

DESIGN OF CONTROL ALGORITHM FOR pH NEUTRALIZATION

By

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FINAL PROJECT REPORT

Submitted to the Electrical & Electronics Engineering Programme

in Partial Fulfilment of the Requirements

for the Degree

Bachelor of Engineering (Hons)

(Electrical & Electronics Engineering)

Universiti Teknologi PETRONAS

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CERTIFICATION OF APPROVAL

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A project dissertation submitted to the
Department of Electrical & Electronics Engineering
Universiti Teknologi PETRONAS
in partial fulfilment of the requirement for the
Bachelor of Engineering (Hons)
(Electrical & Electronics Engineering)

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SEPTEMBER 2012

CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

AQILAH JUANI BINTI ABDUL HALIM

ABSTRACT

Neutralization is an essential part in a process plant which handles wastewater treatment, chemical process, and biotechnology. The main purpose of neutralization is to adjust the pH value of the effluent streams to meet the environmental requirements and quality standards which are continuously being revised. However, pH processes are extremely difficult to control due to their intrinsic nonlinear properties, high sensitivity around neutralization point, and time variant characteristics. Therefore, a robust control algorithm is needed for the utilization of the system. A semi batch event to handle pH process introduced different approach to linearize the complex system. A mathematical model of a specific chemical process is designed as a basis to the development of conventional and advanced controllers. PID controller is developed and the performances are compared through the subsequent objective to familiarize with mathematical modelling in MATLAB's Simulink environment. From the simulation study, conventional controller is inadequate to demonstrate satisfactory control performances. Future works can be carried out using feedback linearization control strategy to accurately regulate the pH values of the model in both simulation as well as pH pilot plant which is built using instrumentation and actuators presently used in the process industries at Block 23, Universiti Teknologi PETRONAS.

ACKNOWLEDGEMENTS

First and foremost, praise to Allah the Almighty for His uncountable graces upon me and for the successful completion of this project in due course of time.

I am heartily thankful to my supervisor Associate Professor Dr. Mohd Noh Karsiti whose encouragement, guidance and support from the initial to the final level enabled me to develop an understanding on the subject thus successfully accomplished my final year project. Also, I would like to express my inmost appreciation to him for giving me the opportunity to be among the students under his excellent supervision.

My respectful gratitude goes to Dr Narahari Marneni, for his friendly approach, constant advice and helpful suggestions to develop more insights in solving problems using numerical methods. Special thanks to a very skilful technician, Mr. Azhar Zainal Abidin for his kindness and assistance while conducting the experiments on the pH pilot plant.

Special thanks to PETRONAS Sponsorship and Talent Sourcing Unit, STS (formerly known as ESU) for all their funding and financial supports during my bachelor studies at Universiti Teknologi PETRONAS. Along with, thanks to my family for their abundant love and care.

Lastly, I offer my regards and blessings to all of those who supported me directly and indirectly in any respect during the completion of the project.

TABLE OF CONTENTS

| | |
|---|-------------|
| ABSTRACT | III |
| ACKNOWLEDGEMENTS..... | IV |
| LIST OF FIGURES | VII |
| LIST OF TABLES | VIII |
| LIST OF ABBREVIATION..... | IX |
| CHAPTER 1 INTRODUCTION | 1 |
| 1.1 Background of Study..... | 1 |
| 1.2 Problem Statement | 2 |
| 1.2.1 Problem Identification | 2 |
| 1.2.2 Significance of Project..... | 2 |
| 1.3 Objective and Scope of the Project | 3 |
| 1.3.1 Objectives | 3 |
| 1.3.2 Scope of Project..... | 3 |
| 1.4 Relevancy of Project | 3 |
| 1.5 Feasibility of Project | 4 |
| CHAPTER 2 LITERATURE REVIEW | 5 |
| 2.1 Neutralization Reaction..... | 5 |
| 2.1.1 Acids and Bases | 5 |
| 2.1.2 Acid-Base Constants..... | 6 |
| 2.1.3 Definition of pH..... | 7 |
| 2.2 Static Modelling | 7 |
| 2.2.1 Titration Curve..... | 7 |
| 2.3 Dynamic Modelling..... | 8 |
| 2.4 pH Control Algorithm | 11 |
| 2.4.1 Importance of Control Algorithm..... | 11 |
| 2.4.2 Fundamentals of pH Neutralization Control Algorithm | 11 |
| 2.4.3 Advanced Forms of Control Algorithm..... | 12 |
| 2.5 Feedback Linearization | 12 |
| 2.5.1 Lie Derivatives..... | 13 |
| 2.5.2 Lie Brackets | 14 |
| CHAPTER 3 METHODOLOGY | 15 |

| | |
|--|-----------|
| 3.1 Research Methodology..... | 15 |
| 3.2 Project Activities | 16 |
| 3.3 Project Duration | 17 |
| 3.4 Tool | 17 |
| CHAPTER 4 RESULTS & DISCUSSIONS..... | 18 |
| 4.1 Model Development of a Semi batch Process..... | 18 |
| 4.1.1 Overview of the Semi batch Process | 18 |
| 4.1.2 Open Loop Experiment..... | 19 |
| 4.1.3 Solving Mathematical Equation using Numerical Method | 21 |
| 4.2 Mathematical Modelling | 23 |
| 4.2.1 Simulink Block Diagram | 23 |
| 4.3 Feedback Control Strategy | 27 |
| 4.3.1 PID Tuning | 29 |
| 4.3.2 PID Implementation..... | 33 |
| CHAPTER 5 CONCLUSIONS & RECOMMENDATIONS..... | 36 |
| 5.1 Conclusions | 36 |
| 5.2 Future Directions..... | 37 |
| 5.2.1 Partial Development of Feedback Linearization | 37 |
| REFERENCES..... | 40 |
| APPENDICES | 42 |

LIST OF FIGURES

| | |
|--|----|
| Figure 1: Titration curves for (a) Monoprotic Acid and (b) Polyprotic Acid | 7 |
| Figure 2: Project Flow Diagram..... | 15 |
| Figure 3: A Semi batch Reactor | 18 |
| Figure 4: pH Pilot Plant at Universiti Teknologi PETRONAS | 19 |
| Figure 5: Flow rate of Alkaline Stream | 19 |
| Figure 6: Process Reaction Curve 1 | 20 |
| Figure 7: Process Reaction Curve 2..... | 20 |
| Figure 8: Graph of H^+ against time (Numerical Method) | 23 |
| Figure 9: Process Equations in Simulink | 24 |
| Figure 10: Graph of H^+ against time (Simulink Block Diagram)..... | 24 |
| Figure 11: Simulation Diagnostics..... | 25 |
| Figure 12: Configuration Parameters | 25 |
| Figure 13: pH Simulink Block Diagram | 26 |
| Figure 14: Logarithmic Subsystem | 26 |
| Figure 15: Saturation Values..... | 26 |
| Figure 16: Graph of pH Characteristics | 27 |
| Figure 17: PID Feedback Control Algorithm..... | 29 |
| Figure 18: Proportional Gain = 15 | 30 |
| Figure 19: Proportional Gain = 40 | 30 |
| Figure 20: Proportional Gain = 54 | 31 |
| Figure 21: PID Block Under Mask | 32 |
| Figure 22: Configuration Parameters for PID Simulink | 33 |
| Figure 23: PID Controller Responses | 34 |

LIST OF TABLES

| | |
|--|----|
| Table 1 : Reviews of Collected Research Papers – Continuous Process in CSTR..... | 9 |
| Table 2 : Reviews of Collected Research Papers – Batch Process | 10 |
| Table 3 : Description of Project Activities..... | 16 |
| Table 4 : List of Tool | 17 |
| Table 5 : List of Variables Involved in Simulation..... | 27 |
| Table 6 : Values of Assumptions | 28 |
| Table 7 : Ziegler-Nichols Closed Loop Parameters..... | 31 |
| Table 8 : Control Performances | 34 |
| Table 9 : Unknowns Involved in Differentiation | 37 |

LIST OF ABBREVIATION

| | |
|------|----------------------------------|
| PI | Proportional Integral |
| FYP | Final Year Project |
| SAE | Strong Acid Equivalent |
| UTP | Universiti Teknologi PETRONAS |
| SISO | Single Input-Single Output |
| CSTR | Continuous Stirred Tank Reactors |

CHAPTER 1

INTRODUCTION

1.1 Background of Study

Pollution of water bodies due to rapid development process, industrialization and infrastructure urges the legislature to mandate stringent pH limits of used effluents before being discharged to the environment. A pH neutralization plant installed at the final stage in process industries should be able to adjust the pH of waste streams within acceptable limits fixed by local, state and federal regulatory organizations to save the environment and to keep the plant equipment in safe condition. In Malaysia, wastewater discharge standard set by the Department of Environment falls within pH value of 5.5 to 9 [1].

Significant researches in controlling pH are widely done because of its industrial importance to measure the state of a reaction. However, the highly nonlinear behaviour of neutralization poses challenges for pH control system [2]. In order to achieve the design purposes, proper control algorithm is introduced. It simply means a control system will be simulated in Simulink in which the complexity of the system can be handled. Control systems are classified into two classes with many dissimilarities and arrangements: logic or sequential controls, and feedback or linear controls [3].

It is necessary to understand the pH characteristics of the reaction and the process plant before designing and implementing the control system in conventional controllers. Advanced forms of control algorithms such as fuzzy, neural network, pole-placement method, and genetic algorithm are often developed for their design simplicity, linear and controllable system which does not need fundamental knowledge of the plant. Despite the alternatives available, it is advisable that the

conventional PID controller is tuned to its optimum level to evaluate on its performance before using an advanced control method.

1.2 Problem Statement

1.2.1 Problem Identification

The pH control is still the scope of work for many researchers. It has been known as problematic due to complexity of the system described as follows [4]:

- i. The high nonlinearity of pH neutralization process around the neutralization point.
- ii. The unknown characteristics of mathematical model of neutralization process.
- iii. The time varying, uncertain and fluctuating characteristics process.

1.2.2 Significance of Project

Control algorithm is designed to fit into a controller regardless of plant complexity while considering the strict concerns on environment and performance requirements. A fed-batch neutralization mathematical equation is modelled and simulated to cater the inherent and severe nonlinearity of a continuous pH neutralization process. It is intended that a linear and stable control system can be constructed to improve the implementation of a pH neutralization plant in the industry.

1.3 Objective and Scope of the Project

1.3.1 Objectives

- i. To model and simulate pH neutralization process considering the plant, the measuring and actuating instruments and control algorithms
- ii. To design and develop control algorithm
- iii. To compare the performances of controller

1.3.2 Scope of Project

This project aims to design and develop control algorithm for a robust and accurate pH neutralization process pilot plant by understanding the neutralization process. Suitable mathematical expressions and derivations will be attained to reflect the behaviour of pH semi batch neutralization process. Through the understanding on the fundamental characteristics of the neutralization process in semi batch, a system is configured to relate the feed and the control structure of each component in a pH neutralization plant. The system is simulated and controlled in MATLAB's Simulink environment. The model will then be used for the execution of desired control strategy.

1.4 Relevancy of Project

Besides the high nonlinearity of the process around the neutralization point, feeds also provoke variation to the pH control. The semi batch neutralization process resolves part of the complex behaviour in pH neutralization contributed by the continuous feeding of base and acid into the reaction tank. The event simplifies the pH control because the second feed is added into a stagnant wastewater slowly until it achieves the allowable pH 5.5 and 9 mandated in Environmental Quality Act, 1974. This is vital because pH range between 6.5 and 7.5 provides an optimum environment for microorganism activity [1]. Wastewaters of pH lower or greater than the range may interrupt the life cycle in an aquatic ecosystem. Therefore, the pH value is controlled by treating alkaline solution into an acidic wastewater, while

alkaline wastewater is reacted with acidic solution in neutralization process to get the desired pH safe for the environment.

1.5 Feasibility of Project

The project will be done in 28 weeks and is allocated into two semesters to realize the control algorithm for the pH neutralization process pilot plant.

First Semester (FYP1):

1. Research on pH neutralization process
2. Modelling and simulation of pH neutralization process pilot plant

In research part, the objectives are to understand the neutralization theories and the working operation of the pilot plant. The modelling and simulation are performed based on the characteristics of the neutralization obtained from the mathematical equations using MATLAB.

Second semester (FYP2):

1. Study on control algorithm
2. Design and implement the control algorithm
3. Analyse and compare performances of control strategy

It continues with further studies on control algorithms wherein suitable control algorithm is developed and used as a controller. System testing and implementation will be aided with software that has an accessibility of the pilot plant in the laboratory. From the explanation given, it is very clear that this project is possible to be carried out within the time frame.

Chapter 2

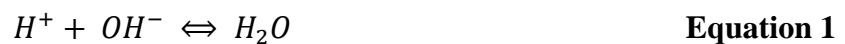
LITERATURE REVIEW

This section summarizes on the literature analysis that has been conducted to provide more understanding on concepts and techniques published in the previous studies concerning pH control of neutralization process. The chapter gives detail context on the characteristics of the pH neutralization process from the papers that have been reviewed in continuous and batch systems. Reviews from the published papers and journals are also included to consider on the results of different controllers that have been put into practiced in the research area.

2.1 Neutralization Reaction

2.1.1 Acids and Bases

From the dissociation of an Arrhenius acids and Arrhenius bases in which the solvent is water, acid produces hydrogen ions while base produces hydroxide ions in that aqueous solution [5].



Water is formed when Arrhenius acids and bases are reacted in neutralization whereas formation of water and a metal salt are produced in acid–alkali reactions.

According to Brønsted–Lowry definition, an acid is a proton donators and a base is a proton acceptors [5]. Thus, acid-base reactions happen when the hydrogen ion is removed from the acid and the hydrogen ion is being added to the base. However, the hydrogen ion H^+ does not actually occur in solution during neutralization.

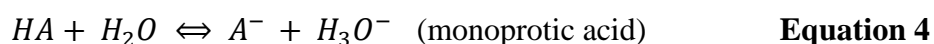


Considering the hydronium ion H_3O^+ , the actual net ionic reaction is:



The number of hydrogen ions H^+ released and received is the factor which classifies whether the acid and base are monoprotic or polyprotic (diprotic, triprotic, etc).

Monoprotic acids have the ability to give only one proton per molecule and have one equivalence point.



In contrast, polyprotic acids are acids that can provide several protons per molecule in a reaction. To illustrate, diprotic acids (sulphuric acid) donate the hydrogen ions in several stages and the reaction can be separated into two unit reactions.



Any bases which receive single hydrogen ions are called monoprotic bases whereas if several ions are accepted, the bases are known as polyprotic.



2.1.2 Acid-Base Constants

The strength of the acids and bases depend on the value of the dissociation constants as discussed in the references [6, 7]. A large value of acid constant indicates a strong acid that gives out and ionizes almost all the protons. Weak acids are denoted by small constant value and only part of the protons that could be freed is actually donated. The dissociation constants for sulphuric acid are as shown in Equation 8-9.

$$K_1 = \frac{[H^+][HSO_4^-]}{H_2SO_4} \quad \text{Equation 8}$$

$$K_2 = \frac{[H^+][SO_4^{2-}]}{HSO_4^-} \quad \text{Equation 9}$$

2.1.3 Definition of pH

The concentration of hydrogen and hydroxide ions determines the pH value of a solution. An acidic solution contains a higher concentration of hydrogen ions than the concentration of hydroxide ions but the concentration of hydroxide ions existing in an alkaline solution is greater than the hydrogen ion. The system considers neutral measurement when the concentrations of both ions are the same. The pH-value is defined as in Equation 10.

$$pH = -\log_{10}[H^+] \quad \text{Equation 10}$$

The pH varies between 1 and 14. Nevertheless, pH which is higher or lower than the range indicated is also possible. At 25°C, it is assumed that the neutrality of a solution is pH 7 for neutral, pH value less than 7 is acidic and greater than 7 is an alkaline.

2.2 Static Modelling

System reaches equilibrium in the acid-base unit reactions can be considered instantaneous and therefore static modelling is practical. Product quality controls as well as indicating state process implement experimental static models to evaluate on the behaviour of the system [6, 7].

2.2.1 Titration Curve

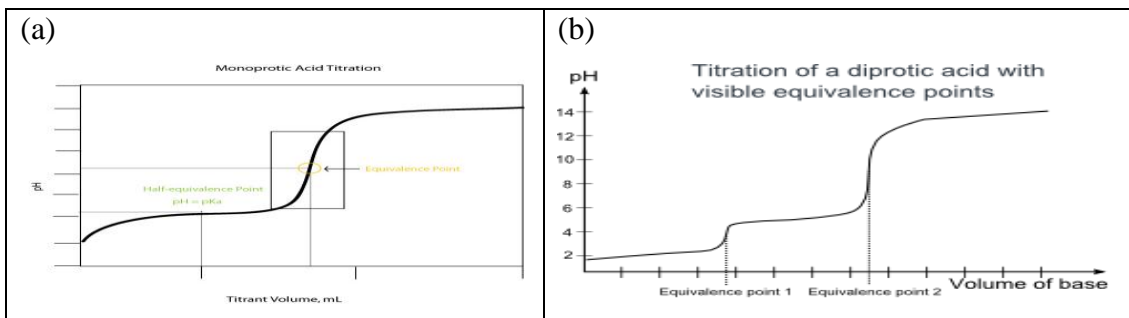


Figure 1: Titration curves for (a) Monoprotic Acid and (b) Polyprotic Acid

A titration curve portrays the characteristics of the acid-base neutralization reaction. The curve becomes the indicator to signify the pH variation around the neutralization point, concerning the types of acids and bases and the total volume or amount of substances been mixed up during the end process. Around the neutralization point, a very quick change in pH is seen upon adding a little amount of reactant into the solution. The region which is further away from the point requires large amounts of reactant to trigger the changes. Thus, the complexity level of the neutralization process in terms of the high-nonlinearity and time varying can be shown using titration curve.

The static part describes the logarithmic term which is a high nonlinearity zone of pH neutralization process as shown in Figure 1 (a) and Figure 1 (b). The solutions have been mixed in right proportions according to the equation if it reaches the "equivalence point". The equivalence points in titration of polyprotic acids, are undefined due to the scattered data plotted on the graph. As such, titration of sulphuric acid shows one equivalence point which is hard to analyse if the acid that has been reacted is unknown initially.

2.3 Dynamic Modelling

Different methods have been proposed to control the system efficiently. McAvoy and his colleagues have stated the basic principle of dynamic modelling [8] which later been generalized for pH process in continuous stirred tank reactor (CSTR) by Gustafsson and Waller [9].

Wright and Kravaris have studied nonlinear form of pH control using Strong Acid Equivalent (SAE) approach implemented on a lab scale process [10]. Likewise, another paper provided a review on the SAE method which the state variable of a reduced model is accessible online from the pH measurements with the insignificant nominal titration curve of the process reaction in neutralization. The formulation of the new approach converts the problematic control into an equivalent linear control problem [11].

It can be further summarized as shown in Table 1:

Table 1 : Reviews of Collected Research Papers – Continuous Process in CSTR

| Author(s) | Title | Findings |
|---|---|---|
| T.McAvoy, E.Hsu and S. Lowenthal (1972) | “Dynamics of pH in Controlled Stirred Tank Reactor” [8] | <ul style="list-style-type: none"> • Presented the underlying process of neutralization in CSTR • Derived the basis of mathematical equations to model in a time-optimal control loop |
| T.K.Gustafsson (1982) | “Calculation of the pH value of a mixture solutions: an illustration of the use of chemical reaction invariants” [9,10] | <ul style="list-style-type: none"> • Studied the estimation of pH value in a chemical reaction. • Mass balances on the invariant species are inherently independent of reaction rates. • Final pH of a solution should consider concentration of all variables involved in reaction. |
| R.A.Wright, M.Soroush, and C.Kravaris (1991) | “Strong acid equivalent control of pH processes: An experimental study” [11] | <ul style="list-style-type: none"> • Strong Acid Equivalent as a pH control algorithm • Requires only nominal inlet titration curve for measurement. • Implemented for lab scale pH process tested on <i>HCl</i> – <i>NaOH</i> and <i>CH₃COOH</i> – <i>NaOH</i>. |
| R.Ibrahim (2008) | “Practical modelling and control implementation studies on a pH neutralization process pilot plant” [12] | <ul style="list-style-type: none"> • Mathematical modelling based on physico-chemical as in McAvoy approach. • Implemented the fuzzy logic controller with the feedback/feedforward control method. |

References [8-12] explained the cases of continuous processes which has become the insight for a batch neutralization process. Few papers which discussed the batch pH process are referred in the Table 2.

Table 2 : Reviews of Collected Research Papers – Batch Process

| Author(s) | Title | Findings |
|---|--|---|
| N.Bharati, E.Sivakumar, J.Shanmugam, M.Chidambaram (2000) | “Control of pH in Fed-batch Neutralisation Processes” [13] | <ul style="list-style-type: none"> • Conveyed the best features of model-based control for a batch process. • Simulated the conventional PI controller and evaluated on the system robustness. |
| A.A.Ishak, M.A.Hussain and E.Elkanzi (2001) | “Modeling and Control Studies of Wastewater Neutralization Process” [14] | <ul style="list-style-type: none"> • Presented the study of dynamics and control of semi batch wastewater neutralization. • The reaction involved a strong base solution and a strong acidic solution. • Modelled and simulated PI control algorithm in MATLAB’s Simulink. |
| J.Barraud, Y.Creff, N. Petit (2008) | “pH Control of a Fed Batch Reactor with Precipitation” [15] | <ul style="list-style-type: none"> • Considered the reactor with occurrence of precipitates. • Conducted experiment on a laboratory reactor in batch nature environment. |

2.4 pH Control Algorithm

2.4.1 *Importance of Control Algorithm*

Deep understanding of the control algorithm is necessary for several purposes [16]. First, the structure of the system and the parameters involved influence the performance of the overall feedback mechanism. The structure which works with the process equipment and instrumentation, are highly unbearable in terms of cost and time. Subsequently, the control performance in the loop must acquire an important degree of flexibility. In a plant, typically only a few algorithms can be implemented and thus, the values of adjustable parameters in the algorithms should be determined.

2.4.2 *Fundamentals of pH Neutralization Control Algorithm*

The most common control method can be generalized into three main types; open loop, feedback, and feedforward. An open loop type of control scheme is used for single-loop systems, known as single-input-single output (SISO) [16]. The implementation lies simultaneously on a process during the start-up and shutdown of each control strategy. The performance of the single loop can be affected when it deals with other loops of a multistage, for instance while controlling the pH to the desired range in the neutralization process.

The second type implies feedback control principles. In neutralization control scheme, the cause and effect relationship can be illustrated between the control valve opening or closing for the flow of reactant and the desired pH value in the process. Consider an acidic solution in a reaction tank where a low pH reading is detected by the sensor. The valve of alkali is opened to counter the pH value until the desired range is attained. In contrast, if the pH value is greater than the set point, the control valve opening is decreased. Thus, this method is commonly known as a corrective control approach. The control action takes place upon the deviation between the process variable and the required set point.

The third control strategy is feedforward control. An input disturbance measurement is taken into account to enhance the control performance. By knowing earlier any

measured disturbance in the system, the error is compensated directly before it affects the whole process. The disturbance that might occur in pH process is the unexpected change of the reactant flow rate. Thus, in feedforward scheme, the controller will counter the changes before the pH value in the reactor tank is significantly affected whenever a disturbance arises. This method can be considered as a preventive control method which is very much faster than the corrective control approach because the disturbance is handled promptly. Combination of feedback and feedforward results in a complete approach of both corrective approach and preventive approach which is more stable and reliable.

2.4.3 Advanced Forms of Control Algorithm

Various forms of advanced controllers are still and have been investigated to obtain the method which is able to handle the vigorous behaviour of pH neutralization process. Pishvae and Shahrokhi (2006) proposed fuzzy modelling in static part to reduce the complicated computational load due to nonlinearities in dynamic part [17]. Fuente, Robles, Casado, Syafiie and Tadeo (2006) also studied the control of pH neutralization process using fuzzy controllers. Instead of PI- fuzzy controller, they concluded that an adequate control performance can be produced using auxiliary variable (AV) to operate on extremely nonlinear processes [18]. Additionally, neural network, observer's design, pole placement method, neuro-fuzzy, predictive control, dynamic matrix control and genetic algorithm are among the alternative control algorithms that can be used to develop on pH neutralization plant.

2.5 Feedback Linearization

Feedback linearization is commonly applied to transform nonlinear system into a linear system described by considering a single-input and single output system [19].

$$\dot{x} = f(x) + g(x)u \quad \text{Equation 11}$$

$$y = h(x) \quad \text{Equation 12}$$

Where f and g are smooth vector fields on \mathbf{R}^n , $x \in \mathbf{R}^n$ is the state vector, and $u, y \in \mathbf{R}$ are the control and output.

A smooth scalar function $h(\mathbf{x})$ of the state \mathbf{x} is represented by partial derivative of ∇h .

$$\nabla h = \frac{\partial h}{\partial \mathbf{x}} \quad \text{Equation 13}$$

Likewise, a vector field $\mathbf{f}(\mathbf{x})$, is denoted by the Jacobian $\nabla \mathbf{f}$

$$\nabla \mathbf{f} = \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \quad \text{Equation 14}$$

2.5.1 Lie Derivatives

The scalar function $h(\mathbf{x})$ and a vector field $\mathbf{f}(\mathbf{x})$ can be defined into a new scalar function known as Lie Derivative explained in Equation 15-21, $L_f h$ of h with respect to vector \mathbf{f} .

$$L_f^0 h = h \quad \text{Equation 15}$$

$$L_f^i h = L_f(L_f^{i-1} h) = \nabla(L_f^{i-1} h)\mathbf{f} \quad \text{for } i = 1, 2, \dots \quad \text{Equation 16}$$

Another vector field \mathbf{g} with a scalar function $L_g L_f h(\mathbf{x})$.

$$L_g L_f h = \nabla(L_f h)\mathbf{g} \quad \text{Equation 17}$$

From the single-output system

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) \quad \text{Equation 18}$$

$$y = h(\mathbf{x}) \quad \text{Equation 19}$$

The derivatives yielded the output into

$$\dot{y} = \frac{\partial h}{\partial \mathbf{x}} \dot{\mathbf{x}} = L_f h \quad \text{Equation 20}$$

$$\ddot{y} = \frac{\partial [L_f h]}{\partial \mathbf{x}} \dot{\mathbf{x}} = L_f^2 h \quad \text{Equation 21}$$

2.5.2 Lie Brackets

Proving the property of Lie bracket

$$\nabla h[\mathbf{f}, \mathbf{g}] = \nabla(L_{\mathbf{g}}h)\mathbf{f} - \nabla(L_{\mathbf{f}}h)\mathbf{g} \quad \text{Equation 22}$$

Expansion of the left-hand side of Equation 22 is shown in Equation 23

$$\nabla h[\mathbf{f}, \mathbf{g}] = \frac{\partial h}{\partial x} \left(\frac{\partial \mathbf{g}}{\partial x} \mathbf{f} - \frac{\partial \mathbf{f}}{\partial x} \mathbf{g} \right) \quad \text{Equation 23}$$

Right-hand side expansion is rewritten in Equation 24 and Equation 25

$$\nabla(L_{\mathbf{g}}h)\mathbf{f} - \nabla(L_{\mathbf{f}}h)\mathbf{g} = \nabla \left(\frac{\partial h}{\partial x} \mathbf{g} \right) \mathbf{f} - \left(\frac{\partial h}{\partial x} \mathbf{f} \right) \mathbf{g} \quad \text{Equation 24}$$

$$\left(\frac{\partial h}{\partial x} \frac{\partial \mathbf{g}}{\partial x} + \mathbf{g}^T \frac{\partial^2 h}{\partial x^2} \right) \mathbf{f} - \left(\frac{\partial h}{\partial x} \frac{\partial \mathbf{f}}{\partial x} + \mathbf{f}^T \frac{\partial^2 h}{\partial x^2} \right) \mathbf{g} = \frac{\partial h}{\partial x} \left(\frac{\partial \mathbf{g}}{\partial x} \mathbf{f} - \frac{\partial \mathbf{f}}{\partial x} \mathbf{g} \right) \quad \text{Equation 25}$$

Chapter 3

METHODOLOGY

3.1 Research Methodology

Figure 2 explains the methodology involved in executing the project.

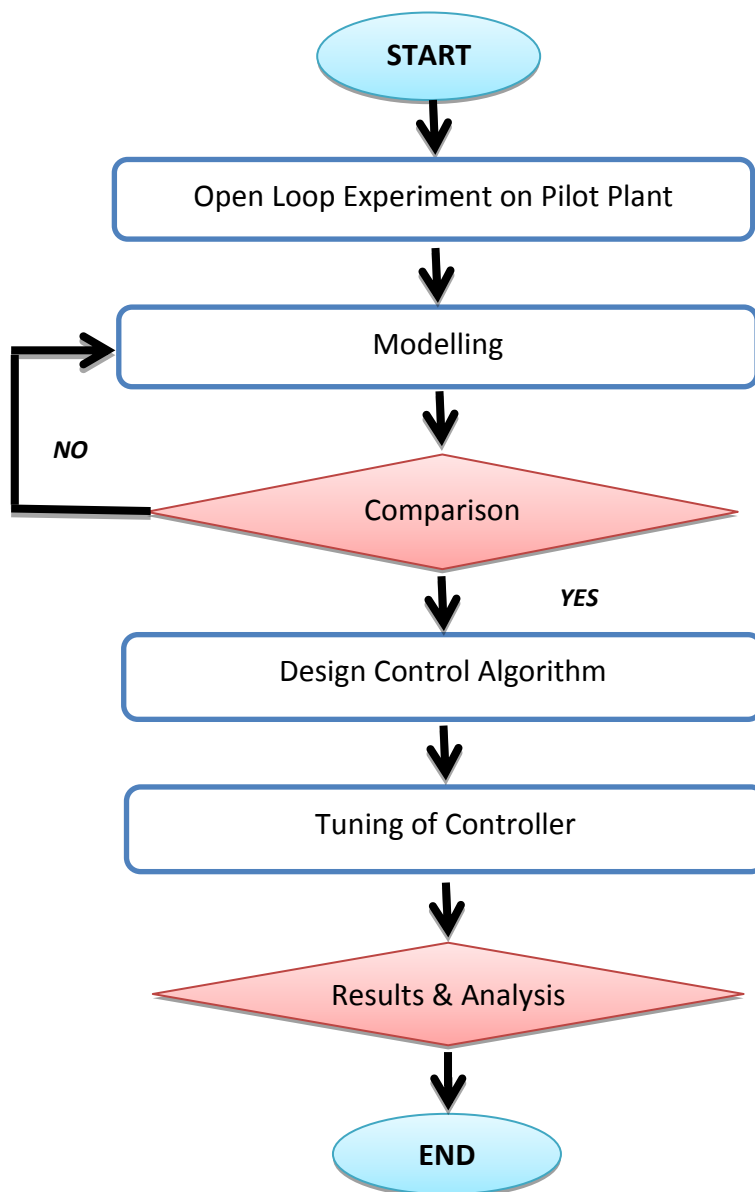


Figure 2: Project Flow Diagram

3.2 Project Activities

Table 3 describes briefly on the steps taken to realize the design of control algorithm for pH neutralization.

Table 3 : Description of Project Activities

| Tasks | Descriptions |
|---|---|
| Open Loop Experiment | <ul style="list-style-type: none"> ▪ Performing an open loop experiment on the laboratory scale pH pilot plant located at Block 23, UTP. ▪ Measurements are taken and will be used for the next step to reflect the actual chemical process in simulation environment. |
| Modelling and simulation of pH neutralization process | <ul style="list-style-type: none"> ▪ Deriving the dynamic model of the neutralization process ▪ Findings on suitable mathematical equations or expressions for semi batch pH process ▪ Creating simulation model using MATLAB's Simulink that represent the pH control characteristics and pH neutralization process of the pilot plant ▪ Evaluating the model by comparing the results obtained from the pilot plant |
| Controller design and development | <ul style="list-style-type: none"> ▪ Designing and developed the control algorithm following the pH neutralization model derived ▪ Research will be done on linearizing the batch pH process ▪ Simulation tests are done to measure the performances |
| Tuning of controller | <ul style="list-style-type: none"> ▪ Tuning the designed control method to obtain desirable parameters which gives a better feedback control. |
| Results and Analysis | <ul style="list-style-type: none"> ▪ The performances of the control strategy will be analysed and the success will be determined by comparing to the output of PID controller also the control objectives. |
| Improvements | <ul style="list-style-type: none"> ▪ Given extra time, an advanced controller will be studied ▪ The performances are analysed and compared |

3.3 Project Duration

In order to effectively monitor the progress of this project, a key milestone consists of 28 weeks duration has been constructed. Refer Appendix A. Alternatively, Gantt chart is planned accordingly to keep up with dateline that has been set by the management for presentations and submission of documentations. Refer Appendix B.

3.4 Tool

Table 4 : List of Tool

| No | Software | Benefits | Associated File |
|----|-------------------------------|--|---|
| 1 | MATLAB R2011a | <ul style="list-style-type: none"> - Develop coding scripts - Fast iteration process in calculation of first order differential equations - Curves plotting - Simulink Block Diagram | <ul style="list-style-type: none"> - rk4ode.m - MATHMODEL.mdl (Figure 13) - closedlooptuning.mdl |
| No | Hardware | Usage | |
| 1 | xPC platform | <ul style="list-style-type: none"> - Allow real time communication between two systems. - Transfer of data and instruction from engineering workstation in the control room and the field instruments on the pH pilot plant. | |
| 2 | pH Neutralization Pilot Plant | <ul style="list-style-type: none"> - Represent the actual industrial neutralization process on a laboratory scale pilot plant. - Reduce consumption of money and time in experiments | |

Chapter 4

RESULTS & DISCUSSIONS

4.1 Model Development of a Semi batch Process

4.1.1 Overview of the Semi batch Process

The system under configuration for this research is a fed batch reactor in which neutralization occurs between a strong acid (Sulphuric Acid), H_2SO_4 and strong base (Sodium Hydroxide), $NaOH$ which is added constantly as a reactant. A semi batch process is neither a continuous nor a batch. For continuous process, the feed is flowing constantly in and the effluent is constantly flowing out. Whereas for batch process, the feed enters into the tank at once and the effluent will be released at once when neutralization completes.

Initially, it is assumed that the wastewater in the reaction tank is an acidic solution which is neutralized by manipulating the flow rate of base to bring the pH to the desired measurement.

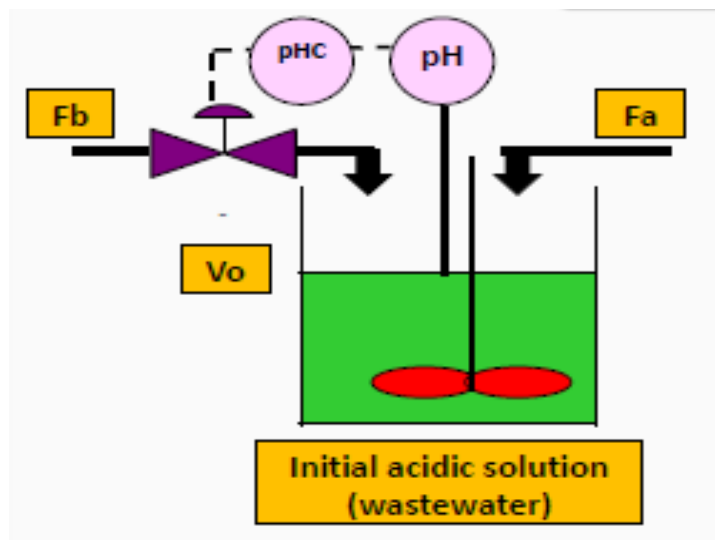


Figure 3: A Semi batch Reactor

4.1.2 Open Loop Experiment

Experimental procedures are carried out to evaluate the pH behaviour in neutralization process. The result in this experimentation will be used to compare with the results obtained from the mathematical modelling simulated in MATLAB.

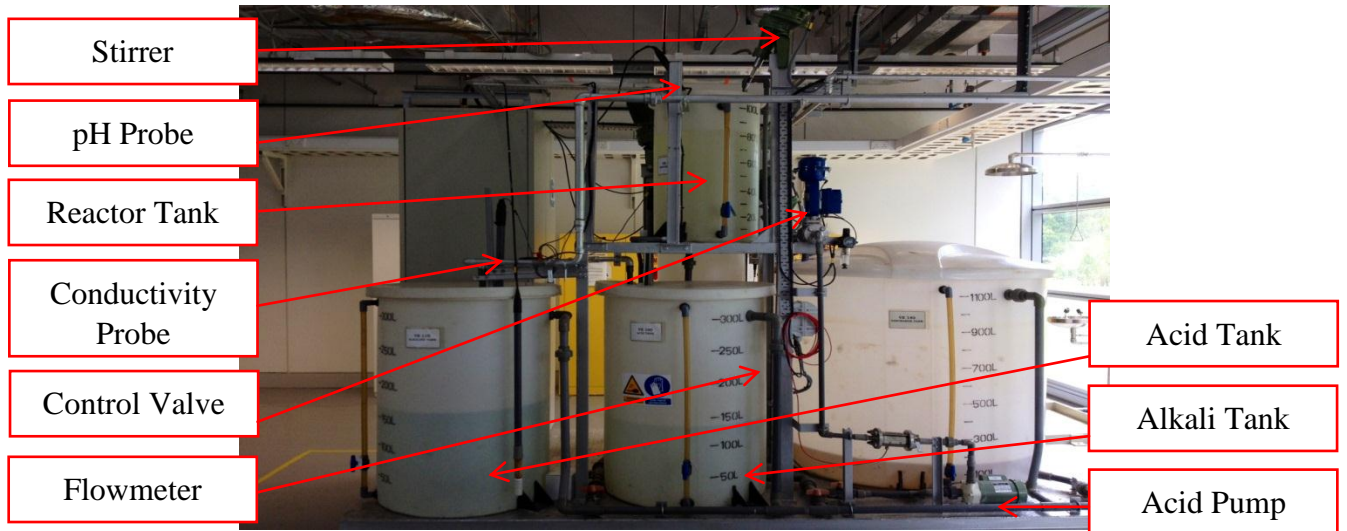


Figure 4: pH Pilot Plant at Universiti Teknologi PETRONAS

To perform the experiment, the initial pH in the reaction tank is set to its lowest possible value since the acidic solution is considered as the wastewater. Both acid and alkaline concentrations are 0.05M. Several trials are made to bring down the wastewater to pH of 3. An acid valve FCV121 corresponds to pump P110. The flow of acid is halted and the process begun by the activation of pump P100 with 50% opening of valve FCV120 at base flows of 25L/h. At $t = 100$ seconds, the valve is opened to 60% and the flow rate is 34.5L/h.

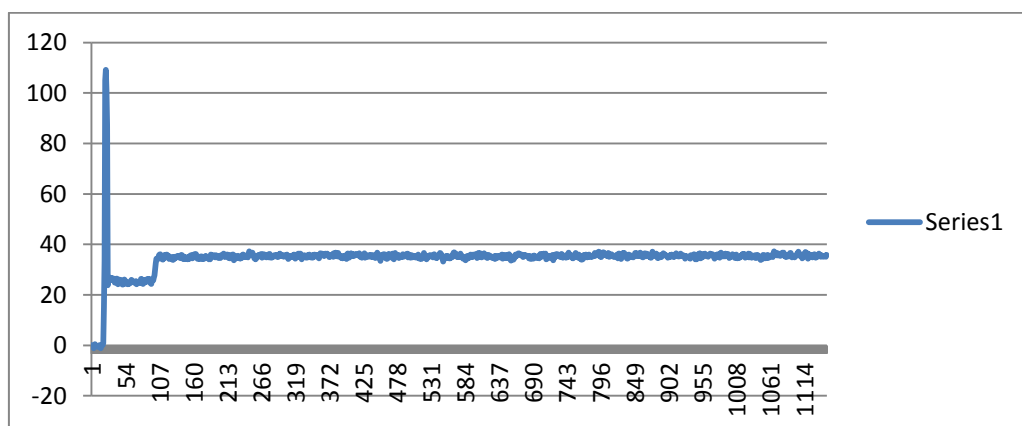


Figure 5: Flow rate of Alkaline Stream

Open loop experiment is also done to show the distinctive behaviour of titration. In the laboratory, two pH meters are installed inside the reaction tank because the available pH meters are no longer able to produce accurate readings as before. Yet, the meters still portrayed appropriate measurements because similar graphs are plotted as shown in Figure 6 and Figure 7.

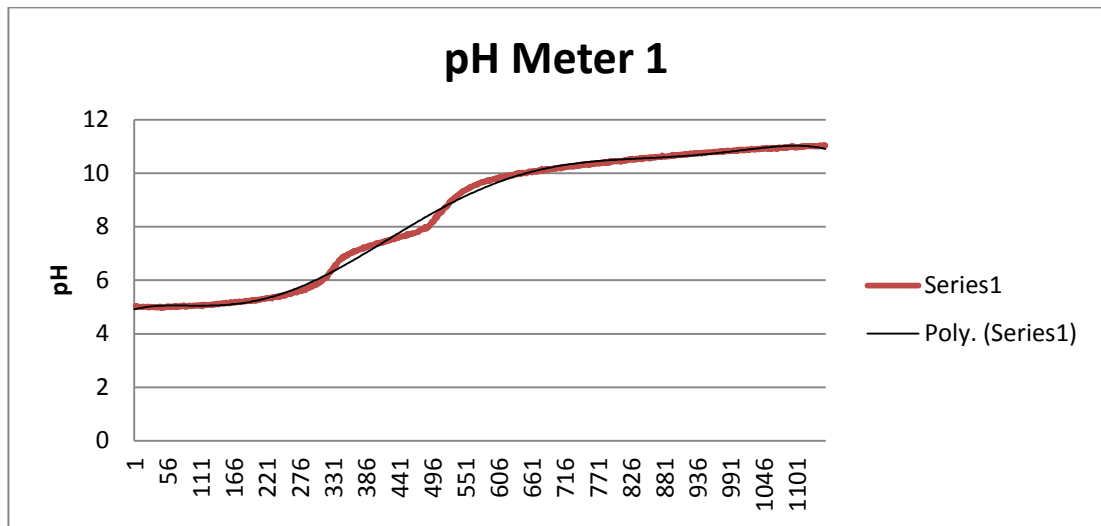


Figure 6: Process Reaction Curve 1

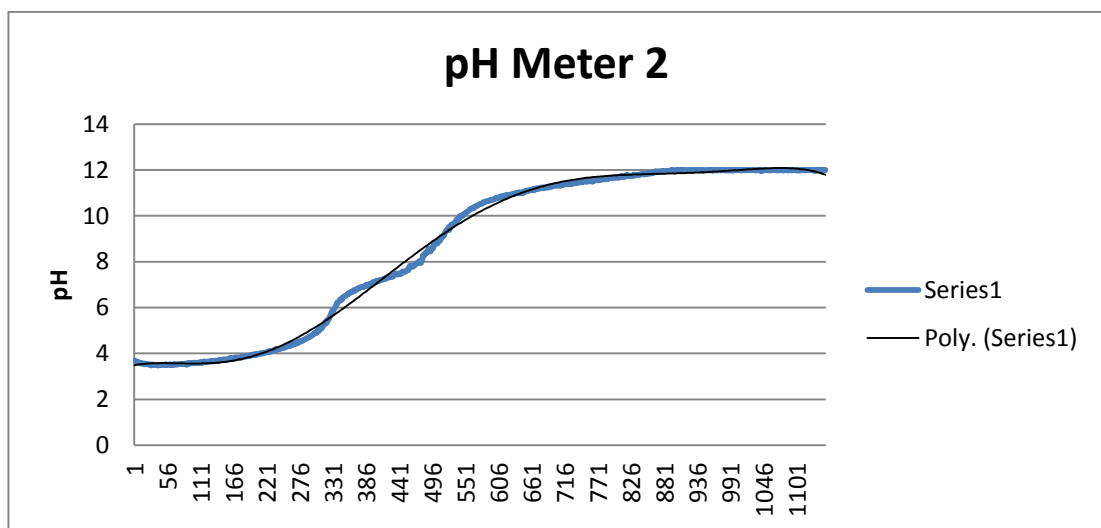


Figure 7: Process Reaction Curve 2

It is noted that the curve has two equivalence points conveying that the the solutions reacted with diprotic acid. The equivalence points can be diminish into S-shaped graphs if polynomially fitted.

4.1.3 Solving Mathematical Equation using Numerical Method

The model equations for the neutralization of Sulphuric Acid, H_2SO_4 and strong base Sodium Hydroxide, $NaOH$ are as stated in the references [20, 21].

Where F_a : flow rate of acid, F_b : flow rate of base, $[SO_4^{2-}]$: the concentration of acid and $[Na^+]$: concentration of base.

The rate of volume changes inside the reaction tank

$$\frac{dV}{dt} = F_a + F_b \quad \text{Equation 26}$$

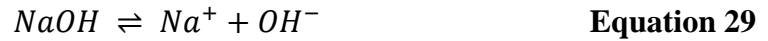
Sodium ion balance

$$\frac{d[Na^+]}{dt} = \left(\frac{F_b}{V}\right)(Na^+) \quad \text{Equation 27}$$

Sulphate ion balance

$$\frac{d[SO_4^{2-}]}{dt} = \left(\frac{F_a}{V}\right)(SO_4^{2-}) \quad \text{Equation 28}$$

Electro neutrality equation



$$2[H^+] + [Na^+] = [SO_4^{2-}] + [OH^-] \quad \text{Equation 31}$$

Water Equilibrium at 25°C

$$[H^+] + [OH^-] = K_W = 10^{-14} \quad \text{Equation 32}$$

Combining Equation 26 to 32 produces

$$\frac{d[H^+]}{dt} = \frac{[H^+]^2}{\{2[H^+]^2 + K_W\}V} [F_a[SO_4^{2-}] - F_b[Na^+]] \quad \text{Equation 33}$$

As shown in Equation 33, pH value can be computed by using Equation 10.

Separation of Variables of First-Order Differential Equations

$$\int dV = \int (F_a + F_b) dt \quad \text{Equation 34}$$

$$V = (F_a + F_b)t + V_0 \quad \text{Equation 35}$$

Substitute Equation 35 into Equation 33

$$\frac{d[y]}{dx} = \frac{[y]^2}{\{2[y]^2 + K_W\}[(F_a + F_b)t + V_0]} [F_a C_a - F_b C_b] \quad \text{Equation 36}$$

Approximating the solution of a first-order initial value problem using Fourth-Order Runge-Kutta method and is represented in the form of

$$y' = f(x, y), y(x_0) = y_0 \quad \text{Equation 37}$$

where y is a function of H^+ and x is a function of time. An M-File for Fourth-Order Runge-Kutta method is developed to solve the first order differential equation.

Assumed an initial amount of wastewater inside the tank $V_0 = 20l$. The following statements are entered in file editor of MATLAB.

```
function [x,y] = rk4ode(dydx,xspan,y0,h)
xi=xspan(1);
xf=xspan(2);
x=(xi:h:xf)';
n=length(x);
y=y0*ones(n,1);
for i=1:n-1
    k1=feval(dydx,x(i),y(i));
    k2=feval(dydx,x(i)+(1/2)*h,y(i)+(1/2)*k1*h);
    k3=feval(dydx,x(i)+(1/2)*h,y(i)+(1/2)*k2*h);
    k4=feval(dydx,x(i)+h,y(i)+k3*h);
    y(i+1)=y(i)+(1/6)*(k1+2*k2+2*k3+k4)*h;
end
```


For sample purposes, the following commands are typed in the command window with an interval of 500s, initial pH value of the wastewater is an acidic with the step size $h = 0.5$.

```
>>dydx=inline
(' (y^2)*(0.01667*0.00005-0.025*0.0005)/((2*y^2+1e-
14)*((0.01667+0.025)*x+20)) ','x','y')
>>[x,y]=rk4ode(dydx,[0 500],0.001,0.5);
```

Results obtained after the iteration are shown in Appendix C. A graph is plotted using command

```
>> plot(y, 'DisplayName', 'y', 'YDataSource', 'y'); figure(gcf)
```

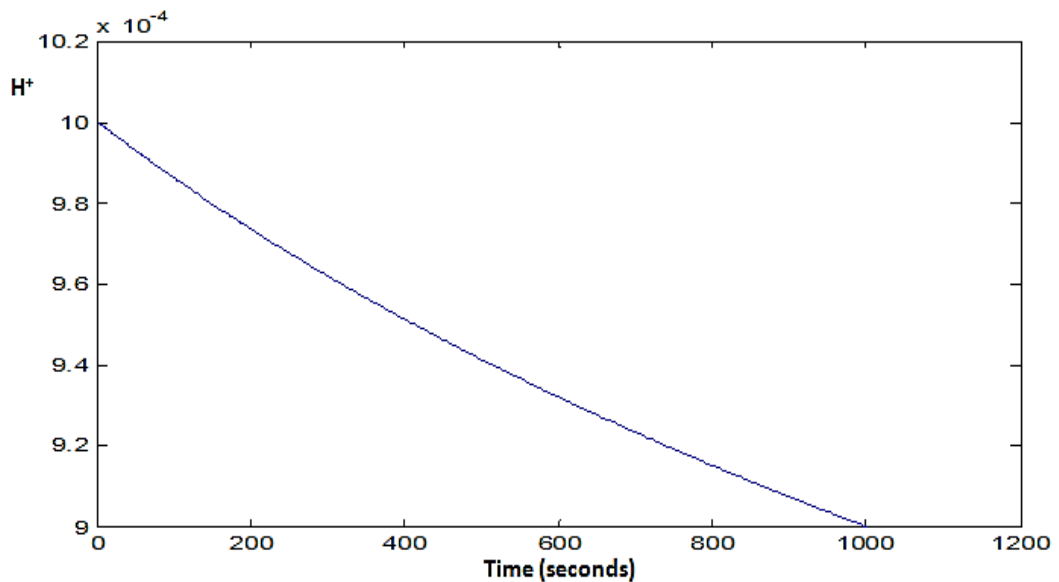


Figure 8: Graph of H^+ against time (Numerical Method)

4.2 Mathematical Modelling

4.2.1 Simulink Block Diagram

Simulink Block Diagram is established in Figure 10. From the results displayed on the scope, a similar graph is produced as in the numerical iteration part using M-File editor. Figure 8 and Figure 10 are compared.

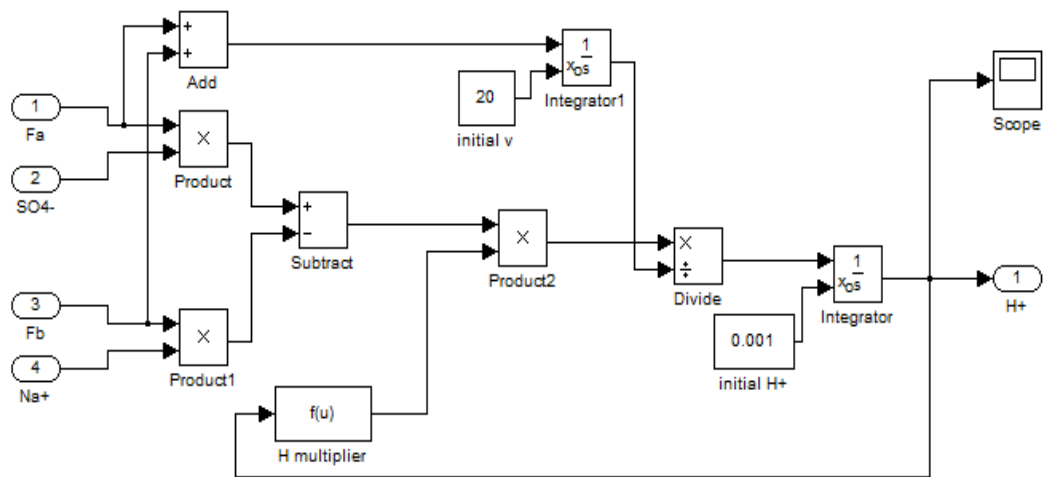


Figure 9: Process Equations in Simulink

Hence it can be interpreted that the solutions of the first order differential equations using Fourth Order Runge-Kutta method can also be explained using the Simulink block diagram which has characterized the chemical process correctly. The block diagram is then used to develop the next stage of the mathematical modelling.

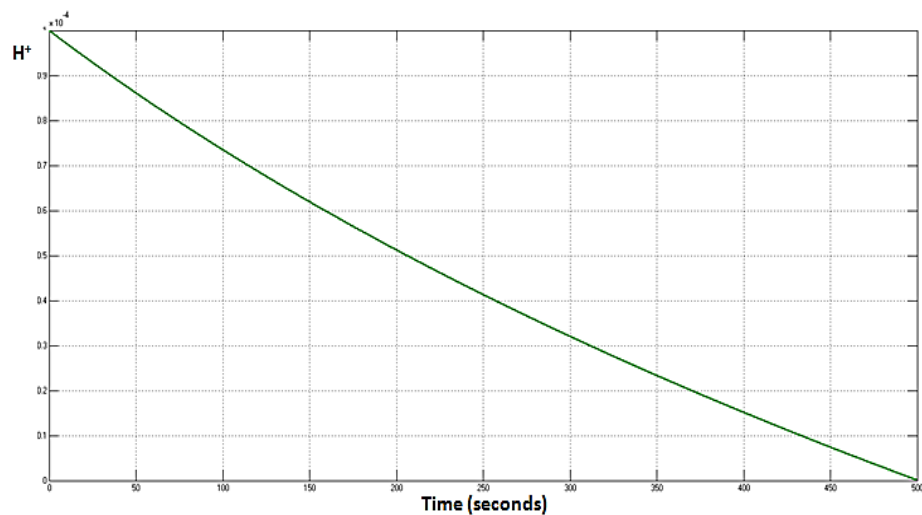


Figure 10: Graph of H^+ against time (Simulink Block Diagram)

Suitable settings must be set beforehand [22] into Configuration Parameters in Simulation tab to evade dialog box of errors in measurement as shown in Figure 11.

Based on Figure 12, type of solver is a variable-step because the process changes with time. Step sizes are set into automatic mode to accommodate the ranges. In order to solve equations using the Fourth Order Runge-Kutta method, `ode45` (Dormand-Prince) is chosen as a solver function. There are actually two main functions to solve for numerical solution and to simulate complex systems namely `ode23` and `ode45`. The differences are in terms of the ability to handle a higher order of calculations and to cater the stiffness of the differential equations.

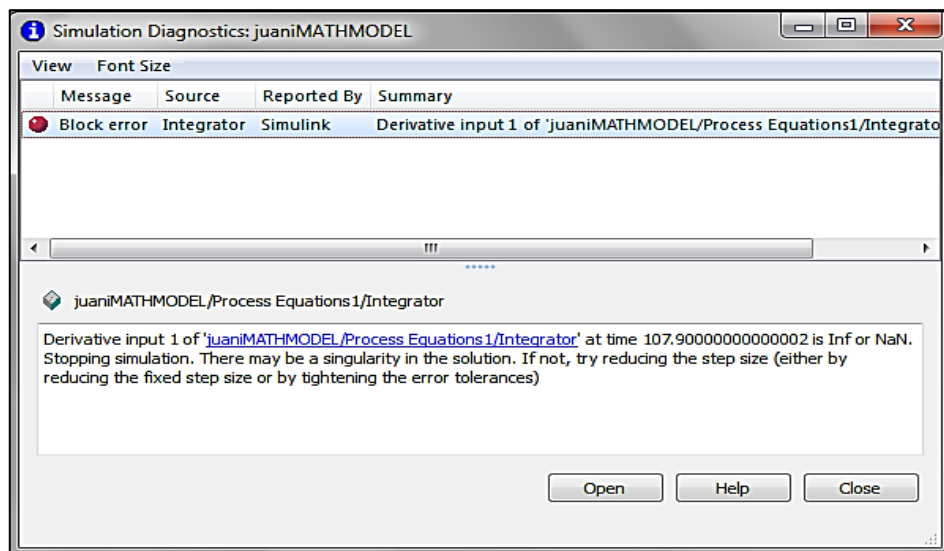


Figure 11: Simulation Diagnostics

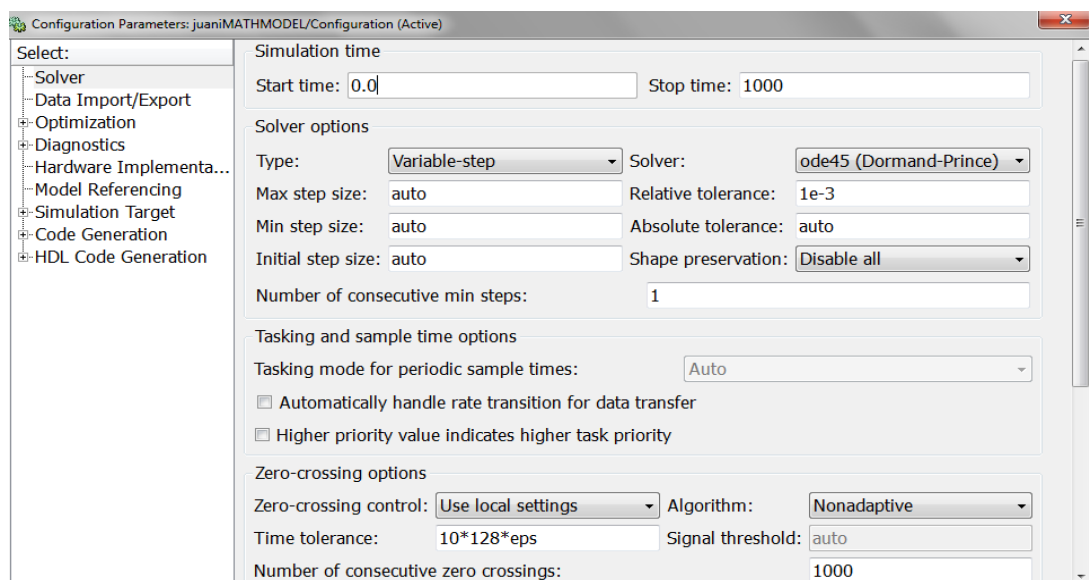


Figure 12: Configuration Parameters

Confirming the process equations are accurate for both in numerical and simulation techniques, the next step is to illustrate the pH characteristics. From Figure 13, the first block represents the mathematical equations derived previously to calculate for H^+ . The second block is the logarithmic function to evaluate on the pH measurement given the flow rates of base and acid.

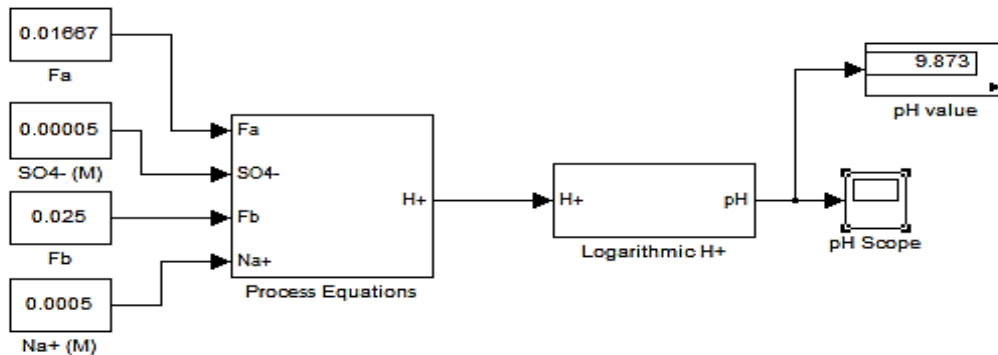


Figure 13: pH Simulink Block Diagram

Saturation block in Figure 14 is placed to create a boundary from pH 3 which is a very acidic condition and to pH 14, a very alkaline condition to measure the neutralization behaviour of a strong acid and a strong base.

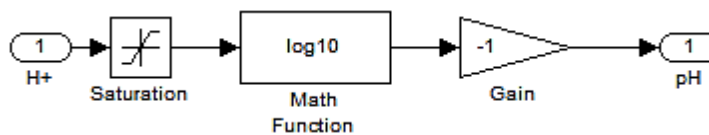


Figure 14: Logarithmic Subsystem

For example, from Equation 10, the value of $pH = -\log(0.001) \Rightarrow 3$.

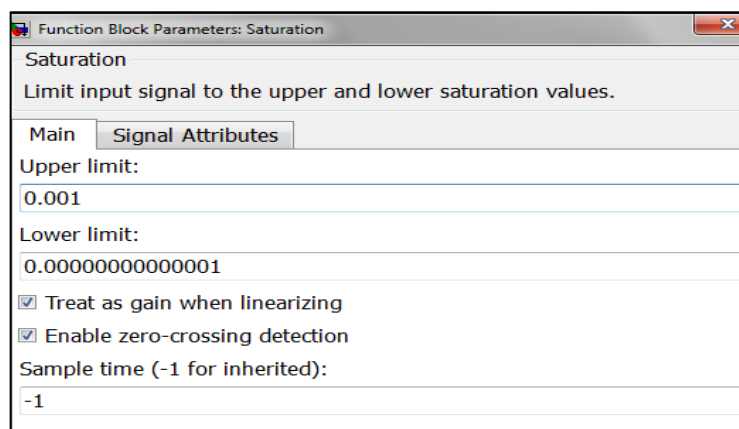


Figure 15: Saturation Values

The program is run for a simulation time of 200 seconds. The shape of the curve in Figure 16 explains the neutralization process. Initially, the pH rises slowly because there is a high concentration of acid to be neutralized by the base. At approximately 106 seconds, the concentrations of H^+ and OH^- are in equal amount thus it has been neutralized and the curve rises sharply. The next portion of curve indicates the excess amount of hydroxide ions by adding base.

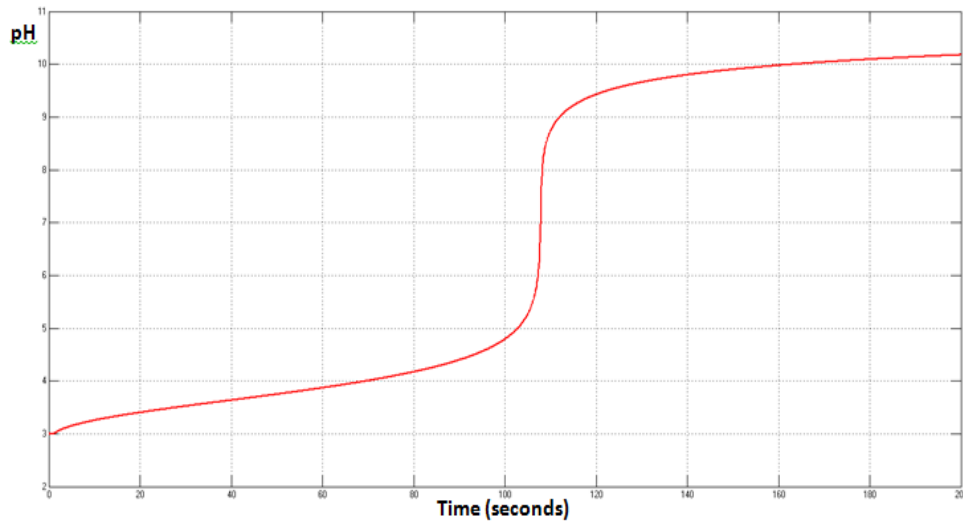


Figure 16: Graph of pH Characteristics

4.3 Feedback Control Strategy

The control objective of this process is to regulate the base flow while maintaining the flow of acid at a constant rate until the pH inside the reactor tank is within the desirable value which is pH 7.

Table 5 : List of Variables Involved in Simulation

| Variables | |
|--------------------------|-----------------------|
| PV, Process Variable | pH-Acidity/Alkalinity |
| MV, Manipulated Variable | Flow of Base |

Values of assumptions are made realizable to signify the real working conditions as shown in Table 6.

Table 6 : Values of Assumptions

| Assumptions | Value |
|---|---------------------|
| F_a , Flow rates of acid | 0.01667l/s |
| F_b , Flow rates of base | <i>to be varied</i> |
| $[SO_4^{2-}]$, Concentration of sulphate | 0.00005M |
| $[Na^+]$, Concentration of sodium | 0.0005M |
| Initial H^+ | 0.001 |
| Initial <i>pH</i> | 3 |
| Initial value of acid in reactor tank | 20l |

The mixing tank is presumed to have a wastewater of acidic state, of pH value less than 7. Hence, the system is to be designed in such a way that the effluent is at pH 7 by controlling only the alkaline valve. The concentrations of hydrogen ions and hydroxide ions are to be made balanced by either increasing or decreasing the amount of acids or bases. The numbers of feeds and measurements to be varied depend on the systems, whether it is single-input and single-output (SISO), multiple-input and single-output (MISO), single-input and multiple-output (SIMO) and multiple-input and multiple-output (MIMO).

In this project, the system evaluated on is SISO system; flow of base as the input, and the indication of pH value as the output. SISO system is chosen because the complexity is much reduced. Neutralization process itself has been identified difficult to control due to the nonlinearity presented from the logarithmic relationship. For this reason, the number of inputs or feeds entering into the reaction tank is reduced to abridge dynamical process.

Figure 17 demonstrates the PID feedback control algorithm in a closed-loop system. In an open-loop system, the feed comes in and the measurement is not compared to the reference point. Nonetheless, a closed-loop control system using a feedback control will compare the output to the reference value. The controller manipulates the input to a system based on the output measurement. The process variable measured is matched to the set point and the deviation between the measured output and

reference is known as error. The error is then adjusted by applying proper corrective action to obtain the preferred effect on the output without affecting the stability of the system.

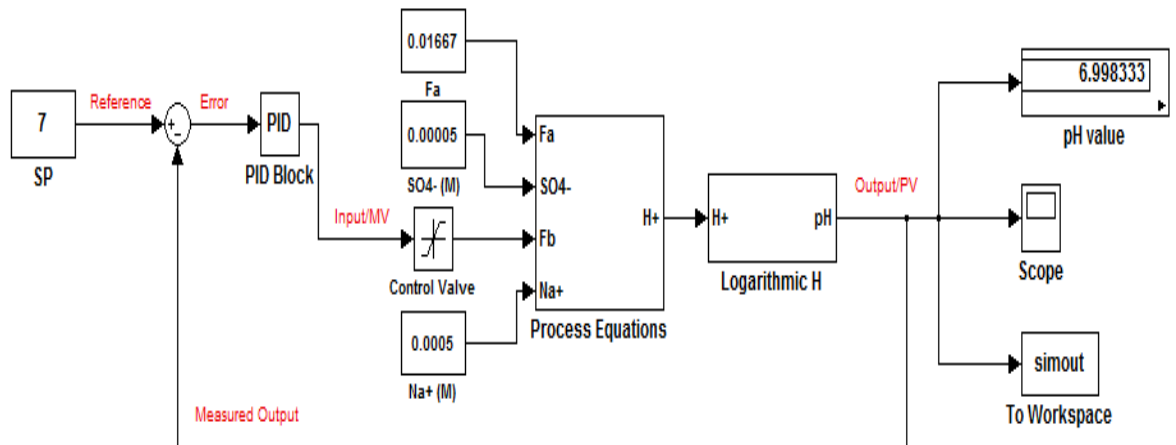


Figure 17: PID Feedback Control Algorithm

4.3.1 PID Tuning

The most widely used controllers in the industrial processes are PID. Tuning of controller is a method to find the most suitable parameters in optimizing the process besides minimizing the divergence between the process variable and the set point. Ziegler-Nichols and Cohen-Coon methods are the common procedure of tuning applied in industries. Controller tuning methods involve a process reaction curve technique and trial and error approach. The Ziegler-Nichols method is applicable to both open loop and closed loop systems. Where else, Cohen-Coon method is merely used for open-loop systems.

From the mathematical model developed, a PID controller is designed and tuned to get the best performance. The Ziegler-Nichols tuning method is selected because the pH system to be assessed is a closed-loop system. Several steps are done to perform tuning:

1. The integral and derivative parts in the PID controller are removed
2. A disturbance such as a step change is introduced to the system
3. The gain is varied till the system oscillates with constant amplitude
4. The ultimate gain, K_u and ultimate oscillation period, P_u are recorded

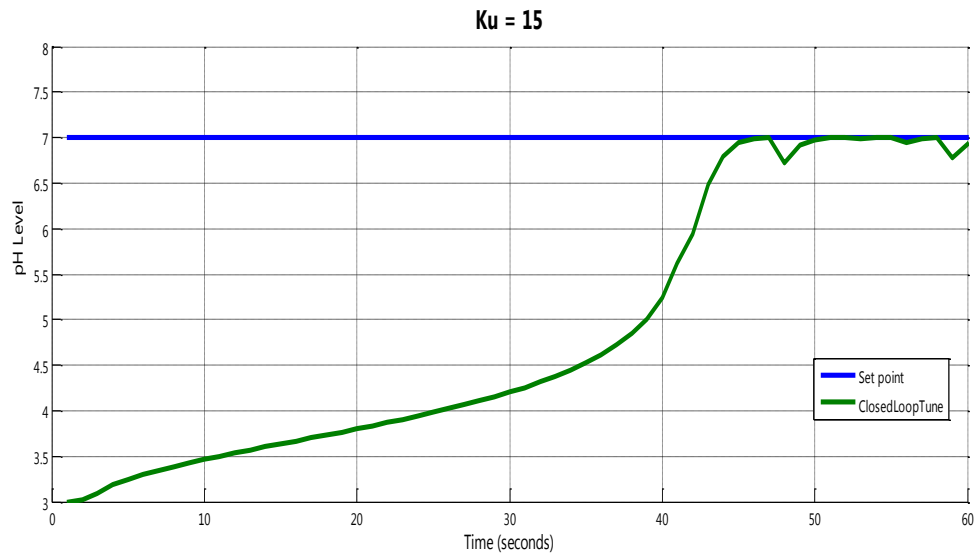


Figure 18: Proportional Gain = 15

The response of the process variable, PV is observed. The proportional gain is adjusted either by increasing or decreasing the value. If it is not oscillatory, the controller gain is increased. If the magnitude of the process variable is increasing, the controller gain is reduced. Several trials are made to find the most suitable proportional gain. With gain of 15 shown in Figure 18, it has no oscillatory effect. Instead, it has spikes up and down in a few regions. This might be due to the noise or the value of the gain is not sufficient enough since the curve has falling spikes. For gain of 40 presented in Figure 19, the curve has displayed oscillation but the shape is fluctuating. Therefore, the gain need to be increased.

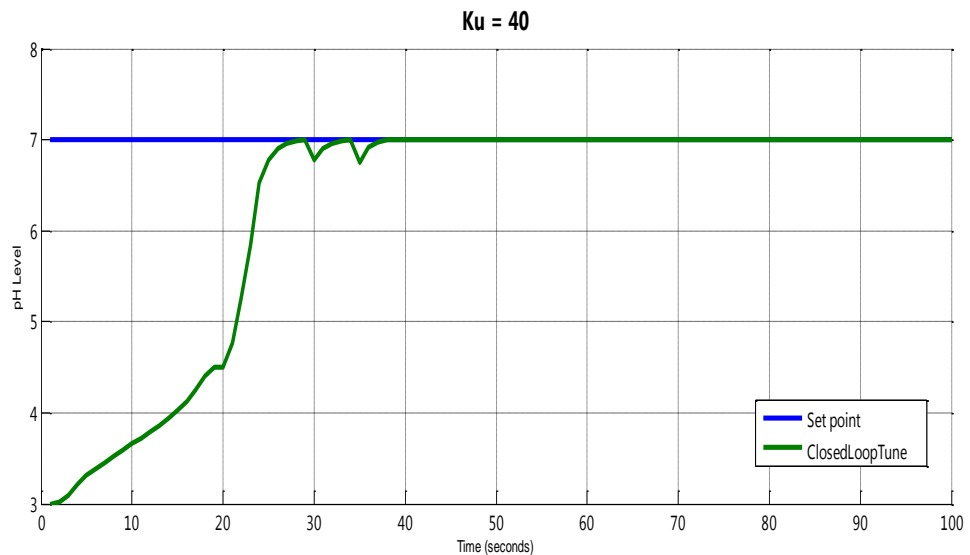


Figure 19: Proportional Gain = 40

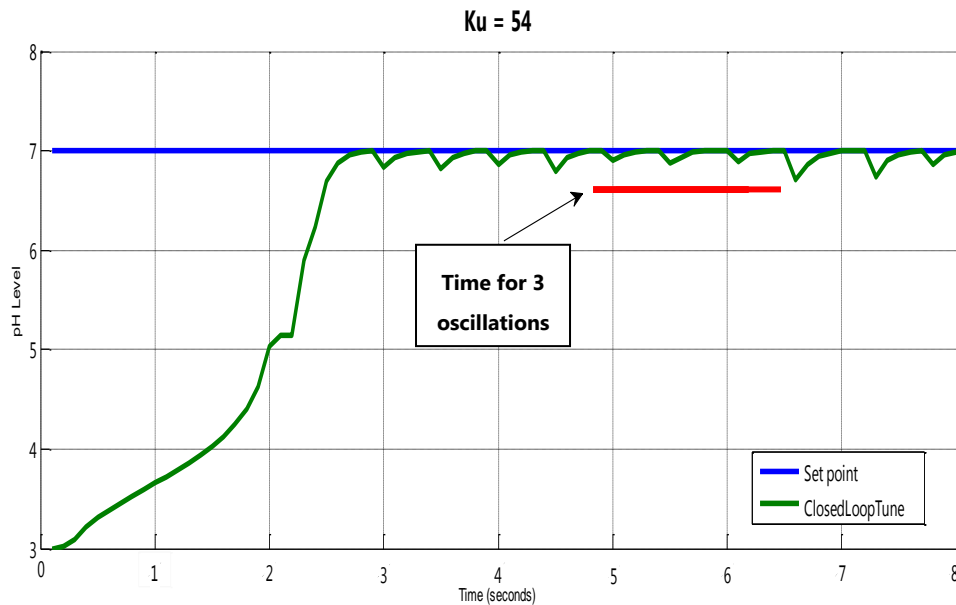


Figure 20: Proportional Gain = 54

An oscillatory response with almost constant amplitude is achieved when the gain is 54. From Figure 20, the proportional gain noted is the ultimate gain, K_u . Next, three or more cycles are allowed in the process and the average period of an oscillation is known as the ultimate period, P_u .

The ultimate gain, K_u and ultimate period of oscillation, P_u are recorded to calculate the parameters for P, PI and PID controller modes tabulated in Table 7. The Ziegler-Nichols Closed Loop Correlations is presented in Appendix D.

Table 7 : Ziegler-Nichols Closed Loop Parameters

| Control Modes | K_c (% valve open) | T_I (sec) | T_D (sec) |
|---------------|----------------------|-------------|-------------|
| P | 27 | - | - |
| P + I | 24.3 | 0.278 | - |
| P + I + D | 32.4 | 0.167 | 0.0417 |

Figure 21 illustrates the structure of PID block theoretically. Equation 38-40 discussed the actions of each controller following the reference in [23].

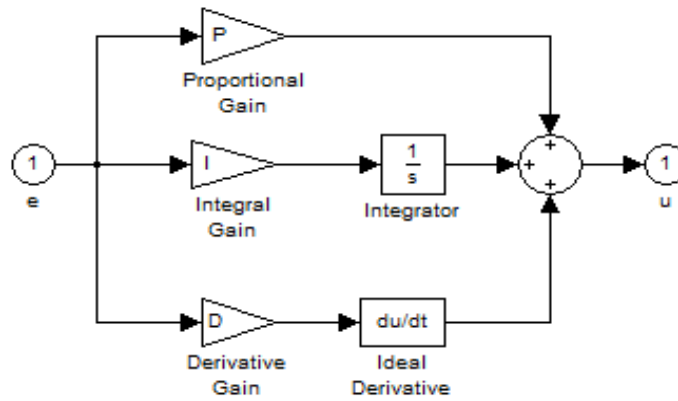


Figure 21: PID Block Under Mask

Proportional mode of a feedback control system introduced controller gain K_c which decreases the rise time of the process. Prior to proportional control only, the system will have steady-state error. Increasing the gain can be established to reduce the error with a trade off of an unstable system behaviour besides longer settling time taken after being disturbed provided that the value is too extreme. The equation describes the proportional controller:

$$U(s) = K_c E(s) \quad \text{Equation 38}$$

The steady state error is compensated with the integral or reset term.

$$U(s) = K_c \left[1 + \frac{1}{T_I s} \right] E(s) \quad \text{Equation 39}$$

Consequently the integral time or reset action combines the controller into a PI controller. The integral time minimizes the steady-state offset in the controlled variable. But then again it has significant effect in destabilizing the system which commonly can be counteracted by K_c adjustments.

Derivative action made the control system response faster. It can be called the rate action which speeds up the controller corresponding to the rate of change of error with time. It is important to note that T_D does not correct the steady state error. The contributions of derivative time lies in anticipating error, preparing for preventive measures before the error affects the system overall and thus increases the stability of the system designed.

$$U(s) = K_c \left[1 + \frac{1}{T_I s} + T_D s \right] E(s) \quad \text{Equation 40}$$

Addition of derivative term commenced the PID controller when combined. Even with the advantages it offers, PID controller is not extensively used as compared to PI controller due to the noise factor initiated by the T_D . Despite the facts, the decision to execute which controller modes is subjective depending on the control objectives and desired control performances of the system.

4.3.2 PID Implementation

Based on the tuning parameters calculated, the values are utilized in the PID block of the mathematical modelling. The simulation is run by combining each controller mode result in a single graph using a multiplexer function block and the considerations involved are set.

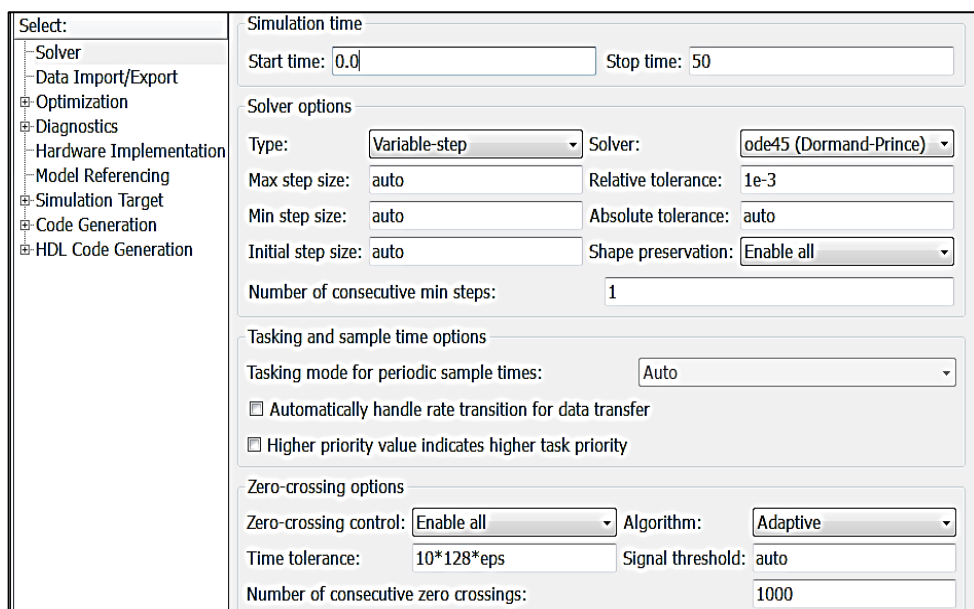
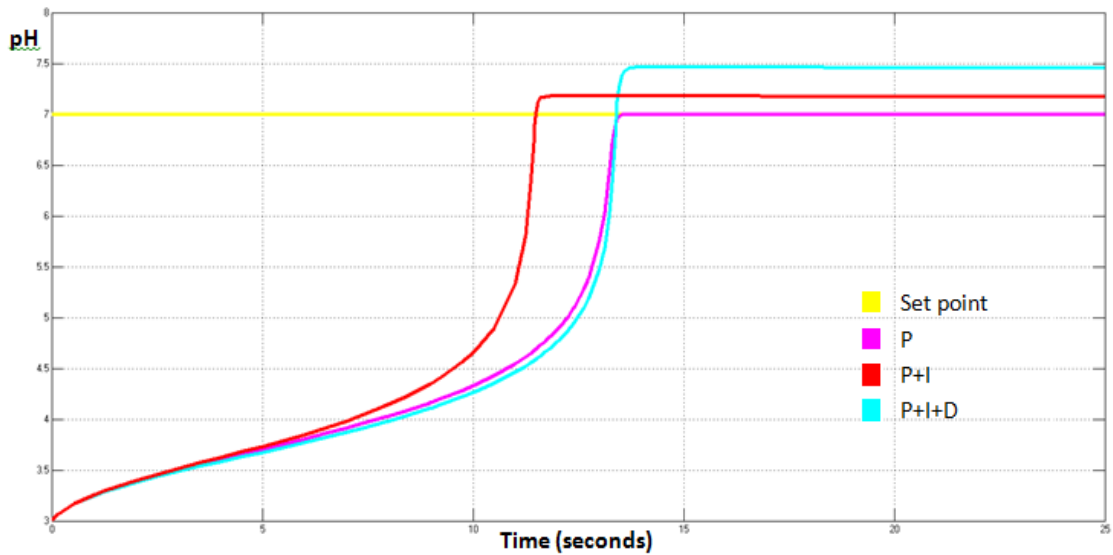


Figure 22: Configuration Parameters for PID Simulink

From Figure 22, it is shown that the shape preservation is enabled to ensure the curves produced are clean from any spikes or noises. For zero-crossing control, it is enabled as violation would not occur which will terminate the simulation procedures when it hits the zero-crossing spot. An adaptive algorithm is preferred rather than non-adaptive because it will modify the system performance following the algorithm.

Graphs of P, PI and PID controller modes plotted are presented in Figure 23:



**Figure 23: PID Controller Responses
(without Time Delay) and (Shape Preservation Enabled)**

Table 8 : Control Performances

| Mode | Steady State Error (%) | Settling Time (sec) | Rise Time (sec) |
|-----------|------------------------|---------------------|-----------------|
| P | 1.00 | 13.31 | 14.00 |
| P + I | 6.43 | 13.38 | 13.43 |
| P + I + D | 2.39 | 11.44 | 11.50 |

Steady state error is defined by the difference between the set point value when it reaches the steady state and the output value of the control system. The error signifies the accuracy of the controller mode. For P mode, it has the smallest steady state error of 1.00%. The addition of integral term however has displayed the highest steady state error among all which is 6.43%.

Time taken for the system to attain an almost constant value between ± 5 percent from the desired value is called as settling time. The short settling time favoured is indicated by the PID mode which takes 11.44 seconds whereas the PI mode shown the longest settling time of 13.38%. Almost near to the time taken to settle down of PI controller is P controller, taking 13.31 seconds to reach $\pm 5\%$ within the steady state range.

Rise time specifies the time it takes starting from the step change in set point until the controlled variable first reaches the new reference point. The response time of PID controller is the fastest with 11.50 seconds. The time taken to rise for P mode is the highest, with 14 seconds than PI mode of 13.43 seconds.

The P only controller despite able to control the system near to the set point value of pH 7, it has high settling time and rise time. The long duration time is unfavoured because implementation in the actual plant processes would lag the running operations. PI mode strays quite a lot from the control objectives of system implementation. It records the highest error, both long rise time and settling time.

As discussed before, the proportional term reduces the rise time of the system. This can be proven according to the table of parameters for Ziegler-Nichols closed loop tuning in Table 7. From Figure 23, the higher gain employed in PID has revealed the shortest rise time compared to P and PI of the smallest $K_c = 24.3$

Generally, the addition of integral term to the conventional controller lessens the steady state offset in most systems. However, this is not the case for this study because nonlinearity occurs in the measurement part with squared term. Nonetheless, the PID controller mode is more suitable to fit the control objectives of the design. Although it has a steady state error slightly higher than P mode only, the error is tolerable since practically pH level beyond 7 is better than less than 7. pH level above 7 but bounded to pH less than 9 indicates alkaline environment which is sufficient for microorganism activities. Also it is chosen because the derivative action dominates the swiftness of the response time. By adding derivative parameter, PID control mode has indicated both the shortest rise time and settling time which is desired in pH neutralization process.

Chapter 5

CONCLUSIONS & RECOMMENDATIONS

5.1 Conclusions

This research addressed the problems related to the field of control system for nonlinear process. A better performance of control algorithm is proposed to benefit the industries and research fields in designing the pH neutralization system which is able to protect and save the environment. The underlying steps involved to obtain the control strategy started with the basic understanding of pH neutralization in terms of its chemical reactions. Outcomes of the chemical equations yielded to the findings on mathematical model representing the process of neutralization.

The mathematical model designed is referred to the semi batch reactor to linearize the dynamical system of the neutralization process. Since there is only one feed regulated into the mixing tank, and only one output which is pH value is considered, the system is much simplified for the single-input and single-output (SISO) system. A feedback loop is added to the SISO system to control the pH value of the process by corrective measures. The simpler version of the system is purposely planned in this project also for understanding reasons before the technique is invented into more complicated systems behaviour.

The familiarization to modelling is exposed to simulate the pH characteristics and compared with the actual pilot plant. The model is necessary to be evaluated before any forms of controllers are applied to the system. Some of the factors which are likely to be contributed from this research are providing a mathematical model which is sufficient to allow development of conventional and advanced control strategies. Besides that, the PID control algorithm is selected with reference to the control objective of the designed system. It is also predicted that the control algorithm can be further implemented on the pH neutralization process pilot plant.

5.2 Future Directions

For the pH pilot plant, new conductivity meters and pH meters are required to measure the pH state of the wastewater before being neutralized and the products produced in the mixing tank after neutralization. Pumps need to be sent for maintenance and services because by times the formation of acids and bases reactions give out by-products such as lump of salts which might get stuck along the pipeline.

Particularly for simulation, pH mathematical model would be the first step in designing control algorithm. Referring to the pH model simulated, the results have shown appropriate behaviour of the neutralization process in a semi batch system by only controlling the flow of base into the mixing tank. Perhaps, multiple-input and single-output (MISO) systems can be designed to expand the wide control options available.

In the next stage, PID controller should be tuned properly to get better and more accurate results. Yet, even with a Ziegler-Nichols closed-loop tuning method the PID control modes has not shown the necessary effects for the applications in a nonlinear system. PID is preferred for its simplicity but the results might be unsatisfactory when applied to complex processes.

5.2.1 Partial Development of Feedback Linearization

Feedback linearization formula is applied to find the transformation which yields to the relative degree of the nonlinear system explained in reference [24].

Table 9 : Unknowns Involved in Differentiation

| | | | |
|----------|----------------------------|---------|-----------------------|
| α | Flow rate of acid | β | Water Constant, K_w |
| h | Concentration of H^+ ion | w | Concentration of acid |
| v | Volume | z | Concentration of base |

Equations 41 to 49 are defined using unknowns in Table 9 to ease the lengthy calculation procedures.

$$\frac{dv}{dt} = \alpha + u \quad \text{Equation 41}$$

$$\frac{dw}{dt} = \frac{\alpha w}{v} \quad \text{Equation 42}$$

$$\frac{dz}{dt} = \frac{uz}{v} \quad \text{Equation 43}$$

$$\frac{dh}{dt} = \frac{h^2}{(2h^2 + \beta)v} (\alpha w - uz) \quad \text{Equation 44}$$

The system is a fourth-order system having the initial values of $v(o), w(o), z(o)$ and $h(o)$.

$$y = \log(h) = h(x) \quad \text{Equation 45}$$

It can be represented in the form of

$$\begin{bmatrix} \frac{dv}{dt} \\ \frac{dw}{dt} \\ \frac{dz}{dt} \\ \frac{dh}{dt} \end{bmatrix} = \begin{bmatrix} \alpha \\ \alpha \left(\frac{w}{v}\right) \\ 0 \\ \frac{h^2}{(2h^2 + \beta)v} \alpha w \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ z/v \\ \frac{-h^2 z}{(2h^2 + \beta)v} \end{bmatrix} u$$

$$\text{Equation 46}$$

Lie derivatives together with the knowledge of quotient rule and product rule are applied to differentiate the above matrices with respect to v, w, z and h .

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -z & 0 & 1 & 0 \\ \frac{1}{v^2} & 0 & \frac{1}{v} & 0 \\ -h^2 z & 0 & -h^2 & \frac{2h^2 v z (\beta)}{v(4h^4 v + \beta^2 v + 4h^2 + 2\beta)} \end{bmatrix} \begin{bmatrix} \alpha \\ \alpha \left(\frac{w}{v}\right) \\ 0 \\ \frac{h^2}{(2h^2 + \beta)v} \alpha w \end{bmatrix}$$

$$\text{Equation 47}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ \propto \left(\frac{w}{v^2}\right) & \frac{\propto}{v} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{-h^2 \propto ew}{(2h^2 + \beta)v^2} & \frac{h^2}{(2h^2 + \beta)v} & \propto 0 & \frac{2hv(h + \beta \propto w - 2h^2 \propto w)}{v(4h^4v + \beta^2v + 4h^2 + 2\beta)} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ z/v \\ \frac{-h^2z}{(2h^2 + \beta)v} \end{bmatrix}$$

Equation 48

Equation 47 subtracts Equation 48 is equal to Equation 49.

$$= \begin{bmatrix} 0 \\ -\propto w \\ \frac{v^2}{v^2} \\ \frac{-z \propto}{v^2} \\ \frac{(2h^2 + \beta)v^2 \{[(4vh^4 + \beta^2v + 4h^2 + 2\beta)(h^2 \propto w - h^2z \propto)] - [(2h^3vz)(h + 2h^2 \propto w)]\}}{v^4(2h^2 + \beta)^2(4h^4v + \beta^2v + 4h^2 + 2\beta)} \end{bmatrix}$$

Equation 49

Unknowns are then substituted with the corresponding values mentioned in Table 6.

$$\Rightarrow \begin{bmatrix} 0 \\ -2.0837 \times 10^{-9} \\ -2.08375 \times 10^{-8} \\ -1.87538 \times 10^{-4} \end{bmatrix}$$

Equation 50

Further study using advanced control strategy which is feedback linearization should be developed in improving the controllability of pH neutralization. Succeeding to the control method explained, the controller should be tested and implemented in MATLAB Simulink's environment as well as on the pH pilot plant to figure out the responses in actual environment.

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APPENDIX A

KEY MILESTONE

| No. | Activities | FYP1 (Week) | | | | | | | | | | | | | | FYP2 (Week) | | | | | | | | | | | | | |
|-----|---|-------------|---|---|---|---|---|---|---|---|----|----|----|----|----|-------------|---|---|---|---|---|---|---|---|----|----|----|----|----|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 1. | Study on pH neutralization process, control algorithms, modelling and simulation of pH plant. | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 2. | Develop suitable model for pH neutralization system | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 3. | Design and develop control algorithm , testing procedures | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 4. | Analysis of results and data | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 5. | Research completion | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

APPENDIX B GANTT CHART

| No. | Tasks | Week No/ Date | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|-----|------------------------------------|---------------|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | |
| 1 | Title Selection | █ | █ | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 2 | Submission of Extended Proposal | | | | | | █ | | | | | | | | | | | | | | | | | | | | | | | |
| 3 | Proposal Defence | | | | | | | | | █ | | | | | | | | | | | | | | | | | | | | |
| 4 | Develop pH Model | | | | | | | | | | | | █ | █ | █ | █ | █ | █ | | | | | | | | | | | | |
| 5 | Submission: Draft Report FYP1 | | | | | | | | | | | | | █ | | | | | | | | | | | | | | | | |
| 6 | Submission: Interim Report | | | | | | | | | | | | | | █ | | | | | | | | | | | | | | | |
| 7 | Develop Control Algorithm | | | | | | | | | | | | | | | | | | | | █ | █ | █ | █ | █ | | | | | |
| 8 | Tuning of Control Algorithm | | | | | | | | | | | | | | | | | | | | | | | | | █ | █ | █ | | |
| 9 | Results & Analysis | | | | | | | | | | | | | | | | | | | | | | | | | | | █ | █ | |
| 10 | Pre-Edx Poster Presentation | | | | | | | | | | | | | | | | | | | | | | | | | | | █ | | |
| 10 | Submission: Draft and Final Report | | | | | | | | | | | | | | | | | | | | | | | | | | | | █ | |
| 11 | Viva | | | | | | | | | | | | | | | | | | | | | | | | | | | | | █ |

APPENDIX C

RUNGE-KUTTA METHOD (ITERATIONS)

```
>> format long
>> dydx=inline('(y^2)*(0.01667*0.00005-0.025*0.00005)/((2*y^2+1e-
14)*((0.01667+0.025)*x+20))','x','y')
[x,y]=rk4ode(dydx,[0 500],0.0001,0.5);
disp([x,y])
```

dydx =

Inline function:

```
dydx(x,y) = (y^2)*(0.01667*0.00005-0.025*0.00005)/((2*y^2+1e-
14)*((0.01667+0.025)*x+20))
```

1.0e+002 *

| | |
|--------------------|--------------------|
| 0 | 0.0000010000000000 |
| 0.0050000000000000 | 0.000000998542447 |
| 0.0100000000000000 | 0.000000997086411 |
| 0.0150000000000000 | 0.000000995631887 |
| 0.0200000000000000 | 0.000000994178873 |
| 0.0250000000000000 | 0.000000992727365 |
| 0.0300000000000000 | 0.000000991277362 |
| 0.0350000000000000 | 0.000000989828858 |
| 0.0400000000000000 | 0.000000988381852 |
| 0.0450000000000000 | 0.000000986936340 |
| 0.0500000000000000 | 0.000000985492319 |
| 0.0550000000000000 | 0.000000984049786 |
| 0.0600000000000000 | 0.000000982608738 |
| 0.0650000000000000 | 0.000000981169172 |
| 0.0700000000000000 | 0.000000979731085 |
| 0.0750000000000000 | 0.000000978294473 |
| 0.0800000000000000 | 0.000000976859335 |
| 0.0850000000000000 | 0.000000975425666 |
| 0.0900000000000000 | 0.000000973993464 |
| 0.0950000000000000 | 0.000000972562726 |
| 0.1000000000000000 | 0.000000971133449 |
| 0.1050000000000000 | 0.000000969705629 |
| 0.1100000000000000 | 0.000000968279265 |
| 0.1150000000000000 | 0.000000966854352 |
| 0.1200000000000000 | 0.000000965430888 |
| 0.1250000000000000 | 0.000000964008870 |
| 0.1300000000000000 | 0.000000962588295 |
| 0.1350000000000000 | 0.000000961169161 |

APPENDIX C (CONT'D)

| | |
|--------------------|-------------------|
| 0.1400000000000000 | 0.000000959751463 |
| 0.1450000000000000 | 0.000000958335200 |
| 0.1500000000000000 | 0.000000956920368 |
| 0.1550000000000000 | 0.000000955506965 |
| 0.1600000000000000 | 0.000000954094987 |
| 0.1650000000000000 | 0.000000952684433 |
| 0.1700000000000000 | 0.000000951275298 |
| 0.1750000000000000 | 0.000000949867580 |
| 0.1800000000000000 | 0.000000948461276 |
| 0.1850000000000000 | 0.000000947056384 |
| 0.1900000000000000 | 0.000000945652900 |
| 0.1950000000000000 | 0.000000944250822 |
| 0.2000000000000000 | 0.000000942850146 |
| 0.2050000000000000 | 0.000000941450871 |
| 0.2100000000000000 | 0.000000940052993 |
| 0.2150000000000000 | 0.000000938656510 |
| 0.2200000000000000 | 0.000000937261418 |
| 0.2250000000000000 | 0.000000935867716 |
| 0.2300000000000000 | 0.000000934475399 |
| 0.2350000000000000 | 0.000000933084466 |
| 0.2400000000000000 | 0.000000931694914 |
| 0.2450000000000000 | 0.000000930306739 |
| 0.2500000000000000 | 0.000000928919940 |
| 0.2550000000000000 | 0.000000927534513 |
| 0.2600000000000000 | 0.000000926150456 |
| 0.2650000000000000 | 0.000000924767767 |
| 0.2700000000000000 | 0.000000923386441 |
| 0.2750000000000000 | 0.000000922006477 |
| 0.2800000000000000 | 0.000000920627873 |
| 0.2850000000000000 | 0.000000919250624 |
| 0.2900000000000000 | 0.000000917874729 |
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| 0.3000000000000000 | 0.000000915126990 |
| 0.3050000000000000 | 0.000000913755140 |
| 0.3100000000000000 | 0.000000912384634 |
| 0.3150000000000000 | 0.000000911015468 |
| 0.3200000000000000 | 0.000000909647639 |
| 0.3250000000000000 | 0.000000908281146 |
| 0.3300000000000000 | 0.000000906915986 |
| 0.3350000000000000 | 0.000000905552155 |
| 0.3400000000000000 | 0.000000904189652 |
| 0.3450000000000000 | 0.000000902828474 |
| 0.3500000000000000 | 0.000000901468618 |
| 0.3550000000000000 | 0.000000900110082 |
| 0.3600000000000000 | 0.000000898752862 |
| 0.3650000000000000 | 0.000000897396958 |
| 0.3700000000000000 | 0.000000896042366 |
| 0.3750000000000000 | 0.000000894689083 |
| 0.3800000000000000 | 0.000000893337107 |
| 0.3850000000000000 | 0.000000891986435 |
| 0.3900000000000000 | 0.000000890637066 |
| 0.3950000000000000 | 0.000000889288996 |
| 0.4000000000000000 | 0.000000887942223 |
| 0.4050000000000000 | 0.000000886596744 |
| 0.4100000000000000 | 0.000000885252557 |
| 0.4150000000000000 | 0.000000883909660 |
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| 0.4250000000000000 | 0.000000881227724 |

APPENDIX C (CONT'D)

| | |
|--------------------|-------------------|
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| 0.4650000000000000 | 0.000000870551069 |
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| 0.4750000000000000 | 0.000000867894581 |
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| 0.4850000000000000 | 0.000000865243124 |
| 0.4900000000000000 | 0.000000863919276 |
| 0.4950000000000000 | 0.000000862596679 |
| 0.5000000000000000 | 0.000000861275331 |
| 0.5050000000000000 | 0.000000859955228 |
| 0.5100000000000000 | 0.000000858636370 |
| 0.5150000000000000 | 0.000000857318753 |
| 0.5200000000000000 | 0.000000856002374 |
| 0.5250000000000000 | 0.000000854687233 |
| 0.5300000000000000 | 0.000000853373326 |
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| 0.5600000000000000 | 0.000000845515676 |
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| 0.6650000000000000 | 0.000000818356124 |
| 0.6700000000000000 | 0.000000817075863 |
| 0.6750000000000000 | 0.000000815796772 |
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| 0.6850000000000000 | 0.000000813242090 |
| 0.6900000000000000 | 0.000000811966496 |
| 0.6950000000000000 | 0.000000810692063 |
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| 0.7100000000000000 | 0.000000806875710 |
| 0.7150000000000000 | 0.000000805605900 |

APPENDIX C (CONT'D)

| | |
|--------------------|-------------------|
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APPENDIX C (CONT'D)

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| 1.2200000000000000 | 0.000000682947551 |
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| 1.2750000000000000 | 0.000000670215335 |
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| 1.2850000000000000 | 0.000000667912777 |
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| 1.2950000000000000 | 0.000000665614000 |

APPENDIX C (CONT'D)

| | |
|--------------------|-------------------|
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| 1.3150000000000000 | 0.000000661027741 |
| 1.3200000000000000 | 0.000000659883520 |
| 1.3250000000000000 | 0.000000658740233 |
| 1.3300000000000000 | 0.000000657597880 |
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| 1.3400000000000000 | 0.000000655315966 |
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| 1.3600000000000000 | 0.000000650763266 |
| 1.3650000000000000 | 0.000000649627401 |
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| 1.3750000000000000 | 0.000000647358432 |
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| 1.3850000000000000 | 0.000000645093135 |
| 1.3900000000000000 | 0.000000643961860 |
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| 1.4000000000000000 | 0.000000641702048 |
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| 1.4200000000000000 | 0.000000637193339 |
| 1.4250000000000000 | 0.000000636068428 |
| 1.4300000000000000 | 0.000000634944419 |
| 1.4350000000000000 | 0.000000633821312 |
| 1.4400000000000000 | 0.000000632699106 |
| 1.4450000000000000 | 0.000000631577798 |
| 1.4500000000000000 | 0.000000630457388 |
| 1.4550000000000000 | 0.000000629337874 |
| 1.4600000000000000 | 0.000000628219255 |
| 1.4650000000000000 | 0.000000627101529 |
| 1.4700000000000000 | 0.000000625984694 |
| 1.4750000000000000 | 0.000000624868750 |
| 1.4800000000000000 | 0.000000623753695 |
| 1.4850000000000000 | 0.000000622639527 |
| 1.4900000000000000 | 0.000000621526245 |
| 1.4950000000000000 | 0.000000620413848 |
| 1.5000000000000000 | 0.000000619302334 |
| 1.5050000000000000 | 0.000000618191702 |
| 1.5100000000000000 | 0.000000617081951 |
| 1.5150000000000000 | 0.000000615973078 |
| 1.5200000000000000 | 0.000000614865084 |
| 1.5250000000000000 | 0.000000613757965 |
| 1.5300000000000000 | 0.000000612651722 |
| 1.5350000000000000 | 0.000000611546352 |
| 1.5400000000000000 | 0.000000610441854 |
| 1.5450000000000000 | 0.000000609338227 |
| 1.5500000000000000 | 0.000000608235469 |
| 1.5550000000000000 | 0.000000607133579 |
| 1.5600000000000000 | 0.000000606032556 |
| 1.5650000000000000 | 0.000000604932399 |
| 1.5700000000000000 | 0.000000603833105 |
| 1.5750000000000000 | 0.000000602734674 |
| 1.5800000000000000 | 0.000000601637104 |
| 1.5850000000000000 | 0.000000600540394 |

APPENDIX C (CONT'D)

| | |
|--------------------|-------------------|
| 1.5900000000000000 | 0.000000599444542 |
| 1.5950000000000000 | 0.000000598349548 |
| 1.6000000000000000 | 0.000000597255410 |
| 1.6050000000000000 | 0.000000596162126 |
| 1.6100000000000000 | 0.000000595069695 |
| 1.6150000000000000 | 0.000000593978116 |
| 1.6200000000000000 | 0.000000592887388 |
| 1.6250000000000000 | 0.000000591797509 |
| 1.6300000000000000 | 0.000000590708477 |
| 1.6350000000000000 | 0.000000589620293 |
| 1.6400000000000000 | 0.000000588532953 |
| 1.6450000000000000 | 0.000000587446458 |
| 1.6500000000000000 | 0.000000586360805 |
| 1.6550000000000000 | 0.000000585275993 |
| 1.6600000000000000 | 0.000000584192022 |
| 1.6650000000000000 | 0.000000583108889 |
| 1.6700000000000000 | 0.000000582026593 |
| 1.6750000000000000 | 0.000000580945134 |
| 1.6800000000000000 | 0.000000579864509 |
| 1.6850000000000000 | 0.000000578784719 |
| 1.6900000000000000 | 0.000000577705760 |
| 1.6950000000000000 | 0.000000576627632 |
| 1.7000000000000000 | 0.000000575550334 |
| 1.7050000000000000 | 0.000000574473865 |
| 1.7100000000000000 | 0.000000573398222 |
| 1.7150000000000000 | 0.000000572323406 |
| 1.7200000000000000 | 0.000000571249414 |
| 1.7250000000000000 | 0.000000570176245 |
| 1.7300000000000000 | 0.000000569103899 |
| 1.7350000000000000 | 0.000000568032373 |
| 1.7400000000000000 | 0.000000566961667 |
| 1.7450000000000000 | 0.000000565891780 |
| 1.7500000000000000 | 0.000000564822709 |
| 1.7550000000000000 | 0.000000563754454 |
| 1.7600000000000000 | 0.000000562687014 |
| 1.7650000000000000 | 0.000000561620387 |
| 1.7700000000000000 | 0.000000560554572 |
| 1.7750000000000000 | 0.000000559489569 |
| 1.7800000000000000 | 0.000000558425374 |
| 1.7850000000000000 | 0.000000557361988 |
| 1.7900000000000000 | 0.000000556299410 |
| 1.7950000000000000 | 0.000000555237637 |
| 1.8000000000000000 | 0.000000554176669 |
| 1.8050000000000000 | 0.000000553116504 |
| 1.8100000000000000 | 0.000000552057142 |
| 1.8150000000000000 | 0.000000550998581 |
| 1.8200000000000000 | 0.000000549940820 |
| 1.8250000000000000 | 0.000000548883857 |
| 1.8300000000000000 | 0.000000547827692 |
| 1.8350000000000000 | 0.000000546772323 |
| 1.8400000000000000 | 0.000000545717750 |
| 1.8450000000000000 | 0.000000544663970 |
| 1.8500000000000000 | 0.000000543610982 |
| 1.8550000000000000 | 0.000000542558787 |
| 1.8600000000000000 | 0.000000541507381 |
| 1.8650000000000000 | 0.000000540456765 |
| 1.8700000000000000 | 0.000000539406936 |
| 1.8750000000000000 | 0.000000538357894 |

APPENDIX C (CONT'D)

| | |
|--------------------|-------------------|
| 1.8800000000000000 | 0.000000537309638 |
| 1.8850000000000000 | 0.000000536262166 |
| 1.8900000000000000 | 0.000000535215478 |
| 1.8950000000000000 | 0.000000534169571 |
| 1.9000000000000000 | 0.000000533124446 |
| 1.9050000000000000 | 0.000000532080100 |
| 1.9100000000000000 | 0.000000531036532 |
| 1.9150000000000000 | 0.000000529993742 |
| 1.9200000000000000 | 0.000000528951728 |
| 1.9250000000000000 | 0.000000527910489 |
| 1.9300000000000000 | 0.000000526870025 |
| 1.9350000000000000 | 0.000000525830332 |
| 1.9400000000000000 | 0.000000524791412 |
| 1.9450000000000000 | 0.000000523753262 |
| 1.9500000000000000 | 0.000000522715881 |
| 1.9550000000000000 | 0.000000521679269 |
| 1.9600000000000000 | 0.000000520643424 |
| 1.9650000000000000 | 0.000000519608344 |
| 1.9700000000000000 | 0.000000518574029 |
| 1.9750000000000000 | 0.000000517540478 |
| 1.9800000000000000 | 0.000000516507690 |
| 1.9850000000000000 | 0.000000515475663 |
| 1.9900000000000000 | 0.000000514444396 |
| 1.9950000000000000 | 0.000000513413889 |
| 2.0000000000000000 | 0.000000512384139 |
| 2.0050000000000000 | 0.000000511355146 |
| 2.0100000000000000 | 0.000000510326910 |
| 2.0150000000000000 | 0.000000509299428 |
| 2.0200000000000000 | 0.000000508272699 |
| 2.0250000000000000 | 0.000000507246723 |
| 2.0300000000000000 | 0.000000506221499 |
| 2.0350000000000000 | 0.000000505197024 |
| 2.0400000000000000 | 0.000000504173299 |
| 2.0450000000000000 | 0.000000503150323 |
| 2.0500000000000000 | 0.000000502128093 |
| 2.0550000000000000 | 0.000000501106609 |
| 2.0600000000000000 | 0.000000500085870 |
| 2.0650000000000000 | 0.000000499065874 |
| 2.0700000000000000 | 0.000000498046622 |
| 2.0750000000000000 | 0.000000497028111 |
| 2.0800000000000000 | 0.000000496010340 |
| 2.0850000000000000 | 0.000000494993309 |
| 2.0900000000000000 | 0.000000493977016 |
| 2.0950000000000000 | 0.000000492961461 |
| 2.1000000000000000 | 0.000000491946642 |
| 2.1050000000000000 | 0.000000490932558 |
| 2.1100000000000000 | 0.000000489919208 |
| 2.1150000000000000 | 0.000000488906591 |
| 2.1200000000000000 | 0.000000487894706 |
| 2.1250000000000000 | 0.000000486883552 |
| 2.1300000000000000 | 0.000000485873127 |
| 2.1350000000000000 | 0.000000484863432 |
| 2.1400000000000000 | 0.000000483854464 |
| 2.1450000000000000 | 0.000000482846224 |
| 2.1500000000000000 | 0.000000481838708 |
| 2.1550000000000000 | 0.000000480831918 |
| 2.1600000000000000 | 0.000000479825851 |
| 2.1650000000000000 | 0.000000478820506 |

APPENDIX C (CONT'D)

| | |
|--------------------|-------------------|
| 2.1700000000000000 | 0.000000477815883 |
| 2.1750000000000000 | 0.000000476811981 |
| 2.1800000000000000 | 0.000000475808797 |
| 2.1850000000000000 | 0.000000474806333 |
| 2.1900000000000000 | 0.000000473804585 |
| 2.1950000000000000 | 0.000000472803554 |
| 2.2000000000000000 | 0.000000471803239 |
| 2.2050000000000000 | 0.000000470803637 |
| 2.2100000000000000 | 0.000000469804749 |
| 2.2150000000000000 | 0.000000468806574 |
| 2.2200000000000000 | 0.000000467809109 |
| 2.2250000000000000 | 0.000000466812355 |
| 2.2300000000000000 | 0.000000465816310 |
| 2.2350000000000000 | 0.000000464820973 |
| 2.2400000000000000 | 0.000000463826343 |
| 2.2450000000000000 | 0.000000462832420 |
| 2.2500000000000000 | 0.000000461839202 |
| 2.2550000000000000 | 0.000000460846688 |
| 2.2600000000000000 | 0.000000459854877 |
| 2.2650000000000000 | 0.000000458863768 |
| 2.2700000000000000 | 0.000000457873361 |
| 2.2750000000000000 | 0.000000456883654 |
| 2.2800000000000000 | 0.000000455894646 |
| 2.2850000000000000 | 0.000000454906336 |
| 2.2900000000000000 | 0.000000453918724 |
| 2.2950000000000000 | 0.000000452931808 |
| 2.3000000000000000 | 0.000000451945587 |
| 2.3050000000000000 | 0.000000450960061 |
| 2.3100000000000000 | 0.000000449975227 |
| 2.3150000000000000 | 0.000000448991087 |
| 2.3200000000000000 | 0.000000448007637 |
| 2.3250000000000000 | 0.000000447024879 |
| 2.3300000000000000 | 0.000000446042809 |
| 2.3350000000000000 | 0.000000445061428 |
| 2.3400000000000000 | 0.000000444080735 |
| 2.3450000000000000 | 0.000000443100728 |
| 2.3500000000000000 | 0.000000442121407 |
| 2.3550000000000000 | 0.000000441142770 |
| 2.3600000000000000 | 0.000000440164817 |
| 2.3650000000000000 | 0.000000439187547 |
| 2.3700000000000000 | 0.000000438210959 |
| 2.3750000000000000 | 0.000000437235051 |
| 2.3800000000000000 | 0.000000436259824 |
| 2.3850000000000000 | 0.000000435285275 |
| 2.3900000000000000 | 0.000000434311404 |
| 2.3950000000000000 | 0.000000433338210 |
| 2.4000000000000000 | 0.000000432365693 |
| 2.4050000000000000 | 0.000000431393850 |
| 2.4100000000000000 | 0.000000430422682 |
| 2.4150000000000000 | 0.000000429452188 |
| 2.4200000000000000 | 0.000000428482365 |
| 2.4250000000000000 | 0.000000427513214 |
| 2.4300000000000000 | 0.000000426544733 |
| 2.4350000000000000 | 0.000000425576923 |
| 2.4400000000000000 | 0.000000424609780 |
| 2.4450000000000000 | 0.000000423643306 |
| 2.4500000000000000 | 0.000000422677498 |
| 2.4550000000000000 | 0.000000421712356 |

APPENDIX C (CONT'D)

| | |
|--------------------|-------------------|
| 2.4600000000000000 | 0.000000420747879 |
| 2.4650000000000000 | 0.000000419784066 |
| 2.4700000000000000 | 0.000000418820917 |
| 2.4750000000000000 | 0.000000417858429 |
| 2.4800000000000000 | 0.000000416896603 |
| 2.4850000000000000 | 0.000000415935437 |
| 2.4900000000000000 | 0.000000414974931 |
| 2.4950000000000000 | 0.000000414015084 |
| 2.5000000000000000 | 0.000000413055894 |
| 2.5050000000000000 | 0.000000412097361 |
| 2.5100000000000000 | 0.000000411139484 |
| 2.5150000000000000 | 0.000000410182261 |
| 2.5200000000000000 | 0.000000409225693 |
| 2.5250000000000000 | 0.000000408269778 |
| 2.5300000000000000 | 0.000000407314516 |
| 2.5350000000000000 | 0.000000406359905 |
| 2.5400000000000000 | 0.000000405405944 |
| 2.5450000000000000 | 0.000000404452634 |
| 2.5500000000000000 | 0.000000403499971 |
| 2.5550000000000000 | 0.000000402547957 |
| 2.5600000000000000 | 0.000000401596590 |
| 2.5650000000000000 | 0.000000400645869 |
| 2.5700000000000000 | 0.000000399695793 |
| 2.5750000000000000 | 0.000000398746362 |
| 2.5800000000000000 | 0.000000397797574 |
| 2.5850000000000000 | 0.000000396849429 |
| 2.5900000000000000 | 0.000000395901925 |
| 2.5950000000000000 | 0.000000394955063 |
| 2.6000000000000000 | 0.000000394008840 |
| 2.6050000000000000 | 0.000000393063257 |
| 2.6100000000000000 | 0.000000392118312 |
| 2.6150000000000000 | 0.000000391174004 |
| 2.6200000000000000 | 0.000000390230333 |
| 2.6250000000000000 | 0.000000389287298 |
| 2.6300000000000000 | 0.000000388344897 |
| 2.6350000000000000 | 0.000000387403131 |
| 2.6400000000000000 | 0.000000386461998 |
| 2.6450000000000000 | 0.000000385521497 |
| 2.6500000000000000 | 0.000000384581628 |
| 2.6550000000000000 | 0.000000383642389 |
| 2.6600000000000000 | 0.000000382703780 |
| 2.6650000000000000 | 0.000000381765800 |
| 2.6700000000000000 | 0.000000380828448 |
| 2.6750000000000000 | 0.000000379891723 |
| 2.6800000000000000 | 0.000000378955624 |
| 2.6850000000000000 | 0.000000378020152 |
| 2.6900000000000000 | 0.000000377085304 |
| 2.6950000000000000 | 0.000000376151080 |
| 2.7000000000000000 | 0.000000375217478 |
| 2.7050000000000000 | 0.000000374284500 |
| 2.7100000000000000 | 0.000000373352142 |
| 2.7150000000000000 | 0.000000372420405 |
| 2.7200000000000000 | 0.000000371489288 |
| 2.7250000000000000 | 0.000000370558790 |
| 2.7300000000000000 | 0.000000369628910 |
| 2.7350000000000000 | 0.000000368699647 |
| 2.7400000000000000 | 0.000000367771001 |
| 2.7450000000000000 | 0.000000366842971 |

APPENDIX C (CONT'D)

| | |
|--------------------|-------------------|
| 2.7500000000000000 | 0.000000365915555 |
| 2.7550000000000000 | 0.000000364988753 |
| 2.7600000000000000 | 0.000000364062565 |
| 2.7650000000000000 | 0.000000363136989 |
| 2.7700000000000000 | 0.000000362212024 |
| 2.7750000000000000 | 0.000000361287670 |
| 2.7800000000000000 | 0.000000360363927 |
| 2.7850000000000000 | 0.000000359440792 |
| 2.7900000000000000 | 0.000000358518266 |
| 2.7950000000000000 | 0.000000357596347 |
| 2.8000000000000000 | 0.000000356675035 |
| 2.8050000000000000 | 0.000000355754330 |
| 2.8100000000000000 | 0.000000354834229 |
| 2.8150000000000000 | 0.000000353914733 |
| 2.8200000000000000 | 0.000000352995840 |
| 2.8250000000000000 | 0.000000352077550 |
| 2.8300000000000000 | 0.000000351159862 |
| 2.8350000000000000 | 0.000000350242775 |
| 2.8400000000000000 | 0.000000349326289 |
| 2.8450000000000000 | 0.000000348410403 |
| 2.8500000000000000 | 0.000000347495115 |
| 2.8550000000000000 | 0.000000346580425 |
| 2.8600000000000000 | 0.000000345666333 |
| 2.8650000000000000 | 0.000000344752837 |
| 2.8700000000000000 | 0.000000343839937 |
| 2.8750000000000000 | 0.000000342927632 |
| 2.8800000000000000 | 0.000000342015921 |
| 2.8850000000000000 | 0.000000341104804 |
| 2.8900000000000000 | 0.000000340194279 |
| 2.8950000000000000 | 0.000000339284346 |
| 2.9000000000000000 | 0.000000338375004 |
| 2.9050000000000000 | 0.000000337466252 |
| 2.9100000000000000 | 0.000000336558090 |
| 2.9150000000000000 | 0.000000335650517 |
| 2.9200000000000000 | 0.000000334743532 |
| 2.9250000000000000 | 0.000000333837134 |
| 2.9300000000000000 | 0.000000332931323 |
| 2.9350000000000000 | 0.000000332026098 |
| 2.9400000000000000 | 0.000000331121457 |
| 2.9450000000000000 | 0.000000330217401 |
| 2.9500000000000000 | 0.000000329313928 |
| 2.9550000000000000 | 0.000000328411038 |
| 2.9600000000000000 | 0.000000327508730 |
| 2.9650000000000000 | 0.000000326607003 |
| 2.9700000000000000 | 0.000000325705857 |
| 2.9750000000000000 | 0.000000324805291 |
| 2.9800000000000000 | 0.000000323905303 |
| 2.9850000000000000 | 0.000000323005894 |
| 2.9900000000000000 | 0.000000322107062 |
| 2.9950000000000000 | 0.000000321208807 |
| 3.0000000000000000 | 0.000000320311129 |
| 3.0050000000000000 | 0.000000319414025 |
| 3.0100000000000000 | 0.000000318517496 |
| 3.0150000000000000 | 0.000000317621541 |
| 3.0200000000000000 | 0.000000316726159 |
| 3.0250000000000000 | 0.000000315831349 |
| 3.0300000000000000 | 0.000000314937111 |
| 3.0350000000000000 | 0.000000314043444 |

APPENDIX C (CONT'D)

| | |
|--------------------|-------------------|
| 3.0400000000000000 | 0.000000313150347 |
| 3.0450000000000000 | 0.000000312257819 |
| 3.0500000000000000 | 0.000000311365861 |
| 3.0550000000000000 | 0.000000310474470 |
| 3.0600000000000000 | 0.000000309583646 |
| 3.0650000000000000 | 0.000000308693389 |
| 3.0700000000000000 | 0.000000307803698 |
| 3.0750000000000000 | 0.000000306914572 |
| 3.0800000000000000 | 0.000000306026010 |
| 3.0850000000000000 | 0.000000305138012 |
| 3.0900000000000000 | 0.000000304250578 |
| 3.0950000000000000 | 0.000000303363705 |
| 3.1000000000000000 | 0.000000302477394 |
| 3.1050000000000000 | 0.000000301591643 |
| 3.1100000000000000 | 0.000000300706453 |
| 3.1150000000000000 | 0.000000299821822 |
| 3.1200000000000000 | 0.000000298937750 |
| 3.1250000000000000 | 0.000000298054236 |
| 3.1300000000000000 | 0.000000297171279 |
| 3.1350000000000000 | 0.000000296288879 |
| 3.1400000000000000 | 0.000000295407035 |
| 3.1450000000000000 | 0.000000294525746 |
| 3.1500000000000000 | 0.000000293645011 |
| 3.1550000000000000 | 0.000000292764831 |
| 3.1600000000000000 | 0.000000291885203 |
| 3.1650000000000000 | 0.000000291006128 |
| 3.1700000000000000 | 0.000000290127604 |
| 3.1750000000000000 | 0.000000289249632 |
| 3.1800000000000000 | 0.000000288372209 |
| 3.1850000000000000 | 0.000000287495337 |
| 3.1900000000000000 | 0.000000286619013 |
| 3.1950000000000000 | 0.000000285743238 |
| 3.2000000000000000 | 0.000000284868010 |
| 3.2050000000000000 | 0.000000283993330 |
| 3.2100000000000000 | 0.000000283119195 |
| 3.2150000000000000 | 0.000000282245606 |
| 3.2200000000000000 | 0.000000281372562 |
| 3.2250000000000000 | 0.000000280500062 |
| 3.2300000000000000 | 0.000000279628106 |
| 3.2350000000000000 | 0.000000278756692 |
| 3.2400000000000000 | 0.000000277885820 |
| 3.2450000000000000 | 0.000000277015490 |
| 3.2500000000000000 | 0.000000276145701 |
| 3.2550000000000000 | 0.000000275276452 |
| 3.2600000000000000 | 0.000000274407743 |
| 3.2650000000000000 | 0.000000273539572 |
| 3.2700000000000000 | 0.000000272671939 |
| 3.2750000000000000 | 0.000000271804844 |
| 3.2800000000000000 | 0.000000270938286 |
| 3.2850000000000000 | 0.000000270072263 |
| 3.2900000000000000 | 0.000000269206777 |
| 3.2950000000000000 | 0.000000268341825 |
| 3.3000000000000000 | 0.000000267477407 |
| 3.3050000000000000 | 0.000000266613522 |
| 3.3100000000000000 | 0.000000265750171 |
| 3.3150000000000000 | 0.000000264887352 |
| 3.3200000000000000 | 0.000000264025064 |
| 3.3250000000000000 | 0.000000263163307 |

APPENDIX C (CONT'D)

| | |
|--------------------|-------------------|
| 3.3300000000000000 | 0.000000262302080 |
| 3.3350000000000000 | 0.000000261441383 |
| 3.3400000000000000 | 0.000000260581215 |
| 3.3450000000000000 | 0.000000259721575 |
| 3.3500000000000000 | 0.000000258862462 |
| 3.3550000000000000 | 0.000000258003877 |
| 3.3600000000000000 | 0.000000257145818 |
| 3.3650000000000000 | 0.000000256288284 |
| 3.3700000000000000 | 0.000000255431276 |
| 3.3750000000000000 | 0.000000254574792 |
| 3.3800000000000000 | 0.000000253718832 |
| 3.3850000000000000 | 0.000000252863395 |
| 3.3900000000000000 | 0.000000252008480 |
| 3.3950000000000000 | 0.000000251154087 |
| 3.4000000000000000 | 0.000000250300216 |
| 3.4050000000000000 | 0.000000249446865 |
| 3.4100000000000000 | 0.000000248594033 |
| 3.4150000000000000 | 0.000000247741722 |
| 3.4200000000000000 | 0.000000246889928 |
| 3.4250000000000000 | 0.000000246038653 |
| 3.4300000000000000 | 0.000000245187896 |
| 3.4350000000000000 | 0.000000244337655 |
| 3.4400000000000000 | 0.000000243487930 |
| 3.4450000000000000 | 0.000000242638721 |
| 3.4500000000000000 | 0.000000241790026 |
| 3.4550000000000000 | 0.000000240941846 |
| 3.4600000000000000 | 0.000000240094180 |
| 3.4650000000000000 | 0.000000239247027 |
| 3.4700000000000000 | 0.000000238400386 |
| 3.4750000000000000 | 0.000000237554257 |
| 3.4800000000000000 | 0.000000236708639 |
| 3.4850000000000000 | 0.000000235863531 |
| 3.4900000000000000 | 0.000000235018934 |
| 3.4950000000000000 | 0.000000234174846 |
| 3.5000000000000000 | 0.000000233331266 |
| 3.5050000000000000 | 0.000000232488195 |
| 3.5100000000000000 | 0.000000231645631 |
| 3.5150000000000000 | 0.000000230803574 |
| 3.5200000000000000 | 0.000000229962024 |
| 3.5250000000000000 | 0.000000229120979 |
| 3.5300000000000000 | 0.000000228280439 |
| 3.5350000000000000 | 0.000000227440403 |
| 3.5400000000000000 | 0.000000226600872 |
| 3.5450000000000000 | 0.000000225761843 |
| 3.5500000000000000 | 0.000000224923318 |
| 3.5550000000000000 | 0.000000224085294 |
| 3.5600000000000000 | 0.000000223247772 |
| 3.5650000000000000 | 0.000000222410750 |
| 3.5700000000000000 | 0.000000221574229 |
| 3.5750000000000000 | 0.000000220738208 |
| 3.5800000000000000 | 0.000000219902685 |
| 3.5850000000000000 | 0.000000219067661 |
| 3.5900000000000000 | 0.000000218233135 |
| 3.5950000000000000 | 0.000000217399107 |
| 3.6000000000000000 | 0.000000216565574 |
| 3.6050000000000000 | 0.000000215732538 |
| 3.6100000000000000 | 0.000000214899998 |
| 3.6150000000000000 | 0.000000214067952 |

APPENDIX C (CONT'D)

| | |
|--------------------|-------------------|
| 3.6200000000000000 | 0.000000213236401 |
| 3.6250000000000000 | 0.000000212405343 |
| 3.6300000000000000 | 0.000000211574779 |
| 3.6350000000000000 | 0.000000210744707 |
| 3.6400000000000000 | 0.000000209915127 |
| 3.6450000000000000 | 0.000000209086039 |
| 3.6500000000000000 | 0.000000208257441 |
| 3.6550000000000000 | 0.000000207429334 |
| 3.6600000000000000 | 0.000000206601716 |
| 3.6650000000000000 | 0.000000205774588 |
| 3.6700000000000000 | 0.000000204947948 |
| 3.6750000000000000 | 0.000000204121796 |
| 3.6800000000000000 | 0.000000203296131 |
| 3.6850000000000000 | 0.000000202470953 |
| 3.6900000000000000 | 0.000000201646261 |
| 3.6950000000000000 | 0.000000200822055 |
| 3.7000000000000000 | 0.000000199998334 |
| 3.7050000000000000 | 0.000000199175098 |
| 3.7100000000000000 | 0.000000198352345 |
| 3.7150000000000000 | 0.000000197530076 |
| 3.7200000000000000 | 0.000000196708289 |
| 3.7250000000000000 | 0.000000195886985 |
| 3.7300000000000000 | 0.000000195066163 |
| 3.7350000000000000 | 0.000000194245821 |
| 3.7400000000000000 | 0.000000193425960 |
| 3.7450000000000000 | 0.000000192606580 |
| 3.7500000000000000 | 0.000000191787678 |
| 3.7550000000000000 | 0.000000190969256 |
| 3.7600000000000000 | 0.000000190151311 |
| 3.7650000000000000 | 0.000000189333845 |
| 3.7700000000000000 | 0.000000188516856 |
| 3.7750000000000000 | 0.000000187700343 |
| 3.7800000000000000 | 0.000000186884306 |
| 3.7850000000000000 | 0.000000186068745 |
| 3.7900000000000000 | 0.000000185253659 |
| 3.7950000000000000 | 0.000000184439048 |
| 3.8000000000000000 | 0.000000183624910 |
| 3.8050000000000000 | 0.000000182811245 |
| 3.8100000000000000 | 0.000000181998054 |
| 3.8150000000000000 | 0.000000181185334 |
| 3.8200000000000000 | 0.000000180373086 |
| 3.8250000000000000 | 0.000000179561310 |
| 3.8300000000000000 | 0.000000178750004 |
| 3.8350000000000000 | 0.000000177939168 |
| 3.8400000000000000 | 0.000000177128801 |
| 3.8450000000000000 | 0.000000176318904 |
| 3.8500000000000000 | 0.000000175509475 |
| 3.8550000000000000 | 0.000000174700514 |
| 3.8600000000000000 | 0.000000173892020 |
| 3.8650000000000000 | 0.000000173083993 |
| 3.8700000000000000 | 0.000000172276432 |
| 3.8750000000000000 | 0.000000171469337 |
| 3.8800000000000000 | 0.000000170662707 |
| 3.8850000000000000 | 0.000000169856542 |
| 3.8900000000000000 | 0.000000169050841 |
| 3.8950000000000000 | 0.000000168245603 |
| 3.9000000000000000 | 0.000000167440829 |
| 3.9050000000000000 | 0.000000166636517 |

APPENDIX C (CONT'D)

| | |
|--------------------|-------------------|
| 3.9100000000000000 | 0.000000165832667 |
| 3.9150000000000000 | 0.000000165029279 |
| 3.9200000000000000 | 0.000000164226352 |
| 3.9250000000000000 | 0.000000163423885 |
| 3.9300000000000000 | 0.000000162621878 |
| 3.9350000000000000 | 0.000000161820330 |
| 3.9400000000000000 | 0.000000161019241 |
| 3.9450000000000000 | 0.000000160218611 |
| 3.9500000000000000 | 0.000000159418438 |
| 3.9550000000000000 | 0.000000158618723 |
| 3.9600000000000000 | 0.000000157819464 |
| 3.9650000000000000 | 0.000000157020662 |
| 3.9700000000000000 | 0.000000156222315 |
| 3.9750000000000000 | 0.000000155424424 |
| 3.9800000000000000 | 0.000000154626987 |
| 3.9850000000000000 | 0.000000153830004 |
| 3.9900000000000000 | 0.000000153033476 |
| 3.9950000000000000 | 0.000000152237400 |
| 4.0000000000000000 | 0.000000151441777 |
| 4.0050000000000000 | 0.000000150646606 |
| 4.0100000000000000 | 0.000000149851887 |
| 4.0150000000000000 | 0.000000149057618 |
| 4.0200000000000000 | 0.000000148263801 |
| 4.0250000000000000 | 0.000000147470433 |
| 4.0300000000000000 | 0.000000146677515 |
| 4.0350000000000000 | 0.000000145885046 |
| 4.0400000000000000 | 0.000000145093026 |
| 4.0450000000000000 | 0.000000144301454 |
| 4.0500000000000000 | 0.000000143510329 |
| 4.0550000000000000 | 0.000000142719652 |
| 4.0600000000000000 | 0.000000141929421 |
| 4.0650000000000000 | 0.000000141139636 |
| 4.0700000000000000 | 0.000000140350297 |
| 4.0750000000000000 | 0.000000139561402 |
| 4.0800000000000000 | 0.000000138772953 |
| 4.0850000000000000 | 0.000000137984947 |
| 4.0900000000000000 | 0.000000137197385 |
| 4.0950000000000000 | 0.000000136410266 |
| 4.1000000000000000 | 0.000000135623590 |
| 4.1050000000000000 | 0.000000134837355 |
| 4.1100000000000000 | 0.000000134051563 |
| 4.1150000000000000 | 0.000000133266211 |
| 4.1200000000000000 | 0.000000132481300 |
| 4.1250000000000000 | 0.000000131696829 |
| 4.1300000000000000 | 0.000000130912798 |
| 4.1350000000000000 | 0.000000130129206 |
| 4.1400000000000000 | 0.000000129346053 |
| 4.1450000000000000 | 0.000000128563338 |
| 4.1500000000000000 | 0.000000127781060 |
| 4.1550000000000000 | 0.000000126999220 |
| 4.1600000000000000 | 0.000000126217816 |
| 4.1650000000000000 | 0.000000125436849 |
| 4.1700000000000000 | 0.000000124656318 |
| 4.1750000000000000 | 0.000000123876221 |
| 4.1800000000000000 | 0.000000123096560 |
| 4.1850000000000000 | 0.000000122317333 |
| 4.1900000000000000 | 0.000000121538540 |
| 4.1950000000000000 | 0.000000120760180 |

APPENDIX C (CONT'D)

| | |
|--------------------|-------------------|
| 4.2000000000000000 | 0.000000119982253 |
| 4.2050000000000000 | 0.000000119204759 |
| 4.2100000000000000 | 0.000000118427696 |
| 4.2150000000000000 | 0.000000117651065 |
| 4.2200000000000000 | 0.000000116874865 |
| 4.2250000000000000 | 0.000000116099095 |
| 4.2300000000000000 | 0.000000115323756 |
| 4.2350000000000000 | 0.000000114548846 |
| 4.2400000000000000 | 0.000000113774366 |
| 4.2450000000000000 | 0.000000113000314 |
| 4.2500000000000000 | 0.000000112226690 |
| 4.2550000000000000 | 0.000000111453494 |
| 4.2600000000000000 | 0.000000110680725 |
| 4.2650000000000000 | 0.000000109908383 |
| 4.2700000000000000 | 0.000000109136467 |
| 4.2750000000000000 | 0.000000108364978 |
| 4.2800000000000000 | 0.000000107593913 |
| 4.2850000000000000 | 0.000000106823274 |
| 4.2900000000000000 | 0.000000106053059 |
| 4.2950000000000000 | 0.000000105283268 |
| 4.3000000000000000 | 0.000000104513901 |
| 4.3050000000000000 | 0.000000103744957 |
| 4.3100000000000000 | 0.000000102976436 |
| 4.3150000000000000 | 0.000000102208337 |
| 4.3200000000000000 | 0.000000101440660 |
| 4.3250000000000000 | 0.000000100673404 |
| 4.3300000000000000 | 0.000000099906569 |
| 4.3350000000000000 | 0.000000099140154 |
| 4.3400000000000000 | 0.000000098374160 |
| 4.3450000000000000 | 0.000000097608585 |
| 4.3500000000000000 | 0.000000096843429 |
| 4.3550000000000000 | 0.000000096078691 |
| 4.3600000000000000 | 0.000000095314372 |
| 4.3650000000000000 | 0.000000094550471 |
| 4.3700000000000000 | 0.000000093786987 |
| 4.3750000000000000 | 0.000000093023920 |
| 4.3800000000000000 | 0.000000092261270 |
| 4.3850000000000000 | 0.000000091499035 |
| 4.3900000000000000 | 0.000000090737216 |
| 4.3950000000000000 | 0.000000089975812 |
| 4.4000000000000000 | 0.000000089214823 |
| 4.4050000000000000 | 0.000000088454249 |
| 4.4100000000000000 | 0.000000087694088 |
| 4.4150000000000000 | 0.000000086934340 |
| 4.4200000000000000 | 0.000000086175006 |
| 4.4250000000000000 | 0.000000085416084 |
| 4.4300000000000000 | 0.000000084657575 |
| 4.4350000000000000 | 0.000000083899477 |
| 4.4400000000000000 | 0.000000083141790 |
| 4.4450000000000000 | 0.000000082384515 |
| 4.4500000000000000 | 0.000000081627649 |
| 4.4550000000000000 | 0.000000080871194 |
| 4.4600000000000000 | 0.000000080115149 |
| 4.4650000000000000 | 0.000000079359513 |
| 4.4700000000000000 | 0.000000078604285 |
| 4.4750000000000000 | 0.000000077849467 |
| 4.4800000000000000 | 0.000000077095056 |
| 4.4850000000000000 | 0.000000076341052 |

APPENDIX C (CONT'D)

| | |
|--------------------|-------------------|
| 4.4900000000000000 | 0.000000075587456 |
| 4.4950000000000000 | 0.000000074834267 |
| 4.5000000000000000 | 0.000000074081484 |
| 4.5050000000000000 | 0.000000073329108 |
| 4.5100000000000000 | 0.000000072577136 |
| 4.5150000000000000 | 0.000000071825571 |
| 4.5200000000000000 | 0.000000071074410 |
| 4.5250000000000000 | 0.000000070323653 |
| 4.5300000000000000 | 0.000000069573300 |
| 4.5350000000000000 | 0.000000068823352 |
| 4.5400000000000000 | 0.000000068073806 |
| 4.5450000000000000 | 0.000000067324664 |
| 4.5500000000000000 | 0.000000066575924 |
| 4.5550000000000000 | 0.000000065827586 |
| 4.5600000000000000 | 0.000000065079650 |
| 4.5650000000000000 | 0.000000064332115 |
| 4.5700000000000000 | 0.000000063584982 |
| 4.5750000000000000 | 0.000000062838249 |
| 4.5800000000000000 | 0.000000062091917 |
| 4.5850000000000000 | 0.000000061345985 |
| 4.5900000000000000 | 0.000000060600453 |
| 4.5950000000000000 | 0.000000059855320 |
| 4.6000000000000000 | 0.000000059110586 |
| 4.6050000000000000 | 0.000000058366251 |
| 4.6100000000000000 | 0.000000057622314 |
| 4.6150000000000000 | 0.000000056878775 |
| 4.6200000000000000 | 0.000000056135634 |
| 4.6250000000000000 | 0.000000055392891 |
| 4.6300000000000000 | 0.000000054650544 |
| 4.6350000000000000 | 0.000000053908595 |
| 4.6400000000000000 | 0.000000053167042 |
| 4.6450000000000000 | 0.000000052425886 |
| 4.6500000000000000 | 0.000000051685126 |
| 4.6550000000000000 | 0.000000050944761 |
| 4.6600000000000000 | 0.000000050204792 |
| 4.6650000000000000 | 0.000000049465219 |
| 4.6700000000000000 | 0.000000048726040 |
| 4.6750000000000000 | 0.000000047987256 |
| 4.6800000000000000 | 0.000000047248868 |
| 4.6850000000000000 | 0.000000046510873 |
| 4.6900000000000000 | 0.000000045773273 |
| 4.6950000000000000 | 0.000000045036067 |
| 4.7000000000000000 | 0.000000044299256 |
| 4.7050000000000000 | 0.000000043562838 |
| 4.7100000000000000 | 0.000000042826814 |
| 4.7150000000000000 | 0.000000042091184 |
| 4.7200000000000000 | 0.000000041355947 |
| 4.7250000000000000 | 0.000000040621105 |
| 4.7300000000000000 | 0.000000039886656 |
| 4.7350000000000000 | 0.000000039152600 |
| 4.7400000000000000 | 0.000000038418939 |
| 4.7450000000000000 | 0.000000037685671 |
| 4.7500000000000000 | 0.000000036952798 |
| 4.7550000000000000 | 0.000000036220319 |
| 4.7600000000000000 | 0.000000035488234 |
| 4.7650000000000000 | 0.000000034756544 |
| 4.7700000000000000 | 0.000000034025249 |
| 4.7750000000000000 | 0.000000033294350 |

APPENDIX C (CONT'D)

| | |
|--------------------|-------------------|
| 4.7800000000000000 | 0.000000032563847 |
| 4.7850000000000000 | 0.000000031833740 |
| 4.7900000000000000 | 0.000000031104031 |
| 4.7950000000000000 | 0.000000030374720 |
| 4.8000000000000000 | 0.000000029645808 |
| 4.8050000000000000 | 0.000000028917295 |
| 4.8100000000000000 | 0.000000028189184 |
| 4.8150000000000000 | 0.000000027461475 |
| 4.8200000000000000 | 0.000000026734170 |
| 4.8250000000000000 | 0.000000026007271 |
| 4.8300000000000000 | 0.000000025280779 |
| 4.8350000000000000 | 0.000000024554697 |
| 4.8400000000000000 | 0.000000023829027 |
| 4.8450000000000000 | 0.000000023103772 |
| 4.8500000000000000 | 0.000000022378936 |
| 4.8550000000000000 | 0.000000021654522 |
| 4.8600000000000000 | 0.000000020930535 |
| 4.8650000000000000 | 0.000000020206980 |
| 4.8700000000000000 | 0.000000019483862 |
| 4.8750000000000000 | 0.000000018761189 |
| 4.8800000000000000 | 0.000000018038968 |
| 4.8850000000000000 | 0.000000017317209 |
| 4.8900000000000000 | 0.000000016595923 |
| 4.8950000000000000 | 0.000000015875122 |
| 4.9000000000000000 | 0.000000015154823 |
| 4.9050000000000000 | 0.000000014435044 |
| 4.9100000000000000 | 0.000000013715808 |
| 4.9150000000000000 | 0.000000012997143 |
| 4.9200000000000000 | 0.000000012279082 |
| 4.9250000000000000 | 0.000000011561668 |
| 4.9300000000000000 | 0.000000010844957 |
| 4.9350000000000000 | 0.000000010129015 |
| 4.9400000000000000 | 0.000000009413934 |
| 4.9450000000000000 | 0.000000008699831 |
| 4.9500000000000000 | 0.000000007986868 |
| 4.9550000000000000 | 0.000000007275266 |
| 4.9600000000000000 | 0.000000006565340 |
| 4.9650000000000000 | 0.000000005857553 |
| 4.9700000000000000 | 0.000000005152608 |
| 4.9750000000000000 | 0.000000004451632 |
| 4.9800000000000000 | 0.000000003756526 |
| 4.9850000000000000 | 0.000000003070730 |
| 4.9900000000000000 | 0.000000002400996 |
| 4.9950000000000000 | 0.000000001761781 |
| 5.0000000000000000 | 0.000000001185436 |

APPENDIX D
ZIEGLER-NICHOLS CLOSED LOOP CORRELATIONS

| Control Modes | K_c | T_I | T_D |
|----------------------|-----------|-----------|---------|
| P only | $0.5K_u$ | - | - |
| P + I | $0.45K_u$ | $P_u/1.2$ | - |
| P + I + D | $0.6K_u$ | $P_u/2$ | $P_u/8$ |

