DETERMINATION OF PRESSURE DROP IN TWO-PHASE, LIQUID-LIQUID SYSTEM IN A HORIZONTAL PIPELINE THROUGH MATLAB SIMULATION

by

Aiman bin Hisham

Dissertation submitted in partial fulfilment of
the requirements for the
Bachelor of Engineering (Hons)
(Chemical Engineering)

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CERTIFICATION OF APPROVAL

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Approved by:

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CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

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AIMAN BIN HISHAM

ABSTRACT

Pressure drop during the flow of fluid is an important parameter to be determined before proceeding with any pipeline design. It is very essential in the determination of the pipe sizing and also the design of related downstream facilities. It will definitely have a huge impact on the optimization of cost and efficiency.

The project aims to develop a model that will determine the pressure drop of a two-phase, liquid-liquid flow in a horizontal pipe through MATLAB simulation as the programming tool. In this modeling work, two-fluid model will be used for the prediction of pressure drop between oil and water. In liquid-liquid systems, the effect of curvature interface must be considered. With the small density differences between the two mediums, the interface configuration can be flat, concave or even convex. The curvature depends on the contact angle, the viscosity and the flow rates of the fluid involved.

This model will be developed using MATLAB programming and will be tested with a few sets of input data. The calculated pressure drop from the developed model from this study will then be compared with experimental data to check for its reliability.

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CHAPTER 1
INTRODUCTION

1.1 Background of Study

The flows of two immiscible liquids are encountered in a diverse range of processes and equipments especially in petroleum industry, where mixtures of oil and water are transported inside pipeline over long distances. Several studies had been done to predict the oil-water flow characteristics, such as flow pattern, water holdup and pressure gradient of flow (Bertola, 2003). In this project, the study will be specifically designed to determine the pressure drop for two-phase, liquid-liquid system inside a horizontal pipeline using MATLAB as the programming tool.

The flow patterns of the immiscible liquids are assumed to be in three different types of interface; concave, convex and flat-shaped interfaces. These types of interface are associated with different contact area between the two fluids and also between the fluids with the pipe wall. Depending on the physical system involved, those variations can have prominent effects on the pressure drop and transport phenomena in the system (Gorelik and Brauner, 1999).

1.2 Problem Statement

1.2.1 Problem Identification

Traditionally, the consideration of interface curvature is usually neglected in a large scale system and is justified in high-density differential systems, such as gas-liquid systems under earth conditions. However, in liquid-liquid systems with small density differences or in reduced gravity systems, surface phenomena maybe dominated which resulted in a curved interface configuration. This curved interface may significantly affect the local and integral two-phase flow characteristics (Brauner, Rovinsky and Moalem Maron, 1996). In order to predict much precise pressure drop, the curvature effect is taken into consideration.

1.2.2 Significance of the Project

The determination of pressure drop in liquid-liquid system is more complex compared to gas-liquid system. This modeling will assist engineers to obtain pressure drop value in liquid-liquid system, to be used as a main basis of their design. It will be very essential as they need to understand the characteristics of the flow system such as the pressure drop in order for them to design the pipeline with a proper and safer size.

1.3 Objective and Scope of Study

1.3.1 Project Objective

The objective of this project is to develop a MATLAB programming code to stimulate and predict the pressure drops in a two-phase, liquid-liquid system in a horizontal pipeline based upon various interfacial configurations. The calculated pressure drop from the simulation will be compared to the experimental data for validation.

1.3.2 Scope of Project

The project involves computer simulation work using MATLAB programming tool to predict the effect of interfacial curvature shapes towards the pressure drop. Based upon two-fluid modeling and the experimental data of liquid heights in the pipeline system, a programming code will be developed that will calculate the differential pressure drops. The calculated pressure drops will be compared with the experimental pressure drop data previously found to validate the findings.

CHAPTER 2
LITERATURE REVIEW

2.1 General Description of Liquid-Liquid Flows: Flow Patterns

Flows of two immiscible liquids are encountered in a diverse range of processes and equipments such as petroleum industry, where mixtures of oil and water are transported through pipes over long distances. Some studies had been conducted to predict oil-water flow characteristics such as flow pattern, water hold-up and pressure gradient; which are very important characteristics in many engineering applications.

Liquid–liquid two-phase flows can appear in quite different topological or morphological configurations. The different structures are usually called flow patterns or flow regimes which are defined as the physical geometry exhibited by a multiphase flow in a conduit; for example, heavier density liquid occupying the bottom of the conduit with the lighter density phase flowing above. In liquid–liquid flow, the phases within the pipe are distributed in several fundamentally different flow patterns or flow regimes, depending primarily on the flow rates of the two phases and the angle of inclination (Brauner and Moalem Maron, 1989).

Two immiscible liquids of different densities tend to stratify when flowing slowly together in a pipe. These separate layers can be classified as stratified flow and stratified flow with mixed layer at the interface as illustrated in Figure 1. The stratified flow is identified when the smooth interface exists; however, as the velocities of the phases increase, the interface becomes more disturbed or wavy and drops of the two liquids may also appear. At that time the flow pattern shifts to the stratified flow with a mixed layer at the interface. There exist water droplets in the oil and oil droplets in the water layer. Both kinds of droplets remain close to the interface. In this case, dynamic and buoyant forces are acting simultaneously on the droplets (Trallero et al, 1997).

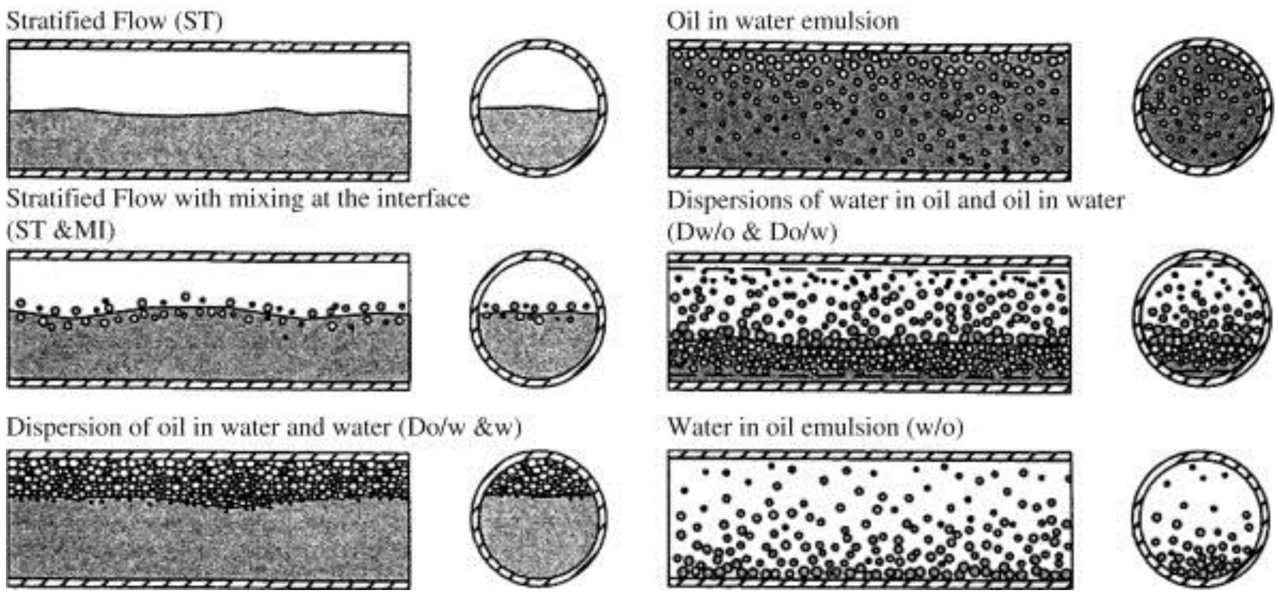

Figure 2.1: Horizontal oil-water flow pattern sketches (Trallero et al, 1997)

2.1.1 Parameters with impact on the flow patterns

Mixture velocity and input water cut:

The effect of mixture velocity and water cut was investigated early by Russell et al. (1959). In general, low mixture velocities allow the flow to be separated or stratified, while high mixture velocity disperses the flow. However, dispersed flows may appear at low velocities provided the water cut is very low or very high.

Viscosity, density and surface tension:

The viscosity effect, studied by Russell et al. (1959), Charles et al. (1961) and Arirachakaran et al. (1989), seems to have little or no effect on the observed flow patterns for oil/water flows. The sequence or the number of observed flow patterns is the same, but transitions from one flow regime to another may appear at different superficial velocities when oils of different viscosities are used. This means that the size of the various flow regime areas on the map can vary slightly. Reports on the isolated effect of density and surface tension are limited. In two-phase flows with high density difference between the phases, stratified flow will generally appear for a larger range of mixture velocities and water fractions than for the case of two-phase flows with low density differences.

Flow geometry and wetting properties:

The flow geometry such as pipe diameter, inlet design and the inclination angle of the pipe are other parameters that can be decisive for which flow pattern that appears (Soleimani et al., 1997). Inlets (i.e. the oil/water mixing unit) can be shaped in a way that it tends to keep the flow stratified.

Temperature and pressure:

Temperature and pressure have influence on the flow patterns in the sense that they influence the physical properties like viscosity, density etc.

2.1.2 Flow pattern transitions

Oil-water flows are gravity dominated. It flows separately when the flow rates of each phase are low, that is, when superficial velocities are low. The interface between the phases is smooth without waves. At higher flow rates waves start to appear at the interface. The wavelength is about twice the pipe diameter (Trallero et al., 1997). Small water droplets exist in the oil layer while small oil droplets appear in the water layer. Both the oil droplets and the water droplets flow close to the interface. Different forces act on the droplets but gravity forces overcome dynamic forces and the droplets are kept close to the interface.

Several investigators have studied the stability of two-phase flows. For gas / liquid flow, it was done by Barnea and Taitel (1993) while for liquid-liquid flow the two contributions from Brauner and Moalem Maron (1992a, b) are central. The latter of Brauner and Moalem Maron's work deals with oil-water flow in particular. The details of their stability analysis are not presented here, but Figure 2 represents the basic propositions from the oil-water study. Continuity and momentum equations of an oil-water system are analyzed and a stability analysis is conducted. Criteria for transition from smooth stratified to other flow regimes are then given. They proposed two transition lines. Both the zero neutral stability (ZNS) line and the zero real characteristics (ZRC) line are shown in Figure 2. The two lines define three zones in the figure. Below the ZNS boundary the flow is smooth stratified.

The "buffer zone" positioned in between the two boundaries includes stratified flows with the existence of interfacial waves. Beyond the ZRC boundary, other flow regimes than stratified wavy exist. Thus, the ZNS boundary represents the transition from stable smooth stratified flow to stratified wavy flow and the ZRC boundary represents the transition from stratified wavy flow to other flow regimes. Also presented in Figure 2 is the effect of density and viscosity ratios on the boundaries. Brauner and Moalem Maron (1992b) found that as the oil-water density ratio was decreased, the area of stable smooth stratified flow also decreased (Figure 2 a and b). A similar effect appeared when the oil-water viscosity ratio was reduced (Figure 2 b and c).

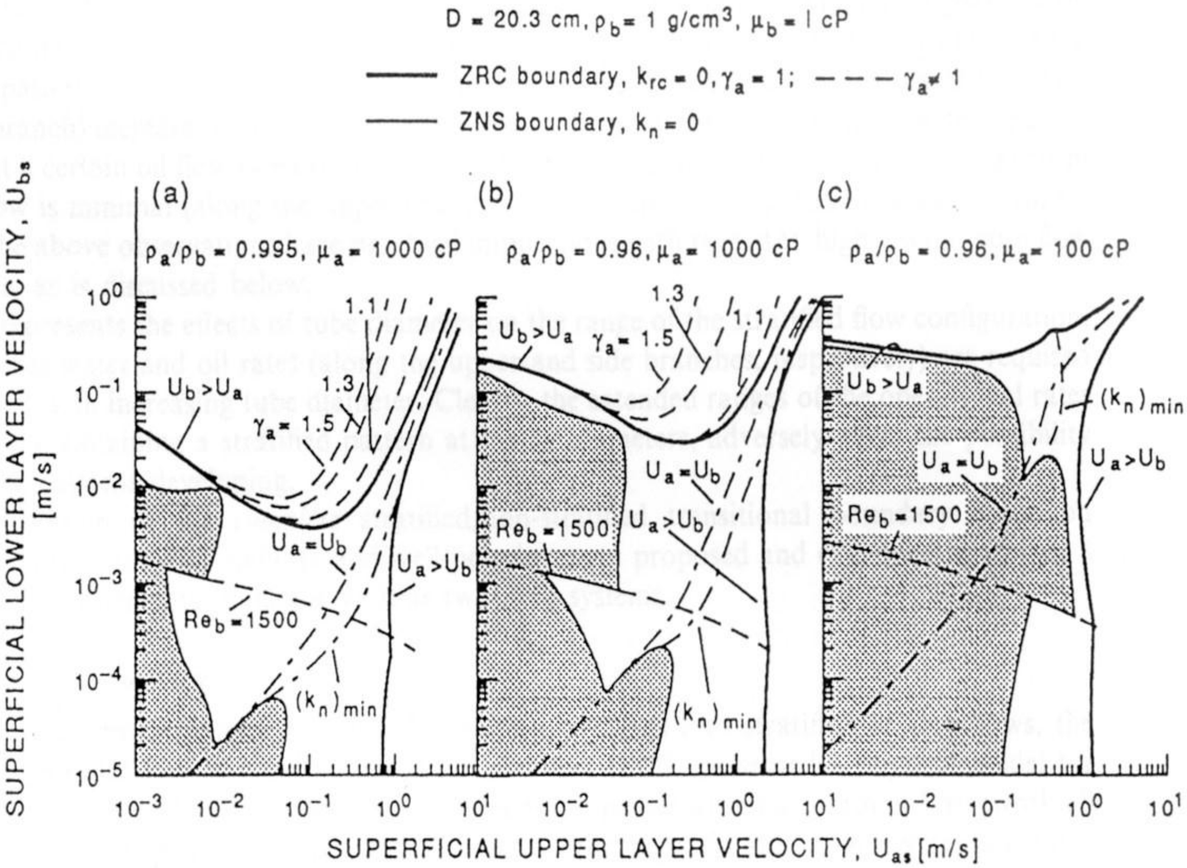

Figure 2.2: The effect of density and viscosity ratios on the location of ZNS and ZRC boundaries (Brauner and Maron, 1992b)

2.1.3 Stratified Flow

The stratified flow configuration is one of the basic and most important distributions during two phase flow through horizontal pipes. Models of stratified flow are needed for predicting the flow characteristics, such as pressure drop and in-situ holdup, and are often used as a starting point in modeling flow patterns transitions. The common assumption is that the interface separating the phases is plane. This assumption is appropriate for gravity dominated systems, such as large scale gas-liquid horizontal flows under earth's gravitation. In reduced gravity systems, capillary systems or in the case of low density differential (such as oil-water systems), surface forces become important. The wetting fluid tends to climb over the tube wall resulting in a curved (convex or concave) interface (Gorelik and Brauner, 1999).

In liquid-liquid flow system which is having a relatively low density difference, surface tension and wetting effects become important, and the interface shape (concave, convex, plane) is an additional field that has to be solved (Brauner, 2003). Figure 3 shows the basic interfacial configurations for a liquid-liquid system in a stratified flow.

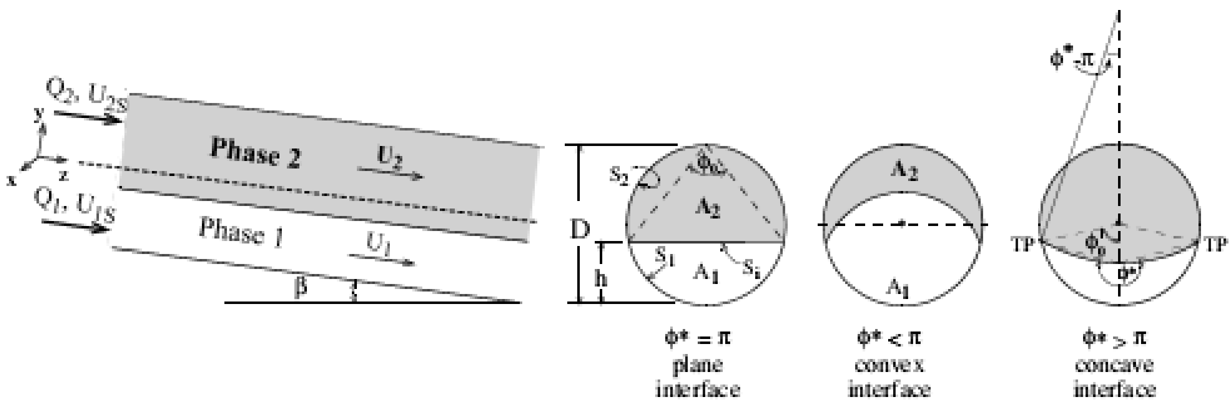

Figure 2.3: Schematic Description of Stratified Flow Configuration (Bertola et al, 2003)

A configuration of a curved interface is associated with a variation in the contact area between the two fluids, and between the fluids and the pipe wall. This variation will significantly affect on the pressure drop and transport phenomena depending on the physical system involved.

2.2 Two-fluid Flow Experiments

Many studies had been conducted in order to understand the characteristics of oil-water two-phase flow in horizontal pipes. Those studies involve pipes that were made from different materials and sizes. The studies were summarized in Table 1.

Table 2.1: Summary of Experimental Systems of Oil-Water Two-Phase Flow

Authors	Pipe Material	Diameter	Length	Superficial Velocity		Temp	Flow Pattern Map Available
Nadler and Mewes (1997)	Perspex	0.059	48	Mixture	0.1 – 1.6	Yes	10 – 30
Beretta et al. (1997a)	Glass	0.003	1	Oil	(5.4 – 55.8)	Yes	15 – 25
				Water	(up to 564)	No	
Trallero et al. (1997)	Acrylic Resin	0.05013	15.54	N/A		N/A	Yes
Beretta et al. (1997b)	Glass	0.003	1	N/A		15 – 25	No
Angeli and Hewitt (1998)	Stainless Steel	0.0243	9.7	Mixture	0.3 – 3.9	No	20
	Perspex	0.024	9.5				
Angeli and Hewitt (2000)	Stainless Steel	0.0243	9.7	Mixture	0.2 – 3.9	Yes	20
	Perspex	0.024	9.5				
Shi et al. (2001)	N/A	0.1	18	Mixture	0.4 – 3	No	25
Ioannou et al. (2005)	Stainless Steel	0.06	16.6	Mixture	3.5–5	No	N/A
	Acrylic Resin	0.032			4–7 small pipe		
Piela et al. (2006)	Acrylic Resin	0.016	6 x 2	Oil	1.35 – 3.5	No	N/A
			4.5 x 2	Water	1 – 3		
Mandal et al. (2007)	PMMA	0.025	2	Oil	0.03 – 1.5	Yes	N/A
		0.012		Water	0.03 – 1.5		

There are various methods applied through the study of the flow patterns of two-phase, liquid-liquid system in horizontal pipe. One of the studies had been done by Liu et al. (2008). They investigated the in-situ phase distribution of the fluids involved in the pipeline by characterizing them by the height of water climbing along the wall circumferentially and the height of water layer at the vertical plane passing the pipe axis, which were measured by two sets of different conductance probes. Each set included parallel chromel wires and parallel ring probes with the spacing of 40 mm. A probe, consisting of two chromel wires, traversed the diameter of the pipe vertically as shown in Figure 4.

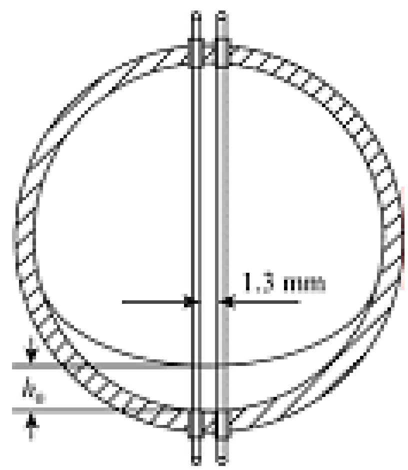

Figure 2.4: Schematic of the parallel chromel wires (Liu et al., 2008)

The parallel wires in the middle of the wire probe behaved like a pair of parallel cylinders separated by a fixed distance of 1.3 mm. One of the wires was excited with a high frequency alternating voltage inducing a current through the probe that was dependent on the height of water layer between the wires. The parallel ring probes, as shown in Figure 5, were composed of a pair of brass rings with the thickness of 4 mm, and these rings were embedded flush with the inner surface of pipe covered by insullac. Nonconductive acrylic resin with the axial thickness of 10 mm was filled between the parallel rings. Both probes were statically calibrated by locating the depth of probes submerged by water.

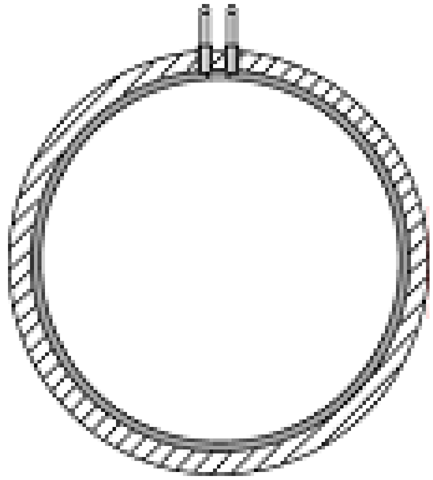

Figure 2.5: Schematic of the parallel ring probes (Liu et al., 2008)

Images of the flow pattern were recorded through the acrylic viewing section using 30 frames per second digital video camera at a position 0.5 m downstream of the inlet. A computer based data acquisition was implemented in the test section.

2.3 Two-fluid Flow Modeling

Two-fluid modeling has been applied in various studies involving both gas-liquid and liquid-liquid systems. Several derived models based upon the two-fluid modeling had been developed.

Brauner et al. (1996) had discovered that the interface curvature in stratified two-phase systems is determined by energy considerations. The discovery is obtained through the use of two-fluid modeling. They revealed that a configuration of a plane interface between two stratified layers is appropriate for two phase systems which are dominated by gravity, as is the case for large scale air-water systems under earth gravitation. However, for a general two-fluid system, the basic *in situ* configuration is stratified layers with a curved interface. The prescription of the characteristic interface curvature is required in order to initiate the solution of the flow problem and the associated transport phenomena. Energy considerations are employed to predict the interface configuration. The effect of the fluid physical properties, *in situ* hold up, tube dimension, wall adhesion and gravitation on the characteristic interface curvature are explored. The prediction of interface curvature provides the closure relation required for a complete solution of stratified flows with curved interfaces for a variety of two-fluid systems (Brauner, Moalem Maron and Rovinsky, 1996).

Two-fluid modeling is also used to solve the momentum equations for a variable interface curvature (Brauner et al, 1998). Energy considerations provide a closure relation for the interface curvature. The analysis identifies all the input dimensionless parameters which determine the solution for the stratified flow pattern. When these are given, a complete solution of the problem is obtained, including the interface shape, in situ hold-up and pressure drop. The validity of the two-fluid model is evaluated by comparing its prediction with available experimental data and with the results of exact analytical solutions for laminar flows with curved interfaces. Thus, the conventional two-fluid model has been extended to tackle stratified flow with curved interfaces and various flow regimes, in which case analytical solutions are complicated and restricted to laminar flows (Brauner, Moalem Maron and Rovinsky, 1998).

Engineers will be able to predict the behavior of the flow regime through this model as it will provide the pressure drop profile prediction. This information will assist them to make a preliminary design in pipeline sizing. Once the pressure drop profile had been discovered to be too high or too low, the engineers are able to do some modification on their preliminary design before finalizing their design to be manufactured later.

CHAPTER 3
METHODOLOGY

3.1 Research Methodology

In this study, a computer simulation work to develop a model that will determine pressure drop for a two-phase, liquid-liquid system in a horizontal pipeline through MATLAB programming will be carried out. Available data and fluid properties from the actual experiment such as height of wire and ring probes; liquid densities, viscosities and flow rates; experimental pressure drops; are obtained from the study previously done by Abdullah (2008, 2009).

Table 3.1: Properties of Fluids used in Study

Properties	Oil	Water
Density, kg/m^3	828.00	1000.00
Viscosity, mPa·s at 25^oC	5.50	1.00
Surface tension, mN/m at 25^oC	39.6	

As stated in the above table, two fluids that are used in the experimental works were oil and water with the stated properties. The pipeline that was chosen in the experimental works to channel the fluids in the calculation of pressure drop has a 14-mm inner diameter with a 50 cm in length. Other properties such as wire-probe height, ring-probe height, oil flow rate and water flow rates were obtained during the experiment and will be used as the input data in this model.

The calculation of pressure drop in the pipeline system with assumption of flat interface and curvature interfacial shape by using a combination of both wire probe and ring probe heights that were obtained from the experimental work. Previous input data are used to determine other parameters or variables required to be used in two-fluid model. A MATLAB programming code will be developed for calculating all parameters in the two-fluid model system. Figure 6 shows the methodology for the development of the project.

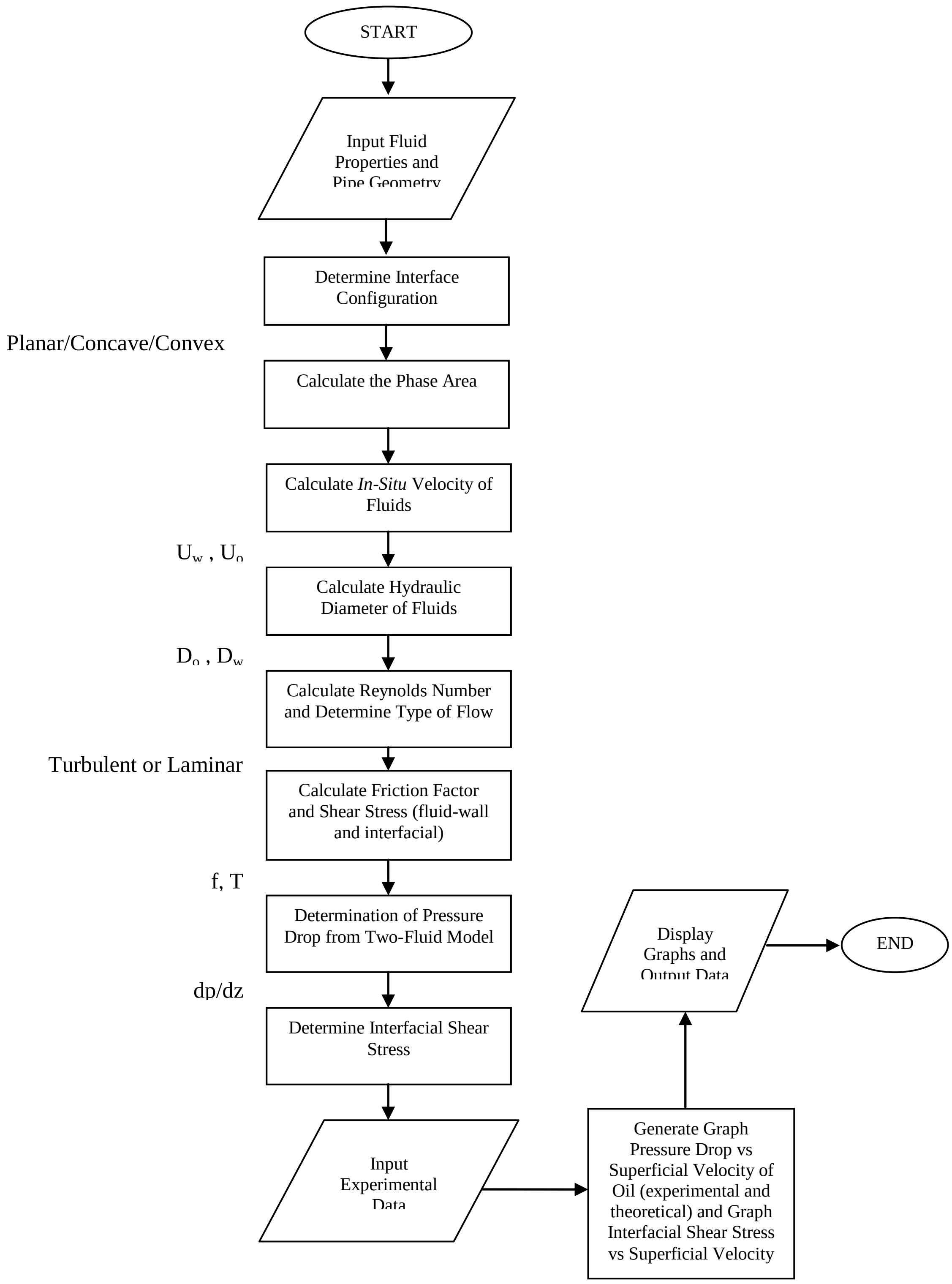

Figure 3.1: Methodology for the Development of the Programming Code of the Two-Fluid Model

3.2 The Two-Fluid Model (TFM)

Two-fluid model was used in this study due to its capability in handling laminar and turbulent flows in horizontal and inclined systems, both in co-current and counter-current stratified flows (Brauner, 2003). By assuming a fully developed stratified flow, the integral form of the momentum equations for the two fluids are shown as below.

$$-A_1\left(\frac{dP}{dz}\right) - \tau_1 S_1 - \tau_i S_i + \rho_1 A_1 g \sin\beta = 0 \qquad (1)$$

$$-A_2\left(\frac{dP}{dz}\right) - \tau_2 S_2 + \tau_i S_i + \rho_2 A_2 g \sin\beta = 0 \qquad (2)$$

Where,

τ_1, τ_2, τ_i = Shear stress for phase 1, 2 and interfacial shear stress;

S_1, S_2, S_i = Liquid wetted-wall perimeter for phase 1, 2 and interfacial perimeter;

A_1, A_2 = Cross sectional area for phase 1 and 2;

$\frac{dP}{dz}$ = Pressure drop in liquid phase;

β = Pipe inclination angle, in degrees;

g = Gravitational acceleration.

If the pipeline is assumed in a horizontal position (no inclination, β = 0), thus equation (1) and (2) can be simplified into:

$$-A_1\left(\frac{dP}{dz}\right) - \tau_1 S_1 - \tau_i S_i = 0 \qquad (3)$$

$$-A_2\left(\frac{dP}{dz}\right) - \tau_2 S_2 + \tau_i S_i = 0 \qquad (4)$$

The perimeter (S) and area (A) of the phase system can be calculated by performing trigonometric derivation equation. The Blasius equation is used to provide the closure laws required for the wall and interfacial shear stresses (τ_1, τ_2, τ_i) in terms of the average velocities (U_1, U_2) and friction factors (f_1, f_2, f_i).

$$\tau_1 = -\frac{1}{2} f_1 \rho_1 U_1 |U_1|; \;\; f_1 = C_1 \left(\frac{\rho_1 D_1 |U_1|}{\mu_1} \right)^{-n_1} \tag{5}$$

$$\tau_2 = -\frac{1}{2} f_2 \rho_2 U_2 |U_2|; \;\; f_2 = C_2 \left(\frac{\rho_2 D_2 |U_2|}{\mu_2} \right)^{-n_2} \tag{6}$$

$$\tau_i = -\frac{1}{2} f_i \rho_i (U_1 - U_2) |U_1 - U_2| \tag{7}$$

Where,

U_1, U_2 = Velocity for phase 1 and 2;

f_1, f_2, f_i = Friction factor for phase 1, 2 and interfacial friction factor;

ρ_1, ρ_2, ρ_i = Density for phase 1, 2 and interfacial density;

μ_1, μ_2 = Viscosity for phase 1 and 2;

C_1, C_2 = Constants for phase 1 and 2 (laminar flow, C = 16; turbulent flow, C = 0.046);

n_1, n_2 = Constants for phase 1 and 2 (laminar flow, n = 1; turbulent flow, n = 0.2)

For separated or stratified two-phase flow, the Reynolds number is defined differently to that of a single-phase flow. Usually Reynolds numbers are defined for each phase according to equations below:

$$Re_1 = \frac{\rho_1 U_1 D_1}{\mu_1} \qquad Re_2 = \frac{\rho_2 U_2 D_2}{\mu_2} \tag{8}$$

Where,

$Re_1, Re_2 =$ Reynolds number for phase 1 and 2;

$D_1, D_2 =$ Hydraulic diameters for phase 1 and 2.

According to the equations above, the Reynolds number of the fluids was determined from the hydraulic diameters, D_1 *and* D_2, which are adjusted according to the relative velocity of the phases. The interface is generally considered as 'stationary' (wetted) for the faster fluid phase whereas the term 'free' is used for the slower fluid phase (Brauner and Moalem Maron, 1989).

When the velocities are of the same order, the interface is considered 'free' with respect to both phases (Brauner, 2003).

$$D_1 = \frac{4A_1}{(S_1 + S_i)};\ D_2 = \frac{4A_2}{S_2};\ \rho = \rho_1\ and\ f_i = F_i f_1 \quad for\ |U_1| > |U_2| \qquad (9)$$

$$D_1 = \frac{4A_1}{S_1};\ D_2 = \frac{4A_2}{(S_2 + S_i)};\ \rho = \rho_2\ and\ f_i = F_i f_2 \quad for\ |U_2| > |U_1| \qquad (10)$$

$$D_1 = \frac{4A_1}{S_1};\ D_2 = \frac{4A_2}{S_2};\ \tau_i \cong 0 \quad for\ U_1 \cong U_2 \qquad (11)$$

Assuming the pressure drops for both liquid phases are equal, equation (3) and (4) can be combined with $\tau_i S_i$ is the same.

$$-A_1\left(\frac{dP}{dz}\right) - \tau_1 S_1 - \tau_i S_i - A_2\left(\frac{dP}{dz}\right) - \tau_2 S_2 + \tau_i S_i = 0$$

$$-A_1\left(\frac{dP}{dz}\right) - \tau_1 S_1 - A_2\left(\frac{dP}{dz}\right) - \tau_2 S_2 = 0$$

$$-\left(\frac{dP}{dz}\right)(A_1 + A_2) - \tau_1 S_1 - \tau_2 S_2 = 0$$

$$-\left(\frac{dP}{dz}\right) = \frac{\tau_1 S_1 + \tau_2 S_2}{(A_1 + A_2)} \tag{12}$$

Equation (12) shows that pressure drop, dP/dz is determined through the ratio of forces due to shear stresses to the total wetted area of the liquids. Hence, the interfacial shear stress is evaluated by substituting the value in either equation (3) or (4)

$$\tau_i = \frac{A_1\left(\frac{dP}{dz}\right) + \tau_1 S_1}{-S_i} \tag{13}$$

3.3 Calculation of Cross Sectional Area and Perimeter

In order to calculate the cross sectional area and wetted perimeter of both oil and water phases, as well as the interface perimeter to be used later in two-fluid model (TFM), basic geometry is considered and calculations are made by using basic trigonometry. The drawing of cross sectional area of the horizontal pipeline is shown as in Figure 3.2.

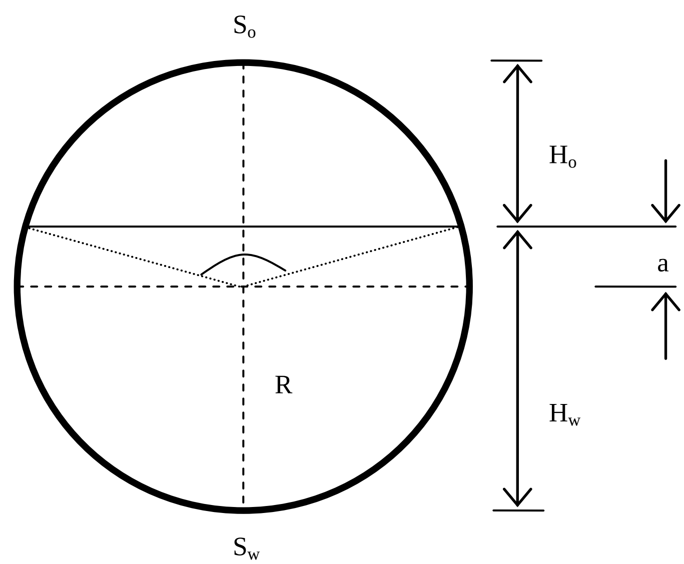

Figure 3.2: Cross sectional area of the pipeline for flat interface

Wire probe and ring probe height data that were obtained from the experiment are important parameters to be used in the calculation of cross sectional area. The derivation of cross sectional area of water and oil, wetted perimeters of oil, water and interface perimeter will be shown in Appendix II section.

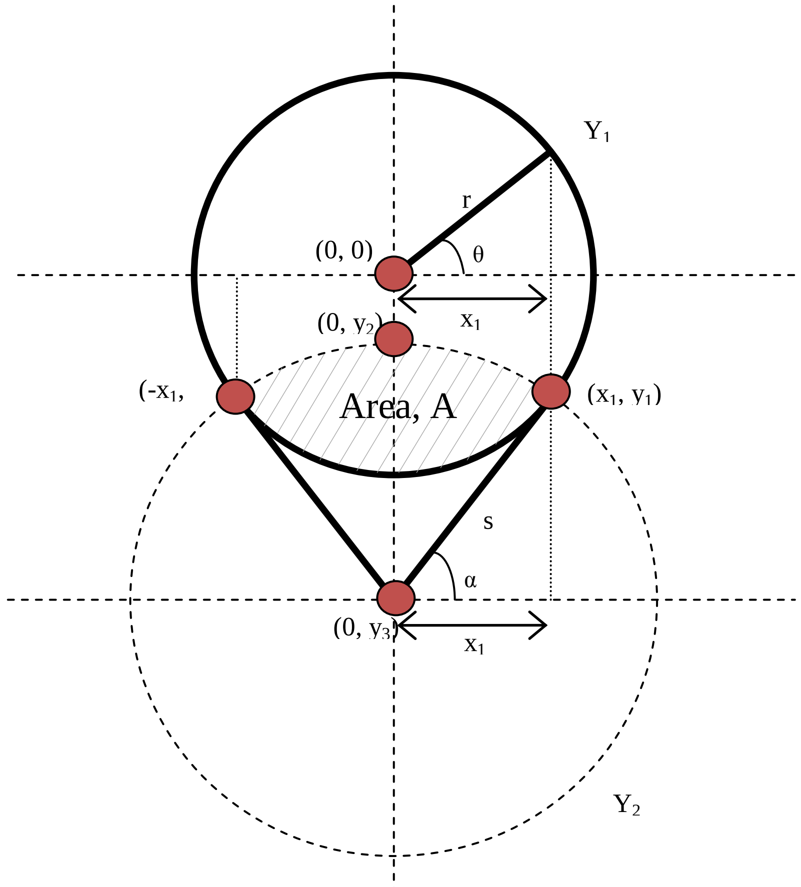

Figure 3.3: The area of lune due to the curved interface in the pipeline

As well as flat interface configuration, curvature interfacial configuration is also derived based upon wire and ring probes' heights obtained from experiment. However, the derivation is more complex compared to flat interface configuration. The basic idea of this derivation is a combination of two circles, one circle represents the pipeline and another imaginary circle represents the curved interface. The interception of these two circles is considered as the cross sectional area of water while the rest of the pipeline will be considered as the cross sectional area of oil. The derivation of cross sectional area and perimeters for phases, oil and water are shown in Appendix III.

CHAPTER 4
RESULT AND DISCUSSION

The comparison between the pressure drops of the oil-liquid, stratified flows in the horizontal pipeline are shown in Figure 4.1 to Figure 4.7. The figures illustrated the results obtained through MATLAB simulation using the planar and curved interfaces, and comparison to the experimental data. The summary of the findings are tabulated in Table 4.1.

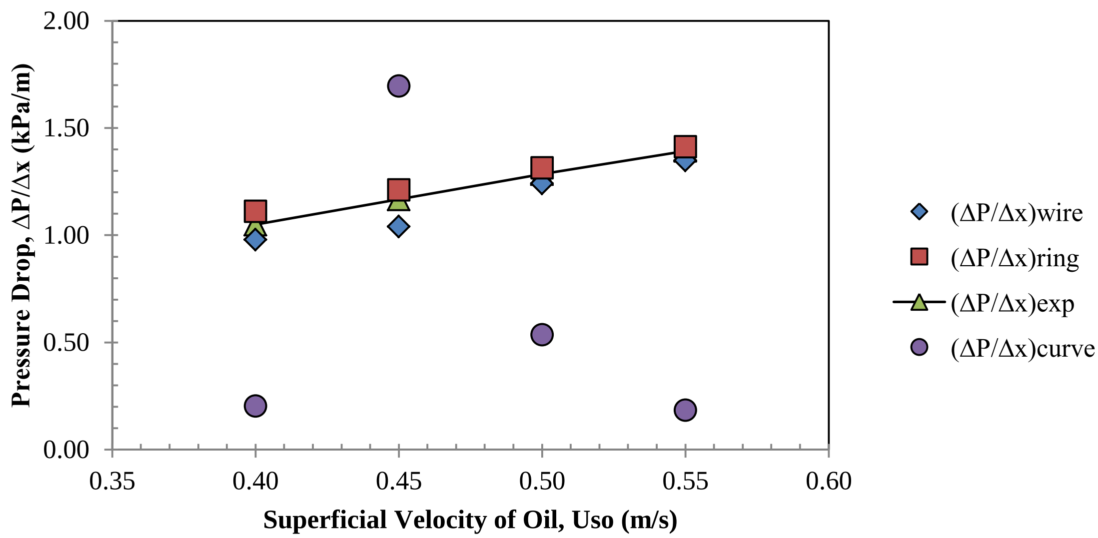

Figure 4.1: Comparison between $(\Delta P/\Delta x)_{calc}$ to $(\Delta P/\Delta x)_{exp}$ for U_{sw} = 0.55 m/s.

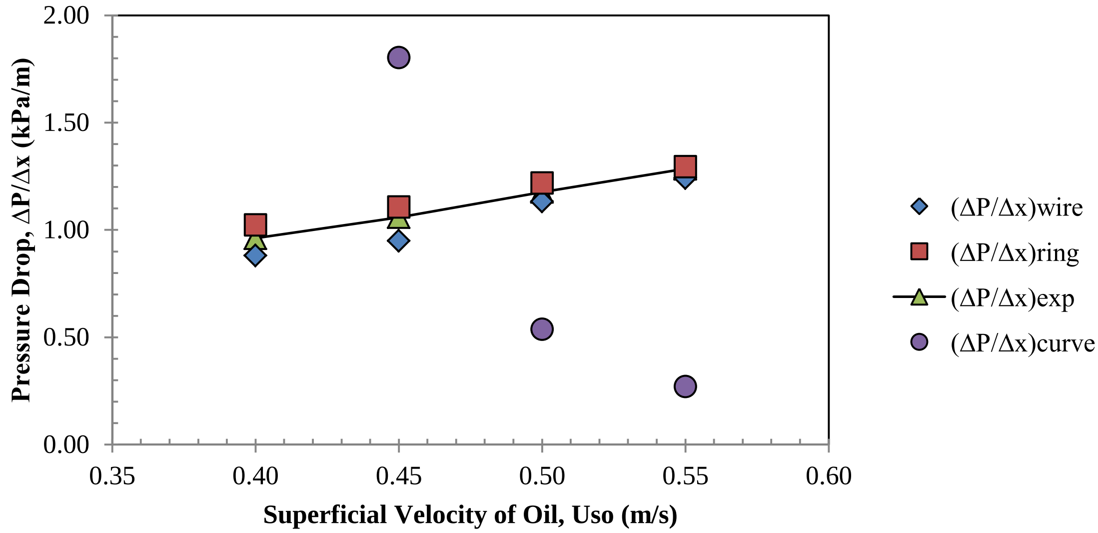

Figure 4.2: Comparison between $(\Delta P/\Delta x)_{calc}$ to $(\Delta P/\Delta x)_{exp}$ for U_{sw} = 0.50 m/s

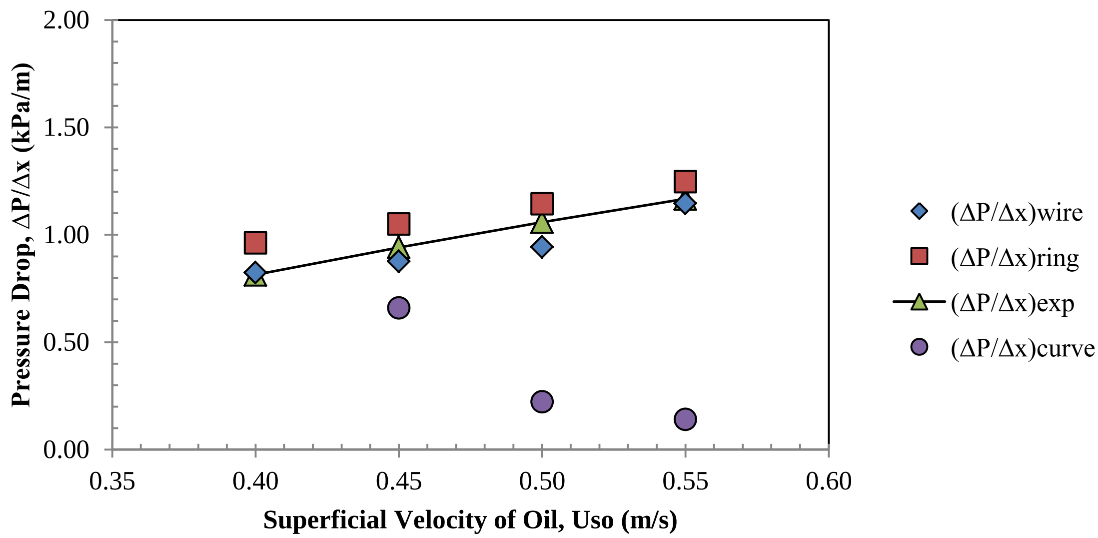

Figure 4.3: Comparison between $(\Delta P/\Delta x)_{calc}$ to $(\Delta P/\Delta x)_{exp}$ for U_{sw} = 0.45 m/s

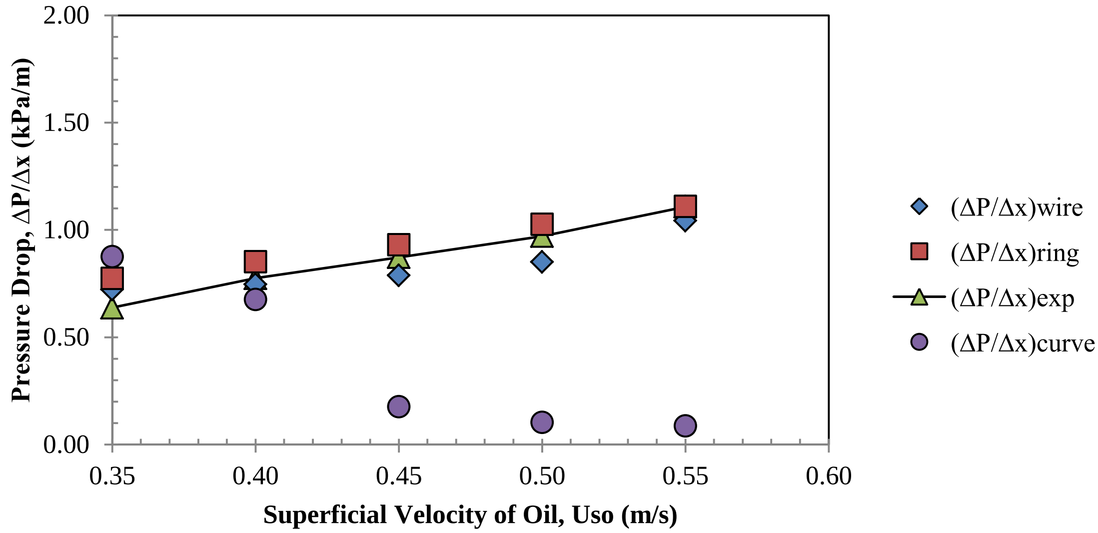

Figure 4.4: Comparison between $(\Delta P/\Delta x)_{calc}$ to $(\Delta P/\Delta x)_{exp}$ for U_{sw} = 0.40 m/s

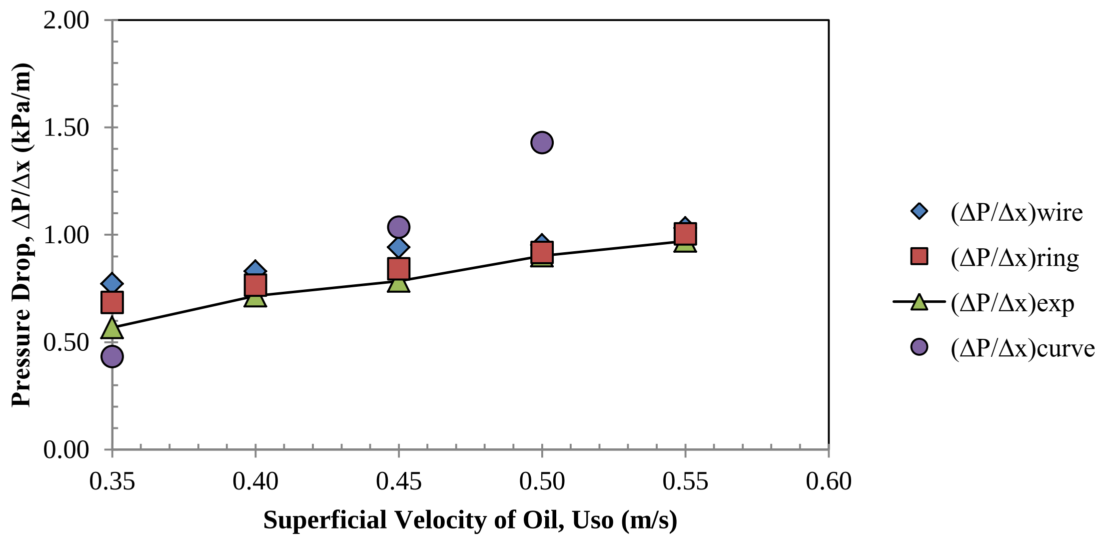

Figure 4.5: Comparison between $(\Delta P/\Delta x)_{calc}$ to $(\Delta P/\Delta x)_{exp}$ for U_{sw} = 0.35 m/s

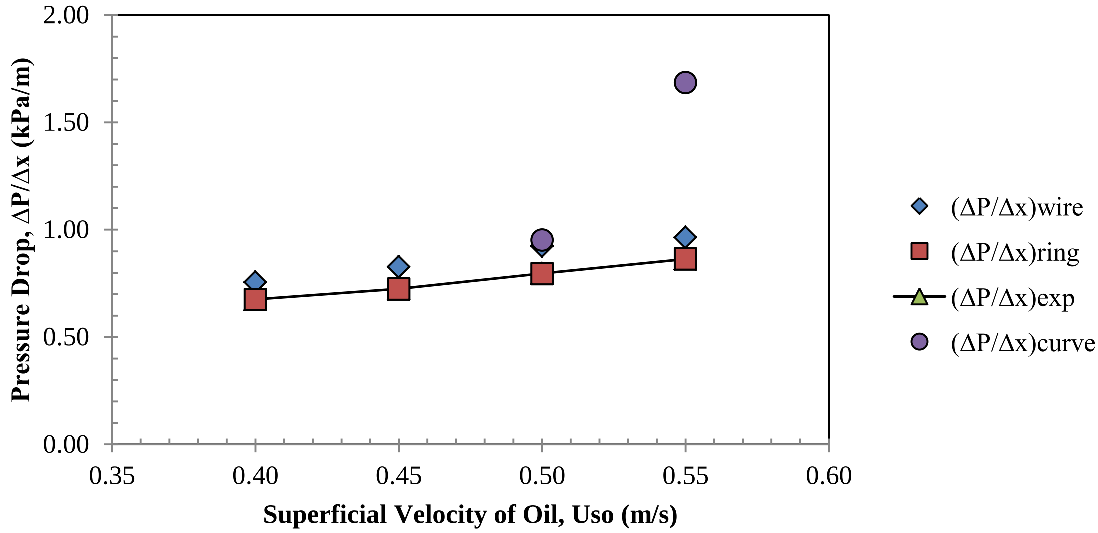

Figure 4.6: Comparison between $(\Delta P/\Delta x)_{calc}$ to $(\Delta P/\Delta x)_{exp}$ for U_{sw} = 0.30 m/s

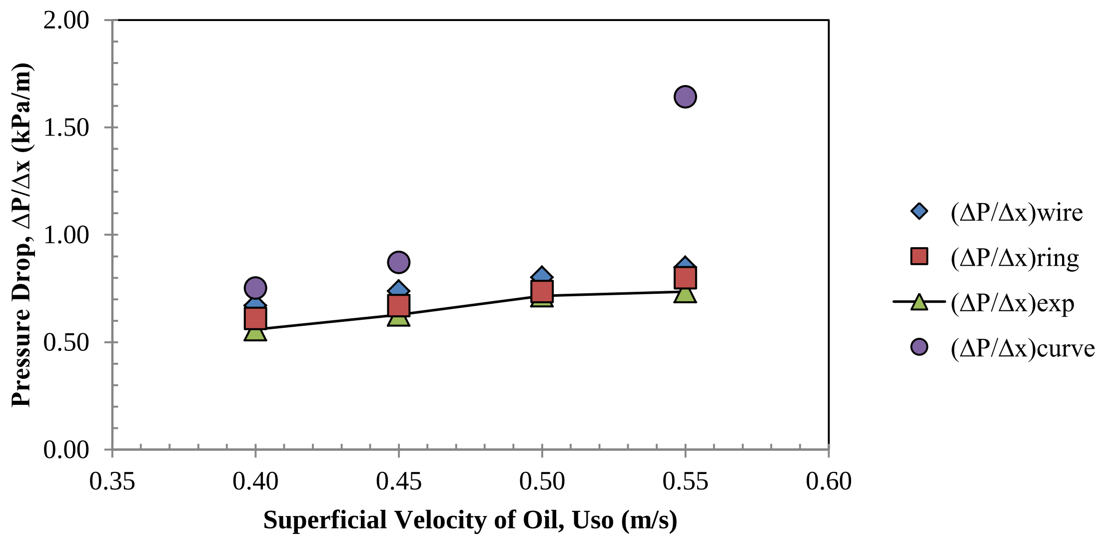

Figure 4.7: Comparison between $(\Delta P/\Delta x)_{calc}$ to $(\Delta P/\Delta x)_{exp}$ for U_{sw} = 0.25 m/s

Table 4.1: Comparison of experimental ΔP/Δx with the ΔP/Δx calculated with flat interface assumption.

Usw	Uso	Height		$\Delta P/\Delta x_{exp}$	Flowrate		$\Delta P/\Delta x_{calc}$			Percentage difference			Shear Stress Interface
		Wire Probe	Ring Probe		Oil	Water	Wire Probe	Ring Probe	Curvature	Wire Probe	Ring Probe	Curvature	
m/s		mm		kPa/m	m^3/s		kPa/m			%			Pa
0.55	0.40	7.19	6.17	1.05	6.16×10^{-5}	8.47×10^{-5}	0.98	1.11	0.20	6.67	5.95	80.58	3.51
	0.45	7.03	5.85	1.17	6.93×10^{-5}	8.47×10^{-5}	1.04	1.21	1.70	10.93	3.74	45.24	9.84
	0.50	6.66	5.53	1.29	7.70×10^{-5}	8.47×10^{-5}	1.24	1.31	0.54	3.44	2.28	58.27	7.94
	0.55	6.52	5.26	1.39	8.47×10^{-5}	8.47×10^{-5}	1.35	1.41	0.18	3.14	1.43	86.74	3.36
0.50	0.40	7.27	5.92	0.96	6.16×10^{-5}	7.70×10^{-5}	0.88	1.02	11.21	8.46	6.53	1066.16	132.10
	0.45	6.89	5.65	1.06	6.93×10^{-5}	7.70×10^{-5}	0.95	1.11	1.80	10.38	4.46	70.21	11.66
	0.50	6.62	5.25	1.18	7.70×10^{-5}	7.70×10^{-5}	1.13	1.22	0.54	3.83	3.56	54.36	7.59
	0.55	6.37	5.06	1.29	8.47×10^{-5}	7.70×10^{-5}	1.24	1.29	0.27	3.42	0.73	78.87	4.23
0.45	0.40	6.69	5.48	0.81	6.16×10^{-5}	6.93×10^{-5}	0.82	0.96	6.22	1.13	18.19	663.86	57.71
	0.45	6.54	5.16	0.94	6.93×10^{-5}	6.93×10^{-5}	0.88	1.05	0.66	6.92	11.51	29.97	8.84
	0.50	6.25	4.84	1.06	7.70×10^{-5}	6.93×10^{-5}	0.94	1.14	0.22	11.01	8.03	78.92	3.43
	0.55	6.01	4.50	1.17	8.47×10^{-5}	6.93×10^{-5}	1.15	1.25	0.14	1.88	6.86	87.85	2.45
0.40	0.35	6.21	5.68	0.64	5.39×10^{-5}	6.16×10^{-5}	0.72	0.77	0.88	13.36	21.51	37.31	3.68
	0.40	6.44	5.34	0.77	6.16×10^{-5}	6.16×10^{-5}	0.75	0.85	0.68	3.77	9.97	12.66	8.86
	0.45	6.43	5.02	0.87	6.93×10^{-5}	6.16×10^{-5}	0.79	0.93	0.18	9.79	6.72	79.69	2.72

0.40	0.50	6.10	4.64	0.97	7.70×10^{-5}	6.16×10^{-5}	0.85	1.03	0.10	12.37	5.64	89.26	1.87
	0.55	5.80	4.35	1.11	8.47×10^{-5}	6.16×10^{-5}	1.04	1.11	0.09	5.92	0.11	92.15	1.71
0.35	0.35	7.02	5.46	0.57	5.39×10^{-5}	5.39×10^{-5}	0.77	0.69	0.43	35.62	20.41	23.70	6.15
	0.40	6.70	5.03	0.72	6.16×10^{-5}	5.39×10^{-5}	0.83	0.77	7.51	15.87	6.92	948.41	62.48
	0.45	7.27	4.68	0.78	6.93×10^{-5}	5.39×10^{-5}	0.94	0.84	1.04	19.91	7.30	32.05	2.70
	0.50	6.33	4.37	0.90	7.70×10^{-5}	5.39×10^{-5}	0.95	0.92	1.43	5.53	1.74	58.30	8.90
	0.55	6.62	4.01	0.97	8.47×10^{-5}	5.39×10^{-5}	1.03	1.00	74.89	6.23	3.42	7611.31	832.19
0.30	0.40	6.54	4.74	0.60	6.16×10^{-5}	4.62×10^{-5}	0.76	0.68	6.74	26.30	12.81	1026.23	52.35
	0.45	6.63	4.60	0.70	6.93×10^{-5}	4.62×10^{-5}	0.83	0.72	2.49	18.79	3.98	256.82	15.68
	0.50	6.94	4.24	0.81	7.70×10^{-5}	4.62×10^{-5}	0.92	0.80	0.95	13.38	2.25	16.91	1.22
	0.55	6.62	3.94	0.88	8.47×10^{-5}	4.62×10^{-5}	0.96	0.86	1.69	9.12	2.18	90.96	11.58
0.25	0.40	5.78	4.11	0.56	6.16×10^{-5}	3.85×10^{-5}	0.67	0.61	0.75	19.92	9.33	34.59	1.31
	0.45	6.05	3.83	0.63	6.93×10^{-5}	3.85×10^{-5}	0.74	0.67	0.87	17.45	6.75	38.95	4.40
	0.50	6.09	3.49	0.72	7.70×10^{-5}	3.85×10^{-5}	0.80	0.74	56.67	11.89	2.73	7813.91	608.07
	0.55	5.71	3.20	0.74	8.47×10^{-5}	3.85×10^{-5}	0.85	0.80	1.64	15.30	8.76	123.32	18.08

Based on Table 4.1, when the superficial velocity of water is at 0.55 m/s, the pressure drop calculated based on wire probe height, $(\Delta P/\Delta x)_{wire}$, is under-predicted than the experimental value; provided that the interface is assumed as flat-shaped. In contrast, the calculated pressure drop based on ring probe, $(\Delta P/\Delta x)_{ring}$, is slightly over-predicted than the experimental value with a maximum percentage difference reaching 5.95%.

As the superficial velocity of water decreases to 0.50 m/s, the $(\Delta P/\Delta x)_{wire}$ predictions were more or less the same as the previous reading with the maximum percentage difference reaches 10.38% compared to 10.93% when the superficial velocity of water is 0.55 m/s. The $(\Delta P/\Delta x)_{ring}$ predictions increased with a small increment in the maximum percentage difference from 5.95% to 6.53%.

At water superficial velocity of 0.45 m/s, the maximum percentage difference reaches until 11.01% as the $(\Delta P/\Delta x)_{wire}$ predictions is again under-predicted than the experimental value. Presumably, the $(\Delta P/\Delta x)_{ring}$ predictions is highly over-predicted with the maximum percentage difference increasing quite rapidly to 18.19%.

As the superficial velocity of water reaches 0.35 m/s and below, both $(\Delta P/\Delta x)_{wire}$ and $(\Delta P/\Delta x)_{ring}$ predictions always gives higher value than the experimental value with the maximum percentage difference of $(\Delta P/\Delta x)_{wire}$ being 35.62% while $(\Delta P/\Delta x)_{ring}$ is only until 20.41%.

Based on the overall view, $(\Delta P/\Delta x)_{ring}$ data gave the closest predictions as compared to $(\Delta P/\Delta x)_{wire}$ data assuming that the interface is flat-shaped. Calculated pressure drop, $(\Delta P/\Delta x)_{calc}$ through ring probe is always greater than the experimental pressure drop, $(\Delta P/\Delta x)_{exp}$ due to lower estimation of cross sectional area of water, A_w, thus make the effects of shear stress of oil, π_{oil} towards $\Delta P/\Delta x$ became superior.

The calculated data is closer to the experimental data for higher velocities of water because the interface shape is near flat whereas for low velocities of water, the calculated data becomes more diverged from the experimental data because the shape of the interface has become more curve-shaped. The interface tends to change shape due to the change in surface tension for both phases with respect to their velocities.

In curvature interface assumption, prediction of pressure drop based upon wire and ring probe heights, $(\Delta P/\Delta x)_{curve}$ between the phases should give better prediction as has been discussed in the previous section. However, through the approach carried out in this project, $(\Delta P/\Delta x)_{curve}$ deviates tremendously from the experimental value with the percentage error ranges from 12.66% to 256.82%. These deviations maybe occurred because of the utilization of incorrect correlation in the formulae derived in the determination of cross-sectional area for both phases. The interface radius function is taken from Brauner & Gorelik, 1999 and then being correlated into a formula of cross-sectional area that had been derived previously by Khalid, 2011. The correlation may not be too reliable as the derived formula has considered the angle of the curve involved. Still, the previous derived formula does give a higher percentage error on the pressure drop. However, this model does give a very large percentage error for certain velocities of both liquids. This is because the shear stress of the interface is high for those cases which are affected by the turbulence of the phases and thus can affect the wettability of the liquids to the wall. The Reynolds number of both liquids differs far from each other and will cause the turbulence of the flow to be not stable and will result in the high shear stress of interface.

Hence, further modifications needed in the correlated function to prove that calculation of pressure drop using curved interface will give better assumption of pressure drop. One way to achieve this is by calculating the cross-sectional area of water by integrating the curve function of the interface radius from one end of the curve to the other end. The cross sectional area of water can be determined from the difference between total area under radius curve function and total area under pipeline curve function as shown in Figure 4.8. From there, cross-sectional area of oil can also be found based on the remaining area of pipeline uncovered by water.

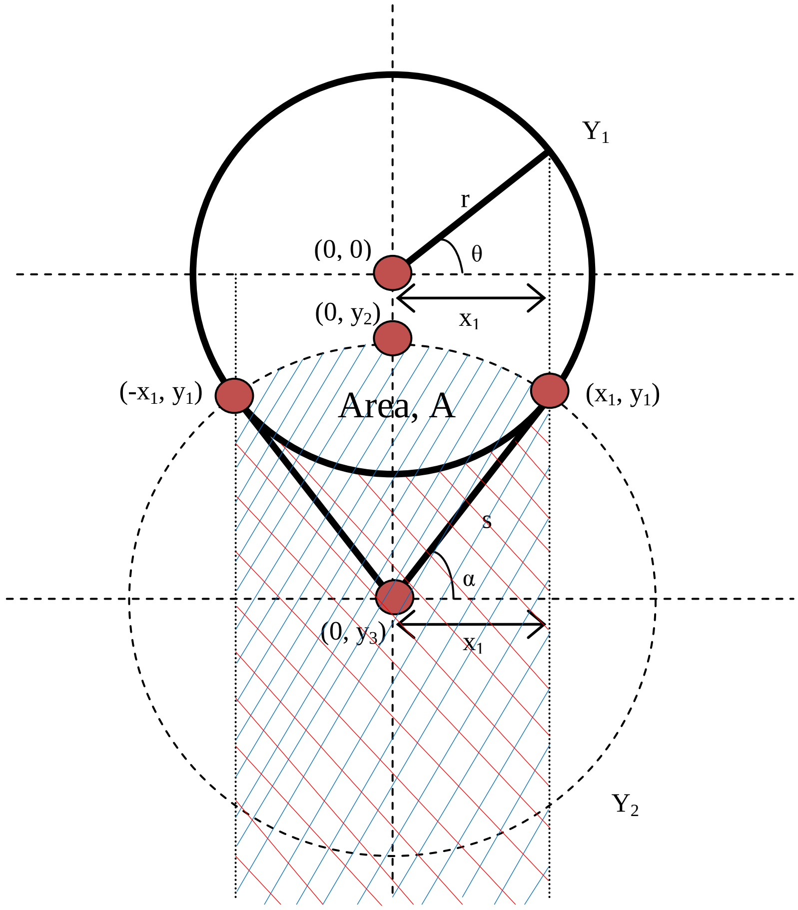

Figure 4.8: Integration of curve function to obtain cross sectional area of water

Another reason of the error is might be due to the interfacial configuration itself. The configuration may not be as curved-shaped as it assumed to be. The interface may not have a perfect plane or obvious curvature configuration as predicted but a combination of both configurations. The interfacial configuration might have a flat interface with a slight curve at the middle point and near the wall as shown in Figure 4.9. As investigated by Lawrence et. al., the parameters which affect the shape of the interface is the Bond number, the holdup and the contact angle of the two fluids and the pipe wall. As the holdup of the denser fluid increases, the interface shape progresses from a convex interface to a flat shape, and later transforming into a concave interface as the pipe is filled through. The fluid–fluid interface becomes flatter with highly curved menisci at the walls as the Bond number increases.

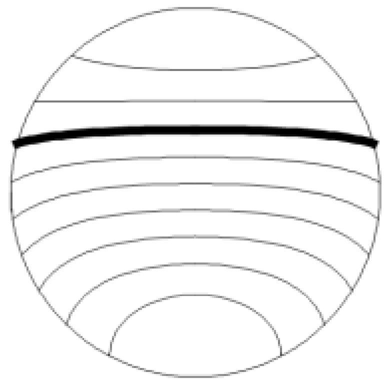

Figure 4.9: Interface shape at different contact angles (Lawrence et. al., 2002)

As mentioned earlier, the shape of the interface is hugely depending on the contact angle between both phases and also the wall. Thus, certain velocities of both fluids will result such an interface as in Figure 4.9. If that is the case, then flat interface assumption will give much more accurate prediction rather than curved interface as it is much closer to becoming flat rather than a curve.

From the above results, it can be observed that the pressure drop increases with respect to the superficial velocities of oil, U_{so} and also water, U_{sw}. This is because the shear stresses for both phases tend to increase with the high velocities. As a result, the pressure drop will increase due to the increasing amount of flow resistance through the pipeline.

CHAPTER 5
CONCLUSION AND RECOMMENDATIONS

4.1 Conclusion

In a nutshell, the focus of the study is to mainly aid engineers in determining the pressure drop of a two-phase, liquid-liquid system in a horizontal pipe based upon various interfacial configurations. A MATLAB programming code developed in this study is able to closely predict the pressure drop for the aforementioned cases. At different configurations, a comparison between calculated and experimental data had proven that the shape of the interface does influence the measurement of pressure drops in an actual scenario. Based on the results obtained, it is shown that flat-shaped interface assumption is not the best assumption for this prediction as the percentage difference is quite large when being compared to experimental data. Curved interface configuration is assumed to give a more accurate prediction; however, that configuration does not provide the expected result. This is due to some errors made in the determination of cross sectional area and wetted perimeter derivation formula used in this model. Modification should be done to improve the prediction and it is still believed that the curved interface configuration assumption will give the closest prediction with respect to the experimental data.

4.2 Recommendations

Some recommendations are suggested in order to improve the reliability of the model. The recommendations are:

- Modifications in the determination of cross sectional area and wetted perimeter derived for the curved interface configuration need to be carried out to improve the calculation of $\Delta P/\Delta x$.
- Improve on the MATLAB coding in order to determine cross sectional area by using integration of the curved interface radius function.

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APPENDICES

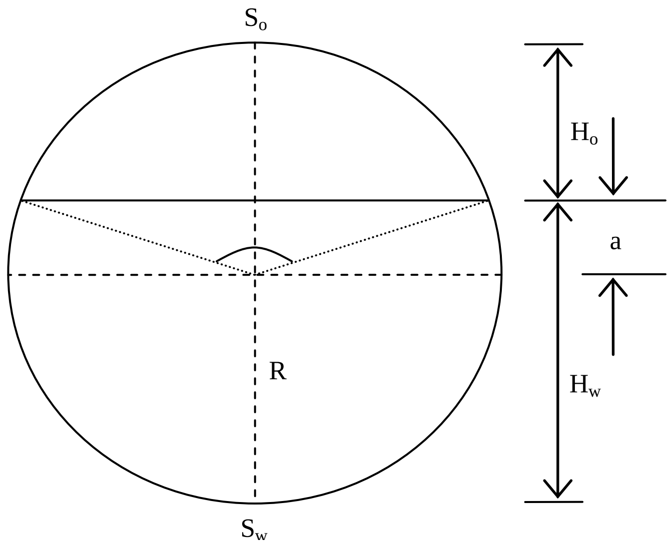
Appendix I: Gantt Chart

No	Detail / Week	1	2	3	4	5	6	7		8	9	10	11	12	13	14
1	Selection of Project Topic								**Mid Semester Break**							
2	Literature review and survey															
3	To understand the concept of two-phase, liquid-liquid flow															
4	Submission of Extended Proposal															
5	Proposal Defense															
6	Determination of Interface Configuration															
7	Mathematical Formula Confirmation															
9	Submission of Interim Draft Report															
10	Submission of Interim Report															

Expected completion

Key Milestone

Appendix II: Area and Perimeter Calculation for Flat Interfacial Configuration



If $H_w > R$

$a = H_w - R$

$H_o = 2R - H_w = R - a$

$$\theta = 2\cos^{-1}\left(\frac{a}{R}\right) = 2\cos^{-1}\left(\frac{H_w - R}{R}\right)$$

$$S_o = R\theta = 2R\cos^{-1}\left(\frac{H_w - R}{R}\right)$$

$$S_w = 2\pi R - R\theta$$

$$= 2R\left[\pi - \cos^{-1}\left(\frac{a}{R}\right)\right]$$

$$= 2R\left[\pi - \cos^{-1}\left(\frac{H_w - R}{R}\right)\right]$$

$$S_i = 2\sqrt{(R^2 - a^2)}$$

$$= 2\sqrt{[R^2 - (H_w - R)^2]}$$

$$A_o = \frac{R^2\theta}{2} - a(R^2 - a^2)^{1/2}$$

$$= R^2 \cos^{-1}\left(\frac{H_w - R}{R}\right) - (H_w - R)[R^2 - (H_w - R)^2]^{1/2}$$

$$A_w = \pi R^2 - A_o$$

$$= \pi R^2 - \left[\frac{R^2\theta}{2} - a(R^2 - a^2)^{1/2}\right]$$

$$= \pi R^2 - R^2 \cos^{-1}\left(\frac{H_w - R}{R}\right) - (H_w - R)[R^2 - (H_w - R)^2]^{1/2}$$

If $H_w < R$

$a = R - H_w$

$Ho = 2R - H_w = R + a$

$$\theta = 2\cos^{-1}\left(\frac{a}{R}\right) = 2\cos^{-1}\left(\frac{R - Hw}{R}\right)$$

$$S_w = R\theta = 2R\cos^{-1}\left(\frac{R - Hw}{R}\right)$$

$$S_o = 2\pi R - R\theta$$

$$= 2R\left[\pi - \cos^{-1}\left(\frac{a}{R}\right)\right]$$

$$= 2R\left[\pi - \cos^{-1}\left(\frac{R - Hw}{R}\right)\right]$$

$$S_i = 2\sqrt{(R^2 - a^2)}$$

$$= 2\sqrt{[R^2 - (R - Hw)^2]}$$

$$A_w = \frac{R^2\theta}{2} - a(R^2 - a^2)^{1/2}$$

$$= R^2 \cos^{-1}\left(\frac{R - Hw}{R}\right) - (R - Hw)[R^2 - (R - Hw)^2]^{1/2}$$

$$A_o = \pi R^2 - A_w$$

$$= \pi R^2 - \left[\frac{R^2\theta}{2} - a(R^2 - a^2)^{1/2}\right]$$

$$= \pi R^2 - R^2 \cos^{-1}\left(\frac{R - Hw}{R}\right) - (R - Hw)[R^2 - (R - Hw)^2]^{1/2}$$

Where,

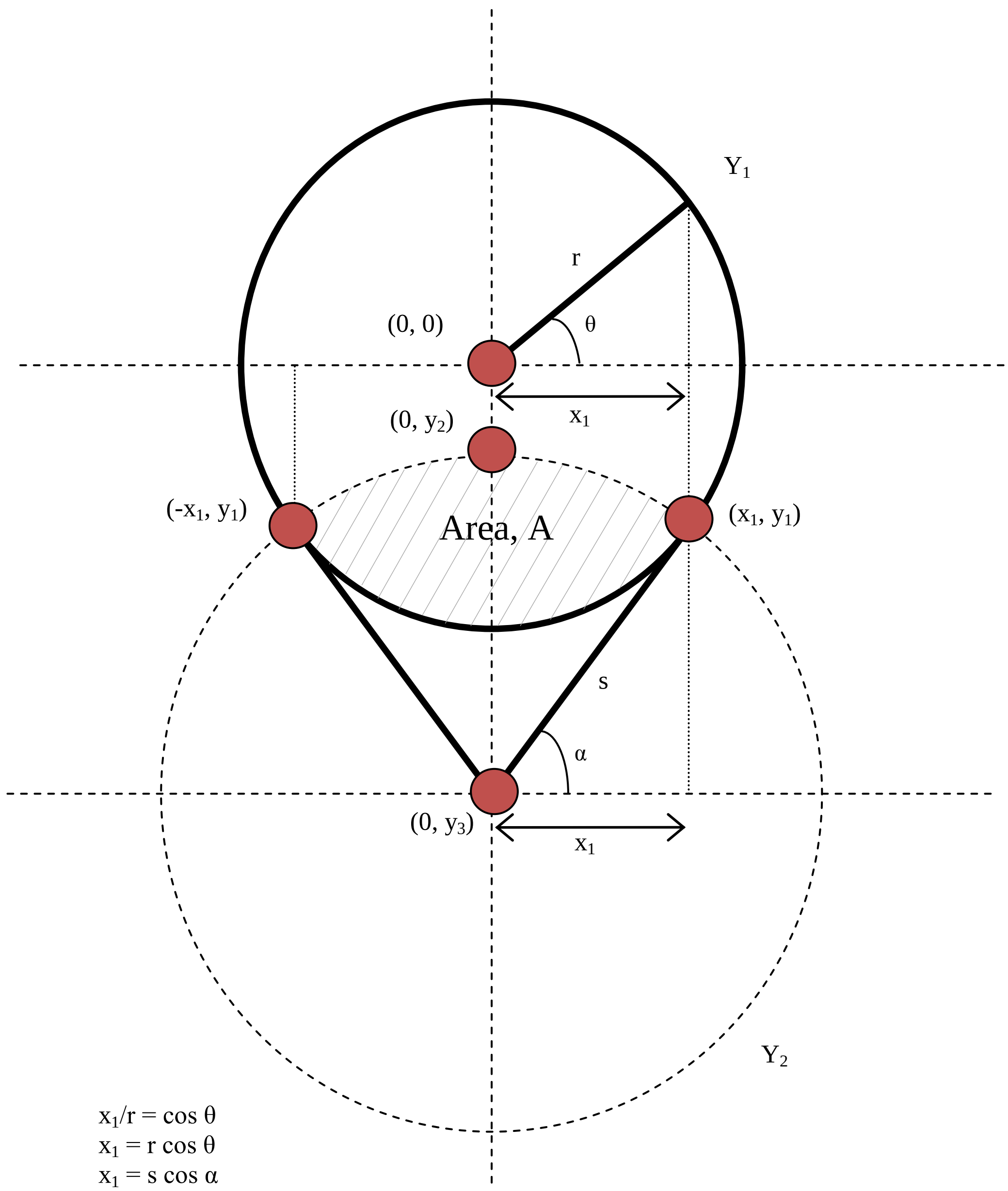
H_w, H_o = Height of oil and water phase;

R = Radius of pipeline;

A_w, A_o = Cross-sectional area for oil and water phase;

S_w, S_o, S_i = Perimeter for oil, water phases and interfacial

Appendix III: Area and Perimeter Calculation for Curvature Interfacial Configuration



$x_1/r = \cos\theta$
$x_1 = r\cos\theta$
$x_1 = s\cos\alpha$
$r\cos\theta = s\cos\alpha$
$\cos\theta = s/r\cos\alpha$
$\theta = \cos^{-1}(s/r\cos\alpha)$

$$\text{Area, A} = \int_{-X_1}^{X_1} Y_1 - Y_2 \, dx = 2\int_{0}^{X_1} Y_1 - Y_2 \, dx$$

$$\text{Area, A} = 2\int_{0}^{X_1} Y_1 - Y_2 \, dx$$

$$= 2\int_{0}^{X_1} \sqrt{r^2 - x^2}\, dx - 2\int_{0}^{x_1} \sqrt{s^2 - x^2} + y_3 dx$$

$$= 2\int_{0}^{X_1} \sqrt{r^2 - x^2}\, dx - 2\int_{0}^{x_1} \sqrt{s^2 - x^2} + y_1 + \frac{{x_1}^2}{y_1} dx$$

$$= 2\int_{\frac{\pi}{2}}^{\cos^{-1}\left(\frac{x_1}{r}\right)} \sqrt{r^2 - r^2\cos^2\theta}\,(-r\sin\theta)\, d\theta$$

$$- 2\int_{\frac{\pi}{2}}^{\cos^{-1}\left(\frac{x_1}{s}\right)} \sqrt{s^2 - s^2\cos^2\alpha}\,(-s\sin\alpha) d\alpha$$

$$- 2\int_{\frac{\pi}{2}}^{\cos^{-1}\left(\frac{x_1}{s}\right)} \left(y_1 + \frac{{x_1}^2}{y_1}\right)(-s\sin\alpha)\, d\alpha$$

$$= -2r^2\int_{\frac{\pi}{2}}^{\cos^{-1}\left(\frac{x_1}{r}\right)} \sin\theta\sin\theta\, d\theta$$

$$+ 2s^2\int_{\frac{\pi}{2}}^{\cos^{-1}\left(\frac{x_1}{s}\right)} \sin\alpha\sin\alpha\, d\alpha + 2s\int_{\frac{\pi}{2}}^{\cos^{-1}\left(\frac{x_1}{s}\right)} \left(y_1 + \frac{{x_1}^2}{y_1}\right)\sin\alpha\;\, d\alpha$$

$$= \frac{-2r^2}{2}\int_{\frac{\pi}{2}}^{\cos^{-1}\left(\frac{x_1}{r}\right)} (1 - \cos 2\theta) d\theta$$

$$+ \frac{2s^2}{2}\int_{\frac{\pi}{2}}^{\cos^{-1}\left(\frac{x_1}{s}\right)} (1 - \cos 2\alpha) d\alpha + 2s\left(y_1 + \frac{{x_1}^2}{y_1}\right)\int_{\frac{\pi}{2}}^{\cos^{-1}\left(\frac{x_1}{s}\right)} \sin\alpha\;\, d\alpha$$

$$= r^2\left[\theta - \frac{\sin 2\theta}{2}\right]_{\frac{\pi}{2}}^{\cos^{-1}\left(\frac{x_1}{r}\right)} + \frac{2s^2}{2}\left[\alpha - \frac{\sin 2\alpha}{2}\right]_{\frac{\pi}{2}}^{\cos^{-1}\left(\frac{x_1}{s}\right)}$$

$$+ 2s\left(y_1 + \frac{{x_1}^2}{y_1}\right)[-\cos\alpha]_{\frac{\pi}{2}}^{\cos^{-1}\left(\frac{x_1}{s}\right)}$$

$$A = -r^2\left\{\left[\cos^{-1}\left(\frac{x_1}{r}\right) - \frac{1}{2}\sin 2\left(\cos^{-1}\left(\frac{x_1}{r}\right)\right)\right] - \left[\frac{\pi}{2} - \frac{\sin\pi}{2}\right]\right\}$$
$$+ s^2\left\{\left[\cos^{-1}\left(\frac{x_1}{s}\right) - \frac{1}{2}\sin 2\left(\cos^{-1}\left(\frac{x_1}{s}\right)\right)\right] - \left[\frac{\pi}{2} - \frac{\sin\pi}{2}\right]\right\}$$
$$+ 2s\left(y_1 + \frac{{x_1}^2}{y_1}\right)\left[-\cos\left(\cos^{-1}\left(\frac{x_1}{s}\right)\right) + \cos\frac{\pi}{2}\right]$$

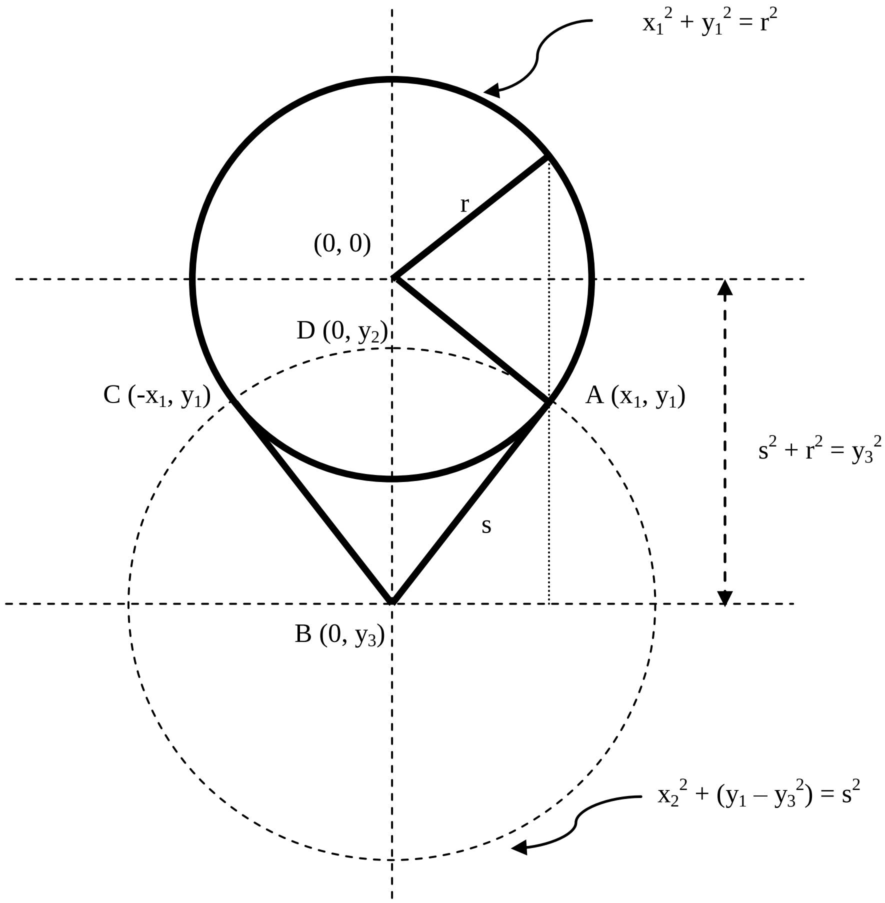
$$A = -r^2\left\{\cos^{-1}\left(\frac{x_1}{r}\right) - \frac{1}{2}2\sin\left(\cos^{-1}\left(\frac{x_1}{r}\right)\right)\cos\left(\cos^{-1}\left(\frac{x_1}{r}\right)\right) - \frac{\pi}{2}\right\}$$
$$+ s^2\left\{\cos^{-1}\left(\frac{x_1}{s}\right) - \frac{1}{2}2\sin\left(\cos^{-1}\left(\frac{x_1}{s}\right)\right)\cos\left(\cos^{-1}\left(\frac{x_1}{s}\right)\right) - \frac{\pi}{2}\right\}$$
$$+ 2s\left(y_1 + \frac{{x_1}^2}{y_1}\right)\left[-\frac{x_1}{s}\right]$$

$$A = -r^2\left\{\cos^{-1}\left(\frac{x_1}{r}\right) - \sin\left(\cos^{-1}\left(\frac{x_1}{r}\right)\right)\left(\frac{x_1}{r}\right) - \frac{\pi}{2}\right\}$$
$$+ s^2\left\{\cos^{-1}\left(\frac{x_1}{s}\right) - \sin\left(\cos^{-1}\left(\frac{x_1}{s}\right)\right)\left(\frac{x_1}{s}\right) - \frac{\pi}{2}\right\} - 2x_1\left(y_1 + \frac{{x_1}^2}{y_1}\right)$$
$$A = -r^2\cos^{-1}\left(\frac{x_1}{r}\right) + r^2\left(\frac{x_1}{r}\right)\sin\left(\cos^{-1}\left(\frac{x_1}{r}\right)\right) + r^2\frac{\pi}{2} + s^2\cos^{-1}\left(\frac{x_1}{s}\right)$$
$$- s^2\left(\frac{x_1}{s}\right)\sin\left(\cos^{-1}\left(\frac{x_1}{s}\right)\right) - s^2\frac{\pi}{2} - 2x_1y_1 - 2\frac{{x_1}^3}{y_1}$$
$$A = -r^2\cos^{-1}\left(\frac{x_1}{r}\right) + rx_1\sin\left(\cos^{-1}\left(\frac{x_1}{r}\right)\right) + r^2\frac{\pi}{2} + s^2\cos^{-1}\left(\frac{x_1}{s}\right)$$
$$- sx_1\sin\left(\cos^{-1}\left(\frac{x_1}{s}\right)\right) - s^2\frac{\pi}{2} - 2x_1y_1 - 2\frac{{x_1}^3}{y_1}$$
$$A = -r^2\cos^{-1}\left(\frac{x_1}{r}\right) + rx_1\sin\left(\cos^{-1}\left(\frac{x_1}{r}\right)\right) + \frac{\pi}{2}(r^2 - s^2) + s^2\cos^{-1}\left(\frac{x_1}{s}\right)$$
$$- sx_1\sin\left(\cos^{-1}\left(\frac{x_1}{s}\right)\right) - 2x_1y_1 - 2\frac{{x_1}^3}{y_1}$$

Where, $s = \sqrt{{x_1}^2 + \frac{{x_1}^4}{{y_1}^2}}$

$$r = \left\{\frac{d}{dx}\frac{\frac{dy}{dx}}{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{1/2}}\right\}^{-1}$$

Interface radius formula obtained from Brauner & Gorelik, 1999



Distance AB; $x_1^2 + (y_1 - y_3)^2 = s^2$

Distance CB; $(-x_1)^2 + (y_1 - y_3)^2 = s^2$

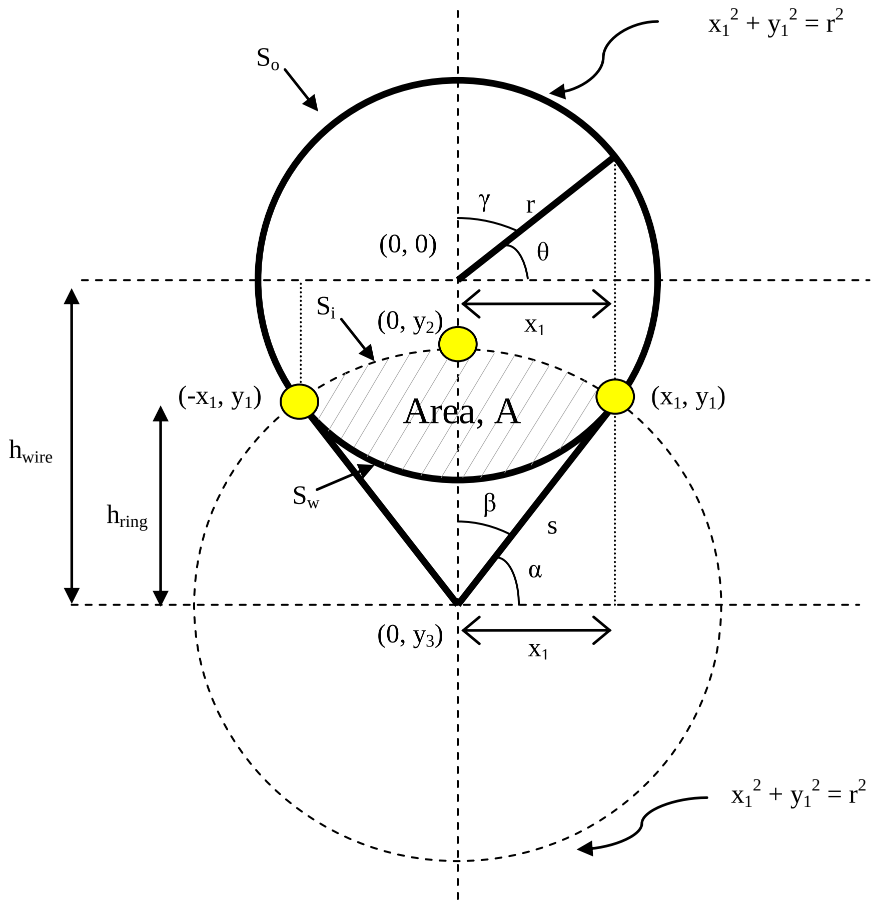
Distance DB; $0^2 + (y_2 - y_3)^2 = s^2$

(1) – (2) $\longrightarrow$ $x_1^2 + (y_1 - y_3)^2 - (y_2 - y_3)^2 = 0$

$x_1^2 + y_1^2 - 2y_1y_3 + y_3^2 - y_2^2 + 2y_2y_3 - y_3^2 = 0$

$x_1^2 + y_1^2 - y_2^2 - 2y_3(y_1 - y_2) = 0$

$$y_3 = \frac{x_1^2 + y_1^2 - y_2^2}{2(y_1 - y_2)}$$



$$y_1 = r - h_{ring}$$

$$y_2 = r - h_{wire}$$

$$x_1 = \sqrt{r^2 - {y_1}^2}$$

$$x_1 = \sqrt{r^2 - \left(r - h_{ring}\right)^2}$$

$$= \sqrt{r^2 - \left(r^2 - 2rh_{ring} + \left(h_{ring}\right)^2\right)}$$

$$= \sqrt{r^2 - r^2 + 2rh_{ring} - \left(h_{ring}\right)^2}$$

$$= \sqrt{h_{ring}\left(2r - h_{ring}\right)}$$

$$\frac{x_1}{s} = \cos\alpha$$

$$\alpha = \cos^{-1}\left(\frac{x_1}{s}\right)$$

$$\beta = \frac{\pi}{2} - \alpha$$

$$2\beta = \pi - 2\alpha$$

$$S_i = s(2\beta) = s\left[\pi - 2\cos^{-1}\left(\frac{x_1}{s}\right)\right]$$

$$\frac{x_1}{r} = \cos\theta$$

$$\theta = \cos^{-1}\left(\frac{x_1}{r}\right)$$

$$\gamma = \frac{\pi}{2} - \theta$$

$$2\gamma = \pi - 2\theta$$

$$S_w = r(2\gamma) = r\left[\pi - 2\cos^{-1}\left(\frac{x_1}{r}\right)\right]$$

$$S_o = 2\pi r - S_w$$

Where,

H_w, H_o = Height of oil and water phase;

R = Radius of pipeline;

A_w, A_o = Cross sectional area for oil and water phase;

S_w, S_o = Perimeter for oil and water phase;

s = Radius for imaginary circle

Appendix IV: MATLAB Programming Code for Flat Interfacial Configuration by using Wire Probe Height

```
fprintf('\n\n');
disp('This program will determine the pressure drop inside the pipeline
for liquid-liquid two phase system by using wire probe height with
assumption that the flat interface between the phases');

%Prompt the user to enter a string character information
wireprobe_height = input('\nPlease enter the set of wire probe heights
in bracket(unit of mm){example = [1 2 3 4]}:')*10^(-3);%array
superficialvelocity_water = input('Please enter the superficial
velocity of water(unit of m/s):');%single number
superficialvelocity_oil = input('Please enter the superficial velocity
of oil in bracket(unit of m/s){example = [1 2 3 4]}:');%array
pipeline_radius = input('Please enter the radius of pipeline (unit of
mm):')*10^(-3);pipeline_length = input('Please enter the length of
pipeline (unit of m):');%single number
density_oil = input('Please enter the density of oil (unit of
kg/m^3):');%single number
density_water = input('Please enter the density of water (unit of
kg/m^3):');%single number
viscosity_oil = input('Please enter the viscosity of oil at 25 degree
Celcius (unit of kg/m.s or Pa.s):');%single number
viscosity_water = input('Please enter the viscosity of water at 25
degree Celcius (unit of kg/m.s or Pa.s):');%single number
Surface_tension = input('Please enter the surface tension at 25 degree
Celcius (unit of kg/s^2 or N/m):');%single number
flowrate_oil = input('Please enter the set of oil flowrate in bracket
for the corresponding wire probe heights(unit of 10^(-5)m^3/s){example
= [1 2 3 4]}:')*10^(-5);%array
flowrate_water = input('Please enter the set of water flowrate in
bracket for the corresponding wire probe heights(unit of 10^(-
5)m^3/s){example = [1 2 3 4]}:')*10^(-5);%array
fprintf('\n\n');

%Set first value to all variables
a = zeros(size(wireprobe_height));
theta = zeros(size(wireprobe_height));
height_water = zeros(size(wireprobe_height));
height_oil = zeros(size(wireprobe_height));
perimeter_oil = zeros(size(wireprobe_height));
perimeter_water = zeros(size(wireprobe_height));
perimeter_interface = zeros(size(wireprobe_height));
area_oil = zeros(size(wireprobe_height));
area_water = zeros(size(wireprobe_height));

k = 1;
while k <= length(wireprobe_height)
if wireprobe_height(k) > pipeline_radius
a(k) = wireprobe_height(k) - pipeline_radius;
theta(k) = 2*(acos(a(k)/pipeline_radius)); %array
```

```
%Calculation of height of water and height of oil in the pipeline
height_water(k) = wireprobe_height(k); %array
height_oil(k) = pipeline_radius - a(k); %array

%Calculation of oil phase perimeter, water phase perimeter and
interface perimeter
perimeter_oil(k) = pipeline_radius*theta(k); %array
perimeter_water(k) = (2*pi*pipeline_radius) -
(pipeline_radius*theta(k)); %array
perimeter_interface(k) = 2*sqrt(pipeline_radius^2 - (a(k)).^2); %array

%Calculation of oil phase area and water phase area
area_oil(k) = ((pipeline_radius^2)*(theta(k))/2) -
((a(k)).*sqrt(pipeline_radius^2) - (a(k)).^2); %array
area_water(k) = (pi*pipeline_radius^2) - area_oil(k); %array

elseif wireprobe_height(k) < pipeline_radius

a(k) = pipeline_radius - wireprobe_height(k);
theta(k) = 2*(acos(a(k)/pipeline_radius)); %array

%Calculation of height of water and height of oil in the pipeline
height_water(k) = wireprobe_height(k); %array
height_oil(k) = pipeline_radius + a(k); %array

%Calculation of oil phase perimeter, water phase perimeter and
interface perimeter
perimeter_oil(k) = (2*pi*pipeline_radius) -
(pipeline_radius*(theta(k))); %array
perimeter_water(k) = pipeline_radius*(theta(k)); %array
perimeter_interface(k) = 2*sqrt(pipeline_radius^2 - (a(k)).^2); %array

%Calculation of oil phase area and water phase area
area_water(k) = ((pipeline_radius^2)*(theta(k))/2) -
((a(k)).*sqrt(pipeline_radius^2) - (a(k)).^2); %array
area_oil(k) = (pi*pipeline_radius^2) - area_water(k); %array

else

disp('Could not determine area and perimeter of the phases');

end

k = k+1;

end

%Calculation of in-situ velocity for water and oil
velocity_oil = flowrate_oil./area_oil;
velocity_water = flowrate_water./area_water;
```

```
%Calculation to determine the hydraulic diameter of water and oil
%Conditions outlined by Moalem-Maron et. al. (1998)

%Set first value to all variables
diameter_oil = zeros(size(wireprobe_height));
diameter_water = zeros(size(wireprobe_height));

k = 1;
while k <= length(wireprobe_height)

    if (velocity_oil(k) > velocity_water(k))

        diameter_oil(k) =
(4*area_oil(k))./(perimeter_oil(k)+perimeter_interface(k));
        diameter_water(k) = (4*area_water(k))./perimeter_water(k);

    elseif (velocity_oil(k) < velocity_water(k))

        diameter_oil(k) = (4*area_oil(k))./perimeter_oil(k);
        diameter_water(k) =
(4*area_water(k))./(perimeter_water(k)+perimeter_interface(k));

    else

        diameter_oil(k) = (4*area_oil(k))./perimeter_oil(k);
        diameter_water(k) = (4*area_water(k))./perimeter_water(k);

    end

    k = k+1;

end

%Calculation of Reynolds number
Reynolds_oil = (density_oil*velocity_oil.*diameter_oil)/viscosity_oil;
Reynolds_water =
(density_water*velocity_water.*diameter_water)/viscosity_water;

%Determination of C and n constant for water phase

%Set first value to all variables
C_water = zeros(size(wireprobe_height));
n_water = zeros(size(wireprobe_height));
flowtype_water = 'ERROR';

k = 1;
while k <= length(wireprobe_height)

    if Reynolds_water(k) < 2000

        C_water(k) = 16;
        n_water(k) = 1;
        flowtype_water = 'Laminar Flow';
```

```
    elseif Reynolds_water(k) > 4000

        C_water(k) = 0.046;
        n_water(k) = 0.2;
        flowtype_water = 'Turbulent Flow';

    else

        C_water(k) = NaN;
        n_water(k) = NaN;
        flowtype_water = 'Transitional Flow';

    end

    k = k+1;

end

disp(flowtype_water);

%Determination of C and n constant for oil phase

%Set first value to all variables
C_oil = zeros(size(Reynolds_water));
n_oil = zeros(size(Reynolds_water));
flowtype_oil = 'ERROR';

k = 1;
while k <= length(Reynolds_water)

    if Reynolds_oil(k) < 2000

        C_oil(k) = 16;
        n_oil(k) = 1;
        flowtype_oil = 'Laminar Flow';

    elseif Reynolds_water(k) > 4000

        C_oil(k) = 0.046;
        n_oil(k) = 0.2;
        flowtype_oil = 'Turbulent Flow';

    else

        C_oil(k) = NaN;
        n_oil(k) = NaN;
        flowtype_oil = 'Transitional Flow';

    end
```

```
    k = k+1;

end

disp(flowtype_oil);

%Calculation of friction factor for oil and water flow
friction_oil =
C_oil.*((density_oil*velocity_oil.*diameter_oil/viscosity_oil).^(-
n_oil));
friction_water =
C_water.*((density_water*velocity_water.*diameter_water/viscosity_water
).^(-n_water));

%Calculation of shear stress for oil and water flow
shearstress_oil = 0.5*(friction_oil*density_oil).*(velocity_oil.^2);
shearstress_water =
0.5*(friction_water*density_water).*(velocity_water.^2);

%Calculation of pressure drop (final stage)
dpdz =
((shearstress_oil.*perimeter_oil)+(shearstress_water.*perimeter_water))
./(-1*(area_oil + area_water));
pressure_drop = abs(dpdz/1000);

%Calculation of shear stress of interface
shearstress_interface = (area_oil.*dpdz +
shearstress_oil.*perimeter_oil)./(-1*perimeter_interface);

%Shows parameters for both oil and water phases
fprintf('\n\n');
disp('By assuming a flat interface between phases, the parameters for
both oil and water phases are obtained');
fprintf('\n');
fprintf('Wire Probe Height\t\tPerimeter of Oil Phase\t\tPerimeter of
Water Phase\t\tPerimeter of Interface\t\tArea of Oil Phase\t\tArea of
Water Phase\n');
table_1 = [wireprobe_height; perimeter_oil; perimeter_water;
perimeter_interface; area_oil; area_water];
fprintf('%10.6f m %25.6f m %25.6f m %27.6f m %23.6f m^2 %20.6f m^2\n',
table_1);
fprintf('\n\n');
fprintf('Velocity of Oil\t\tVelocity of Water\t\tDiameter of
Oil\t\tDiameter of Water\t\tReynolds Number of Oil\t\tReynolds Number
of Water\n');
table_2 = [velocity_oil; velocity_water; diameter_oil; diameter_water;
Reynolds_oil; Reynolds_water];
fprintf('%10.6f m/s %15.6f m/s %20.6f m %17.6f m %28.6f %25.6f \n',
table_2);
fprintf('\n\n');
fprintf('Friction Factor of Oil\t\tFriction Factor of Water\t\tShear
Stress of Oil\t\tShear Stress of Water\t\tPressure Drop\t\tShear Stress
of Interface\n');
table_3 = [friction_oil; friction_water; shearstress_oil;
shearstress_water; pressure_drop; shearstress_interface];
```

```
fprintf('%15.6f %25.6f %30.6f Pa %20.6f Pa %21.6f kPa/m %20.6f Pa\n',
table_3);
fprintf('\n\n');

%Graph Plot
disp('Graph Comparison of Theoretical Data vs Experimental Data');
experimental_pressuredrop = input('\nPlease enter the set of
experimental data in bracket(unit of kPa){example = [1 2 3 4]}:');
%array

subplot(2,1,1),plot(superficialvelocity_oil,pressure_drop,'-
or',superficialvelocity_oil,experimental_pressuredrop./pipeline_length,
'-xk')
title('Graph of Pressure Drop vs Superficial Velocity of Oil')
xlabel('Superficial Velocity of Oil, m/s')
ylabel('Pressure Drop, kPa/m')
legend('Theoretical Value (wire probe)', 'Experimental Value')
grid on

%Graph Plot
fprintf('\n\n');
disp('Graph Shear Stress of Interface vs Superficial Velocity of Oil');

subplot(2,1,2), plot(superficialvelocity_oil, shearstress_interface,'-
or')
title('Graph of Shear Stress of Interface vs Superficial Velocity of
Oil')
xlabel('Superficial Velocity of Oil, m/s')
ylabel('Shear Stress of Interface, Pa')
grid on
```

Appendix V: MATLAB Programming Code for Flat Interfacial Configuration by using Ring Probe Height

```
fprintf('\n\n');
disp('This program will determine the pressure drop inside the pipeline
for liquid-liquid two phase system by using ring probe height with
assumption that the flat interface between the phases');

%Prompt the user to enter a string character information
ringprobe_height = input('\nPlease enter the set of ring probe heights
in bracket(unit of mm){example = [1 2 3 4]}:')*10^(-3); %array
superficialvelocity_water = input('Please enter the superficial
velocity of water(unit of m/s):'); %single number
superficialvelocity_oil = input('Please enter the superficial velocity
of oil in bracket(unit of m/s){example = [1 2 3 4]}:'); %array
pipeline_radius = input('Please enter the radius of pipeline (unit of
mm):')*10^(-3);pipeline_length = input('Please enter the length of
pipeline (unit of m):'); %single number
density_oil = input('Please enter the density of oil (unit of
kg/m^3):'); %single number
density_water = input('Please enter the density of water (unit of
kg/m^3):'); %single number
viscosity_oil = input('Please enter the viscosity of oil at 25 degree
Celcius (unit of kg/m.s or Pa.s):'); %single number
viscosity_water = input('Please enter the viscosity of water at 25
degree Celcius (unit of kg/m.s or Pa.s):'); %single number
Surface_tension = input('Please enter the surface tension at 25 degree
Celcius (unit of kg/s^2 or N/m):'); %single number
flowrate_oil = input('Please enter the set of oil flowrate in bracket
for the corresponding ring probe heights(unit of 10^(-5)m^3/s){example
= [1 2 3 4]}:')*10^(-5); %array
flowrate_water = input('Please enter the set of water flowrate in
bracket for the corresponding ring probe heights(unit of 10^(-
5)m^3/s){example = [1 2 3 4]}:')*10^(-5); %array
fprintf('\n\n');

%Set first value to all variables
a = zeros(size(ringprobe_height));
theta = zeros(size(ringprobe_height));
height_water = zeros(size(ringprobe_height));
height_oil = zeros(size(ringprobe_height));
perimeter_oil = zeros(size(ringprobe_height));
perimeter_water = zeros(size(ringprobe_height));
perimeter_interface = zeros(size(ringprobe_height));
area_oil = zeros(size(ringprobe_height));
area_water = zeros(size(ringprobe_height));

k = 1;
while k <= length(ringprobe_height)
if ringprobe_height(k) > pipeline_radius
a(k) = ringprobe_height(k) - pipeline_radius;
theta(k) = 2*(acos(a(k)/pipeline_radius)); %array
```

```
%Calculation of height of water and height of oil in the pipeline
height_water(k) = ringprobe_height(k); %array
height_oil(k) = pipeline_radius - a(k); %array

%Calculation of oil phase perimeter, water phase perimeter and
interface perimeter
perimeter_oil(k) = pipeline_radius*theta(k); %array
perimeter_water(k) = (2*pi*pipeline_radius) -
(pipeline_radius*theta(k)); %array
perimeter_interface(k) = 2*sqrt(pipeline_radius^2 - (a(k)).^2); %array

%Calculation of oil phase area and water phase area
area_oil(k) = ((pipeline_radius^2)*(theta(k))/2) -
((a(k)).*sqrt(pipeline_radius^2) - (a(k)).^2); %array
area_water(k) = (pi*pipeline_radius^2) - area_oil(k); %array

elseif ringprobe_height(k) < pipeline_radius

a(k) = pipeline_radius - ringprobe_height(k);
theta(k) = 2*(acos(a(k)/pipeline_radius)); %array

%Calculation of height of water and height of oil in the pipeline
height_water(k) = ringprobe_height(k); %array
height_oil(k) = pipeline_radius + a(k); %array

%Calculation of oil phase perimeter, water phase perimeter and
interface perimeter
perimeter_oil(k) = (2*pi*pipeline_radius) -
(pipeline_radius*(theta(k))); %array
perimeter_water(k) = pipeline_radius*(theta(k)); %array
perimeter_interface(k) = 2*sqrt(pipeline_radius^2 - (a(k)).^2); %array

%Calculation of oil phase area and water phase area
area_water(k) = ((pipeline_radius^2)*(theta(k))/2) -
((a(k)).*sqrt(pipeline_radius^2) - (a(k)).^2); %array
area_oil(k) = (pi*pipeline_radius^2) - area_water(k); %array

else

disp('Could not determine area and perimeter of the phases');

end

k = k+1;

end

%Calculation of in-situ velocity for water and oil
velocity_oil = flowrate_oil./area_oil;
velocity_water = flowrate_water./area_water;
```

```
%Calculation to determine the hydraulic diameter of water and oil
%Conditions outlined by Moalem-Maron et. al. (1998)

%Set first value to all variables
diameter_oil = zeros(size(ringprobe_height));
diameter_water = zeros(size(ringprobe_height));

k = 1;
while k <= length(ringprobe_height)

    if (velocity_oil(k) > velocity_water(k))

        diameter_oil(k) =
(4*area_oil(k))./(perimeter_oil(k)+perimeter_interface(k));
        diameter_water(k) = (4*area_water(k))./perimeter_water(k);

    elseif (velocity_oil(k) < velocity_water(k))

        diameter_oil(k) = (4*area_oil(k))./perimeter_oil(k);
        diameter_water(k) =
(4*area_water(k))./(perimeter_water(k)+perimeter_interface(k));

    else

        diameter_oil(k) = (4*area_oil(k))./perimeter_oil(k);
        diameter_water(k) = (4*area_water(k))./perimeter_water(k);

    end

    k = k+1;

end

%Calculation of Reynolds number
Reynolds_oil = (density_oil*velocity_oil.*diameter_oil)/viscosity_oil;
Reynolds_water =
(density_water*velocity_water.*diameter_water)/viscosity_water;

%Determination of C and n constant for water phase

%Set first value to all variables
C_water = zeros(size(ringprobe_height));
n_water = zeros(size(ringprobe_height));
flowtype_water = 'ERROR';

k = 1;
while k <= length(ringprobe_height)

    if Reynolds_water(k) < 2000

        C_water(k) = 16;
        n_water(k) = 1;
        flowtype_water = 'Laminar Flow';
```

```
    elseif Reynolds_water(k) > 4000

        C_water(k) = 0.046;
        n_water(k) = 0.2;
        flowtype_water = 'Turbulent Flow';

    else

        C_water(k) = NaN;
        n_water(k) = NaN;
        flowtype_water = 'Transitional Flow';

    end

    k = k+1;

end

disp(flowtype_water);

%Determination of C and n constant for oil phase

%Set first value to all variables
C_oil = zeros(size(Reynolds_water));
n_oil = zeros(size(Reynolds_water));
flowtype_oil = 'ERROR';

k = 1;
while k <= length(Reynolds_water)

    if Reynolds_oil(k) < 2000

        C_oil(k) = 16;
        n_oil(k) = 1;
        flowtype_oil = 'Laminar Flow';

    elseif Reynolds_water(k) > 4000

        C_oil(k) = 0.046;
        n_oil(k) = 0.2;
        flowtype_oil = 'Turbulent Flow';

    else

        C_oil(k) = NaN;
        n_oil(k) = NaN;
        flowtype_oil = 'Transitional Flow';

    end
```

```
    k = k+1;

end

disp(flowtype_oil);

%Calculation of friction factor for oil and water flow
friction_oil =
C_oil.*((density_oil*velocity_oil.*diameter_oil/viscosity_oil).^(-
n_oil));
friction_water =
C_water.*((density_water*velocity_water.*diameter_water/viscosity_water
).^(-n_water));

%Calculation of shear stress for oil and water flow
shearstress_oil = 0.5*(friction_oil*density_oil).*(velocity_oil.^2);
shearstress_water =
0.5*(friction_water*density_water).*(velocity_water.^2);

%Calculation of pressure drop (final stage)
dpdz =
((shearstress_oil.*perimeter_oil)+(shearstress_water.*perimeter_water))
./(-1*(area_oil + area_water));
pressure_drop = abs(dpdz/1000);

%Calculation of shear stress of interface
shearstress_interface = (area_oil.*dpdz +
shearstress_oil.*perimeter_oil)./(-1*perimeter_interface);

%Shows parameters for both oil and water phases
fprintf('\n\n');
disp('By assuming a flat interface between phases, the parameters for
both oil and water phases are obtained');
fprintf('\n');
fprintf('Ring Probe Height\t\tPerimeter of Oil Phase\t\tPerimeter of
Water Phase\t\tPerimeter of Interface\t\tArea of Oil Phase\t\tArea of
Water Phase\n');
table_1 = [ringprobe_height; perimeter_oil; perimeter_water;
perimeter_interface; area_oil; area_water];
fprintf('%10.6f m %25.6f m %25.6f m %27.6f m %23.6f m^2 %20.6f m^2\n',
table_1);
fprintf('\n\n');
fprintf('Velocity of Oil\t\tVelocity of Water\t\tDiameter of
Oil\t\tDiameter of Water\t\tReynolds Number of Oil\t\tReynolds Number
of Water\n');
table_2 = [velocity_oil; velocity_water; diameter_oil; diameter_water;
Reynolds_oil; Reynolds_water];
fprintf('%10.6f m/s %15.6f m/s %20.6f m %17.6f m %28.6f %25.6f \n',
table_2);
fprintf('\n\n');
fprintf('Friction Factor of Oil\t\tFriction Factor of Water\t\tShear
Stress of Oil\t\tShear Stress of Water\t\tPressure Drop\t\tShear Stress
of Interface\n');
table_3 = [friction_oil; friction_water; shearstress_oil;
shearstress_water; pressure_drop; shearstress_interface];
```

```
fprintf('%15.6f %25.6f %30.6f Pa %20.6f Pa %21.6f kPa/m %20.6f Pa\n',
table_3);
fprintf('\n\n');

%Graph Plot
disp('Graph Comparison of Theoretical Data vs Experimental Data');
experimental_pressuredrop = input('\nPlease enter the set of
experimental data in bracket(unit of kPa){example = [1 2 3 4]}:');
%array

subplot(2,1,1),plot(superficialvelocity_oil,pressure_drop,'-
or',superficialvelocity_oil,experimental_pressuredrop./pipeline_length,
'-xk')
title('Graph of Pressure Drop vs Superficial Velocity of Oil')
xlabel('Superficial Velocity of Oil, m/s')
ylabel('Pressure Drop, kPa/m')
legend('Theoretical Value (ring probe)', 'Experimental Value')
grid on

%Graph Plot
fprintf('\n\n');
disp('Graph Shear Stress of Interface vs Superficial Velocity of Oil');

subplot(2,1,2), plot(superficialvelocity_oil, shearstress_interface,'-
or')
title('Graph of Shear Stress of Interface vs Superficial Velocity of
Oil')
xlabel('Superficial Velocity of Oil, m/s')
ylabel('Shear Stress of Interface, Pa')
grid on
```

Appendix VI: MATLAB Programming Code for Curvature Interfacial Configuration

```
fprintf('\n\n');
disp('This program will determine the pressure drop inside the pipeline
for liquid-liquid two phase system by using wire probe height and ring
probe height with assumption that the curve interface between the
phases');

%Prompt the user to enter a string character information
wireprobe_height = input('\nPlease enter the set of wire probe heights
in bracket(unit of mm){example = [1 2 3 4]}:')*10^(-3); %array
ringprobe_height = input('\nPlease enter the set of ring probe heights
in bracket(unit of mm){example = [1 2 3 4]}:')*10^(-3); %array
superficialvelocity_water = input('Please enter the superficial
velocity of water(unit of m/s):'); %single number
superficialvelocity_oil = input('Please enter the superficial velocity
of oil in bracket(unit of m/s){example = [1 2 3 4]}:'); %array
pipeline_radius = input('Please enter the radius of pipeline (unit of
mm):')*10^(-3); %single number
pipeline_length = input('Please enter the length of pipeline (unit of
m):'); %single number
density_oil = input('Please enter the density of oil (unit of
kg/m^3):'); %single number
density_water = input('Please enter the density of water (unit of
kg/m^3):'); %single number
viscosity_oil = input('Please enter the viscosity of oil at 25 degree
Celcius (unit of kg/m.s or Pa.s):');%single number
viscosity_water = input('Please enter the viscosity of water at 25
degree Celcius (unit of kg/m.s or Pa.s):'); %single number
surface_tension = input('Please enter the surface tension at 25 degree
Celcius (unit of kg/s^2 or N/m):'); %single number
flowrate_oil = input('Please enter the set of oil flowrate in bracket
for the corresponding wire probe heights(unit of 10^(-5)m^3/s){example
= [1 2 3 4]}:')*10^(-5); %array
flowrate_water = input('Please enter the set of water flowrate in
bracket for the corresponding wire probe heights(unit of 10^(-
5)m^3/s){example = [1 2 3 4]}:')*10^(-5); %array
fprintf('\n\n');

%zeros(Set first value to all variables)
y1 = zeros(size(ringprobe_height));
y2 = zeros(size(wireprobe_height));

%Calculation of oil phase area and water phase area

k = 1;
while k <= length(ringprobe_height)

    if (pipeline_radius > ringprobe_height(k))
        y1(k) = 0 - (pipeline_radius - ringprobe_height(k));
    elseif (pipeline_radius < ringprobe_height(k))
        y1(k) = 0 + (ringprobe_height(k) - pipeline_radius);
    else
```

```
            y1(k) = 0;
        end

        k = k+1;

    end

    k = 1;
    while k <= length(wireprobe_height)

        if (pipeline_radius > wireprobe_height(k))
            y2(k) = 0 - (pipeline_radius - wireprobe_height(k));
        elseif (pipeline_radius < wireprobe_height(k))
            y2(k) = 0 + (wireprobe_height(k) - pipeline_radius);
        else
            y2(k) = 0;
        end

        k = k+1;

    end

    x1 = sqrt((pipeline_radius.^2) - (y1.^2));
    y3 = (x1.^2 + y1.^2 - y2.^2)/(2*(y1 - y2));
    s = sqrt(y3.^2 - (pipeline_radius^2));

    %Curve function of interface radius is obtained from Brauner & Gorelik
    (1999)
    interface_radius = ((x1.*((2.*x1)./(s.^2 - x1.^2) + (2.*x1.^3)./(s.^2 -
    x1.^2).^2))./(2.*(x1.^2./(s.^2 - x1.^2) + 1).^(3/2).*(s.^2 -
    x1.^2).^(1/2)) - x1.^2./((x1.^2./(s.^2 - x1.^2) + 1).^(1/2).*(s.^2 -
    x1.^2).^(3/2)) - 1./((x1.^2/(s.^2 - x1.^2) + 1).^(1/2).*(s.^2 -
    x1.^2).^(1/2)).^(-1));

    area_water = ((-1*(interface_radius.^2)).*(acos(x1/interface_radius)))
    + (interface_radius.*x1.*(sin(acos(x1/interface_radius)))) +
    ((pi/2)*((interface_radius.^2) - (s.^2))) + ((s.^2).*acos(x1./s)) -
    (s.*x1.*sin(acos(x1./s))) - (2*x1.*y1) - (2*(x1.^3)./y1);
    area_oil = (pi*pipeline_radius.^2) - area_water;

    theta = acos(x1/pipeline_radius);
    ghama = (pi/2) - theta;
    perimeter_water = pipeline_radius*(2*ghama);
    perimeter_oil = (2*pi*pipeline_radius) - perimeter_water;
    alpha = acos(x1./s);
    betha = (pi/2) - alpha;
    perimeter_interface = s.*(2*betha);

    %Calculation of in-situ velocity for water and oil
    velocity_oil = flowrate_oil./area_oil;
    velocity_water = flowrate_water./area_water;
```

```
%Calculation to determine the hydraulic diameter of water and oil
%Conditions outlined by Moalem-Maron et. al. (1998)

%Set first value to all variables
diameter_oil = zeros(size(wireprobe_height));
diameter_water = zeros(size(wireprobe_height));

k = 1;
while k <= length(wireprobe_height)

    if (velocity_oil(k) > velocity_water(k))

        diameter_oil(k) = (4*area_oil(k))./(perimeter_oil(k) +
perimeter_interface(k));
        diameter_water(k) = (4*area_water(k))./perimeter_water(k);

    elseif (velocity_oil(k) < velocity_water(k))

        diameter_oil(k) = (4*area_oil(k))./perimeter_oil(k);
        diameter_water(k) = (4*area_water(k))./(perimeter_water(k) +
perimeter_interface(k));

    end

    k = k+1;

end

%Calculation of Reynolds number
Reynolds_oil = (density_oil*velocity_oil.*diameter_oil)/viscosity_oil;
Reynolds_water =
(density_water*velocity_water.*diameter_water)/viscosity_water;

%Determination of C and n constant for water phase
%Set first value to all variables
C_water = zeros(size(wireprobe_height));
n_water = zeros(size(wireprobe_height));
flowtype_water = 'ERROR';

k = 1;
while k <= length(wireprobe_height)

    if Reynolds_water(k) < 2000

        C_water(k) = 16;
        n_water(k) = 1;
        flowtype_water = 'Laminar Flow';

    elseif Reynolds_water(k) > 4000

        C_water(k) = 0.046;
        n_water(k) = 0.2;
        flowtype_water = 'Turbulent Flow';
```

```
    else

        C_water(k) = NaN;
        n_water(k) = NaN;
        flowtype_water = 'Transitional Flow';

    end

    k = k+1;

end

disp(flowtype_water);

%Determination of C and n constant for oil phase
%Set first value to all variables
C_oil = zeros(size(Reynolds_water));
n_oil = zeros(size(Reynolds_water));
flowtype_oil = 'ERROR';

k = 1;
while k <= length(Reynolds_water)

    if Reynolds_oil(k) < 2000

        C_oil(k) = 16;
        n_oil(k) = 1;
        flowtype_oil = 'Laminar Flow';

    elseif Reynolds_water(k) > 4000

        C_oil(k) = 0.046;
        n_oil(k) = 0.2;
        flowtype_oil = 'Turbulent Flow';

    else

        C_oil(k) = NaN;
        n_oil(k) = NaN;
        flowtype_oil = 'Transitional Flow';

    end

    k = k+1;

end

disp(flowtype_oil);
```

```
%Calculation of friction factor for oil and water flow
friction_oil =
C_oil.*((density_oil*velocity_oil.*diameter_oil/viscosity_oil).^(-
n_oil));
friction_water =
C_water.*((density_water*velocity_water.*diameter_water/viscosity_water
).^(-n_water));

%Calculation of shear stress for oil and water flow
shearstress_oil = 0.5*(friction_oil*density_oil).*(velocity_oil.^2);
shearstress_water =
0.5*(friction_water*density_water).*(velocity_water.^2);

%Calculation of pressure drop (final stage)
dpdz =
((shearstress_oil.*perimeter_oil)+(shearstress_water.*perimeter_water))
./(-1*(area_oil + area_water));
pressure_drop = abs(dpdz/1000);

%Calculation of shear stress of interface
shearstress_interface = (area_oil.*dpdz +
shearstress_oil.*perimeter_oil)./(-1*perimeter_interface);

%Shows parameters for both oil and water phases
fprintf('\n\n');
disp('By assuming a curve interface between phases, the parameters for
both oil and water phases are obtained');
fprintf('\n');
fprintf('Ring Probe Height\t\tWire Probe Height\t\tPerimeter of Oil
Phase\t\tPerimeter of Water Phase\t\tPerimeter of Interface\t\tArea of
Oil Phase\t\tArea of Water Phase\n');
table_1 = [ringprobe_height; wireprobe_height; perimeter_oil;
perimeter_water; perimeter_interface; area_oil; area_water];
fprintf('%10.6f m %25.6f m %25.6f m %25.6f m %27.6f m %23.6f m^2 %20.6f
m^2\n', table_1);
fprintf('\n\n');
fprintf('Velocity of Oil\t\tVelocity of Water\t\tDiameter of
Oil\t\tDiameter of Water\t\tReynolds Number of Oil\t\tReynolds Number
of Water\n');
table_2 = [velocity_oil; velocity_water; diameter_oil; diameter_water;
Reynolds_oil; Reynolds_water];
fprintf('%10.6f m/s %15.6f m/s %20.6f m %17.6f m %28.6f %25.6f \n',
table_2);
fprintf('\n\n');
fprintf('Friction Factor of Oil\t\tFriction Factor of Water\t\tShear
Stress of Oil\t\tShear Stress of Water\t\tPressure Drop\t\tShear Stress
of Interface\n');
table_3 = [friction_oil; friction_water; shearstress_oil;
shearstress_water; pressure_drop; shearstress_interface];
fprintf('%15.6f %25.6f %30.6f Pa %20.6f Pa %21.6f kPa/m %20.6f Pa\n',
table_3);
fprintf('\n\n');
```

```
%Graph Plot
disp('Graph Comparison of Theoretical Data vs Experimental Data');
experimental_pressuredrop = input('\nPlease enter the set of
experimental data in bracket(unit of kPa){example = [1 2 3 4]}:');
%array


subplot(2,1,1), plot(superficialvelocity_oil, pressure_drop,'-or',
superficialvelocity_oil, experimental_pressuredrop./pipeline_length, '-
xk')
title('Graph of Pressure Drop vs Superficial Velocity of Oil')
xlabel('Superficial Velocity of Oil, m/s')
ylabel('Pressure Drop, kPa/m')
legend('Theoretical Value (wire probe)', 'Experimental Value')
grid on

%Graph Plot
fprintf('\n\n');
disp('Graph Shear Stress of Interface vs Superficial Velocity of Oil');

subplot(2,1,2), plot(superficialvelocity_oil, shearstress_interface,'-
or')
title('Graph of Pressure Drop vs Superficial Velocity of Oil')
xlabel('Superficial Velocity of Oil, m/s')
ylabel('Shear Stress of Interface, Pa')
grid on
```