisor, **Dr. Lemma Dendena Tuffa** for his guidance and care throughout the completion of this project. Despite having a busy schedule as a lecturer, he never hesitates to assist me in the very best way he can. Thanks for his valuable advice, encouragement and patience, I am able to improve in so many ways especially in the control and dynamics field and also in operating the MATLAB software. I also would like to express my deepest appreciation to anyone who got involved directly or indirectly in the completion of this project.

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ABBREVIATIONS AND NOMENCLATURE

CSTR	Continuous Stirred-Tank Reactor
DS	Direct Synthesis
IAE	Integral of the Absolute value of the Error
ISE	Integral of the Squared Error
ITAE	Integral of Time-weighed Absolute Error
PI	Proportional and Integral
PID	Proportional, Integral and Derivative
RC	Recycle Compensator
SIMC	Simple Internal Model Control
Z-N	Ziegler-Nichols

CHAPTER 1

INTRODUCTION

1.1 Background of Study

In a process plant, the main objective is to convert the feed or reactant into a desired product. However, in the real-life situation, typically the reactor used in a process plant does not completely convert all of the reactant into the product. The unreacted feed is then purged out of the reactor along with the product. Not only this is economically unviable as the reactant is wasted, but this also causes difficulties later on in the process as the unreacted feed is required to be separated from the final product and treated before it is purged as a waste.

In order to prevent problems in the latter stages and to increase the economic efficiency, a recycle stream is introduced to the reactor system to recycle the unreacted feed. A recycle loop coupled with a reactor will generally contain a separation process in which unused reactants are separated from products. These reactants are then fed back into the reactor along with the fresh feed.

Reactions with recycle are very useful for a number of reasons, most notably because they can be used to improve the selectivity of multiple reactions occurring in the reactor, push a reaction beyond its equilibrium conversion, or speed up a catalytic reaction by removing the products. However, in a process control and dynamics point of view reaction with recycle could cause lots of problems.

Through the years, many controllers have been found to control a certain dynamics problems. In the recycle system, the most commonly used control strategy is by using recycle compensator. It is said that the recycle compensator can help to reduce the negative impact of the recycle stream to the dynamics of the plant. With the existence of the found controllers, comparisons and combinations of the controllers can be done to further improve the control of recycle process.

1.2 Problem Statement

Recycle process is important to increase the conversion of the feed. It is also required due to some environmental and economic constraints. However, although recycling has so many advantages, it also has negative impact on the whole process. Recycle streams causes various problems and dynamic phenomena such as the snowball effect and extremely slow response. Various studies have been conducted to generate a controller strategy to solve the problem with the recycle process. Control strategies to stabilize the problems caused by the recycle process are important to ensure the recycle stream can be used in the plant without causing problem to the whole dynamics. This way, the advantages of the recycle system can be fully utilized without worrying about the drawbacks that can be caused in the control and dynamics point of view. Studies have been conducted to control these kind of problems. The commonly used control strategy that used to counter the recycle dynamics problem is by using the recycle compensator. However, in this research comparison of this method to other controller and possibly a combination could be done to further improve the control of recycle process.

1.3 Objectives and Scope of Study

The objectives of this study are:

1.3.1 To compare the performance of various controller strategies based on for processes with recycle stream in a 2 CSTR system with recycle.

1.3.2 To analyse the performance of the controller strategies using on Integral of the absolute value of the error, Integral of the squared error, Integral of time-weighed absolute error.

The scope of this study is to compare the performance of various controller strategies to stabilize the problems caused by recycle stream. The controller strategies that will be studied are the recycle compensator, Ziegler-Nichols tuning method and

Skogestad tuning method. To compare the performance, some integral error analysis is used. This will be assisted by using simulation on the MATLAB software. To gain understanding on the controller strategies used by previous studies for processes with recycle stream, literature research is done.

CHAPTER 2

LITERATURE REVIEW

In this section, the effect of the recycle system to the dynamics of the whole process is explained in details. Several control strategies have been found through the literature research. The theory and derivation method of the reviewed control strategies are studied and explained. After the controller strategies have been reviewed, a case study is reviewed to pose as the process model for the testing of the controllers.

2.1 Recycle Process

The recycle stream is usually introduced to the output of a reactor to send back the unreacted feed back to the fresh feed stream.

Figure 2.1 shows an example of a block diagram of a plant with recycle. In the figure, 'G' represents the direct of forward path transfer function that describes the relationship between the output and the input. 'G_r' represents the recycle path transfer function and 'G_d' represents the disturbance transfer function. Disturbance is a variable in the process plant that can affect the process but the variable itself is not affected by the process and cannot be controlled.

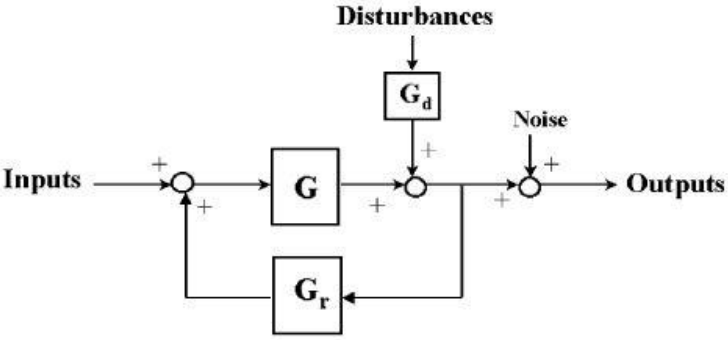


Figure 2.1 An example of a block diagram of a plant with recycle

Denoting the output as 'Y' and the input as 'U', the transfer function of the plant, according to the general expression of feedback control systems is as follows:

$$\frac{Y(s)}{U(s)} = \frac{G}{1 - GG_r} \quad (2.1)$$

Where $G = \frac{K_1 e^{-t_1 s}}{\tau_1 s + 1}$ and $G_r = \frac{K_2 e^{-t_2 s}}{\tau_2 s + 1}$, assuming the stable first order transfer function.

Substituting G and G_r into equation (2.1):

$$\frac{Y(s)}{U(s)} = \frac{K_1 e^{-t_1 s}(\tau_2 s + 1)}{(\tau_1 s + 1)(\tau_2 s + 1) - K_1 K_2 e^{-(t_1 + t_2)s}} \quad (2.2)$$

The presence of the deadtime term in in the denominator makes it difficult to design the controller. Most analytical control design algorithms cannot be applied to systems that do not have a rational denominator. For the case if the time delays t_1 and t_2 are zero, it is observed that when K_1K_2 is more than 1, the system becomes open-loop unstable (Lakshminarayanan & Takada, 2001).

2.2 Negative Impact of the Recycle Stream

The recycle process is found to cause lots of negative impacts on the whole system. Some of the identified impacts are as follows:

2.2.1 Increasing the response time, thus resulting in extremely slow response (Taiwo & Krebs, 1994) (Lakshminarayanan & Takada, 2001)
2.2.2 A small change in the disturbance variable causes a large change in the manipulated variable, also known as the “snowball effect” (Tremblay *et al.*, 2006) (Lakshminarayanan & Takada, 2001)
2.2.3 Higher process steady state sensitivity (Taiwo & Krebs, 1994)
2.2.4 Limit cycles (Madhukar *et al.*, 2005) (Lakshminarayanan & Takada, 2001)
2.2.5 Instability (Madhukar *et al.*, 2005) (Lakshminarayanan & Takada, 2001)

2.3 Controller Strategies for Processes with Recycle Stream

In order to solve the problem caused by the recycle stream, some controller strategies can be applied to the process. In this study, three strategies are studied which are the recycle compensator, Skogestad's tuning method and the direct synthesis method.

2.3.1 Recycle Compensator

According to Mészáros & Čirka (2009), a recycle compensator is a part of the regulator used to minimize or eliminate the effects of the recycle by suppressing the negative feedbacks. The compensator is introduced to eliminate recycle loop time constant. Figure 2.2 shows a block diagram of a system of a plant with recycle including a recycle compensator $\mathbf{G_{RC}}$. The variables **y**, **u**, **d** are controlled, manipulated, disturbance controller outputs respectively. $\mathbf{G_r}$***** is the transfer function of the recycle process.

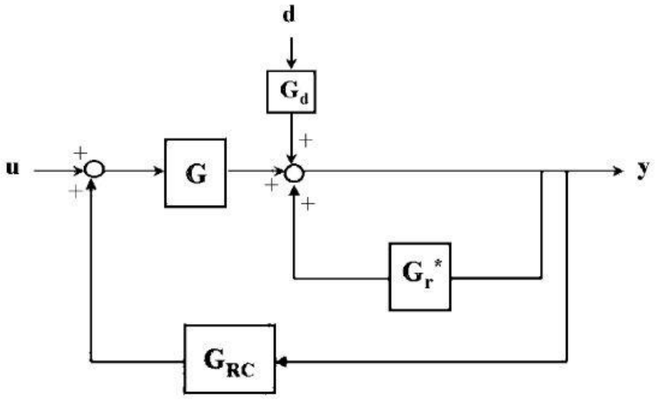


Figure 2.2 Block diagram of a system consisting of a plant with recycle and a recycle compensator

The open-loop plant output is given by

$$y(s) = \frac{G}{(I - G_r *)} u(s) + \frac{G_d}{(I - G_r *)} d(s) (2.3)$$

The compensator that totally cancels the negative effect of the recycle is known as the perfect recycle compensator (Taiwo & Krebs, 1994). Such compensator revert the transfer function into its dynamically favourable state, which is the original transfer function without recycle, that is

$$y(s) = Gu(s) + G_d d(s) (2.4)$$

To specify the recycle compensator, block diagram algebra is applied to the inner loop G in Figure 2.2 to give

$$y(s) = \frac{G}{1 - GG_{RC} - G_r *} u(s) + \frac{G_d}{1 - GG_{RC} - G_r *} d(s)(2.5)$$

By choosing G_{RC} to be $G_{RC} = \frac{-G_r *}{G}$ the expression$y(s) = Gu(s) + G_d d(s)$ is obtained. Therefore the design expression for the recycle compensator is given by

$$G_{RC} = \frac{-G_r *}{G} \ (2.6)$$

The recycle compensator is to handle internally generated disturbances to the process (Lakshminarayanan & Takada, 2001).

2.3.2 Skogestad Tuning Method

Skogestad's method is model-based, assuming that the mathematical model of the process which is the transfer function model is available. The controller parameters are expressed as functions of the process model parameters (Haugen, 2010).

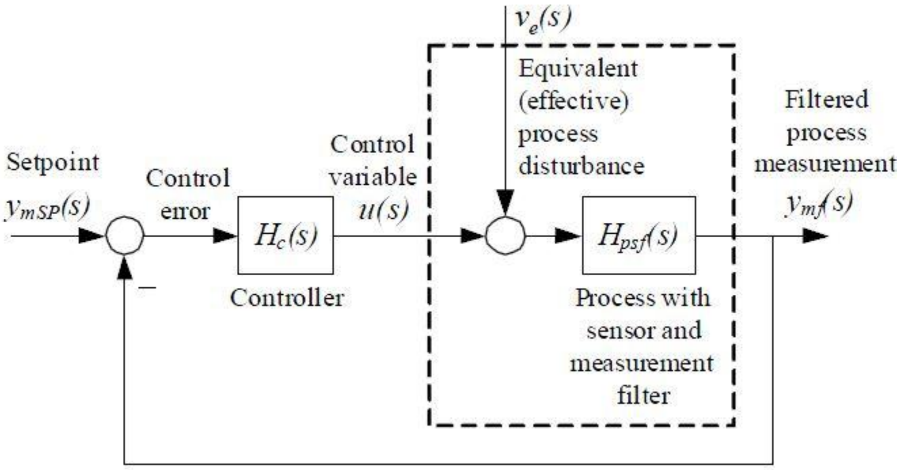


Figure 2.3 Block diagram of the control system in PID tuning with Skogestad's method

Figure 2.3 shows a block diagram of the control system in PID tuning with Skogestad's method. The transfer function H_{psf} (s) is a combined transfer function of the process, the sensor, and the measurement lowpass filter. It represents all the dynamics that affect the controller. For simplicity, this transfer function denoted as the "process transfer function".

The design principle of Skogestad's method includes the control system tracking transfer function T(s), which is the transfer function from the set point to the process measurement, is specified as a first order transfer function with time delay:

$$T(s) = \frac{y_{mf}(s)}{y_{mSP}(s)} = \frac{1}{T_C s + 1} e^{-\tau s} \quad (2.7)$$

Where T_C is the specified time constant of the control system and τ is the process time delay given by the process model. From the block diagram in Figure 2.3, the tracking transfer function is derived as:

$$T(s) = \frac{H_C(s)H_{psf}(s)}{1 + H_C(s)H_{psf}(s)} \quad (2.8)$$

Equating the equations (2.7) and (2.8):

$$\frac{H_C(s)H_{psf}(s)}{1 + H_C(s)H_{psf}(s)} = \frac{1}{T_C s + 1} e^{-\tau s} \quad (2.9)$$

In this equation, it is found that the only unknown is the controller transfer function H_c(s). By making some proper simplifying approximations to the time delay term, the controller becomes PID controller or a PI controller for the process transfer function assumed (Haugen, 2010).

2.3.3 Direct Synthesis Method

In this method, the controller design is based on a process model and a desired closed-loop transfer function. The Direct Synthesis (DS) approach provides valuable insights into the relationship between the process model and the resulting controller (Seborg *et al.*, 2004). Figure 2.4 shows a block diagram for standard feedback control system.

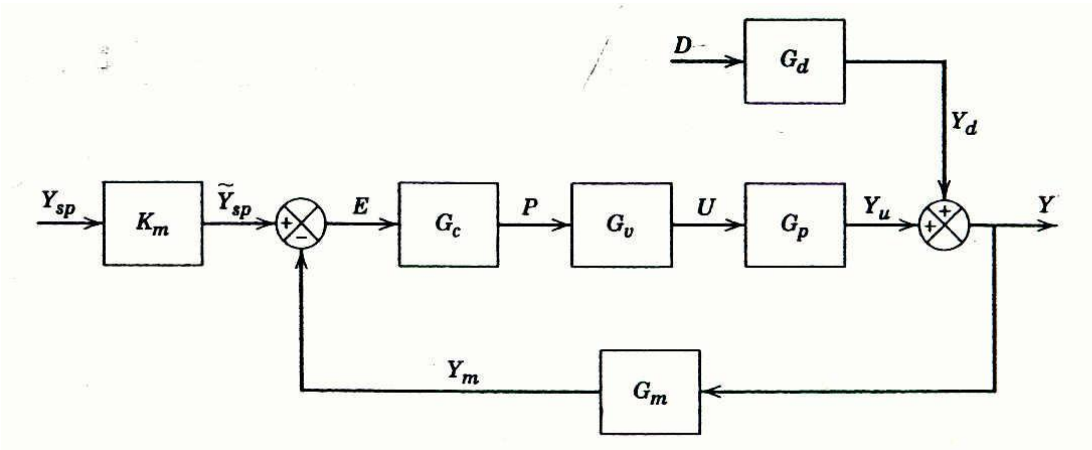


Figure 2.4 Block diagram for a standard feedback control system

The close-loop transfer function for set-point changes is derived as:

$$\frac{Y}{Y_{SP}} = \frac{K_m G_c G_v G_p}{1 + G_c G_v G_p G_m} \quad (2.10)$$

To make it simple, let $G = G_c G_v G_p$ and assume that $G_m = K_m$

$$\frac{Y}{Y_{SP}} = \frac{G_c G}{1 + G_c G} \quad (2.11)$$

Expressing equation (2.11) in terms of G_C:

$$G_c = \frac{1}{G}\left(\frac{\frac{Y}{Y_{SP}}}{1 - \frac{Y}{Y_{SP}}}\right) \quad (2.12)$$

Replacing G by $\tilde{G}$ and $\frac{Y}{Y_{SP}}$ by a desired closed-loop transfer function, $\left(\frac{Y}{Y_{SP}}\right)_d$ into equation (12)

$$G_c = \frac{1}{\tilde{G}}\left(\frac{\left(\frac{Y}{Y_{SP}}\right)_d}{1 - \left(\frac{Y}{Y_{SP}}\right)_d}\right) \quad (2.13)$$

Ideally, $\left(\frac{Y}{Y_{SP}}\right)_d = 1$ is desirable so that the controlled variable tracks set-point changes instantaneously without any error. This is also known as perfect control and cannot be achieved by feedback control. For processes without time delays, the first order model is more reasonable (Seborg *et al.*, 2004):

$$\left(\frac{Y}{Y_{SP}}\right)_d = \frac{1}{\tau_C s + 1} \quad (2.14)$$

Where τ_C is the desired closed-loop time constant. By substituting this into the previous equation

$$G_C = \frac{1}{\tilde{G}} \frac{1}{\tau_C s} \quad (2.14)$$

The term $\frac{1}{\tau_C s}$ provides integral control action and eliminates offsets (Seborg *et al.*, 2004). If the process transfer function contains a time delay θ, the desired closed-up transfer function is

$$\left(\frac{Y}{Y_{SP}}\right)_d = \frac{e^{-\theta s}}{\tau_C s + 1} \quad (2.15)$$

Combining equations (2.14) and (2.15):

$$G_C = \frac{1}{\tilde{G}} \frac{e^{-\theta s}}{\tau_C s + 1 - e^{-\theta s}} \quad (2.16)$$

By using Taylor series expansion$e^{-\theta s} \approx 1 - \theta s$:

$$G_C = \frac{1}{\tilde{G}} \frac{e^{-\theta s}}{(\tau_C + \theta)s} \quad (2.17)$$

From equation (2.17), it is found to have integral control action. For first order plus time delay model, equation (2.18) is substituted into equation (2.17)

$$\tilde{G} = \frac{K e^{-\theta s}}{\tau_C s + 1} \quad (2.18)$$

The substitution gives the following first order plus time delay model (2.19):

$$G_C = K_C \left(1 + \frac{1}{\tau_I s}\right) (2.19)$$

Where $K_C = \frac{1}{K} \frac{\tau}{\theta + \tau_C}$ and $\tau_I = \tau$

2.4 Case study

In order to compare the control strategies reviewed before, one case study is needed to ensure the comparison is feasible and legitimate. In this study, the case study chosen is two CSTR systems with recycle. Figure 2.5 shows the schematics of 2 CSTR systems with recycle. The two reactors are connected in series with the outlet of the second reactor recycled into the inlet of the first reactor. The two reactors are assumed to be well mixed by which first order irreversible reactions not accompanied by any heat effects occur.

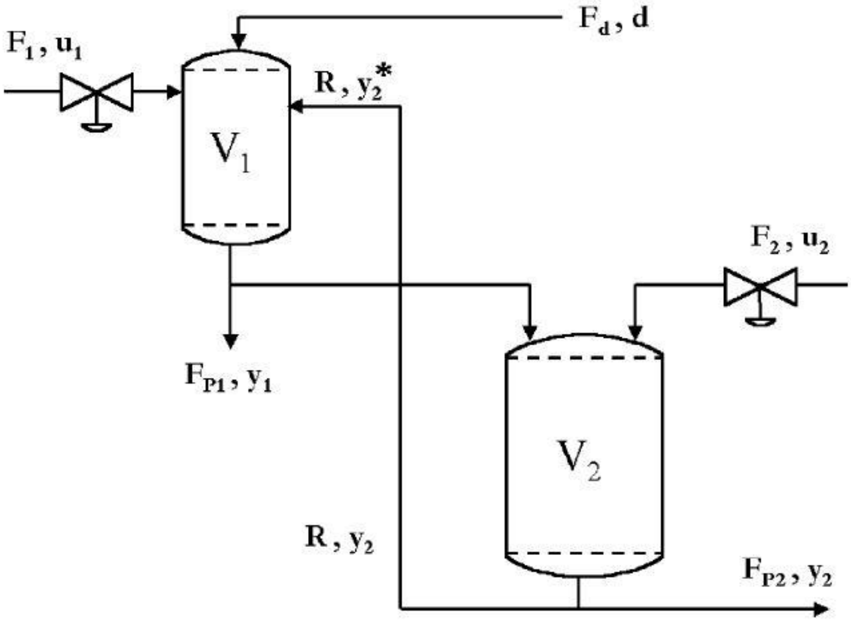


Figure 2.5 Schematic of 2 CSTR systems with recycle

The levels and flow rates are assumed to be constant, only the effect of the composition is taken into consideration. The control objective of this system is to maintain the reactor outlet compositions y_1 and y_2 at specified levels by manipulating the two feed compositions, u_1 and u_2. The main disturbance is the composition d of the stream entering the first reactor. The description of the variables and the values of the operating parameters are given in Table 2.1 (Lakshminarayanan & Takada, 2000).

Table 2.1 Variables and parameters for the 2 CSTR with recycle system

Parameter	Symbol	Value
Feed flow rate into reactor 1	F_1	1 m^3/min
Feed flow rate into reactor 2	F_2	0.5 m^3/min
Recycle flow rate	R	10 m^3/min
Disturbance flow rate	F_d	0.5 m^3/min
Product removal rate form Reactor 1	F_{p1}	1 m^3/min
Product removal rate from Reactor 2	F_{p2}	1 m^3/min
Composition of stream F_1	u_1	2 kmol/min
Composition of stream F_2	u_2	3 kmol/min
Composition of disturbance stream	d	1 kmol/min
Reactor 1 outlet composition	y_1	1 kmol/min
Reactor 2 outlet composition	y_2	1 kmol/min
Recycle stream composition at entrance of Reactor 1	y_2^*	1 kmol/min
Volume of Reactor 1	V_1	1 m^3
Volume of Reactor 2	V_2	10 m^3
Measurement delay in composition sensors	θ_m	1 min
Recycle delay (outlet of Reactor 2 to inlet of Reactor 1)	θ_r	2 min
Kinetic rate constant (Reactor 1)	k_1	1 min^{-1}
Kinetic rate constant (Reactor 2)	K_2	0.1 min^{-1}

Main hypotheses of the process are (Scali & Ferrari, 1999):

- The two reactors are perfectly stirred and a first order irreversible reaction, with kinetic constant k_1 and k_2.
- Levels, temperatures and flow rates are constant.
- The control objective is to maintain constant output of the two compositions y_1 and y_2, by manipulating the feed compositions u_1 and u_2.
- The composition d of the uncontrolled flow to the first reactor is the main disturbance.

- Assuming perfect measurement of composition, with time delay of θ_m.

Line (1) is without recycle and line (2) is with recycle (Scali & Ferrari, 1999). It is found that the recycle stream greatly affects the dynamics of the plant. Therefore, this case study is suitable to be used in this study to compare the controller strategies that has been reviewed.

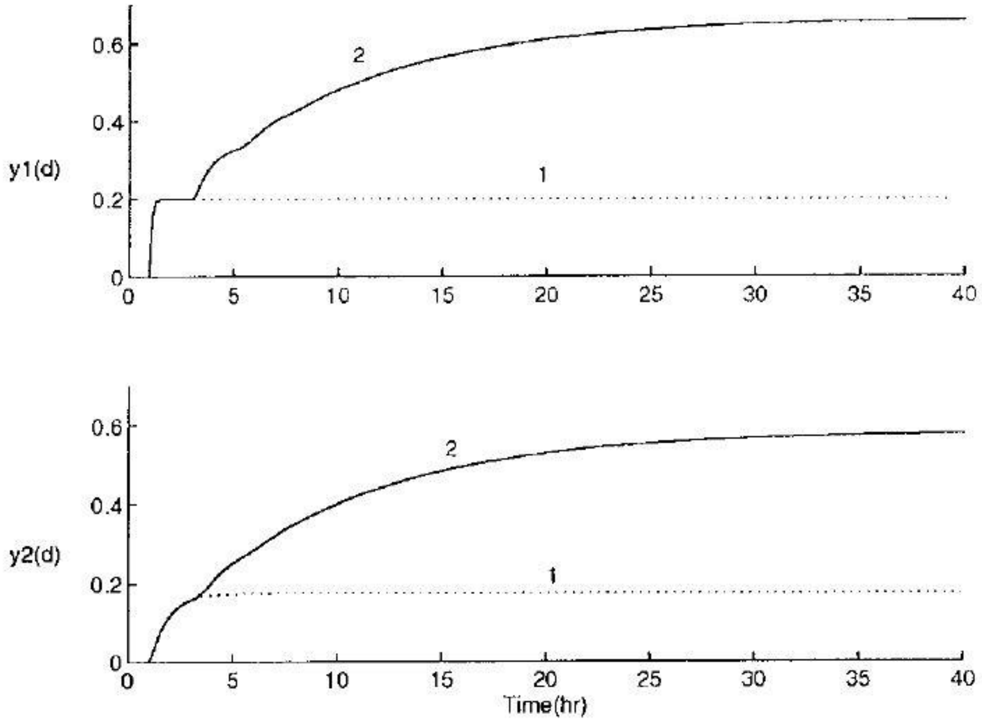
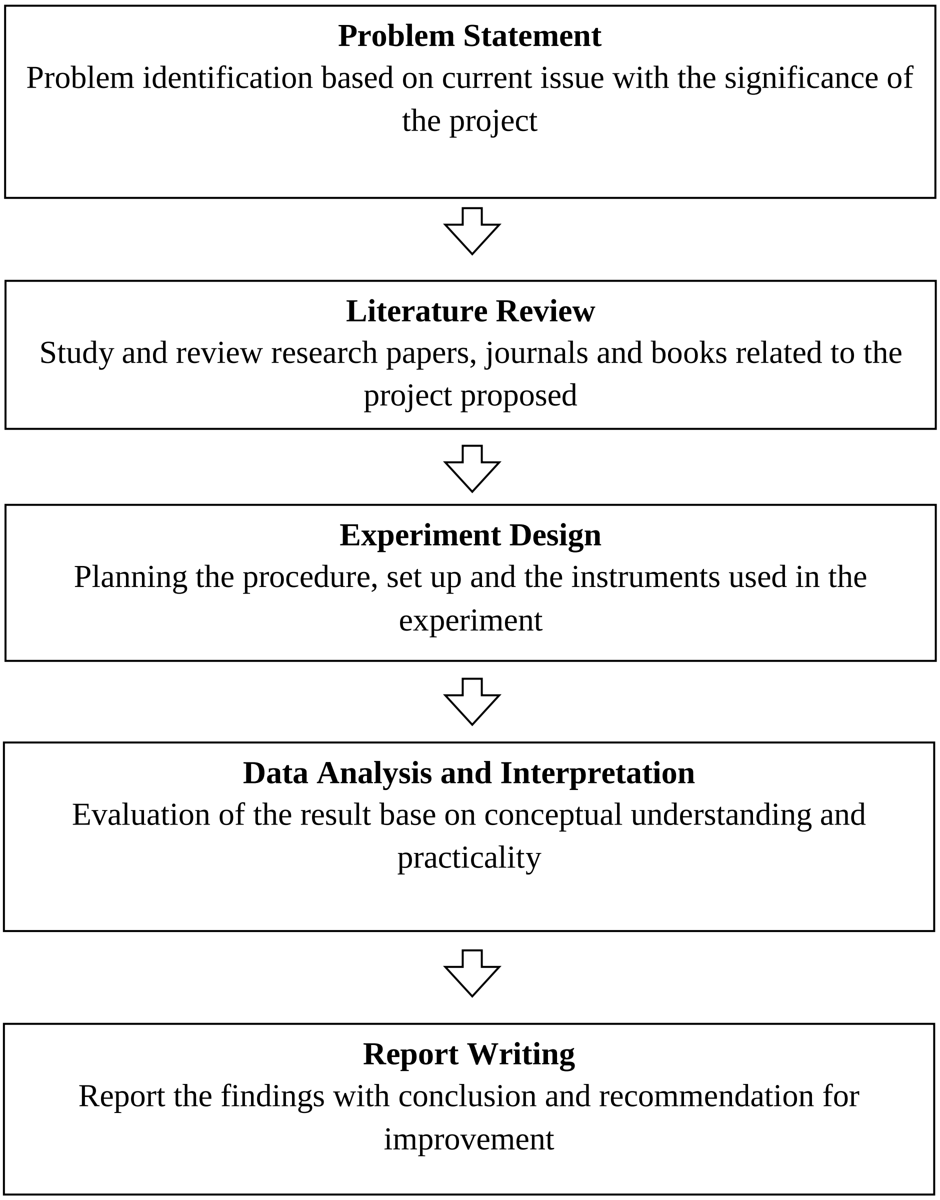


Figure 2.6 Open loop response of the two output concentrations (1) without recycle; (2) with recycle

CHAPTER 3

METHODOLOGY

3.1 General Research Methodology



Problem Statement
Problem identification based on current issue with the significance of the project

Literature Review
Study and review research papers, journals and books related to the project proposed

Experiment Design
Planning the procedure, set up and the instruments used in the experiment

Data Analysis and Interpretation
Evaluation of the result base on conceptual understanding and practicality

Report Writing
Report the findings with conclusion and recommendation for improvement

3.2 Project Activities

The study will be conducted using the MATLAB simulation. Various controller strategies will be selected from the literature review and their performance will be tested using simulation of 2 CSTR systems with recycle.

Firstly, the transfer function of the system is derived based on the case study found in literature review. After the transfer function is derived, the block diagram is generated and the system is tested to find out the significance of the effect of the recycle stream to the dynamics of the system. If the recycle stream shows a significant effect, the system will be used. If otherwise, other system will be searched through literature.

After the case study block diagram and transfer function is derived, the system will be added with the derived control strategies to eliminate the effect of recycle stream on the system. The best control strategies will be selected based on their performance in the simulation.

In order to compare the three control strategies in a more precise manner, they are compared by using tuning relations based on integral error criteria. Three integral control criteria will be used in this project are (Seborg *et al.*, 2004):

3.2.1 Integral of the absolute value of the error (IAE)

$$IAE = \int_0^\infty |e(t)|dt$$

3.2.2 Integral of the squared error (ISE)

$$ISE = \int_0^\infty e(t)^2 dt$$

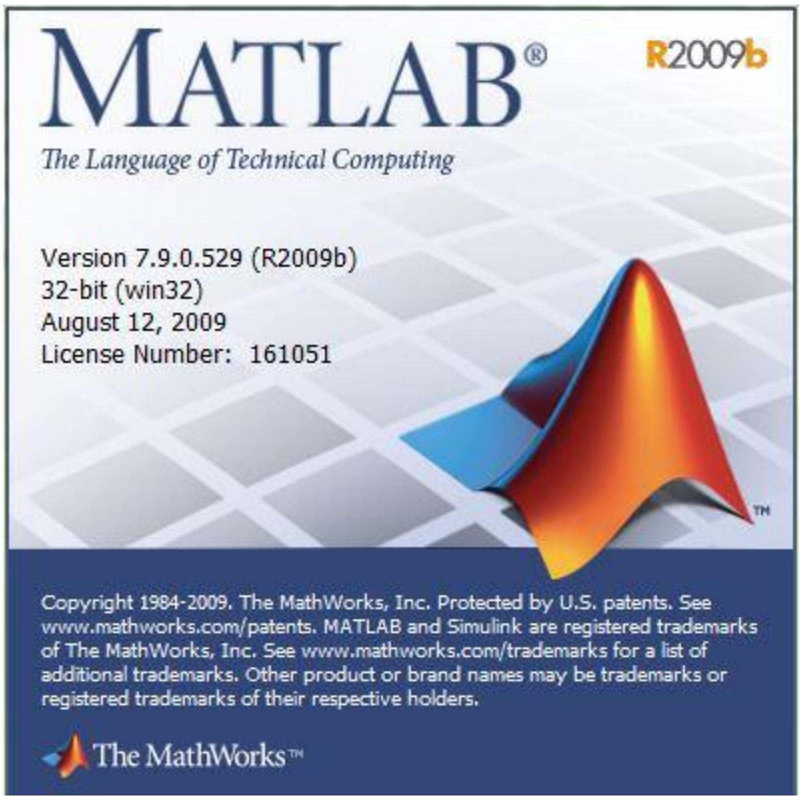
3.2.3 Integral of time-weighed absolute error (ITAE)

$$ITAE = \int_0^\infty t|e(t)|dt$$

These integral control criteria will be used to compare the performance of the recycle compensator, Ziegler-Nichols tuning method and Skogestad tuning method. The integral error values are compared and discussed for any improvement and modifications.

3.3 Tools/software

In this project, most of the works that will be done involves simulating the transfer functions. To assist the simulation, the software that will be used is the MATLAB Version 7.9.0.529 (R2009b).



Version 7.9.0.529 (R2009b)
32-bit (win32)
August 12, 2009
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Figure 3.1 the MATLAB software

3.4 Gantt Chart

Table 3.1 Gantt chart of Final Year Project (FYP)

		FYP 1															FYP 2														
No	Detail/Week	1	2	3	4	5	6	7		8	9	10	11	12	13	14	1	2	3	4	5	6	7		8	9	10	11	12	13	14
1	Selection of project topic	🟦	🟦						Mid-semester break															Mid-semester break							
2	Preliminary research work		🟦	🟦	🟦	🟦																									
3	Submission of Extended Proposal Defence						🟦																								
4	Preparation for proposal defence						🟦	🟦		🟦																					
5	Proposal Defence									🟦	🟦																				
6	Selection of case study											🟦	🟦	🟦																	

7	Review on integral control criteria to compare the performance of control strategies											🟦	🟦	🟦																	
8	Preparation of Interim Report													🟦																	
9	Submission of Interim Draft Report														🟦																
10	Submission of Interim Report															🟦															
11	Derivation of system transfer function and block diagram																🟨	🟨	🟨												

12	Derivation of recycle compensator and Z-G tuning method																			🟨	🟨	🟨	🟨								
13	Submission of Progress report																								🟨						
14	Derivation of SIMC tuning method parameters																									🟨					
15	SIMC testing																									🟨	🟨	🟨			
16	Integral error analysis and comparison of control strategies																											🟨	🟨		
17	Report-writing																													🟨	
18	Submission of dissertation																														🟨

CHAPTER 4

RESULT AND DISCUSSION

4 RESULTS AND DISCUSSION

4.1 Derivation of Transfer Functions and Block Diagram

From Figure 2.5, the mass balance of the two reactors is expressed as:

$$F_1 u_1 + F_d d + R y_2 = (F_1 + F_d + R) y_1 + V_1 k_1 y_1 + V_1 \frac{dy_1}{dt} \text{ (4.1)}$$

$$F_2 u_2 + \left(F_1 + F_d + R - F_{p1}\right) y_1 = (F_{p2} + R) y_2 + V_2 k_2 y_2 + V_2 \frac{dy_2}{dt} \text{ (4.2)}$$

The two concentrations in time can be expressed in the following equations:

$$\frac{dy_1}{dt} = \frac{F_1}{V_1} u_1 + \frac{F_d}{V_1} d + \frac{R}{V_1} y_2 - \left(\frac{(F_1 + F_d + R)}{V_1} + k_1\right) y_1 \text{ (4.3)}$$

$$\frac{dy_2}{dt} = \frac{F_2}{V_2} u_2 + \frac{\left(F_1 + F_d + R - F_{p1}\right)}{V_2} y_1 - \left(\frac{\left(F_{p2} + R\right)}{V_2} + k_2\right) y_2 \text{ (4.4)}$$

The values of the variables and parameters in Table 2.1 are substituted in equations (4.3) and (4.4) to give the following equations:

$$\frac{dy_1}{dt} = u_1 + 0.5d + 10y_2 - 12.5y_1 \text{ (4.5)}$$

$$\frac{dy_2}{dt} = 0.05u_2 + 1.05y_1 - 1.2y_2 \text{ (4.6)}$$

Laplace transform is applied on equation (4.5) and further simplified to give:

$$sY_1(s) = U_1(s) + 0.5D(s) + 10Y_2(s) - 12.5Y_1(s)$$

$$(s + 12.5)Y_1(s) = U_1(s) + 0.5D(s) + 10Y_2(s)$$

$$Y_1(s) = \frac{1}{(s + 12.5)} U_1(s) + \frac{0.5}{(s + 12.5)} D(s) + \frac{10}{(s + 12.5)} Y_2(s)$$

$$Y_1(s) = \frac{0.08}{(0.08s + 1)} U_1(s) + \frac{0.04}{(0.08s + 1)} D(s) + \frac{0.8}{(0.08s + 1)} Y_2(s) \text{ (4.7)}$$

Equation (4.6) is applied with Laplace transform and simplified to give:

$$sY_2(s) = 0.05U_2(s) + 1.05Y_1(s) - 1.2Y_2(s)$$

$$(s+1.2)Y_2(s) = 0.05U_2(s) + 1.05Y_1(s)$$

$$Y_2(s) = \frac{0.05}{(s+1.2)}U_2(s) + \frac{1.05}{(s+1.2)}Y_1(s)$$

$$Y_2(s) = \frac{0.0417}{(0.8333s+1)}U_2(s) + \frac{0.875}{(0.8333s+1)}Y_1(s) \; (4.8)$$

With added time delays, the transfer equations (4.7) and (4.8) become:

$$Y_1(s) = \frac{0.08}{(0.08s+1)}e^{-s}U_1(s) + \frac{0.04}{(0.08s+1)}e^{-s}D(s) + \frac{0.8}{(0.08s+1)}e^{-2s}Y_2(s) \; (4.9)$$

$$Y_2(s) = \frac{0.0417}{(0.8333s+1)}e^{-s}U_2(s) + \frac{0.875}{(0.8333s+1)}Y_1(s) \; (4.10)$$

By inserting equation (4.9) into (4.10), the transfer function of the whole system is:

$$Y_2(s) = \frac{0.07}{(0.0667s^2 + 0.9133s + 0.3)e^{-s}}U_1(s) + \frac{0.0417(0.08s+1)}{(0.0667s^2 + 0.9133s + 0.3)e^{-s}}U_2(s) + \frac{0.035}{(0.0667s^2 + 0.9133s + 0.3)e^{-s}}D(s) \; (4.11)$$

The block diagram of these transfer equations are expressed in the block diagram shown in figure 4.1. Equation 4.11 shows the transfer function of the process. Here, the transfer function explains the relations between inputs U_1, U_2 and D and output Y_2.

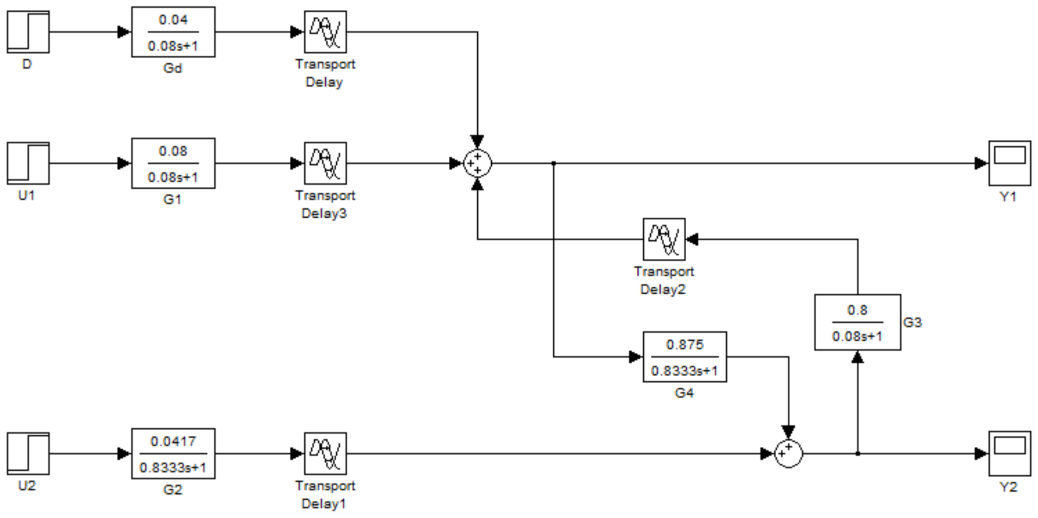


Figure 4.1 Block diagram of 2 CSTR with recycle stream

4.2 Step Test on the Recycle System

In order to test the significance of the recycle stream on the whole dynamics of the plant, a step test is conducted. The cases of the tests are shown in Table 4.1. To test the dynamics of the recycling system, the inputs U1, U2 and disturbance D are manipulated as follows:

Table 4.1 Cases for step test

Case	Step values			Observed output
	U1	U2	D	
1	1	0	0	Y1
2	1	0	0	Y2
3	0	1	0	Y1
4	0	1	0	Y2
5	0	0	1	Y1
6	0	0	1	Y2

The graph resulting from the listed cases in Table 4.1 is shown in Figures 4.2, 4.3, 4.4, 4.5, 4.6 and 4.7.

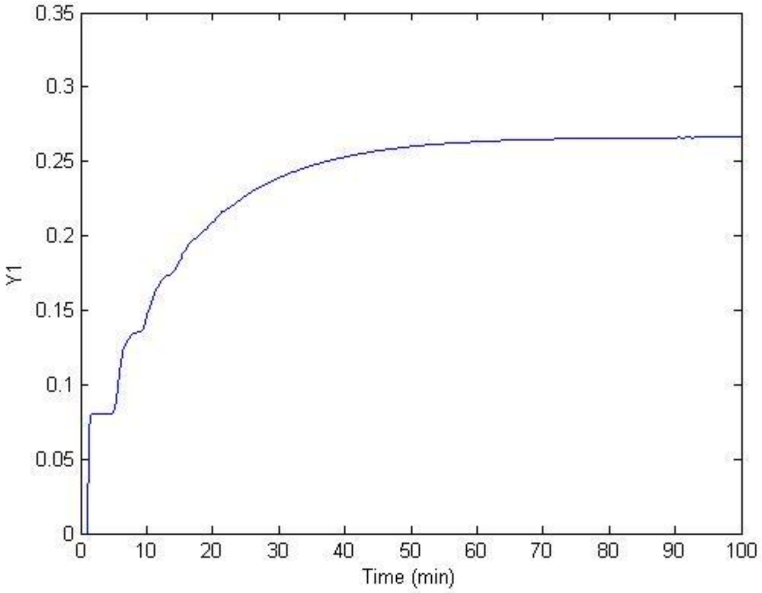


Figure 4.2 Case 1

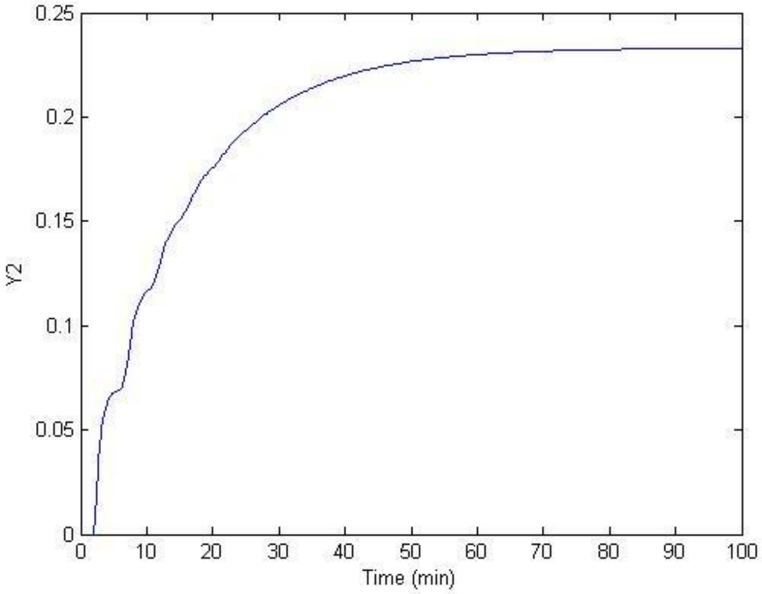


Figure 4.3 Case 2

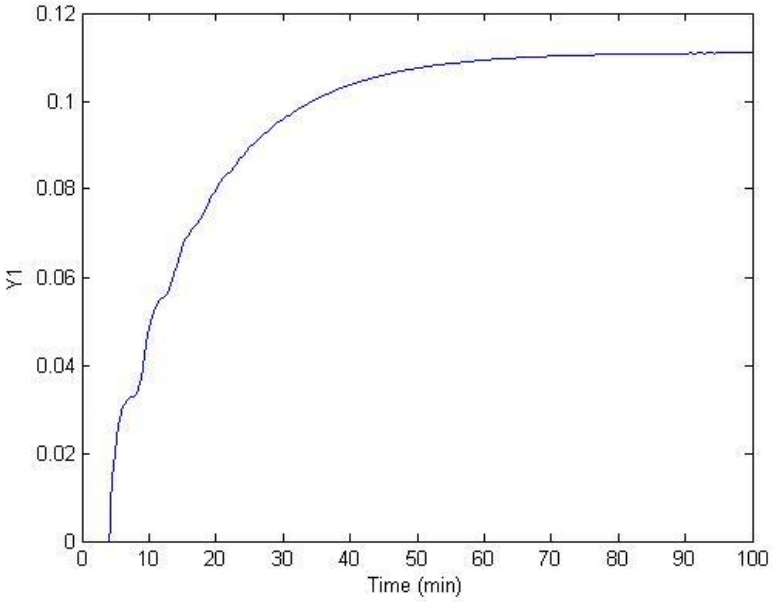


Figure 4.4 Case 3

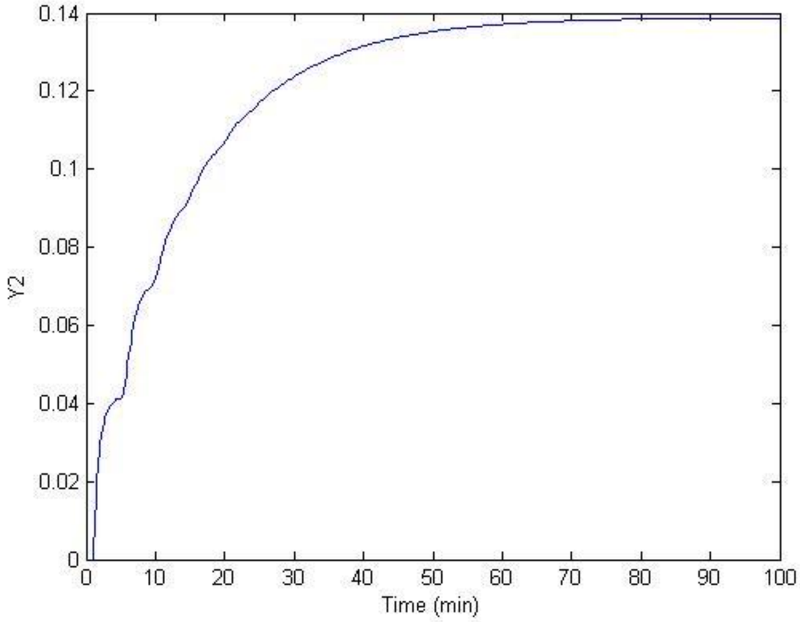


Figure 4.5 Case 4

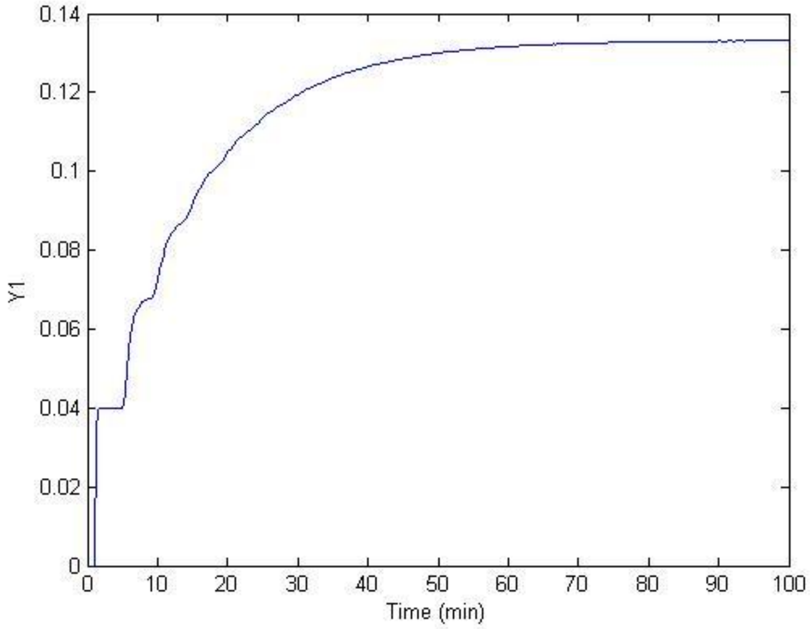


Figure 4.6 Case 5

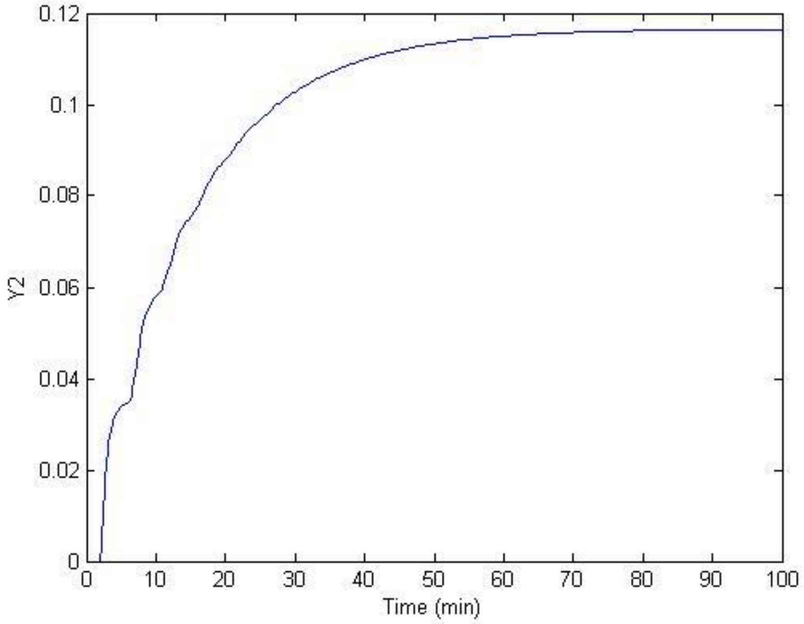


Figure 4.7 Case 6

The system is also tested without the recycle stream to compare and identify the effect of the recycle stream on the whole dynamics of the plant. Figure 4.8 and Figure 4.9 show the disturbance step change response for system with and without recycle for outputs Y1 and Y2 respectively.

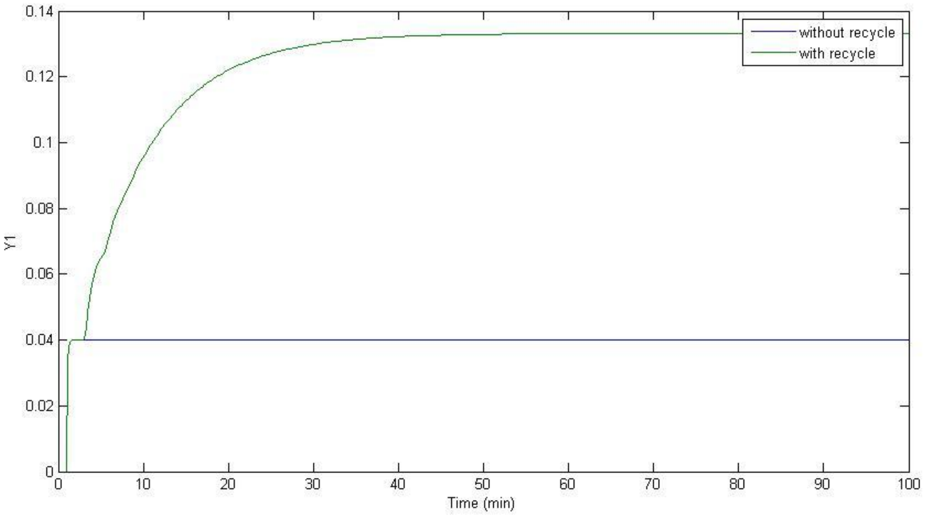


Figure 4.8 Response in disturbance step change for output Y1 for process with and without recycle

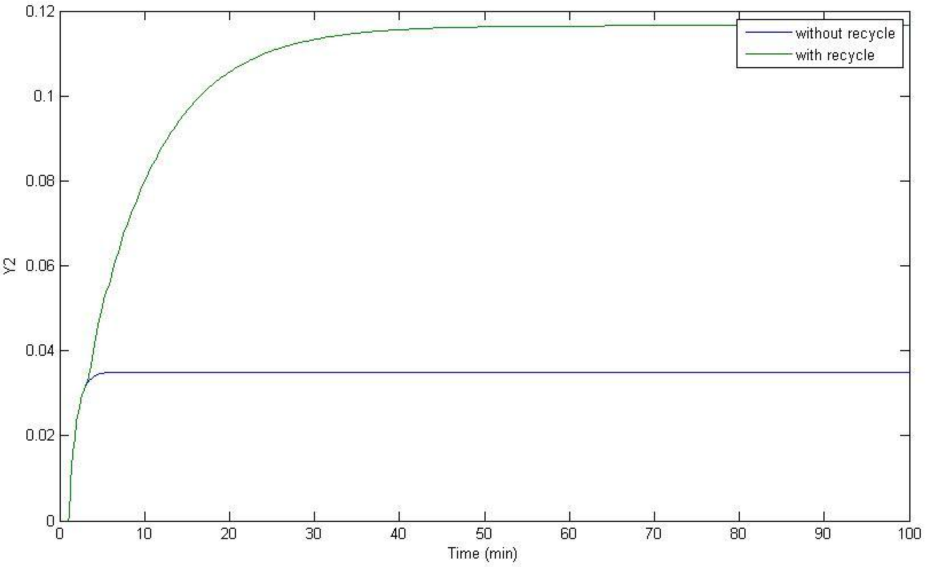


Figure 4.9 Response in disturbance step change for output Y2 for process with and without recycle

From Figures 4.8 and 4.9, it is found that the recycle stream affects the dynamics greatly in terms of offset and response time. From figure 4.9, it is observed that process with recycle shows a longer response time at about 40 minutes compared to without recycle at about 4 minutes. This shows that the recycle stream affect the dynamics of the whole process greatly and some control strategies are needed to counter this effect.

4.3 The Recycle Compensator

The first control strategy that will be used to overcome the problems caused by the recycle system is by using the recycle compensator. The recycle compensator uses a mathematical term that could simplify the transfer function of the recycle system as if the recycle stream does not exist. This will eventually cancel out the effects of the recycle stream on the dynamics of the whole plant.

4.3.1 Derivation and Block Diagram

In order to derive the transfer function, the block diagram of the system with recycle compensator is drawn. The transfer function is then derived based on the diagram. Figure 4.10 shows the block diagram of the system with recycle and recycle compensator.

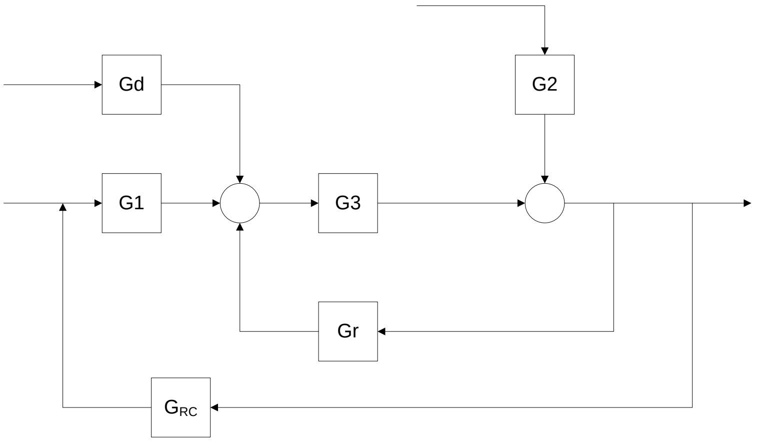


Figure 4.10 Block diagram of process with recycle and recycle compensator

In order to ease the derivation, the block diagram is further simplified into Figure 4.11, where $G_R = \frac{G_3}{1-G_r G_3}$

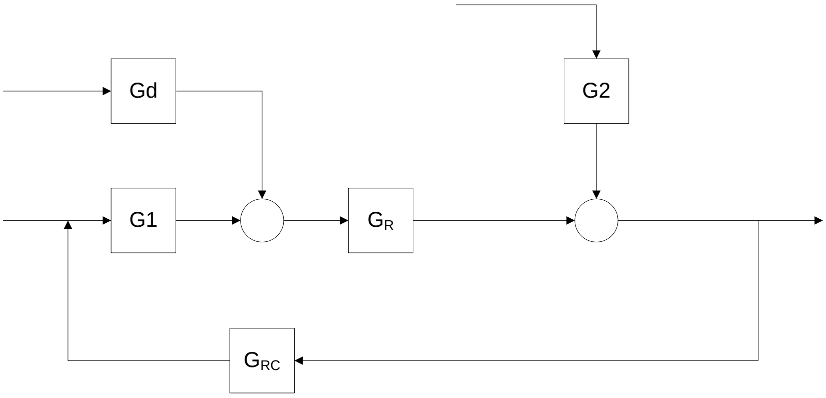


Figure 4.11 Simplified block diagram of process with recycle and recycle compensator

From Figure 4.11, the derived closed-loop transfer function is as follows:

$$Y_2(s) = \frac{G_1 G_R}{1 - G_1 G_R G_{RC}} U_1(s) + \frac{G_d G_R}{1 - G_1 G_R G_{RC}} D(s) + \frac{G_2(1 - G_r G_3)}{1 - G_1 G_R G_{RC}} U_2(s) \ (4.12)$$

Where $G_R = \frac{G_3}{1 - G_r G_3}$

By substituting the value of GR into equation (4.12) to give the following equation (4.13):

$$Y_2(s) = \frac{G_1 G_3}{1 - G_r G_3 - G_1 G_3 G_{RC}} U_1(s) + \frac{G_3 G_d}{1 - G_r G_3 - G_1 G_3 G_{RC}} D(s) + \frac{G_2(1 - G_r G_3)^2}{1 - G_r G_3 - G_1 G_3 G_{RC}} U_2(s) \ (4.13)$$

By choosing$G_{RC} = -\frac{G_r}{G_1}$, the denominator of the closed-loop transfer function can be cancelled out to 1, converting equation (4.12) into the general form without the effect of the recycle stream, as shown below:

$$if\ G_{RC} = -\frac{G_r}{G_1}$$

$$1 - G_r G_3 - G_1 G_3 G_{RC}$$

$$1 - G_r G_3 - G_1 G_3 \left(-\frac{G_r}{G_1}\right) = 1$$

By substituting the values of G_r and G_1 from equation (4.9), the recycle compensator is obtained as:

$$G_{RC} = -\frac{G_r}{G_1}$$

$$G_{RC} = -\frac{\dfrac{0.8}{0.08s + 1}e^{-2s}}{\dfrac{0.08}{0.08s + 1}e^{-s}} = -10e^{-s}$$

4.3.2 Step Test

In order to test the performance of the compensator, a series of step tests are conducted and the response of the system with recycle compensator is compared with system without recycle compensator, system with feedback controller and system with feedback controller and recycle compensator.

Figures 4.12 and 4.13 shows the graph of disturbance step change response for outputs y_1 and y_2 comparing for system with and without recycle compensator.

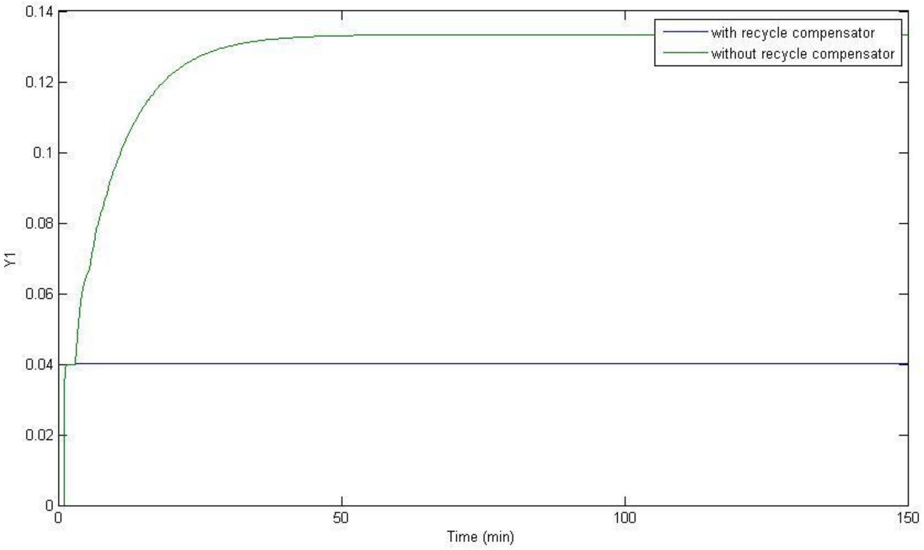


Figure 4.12 Disturbance step change for system with recycle compensator and without recycles compensator for output y_1

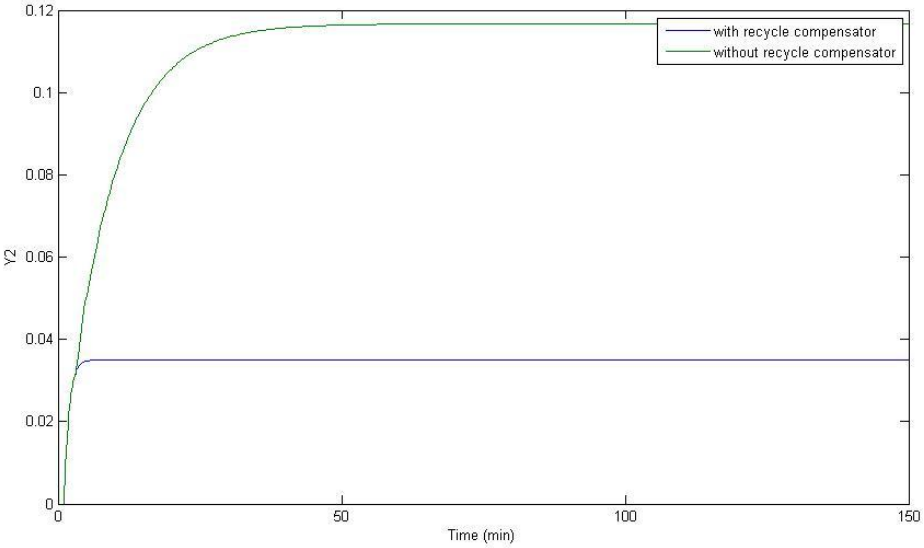


Figure 4.13 Disturbance step change for system with recycle compensator and without recycle compensator for output y_2

From Figures 4.12 and 4.13, it is found that the recycle compensator completely cancels out the effect of the recycle stream. It is also observed that the graph for system with recycle compensator is very similar to the system without recycle as shown in Figures 4.8 and 4.9.

4.4 Ziegler-Nichols Tuning Method

To further investigate the effect of the recycle compensator on the dynamics of the system, a comparison is done in case where a feedback controller is included in the system. This is to study the effect of the recycle compensator on the dynamics of a recycle system with feedback control systems. The values of K_c and τ_I are obtained from a paper by Lakshminarayanan & Takada (2001) as shown in the Table 4.2. The tuning method used is the Ziegler-Nichols tuning rules with fundamental model.

Table 4.2 Controller parameters for the two CSTR with recycle system (Lakshminarayanan & Takada, 2001)

Control configuration	Feedback controller 1		Feedback controller 2	
	K_{c1}	τ_{I1}	K_{c2}	τ_{I2}
Feedback + recycle compensator	3.6	0.58	16	1.33
Feedback	4.63	1.89	17	2.29

Figure 4.14 shows the block diagram of the two CSTR with recycle system with feedback controllers and recycle compensator in the MATLAB software. The parameters used for the feedback controller follows the values in Table 4.2. From this system, the performance is tested with a series of step tests.

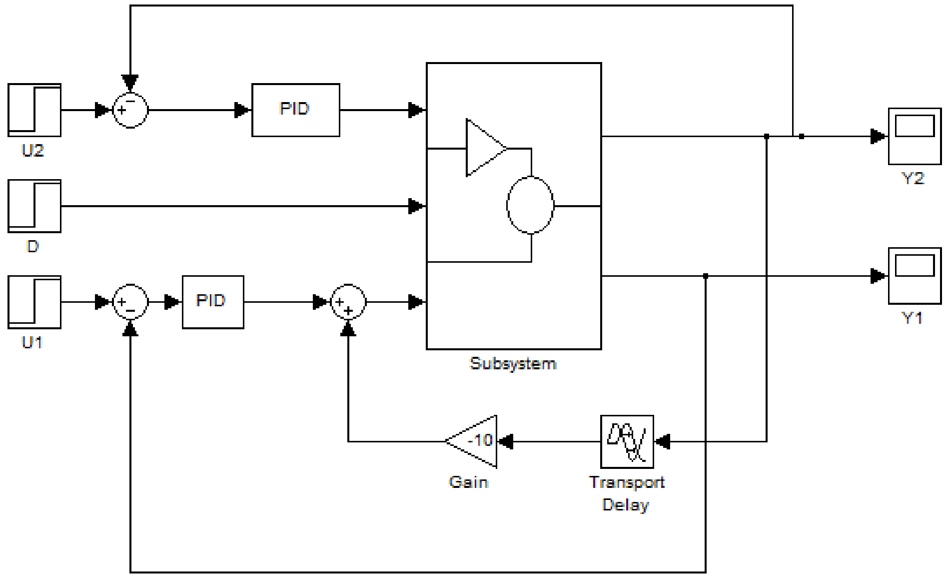


Figure 4.14 block diagram of the two CSTR with recycle system with feedback controllers and recycle compensator in the MATLAB software

Figures 4.15 and 4.16 show the closed-loop performance for a step change in disturbance with the control setting shown in Table 4.2. From the figures, it is found that the feedback controlled system without recycle compensator for both outputs y_1 and y_2 show more oscillation than the feedback control system with recycle compensator. The feedback control system with recycle compensator also shows faster response time and it appears to reach the desired set point input faster than that the feedback controlled system without recycle compensator. However, it is also found that the feedback controlled system with recycle compensator undergo a slightly larger offset than feedback controlled system without recycle compensator before reaching the set point value.

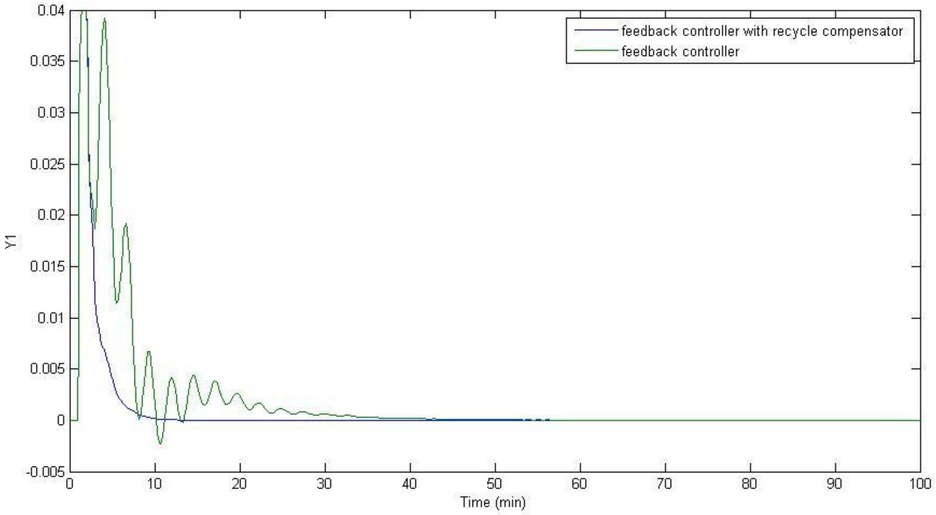


Figure 4.15 Performance for disturbance step change for output Y1

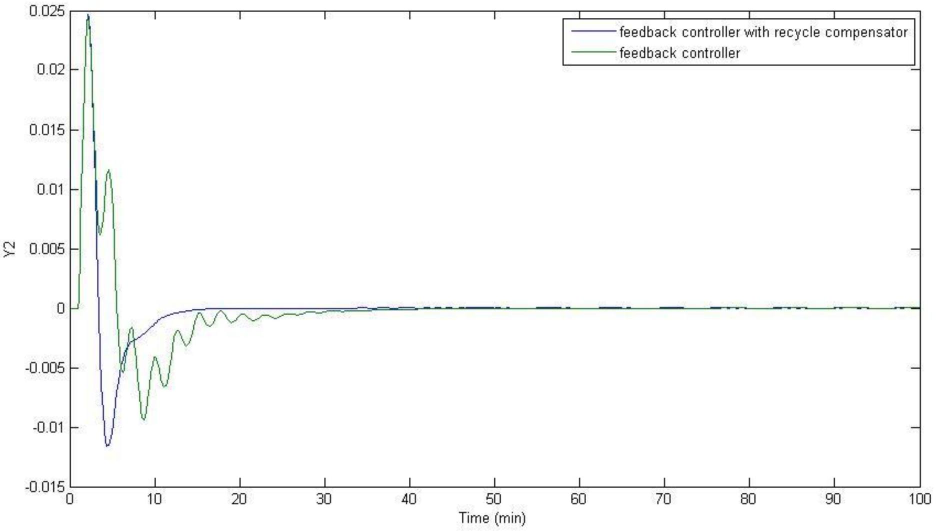


Figure 4.16 Performance for disturbance step change for output Y2

4.4.1 Conclusion

From the tests that have been conducted, it is found that the recycle compensator did a very good job in eliminating the effects caused by the recycle stream. It is found that by including the recycle compensator to the system, the dynamics of the system is completely the same as if the recycle stream is not in the system. For systems with feedback controllers with Ziegler-Nichols tuning, it is found that the recycle compensator has significantly improved the dynamics by reducing the oscillation of the response as well as decreasing the response time.

4.5 Skogestad Tuning Method

4.5.1 Tuning of feedback controllers

Based on the Skogestad Tuning Method or also known as simple internal model controller (SIMC) tuning method, the calculated parameters of the feedback controllers are obtained as in Table 4.3.

Table 4.3 Parameters for feedback controllers using SIMC

	$K_{C,1}$	$\tau_{I,1}$	$\tau_{D,1}$	$K_{C,2}$	$\tau_{I,2}$	$\tau_{D,2}$
PI	0.04	0.08	-	0.0209	0.0417	-
PID	0.3867	0.58	0.0690	0.3611	0.5417	0.3849

4.5.2 Comparison of PI and PID controllers

Figures 4.17 and 4.18 shows the disturbance step change of the SIMC tuning using PI and PID controllers for the outputs Y1 and Y2 respectively. From the graph, it is found that PID controller shows a slightly better response. The PID controller produces less offset for Y1 output, but higher offset for output Y2. In both of the outputs, it is found that the PID controller shows a slightly faster response compared to the PI controller.

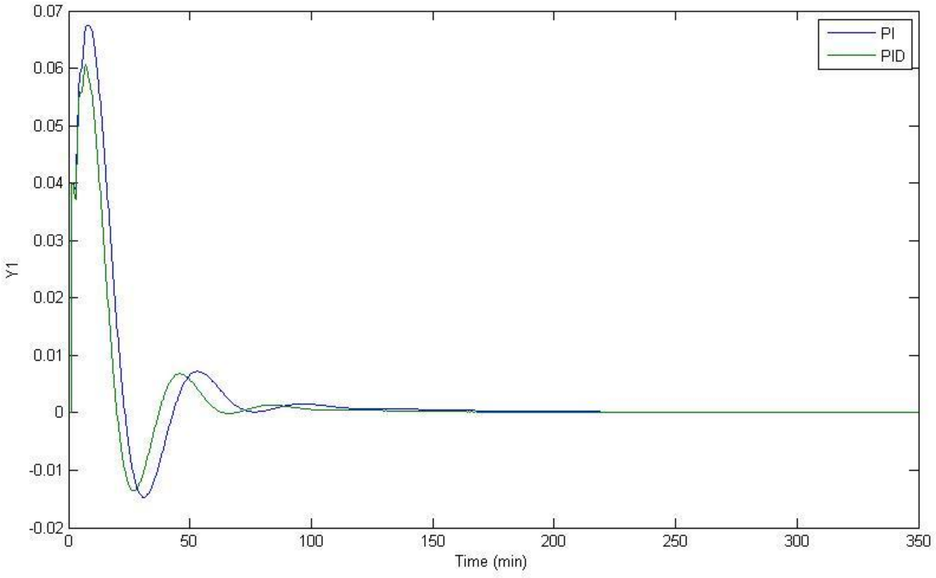


Figure 4.17 SIMC tuning for PI and PID for output Y1

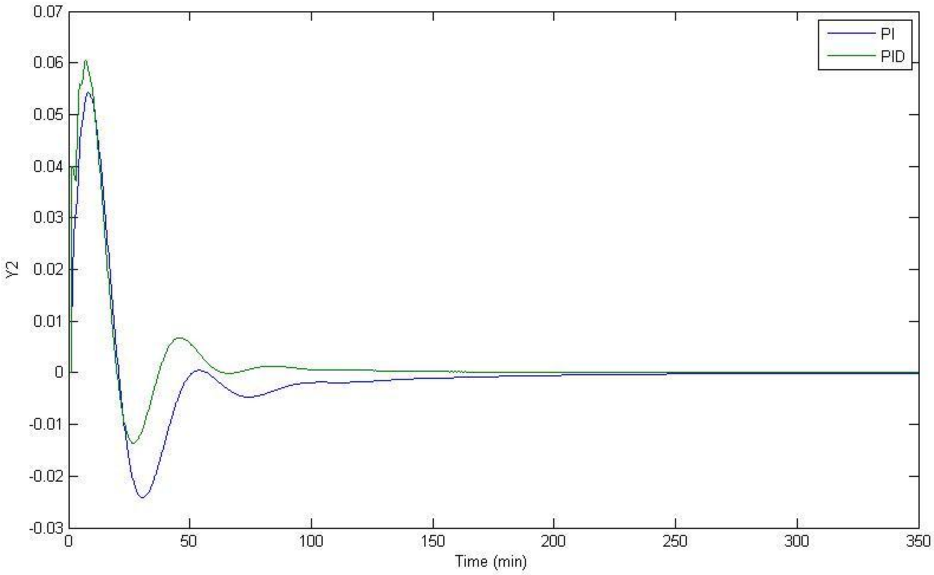


Figure 4.18 SIMC tuning for PI and PID for output Y2

From the comparison, it is found that the PID controller with SIMC tuning shows better response than PI controller with SIMC tuning.

4.5.3 SIMC Tuning Method with Recycle compensator

Based on the previous tuning method, it is found that by adding the recycle compensator to the plant model, the response significantly improved. In order to further investigate the effect of the recycle compensator to the dynamic of the plant model, the recycle compensator is added.

4.5.3.1 PI controller

Figures 4.19 and 4.20 show the disturbance step change response for PI SIMC tuning method with and without recycle compensator for the output Y1 and Y2 respectively. From the graph, it is found that the response time of PI SIMC tuning method is significantly longer than the Ziegler-Nichols tuning method, which is about 150 minutes. However, SIMC tuning methods shows fewer oscillations, but a larger offset. The addition of the recycle compensator slightly improves the response by reducing the oscillation, response time and offset for both outputs Y1 and Y2.

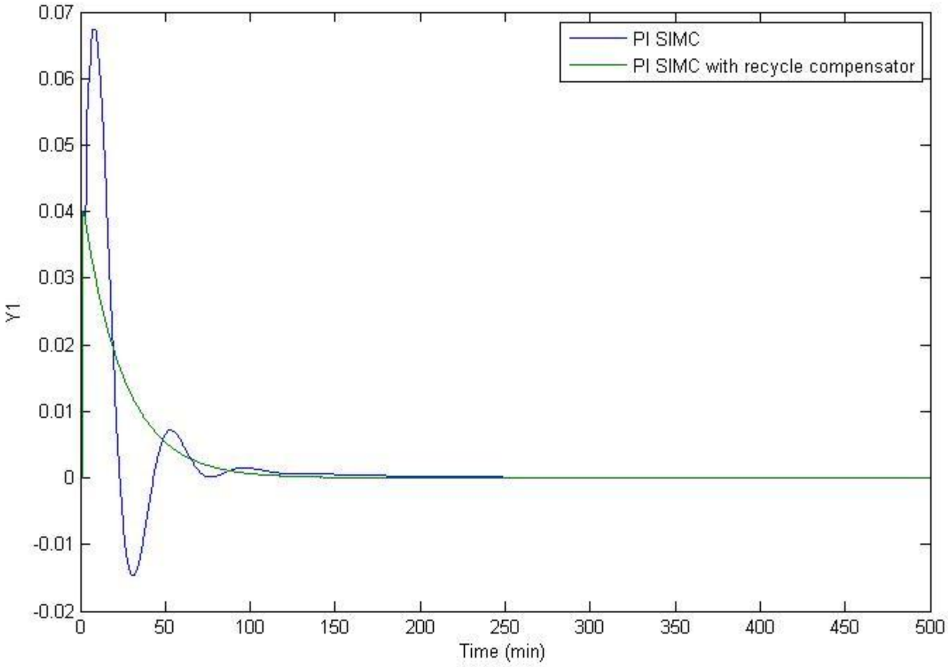


Figure 4.19 Disturbance step changes for PI controller using SIMC tuning method with and without recycle compensator for output Y1

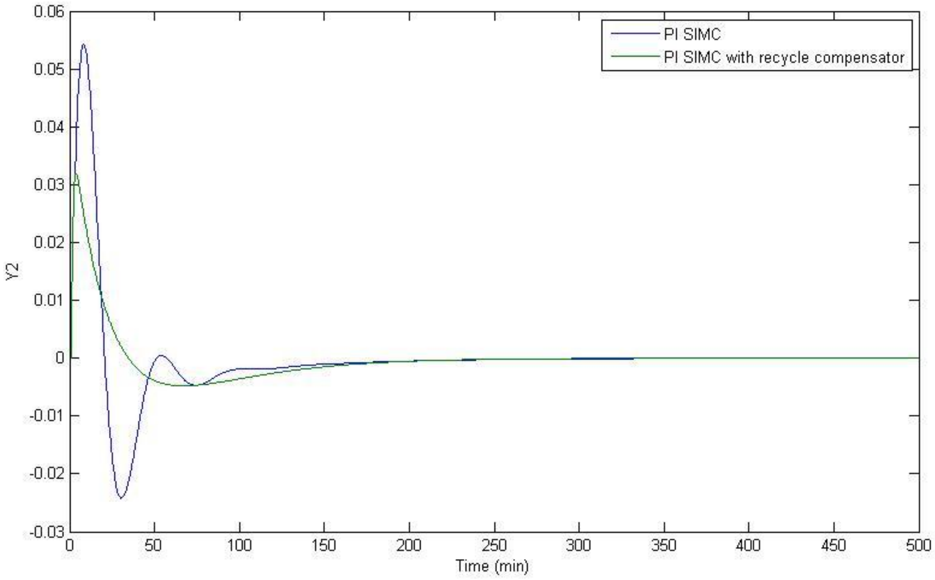


Figure 4.20 Disturbance step changes for PI controller using SIMC tuning method with and without recycle compensator for output Y2

4.5.3.2 PID controller

Figures 4.21 and 4.22 show the disturbance step change response for PID SIMC tuning method with and without recycle compensator for the output Y1 and Y2 respectively. From the graph, it is found that the response time of PID SIMC tuning method is slightly better compared to PI SIMC in terms of response time, offset and number of oscillations. Compared to Ziegler-Nichols tuning method, the PID SIMC tuning method shows a longer response time, larger offset but less oscillation. The addition of the recycle compensator slightly improves the response by reducing the oscillation, response time and offset for both outputs Y1 and Y2.

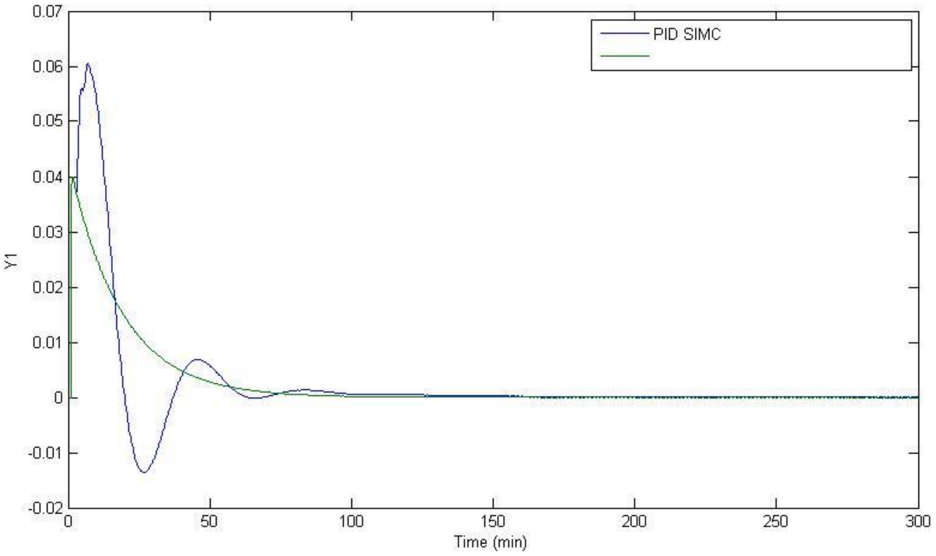


Figure 4.21 Disturbance step changes for PID controller using SIMC tuning method with and without recycle compensator for output Y1

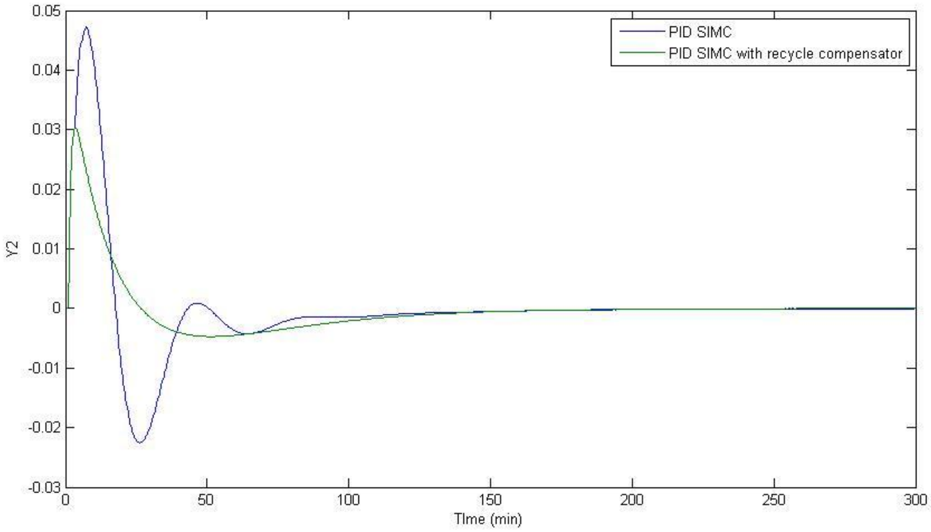


Figure 4.22 Disturbance step changes for PID controller using SIMC tuning method with and without recycle compensator for output Y2

4.5.3.3 Conclusion

It is found that the Ziegler-Nichols tuning method handles the instability of the recycle system better than SIMC tuning method. From the response graph, it is found that the SIMC tuning method shows quite a long response time compared to Ziegler-Nichols tuning method. The offset value is slightly larger, but the oscillation is SIMC tuning method is found to be lesser than of Ziegler-Nichols’. By adding the recycle compensator to the system, the response slightly improved by reducing the oscillations, offset and the response time for both the PI and PID controllers.

4.6 Comparison of control strategies

The integral error analysis is performed on the response graph of each of the tested control strategy. This is to obtain a more accurate comparison to get better understanding on the respective performance.

4.6.1 Integral error analysis

The calculations of the integral errors are done by using the MATLAB software. The coding used is attached in the appendix. Table 4.4 and 4.5 shows the obtained values of IAE, ISE and ITAE for outputs Y1 and Y2 respectively. The values calculated represent the amount of error produced for each control strategy. The lesser the value, the better the performance of the control strategy.

Table 4.4 Integral Error Analysis of controller strategies for output Y1

Control Strategy	**IAE**	**ISE**	**ITAE**
Ziegler-Nichols	0.2072	0.0047	1.4677
Ziegler-Nichols + recycle compensator	0.0805	0.0021	0.2091
SIMC (PI)	1.3634	0.0537	36.6758
SIMC (PID)	1.0417	0.0367	23.0929
SIMC (PI) + recycle compensator	1.0016	0.0207	25.8344
SIMC (PID) + recycle compensator	0.7513	0.0153	14.8013

Table 4.5 Integral Error Analysis of controller strategies for output Y2

Control Strategy	**IAE**	**ISE**	**ITAE**
Ziegler-Nichols	0.1077	0.0011	0.8957
Ziegler-Nichols + recycle compensator	0.0701	0.0009	0.2950
SIMC (PI)	1.3661	0.0370	54.8441
SIMC (PID)	1.0179	0.0246	32.3198
SIMC (PI) + recycle compensator	0.8949	0.0107	51.7417
SIMC (PID) + recycle compensator	0.6668	0.0078	29.4250

From tables 4.4 and 4.5, it is confirmed that the Ziegler Nichols shows the best performance is stabilizing the problems caused by the recycle stream as it shows the least value of IAE, ISE and ITAE for both outputs Y1 and Y2 compared to PI SIMC and PID SIMC. It is also confirmed that the addition of the recycle compensator to each controller improves the performance of the controller. It is observed that the values IAE, ISE and ITAE decreased significantly when the recycle compensator is included. The ITAE of SIMC tuning method shows a significantly high value due to the long response time exhibited by the controller.

4.6.2 Effect of recycle compensator on integral error values

Figures 4.23 and 4.24 shows the values of IAE, ISE and ITAE with and without the recycle compensator. The trends show that the addition of the recycle compensator reduces the integral error values of the controllers. The recycle compensator greatly affects the ITAE of PI SIMC and PID SIMC tuning method but shows very small effect on the integral error of the Ziegler-Nichols tuning method.

4.6.3 Conclusion

From the studies that has been conducted. It is concluded that the Ziegler-Nichols tuning method with recycle compensator is the best control strategy that can be used to control the recycle process. This control strategy has the shortest response time of 15 minutes and the smallest offset values of 0.025 for the Y2 output. The integral error values of this control strategy is also the smallest compared to the other control strategies, with values of IAE, ISE and ITAE of 0.0701, 0.0009 and 0.2950 respectively for the Y2 output.

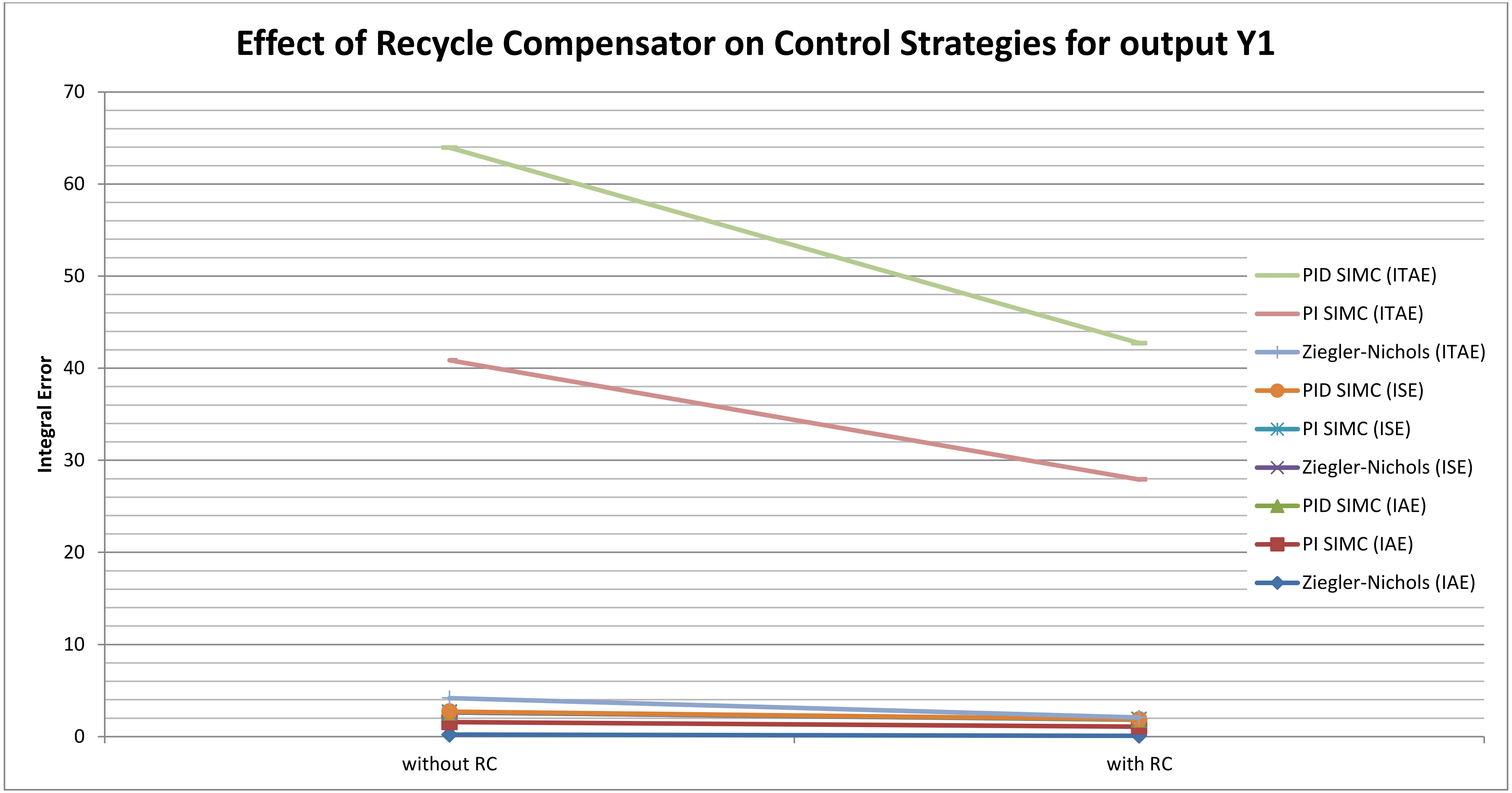


Figure 4.23 Effect of Recycle Compensator on Control Strategies for output Y1

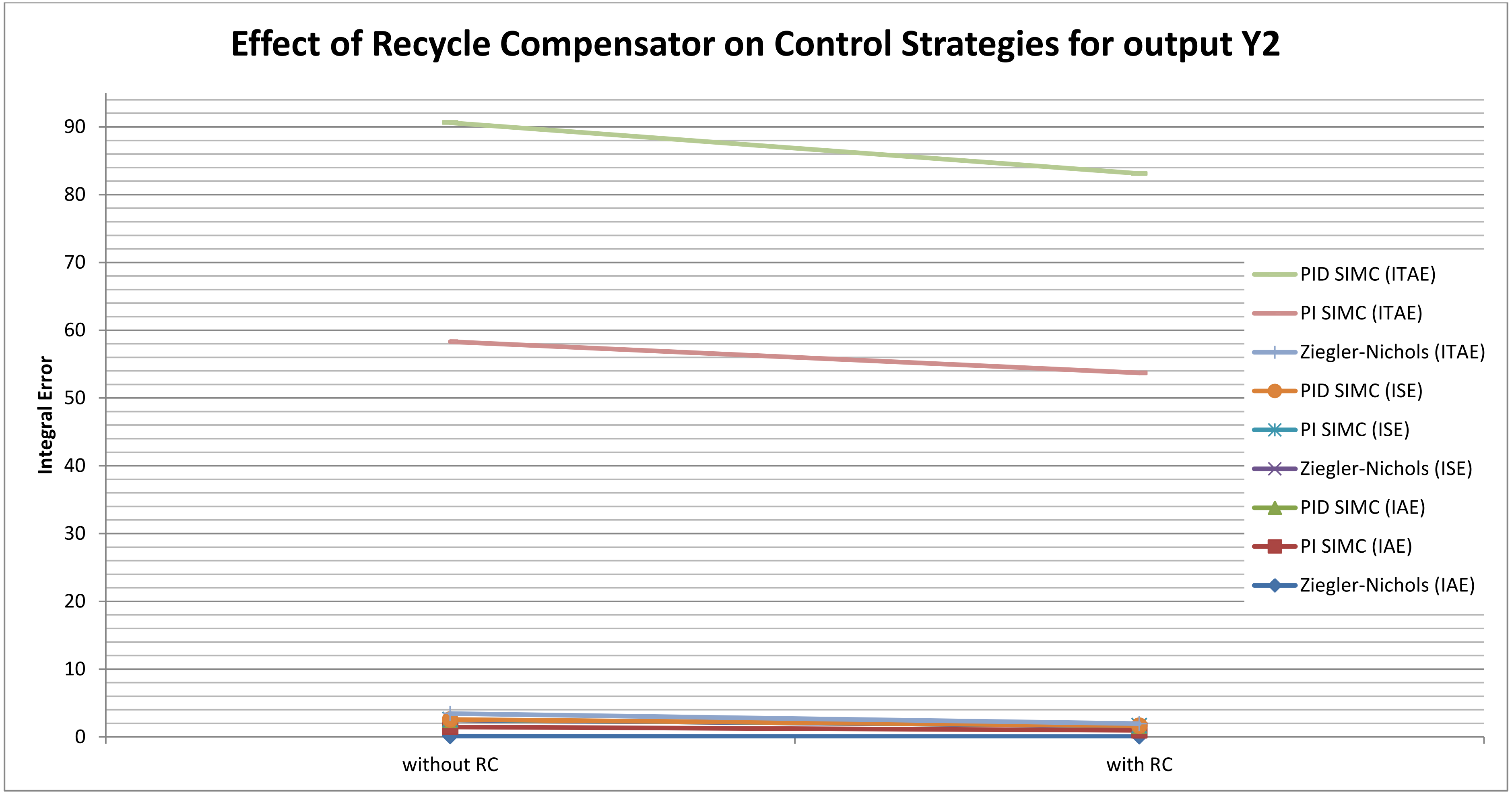


Figure 4.24 Effect of Recycle Compensator on Control Strategies for output Y2

CHAPTER 5

CONCLUSION AND RECOMMENDATIONS

From the studies and test that have been conducted, after comparing the performance of the three controller strategies based on for processes with recycle stream in a 2 CSTR system with recycle, the best control strategy is found to be the combination of the Ziegler-Nichols tuning method and the recycle compensator. This control strategy shows the shortest response time and the least value of offset compared to the other control strategies and their combinations.

In order to compare the control strategies in a more accurate manner, the performance of the controller strategies are tested using the integral error analysis, which are IAE, ISE and ITAE. The value calculated using these method represents the error produced by the control strategies. The combination of Ziegler-Nichols and recycle compensator shows the lowest value of IAE, ISE and ITAE therefore concluding that this control strategy is the best one tested in this study.

For future studies, it is recommended that more control strategies should be tested to control the recycle process. With the existing control strategies and the possible inventions in the future, it is possible to produce quite a number of control strategy combinations to further improve the control of recycle process.

REFERENCES

Bequette, B. W. (1998). *Process Dynamics: Modelling, Analysis and Simulation*. Upper Saddle River, NJ: Prentice Hall PTR.

Chau, P. C. (2002). *Process Control: A First Course with MATLAB*. NY: Cambridge University Press.

Haugen, F. (2010). Tuning of PID Controller. In *Basic Dynamics and Control* (pp. 129-155). Skien, Norway: TechTeach. Retrieved June 26, 2012 from http://techteach.no/publications/books/dynamics_and_control/tuning_pid_controller.pdf

Lakshminarayanan, S. & Takada, H. (2000). Empirical Modeling and Control of Processes with Recycle: Some Insights via Case Studies. *Chemical Engineering Science* 56, 3327-3340.

Madhukar, G.M. *et al.* (2005). Feedback Control of Processes with Recycle A Control Loop Performance Perspective. *Chemical Engineering Research and Design,* 83(A2): 143-152.

Mészáros, A. & Čirka, L. (2009). Control Analysis for Processes with Internal Recycle. *Chemical Engineering Transactions* 18

Roffel, B & Betlem, B. (2006). *Process Dynamics and Control: Modeling for Control and Prediction*. West Sussex, England: John Wiley & Sons Ltd.

Scali, C. & Ferrari, F. (1999). Performances of control systems based on recycle compensators in integrated plants. *Journal of Process Control* 9, 425-437.

Seborg, D. E. *et al.* (2004). PID Controller Design, Tuning, and Troubleshooting. In *Process Dynamics and Control* (2nd ed.). (pp. 300-323). Hoboken, NJ: John Wiley & Sons, Inc.

Taiwo, O. & Krebs, V. (1994). Robust Control System Design for Plant with Recycle. *The Chemical Engineering Journal*, 61. 1-6.

Tremblay, E. *et al.* (2006). Control of a Process with Recycle: Robustness of a Recycle Compensator. *Ind. Eng. Chem. Res.* 45, 1924-1934.

APPENDIX 1 – Simple IMC controller setting derivation

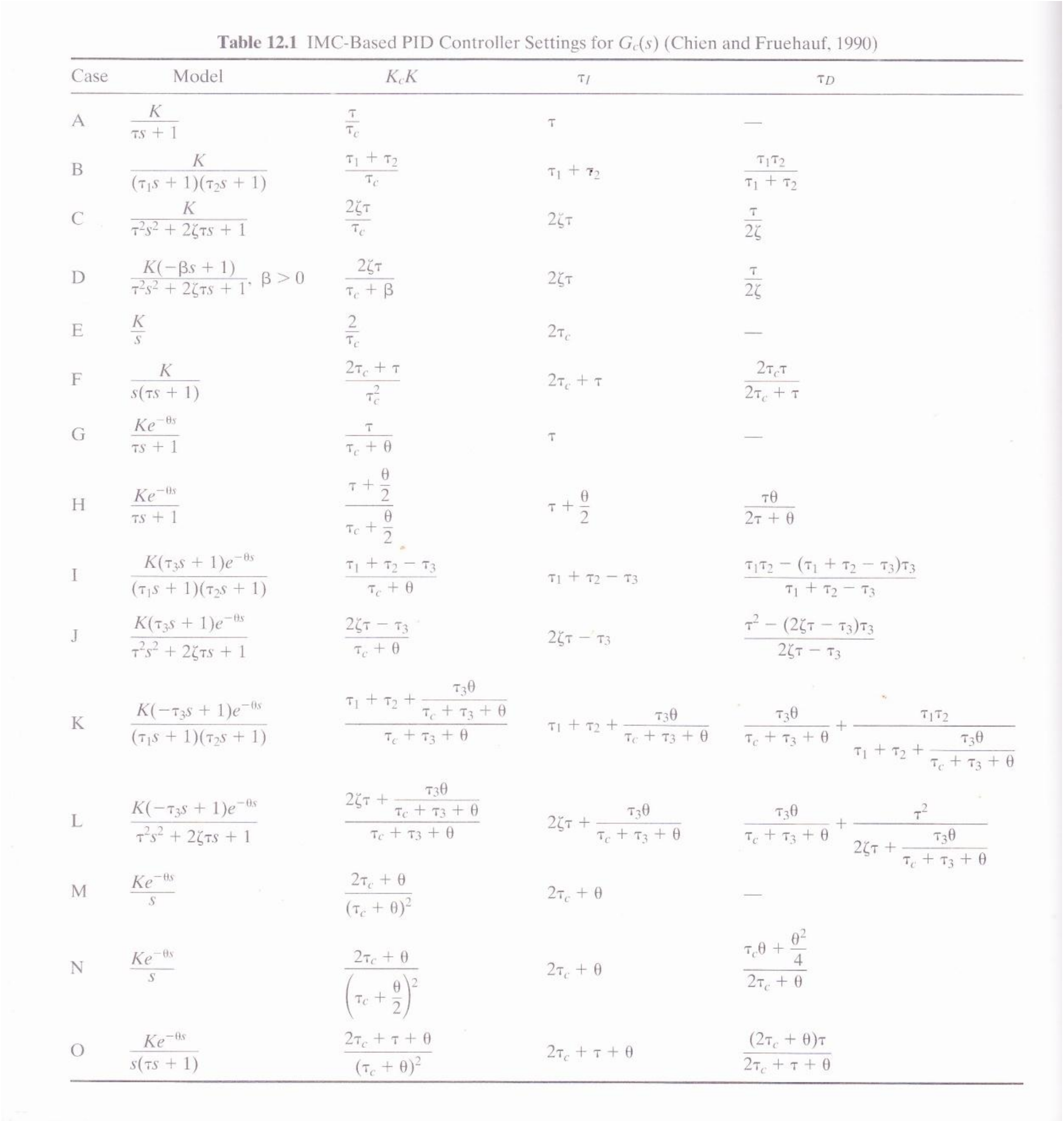
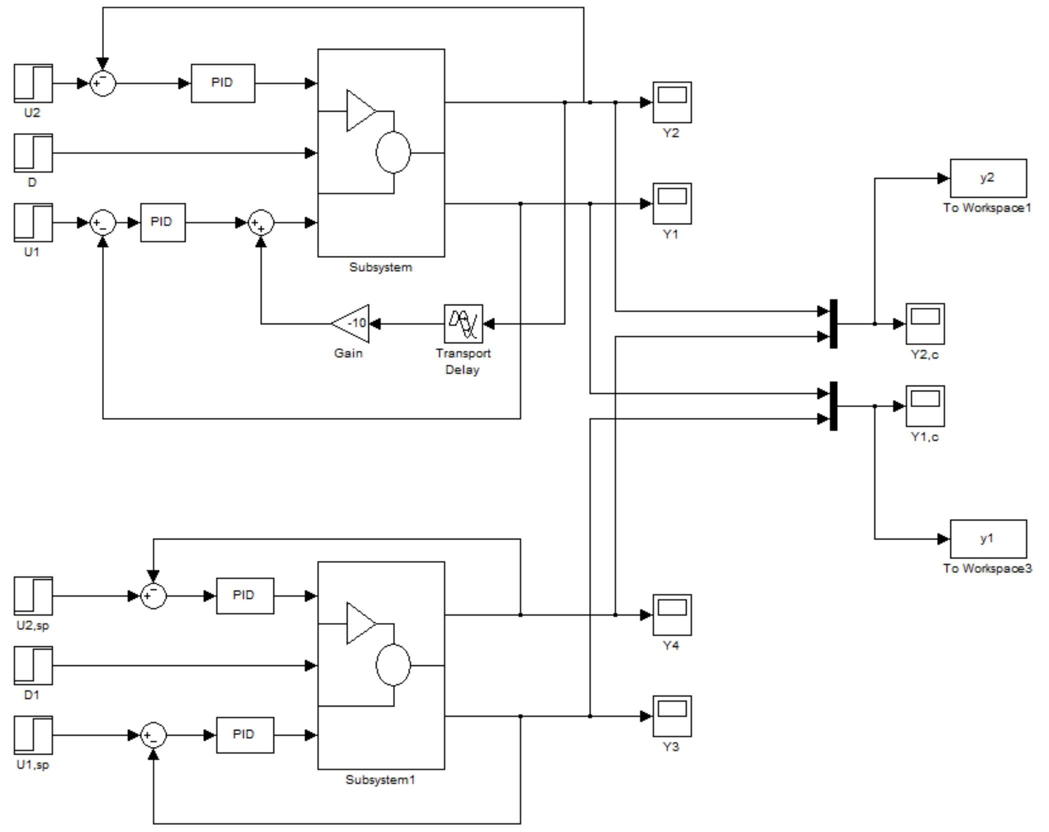


Table 12.1 IMC-Based PID Controller Settings for $G_c(s)$ (Chien and Fruehauf, 1990)

Case	Model	K_cK	τ_I	τ_D
A	$\frac{K}{\tau s+1}$	$\frac{\tau}{\tau_c}$	τ	—
B	$\frac{K}{(\tau_1 s+1)(\tau_2 s+1)}$	$\frac{\tau_1+\tau_2}{\tau_c}$	$\tau_1+\tau_2$	$\frac{\tau_1\tau_2}{\tau_1+\tau_2}$
C	$\frac{K}{\tau^2 s^2+2\zeta\tau s+1}$	$\frac{2\zeta\tau}{\tau_c}$	$2\zeta\tau$	$\frac{\tau}{2\zeta}$
D	$\frac{K(-\beta s+1)}{\tau^2 s^2+2\zeta\tau s+1}$, $\beta>0$	$\frac{2\zeta\tau}{\tau_c+\beta}$	$2\zeta\tau$	$\frac{\tau}{2\zeta}$
E	$\frac{K}{s}$	$\frac{2}{\tau_c}$	$2\tau_c$	—
F	$\frac{K}{s(\tau s+1)}$	$\frac{2\tau_c+\tau}{\tau_c^2}$	$2\tau_c+\tau$	$\frac{2\tau_c\tau}{2\tau_c+\tau}$
G	$\frac{Ke^{-\theta s}}{\tau s+1}$	$\frac{\tau}{\tau_c+\theta}$	τ	—
H	$\frac{Ke^{-\theta s}}{\tau s+1}$	$\frac{\tau+\frac{\theta}{2}}{\tau_c+\frac{\theta}{2}}$	$\tau+\frac{\theta}{2}$	$\frac{\tau\theta}{2\tau+\theta}$
I	$\frac{K(\tau_3 s+1)e^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)}$	$\frac{\tau_1+\tau_2-\tau_3}{\tau_c+\theta}$	$\tau_1+\tau_2-\tau_3$	$\frac{\tau_1\tau_2-(\tau_1+\tau_2-\tau_3)\tau_3}{\tau_1+\tau_2-\tau_3}$
J	$\frac{K(\tau_3 s+1)e^{-\theta s}}{\tau^2 s^2+2\zeta\tau s+1}$	$\frac{2\zeta\tau-\tau_3}{\tau_c+\theta}$	$2\zeta\tau-\tau_3$	$\frac{\tau^2-(2\zeta\tau-\tau_3)\tau_3}{2\zeta\tau-\tau_3}$
K	$\frac{K(-\tau_3 s+1)e^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)}$	$\frac{\tau_1+\tau_2+\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}}{\tau_c+\tau_3+\theta}$	$\tau_1+\tau_2+\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}$	$\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}+\frac{\tau_1\tau_2}{\tau_1+\tau_2+\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}}$
L	$\frac{K(-\tau_3 s+1)e^{-\theta s}}{\tau^2 s^2+2\zeta\tau s+1}$	$\frac{2\zeta\tau+\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}}{\tau_c+\tau_3+\theta}$	$2\zeta\tau+\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}$	$\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}+\frac{\tau^2}{2\zeta\tau+\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}}$
M	$\frac{Ke^{-\theta s}}{s}$	$\frac{2\tau_c+\theta}{(\tau_c+\theta)^2}$	$2\tau_c+\theta$	—
N	$\frac{Ke^{-\theta s}}{s}$	$\frac{2\tau_c+\theta}{\left(\tau_c+\frac{\theta}{2}\right)^2}$	$2\tau_c+\theta$	$\frac{\tau_c\theta+\frac{\theta^2}{4}}{2\tau_c+\theta}$
O	$\frac{Ke^{-\theta s}}{s(\tau s+1)}$	$\frac{2\tau_c+\tau+\theta}{(\tau_c+\theta)^2}$	$2\tau_c+\tau+\theta$	$\frac{(2\tau_c+\theta)\tau}{2\tau_c+\tau+\theta}$

APPENDIX 2 – Block Diagram comparing Z-G tuning method with and without recycle compensator



APPENDIX 3 – MATLAB coding for calculation of integral errors

```
function [IAE, ISE, ITAE]=perform_3 (y,ysp,t)

%insert block diagram file name here
simOut = sim( 'compensatorwithcontroller' )

%formula for IAE
e=abs(y-ysp);
IAE=trapz(t,e)

%formula for ISE
e2=e.^2;
ISE=trapz(t,e2)

%formula for ITAE
e3=e.*t;
ITAE=trapz(t,e3)

simOut =

         0                  3.6375                 10.7641
    0.0000                  3.7778                 11.0070
    0.0002                  3.9181                 11.2570
    0.0012                  4.0677                 11.5371
    0.0062                  4.2450                 11.8006
    0.0313                  4.4223                 12.0368
    0.0842                  4.5958                 12.2708
    0.1472                  4.7925                 12.5274
    0.2222                  4.9764                 12.8018
    0.3110                  5.1467                 13.0737
    0.4177                  5.3270                 13.3260
    0.5492                  5.5125                 13.5698
    0.7174                  5.6983                 13.8266
    0.9421                  5.8924                 14.1008
    0.9811                  6.0993                 14.3718
    1.0201                  6.3175                 14.6271
    1.0623                  6.5176                 14.8752
    1.1114                  6.7150                 15.1332
    1.1831                  6.8991                 15.4065
    1.2802                  7.0918                 15.6795
    1.3749                  7.2890                 15.9392
    1.4864                  7.5062                 16.1902
    1.6191                  7.7266                 16.4477
    1.7841                  7.9471                 16.7193
    1.9985                  8.1856                 16.9953
    2.1621                  8.4404                 17.2605
    2.3258                  8.7037                 17.5138
    2.4863                  8.9618                 17.7677
    2.6357                  9.2157                 18.0345
    2.7763                  9.5023                 18.3111
    2.9325                  9.7887                 18.5819
    3.1071                 10.0413                 18.8391
    3.3030                 10.2696                 19.0910
    3.4674                 10.5065                 19.3521
```

```
19.6266                  34.4601                  49.2563
19.9022                  34.7133                  49.5023
20.1653                  34.9588                  49.7759
20.4175                  35.2189                  50.0671
20.6731                  35.4997                  50.3435
20.9428                  35.7818                  50.5914
21.2206                  36.0442                  50.8308
21.4904                  36.2898                  51.0909
21.7459                  36.5406                  51.3785
21.9976                  36.8133                  51.6657
22.2606                  37.0987                  51.9252
22.5371                  37.3717                  52.1639
22.8128                  37.6225                  52.4116
23.0742                  37.8673                  52.6892
23.3251                  38.1293                  52.9816
23.5813                  38.4124                  53.2550
23.8530                  38.6944                  53.4994
24.1320                  38.9544                  53.7388
24.4009                  39.1982                  54.0025
24.6545                  39.4499                  54.2933
24.9056                  39.7253                  54.5791
25.1701                  40.0119                  54.8345
25.4486                  40.2831                  55.0712
25.7244                  40.5314                  55.3213
25.9840                  40.7758                  55.6032
26.2334                  41.0401                  55.8965
26.4902                  41.3256                  56.1660
26.7640                  41.6071                  56.4068
27.0441                  41.8644                  56.6467
27.3118                  42.1065                  56.9149
27.5635                  42.3594                  57.2087
27.8141                  42.6378                  57.4923
28.0803                  42.9253                  57.7431
28.3607                  43.1943                  57.9782
28.6363                  43.4400                  58.2316
28.8940                  43.6845                  58.5181
29.1419                  43.9514                  58.8117
29.3994                  44.2391                  59.0766
29.6755                  44.5195                  59.3137
29.9566                  44.7739                  59.5549
30.2228                  45.0147                  59.8283
30.4724                  45.2694                  60.1249
30.7228                  45.5508                  60.4051
30.9908                  45.8387                  60.6508
31.2731                  46.1051                  60.8849
31.5483                  46.3483                  61.1426
31.8038                  46.5933                  61.4342
32.0504                  46.8634                  61.7271
32.3090                  47.1529                  61.9863
32.5874                  47.4317                  62.2199
32.8691                  47.6829                  62.4633
33.1336                  47.9228                  62.7428
33.3812                  48.1799                  63.0420
33.6316                  48.4643                  63.3175
33.9018                  48.7522                  63.5575
34.1858                  49.0154                  63.7913
```

```
   64.0546
   64.3517
   64.6425
   64.8951
   65.1252
   65.3724
   65.6591
   65.9601
   66.2294
   66.4632
   66.6975
   66.9681
   67.2710
   67.5580
   67.8025
   68.0297
   68.2824
   68.5776
   68.8795
   69.1403
   69.3675
   69.6039
   69.8838
   70.1931
   70.4737
   70.7083
   70.9333
   71.1940
   71.4997
   71.8000
   72.0496
   72.2698
   72.5108
   72.8031
   73.1183
   73.3880
   73.6106
   73.8351
   74.1081
   74.4278
   74.6619
   74.8960
   75.1778
   75.4962
   75.7851
   76.0177
   76.2338
   76.4875
   76.7983
   77.0511
   77.3039
   77.5823
   77.8732
   78.1449
   78.3891
   78.6298
   78.8951
   79.1873
   79.4738
   79.7266
   79.9607
   80.2110
   80.4971
   80.7977
   81.0656
   81.2988
   81.5339
   81.8064
   82.1137
   82.4001
   82.6396
   82.8635
   83.1181
   83.4207
   83.7295
   83.9844
   84.2019
   84.4354
   84.7233
   85.0548
   85.2717
   85.4885
   85.7723
   86.1177
   86.3424
   86.5670
   86.8548
   87.1894
   87.4137
   87.6379
   87.9217
   88.2517
   88.4795
   88.7074
   88.9906
   89.3166
   89.5466
   89.7765
   90.0588
   90.3809
   90.6125
   90.8440
   91.1251
   91.4435
   91.7332
   91.9664
   92.1825
   92.4356
   92.7452
   93.0631
   93.3183
   93.5291
   93.7560
   94.0435
   94.3773
   94.5972
   94.8170
   95.1013
   95.4431
   95.6649
   95.8867
   96.1721
   96.5069
   96.7329
   96.9588
   97.2426
   97.5703
   97.7987
   98.0272
   98.3095
   98.6331
   98.8642
   99.0953
   99.3768
   99.6961
   99.9270
  100.0000


IAE =

    0.0702


ISE =

  8.8942e-004


ITAE =

    0.3023
```