MODAL ANALYSIS OF AN AIRCRAFT LANDING GEAR

By

Muhammad Amsyar Bin Zulkifly

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Universiti Teknologi PETRONAS,

Bandar Seri Iskandar,

31750 Tronoh,

Perak Darul Ridzuan.

CERTIFICATION OF APPROVAL

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in partial fulfilment of the requirement for the

BACHELOR OF ENGINEERING (Hons)

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Approved by,

____

(Ir Idris Bin Ibrahim)

UNIVERSITI TEKNOLOGI PETRONAS

TRONOH, PERAK

September 2012

CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

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MUHAMMAD AMSYAR BIN ZULKIFLY

ABSTRACT

During the course of an aircraft landing and take-off, the landing gear is subjected to a very massive force that needs to be diverted so that it will not affect the aircraft. Some of the dynamic characteristics that are caused by the force that is subjected to the landing gear are gear walk, shimmy, and brake-induced vibration. Stiffness and dampers are two most important elements of the landing gear to provide smooth motion of the aircraft. However, the data is not available to the public for research purpose. This project is undertaken to establish design data of an aircraft landing gear and study its dynamic characteristics using modal analysis method where the project is carried out using ANSYS software. Modal analysis is basically the study of the dynamic characteristic of a structure when subjected to a vibrating excitation where in this case, the structure to be analysed is the landing gear of the aircraft. In this project, a specific landing gear of an aircraft from Embraer is used as the model for the analysis. After the modelling is done, all the boundary conditions such as the mechanical properties and the connections are properly defined. Modal analysis is then performed in order to obtain the deformations, mode shapes, and the natural frequencies of the landing gear. From the result, five different mode shapes are taken into account and natural frequencies of the mode shapes are in the range of 129 – 379 Hz. The obtained result is discussed and compared to the closest possible research for its validity and it can be concluded that the project is successful where the mode shapes and natural frequencies that are obtained have the same pattern as the result in other researches.

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CHAPTER 1: INTRODUCTION

1.1 Background

In an aircraft, the typical main components are the fuselage, wings, powerplant, empennage, and landing gear. The fuselage is the body of the aircraft where the controls, cabin and storage space are located. The wings are the parts that help the lifting of the aircraft. Powerplant provides the thrust and it is consisted of engine and propeller. Landing gear, which is the main focus in this paper, helps the movement of the aircraft itself on land.

In manoeuvring the aircraft on the land, the landing gear (sometimes called as aircraft undercarriage) plays a very important role so that it runs smoothly where it absorbs energy during landing. Even though the landing gear mentions the word 'landing', it also has important function during taxiing and takeoff while helping movement and provides support at the same time. There are three types of landing gear that are commonly used and they are; conventional, tricycle and tandem.

A typical aircraft landing gear is consists of tires, wheels, drag brace, energy absorption mechanism, and brakes. There are also some other components such as door panels, retracting mechanisms, steering devices, and shimmy dampers. Nowadays, most of the landing gears of the aircrafts are retractable into the fuselages in order to reduce the air drag. For clearer understanding of the components of a landing gear, please refer to Figure 1-1.

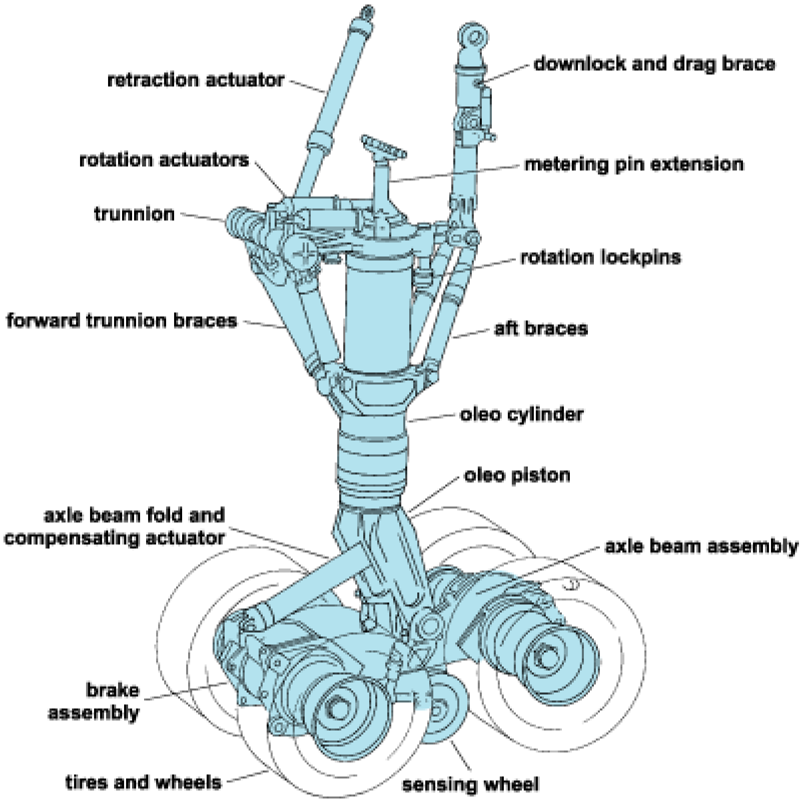

Figure 1-1: Components of a landing gear [1]

1.2 Problem statement

An aircraft landing gear is one of the important components in the aircraft and it is usually huge impact load especially during take-off and landing. Stiffness and dampers are two most important elements of the landing gear to provide smooth motion of the aircraft. However, the data is not available to the public for research purpose. This project is undertaken to establish design data of an aircraft landing gear and study its dynamic characteristics using modal analysis method.

1.3 Objectives

i. To develop modal parameters of an aircraft landing gear using finite element analysis modelling method.
ii. To study the dynamic characteristic (deformation) of landing gear.

1.4 Scope of study

This project examines the dynamic characteristics of an aircraft landing gear at two operating conditions which is when the aircraft is landing and taking off.

i. The aircraft landing gear for the selected aircraft model will be studied and design data will be collected.
ii. Develop finite element analysis model for the design.
iii. Study the dynamic characteristic especially on the deformations of the landing gear at different frequencies of excitation.

CHAPTER 2: LITERATURE REVIEW

2.1 Review of previous work

In a presentation of Experimental Modal Analysis of Landing Gears by Alvin Fong P. Eng, it was said that during rapid landing, the load of the landing gear can be influenced by the landing gear modal characteristics. It also affects one of the concerns in landing gear dynamics which is called as shimmy. Shimmy of a landing gear system is a violent self-excited oscillation driven by the interaction between tire and the ground and this motion interacts with the landing gear structure. Shimmy can occur during taxiing, take-off, and landing and the effect could be detrimental to the landing gear structure or the aircraft [2].

In Simulation of Aircraft Landing Gear Dynamics Using Flexible Multibody Dynamics Methods in SIMPACK by Prashant Dilip Khapane. The oscillations of the landing gear is simulated using SIMPACK software. The multibody of the landing gear is then represented by simple body elements such as main fitting, the shock tube, and two/four wheels respectively [3]. Using FEA tool in the SIMPACK, an elastic body is set up and it is transferred into the software using modal approach. Two types of modal simulations were done; braking and reduced friction-induced vibration. In braking simulation, it was compared between two types of brakes; ABS and dynamic braking. It was found that ABS is better in providing passenger comfort and reduced friction-induced vibration. While for the second simulation, flexible landing gear is used at the attachment point and deflection were reduced to almost zero.

In this book also, modal analysis done where three different models were modelled. The first one is the form of a beam model which can be referred in Figure 2-1.

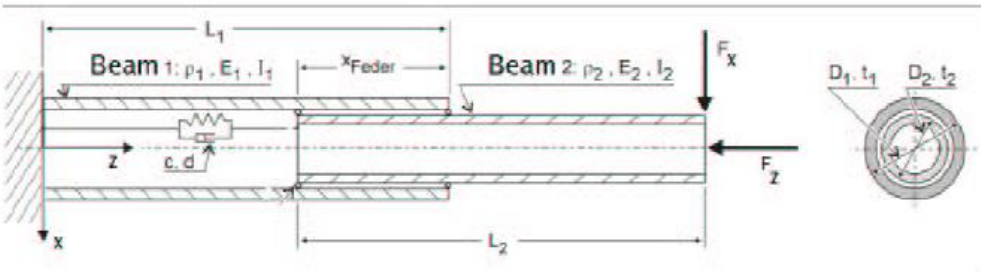

Figure 2-1: Landing gear in the form of beam model [4]

While the other two models are in the form of proper landing gear components as in Figure 2-2. The difference between the models is the stroke of the strut during the analysis where the first one is considered as a constant while the other is considered as variable.

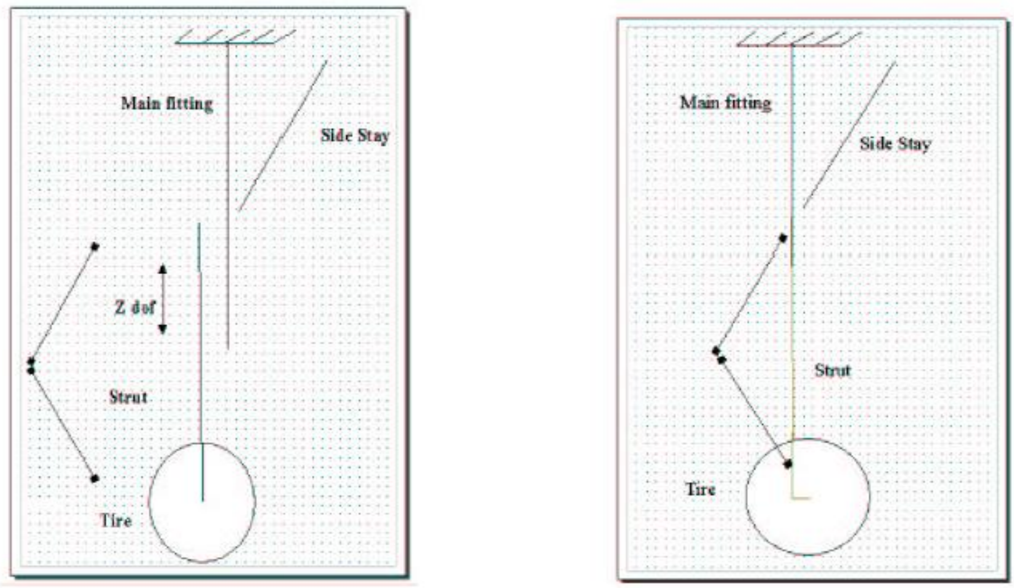

Figure 2-2: Proper landing gear model for the modal analysis [4]

2.2 Theoretical background

The modal analysis in this project will be focused during landing and takeoff of the aircraft.

Modal analysis is the study of dynamic characteristics of structures. It is used to determine the natural frequencies and mode shapes of a structure where the results are used to help design the structural system of the structure for vibrations applications [5]. The analysis is usually used using Finite Element Method (FEM) and can be performed using ANSYS.

Modes are inherent properties of a structure, and are determined by the material properties (mass, damping, and stiffness), and boundary conditions of the structure [6]. Each of the modes differs from the other through modal damping, mode shape and resonant frequency.

Basic equation for Multi DOF system:

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{F\}$$

- The mass, damping and stiffness matrices are constant with time
- The unknown nodal displacements vary with time

A continuous structure has an infinite number of degrees of freedom. The finite element method approximates the real structure with a finite number of DOFs. In this project, the result from the analysis is compared to the closest analysis that had been done by other study.

CHAPTER 3: METHODOLOGY

3.1 Research Methodology

As mentioned before, the project is undertaken to establish design data of an aircraft landing gear and study its dynamic characteristics using modal analysis method. The research will be made available to the public for further research. For the project to start, literature review is first done for thorough understanding of landing gear and modal analysis. The dimension of a landing gear from a specific aircraft is taken as the reference for modelling purpose. The properties of the parts of the landing gear such as the Young's modulus and Poisson's ratio will be used from the literature resource as reference. If the required properties are not provided, the most common material and properties used in landing gears is applied in the modal analysis.

After literature review is done, modelling phase is performed using CATIA and the model is transferred into ANSYS for simulation in two conditions: landing and take-off. The design data for the landing gear is taken from an aircraft of Embraer.

Simulation is done on the effect of deformation on the landing gear. Among the modal parameter to be studied are the mode shapes, deformation, and natural frequencies of the mode shapes.

After the simulation is performed, the results are compared to other researches that are similar in term of the study area. The results that will be compared are the mode shapes and the natural frequencies of the landing gear. If the results differ too much from the other researches, the project will start again from the modelling phase. After satisfying results are obtained, the project will be completed with a report for submission.

3.2 Project Flow Chart

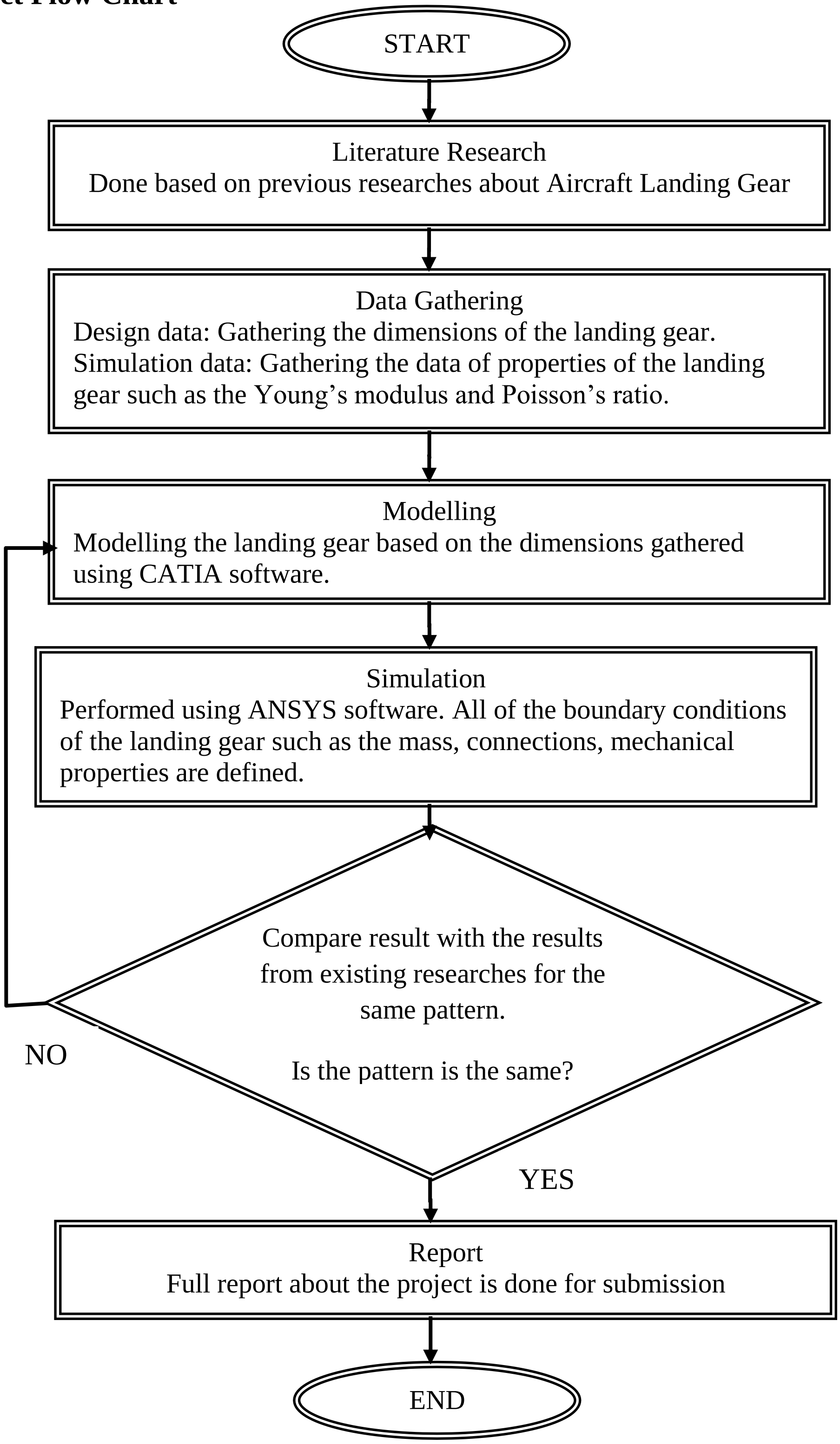

Figure 3-1: Flow chart of the project

3.3 Modelling

3.3.1 Landing Gear Model

The model of the landing gear in this project is based on an aircraft from Embraer that uses hydraulically actuated type. The landing gear model is made using CATIA and then is transferred into ANSYS for simulation. The landing gear is consisted of a few rigid body elements which are main-fitting, shock tube, and wheels with additional bogie. Each part only has one degree of freedom. Figure 3-2 represents the schematic of the landing gear.

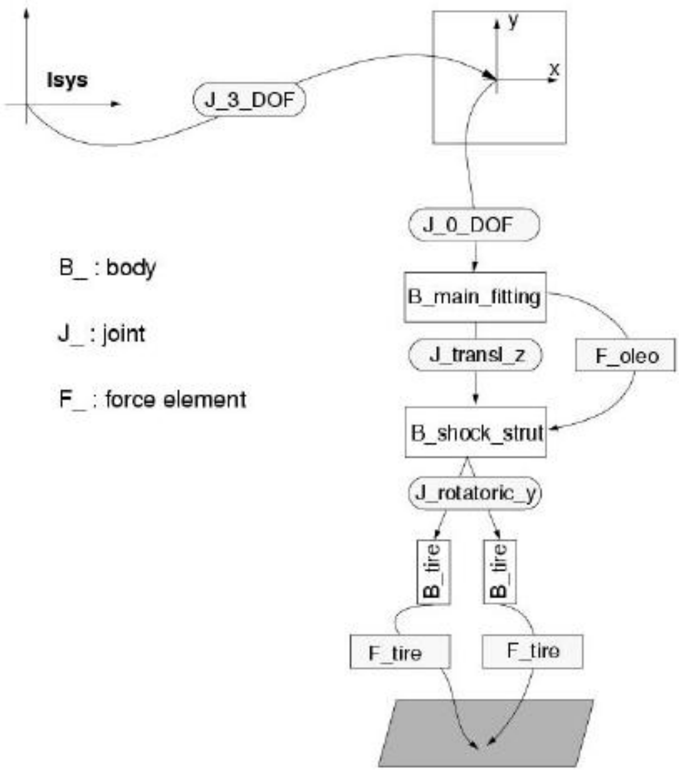

Figure 3-2: Schematic diagram of the landing gear [7]

The model of the landing gear produced using CATIA basically is basically based on the Figure 3-3 where the parts that should be taken into account are the main fitting, shock strut and the tire.

Figure 3-3: Landing gear model reference [7]

The modelling of the aircraft landing gear is based on the design taken from Simulation of Aircraft Landing Gear which is based on the Embraer regional aircraft. The design is combined into simpler design in order to ease the process of analysis using ANSYS as the usual analysis in this software involves only the critical part of a structure without including the details of a structure. If the design is too complicated, it will consume more time thus affecting the process of getting proper result and also the process of analysing the result.

In performing the modal analysis, the design needs to be defined properly such as for the connections and physical and material properties. The connections can be either between body to body or body to ground. There is also contact option between the bodies. As for the material properties, there are preset materials to be chosen from. If the material needed is not available, it can be added to the database. As for this analysis, based on the provided Young's modulus and Poisson's ratio, the material of the landing gear can be concluded that it is made from stainless steel**.** After a few try and error, it was found that the process is complicated and everything is interconnected with each other. If there is one mistake occur at one part, the whole result will be affected.

After further discussion, it was decided that the design of the landing gear needed to be simplified into a single body as the time frame to perform the analysis is short. Below is the first model of the simplified landing gear.

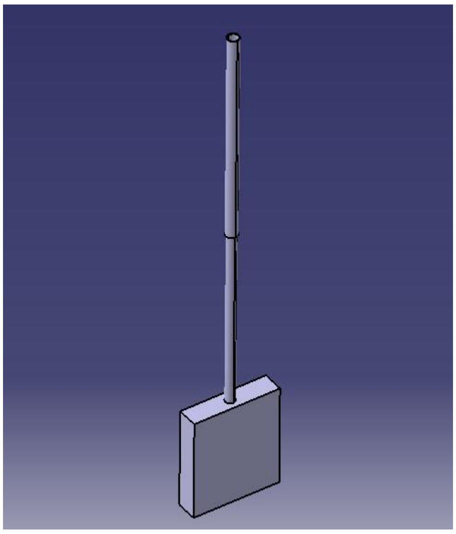

Figure 3-4: Simplified model of the landing gear

However, after a few times of analysis, it was found that the required result could be obtained. This was because for a single body, only a limited constraints can be defined thus making the option not feasible. It was then remodelled into separated bodies as in Figure 3-5.

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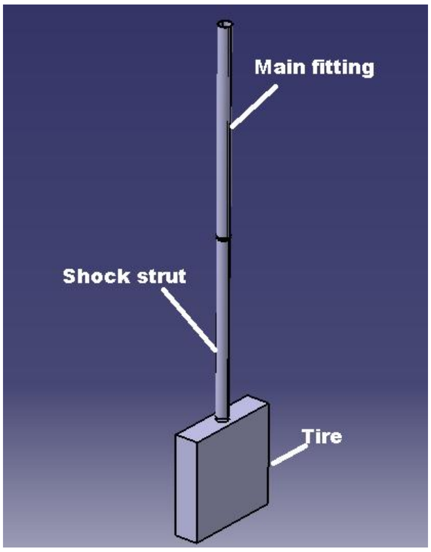

Figure 3-5: Simplified model with separated bodies

The model is basically the shape same as the previous model but it is separated into three parts which are the main fitting, shock strut, and tire. After the modal analysis was perform, the mode shape that was required started to show the required shape. The mode shape can be referred in the result section. For further refinement of the result, the model is remodelled into a new model that has more resemblance to the original landing gear and this is the final model that is used in this project and for the analysis. Figure 3-6 is the final model of the landing gear for the modal analysis project.

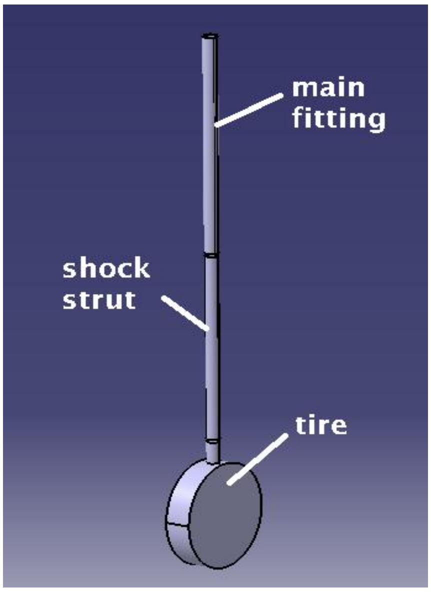

Figure 3-6: Modified model of the landing gear

After modelling in CATIA, the model is then transferred in ANSYS for the modal analysis. The model is combined and defined according to its joints, contacts and support. All of the constraints needed to be defined properly in order to avoid error thus affecting the results obtained. The figure A-1 in Appendix A is the model of the landing gear after assembled according to its joints. At this point, all of the properties of the landing gear such as the materials and mass are defined. The letter A, B and C are the point masses of the bodies of the landing gear.

The model is then meshed in separate elements (this is the part where the finite element analysis come into the picture) before solved by the software. Refer to figure A-2 in Appendix A for the meshed model of the landing gear.

As mentioned before, the first aim was to perform the analysis as a single body landing gear. However, due to the limitations of the single body analysis such as the redundancy of the constraints, the analysis needed to be altered into multi-body analysis. It needed to be given attention that in modal analysis, the forces and the damping that act on the bodies are to be ignored. This is because modal analysis is a vibration analysis where the aim is to find the natural frequencies and mode shapes of a structure (in this case, the landing gear) when given some frequencies of excitation. This will be explained later in this section.

The basic working principle of the landing gear in this modal analysis is basically the same as a beam where both of the ends are fixed. One end is fixed to the aircraft fuselage and the other end is fixed to the ground. The other end is fixed to the ground to mimic the situation during landing and when the aircraft is about take-off for flight.

The basic parameters that are needed in this analysis are mass, moment inertia of the axle, Poisson’s ratio, Young’s modulus and the frictional coefficient. The frictional coefficient is used as the replacement of the damping effect for the landing gear.

3.3.2 Landing Gear Data

The data for the modelling of the landing gear is taken from Simulation of Landing Gear Dynamics and Brake-Gear Interaction and below is the data used for the analysis:

Table 3-1: Dimensions of the landing gear [7]

	Main Fitting	Shock Strut	Tire
Outer Diameter	9.4 cm	8.0 cm	70.0 cm
Inner Diameter	8.6 cm	7.0 cm	-
Length	1.469 m	1.24 m	17 cm
Weight	122 kg	53 kg	180 kg per tire

Table 3-2: Mass and moment of inertia used for the landing gear [7]

	Mass (kg)	I_x	I_y	I_z
Main Fitting	122	-	-	-
Shock strut	53	-	-	-
Axle	70	1.74	1.744	6.285
Tire	50	11.86	11.86	6.86

3.4 Gantt Chart

Table 3-3: Gantt chart for FYP I & I

		FYP 1														FYP 2														
No.	Detail/ Week	1	2	3	4	5	6	7	8	9	10	11	12	13	14	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	Selection of Project Title	■	■																											
	-Selection from enlisted titles by lecturers																													
2	Preliminary Research Work / Literature Review		■	■	■	■																								
	-Study of basic principles and previous works of the project																													
3	Submission of Extended Proposal						●																							
4	Extensive Research on shimmy of aircraft landing gear							■	■																					
	-Preparation for proposal defense and FYP 2																													
5	Proposal Defense								■	●																				
7	Submission of Interim Draft Report													●																
8	Submission of Interim Report														●															
9	Design of the landing gear										■	■	■	■	■	■	■	■												
	-Designed using CATIA																													
10	Modelling and Simulation																	■	■	■	■	■	■	■						
	-Simulation in ANSYS using the design landing gear in CATIA																													
11	Analysis of Data																						■	■	■					
	-Analysis of obtained results from simulation																													
12	Submission of Progress Report																								●					
13	Conclusion and Recommendation																									■	■			
14	Submission of Draft Report																											●		
15	Submission of Technical Paper and Dissertation																											●		
17	Oral presentation																												●	
	-Present the findings on the project for evaluation																													
18	Submission of project dissertation																													●

Legends:

■ Project Activity

● Key Milestone

CHAPTER 4: RESULTS AND DISCUSSION

4.1 Results and Discussion

These are the results for the modal analysis of the landing gear. Five mode shapes of total deformation are taken into record from Figure 4-1 until Figure 4-5. For Figure 4-1 which is mode shape 1, the shape of the deformation is in torsion and lateral shape. Mode shape 2 shows fore-aft pattern and mode shape 3 also shows deformation in torsion shape. As for mode shape 4, the deformation is in second lateral shape and for mode shape 5; the deformation is in the vertical motion of the wheels. From Figure 4-1, the deformation is focused at the shock strut where the deformation is approximately 0.11m when the frequency of excitation is equal to the natural frequency of the mode shape 1 which is at 129.5 Hz. For Figure 4-2, the deformation is focused at both main fitting and shock strut in Y-direction with deformation of 0.1m with natural frequency of 135.86Hz. The deformation for Figure 4-3 and 4-4 are 0.19m and 0.2m with natural frequencies of 205.31 Hz and 210.79 Hz respectively in X-direction. Finally, the deformation for Figure 4-5 is in Z-direction where it is focused at the main fitting at the value of 0.09m at the natural frequency of 378.01 Hz.

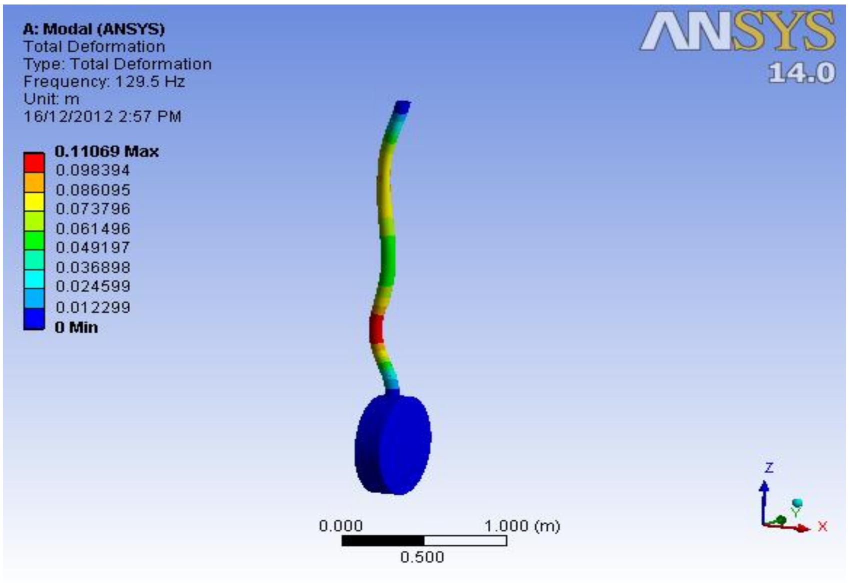

Figure 4-1: Mode Shape 1

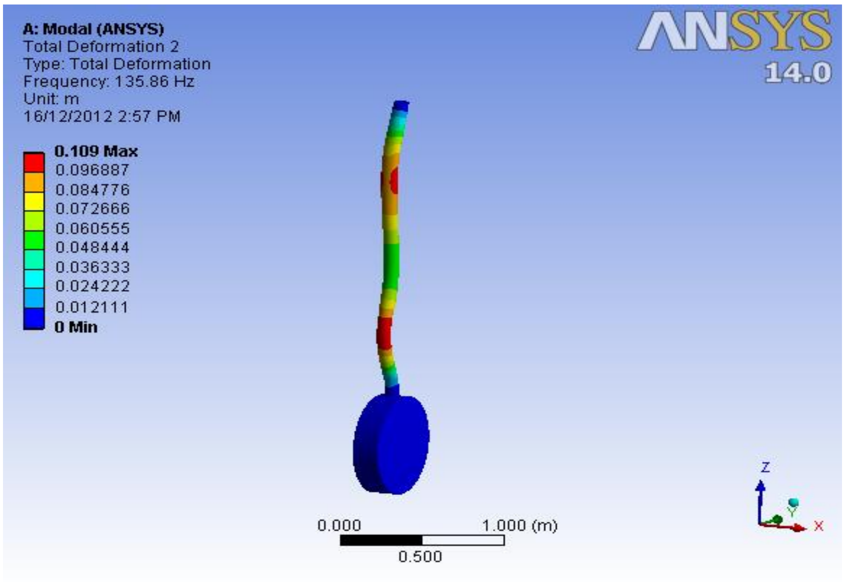

Figure 4-2: Mode Shape 2

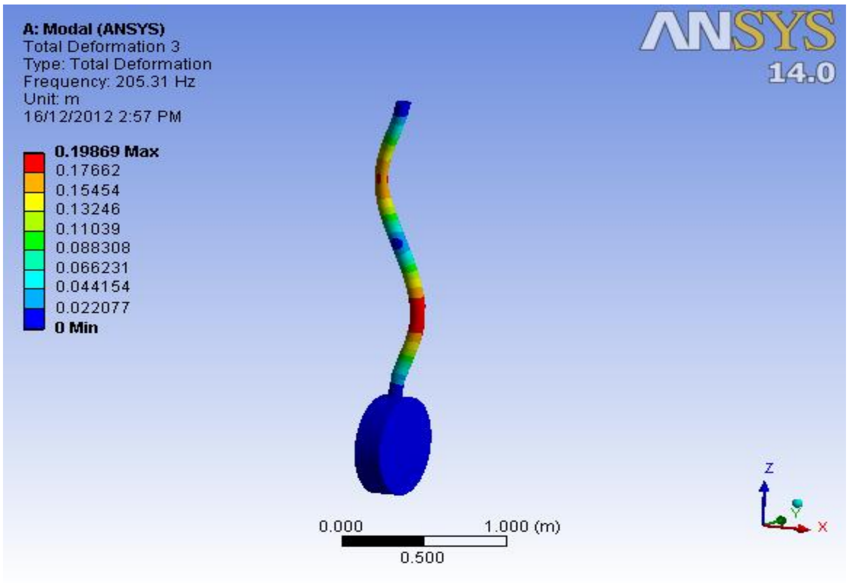

Figure 4-3: Mode Shape 3

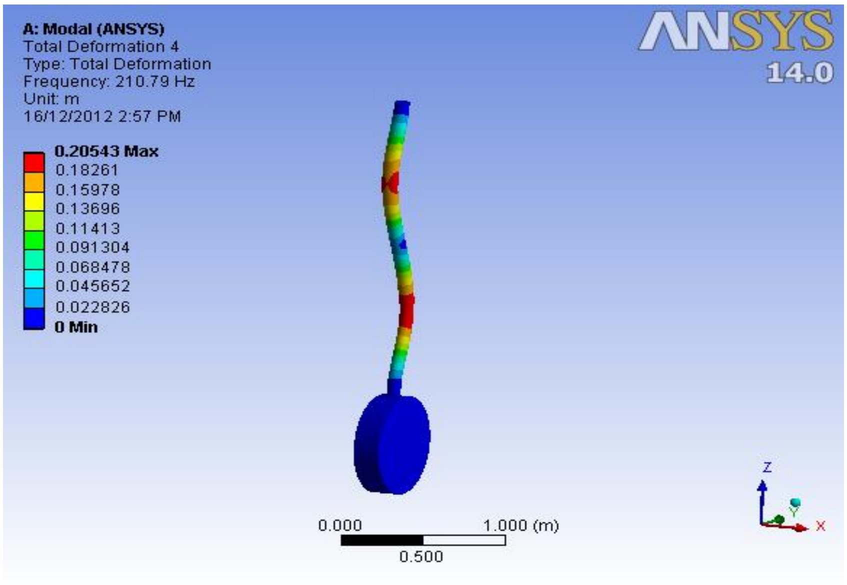

Figure 4-4: Mode Shape 4

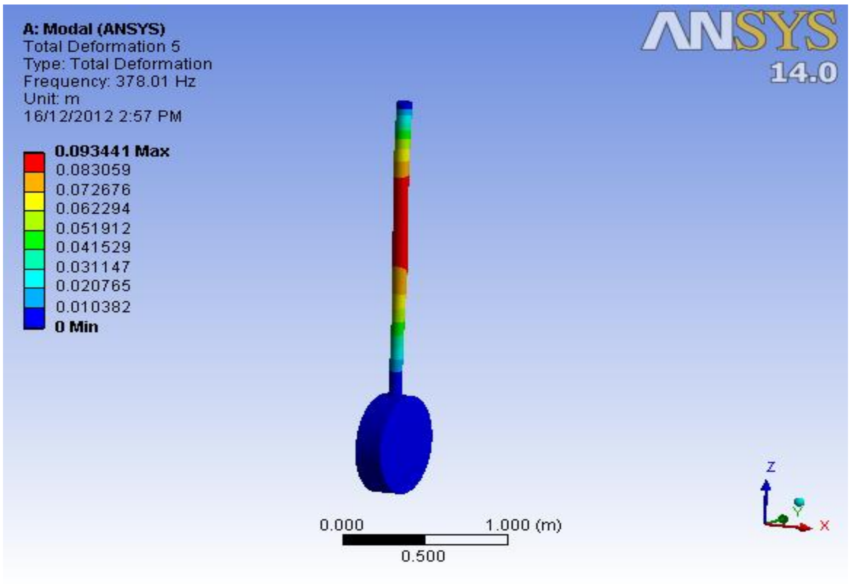

Figure 4-5: Mode Shape 5

The difference between the mode shapes is due to the difference between the frequencies of excitation to the landing gear. In the results, there are 5 different mode shapes. Mode shape 1 is the motion of the landing gear when the structure vibrated in half of sine curve shape. While mode shape 2 is the motion of the landing gear when it vibrated at full of the sine curve. The other mode shapes are also explained based on the same principle. Table 4-1 are the natural frequencies of the mode shapes analysed during the analysis.

Table 4-1: Natural frequencies of the mode shapes

Mode	Frequency (Hz)
1	129.5
2	135.86
3	205.31
4	210.79
5	378.01

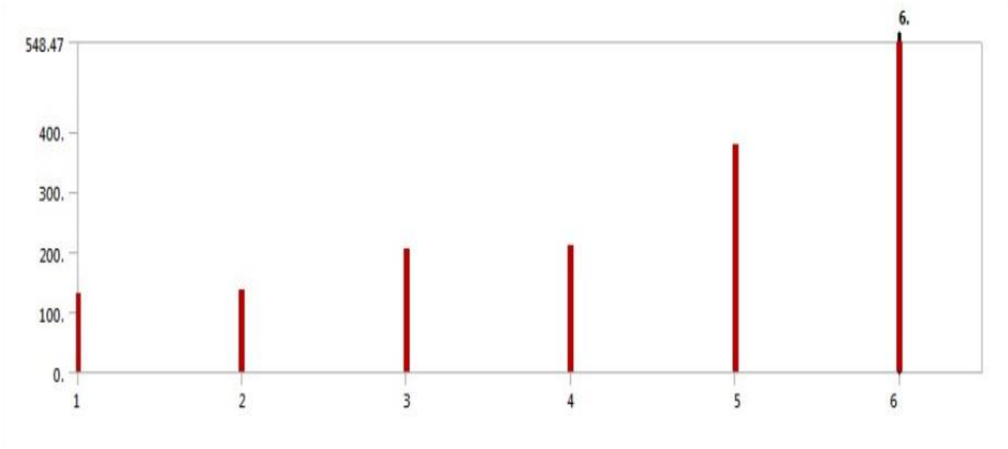

Figure 4-6: Plotted natural frequencies

The followings are the result from the modal analysis performed in the Simulation of Landing Gear Dynamics and Brake-Gear Interaction book using their model.

Table 4-2: result from the modal analysis performed in the Simulation of Landing Gear Dynamics and Brake-Gear Interaction [7]

Mode number	Eigen Frequency, Hz
1	10.10
2	11.22
3	13.25
4	45.69
5	62.31

While the result is comparatively is different, basically the system have the same pattern of natural frequencies and mode shapes (refer to Figure 4-7). The difference in values is due to the difference of condition of the analysis and the design of the landing gear.

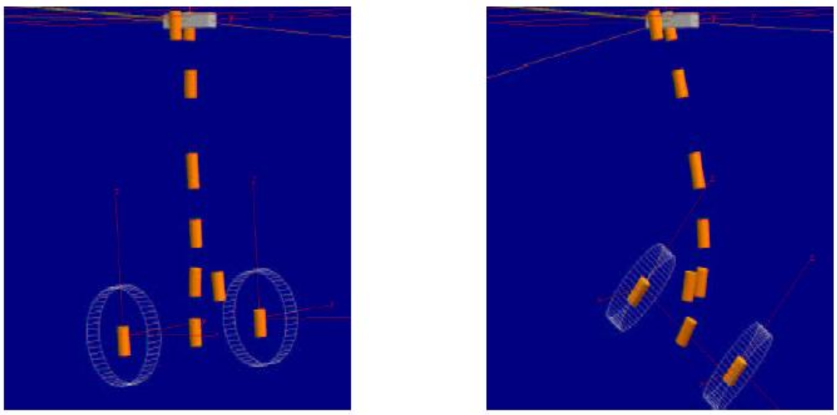

Figure 4-7: Mode shape of a landing gear [7]

From the modal analysis, the obtained results are the mode shapes and the natural frequencies of the mode shapes, which are also the dynamic properties of the landing gear. During this analysis, the structure is given different frequencies of excitation at a constant force. At specific frequencies (which in this case, the natural frequencies), the landing gear structure will start to have a response where it undergone a motion that occurs naturally hence explaining the mode shapes as in Figure 4-1 until 4-5. This situation is the reason that causes the deformations of the structure showed by the mode shapes. These frequencies are really important as they might cause damage to the structure such as fatigue or permanent failure.

In order to prevent the structure from vibrating at its natural frequencies, damping system is introduced as an alternative dispersion mean for the energy that is produced without compromising the structure itself.

Modal analysis can also be done experimentally where the landing gear can be give frequencies of excitation using shaker at a constant force. The response is then recorded by an accelerometer that is attached at the other end of the landing gear.

Modal analysis is a preferable analysis for a vibration study of a structure since it helps to find the problems that are caused by vibrations. Moreover, it tackles the problems before it happens during the real working condition and it helps to optimize the landing gear design and improves its performance.

CHAPTER 5: CONCLUSION AND RECOMMENDATION

5.1 Conclusion

The modal analysis of the aircraft landing gear is successfully performed where the aim of developing modal parameters using finite element analysis is achieved in which mode shapes and natural frequencies and the deformations of the landing gear are obtained. Five mode shapes were taken into representing the condition of landing and taking off of the aircraft. Natural frequencies between the ranges of 129 – 379 Hz were obtained and whenever the frequency of excitation approaches one of the natural frequencies, the landing gear will undergo a drastic deformation up to 0.2m which can be detrimental if it occurs.

5.2 Recommendation

The project can be further improved with experimental analysis of the landing gear by making a simple model of the landing gear. A shaker can be attached to the structure and the response can be recorded. The result can be compared with the results obtained from the simulation for validation. Extensive research can also be done such as utilising the results that are obtained for applications that involves the vibrations of the landing gear. The mode shapes and the natural frequencies can be used as reference for study of the effect when a landing gear vibrates nearing the natural frequencies and it can also be used as reference for improving the design and performance of the landing gear.

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APPENDICES

Appendix A

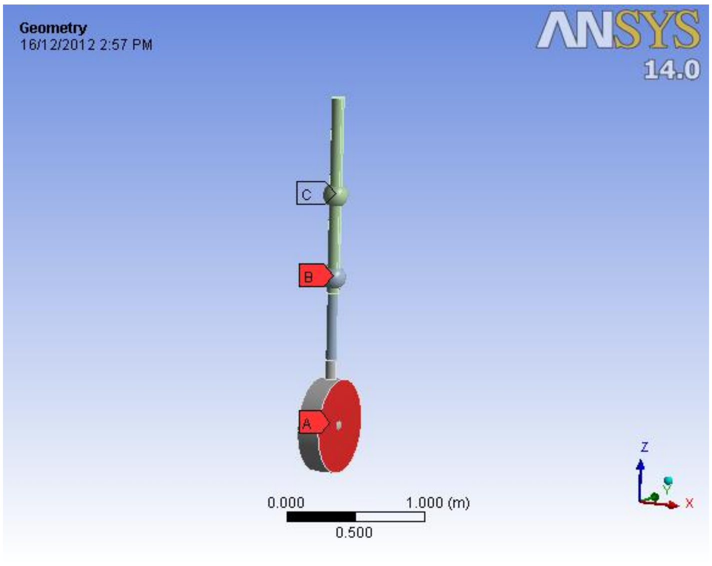

Figure A-1

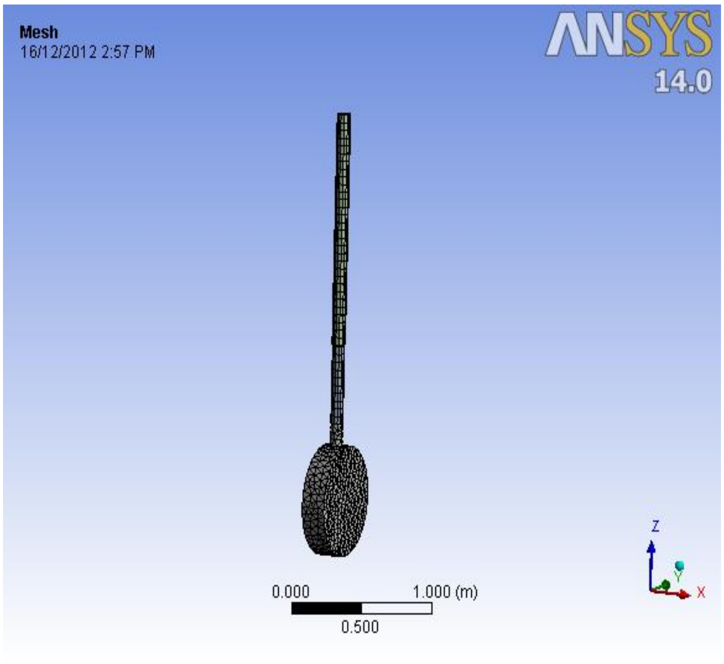

Figure A-2

Appendix B

***** PARTICIPATION FACTOR CALCULATION ***** X DIRECTION

MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	CUMULATIVE MASS FRACTION	RATIO EFF.MASS TO TOTAL MASS
1	129.501	0.77219E-02	-14.965	0.943230	223.938	0.470572	0.251495
2	135.858	0.73606E-02	-0.83455E-01	0.005260	0.696470E-02	0.470587	0.782175E-05
3	205.310	0.48707E-02	0.48388	0.030500	0.234144	0.471079	0.262957E-03
4	210.787	0.47441E-02	0.93969E-02	0.000592	0.883012E-04	0.471079	0.991672E-07
5	378.009	0.26454E-02	-0.13063E-01	0.000823	0.170632E-03	0.471079	0.191629E-06
6	548.470	0.18233E-02	15.865	1.000000	251.705	1.00000	0.282679
sum					475.884		0.534444

***** PARTICIPATION FACTOR CALCULATION ***** Y DIRECTION

MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	CUMULATIVE MASS FRACTION	RATIO EFF.MASS TO TOTAL MASS
1	129.501	0.77219E-02	0.77710E-01	0.005504	0.603889E-02	0.302906E-04	0.678201E-05
2	135.858	0.73606E-02	-14.119	1.000000	199.352	0.999964	0.223883
3	205.310	0.48707E-02	0.59431E-02	0.000421	0.353204E-04	0.999964	0.396668E-07
4	210.787	0.47441E-02	-0.35062E-01	0.002483	0.122937E-02	0.999970	0.138065E-05
5	378.009	0.26454E-02	-0.76605E-01	0.005426	0.586829E-02	1.00000	0.659041E-05
6	548.470	0.18233E-02	0.86149E-02	0.000610	0.742165E-04	1.00000	0.833493E-07
sum					199.365		0.223898

***** PARTICIPATION FACTOR CALCULATION ***** Z DIRECTION

MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	CUMULATIVE MASS FRACTION	RATIO EFF.MASS TO TOTAL MASS
1	129.501	0.77219E-02	-0.21494E-02	0.000150	0.461979E-05	0.226184E-07	0.518828E-08
2	135.858	0.73606E-02	-0.71326E-01	0.004993	0.508746E-02	0.249307E-04	0.571350E-05
3	205.310	0.48707E-02	0.43003E-02	0.000301	0.184927E-04	0.250213E-04	0.207683E-07
4	210.787	0.47441E-02	-0.37433	0.026204	0.140120	0.711048E-03	0.157363E-03
5	378.009	0.26454E-02	14.285	1.000000	204.057	0.999769	0.229167
6	548.470	0.18233E-02	0.21713	0.015200	0.471452E-01	1.00000	0.529466E-04
sum					204.249		0.229383

Figure B-1

***** PARTICIPATION FACTOR CALCULATION *****ROTX DIRECTION

MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	CUMULATIVE MASS FRACTION
1	129.501	0.77219E-02	0.75989E-01	0.005599	0.577439E-02	0.285940E-04
2	135.858	0.73606E-02	-13.572	1.000000	184.195	0.912137
3	205.310	0.48707E-02	-0.13876E-01	0.001022	0.192544E-03	0.912138
4	210.787	0.47441E-02	4.2121	0.310357	17.7420	0.999994
5	378.009	0.26454E-02	0.28239E-01	0.002081	0.797422E-03	0.999998
6	548.470	0.18233E-02	0.20253E-01	0.001492	0.410172E-03	1.00000
sum					201.944	

***** PARTICIPATION FACTOR CALCULATION *****ROTY DIRECTION

MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	CUMULATIVE MASS FRACTION
1	129.501	0.77219E-02	15.557	0.430585	242.033	0.152970
2	135.858	0.73606E-02	0.85729E-01	0.002373	0.734944E-02	0.152974
3	205.310	0.48707E-02	-5.8945	0.163144	34.7456	0.174934
4	210.787	0.47441E-02	-0.36530E-01	0.001011	0.133443E-02	0.174935
5	378.009	0.26454E-02	0.30145E-01	0.000834	0.908732E-03	0.174936
6	548.470	0.18233E-02	-36.131	1.000000	1305.44	1.00000
sum					1582.23	

***** PARTICIPATION FACTOR CALCULATION *****ROTZ DIRECTION

MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	CUMULATIVE MASS FRACTION
1	129.501	0.77219E-02	0.62880E-04	0.016340	0.395395E-08	0.236435E-03
2	135.858	0.73606E-02	0.94569E-03	0.245742	0.894333E-06	0.537150E-01
3	205.310	0.48707E-02	-0.25900E-03	0.067303	0.670834E-07	0.577264E-01
4	210.787	0.47441E-02	-0.97274E-03	0.252771	0.946230E-06	0.114308
5	378.009	0.26454E-02	0.45671E-04	0.011868	0.208585E-08	0.114433
6	548.470	0.18233E-02	-0.38483E-02	1.000000	0.148095E-04	1.00000
sum					0.167232E-04	

Figure B-2