# CHAPTER 1 INTRODUCTION

### 1.1 Vibration

Vibration refers to mechanical oscillations about an equilibrium point. The oscillations may be periodic such as the motion of a pendulum or random such as the movement of a tire on a gravel road.

Vibration is occasionally "desirable". For example the motion of a tuning fork, the reed in a woodwind instrument or harmonica, or the cone of a loudspeaker is desirable vibration, necessary for the correct functioning of the various devices.

More often, vibration is undesirable, wasting energy and creating unwanted sound which is noise. For example, the vibration motions of engines, electric motors, or any mechanical device in operation are typically unwanted. Such vibrations can be caused by imbalances in the rotating parts, uneven friction, the meshing of gear teeth, etc. Careful designs usually minimize unwanted vibrations.

### **1.2 Background of the Project**

Machinery with moving parts can often cause vibration problems. This may be noise, excessive deflection, wear, fatigue, etc. A common solution is called vibration. The machine vibrates because it is being acted upon by a force fluctuating at frequency. The

dynamic response of the machine is dependent on the magnitude of the force, but also on the ratio of the forcing frequency to the machines natural frequency. Generally we want this ratio to be as high as possible giving very low dynamic response to the fluctuating force.

The force is often due to a physical effect, such as gear tooth meshing, fan blade passing, pump reciprocation, cutting tool speed, etc. There is little or nothing that can be done to affect these. The frequency of the forcing is usually due to the speed that the machine is operating at. Again, that cannot usually be altered.

A floating raft to isolate vibration of machines placed on it will be studied. After deriving mathematical model, the concepts and relationships of spring, damper and configuration of the spring and damper will be discussed. The best configuration characteristic of the system will be investigated when minimization of vibration or frequency is applied. Through numerical simulation, the control efficiencies will be compared to obtain some general design principles of the floating raft systems.

### **1.3 Problem Statement**

With the development of the vibration control techniques and increasing strict requirement for the vibration isolation in the industry and everyday life, classical one-stage isolation systems exhibit poor performances. To achieve more efficient vibration cancellation, some two-stage even multi stage isolation systems have received increasingly research attention in recent years and active control techniques of two-stage isolation systems are even discussed.

Vibration isolation techniques are able to dynamically adapt the characteristic parameters of the systems or structures in order to meet the strict requirements of

vibration isolation. Because of their great adaptive capacity, much attention has been devoted to active isolation systems. Substantial works on active vibration control have been published where the power flow transmitted to flexible foundations or receivers has been considered as the cost function to maximizing the cancellation of vibration. However, in these researches, the theoretical models of the isolation systems are mostly of one-stage, and more complicated two-stage active isolate on systems, such as floating raft systems, have not been dealt with.

Based on perspective above, a novel analytical model is developed to describe floating raft isolation systems. The mobility or impedance matrix technique is used to derive the mobility matrices of the subsystems, respectively, such as machines, mounts, floating raft and plate foundation. With the general mobility matrices of the subsystems, the passive system will be solved. The results will provide important instructions for the vibration design of active floating raft systems.

#### 1.4 Objective

The main objective of this paper is to highlight on the vibration isolation of floating raft system by controlling transmission of the power flow. Fine points of the objective are as follow:

- To develop mathematical model of a floating raft isolation system consisting of machines, upper mounts, raft, lower mounts and foundation.
- To study parametrically and identify the best configuration of isolation raft system consisting of spring and damper by using MATLAB software.

• To design the floating rafts system with requirement maximum displacement less than 3.5 mm.

### 1.5 Scope of Work

This one year project includes literature review of the researches, articles, and information related to the topic. Apart from that, mathematical modeling, writing computer coding and computer simulation will be carried out to propose the best vibration isolation system configuration for floating raft without exceed the requirement needed.

# CHAPTER 2 LITERATURE REVIEW

#### 2.1 Degree of Freedom

The minimum number of independent coordinate required to determine completely the positions of all parts of a system at any instant of time defines the degree of freedom of the system. In mechanics, degrees of freedom (DOF) are the set of independent displacements that specify completely the displaced or deformed position of the body or system. This is a fundamental concept relating to systems of moving bodies in mechanical engineering, aeronautical engineering, robotics, automotive engineering, locomotive engineering, structural engineering, etc.

### 2.2 Vibration Isolation

Vibration isolation is the process of isolating an object, such as a piece of equipment, from the source of vibrations. There are two types of two-stage vibration isolation system, namely, active vibration isolation and passive vibration isolation.

In the passive vibration isolation system, however, the vibrant object is the base and the purpose of the control system is to keep the displacement of the upper object stable. Passive vibration isolation systems consist essentially of a mass, spring and damper (dash-pot). For the active vibration isolation system, the objective is to decrease the forces transmitted to the foundation. Active vibration isolation systems contain, along with the spring, a feedback circuit which consists of a piezoelectric accelerometer, a controller, and an electromagnetic transducer. The acceleration (vibration) signal is processed by a control circuit and amplifier. Then it feeds the electromagnetic actuator, which amplifies the signal. As a result of such a feedback system, a considerably stronger suppression of vibrations is achieved compared to ordinary damping. For example, the floating raft isolation system in the ship is used to decrease the forces produced by the motors in order to make the base of the ship stable.

Below is some related research of vibration isolation system which entitles:

'Research on performance indices of vibration isolation system' By H.L. Sun, H.B. Chen, K. Zhang and P.Q. Zhang

### Abstract

Vibration isolators have been widely used to reduce the vibration and noise transmitted between the components of mechanical systems. The most elementary form of a vibration isolator can be considered as a resilient member with energy-dissipating means connecting the equipment and foundation. There are two classes of vibration isolation problem: one is to isolate vibrating equipment from a foundation and the other to isolate equipment from foundation vibration. Two corresponding typical performance criteria are based on motion and force transmissibility, respectively. Another performance index is isolator effectiveness or insertion loss which is defined as the ratios of (un-isolated) motion response or force to the corresponding motion response or force when an isolator is interposed between equipment and foundation.

### 2.3 Mathematical Modeling

The purpose of mathematical modeling is to represent all the important features of the system for the purpose of deriving the mathematical (or analytical) equation governing the system's behavior. The mathematical model should include enough detail to be able to describe the system in terms of equation without making too complex. The mathematical model may be linear or non-linear, depending on the behavior of the system's component.

or

#### 2.4 D'Alembert Principle

The equation of motion derive by Newton second law of motion

$$F(t) = m\ddot{x} \tag{2.1}$$

$$M(t) = J\ddot{\theta} \tag{2.2}$$

Or can be written as

$$F(t) - m\ddot{x} = 0 \tag{2.3}$$

$$M(t) - J\ddot{\theta} = 0 \tag{2.4}$$

These equations can be considered equilibrium equation provided that  $-m\ddot{x}$  and  $-J\ddot{\theta}$  are treated as a force and a moment. This fictitious force or moment is known as inertia force or inertia moment and the artificial state of equilibrium is known as dynamic equilibrium. This principle is called D'Alembert's principle.

### **CHAPTER 3**

### **METHODOLOGY/ PROJECT WORK**

### 3.1 **Procedure Identification**

The estimated work flow throughout this project is summarized in the schematic flow diagram as shown below:



### 3.2 The Dynamic Analysis of the Machines

The dynamic of the control system is studied using a mobility matrix technique. The governing equation of mobility matrix can be express as

$$\begin{cases} V_{At} \\ V_{Ab} \end{cases} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{cases} F_{At} \\ F_{Ab} \end{cases}$$
(3.1)

Where  $F_{At}$ ,  $F_{Ab}$ ,  $V_{At}$ ,  $V_{Ab}$  are, respectively, the upper and the lower forces and their corresponding velocities of the subsystem A, the abbreviation of bottom, b denotes the bottom output, and the abbreviation of top, t, indicates the top output of the corresponding subsystem.

Also, from the two degree of freedom systems of vibration, the equations can be written in matrix form as

$$[m] \ddot{x}(t) + [c] \dot{x}(t) + [k] x(t) = F(t)$$
(3.2)

For the analysis of the dynamic of the machine, the derivation has been made to derive the equation of the system. Following are derivation by applying Newton's second law of motion to each mass or rigid body shown by the free-body-diagram as

$$m_i \ddot{x}_i = \sum_j F_{ij} \text{ (for mass,} m_i) \tag{3.3}$$

Or

$$J_i \ddot{\theta}_i = \sum_j M_{ij} \text{ (for rigid body of inertia } J_i)$$
(3.4)

Where  $\sum_{j} F_{ij}$  denotes the sum of all forces acting on mass  $m_i$  and  $\sum_{j} M_{ij}$  indicates the sum of moments of all forces (about suitable axis) acting on the rigid body of mass moment of inertia,  $J_i$ .

### 3.3 Floating Raft System



Figure 3.1 below shows an analytical model of floating raft isolation system.

Figure 3.1: Analytical model of a floating raft isolation system.

As an advanced isolation system, the floating raft isolation system is more complicated than most general two-stage isolation systems and it provides much better vibration reduction than the latter. Two or more machines are mounted on a single intermediate raft structure. The overall isolation system can be divided into five subsystems which is machines A, upper mount system B, passive isolators, intermediate raft R, lower mount system D including passive isolators and foundation C. The intermediate raft structure is considered as a rigid block and the flexible foundation is modeled as a thin rectangular plate simply supported at its four edges. These five subsystems are connected at a finite number of junctions by the mounts.

In engineering, the vertical vibration energy is more significant than that of other directions especially in low-frequency band, so only the vertical forces and the resulting

motions of the system are concerned in the presented model as showed in the figure 3.2 below.



Figure 3.2: Configuration of spring and damper of floating raft isolation system

### 3.3.1 Floating Raft Isolation System Configuration

The floating raft isolation system in this project considered is composed of two upper machines with raft in the middle connected to the foundation as shown in the figure 3.2. Each end machine is connected with suspension consist of one spring and one damper. The length of the intermediate raft in this project is 10 meters. In this project, the author just considers the vertical forces. So, the width is not important in this project. For the both machines the length are about 3 meters each.

### **3.4** Equation of Motion

### 3.4.1 Summation Force on the Intermediate Raft

From the analytical model of a floating raft system in figure 3.1, there is two (2) degree of freedoms for each machine which is bounce and pitch motions as well as the floating raft. Overall, six (6) degree of freedoms will be developed in this model. The force on the upper machine is developed from the fluctuating machine. Using the d' Alembert force method, the equation of motion derived as follows from each of free body diagrams. Figure 3.3 below shows the free body diagram of the intermediate raft.



Figure 3.3: Free body diagram of the intermediate raft.

Total force at the Z-axis on the intermediate raft

$$\sum F_{z} = M_{R}\ddot{z}_{R}$$

$$= F_{d_{11}} + F_{s_{11}} + F_{d_{12}} + F_{s_{12}} + F_{d_{21}} + F_{s_{21}} + F_{d_{22}} + F_{s_{22}} - (F_{d_{R1}} + F_{sR1} + F_{dR2} + F_{sR2} + F_{dR3} + F_{sR3}$$
(3.5)

Total of moment acting at center of gravity of the intermediate raft

$$\sum M_{CG_R} = J_R \ddot{\theta_R}$$

$$= -F_{d_{11}} x_{R1} - F_{s_{11}} x_{R1} - F_{d_{12}} x_{R2} - F_{s_{12}} x_{R2} + F_{d_{21}} x_{R3} + F_{s_{21}} x_{R3} + F_{d_{22}} x_{R4} + F_{s_{22}} x_{R4} + F_{d_{R1}} x_{R5} + F_{s_{R1}} x_{R5} + F_{d_{R2}} x_{R6} + F_{s_{R2}} x_{R6} - F_{d_{R3}} x_{R7} - F_{s_{R3}} x_{R7}$$

$$(3.6)$$

### 3.4.2 Summation Force on the Upper Machine



Figure 3.4: FBD of Machine 1



Figure 3.5: FBD of Machine 2

Total force at the Z-axis for the Machine 1:

$$\sum F_z = M_1 \ddot{z}_1$$

$$= -F_1 - F_{d_{11}} - F_{s_{11}} - F_{d_{12}} - F_{s_{12}}$$
(3.7)

Total of moment acting at center of gravity of the Machine 1:

$$\sum M_{CG_1} = J_1 \ddot{\theta}_1$$

$$= F_{d_{11}} x_1 + F_{s_{11}} x_1 - F_{d_{12}} x_2 - F_{s_{12}} x_2$$
(3.8)

Total force at the Z-axis for Machine 2:

$$\sum F_z = M_2 \ddot{z}_2$$

$$= -F_1 - F_{d_{21}} - F_{s_{21}} - F_{d_{22}} - F_{s_{22}}$$
(3.9)

Total of moment acting at center of gravity of the Machine 2:

$$\sum M_{CG_2} = J_2 \ddot{\theta_2}$$

$$= F_{d_{21}} x_3 + F_{s_{21}} x_3 - F_{d_{22}} x_4 - F_{s_{22}} x_4$$
(3.10)

Forces between the machines and the intermediate raft:

$$F_{s_{11}} = k_{11} \Big[ (z_1 - x_1 \theta_1) - (z_R - x_{R_1} \theta_R) \Big]$$
(3.11)

$$F_{d_{11}} = c_{11} \left[ \left( \dot{z}_1 - x_1 \dot{\theta}_1 \right) - \left( \dot{z}_R - x_{R_1} \dot{\theta}_R \right) \right]$$
(3.12)

$$F_{s_{12}} = k_{12} \Big[ (z_1 + x_2 \theta_1) - (z_R - x_{R_2} \theta_R) \Big]$$
(3.13)

$$F_{d_{12}} = c_{12} \left[ \left( \dot{z}_1 - x_2 \dot{\theta}_1 \right) - \left( \dot{z}_R - x_{R_2} \dot{\theta}_R \right) \right]$$
(3.14)

$$F_{s_{21}} = k_{21} [(z_2 - x_3 \theta_2) - (z_R + x_{R_3} \theta_R)]$$
(3.15)

$$F_{d_{21}} = c_{21} \left[ \left( \dot{z}_2 - x_3 \dot{\theta}_2 \right) - \left( \dot{z}_R + x_{R_3} \dot{\theta}_R \right) \right]$$
(3.16)

$$F_{s_{22}} = k_{22} [(z_2 + x_4 \theta_2) - (z_R + x_{R_4} \theta_R)]$$
(3.17)

$$F_{d_{22}} = c_{22} \left[ \left( \dot{z}_2 + x_4 \dot{\theta}_2 \right) - \left( \dot{z}_R + x_{R_4} \dot{\theta}_R \right) \right]$$
(3.18)

Forces between the intermediate raft and the foundation:

$$F_{S_{R1}} = k_{R1} \Big( z_R - x_{R_5} \theta_R \Big) \tag{3.19}$$

$$F_{d_{R1}} = c_{R1} \left( \dot{z}_R - x_{R_5} \dot{\theta}_R \right) \tag{3.20}$$

$$F_{s_{R2}} = k_{R2} \Big( z_R - x_{R_6} \theta_R \Big) \tag{3.21}$$

$$F_{d_{R2}} = c_{R2} \left( \dot{z}_R - x_{R_6} \dot{\theta}_R \right) \tag{3.22}$$

$$F_{S_{R3}} = k_{R3} \Big( z_R + x_{R_7} \theta_R \Big)$$
(3.23)

$$F_{d_{R3}} = c_{R3} \left( \dot{z}_R + x_{R_7} \dot{\theta}_R \right) \tag{3.24}$$

### 3.4.3 Overall Equation of Motion

Below are overall equations of motion in isolation raft system after the equations 3.11 to 3.24 have been put into the equations 3.5 until 3.10. The equations show that the motion of the each mass will influence the motion of mass next to it. Therefore the equations can be written in matrix form.

$$\sum F_{z} = M_{R} \ddot{z}_{R}$$

$$(3.25)$$

$$c_{11} \dot{z}_{1} - c_{11} x_{1} \dot{\theta}_{1} - c_{11} \dot{z}_{R} + c_{11} x_{R_{1}} \dot{\theta}_{R} + c_{12} \dot{z}_{1} + c_{12} x_{2} \dot{\theta}_{1} - c_{12} \dot{z}_{R} + c_{12} x_{R_{2}} \dot{\theta}_{R} + c_{21} \dot{z}_{2} + c_{21} x_{3} \dot{\theta}_{2} - c_{21} \dot{z}_{R} - c_{21} x_{R_{3}} \dot{\theta}_{R} + c_{22} \dot{z}_{2} + c_{22} x_{4} \dot{\theta}_{2} - c_{22} \dot{z}_{R} - c_{22} x_{R_{4}} \dot{\theta}_{R} + k_{11} z_{1} - k_{11} x_{1} \theta_{1} - k_{11} z_{R} + k_{11} x_{R_{1}} \theta_{R} + k_{12} z_{1} + k_{12} x_{2} \theta_{1} - k_{12} z_{R} + k_{12} x_{R_{2}} \theta_{R} + k_{21} z_{2} - k_{21} x_{3} \theta_{2} - k_{21} z_{R} - k_{21} x_{R_{3}} \theta_{R} + k_{22} z_{2} + k_{22} x_{4} \theta_{2} - k_{22} z_{R} - k_{22} x_{R_{4}} \theta_{R} - c_{R_{1}} \dot{z}_{R} + c_{R_{1}} x_{R_{5}} \dot{\theta}_{R} - c_{R_{2}} \dot{z}_{R} + c_{R_{2}} x_{R_{6}} \dot{\theta}_{R} - c_{R_{3}} \dot{z}_{R} - c_{R_{3}} x_{R_{7}} \dot{\theta}_{R} - k_{R_{1}} z_{R} + k_{R_{1}} x_{R_{5}} \theta_{R} - k_{R_{2}} z_{R} + k_{R_{2}} x_{R_{6}} \theta_{R} - k_{R_{3}} z_{R} - k_{R_{3}} x_{R_{7}} \theta_{R}$$

$$\sum M_{CG_R} = J_R \ddot{\theta}_R$$

$$= -c_{11} x_{R1} \dot{z}_1 + c_{11} x_{11} x_{R1} \dot{\theta}_1 + c_{11} x_{R1} \dot{z}_R - c_{11} x_{R1} x_{R1} \dot{\theta}_R - c_{12} x_{R2} \dot{z}_1 - c_{12} x_{22} x_{R2} \dot{\theta}_1 + c_{12} x_{R2} \dot{z}_R - c_{12} x_{R2} x_{R2} \dot{\theta}_R + c_{21} x_{R3} \dot{z}_2 - c_{21} x_{3} x_{R3} \dot{\theta}_2 - c_{21} x_{R3} \dot{z}_R - c_{21} x_{R3} \dot{x}_R \dot{\theta}_R + c_{22} x_{R4} \dot{z}_2 + c_{22} x_4 x_{R4} \dot{\theta}_2 - c_{22} x_{R4} \dot{z}_R - c_{22} x_{R4} \dot{x}_R \dot{\theta}_R + c_{R1} x_{R5} \dot{z}_R - c_{R1} x_{R5} \dot{x}_R \dot{\theta}_R + c_{R2} x_{R6} \dot{z}_R - c_{R2} x_{R6} \dot{\theta}_R - c_{R3} x_{R7} \dot{z}_R - c_{R3} x_{R7} \dot{\theta}_R - k_{11} x_{R1} z_1 + k_{11} x_{R1} a_R - k_{11} x_{R1} a_R - k_{12} x_{R2} z_1 - k_{12} x_{2} x_{R2} \theta_1 + k_{12} x_{R2} z_R - k_{12} x_{R2} \theta_R + k_{21} x_{R3} z_2 - k_{21} x_{3} x_{R3} \theta_2 - k_{21} x_{R3} x_R \theta_R + k_{22} x_{R4} z_2 + k_{22} x_{4} x_{R4} \theta_2 - k_{22} x_{R4} z_R - k_{22} x_{R4} x_{R4} \theta_R + k_{R1} x_{R5} z_R - k_{R1} x_{R5} \theta_R + k_{R2} x_{R6} z_R - k_{R2} x_{R6} \theta_R - k_{R3} x_{R7} z_R - k_{R1} x_{R5} x_{R5} \theta_R + k_{R2} x_{R6} z_R - k_{R3} x_{R7} z_R - k_{R3} x_{R7} z_R - k_{R1} x_{R5} z_R - k_{R1} x_{R5} x_{R5} \theta_R + k_{R2} x_{R6} z_R - k_{R3} x_{R7} z_R - k_{R3}$$

$$\sum F_{z} = M_{1} \ddot{z}_{1}$$

$$= -F_{1} - c_{11} \dot{z}_{1} + c_{11} x_{1} \dot{\theta}_{1} + c_{11} \dot{z}_{R} - c_{11} x_{R_{1}} \dot{\theta}_{R} - k_{11} z_{1} + k_{11} x_{1} \theta_{1} + k_{11} z_{R} - k_{11} x_{R_{1}} \theta_{R} - c_{12} \dot{z}_{1} - c_{12} x_{2} \dot{\theta}_{1} + c_{12} \dot{z}_{R} - c_{12} x_{R_{2}} \dot{\theta}_{R} - k_{12} z_{1} - k_{12} x_{2} \theta_{1} + k_{12} z_{R} - k_{12} x_{R_{2}} \theta_{R}$$

$$(3.27)$$

$$\sum M_{CG_1} = J_1 \dot{\theta}_1$$

$$= c_{11} x_1 \dot{z}_1 - c_{11} x_1 x_1 \dot{\theta}_1 - c_{11} x_1 \dot{z}_R + c_{11} x_{R_1} x_1 \dot{\theta}_R + k_{11} x_1 z_1 - k_{11} x_1 x_1 \theta_1 - k_{11} x_1 z_R + k_{11} x_{R_1} x_1 \theta_R - c_{12} x_2 \dot{z}_1 - c_{12} x_2 x_2 \dot{\theta}_1 + c_{12} x_2 \dot{z}_R - c_{12} x_{R_2} x_2 \dot{\theta}_R - k_{12} x_2 z_1 - k_{12} x_2 x_2 \theta_1 + k_{12} x_2 z_R - k_{12} x_{R_2} x_2 \theta_R$$

$$(3.28)$$

$$\sum F_{z} = M_{2}\ddot{z}_{2}$$

$$= -F_{1} - c_{21}\dot{z}_{2} + c_{21}x_{3}\dot{\theta}_{2} + c_{21}\dot{z}_{R} + c_{21}x_{R_{3}}\dot{\theta}_{R} - k_{21}z_{2} + k_{21}x_{3}\theta_{2} + k_{21}z_{R} + k_{21}x_{R_{3}}\theta_{R} - c_{22}\dot{z}_{2} - c_{22}x_{4}\dot{\theta}_{2} + c_{22}\dot{z}_{R} + c_{22}x_{R_{4}}\dot{\theta}_{R} - k_{22}z_{2} - k_{22}x_{4}\theta_{2} + k_{22}z_{R} + k_{22}x_{R_{4}}\theta_{R}$$

$$(3.29)$$

$$\sum M_{CG_2} = J_2 \ddot{\theta}_2$$

$$= c_{21} x_3 \dot{z}_2 - c_{21} x_3 x_3 \dot{\theta}_2 - c_{21} x_3 \dot{z}_R - c_{21} x_{R_3} x_3 \dot{\theta}_R + k_{21} x_3 z_2 - k_{21} x_3 x_3 \theta_2$$

$$- k_{21} x_3 z_R - k_{s_{21}} x_{R_3} x_3 \theta_R - c_{22} x_4 \dot{z}_2 - c_{22} x_4 x_4 \dot{\theta}_2 + c_{22} x_4 \dot{z}_R$$

$$+ c_{22} x_{R_4} x_4 \dot{\theta}_R - k_{22} x_4 z_2 - k_{22} x_4 x_4 \theta_2 + k_{22} x_4 z_R + k_{22} x_{R_4} x_4 \theta_R$$
(3.30)

## 3.5 Equations in Matrix Form

$\begin{bmatrix} 0 & 0 & 0 & 0 & M_2 & 0 \\ 0 & 0 & 0 & 0 & J_2 \end{bmatrix} \begin{bmatrix} z_2 \\ \dot{\theta}_2 \end{bmatrix}$	$\begin{bmatrix} M_R \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$	0 / <sub>R</sub> 0 0 0 0	$     \begin{array}{c}       0 \\       0 \\       M_1 \\       0 \\       0 \\       0 \\       0 \end{array} $	$egin{array}{c} 0 \\ 0 \\ 0 \\ J_1 \\ 0 \\ 0 \end{array}$	$     \begin{array}{c}       0 \\       0 \\       0 \\       M_2 \\       0     \end{array} $	$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ J_2 \end{bmatrix}$	$\ddot{\mathcal{B}}_{R}$ $\ddot{\mathcal{B}}_{R}$ $\ddot{\mathcal{B}}_{1}$ $\ddot{\mathcal{B}}_{2}$	+
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$$\begin{bmatrix} c_{11} + c_{12} + c_{21} + c_{22} + c_{R1} + c_{R2} + c_{R3} \\ -c_{11}x_{R1} - c_{12}x_{R2} + c_{21}x_{R3} + c_{22}x_{R4} - c_{R1}x_{R5} - c_{R2}x_{R6} + c_{R3}x_{R7} \\ -c_{11} - c_{12} \\ c_{11}x_1 - c_{12}x_2 \\ -c_{21} - c_{22} \\ c_{21}x_3 - c_{22}x_4 \end{bmatrix}$$

$$-c_{11} x_{R_1} - c_{12} x_{R_2} + c_{21} x_{R_3} + c_{22} x_{R_4} - c_{R_1} x_{R_5} - c_{R_2} x_{R_6} - c_{R_3} x_{R_7}$$

$$c_{11} x_{R_1} x_{R_1} + c_{12} x_{R_2} x_{R_2} + c_{21} x_{R_3} x_{R_3} + c_{22} x_{R_4} x_{R_4} + c_{R_1} x_{R_5} x_{R_5} + c_{R_2} x_{R_6} x_{R_6} - c_{R_3} x_{R_7} x_{R_7}$$

$$c_{11} x_{R_1} + c_{12} x_{R_2}$$

$$-c_{11} x_{R_1} x_1 + c_{12} x_{R_2} x_2$$

$$-c_{21} x_{R_3} - c_{22} x_{R_4}$$

$$c_{21} x_{R_3} x_3 - c_{22} x_{R_4} x_4$$

$$\begin{bmatrix} \dot{z}_{R} \\ \dot{\theta}_{R} \\ \dot{z}_{1} \\ \dot{\theta}_{1} \\ \dot{z}_{2} \\ \dot{\theta}_{2} \end{bmatrix} + \begin{pmatrix} k_{11} + k_{12} + k_{21} + k_{22} + k_{R1} + k_{R2} + k_{R3} \\ -k_{11}x_{R1} - k_{12}x_{R2} + k_{21}x_{R3} + k_{22}x_{R4} - k_{R1}x_{R5} - k_{R2}x_{R6} + k_{R3}x_{R7} \\ -k_{11} - k_{12} \\ k_{11}x_{1} - k_{12}x_{2} \\ -k_{21} - k_{22} \\ k_{21}x_{3} - k_{22}x_{4} \end{pmatrix}$$

$$\begin{aligned} -k_{11}x_{R1} - k_{12}x_{R2} + k_{21}x_{R3} + k_{22}x_{R4} - k_{R1}x_{R5} - k_{R2}x_{R6} + k_{R3}x_{R7} \\ k_{11}x_{R_1}x_{R1} + k_{12}x_{R_2}x_{R2} + k_{21}x_{R_3}x_{R3} + k_{22}x_{R_4}x_{R4} + k_{R1}x_{R_5}x_{R5} + k_{R2}x_{R_6}x_{R6} + k_{R3}x_{R_7}x_{R7} \\ k_{11}x_{R_1} + k_{12}x_{R_2} \\ -k_{11}x_{R_1}x_1 + k_{12}x_{R_2}x_2 \\ -k_{21}x_{R_3} - k_{22}x_{R_4} \\ k_{21}x_{R_3}x_3 - k_{22}x_{R_4}x_4 \end{aligned}$$

$$\begin{bmatrix} Z_R \\ \theta_R \\ Z_1 \\ \theta_1 \\ Z_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ -F_1 \\ \mathbf{0} \\ -F_1 \\ \mathbf{0} \end{bmatrix}$$

### 3.6 Runge Kutta Form

In order to solve 6x6 matrices, Runge-Kutta fourth order is applied so that the matrix equations of motion are used to express the acceleration vector as

$$\vec{x}(t) = [m]^{-1} \left( \vec{F}(t) - [c] \vec{x}(t) - [k] \vec{x}(t) \right)$$
(3.31)

Assuming the displacements and velocities as unknowns, a new vector is defined as

$$\vec{X}(t) = \begin{cases} \vec{x}(t) \\ \vec{x}(t) \end{cases}$$
(3.32)

So that,

$$\vec{x}(t) = \begin{cases} \vec{x}(t) \\ \vec{x}(t) \end{cases} = \begin{cases} \vec{x} \\ [m]^{-1} (\vec{F}(t) - [c] \vec{x} - [k] \vec{x}) \end{cases}$$
(3.33)

Rearrange the equation above to obtain

$$\vec{x}(t) = \begin{bmatrix} [0] & [I] \\ -[m]^{-1}[k] & -[m]^{-1}[c] \end{bmatrix} \begin{cases} \vec{x}(t) \\ \vec{x}(t) \end{cases} + \begin{cases} 0 \\ [m]^{-1}\vec{F}(t) \end{cases}$$
(3.34)

That is,

$$\vec{\dot{X}}(t) = \vec{f}(\vec{X}, t) \tag{3.35}$$

Where

$$\vec{\dot{X}}(t) = \vec{f}(\vec{X}, t) = [A]\vec{X}(t) + \vec{F}(t)$$
(3.36)

$$[A] = \begin{cases} [0] & [I] \\ -[m]^{-1}[k] & -[m]^{-1}[c] \end{cases}$$
(3.37)

$$\vec{F}(t) = \begin{cases} 0\\ [m]^{-1}\vec{F}(t) \end{cases}$$
(3.38)

With this, the recurrence formula to evaluate  $\vec{X}(t)$  at different grid points  $t_i$  according to the fourth order Runge Kutta method becomes

$$\vec{X}_{i+1} = X_i + \frac{1}{6} \left[ \vec{K}_1 + 2\vec{K}_2 + 2\vec{K}_3 + \vec{K}_4 \right]$$
(3.39)

Where,

$$\vec{K}_{1} = h\vec{f}(\vec{X}_{i}, t_{i})$$
$$\vec{K}_{2} = h\vec{f}(\vec{X}_{i} + \frac{\vec{K}_{1}}{2}, t_{i} + \frac{h}{2})$$
$$\vec{K}_{3} = h\vec{f}(\vec{X}_{i} + \frac{\vec{K}_{2}}{2}, t_{i} + \frac{h}{2})$$
$$\vec{K}_{4} = h\vec{f}(\vec{X}_{i} + \vec{K}_{3}, t_{i+1})$$

A numerical procedure used is Runge-Kutta method which solved the matrix equation computationally using MATLAB software.

(Refer to appendix 1 for the MATLAB coding)

# CHAPTER 4 RESULTS AND DISCUSSION

### 4.1 Case 1 – Configuration of Spring and Damper

For case 1, the project is run to determine how configuration of spring and damper effects the vibration of floating raft system. The duration of MATLAB simulation is set to be 10 second. The weight of each machine is 200kg and the weight of floating raft is 100kg. The parameters of both upper machines are 400 N for the external force on each machine. All springs and dampers are set to 100000 N/m and 400 Ns/m each. For this case, we are just looking at the one suspension under the floating raft which is the variable distances from the datum or classified as 'xr6'.

4.1.1	Case 1.1 – Machines 1	l and 2 O	perate at th	ie Same Fre	equency
-------	-----------------------	-----------	--------------	-------------	---------

Innut	'urb'	Max Displacement	Dignlagoment	Time telen
mput	XIO	Max. Displacement	Displacement	Time taken
	(m)	before stable, $z_{i1}$	when stable, $z_1$	to stabilize
		(mm)	(mm)	(second)
	-6	6.893	2.536	3
$F_1 = F_2$	-5	6.887	2.612	3
= 400 N	-4	6.871	2.710	3
	-3	6.884	2.770	3
	-2	6.888	2.861	3
$\omega_1 = \omega_2 =$	-1	6.925	2.957	3
300rpm	0	6.993	3.046	3
-	1	7.027	3.115	3
	2	7.031	3.194	3
	3	6.995	3.280	3

Table 4.1: Displacement of Machine 1,  $z_1$  in Case 1.1

4	6.925	3.368	3
5	6.825	3.440	3
6	6.720	3.517	3

'xr6' Max. Displacement Input Displacement Time taken before stable,  $z_{i2}$ when stable,  $z_2$ to stabilize (m) (second) (mm) (mm) 6.720 -6 3.517 3  $F_1 = F_2$ = 400 N-5 3.440 3 6.825 6.925 3.368 3 -4 -3 3 6.995 3.280  $\omega_1 = \omega_2 = 300$ rpm -2 7.031 3.194 3 3 -1 7.027 3.115 6.993 3.046 3 0 3 6.925 2.957 1 2 2.861 3 6.888 3 3 2.770 6.884 3 2.710 4 6.871 5 6.887 2.612 3 3 2.536 6 6.893

Table 4.2: Displacement of Machine 2,  $z_2$  in Case 1.1

Table 4.3: Displacement of Floating Raft,  $z_R$  in Case 1.1

Input	ʻxr6'	Max. Displacement	Displacement	Time taken
	(m)	before stable, $z_{iR}$	when stable, $z_R$	to stabilize
		(mm)	(mm)	(second)
	-6	4.242	2.010	3
$F_1 = F_2$	-5	4.277	2.017	3
=400 N	-4	4.310	2.025	3
	-3	4.345	2.035	3
	-2	4.374	2.044	3
$\omega_1 = \omega_2 =$	-1	4.389	2.066	3
300rpm	0	4.395	2.074	3
1	1	4.389	2.066	3
	2	4.374	2.044	3
	3	4.345	2.035	3
	4	4.310	2.025	3
	5	4.277	2.017	3
	6	4.242	2.010	3

## 4.1.2 Case 1.2 – Machines 1 Operates and Machine 2 is Off

Input	ʻxr6'	Max. Displacement	Displacement	Time taken
	(m)	before stable, $z_{i1}$	when stable, $z_1$	to stabilize
		(mm)	(mm)	(second)
	-6	6.906	2.902	3
$F_1 = 400 N$	-5	6.973	2.932	3
-	-4	7.029	2.929	3
$F_2 = 0 N$	-3	7.073	2.964	3
2	-2	7.102	3.049	3
$\omega_1 = 300$ rpm	-1	7.164	3.074	3
1 1	0	7.225	3.144	3
$\omega_2 = 0$	1	7.266	3.232	3
2	2	7.277	3.334	3
	3	7.253	3.455	3
	4	7.201	3.578	3
	5	7.193	3.174	3
	6	7.144	3.885	3

Table 4.4: Displacement of Machine 1,  $z_1$  in Case 1.2

Table 4.5: Displacement of Machine 2,  $z_2$  in Case 1.2

Input	ʻxr6'	Max. Displacement	Displacement	Time taken
	(m)	before stable, $z_{i2}$	when stable, $z_2$	to stabilize
		(mm)	(mm)	(second)
	-6	1.427	0.367	3.5
$F_1 = 400 N$	-5	1.196	0.291	3.5
-	-4	0.973	0.225	3.5
$F_2 = 0 N$	-3	0.691	0.173	3.5
-	-2	0.414	0.137	3.5
$\omega_1 = 300$ rpm	-1	0.236	0.116	3.5
	0	0.204	0.111	3.5
$\omega_2 = 0$	1	0.236	0.116	3.5
2	2	0.414	0.137	3.5
	3	0.691	0.173	3.5
	4	0.973	0.225	3.5
	5	1.196	0.291	3.5
	6	1.427	0.367	3.5

Input	'xr6'	Max. Displacement	Displacement	Time taken
	(m)	before stable, $z_{iR}$	when stable, $z_R$	to stabilize
		(mm)	(mm)	(second)
	-6	2.488	1.023	3
$F_1 = 400 N$	-5	2.440	1.014	3
*	-4	2.391	1.011	3
$F_2 = 0 N$	-3	2.339	1.009	3
2	-2	2.286	1.008	3
$\omega_1 = 300$ rpm	-1	2.233	1.007	3
1 1	0	2.197	1.007	3
$\omega_2 = 0$	1	2.155	1.007	3
2	2	2.106	1.007	3
	3	2.050	1.006	3
	4	1.989	1.006	3
	5	1.926	1.005	3
	6	1.864	1.004	3

Table 4.6: Displacement of Floating Raft,  $z_R$  in Case 1.2

### 4.1.3 Case 1.3 – Machines 1 and 2 Operate with Different Frequency

Input	'xr6'	Max. Displacement	Displacement	Time taken
_	(m)	before stable, $z_{i1}$	when stable, $z_1$	to stabilize
		(mm)	(mm)	(second)
	-6	7.049	2.882	3
$F_1 = 400 N$	-5	7.060	2.901	3
-	-4	7.069	2.921	3
$F_2 = 400 N$	-3	7.074	2.952	3
	-2	7.078	3.011	3
$\omega_1 = 300$ rpm	-1	7.099	3.067	3
	0	7.157	3.140	3
$\omega_2 = 600 \text{rpm}$	1	7.200	3.231	3
	2	7.222	3.329	3
	3	7.214	3.451	3
	4	7.173	3.578	3
	5	7.151	3.718	3
	6	7.126	3.871	3

Table 4.7: Displacement of Machine 1,  $z_1$  in Case 1.3

Input	ʻxr6'	Max. Displacement	Displacement	Time taken
-	(m)	before stable, $z_{i2}$	when stable, $z_2$	to stabilize
		(mm)	(mm)	(second)
	-6	1.380	0.836	3.5
$F_1 = 400 N$	-5	1.295	0.756	3.5
-	-4	1.412	0.694	3.5
$F_2 = 400 N$	-3	1.522	0.648	3.5
2	-2	1.596	0.614	3.5
$\omega_1 = 300$ rpm	-1	1.634	0.593	3.5
1 1	0	1.648	0.583	3.5
$\omega_2 = 600$ rpm	1	1.634	0.587	3.5
2 1	2	1.602	0.599	3.5
	3	1.947	0.623	3.5
	4	2.282	0.662	3.5
	5	2.514	0.722	3.5
	6	2.600	0.799	3.5

Table 4.8: Displacement of Machine 2,  $z_2$  in Case 1.3

Table 4.9: Displacement of Floating Raft,  $z_R$  in Case 1.3

Input	ʻxr6'	Max. Displacement	Displacement	Time taken
	(m)	before stable, $z_{iR}$	when stable, $z_R$	to stabilize
		(mm)	(mm)	(second)
	-6	2.843	1.153	3
$F_1 = 400 N$	-5	2.836	1.160	3
	-4	2.832	1.162	3
$F_2 = 0 N$	-3	2.830	1.171	3
2	-2	2.828	1.176	3
$\omega_1 = 300$ rpm	-1	2.822	1.181	3
	0	2.812	1.184	3
$\omega_2 = 0$	1	2.795	1.193	3
2	2	2.769	1.198	3
	3	2.736	1.204	3
	4	2.705	1.210	3
	5	2.668	1.217	3
	6	2.627	1.228	3

### 4.2 Case 2 – Availability of Spring or Damper

For case 2, the objective is to determine how much effect of spring and damper to the floating raft system. The duration of MATLAB simulation is set to be 10 second. The weight of each machine is 200kg and the weight of floating raft is 100kg. The parameters of both upper machines are 400 N for the external force and 300rpm for the natural frequency of each machine while the machines were operated. All springs and dampers are set to 100000 N/m and 400 Ns/m each. For this case, we are looking into several cases when the value of spring and damper has been changed. The configuration of the spring and damper with the length 'xr6' meter from datum is set to be 0 meter which means the spring and damper is located exactly on the datum. So, the effect of the availability of spring and damper will not be affected by the configuration of the spring and damper. The duration of MATLAB simulation is set to be 10 second.





Figure 4.1: Both Machines 1 and 2 in case 2.1 (with damper)

Figure 4.1 shows the maximum displacement that both machine 1 and 2 experienced are 6.99mm and the time taken for the machines to stable is about 3 second before it run with 3.06mm of displacement.



Figure 4.2: Floating raft in case 2.1 (with damper)

Figure 4.2 shows the maximum displacement that floating raft experienced is 4.40mm and the time taken for the floating raft to stable is about 3 second before it run with 2.07mm of displacement.

4.2.2 Case 2.2 - The System Operates with without Damper



Figure 4.3: Both Machine 1 and 2 in Case 2.2 (without damper)

Figure 4.3 shows the maximum displacement that machines 1 and 2 experienced is 7.947mm. From the graph, the machine 1 and 2 will be facing random vibration. The displacement is not stable and keeps changing every second.



Figure 4.4: Floating Raft in Case 2.2 (without damper)

Figure 4.4 shows the maximum displacement that floating raft experienced which is 4.907mm. From the graph, the floating raft will be facing random vibration. The displacement is not stable and keeps changing every second.

### 4.3 Summary of Case 1

For case 1, the results show that there are relationship between the configuration of spring and damper with the displacement of machines. The configuration of spring and damper played very important role to maintain the stability of the machines supported by floating raft.



Figure 4.5 below show the summary of the result of case 1.1 which is the machines 1 and 2 operate at the same frequency.

Figure 4.5: Summary of case 1.1

For case 1.1, from the result shows in the figure 4.5, the highest displacement of machine 1 is when the spring and damper is place at the range of 2 meter from the datum which is 7.03mm. The highest displacement of machine 2 is when the spring and damper is place at the range of -2 meter from the datum which is the same as machine 1, 7.03mm. For this case, both machines can be placed between the ranges of -5 meter to 5 meter from the datum to satisfy the control limit of maximum displacement after stable which is less than 3.5mm. The range of 6 and -6 meter from datum will give displacement of machine 1 and 2 more than 3.5mm. So, it will go beyond the control limit or the requirement.



Figure 4.6 below show the summary of the result of case 1.2 which is the machines 1 operates and machine 2 is off.

Figure 4.6: summary for Case 1.2

For case 1.2, from the result shows in the figure 4.6, the highest displacement of machine 1 is when the spring and damper is place 2 meter from the datum which is 7.28mm and the highest displacement of machine 2 is when the spring and damper is place at 6 or -6 meter from the datum which is 1.427mm. the spring and damper should be placed between the ranges of -6 meter to 3 meter from the datum to satisfy the control limit of maximum displacement of 3.5 mm. from the graph, when the spring and damper has been put at the range of 4 meter to 6 meter from the datum, the machine 1 will give displacement more than 3.5 mm. So, it will go beyond the control limit or the requirement.

Figure 4.7 below show the summary of the result of case 1.3 which is the machines 1 and 2 operate with different frequency.



Figure 4.7: Summary for Case 1.3

For case 1.3, from the result shows in the figure 4.7, the highest displacement of machine 1 is when the spring and damper is place 2 meter from the datum which is 7.22mm and the highest displacement of machine 2 is when the spring and damper is place 6 meter from the datum which is 2.6mm. The spring and damper can be placed between the ranges of -6 meter to 3 meter from the datum to satisfy the control limit of maximum displacement of 3.5 mm. from the graph, when the spring and damper has been put at the range of 4 meter to 6 meter from the datum, the machine 1 will give displacement more than 3.5 mm. So, it will go beyond the control limit or the requirement. Even though the frequency of machine 2 is increased by 2 times, the displacement of machine 2 is still lower than the displacement of machine 1.

# CHAPTER 5 CONCLUSION

### 5.1 Conclusion

In this project, there are two cases have been studied which are case 1 and case 2. In the case 1 which is the configuration of the spring and damper under the floating raft, there are three minor sub cases. The sub cases are, the machines operate with the same frequency, the machine 1 operates and machine 2 is off and the machine operate with different frequency. For the case 2, availability of spring and damper is studied. In this case, there are two sub cases which are the machines run with spring and damper place between the machine and floating raft and also floating raft with foundation, and the machines run only consist of spring without damper placed in any part.

### 5.2 Recommendation

As conclusion, in this study, an analytical model of the two stage floating raft isolation system is presented. The mobility matrices of subsystems are derived by the substructure mobility technique. The action of spring, damper and its configuration are discussed. In order to identify the best parameters for the selected configuration, the most important key factor here is the requirement for the maximum vibration of machines. In order to come up with an effective solution and satisfy the industrial requirement, understanding of the problems and other constraints must be considered.

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## **APPENDICES**

Appendix 1: MATLAB Coding

Appendix 2: Gantt chart – First Semester

Appendix 3: Gantt chart – Second Semester

Gantt chart – First Semester



Gantt Chart - Second Semester

2	P.44-21/30:-1-	-	•	•	-	•	2	r	•	•	10	:	-	-	
.0V	Detail/ Week	I	7	c	4	e	0	1	0	۷	TO	11	14	13	14
-	Project Work Continue														
	-Writing the mathematical equation in MATLAB														
2	Submission of Progress Report 1														
ŝ	Project Work Continue														
4	Submission of Progress Report 2														
S	Seminar (compulsory)														
S	Project work continue														
	-MATLAB programming														
	-Report Writing														
9	Poster Exhibition														
7	Submission of Dissertation (soft bound)														
8	Oral Presentation														
6	Submission of Project Dissertation (Hard Bound)														
				,											

Suggested milestone Process

#### **Main Function**

```
clear all
Mr=100;
Jr=100*10^2/12;
M1=200;
M2 = 200;
J1=200*2^2/12;
J2=200*2^2/12;
F1=400;
F2=400;
w1=300*2*3.142/60;
w2=300*2*3.142/60;
c11=400;
c12=400;
c21=400;
c22=400;
cr1=400;
cr2=400;
cr3=400;
k11=100000;
k12=100000;
k21=100000;
k22=100000;
kr1=100000;
kr2=100000;
kr3=100000;
x1=1;
x2=1;
x3=1;
x4=1;
xr1=7;
xr2=5;
xr3=5;
xr4=7;
xr5=7;
xr6=6;
xr7=7;
tstart=0;
                %assumption value
zr=0;
                %assumption value
z1=0;
z2=0;
                %assumption value
                %assumption value
tetar=0;
teta1=0;
                %assumption value
teta2=0;
                %assumption value
                %assumption value
zldot=0;
```

```
%assumption value
z2dot=0;
                 %assumption value
zrdot=0;
tetardot=0;
                 %assumption value
teta1dot=0;
                 %assumption value
teta2dot=0;
                 %assumption value
u0(1:12,1:1)=0;
u1(1:12,1:1)=0;
u2(1:12,1:1)=0;
u3(1:12,1:1)=0;
u4(1:12,1:1)=0;
u0(1,1)=zr;
u0(2,1) = tetar;
u0(3,1)=z1;
u0(4,1)=teta1;
u0(5,1)=z2;
u0(6,1)=teta2;
u0(7,1)=zrdot;
u0(8,1)=tetardot;
u0(9,1) = z1dot;
u0(10,1)=teta1dot;
u0(11,1) = z2dot;
u0(12,1)=teta2dot;
% making m
m(1:6, 1:6) = 0;
m(1,1)=Mr;
m(2,2) = Jr;
m(3,3) = M1;
m(4, 4) = J1;
m(5, 5) = M2;
m(6, 6) = J2;
%making c
c(1:6, 1:6) = 0;
c(1,1)=c11 + c12 + c21 + c22 + cr1 + cr2 + cr3;
c(2,1)=-c11*xr1 - c12*xr2 + c21*xr3 + c22*xr4 - cr1*xr5 - cr2*xr6 +
cr3*xr7;
c(3,1) = -c11 - c12;
c(4,1) = c11 \times 1 - c12 \times 2;
c(5,1) = -c21 - c22;
c(6,1)=c21*x3 - c22*x4;
c(1,2)=-c11*xr1 - c12*xr2 + c21*xr3 + c22*xr4 - cr1*xr5 - cr2*xr6 +
cr3*xr7;
c(2,2)=c11*xr1*xr1 + c12*xr2*xr2 + c21*xr3*xr3 + c22*xr4*xr4 +
cr1*xr5*xr5 + cr2*xr6*xr6 + cr3*xr7*xr7;
c(3,2)=c11*xr1 + c12*xr2;
c(4,2)=-c11*xr1*x1 + c12*xr2*x2;
c(5,2)=-c21*xr3 - c22*xr4;
c(6,2)=c21*xr3*x3 - c22*xr4*x4;
c(1,3) = -c11 - c12;
```

```
c(2,3)=c11*xr1 + c12*xr2;
c(3,3)=c11 + c12;
c(4,3) = -c11 \times 1 + c12 \times 2;
c(5,3)=0;
c(6,3)=0;
c(1, 4) = c11 \times x1 - c12 \times x2;
c(2,4) = -c11 \times 1 \times r1 + c12 \times 2 \times r2;
c(3,4)=-c11*x1 + c12*x2;
c(4,4)=c11*x1*x1+c12*x2*x2;
c(5, 4) = 0;
c(6, 4) = 0;
c(1,5) = -c21 - c22;
c(2,5)=-c21*xr3 - c22*xr4;
c(3,5)=0;
c(4,5)=0;
c(5,5)=c21 + c22;
c(6,5) = -c21 \times 3 + c22 \times 4;
c(1, 6) = c21 \times x3 - c22 \times x4;
c(2,6)=c21*x3*xr3 - c22*x4*xr4;
c(3, 6) = 0;
c(4, 6) = 0;
c(5,6) = -c21 \times 3 + c22 \times 4;
c(6, 6) = c21 \times 3 \times 3 + c22 \times 4 \times 4;
%c(1:6,1:6)=0;
% making k -----
k(1:6, 1:6) = 0;
k(1,1) = k11 + k12 + k21 + k22 + kr1 + kr2 + kr3;
k(2,1)=-k11*xr1 - k12*xr2 + k21*xr3 + k22*xr4 - kr1*xr5 - kr2*xr6 +
kr3*xr7;
k(3,1) = -k11 - k12;
k(4,1) = k11 \times 1 - k12 \times 2;
k(5,1) = -k21 - k22;
k(6,1) = k21 \times 3 - k22 \times 4;
k(1,2)=-k11*xr1 - k12*xr2 + k21*xr3 + k22*xr4 - kr1*xr5 - kr2*xr6 +
kr3*xr7;
k(2,2)=k11*xr1*xr1 + k12*xr2*xr2 + k21*xr3*xr3 + k22*xr4*xr4 +
kr1*xr5*xr5 + kr2*xr6*xr6 + kr3*xr7*xr7;
k(3,2)=k11*xr1 + k12*xr2;
k(4,2)=-k11*xr1*x1 + k12*xr2*x2;
k(5,2)=-k21*xr3 - k22*xr4;
k(6,2)=k21*xr3*x3 - k22*xr4*x4;
k(1,3) = -k11 - k12;
k(2,3)=k11*xr1 + k12*xr2;
k(3,3) = k11 + k12;
k(4,3) = -k11 \times 1 + k12 \times 2;
k(5,3)=0;
k(6,3)=0;
k(1,4)=k11*x1 - k12*x2;
k(2,4)=-k11*x1*xr1 + k12*x2*xr2;
k(3,4) = -k11 \times 1 + k12 \times 2;
k(4,4)=k11*x1*x1+k12*x2*x2;
```

```
k(5,4)=0;
k(6,4)=0;
k(1,5)=-k21 - k22;
k(2,5)=-k21*xr3 - k22*xr4;
k(3,5)=0;
k(4,5)=0;
k(5,5)=k21 + k22;
k(6,5)=-k21*x3 + k22*x4;
k(1,6)=k21*x3 - k22*x4;
k(2,6)=k21*x3*xr3 - k22*x4*xr4;
k(3,6)=0;
k(4,6)=0;
k(5,6)=-k21*x3 + k22*x4;
k(6,6)=k21*x3*x3 + k22*x4*x4;
%k(1:6,1:6)=0;
```

```
%making A
```

```
A(1:12,1:12)=0;
A(1,7)=1;
A(2,8)=1;
A(3,9)=1;
A(4,10)=1;
A(5,11)=1;
A(6,12)=1;
A(7:12,1:6)=-inv(m)*k;
A(7:12,7:12)=-inv(m)*c;
```

```
tend=10;
h=0.01;
nstep=(tend-tstart)/h;
tn(1:nstep)=0;
uln(1:nstep)=0;
u2n(1:nstep)=0;
u3n(1:nstep)=0;
u4n(1:nstep)=0;
u5n(1:nstep)=0;
u6n(1:nstep)=0;
u7n(1:nstep)=0;
u8n(1:nstep)=0;
u9n(1:nstep)=0;
u10n(1:nstep)=0;
ulln(l:nstep)=0;
u12n(1:nstep)=0;
```

```
%add as example from sv
itime=1;
tn(itime)=tstart;
uln(itime)=u0(1,1);
u2n(itime)=u0(2,1);
u3n(itime)=u0(3,1);
u4n(itime)=u0(4,1);
```

```
u5n(itime)=u0(5,1);
u6n(itime)=u0(6,1);
u7n(itime)=u0(7,1);
u8n(itime)=u0(8,1);
u9n(itime)=u0(9,1);
u10n(itime)=u0(10,1);
ulln(itime)=u0(11,1);
u12n(itime)=u0(12,1);
t=tstart;
%integration
for n=1:1:(nstep-1)
    u1=u0;
    d1dot=u1(3,1);
    [Ft]=f1(F1,F2,w1,w2,t);
    [Fx]=fx(A,u1,m,Ft);
    K1=h*Fx;
    t2=t+h/2;
    u2=u0+0.5*K1;
    dldot=u2(3,1);
    [Ft]=f1(F1,F2,w1,w2,t2);
    [Fx]=fx(A,u2,m,Ft);
    K2=h*Fx;
    t3=t+h/2;
    u3=u0+0.5*K2;
    d1dot=u3(3,1);
    [Ft]=f1(F1,F2,w1,w2,t3);
    [Fx]=fx(A,u3,m,Ft);
    K3=h*Fx;
    t4=t+h;
    u4=u0+K3;
    d1dot=u4(3,1);
    [Ft]=f1(F1,F2,w1,w2,t4);
    [Fx]=fx(A,u4,m,Ft);
    K4=h*Fx;
    t=t+h;
    u0=u0+(K1+2*K2+2*K3+K4)/6;
    itime=itime+1;
    tn(itime)=t;
    uln(itime)=u0(1,1);
    u2n(itime) = u0(2,1);
    u3n(itime)=u0(3,1);
    u4n(itime)=u0(4,1);
    u5n(itime)=u0(5,1);
    u6n(itime)=u0(6,1);
    u7n(itime)=u0(7,1);
```

```
u8n(itime)=u0(8,1);
u9n(itime)=u0(9,1);
u10n(itime)=u0(10,1);
u11n(itime)=u0(11,1);
u12n(itime)=u0(12,1);
```

end
plot (tn,u3n)

```
xlabel('s(second)')
ylabel('x(meter)')
```

### **Fx Function**

```
function [Fx]=fx(A,u,m,Ft)
Fx1(1:12,1:1)=0;
Fx1(7:12,1:1)=inv(m)*Ft;
Fx=A*u+Fx1;
```

### **Input Function**

```
function [Ft]=f1(F1,F2,w1,w2,t)
Fz1=F1*sin(w1*t);
Fz2=F2*sin(w2*t);
Ft(1:6,1:1)=0;
Ft(3,1)=-Fz1;
Ft(5,1)=-Fz2;
```

All of these codes are written in M-File editor in the MATLAB software.